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STUDIES OF TANDEM MIRROR CONFINEMENT*

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ABSTRACT

This paper describes a number of physics studies relevant to tandem mirror confinement. We give the basic axial confinement laws and show that $T_e$ depends critically upon end loss. Sufficient central-cell end loss can stabilize the drift-cyclotron loss-cone mode in the plugs, although the resultant $T_e$ scaling is too slow for reactors. Minimum-B plugs stabilize flute MHD- and rotation-driven modes; local ballooning sets limits on $\beta \geq 0.5$. Proper magnetic symmetry is important for good drift confinement. For small increases in the total injected power, supplementary ion-cyclotron r.f. heating can halve the neutral-beam energy required to maintain plug densities.
I INTRODUCTION

A tandem mirror (TM) system [1,2] has three collinear mirror plasmas: two small, magnetically confined, hot end-cells (plugs) of average energy \( E_p \) and density \( n_p \); and a large-volume, less hot central cell of temperature \( T_c \) and density \( n_c < n_p \). Electrons of temperature \( T_e \) are confined in the overall ambipolar potential. Central-cell ions with \( T_c < \Phi_c \) are confined by the potential barrier

\[
e\Phi_c = T_e \ln(n_p/n_c)
\]  

(1)

Important physics questions addressed in the first generation of TM experiments will include microstability of the plugs, MHD curvature- and rotation-driven flute and ballooning stability, ion-drift confinement, and ion-electron confinement and energetics under neutral-beam and ion-cyclotron heating and direct electron heating. We discuss topics relevant to these for conditions typical of experiments under construction, e.g., the Tandem Mirror Experiment (TMX) at the Lawrence Livermore Laboratory [3], and conceptual reactors [4].

II MICROSTABILITY OF THE PLUGS

TM plugs are susceptible to loss-cone modes such as the drift-cyclotron loss-cone (DCLC) mode [5] thought to dominate all current high-density \( \nu_{pi}/\nu_{ci} >> 1 \) mirror machines. The conditions for plug stability can constrain system parameters; therefore, they must be granted primary consideration in a TM system design. Because of the small plug length in units of gyroradii, instabilities such as the high frequency convective mode [5] and the Alfvén ion-cyclotron mode are calculated not to be significant [6].

Several means for stabilizing the DCLC modes have proven successful in low energy mirror machines [7,8], and others have been proposed [9,10,11]. In energetic neutral-beam-powered mirror plasmas, only stabilization by a low-energy stream has proved routinely successful to date [12], although there is evidence [13] for the predicted reduction in the stream required with increasing radial scale length [14]. Both theoretically [14] and experimentally [12], the flux of ions found necessary to stabilize 2XIIB (radius ~ 2 to 3 ion gyroradii) is given by

\[
\dot{j}_{\text{stab}} = \frac{3.2 \times 10^7 \lambda T_e^{3/2}(\text{keV})_p (\text{cm}^{-3})}{(R - 1)A_p^{1/2}E_p(\text{keV})L_p(\text{cm})} \text{cm}^{-3} \text{s}^{-1}
\]  

(2)

where \( L_p \), and \( R_p \), and \( A_p \) are the plug length, \( \varepsilon \)-enhanced mirror ratio, and ion atomic number, respectively, and \( \lambda \) is a coefficient, theoretically about unity, which is to be determined from experiment. A value of \( \lambda \approx 0.5 \) is inferred from measured fluxes [15].

The minimum flux cited above was calculated from the requirement of a minimum density of low-energy ions needed to partially fill the loss cone, coupled with the fact that these ions are confined only for an axial transit time. We calculate this minimum density and consider other effects of low-density, low-energy plasma in combination with the hot loss-cone plasma by numerically integrating a differential equation.
described earlier [16]. This equation along B allows finite β and
variation of B and density. In particular, we have modeled a Maxwellian
central-cell plasma that is blocked by the ambipolar potential of a
mirror plasma. For β >> m_e/m_i, we do not find that the combination
of two adjacent plasmas is stable by virtue of a flute average [17].
Instead, unless the axial extent of the hot plasma is < c/ω_p, we find
that warm plasma is required at the midplane. Because in the TM appli-
cation such warm plasma must be considered lost from the central cell,
its lifetime in the plug is only a transit time; and we recover Eq. (2)
as the required flux through the plug.

