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MULTIPLE-COMPARISON COMPUTER PROGRAM USING THE BONFERRONI t STATISTIC

Eric E. Johnson

MASTER

November 1980

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DEPARTMENT OF ENERGY

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ABSTRACT

To ascertain the agreement among laboratories, samples from a single batch of material are analyzed by the different laboratories and results are then compared. A graphical format was designed for presenting the results and for showing which laboratories have significantly different results. The appropriate statistic for simultaneously testing the significance of the differences between several means is Bonferroni t. A computer program was written to make the tests between means based on Bonferroni t and also to make multiple comparisons of the standard deviations associated with the means. The program plots the results and indicates means and standard deviations which are significantly different.

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SUMMARY

A multiple-comparison procedure is presented for simultaneously testing the differences between several sample means taken in all possible pairs. A computer program implementing the multiple-comparison procedure is listed.

The procedure has been applied to the comparison of analyses by different laboratories on a single batch and to the comparison of results by different methods on a single batch analyzed by one laboratory. An example of the output gives a graphical presentation of results.

Empirical equations which were devised for computing Bonferroni t tabular values are also given.

INTRODUCTION

A multiple-comparison computer program was devised at the Oak Ridge Y-12 Plant^(a) in 1976 for making a graphical comparison of control-batch analyses obtained by four laboratories in the National Uranium Resources Evaluation (NURE) program. Since the inception of the computer procedure, other interlaboratory comparisons are being made using this procedure. This report presents a description of the multiple-comparison output, the equations used in making the comparisons, an operational guide, and a listing of the computer program.

(a) Operated by the Union Carbide Corporation, Nuclear Division, for the Department of Energy.

MULTIPLE-COMPARISON COMPUTER PROGRAM

OUTPUT AND PROGRAM DESCRIPTION

The program is set up to compare two to six means with each other. The illustration in Figure 1 shows the comparison of four means.

Y-80-489

MULTIPLE COMPARISON OF MEANS AND STANDARD DEVIATIONS - MG/L NITRATE
ANALYSES OF A MEASUREMENT CONTROL BATCH OF WATER FOR ENVIRONMENTAL STUDIES

	STANDARD DEVIATION	LINES: STD DEV NOT DIFF APPROX <u>95% CONF</u>
X	0.21	
Y	0.43	
P	0.63	
K	1.68	

Figure 1. MULTIPLE COMPARISON OF MEANS AND STANDARD DEVIATIONS.

The left-side tabular portion in Figure 1 presents the data source, mean based on the specified number of results, and the 95% confidence limit of the mean.

The mean, number of results, and standard deviation for each data source are entered into the computer for the presentation in the left-side tabular portion. The raw data are computed by the following equations (not included in the computer program under discussion).

$$\text{Mean, } \bar{x} = \frac{\sum x}{N}, \quad (1)$$

where N is the number of results.

Standard deviation of a single determination,

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{N - 1}} \quad (2)$$

$$\text{The computer program calculates the 95\% confidence limit} = \frac{tS}{\sqrt{N}} \quad (3)$$

Student's t for the 95% confidence level based on N - 1 degrees of freedom is computed by the empirical equation:¹

$$\text{Student's } t = \frac{1.8148 (N - 1) + 0.2500}{0.9259 (N - 1) - 1.0000} \quad (4)$$

If available, the following items are fed into the computer prior to computation: the standard value for the measurements under consideration, the number of results, and the standard deviation of a single determination of the standard value. If the number of results for the standard value's standard deviation is not available, a value of 10 is assumed.

Symbols for describing the test to determine whether or not the mean of the measurements is significantly different from the standard value at the 95% confidence level are as follows:

S_1 = standard deviation of the measurements

N_1 = number of measurements

S_0 = standard deviation of a single determination of the standard value

N_0 = number of results for computing the standard value.

The t' test statistic, its associated degrees of freedom DF, and the t distribution value at 95% confidence are computed according to the following:²

$$t' = \frac{|\text{Mean} - \text{Standard Value}|}{\sqrt{S_1^2/N_1 + S_0^2/N_0}}, \quad (5)$$

$$DF = \frac{(S_1^2/N_1 + S_0^2/N_0)^2}{\frac{(S_1^2/N_1)^2}{N_1 - 1} + \frac{(S_0^2/N_0)^2}{N_0 - 1}} \quad (6)$$

$$t \text{ distribution value at 95\% confidence level} = \frac{1.8148 DF + 0.2500}{0.9259 DF - 1.0000} \quad (7)$$

The t' and the t distribution value are then compared. If t' is greater than the t distribution value, the null hypothesis that Mean = Standard Value is rejected. For this case, an asterisk is printed after the mean indicating that the mean is significantly different from the standard value at the 95% confidence level.

