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MEASUREMENT OF LOW ENERGY NEUTRINO ABSORPTION PROBABILITY IN THALLIUM 205

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Abstract

A major aspect of the P-P solar neutrino flux determination using ^{205}Tl is the very difficult problem of experimentally demonstrating the neutrino reaction cross section with about 10 percent accuracy. Based on ideas originating in discussions of P. Kienle, W. Henning and W. Kutschera, such measurements now become possible. Moreover, the proposed measurements of weak interaction rates are in the isotopic states of interest; hence they will be much more readily acceptable than would indirect measurements via strong or electromagnetic interactions or theoretical calculations.

One will soon be able to completely strip the electrons from atomic ^{205}Tl and to maintain the bare nucleus in this state in the heavy ion storage ring to be built at GSI Darmstadt. This nucleus can decay by emitting a beta-minus particle into the bound K-level of the daughter $^{205}_{82}\text{Pb}^{81+}_{1/2-}$ ion as the only energetically open decay channel, (plus, of course, an antineutrino). This single channel beta decay explores the same nuclear wave functions of initial and final states as does the neutrino capture in atomic ^{205}Tl , and thus its probability or rate is governed by the same nuclear matrix elements that affect both weak interactions:

$$^{205}_{81}\text{Tl}^{81+}_{1/2+} + ^{205}_{82}\text{Pb}^{81+}_{1/2-} + \bar{\nu} \quad (1)$$

$$\nu + ^{205}_{81}\text{Tl}^0_{1/2+} + ^{205}_{82}\text{Pb}^{1+}_{1/2-} + e^{-}$$

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Measuring the rate of accumulation of $^{205}\text{Pb}^{81+}$ ions in the circulating beam of $^{205}\text{Tl}^{81+}$ ions gives directly the cross section of reaction (2). The calculations of the expected rates under realistic experimental conditions will be shown to be very favorable for the measurement, e.g., $\sim 10^4$ ions per hour in a 100 ma. beam. The great sensitivity of the rate to the ^{205}Pb - ^{205}Tl mass difference will appear (12%/keV), showing the need to materially improve the uncertainty in this value, and several experiments will be proposed. A special calibration experiment to verify the method and check the theoretical calculations will be suggested. Finally the neutrino cross section calculation based on the observed rate of reaction (1) will be shown. Demonstrating bound state beta decay may be the first verification of the theory of this very important process that influences beta decay rates of several isotopes in stellar interiors, e.g., ^{187}Re , that play important roles in geologic and cosmologic dating and nucleosynthesis.

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I. INTRODUCTION

We consider a proposed determination of the low energy neutrino absorption cross section of ^{205}Tl by measuring the rate of bound ls state beta-minus decay of bare $^{205}\text{Tl}^{81+}$ nuclei circulating in a beam in a storage ring. This scheme is based on earlier ideas of Paul Kienle, Walter Henning and Walter Kutschera on the relative shifts of the energies of the whole system "nucleus + atomic electrons" in highly stripped ions and their effect on parent-daughter "nucleus-atom" spontaneous transitions including beta and other decays.

Neutrino absorption in $^{205}\text{Tl}_{1/2+}$ to the 2.3 keV, 1/2- state in ^{205}Pb , of interest as an integrating detector of low energy solar neutrinos¹, is the inverse of the experimentally unobservable L+M+N... shell electron capture of the $^{205}\text{Pb}_{1/2-}$ state. (The 1/2- state in ^{205}Pb decays primarily by outer shell internal conversion, with an undetectable electron capture branch of order 10^{-15}). Hence the relative rates of these inverse decays are, apart from known phase space factors, governed by the same set of first forbidden non unique nuclear matrix elements.

One knows from shell model considerations that the inverse electron capture transition is, with high probability a quite fast parity forbidden transition. Moreover, the high Z and low disintegration energy $Q = (44 \text{ keV}/27.2 \text{ eV}) = 1620$ atomic units make $\xi = Z/QR \approx 380 \gg 1$, (R =nuclear radius = 1.34×10^{-4} atomic units) thus satisfying the " ξ " approximation. Both of these facts indicate that the few usually larger energy independent nuclear matrix elements suffer no significant cancellations and thus they dominate the transition rate.