As a result of these considerations, TMX has been designed so that in
its minimum operating mode the flux of the central-cell loss flowing
through the plugs equals j_{stab} in Eq. (2). This condition is used to
determine the operating parameters given in the next section. Should
alternate plug stabilization techniques prove successful in reducing the
required central-cell loss flux, those operating parameters will improve
accordingly.

We note that because this high end loss lowers T_e, TM reactors must
rely upon some other means for DCLC stability, except possibly for a
fusion-fission hybrid [4]. It is important, therefore, to demonstrate
the effectiveness of one of the means in Refs. [7-11] in a hot, neutral-
beam-sustained mirror machine.

III PARTICLE AND ENERGY AXIAL CONFINEMENT

Because both electrons and thermalized central-cell ions are magneti-
cally and potentially confined, the procedure for calculating their loss
rates is that of Pastukhov [18]. His results have been generalized to
(multiple) ion species and bounce-averaged electron motion in the three-
cell system [19]. Here, we assume a single hydrogenic ion species and
neglect electron scattering in the plugs relative to that in the (large
volume) central cell. For the species label a = e or c (c for central-
cell ions), temperature T_a, 90°-scattering rate

\[
\tau_a^{-1} = \frac{4}{\sqrt{2}} e \lambda_a \tau_c \tau_a^{-1/2} T_a^{-3/2} \tag{3}
\]

(where λ_a is the Coulomb logarithm), confining potential \( \Phi_a \) for each
species defined as positive, and mirror ratio R_c, we write the Pastukhov
formulae for \( e\Phi_a/T_a = 2 \) as

\[
\frac{dn_a}{dt}_{\text{loss}} = -\frac{n_c e_a}{\tau_a \gamma(e R_c)} \left( \frac{T_a}{e\Phi_a} \right) \exp \left( -\frac{T_a}{e\Phi_a} \right), \tag{4}
\]

\[
\frac{3}{2} \frac{d(n_a T_a)}{dt}_{\text{loss}} = (T_a + e\Phi_a) \frac{dn_a}{dt}_{\text{loss}} \tag{5}
\]

where \( e = 2, \epsilon_c = 1, e > 0, \) and
We define the particle flux per unit volume into the central cell and each plug, $J_c$, $J_p$, and the respective volumes per unit area $L_c/B_c$, $L_p/B_p$, with lengths $L_c$ and $L_p$ and magnetic field strengths $B_c$, $B_p$. Although for a reactor $J_c L_c/B_c \approx 2 J_p L_p/B_p$, we assume here that $J_c L_c/B_c \gg 2 J_p L_p/B_p$ as it is for TMX operation due to the requirement of DCLC stability (see Sec. II). The ratio $e \phi_e/T_e$ is determined by equality of loss rates from Eq. (4),

$$\frac{d n_c}{d t} \bigg|_{\text{loss}} = \frac{d n_e}{d t} \bigg|_{\text{loss}} = j_c ;$$  \hspace{1cm} (7)

for typical TMX confinement $e \phi_e/T_e \approx 5.5$; for a reactor, $\approx 6.5$. Because gas-fed ions are lost from the central cell by heating, we must consider energy balance in conjunction with confinement. Electrons are heated by the plug ions and cooled by the central-cell ions and electron loss; again for $J_c L_c/B_c \gg 2 J_p L_p/B_p$ this gives

$$\frac{3}{2} \frac{d}{d t} (n_c T_e^2) = \frac{L_c B_c n_e^2}{L_c B_c (n \tau)_{eq}} + P_{e-aux} - \frac{3}{2} \frac{n_c^2 (T_e - T_c)}{(n \tau)_{eq}} - (e \phi_e + T_e) j_c ,$$  \hspace{1cm} (8)