The center plot of results in Figure 1 presents the means designated by Os and their 95% confidence limits which are shown as bars. The limits are based on Bonferroni t which will be explained. The means are presented in the order from lowest to highest so that the lines of nonsignificance at the right side of the figure can be drawn correctly.

The plot of means and their limits and the right-hand nonsignificance lines is basically a test of multiple hypothesis in which all possible "pairwise" comparisons of means are made. The appropriate statistic for use in computing the plotted limits and for making the multiple comparisons is the Bonferroni t .^{3,4}

The value of the Bonferroni t statistic depends on the number of means being compared. The initial step in computing Bonferroni t is to note how many means are being compared and then to look up the appropriate Bonferroni t equation given in Table 1. These equations (based on Bonferroni t values in Reference 4, Table II) were devised by the hyperbolic equation-fitting technique.⁵

Table 1
EMPIRICAL EQUATIONS FOR BONFERRONI t
95% CONFIDENCE, TOTAL IN TWO EQUAL
TAILS, INCLUDING STUDENT'S t FOR
TWO-MEAN COMPARISON

Number of Means Being Compared	Bonferroni t Except where Noted
	DF = degrees of freedom
2	$\frac{1.8148 \text{ DF} + 0.2500}{0.9259 \text{ DF} - 1.0000}$ (Student's t)
3	$\frac{1.5524 \text{ DF} + 0.1538}{0.6467 \text{ DF} - 1.0000}$
4	$\frac{1.6521 \text{ DF} + 0.7559}{0.6271 \text{ DF} - 1.0000}$
5	$\frac{1.5796 \text{ DF} + 0.7810}{0.5628 \text{ DF} - 1.0000}$
6	$\frac{1.5994 \text{ DF} + 1.1069}{0.5464 \text{ DF} - 1.0000}$

The 95% confidence limits of the mean are computed by the following formula:

$$\text{Mean} \pm \frac{(\text{Bonferroni } t) S}{\sqrt{N}} \quad (8)$$

where Bonferroni t is based on $DF = N - 1$.

These limits are printed as vertical bars around the 0 mean value. See example in Figure 1.

Tests of hypotheses that the pairs of means are not significantly different at the 95% confidence level are made by the t' test. All possible pairs of means are examined in the computer program. Subscripts 1 and 2 in the following equations refer to any two means being tested.

$$t' = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{S_1^2/N_1 + S_2^2/N_2}} \quad , \text{ and} \quad (9)$$

$$DF = \frac{(S_1^2/N_1 + S_2^2/N_2)^2}{\frac{(S_1^2/N_1)^2}{N_1 - 1} + \frac{(S_2^2/N_2)^2}{N_2 - 1}} \quad (10)$$

Bonferroni t value is based on this DF value.

The t' and Bonferroni t are then compared for the pair of means under consideration. If t' is greater than Bonferroni t, the difference between the two means is significant at the 95% confidence level. If t' is equal to or less than Bonferroni t, the difference between the two means is not significantly different at the 95% confidence level. At the right side of the output as shown in the example in Figure 1, a vertical line is drawn for each pair of means having a difference which is not significantly different from zero. In other words, the lines show means which agree with each other and the absence of a line indicates disagreement.

The bottom of the output sheet, as shown in the example in Figure 1, presents the standard deviations for the various sources in order from the smallest to the largest. Lines showing standard deviations not significantly different are printed to the right of the standard deviations. These lines are based on all possible pairwise comparisons of the variances.

The F value for each pair of variances is computed according to the following:

$$F = \frac{\text{larger variance}}{\text{smaller variance}} = \frac{\text{larger } S^2}{\text{smaller } S^2} \quad (11)$$

DF (larger) = degrees of freedom for larger variance,

DF (smaller) = degrees of freedom for smaller variance.

The formula for degrees of freedom is $N - 1$, where N is the number of results.

This calculated F value is compared with the appropriate tabular F value for the given degrees of freedom of the larger and smaller variances. The procedure for choosing the appropriate F table is described in the next paragraph. If the calculated F value is greater than the tabular F value, the two variances being tested are significantly different. If the calculated F value is equal to or less than the tabular F value, the two variances being tested are not significantly different. A vertical line is printed for each pair of variances which shows nonsignificance.

Tables of F values (for multiple comparisons at the 0.05 probability level) similar to the Bonferroni tables are not available. To circumvent this deficiency, the appropriate F table which is available is chosen according to the number of variances being compared to give a confidence which is close to 95%.⁶ Table 2 which is programmed into the computer presents the F table to be associated with the number of variances being compared.