In this ξ approximation the smaller lepton wave function combinations (by factor ξ) are ignored. Then the relative transition rates for various weak

interaction decay channels between these nuclear states in the same Z-
increasing (-or decreasing) sense, e.g., for the $(\lambda_{M1}/\lambda_{L1})$ ratio of electron
capture rates from the M1 and L1 orbitals in

are independent of the values of the nuclear matrix elements². In such a case
these larger matrix element terms all share a common dominant lepton-wave-
function-combination factor as coefficient and hence they cancel in the ratio.

However, in the comparison of the reduced (i.e., phase space factor
independent) transition rates of two transitions inverse to each other, i.e.,
 $Z+1 \rightarrow Z$ and $Z \rightarrow Z+1$, e.g.,

in the special case of first forbidden non-unique transitions the cross
product terms involving some of the nuclear matrix elements change sign
between $Z \rightarrow Z+1$ and $Z+1 \rightarrow Z$, so the cancellation is not complete. Thus the
ratio of forward-to-backward reduced rates is not independent of the nuclear
matrix elements, so measuring the ft value of one does not exactly determine
that of the inverse transition. Takahashi and Yokoi³ state that such "sign
changes mostly cancel out once conserved vector current and approximate
relations between the matrix elements are adopted". Thus such a forward-
backward comparison can be a useful guide, but for an exact comparison of a
measured to an inferred rate both should be in the same Z-direction between the
same states. Both types of comparison are used here, and it turns out that the
comparisons of three ways of calculating the bound state beta decay rate

brings out several useful features not evident in any of the methods alone.

II. ENERGETICS OF BOUND STATE BETA-MINUS DECAY OF TOTALLY STRIPPED $^{205}\text{Tl}^{81+}$

Refer to Fig. 1.

Let Q_{ec} be the bound state electron capture Q value of the 2.3 keV, 1/2- excited state of ^{205}Pb ; $Q_{ec} = 44 \pm 1$ keV from the relative subshell capture ratios⁴ in $^{205}\text{Pb}_{5/2-}$ ground state decay, or, $Q_{ec,W} = 56 \pm 1.6$ keV from Wapstra's⁵ mass evaluation. We use the former value here and discuss this serious disparity in the Appendix.

Let $B.E._{Tl} = 551.0$ keV be the total binding energy of the 81 electrons in Tl, and $B.E._{Pb} = 568.3$ keV be the total binding energy of the 82 electrons in Pb. Both values come from the relativistic Dirac-Fock calculations of Desclaux⁶ including the Breit interaction. Their difference, 17.3 keV, should be accurate to ± 0.1 keV; the similar $\Delta B.E.$ from the pair $^{94}\text{Pu}-^{95}\text{Am}$ is 22.42 keV from the Desclaux code and 22.32 keV from the Oxford Dirac-Fock code⁷. Stripping both atoms raises the ion energies by these B.E. amounts.

Let $B.E._{1s}(\text{Pb}^{81+}) = 101.34$ keV be the hydrogenic 1s binding energy in Pb^{81+} , obtained from the Oxford Dirac-Fock relativistic code with Breit interaction, Lamb shift and QED corrections included. A similar calculation of Johnson and Soff⁸ obtained 101.34 keV. With a point nucleus instead of a uniformly charged nucleus, $B.E._{1s} = 101.41$ keV. Such codes are experimentally realistic; a similar code matched the experimental binding energy of the 1s orbital in ^{100}Fm , 141.967 ± 0.013 keV to ± 0.025 keV⁹.

Then the $Q_{\beta-,1s}$ bound state beta-minus decay energy from the stripped $^{205}\text{Tl}^{81+}$ ion to hydrogenic lead

$$Q_{\beta-,1s} = -Q_{ec} + (B.E._{Tl} - B.E._{Pb}) + B.E._{1s}(\text{Pb}^{81+}) = 40.0 \pm 1 \text{ keV}, \quad (1)$$

where the uncertainty is the stated experimental error of Q_{ec} . It will be shown that the $1s$ bound state beta decay rate (hence the neutrino cross section) is very sensitive to the $Q_{\beta-,1s}$ value, about 11% per keV, placing severe requirements on the accuracy of Q_{ec} . The disagreement with the Wapstra⁵ mass difference must be explored. For later comparison $Q_{\beta-,1s}$ based on Wapstra's $Q_{ec,W}$ is $Q_{\beta-,1s,W} = 28.0 \pm 1.6 \text{ keV}$.