where $(n \tau)_{eq} = 1.4 \times 10^{12} T_e^{3/2}$(keV) (for D$^+$) is the electron-ion equipartition time. $P_{e-aux}$ represents any auxiliary power into the electrons, e.g., that fraction deposited either by neutral injection into the central cell or by direct electron heating. The central-cell ion power flow is similar, except that by charge exchange a gas-feed source introduces a power loss but no particle loss. For a ratio of charge-exchange to ionization cross sections $\sigma_x/\sigma_i$, the ion power balance becomes

$$\frac{3}{2} \frac{d}{d t} (n_c T_c^2) = \frac{3}{2} \frac{n_c^2 (T_e - T_c)}{(n \tau)_{eq}} + P_{c-aux} - \left[ e \phi_c + T_c (1 + \frac{3}{2} \frac{\sigma_x}{\sigma_i}) \right] j_c ,$$  \hspace{1cm} (9)

If Eqs. (8) and (9) are added, we obtain

$$\frac{3}{2} \frac{d}{d t} n_c (T_e + T_c) = \frac{L_c B_c n_e^2}{L_c B_c (n \tau)_{eq}} + P_{e-aux} + P_{c-aux} - \eta T_e^2 j_c ,$$  \hspace{1cm} (10)

where we introduce

$$\eta = \frac{T_e^{-1}}{T_c} \left[ e \phi_c + T_e + e \phi_e + T_c (1 + \frac{3}{2} \frac{\sigma_x}{\sigma_i}) \right]$$  \hspace{1cm} (11)

as the energy, in units of $T_e$, carried off by each injected electron and its associated ion; $\eta$ is a slowly varying parameter, dominated by $e \phi_e/T_e$, having a value about 8.

For fixed plug parameters $n_p, E_p$, these rate equations describe classical TM confinement for arbitrary $j_c$. Their solutions have been compared with solutions of the full Fokker-Planck equation [19], giving $\pm 20\%$ agreement for $R_c \geq 3$ and $e \phi_c/T_c \geq 2$. 

-4-
As discussed in Sec. II, the TMX plugs will be stabilized to DCLC by adjusting sources for the central cell according to Eq. (2). With volume weighting, stability requires

\[
j_c \geq 2.3 \times 10^7 \frac{\lambda B_c}{B (R - 1)} \frac{T_{e}^{3/2} (\text{keV}) n_p (\text{cm}^{-3})}{E (\text{keV}) L_c (\text{cm})},
\]

(12)
in which case for no auxiliary power Eqs. (10) and (12) give

\[
T_e (\text{keV}) = 0.11 \left[ \frac{\beta_p^2 (T_e) E (\text{keV}) (R - 1) L (\text{cm})}{\eta \lambda} \right]^{1/4}
\]

(13)
where \(\beta_p\) is the plug beta. As in 2XIIB, the loss flux required to stabilize the plugs lowers \(T_e\) well below its classical value. As long as \(T_e\) is held to this scaling, improvement with plug parameters is slow.

Based on these equations, we have developed a rate code to study transient behavior [20] such as startup in TMX. To model the TMX plugs subject to DCLC, we set

\[
\frac{dn_p}{dt} = \max \left\{ \frac{n}{\tau_p}; j_{\text{stab}} - j_c \right\}
\]

where \(\tau_p \approx \tau_{eq}\), which approximates very well the full quasi-linear code [14]. We find that reaching a steady state in TMX within the 25-ms beam pulse requires a programmed central-cell gas feed, with a short, initial pulse 5 times the steady state. We have also studied transient reactor problems, such as purging of thermalized plug alphas, replacing Eq. (14) with an improved model for classical plug confinement [20].

IV LOW-FREQUENCY STABILITY

Both rotation and local bad curvature in the transitions between the central cell and the plugs can drive low-frequency modes in a TM system. The plugs are generally rigid due to their high pressure and minimum-B, with line-bending displacements accordingly excluded from them.