Table 2
SELECTION OF F TABLE FOR MULTIPLE COMPARISONS OF VARIANCES

Number of Variances Being Compared	Number of Independent Hypotheses Being Tested (number of contrasts)	F Table Needed To Attain Desired Probability (percentage point table heading)(1)	Percent Confidence That All Hypotheses Will Be Accepted When They Are True
2	1	0.975	95
3	3	0.990	94
4	6	0.995	94
5	10	0.995	90
6	15	0.999	97

(1) Reference 7.

OPERATION OF FORTRAN IV PROGRAM

The program has been designed to compare two to six means with each other and then make all possible comparisons of the variances associated with the means. If a standard value and its standard deviation are available, the program compares the mean values with the standard value.

The input for the program for each item being compared consists of a data set. The cards in each data set are as follows:

Three Heading Cards - Cards 1, 2, and 3

Column Contents

1-80 A user-specified title. In the Figure 1 example, the top three lines were entered on these cards.

Symbol-Identification Card - Card 4

Column Contents

1-80 1 to 3 letter or number identifiers for the items being compared. In the Figure 1 example, the laboratory symbols K, P, X, and Y are defined.

Decimal Place Card - Card 5

Column Contents

3 Number of places to right of decimal in data values: 0 for no places to the right to 5 for five places to the right (i.e., x.xxxxx). The program as written will not handle six or more.

4-80 Additional heading or symbol-identification information can be entered here. For example, single letter identifiers could be defined for different methods, such as C for colorimetric and G for gravimetric.

Data Cards - Card 6, Card 7, . . . , Card (last item in comparison)

Each card contains the data from only one source (i.e., the item being compared). In the example, each card contains one laboratory's set of data.

Column	Contents
1-2	A user-specified notation for filing and sorting cards. This field can be left blank.
6-8	Symbol identifier defined in Cards 4 and 5. For example, Y-C could be used to identify Y-12 colorimetric data.
9-15	Number of determinations for standard value, right justified. If the value is not known, the field is left blank and a value of 10 is assumed.
16-25	Standard value (mean of number of results specified in Columns 9-15) right justified. If the value is not known, the field is left blank.
26-35	Standard deviation of a standard value right justified. If a standard value entry is made, a standard deviation value must be entered for making valid comparisons with the standard value.
36-40	Number of measurements in the data set from one source, right justified.
41-50	Mean for the number of measurements in the data set from one source (with decimal point, if real variable; right justified, if integer variable).
51-60	Standard deviation for the number of measurements in the data set from one source (with decimal point, if real variable; right justified, if integer variable).

Data cards must be sorted on Columns 41-50 so that the mean values are in order from low to high.

Blank Card - a blank card is the last card in each data set.

EXAMPLE

The data in Figure 1 are from an interlaboratory control program currently being run at the four Union Carbide Nuclear Division laboratories: Oak Ridge Gaseous Diffusion Plant (K-25), Oak Ridge National Laboratory (X-10), Paducah Gaseous Diffusion Plant (PGDP), and the Oak Ridge Y-12 Plant (Y-12).

The vertical lines in Figure 1 at the right of the mean plot and at the bottom comparison of standard deviations have the following meaning. Pairs not found to be significantly different are connected with a line. Pairs not connected with a line were found to be significantly different.

In Figure 1 mean comparison, no line between X and K indicates the X and K means are significantly different. In the standard deviation comparison, no lines between X and P and between K and each of the other three laboratories indicates the variances for X and P are significantly different and the variance for K is significantly different from the variances of the other laboratories.

The computer program also gives a printed output of a t and F statistics table corresponding to the figure. The output relating to Figure 1 is presented in Figure 2. Calculated values of t and F for all of the paired comparisons are presented in Figure 2. In case comparisons are desired at the confidence levels other than 95%, t and F values at the desired levels can be looked up in tables for comparison with the calculated t and F values.

Y-80-490

XBAR: Item Mean Number in (): Items 1 (top) Through 4 (bottom) As Listed in Mean Comparison					T': t Calculated T: t Tabular Value	
XBAR(1)=	7.2200	XBAR(2)=	7.5000	T'=	1.48	T(12.96)= 3.11
XBAR(1)=	7.2200	XBAR(3)=	7.5200	T'=	2.21	T(15.16)= 3.03
XBAR(1)=	7.2200	XBAR(4)=	8.9200	T'=	3.48	T(11.28)= 3.19
XBAR(2)=	7.5000	XBAR(3)=	7.5200	T'=	0.09	T(19.42)= 2.94
XBAR(2)=	7.5000	XBAR(4)=	8.9200	T'=	2.74	T(14.03)= 3.07
XBAR(3)=	7.5200	XBAR(4)=	8.9200	T'=	2.80	T(12.44)= 3.13
X,SX: Item Mean, Standard Deviation (means in order low to high)						
Y,SY: Standard Value, Standard Deviation						
X,SX,Y,SY=	7.2200	0.2100	7.3000	0.1400	T'= 1.1429	T(22.99)= 2.07
X,SX,Y,SY=	7.5000	0.6300	7.3000	0.1400	T'= 1.0685	T(12.29)= 2.17
X,SX,Y,SY=	7.5200	0.4300	7.3000	0.1400	T'= 1.6693	T(13.71)= 2.15
X,SX,Y,SY=	8.9200	1.6800	7.3000	0.1400	T'= 3.3266	T(11.18)= 2.20
NY = 10 Number of Results in Standard Value						
1	3	2	4	Standard Deviation Order According to (1) Through (4) Listing for Means		
SX: Item Standard Deviation						
Number in (): Items 1 (top) Through 4 (bottom) As Listed in Standard Deviation Comparison					F': F Calculated F: Tabular Value DF and Probability Level in ()	
SX(1)=	0.2100	SX(2)=	0.4300	F'= 4.19	F(11,14,0.995)=	4.5086
SX(1)=	0.2100	SX(3)=	0.6300	F'= 9.00	F(11,14,0.995)=	4.5086
SX(1)=	0.2100	SX(4)=	1.6800	F'= 64.00	F(11,14,0.995)=	4.5086
SX(2)=	0.4300	SX(3)=	0.6300	F'= 2.15	F(11,11,0.995)=	5.3195
SX(2)=	0.4300	SX(4)=	1.6800	F'= 15.26	F(11,11,0.995)=	5.3195
SX(3)=	0.6300	SX(4)=	1.6800	F'= 7.11	F(11,11,0.995)=	5.3195

Figure 2. THE t AND F STATISTICS.

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APPENDIX

PROGRAM LISTING

```

REAL*8 BLANK8/'      '/
INTEGER NBLANK/'      '/,NZER/'      0'/
DIMENSION NUMD(30),DAVE(30),NSTD(30)
DIMENSION BUF(5)
DIMENSION IR(10),IW(10)
REAL*8 ALAB(30)      ,EMS
DIMENSION T(5)      ,TITLE(20)      ,T1(20)
DIMENSION B(10),AVE(10), NUM(10), CONF(10), STD(10),AL(41)
REAL AST/'*'/,AS(10)
REAL BLANK/'      '/,ADIF(15)/.2,.4,.8,4,.8,.20,.40,.80,.200,.400.,
1 800./, AINC(15)/.05,.1,.2,.5,1.,2.,5.,10.,20.,50.,100.,200./,
2 VERT/'*'/,XX/'0'/
DIMENSION NPTR (10)
DIMENSION ST(10), NU(10) ,AV(10)
DIMENSION XLIN(10,10)
DIMENSION IRES(10,10)
NRES = 10
5 CONTINUE
READ (5,10,END=1000) TITLE
10 FORMAT (20A4)
WRITE (8,20)
20 FORMAT ('1'//////// )
WRITE(8,30) TITLE
30 FORMAT (' ',20A4)
WRITE (6,31) TITLE
31 FORMAT ('1',20A4/33X,'T & F STATISTICS')
READ (5,10) T1
WRITE(8,30) T1
READ (5,10) T1
WRITE(8,30) T1
WRITE(8,40)
40 FORMAT (' ')
READ (5,10) T1
WRITE(8,30) T1
READ (5,11)IGD, T1
11 FORMAT (I3,19A4,A1)
WRITE (8,12) T1
12 FORMAT (' ',19A4,A1)
WRITE (8,40)
WRITE (8,50)
50 FORMAT ( T28,' MEANS (0) & BONFERRONI 95% CONF. INTERVALS')
I=1
75 READ (5,70) B(I),ALAB(I),MSTD ,DSV,ERS , NUM(I),AVE(I),STD(I)
70 FORMAT (A2,3X,A3,2X,I5,2F10.0,I5,2F10.0)
NSTD(I) = MSTD
IF ( MSTD .EQ. 0) NSTD(I) = 10
DAVE(I) = AVE(I)
NUMD(I) = NUM(I)
IF (B(I) .EQ. BLANK .AND. ALAB(I) .EQ. BLANK8) GO TO 80
NSD = MSTD
SV = DSV
ERSV = ERS
I=I+1
GO TO 75