We investigate the stability to both flute and ballooning interchange modes using a differential-eigenvalue equation along \(B\), which is localized by an eikonal approximation normal to \(B\). With \(B = Bb = \psi \times \phi\), \(d\xi = b \cdot d\tau\), \(E_\perp = -B\psi U\), and \(U(\psi, \phi, \lambda) = \tilde{U}(\lambda) \exp S(\psi, \phi)\), we use an equation obtained by Newcomb [21]:

\[
\frac{d}{d\xi} \left[ \frac{|\psi S|^2}{B} \left[ 1 + \frac{4\pi}{B^2} (p_\perp - p_n) \right] \frac{d \tilde{U}}{d\xi} \right] + \frac{4\pi}{B} \left\{ \frac{|\psi S|^2}{B^2} \frac{M}{2} \right. \\
+ \frac{3}{3\psi} (p_\perp + p_n) \left[ \frac{3S}{3\psi} K_\psi - \frac{3S}{3\phi} K_\phi \right] \right\} \tilde{U} = 0,
\]

(15)
where the curvature has been written \(K = b \cdot \nabla b = K_\psi \psi + K_\phi \phi\), and \(M\) is the mass density. Except for the allowance for anisotropic pressure, this is the equation used to study ballooning modes in tokamaks [22].
In solving Eq. (15), we have used quasi-analytic models for the vacuum field and the pressures in finite-β equilibria. In the paraxial approximation, the flux coordinates are

\[
\psi = \frac{1}{2} \hat{B}(z) \left\{ x^2 \exp[-2C(z)] + y^2 \exp[2C(z)] \right\},
\]

\[
0 = \tan^{-1}\left\{ \frac{y}{x} \exp[2C(z)] \right\},
\]

where \( \hat{B}_{\text{vac}}(z) \) and \( C(z) \) are chosen to fit numerical vacuum field calculations. As a model, we use the finite pressure depression of \( \hat{B} \)

\[
\hat{B}^2 + 8\pi p_\perp(\hat{B},\psi) = \hat{B}_{\text{vac}}^2(z)
\]

both in the plugs and in the central cell including electron pressure, but we assume that \( C(z) \) is unaltered. In the plugs, we have used the ion pressure profiles

\[
p_i(B,\psi) = \frac{1}{2} B(B_1 - B)^2 \text{ and } p_\perp(B,\psi) = \frac{1}{2} B_1^2 (B_1 - B)^{\nu-1},
\]

where \( B_1 \) and \( \nu \) are parameters chosen to fit the injected profiles, and we have verified the mirror and firehose stability criteria,

\[
\frac{\partial}{\partial B} \left[ B^2 + 8\pi p_\perp \right] > 0 \quad \text{and} \quad B^2 + 4\pi(p_\perp - p_i) > 0
\]

for the chosen parameters. In the central cell, the ions are isotropic, as are the electrons everywhere. These models, together with the parameter \( \xi = 2\psi(\partial S/\partial \psi)/(\partial S/\partial \theta) \), fix the coefficients in Eq. (15). Alternate field or pressure models can also be used.

Equation (15) is solved subject to either of two boundary conditions,

\[
\frac{dU}{d\xi} \big|_{\pm L/2} = 0 \quad \text{(free)} \quad \text{or} \quad U(\pm L/2) = 0 \quad \text{(line tied)},
\]

where for most purposes \( L \) is the overall length. To solve, \( \omega^2 \) is approximated by a Rayleigh-Ritz method as a first guess for a shooting technique. For fixed plug parameters and vacuum field, the parameter space \( \psi(\psi,\theta,\xi) \) is searched for the maximum central cell \( B_C \) for which \( \omega^2 > 0 \) everywhere.

There are no flute solutions of Eq. (15) provided the line integral of the \( \partial(p_\perp + p_i)/\partial \psi \) -term is positive. This depends upon the ratio \( p_i/p_n \), occurs for a sufficiently large ratio of plug to central-cell pressure, and results from a pressure-weighted flux-tube volume that decreases with \( \psi \). If we assume this to be the case, as in TMX, the \( B_C \) limit estimated from equality of the MHD growth time with the Alfvén time through the bad-curvature region is of order unity. Numerically, we find \( B_C \)-limits in the range 0.5 to 0.6 for the TMX magnet set.