```

```

80 CONTINUE
  CALL CORE(BUF,20)
  WRITE(49,41) NSD
  CALL CORE(BUF,20)
  READ (49,42) NSD
41 FORMAT (I4)
42 FORMAT (A4)
  IF (NSD .EQ. NZER) NSD = NBLANK
  NPL = I-1
  FMS = 0D0
  NDF = 0
  NP1 = NPL-1
  DO 85 I=1,NPL
    NM1 = NUM(I) -1
    NDF = NDF + NM1
    EMS = EMS + STD(I)*STD(I)*NM1
85 CONTINUE
  EMS = EMS/NDF
  CALL PSORTM(4,AVE,1,NPL,NPTR)
  DO 101 I=1,NPL
    AV(I) = AVE(NPTR(I))
    ST(I) = STD(NPTR(I))
    NU(I) = NUM(NPTR(I))
101 CONTINUE
  CALL BONFT (AV,ST,NU,NPL,IRES,NRES)
98 CONTINUE
  CALL RANGE(NPL,IRES,NRES,IR,IW)
  CALL LINES(IR,XLIN,NPL)
C
C   MAY NEED TO SORT HERE
  IF (SV .EQ. 0) GO TO 62
  GO TO (91,92,93,94,95), IGO
91 WRITE (8,291) SV,ERSV
291 FORMAT(T32,' STD VALUE (VERTICAL LINE) =',F9.1,T74,' LINES: '//,
1 T18,' STUDENT'S',T32,' STD DEV OF STD',12X,'= ',
2 F7.1,T74,' MEANS')
  GO TO 96
92 WRITE (8,292) SV,ERSV
292 FORMAT(T32,' STD VALUE (VERTICAL LINE) =',F9.2,T74,' LINES: '//,
1 T18,' STUDENT'S',T32,' STD DEV OF STD',12X,'= ',
2 F7.2,T74,' MEANS')
  GO TO 96
93 WRITE (8,293) SV,FRSV
293 FORMAT(T32,' STD VALUE (VERTICAL LINE) =',F9.3,T74,' LINES: '//,
1 T18,' STUDENT'S',T32,' STD DEV OF STD',12X,'= ',
2 F7.3,T74,' MEANS')
  GO TO 96
94 WRITE (8,294) SV,ERSV
294 FORMAT(T32,' STD VALUE (VERTICAL LINE) =',F9.4,T74,' LINES: '//,
1 T18,' STUDENT'S',T32,' STD DEV OF STD',12X,'= ',
2 F7.4,T74,' MEANS')
  GO TO 96
95 WRITE (8,295) SV,ERSV
295 FORMAT(T32,' STD VALUE (VERTICAL LINE) =',F9.5,T74,' LINES: '//,
1 T18,' STUDENT'S',T32,' STD DEV OF STD',12X,'= ',
2 F7.5,T74,' MEANS')
96 CONTINUE
  GO TO 1197

```

```

62 CONTINUE
   WRITE (8,1198)
1198 FORMAT (T74,' LINES:'/T18,' STUDENT'S',T74,' MEANS')
1197 CONTINUE
   WRITE (8,1199)
1199 FORMAT (T10,' MEAN & 95% CONF',T74,' NOT')
   IF (SV .NE. 0.) WRITE (8,1200) NSD
1200 FORMAT('+', T32,' N =',A4,2X,'(IF BLANK, 10 IS ASSUMED)')
   WRITE (8,1201)
1201 FORMAT(' ',2X,' (NO. OF LIM',T74,' DIFF**'/
2' ',2X,' RESULTS) +/-',T73,' 95% CONF'
3 /'+',T73,' _____')
C
   WRITE (6,1)
1 FORMAT ('0',//)
   IFAST = 0
   DO 105 I=1,NPL
   AS(I) = BLANK
   ID = 0
   CALL MEAND(AVE(I),STD(I),NUM(I),NPL,SV,ERSV,NSTD,CONF(I),ID)
   IF (SV .EQ. 0. .AND. ERSV .EQ. 0.) GO TO 105
   IF (ID .NE. 0) IFAST = 1
   IF (ID .NE. 0) AS(I) = AS(I)
105 CONTINUE
   WRITE (6,106) NSTD(1)
106 FORMAT('ONLY =',I5)
   ALL = AVE(1)-CONF(1)
   AU = AVE(NPL) + CONF(NPL)
   DIFF = AU-ALL
   ASV = ALL+ DIFF/2.
   DIFF = DIFF + DIFF/5.
   DO 110 I=1,12
   K = I
   IF (DIFF .LT. ADIF (I)) GO TO 120
110 CONTINUE
120 CONTINUE
   AC = AINC(K)
   T(3) = IFIX((ASV + AC)/AC)*AC
   IF (T(3) -2.*AC .GT. ALL) T(3) = T(3) - AC
   IF (T(3) + 2.*AC .LT. AU) T(3) = T(3) + AC
   IF (T(3) - 2.*AC .LE. ALL) GO TO 121
   K = K + 1
   GO TO 120
121 CONTINUE
   T(1) = T(3) - 2.*AC
   T(2) = T(3) - AC
   T(4) = T(3) + AC
   T(5) = T(3) + 2.*AC
   WRITE (8,130) (T(I),I=1,5)
130 FORMAT (' ',T25,F7.2,4(3X,F7.2))
   WRITE (8,140)
140 FORMAT (' T28,' |',4(9X,'|'))
   INC = K
   TI = T(1)
   DIFF = T(5) - TI
   KSV = (SV-TI)/DIFF*40. + 1.5
   DO 150 I=1,NPL
   DU 160 J=1,41

```

```

160 AL(J) = BLANK
    IF (SV .NE. 0.) AL(KSV) = VERT
    WRITE (8,145) (AL(J),J=1,41),(XLIN(J,I),J=1,NP1)
145 FORMAT (T28,' ',41A1,4X,6(A1,2X))
    K = (AVE(I) - TI)/DIFF * 40. + 1.5
    AL(K) = XX
    K = (AVE(I) - CONF(I) - TI)/DIFF * 40. + 1.5
    IF (AL(K) .NE. BLANK) K = K-1
    AL(K) = VERT
    K = (AVE(I) + CONF(I) - TI)/DIFF * 40. + 1.5
    IF (AL(K) .NE. BLANK) K = K+1
    AL(K) = VERT
    GO TO (171,172,173,174,175), IGO
171 WRITE (8,1810) ALAB(I), AVE(I), AS(I), NUM(I), CONF(I),
    1 (AL(J),J=1,41),(XLIN(J,I), J=1,NP1)
1810 FORMAT(2X,A4,F7.1,A1,'(',I2,')',F7.1,3X, 41A1,4X,6(A1,2X))
    GO TO 186
172 WRITE (8,182) ALAB(I), AVE(I), AS(I), NUM(I), CONF(I),
    2 (AL(J),J=1,41),(XLIN(J,I), J=1,NP1)
182 FORMAT(2X,A4,F7.2,A1,'(',I2,')',F7.2,3X, 41A1,4X,6(A1,2X))
    GO TO 186
173 WRITE (8,183) ALAB(I), AVE(I), AS(I), NUM(I), CONF(I),
    3 (AL(J),J=1,41),(XLIN(J,I), J=1,NP1)
183 FORMAT(2X,A4,F7.3,A1,'(',I2,')',F7.3,3X, 41A1,4X,6(A1,2X))
    GO TO 186
174 WRITE (8,184) ALAB(I), AVE(I), AS(I), NUM(I), CONF(I),
    4 (AL(J),J=1,41),(XLIN(J,I), J=1,NP1)
184 FORMAT(2X,A4,F7.4,A1,'(',I2,')',F7.4,3X, 41A1,4X,6(A1,2X))
    GO TO 186
175 WRITE (8,185) ALAB(I), AVE(I), AS(I), NUM(I), CONF(I),
    5 (AL(J),J=1,41),(XLIN(J,I), J=1,NP1)
185 FORMAT(2X,A4,F7.5,A1,'(',I2,')',F7.5,3X, 41A1,4X,6(A1,2X))
186 CONTINUE
150 CONTINUE
    WRITE (8,1800)
1800 FORMAT (' ')
    IF (IFAST .EQ. 1 .AND. SV .NE. 0.) WRITE (8,180)
180 FORMAT (' ',T11,' *SIGNIFICANTLY DIFFERENT FROM STD VALUE BASED ON
    1 STUDENT'S T.' /' ',T12,' THIS TEST OVERRIDES PLOT WHERE INTERVAL
    2 INCLUDES STD.')
    WRITE (8,181)
181 FORMAT (' ',T10,' **BASED ON BONFERRONI T.')
    CALL PSORTM (4,STD,1,NPL,NPTR)
    WRITE (6,196) (NPTR(I),I=1,NPL)
196 FORMAT ('0',10I5)
    DO 195 I=1,NPL
    ST(I) = STD(NPTR(I))
    NU(I) = NUM(NPTR(I))
195 CONTINUE
C   IF (IBART .EQ. 1) GO TO 197
C   IR(1) = NPL
C   DO 199 I=2,NPL
C 199 IR(I) = 0
C   GO TO 198