Rotation-driven modes of a straight column having low azimuthal and radial mode numbers \( m \) and \( n \), respectively have been described by Freidberg and Pearlstein (F-P) [23] in the context of finite Larmor radius fluid equations. For rigid-rotor equilibria, they find two classes of modes. The first, having \( m = 2, n = 0 \), is most unstable for \( k_z = 0 \); the second, having \( m = 1, n = 0 \), requires \( k_z \neq 0 \). The former has the larger growth rates, particularly \( m = 2, n = 0 \); however, the latter has the lower threshold for rotation, arbitrarily small in the ion diamagnetic direction.
We first apply their results to ballooning in the central cell. Assuming that the radial electric field is given by Boltzmann-distributed electrons \[ e^c(r) \approx \text{constant} + T_e \ln n_c(r) \], the F-P result for the growth rate of the \( m = 2, n = 0 \) mode when \( T_e > T_c \) is given by

\[
\gamma_{\text{rot}} \approx \Omega_s \left[ \frac{T_e}{T_c} \left( \frac{T_e}{T_c} + 1 \right) \right]^{1/2},
\]

where \( \Omega_s = \frac{2 \rho_c}{r_c} \left( 1 + (1 - \beta_c)^{1/2} \right)^{-1} \) (20)

\( \rho_c, \Omega_c \) are the vacuum ion gyroradius and frequency, and \( \beta_c \) is the on-axis beta. This mode is stabilized for \( k_z V_a \geq \gamma_{\text{rot}} \), where \( V_a \) is the Alfven speed. We take \( k_z = \pi/L_c \) to obtain the condition

\[
\frac{\beta_c}{1 + (1 - \beta_c)^{1/2}} < \pi^2 \frac{T_c}{T_e} \left( \frac{r_c^2}{L_c \rho_c} \right)^2.
\]

(21)

For both TMX and reactors, the ratio of lengths is nearly unity; so that this condition is easily satisfied. For our assumed electric field model and \( T_e > T_c \), \( m = 2, n = 0 \) requires the largest \( k_z \) for stability; so Eq. (22) is a sufficient condition for ballooning stability of all the F-P rotation modes.

Satisfaction of Eq. (21) does not ensure stability to a rotation-driven flute having no finite-\( k_z \) stabilization. The F-P modes were derived for a cylindrical plasma. For flute stability in a TM, the minimum-B plugs must stabilize the rotating central cell; therefore this question must be addressed allowing for both the nonaxisymmetry and the axial variation. As a tool for including axial variation, Eq. (15) assumed large \( m \) and is local in angle. It is therefore unsuitable, especially for the F-P \( m = 2, n = 0 \) mode having the largest \( k_z = 0 \) growth rate. A flute-averaged \( r, \theta \) description of this mode will be developed in the future; here we estimate its effect.

The rotational drive is potentially dangerous because it acts over the entire central-cell length \( L_c \), whereas curvature is limited to the two transitions, each of length \( L_{\text{tr}} \). To compare the two, we compare the weighted growth rates, given by Eq. (20) and \( \gamma_{\text{MHD}} = \gamma_{\text{MHD}} \rho_c^2 L_{\text{tr}}^2 \), where \( R_c \) is the central-cell mirror ratio. For rotation not to dominate curvature requires \( L_c \gamma_{\text{rot}}^2 \leq 2 L_{\text{tr}} \gamma_{\text{MHD}} \), or

\[
\frac{T_e}{T_c} \left( \frac{T_e}{T_c} + 1 \right) \leq 2 \frac{\rho_c^4}{\rho_c^2 L_{\text{tr}}^2 L_c}.
\]

(22)

For both TMX and one set of reactor parameters [4], this inequality is barely satisfied, showing that rotation is an important consideration for flute stability but that it will not severely alter results that neglect it.