```

```

197 CONTINUE
  CALL PARVAR(ST,NI,NPL,IRES,NRES)
  CALL RANGE(NPL,IRES,NRES,IR,IW)
  99 FORMAT (' ',5I5)
198 CONTINUE
  CALL LINES(IR,XLIN,NPL)
  WRITE (8,200)
200 FORMAT ('0'//T22,' LINES:'//T22,' STD DEV' /' ',2X,
1' STANDARD',T22,' NOT DIFF'/' ',2X,' DEVIATION',
2 T22,' APPROX' / T22,' 95% CONF'/'+',T22,' _____'//)
  DO 250 I=1,NPL
    GO TO (211,212,213,214,215),IGO
211 WRITE (8,221) ALAB(NPTR(I)),ST(I),(XLIN(J,I),J=1,NP1)
221 FORMAT (' ',1X,A8,F8.1,4X, 5(A1,2X))
    GO TO 216
212 WRITE (8,222) ALAB(NPTR(I)),ST(I),(XLIN(J,I),J=1,NP1)
222 FORMAT (' ',1X,A8,F8.2,4X, 5(A1,2X))
    GO TO 216
213 WRITE (8,223) ALAB(NPTR(I)),ST(I),(XLIN(J,I),J=1,NP1)
223 FORMAT (' ',1X,A8,F8.3,4X, 5(A1,2X))
    GO TO 216
214 WRITE (8,224) ALAB(NPTR(I)),ST(I),(XLIN(J,I),J=1,NP1)
224 FORMAT (' ',1X,A8,F8.4,4X, 5(A1,2X))
    GO TO 216
215 WRITE (8,225) ALAB(NPTR(I)),ST(I),(XLIN(J,I),J=1,NP1)
225 FORMAT (' ',1X,A8,F8.5,4X, 5(A1,2X))
    GO TO 216
216 CONTINUE
  WRITE (8,300) (XLIN(J,I),J=1,NP1)
  300 FORMAT (' ',21X,5(A1,2X))
250 CONTINUE
  GO TO 5
1000 CONTINUE
  RETURN
  END
  SUBROUTINE MEAND(X,S,N,NPL,Y,S2,NY,LUNF,IO)
    T(DF) = (1.8148*DF + .25)/(1.9259*DF - 1.0)
    DF = N-1
    SX = S/SQRT(FLOAT(N))
    SY = S2/SQRT(FLOAT(NY))
    TSTAT = T(DF)
    CONF = TSTAT*SX
    D = (SX*SX + SY*SY)**2/((SX*SX)**2/(N-1) +
1 (SY*SY)**2/(NY-1))
    TS = T(D)
    TPR = ABS(X-Y)/SQRT(S*S/N + S2*S2/NY)
    ID = 0
    IF(TPR.GT. TS) ID = 1
    TPR = ABS(TPR)
    TS = ABS(TS)
    WRITE (6,10) X,S ,Y,S2,TPR,D ,TS
10 FORMAT('0X,SX,Y,SY=',4F9.4,5X,'T'=' ',F9.4,4X,'T(' ',F5.2,' )=' ',F9.2)
    RETURN
  END
  SUBROUTINE RANGE(M,IRES,NRES,IR,IW)
    DIMENSION IRES(NRES,1),IR(1),IW(1)
    MN1 = M-1
    M1 = M+1

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      DO 10 I=1,M
      IR(I) = 0
      IW(I) = 0
10  CONTINUE
      DO 125 I=1,MN1
      J= M1-I
      L = 1
      JM1 = J-1
      DO 125 K=1,I
      JPL = L + JM1
      IF (IW(L) + IW(JPL) .GT. 1) GO TO 125
      IF(IRES(L,JPL) .GT.0) GO TO 125
      IR(L) = J
      DO 120 II=L,JPL
      IW(II) = 1
120 CONTINUE
125 L = L + 1
      RETURN
      END
      SUBROUTINE LINES(IR,XLIN,NPL)
      REAL BLANK/' ',VERT/'|'/
      DIMENSION IR(1),XLIN(10,10)
      NP1 = NPL-1
      DO 230 I=1,NP1
      DO 225 J=1,5
225 XLIN(I,J) = BLANK
      IF (IR(I) .EQ. 0) GO TO 230
      LIM = I + IR(I) -1
      DO 240 J=I,LIM
      XLIN(I,J) = VERT
240 CONTINUE
230 CONTINUE
      RETURN
      END
      SUBROUTINE BONFT (XBAR,STD,NP,N,IRES,NRES )
      T2(DF) = (1.8148*DF + 0.2500)/(0.9259*DF - 1.0000)
      T3(DF) = (1.5524*DF + 0.1538)/(0.6467*DF - 1.0000)
      T4(DF) = (1.6521*DF + 0.7559)/(0.6271*DF - 1.0000)
      T5(DF) = (1.5796*DF + 0.7810)/(0.5628*DF - 1.0000)
      T6(DF) = (1.5994*DF + 1.1069)/(0.5464*DF - 1.0000)
      DIMENSION XBAR(1),STD(1),NP(1),IRES(NRES,1),SQN(10)
      WRITE (6,5)
5  FORMAT ('0'//)
      DO 10 I=1,N
      SQN(I) = STD(I)*STD(I)/NP(I)
      DO 10 J=1,N
      IRES(I,J)= 0
10  CONTINUE
      NM1 = N-1
      DO 20 I=1,NM1
      IP1 = I+1
      DO 20 J=IP1,N
      T = (XBAR(I)- XBAR(J))/SQRT (SQN(I) + SQN(J))
      DF = (SQN(I) + SQN(J))*2 / (SQN(I)*SQN(I)/(NP(I)-1)
      + SQN(J)*SQN(J)/(NP(J)-1))
1  NGO = N-1
      GO TO (2 ,30,40,50,6 ), NGO

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```

2 CONTINUE
  TCRIT = T2(DF)
  GO TO 60
30 CONTINUE
  TCRIT = T3(DF)
  GO TO 60
40 CONTINUE
  TCRIT = T4(DF)
  GO TO 60
50 CONTINUE
  TCRIT = T5(DF)
  GO TO 60
6 CONTINUE
  TCRIT = T6(DF)
  GO TO 60
60 CONTINUE
  IF ( ABS(T) .GT. TCRIT) IRES(I,J)= 1
  TCRIT = ABS(TCRIT)
  T = ABS(T)
  WRITE (6,70) I, XBAR(I), J,XBAR(J), T, DF,TCRIT
70 FORMAT ('0', 2('XBAR(',I1,')= ',F8.4,5X), 5X,'T'= ',F8.2,5X,
1 'T(',F5.2,')= ',F8.2)
20 CONTINUE
  RETURN
  END
  SUBROUTINE PARVAR (STD,NUM,N, IRES,NRES)
  REAL PI(6)/ .975,.975,.990,.995,.995,.999/
  DIMENSION STD(1), NUM(1), IRES(NRES,1), VAR(10)
  WRITE (6,5)
5 FORMAT ('0'//)
  NM1 = N-1
  P = PI(N)
  DO 10 I=1,N
  VAR(I) = STD(I)*STD(I)
  DO 10 J=1,N
  IRES(I,J) = 0
10 CONTINUE
  DO 20 I=1,NM1
  IP1 = I+1
  DO 20 J= IP1,N
  DF1 = NUM(J) - 1
  DF2 = NUM(I) - 1
  NF1 = DF1
  NF2 = DF2
  F = VAR(J)/VAR(I)
  CALL MDFT (P,DF1,DF2,FCRIT,IER )
  IF (F .GT. FCRIT) IRES(I,J)=1
  WRITE (6,30) I, STD(I), J, STD(J), F,NF1,NF2,P,FCRIT
30 FORMAT ('0',2(' SX(',I1,')= ',F8.4,5X),'F'= ',F8.2,5X,
1 'F(',I3,',' ,I3,',' ,F5.3,')= ', F8.4)
20 CONTINUE
  RETURN
  END

```

C PSORTM - MULTIPLE BYTE, LINEAR SELECTION, POINTER SORT
 C PROGRAM AUTHOR A. A. BROOKS
 C COMPUTING TECHNOLOGY CENTER, UNION CARBIDE CORP., NUCLEAR DIV.,
 C OAK RIDGE, TENN.

```

C      NUMBER OF COMPARISONS==**2/2
C      NBYTES=NUMBER OF BYTES FOR SEARCHKEY LE 4*NDL1
C      LIST=DOUBLY DIMENSIONED ARRAY TO BE SORTED
C      NDL1=FIRST DIMENSION OF LIST
C      NDL2=INDEX (SECOND DIMENSION) OF LAST ELEMENT OF LIST
C      NPTR=POINTER ARRAY , DIMENSION GE NDL2
C      *****
C      SUBROUTINE PSORTM(NBYTES,LIST,NDL1,NDL2,NPTR)
C      DIMENSION LIST(NDL1,NDL2),NPTR(NDL2)
COMMENT CHECK INITIAL ORDER
      I1=NPTR(1)
      IF (NDL2.LE.1.AND.I1.EQ.1) RETURN
      IF (I1.LT.1.OR.I1.GT.NDL2) GO TO 30
      DO 20 I=2,NDL2
        I2=NPTR(I)
        IF (I1.EQ.I2) GO TO 30
        IF (I2.LT.1.OR.I2.GT.NDL2) GO TO 30
        IF (ICOMPA(LIST(1,I1),LIST(1,I2),NBYTES)) 10,10,30
      10  I1=I2
      20  CONTINUE
      RETURN
C      *****
COMMENT SET UP POINTER ARRAY
      30 DO 40 I=1,NDL2
        40  NPTR(I)=I
C      *****
      IF (NDL2.LE.1) RETURN
      IUP=NDL2-1
      DO 70 I=1,IUP
        JHOLD=I
        JLOW=I+1
        DO 60 J=JLOW,NDL2
          IF (ICOMPA(LIST(1,NPTR(J)),LIST(1,NPTR(JHOLD)),NBYTES)) 50,
            > 60,60
        50  JHOLD=J
        60  CONTINUE
        IF (JHOLD.EQ.I) GO TO 70
        NPTRIH=NPTR(I)
        NPTR(I)=NPTR(JHOLD)
        NPTR(JHOLD)=NPTRIH
      70  CONTINUE
      80 RETURN
      END

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