V VACUUM DRIFT SURFACES

The minimum dependence of drift surface on pitch angle, called omni-
genility, relies on the symmetry about the central cell midpoint \( z = -z, \theta = \pi/2 - \theta \). Particles of constant \( \epsilon, \mu \) drift on surfaces of
\[ J = \sqrt{2} \int d\xi \sqrt{\mathcal{C} - \frac{\mathcal{B}}{m} - \frac{q}{m} \phi} = \text{constant.} \]  

In the paraxial expansion [see Eq. (16)], \( d\xi = dz \) and

\[ B(\psi, \theta, \zeta, z) = \hat{B}(z) + \psi [G_1(z) + G_2(z) \cos 2\theta] + O(\psi^2), \]  

where

\[
\begin{bmatrix}
G_1 \\
G_2
\end{bmatrix} = \left[ (C^2 - \frac{1}{\mathcal{B}})q \mathcal{B} \right]^{1/2} \begin{bmatrix}
\cosh 2C \\
\sinh 2C \\
\cosh 2C \\
\sinh 2C
\end{bmatrix},
\]

and a similar expansion holds for \( \phi \). For the above symmetry, \( G_1 \) is even in \( z \) and \( G_2 \) is odd, so that

\[ J = J_0(e, \mu, \psi) + \psi^2 \cos 4\theta \, J_4(e, \mu) + O(\psi^4), \]  

with \( \theta \) -dependence appearing only in order \( \psi^2 \). Drift orbits follow

\[ \frac{d\psi}{d\theta} = 4\psi^2 \frac{J_1}{\partial J_0/\partial \psi} \sin 4\theta. \]  

For vacuum fields, \( \partial J_0/\partial \psi \approx \text{constant} \), and Eq. (27) is in excellent agreement with numerically calculated fields and orbits. Violation of the above symmetry gives \( \Delta \psi/\psi = O(1) \) [25] and much enhanced radial mixing. For finite \( \beta \) and \( \phi \neq 0 \), Eq. (27) can lead to banana orbits due to \( \partial J_0/\partial \psi \) vanishing locally, which has numerous implications for transport [24].

The splitting of drift surfaces of particles with positive and negative parallel velocity results from \( G_2(z) \neq 0 \) in Eq. (24). As shown in Ref. [24], this leads to an enhanced transport when the axial bounce and rotational frequencies of the particles are comparable. This splitting can be reduced to higher order in \( \psi \) provided \( G_2 \) can be made to vanish. If the vacuum field is designed to satisfy this condition, the actual field and \( \phi \) will also satisfy it.

VI ION-CYCLOTRON HEATING OF PLUGS

We have begun investigation of ion-cyclotron resonance heating (ICRH) in TM plugs as a means for reducing required neutral-beam energies or currents. Although many topics bear on a final evaluation, we have focused initially on the effects of velocity space transport induced by ICRH in the diffusive approximation, modifying an existing Fokker-Planck code [26]. We consider second-harmonic and fundamental heating, in which is included parallel cooling when \( \omega \leq \omega_{ci} \) (local) and \( k_n \neq 0 \) [27]. The two frequencies behave qualitatively alike, although the harmonic generates a more energetic tail.

In considering ICRH as a method for supplementing reduced energy neutral beams, we find similar confinement for the same total net power, provided neutrals are injected with energies at least twice the ambipolar cutoff \( e\psi/(R - 1) \). For example, under reactor conditions, a constant density can be maintained with halved injection energy, doubled current,
and 25% ICRH-power. For laboratory plasmas, we also find that confinement improves with ICRH; however, due to the increased ion energy, there can be an important power loss through charge exchange [28]. These studies will continue and will be reported at a later date.

Finally, we are studying the feasibility of using ICRH to trap end-lost central-cell ions in the plugs [28]. If successful, this would avoid use of many or all neutral beams, but the possible deleterious effects on confinement in the central cell must be carefully assessed.

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