The $1s$ bound state beta-minus decay of ${}^{205}\text{Tl}^{81+}$ is its only open beta decay channel.

III. CALCULATION OF THE $1s$ BOUND STATE BETA-MINUS DECAY RATE OF ${}^{205}\text{Tl}^{81+}$

Three ways of calculating the rate will be shown. Intercomparison of their differing logical structure leads to a partial reconciliation of their numerical results and to a proposed experimental way of resolving the residual disagreement. Without any one of the three such reconciliation could not be obtained.

The fundamental assumption underlying all methods is this: in allowed or forbidden unique transitions, and for first forbidden non unique transitions where the ξ approximation applies, the ft values pertaining to all parallel channels of weak interaction processes connecting a specific level in nucleus Z with a specific level in nucleus $Z+1$ are equal. One does not find this stated explicitly in any literature known to me, but it is implicit in numerous applications, most often seen in the simple equations giving the relative (partial) rates of electron capture from different orbitals, e.g.,

$$\frac{\lambda_{M1}}{\lambda_{L1}} = \frac{q_{\nu, M1}^2 \phi^2(R)_{M1} B_{M1}}{q_{\nu, L1}^2 \phi^2(R)_{L1} B_{L1}} \quad (2)$$

where the q_{ν} are the neutrino momenta, the $\phi(R)$ are the bound state electron radial wave function amplitudes evaluated at the nuclear surface and the B are factors near unity correcting for the effects of imperfect wave function overlap of parent and daughter orbitals and for the contributions of exchange transitions. For electron capture the expressions in numerator and denominator are defined as the Fermi function f , that which appears familiarly in $\log ft$, in complete analogy to the Fermi function integral-over-phase-space in beta-minus continuum decay. As the λ 's are inverse lifetimes one sees the identity of ft values for the two branches. Such identity of ft values follows, of course, from the relation that the ft value is inversely proportional to the square of the sum of nuclear matrix elements connecting the states.

All methods of calculating the bound state beta decay rate are, explicitly or implicitly relative methods, based on a measured or, in our case necessarily assumed, ft value or nuclear matrix element combination value. The measured bound state decay rate will then determine the true ft (or matrix element) value, which, in turn, will be used to calculate the neutrino interaction cross section with ^{205}Tl .

Since the purpose of the present calculation is to determine whether the proposed method of measuring the bound state beta decay rate is practical and realistic, I assume a value for $\log ft$ based on measured values of $\log ft = 5.3$ for several "analogous" decays between $1/2+ \leftrightarrow 1/2-$ states in nearby isotopes¹, but with the transition rate reduced by a factor of 4 to reflect the fact that the shell model orbital in ^{205}Tl which feeds the decay is

estimated to constitute only about one quarter of the ground state wave function.¹⁰ Thus the value of $\log ft$ used in all three calculations is 5.90 (t = half life in seconds) and all methods should give consistent results, with the qualifications to be shown for each.

Method 1. Relative single channel decay rates.

In complete analogy to equation (2) I write the ratio of the rate of the 1s bound state beta decay of the stripped $^{205}\text{Tl}^{81+}$ ion to the inverse $(Z+1+Z)$ L1 electron capture rate of the neutral $^{205}_{81}\text{Pb}_{1/2-}$ atom:

$$\frac{\lambda_{\beta-, 1s}}{\lambda_{ec, 2s}} = \frac{W_{\nu}^2 [\psi_{1s}^{81+}(R)/R]^2 B_{1s}^{81+}}{2W_{\nu}^2 [\psi_{2s}^0(R)/R]^2 B_{2s}^0} \quad (3)$$

where the W_{ν} or $W_{\bar{\nu}}$ are the neutrino (antineutrino) energies or momenta, the $[\psi(R)/R]$ are the appropriate bound state wave function amplitudes evaluated at the nuclear surface (actually the small components of the relativistic wave functions, consistent with both transitions connecting nuclear states of opposite parity) and the B correction factors are defined as for equation (2). The factor 2 in the denominator accounts for 2 initial 2s electrons, relative to 1 (numerator) nuclear-produced electron entering the 1s orbital. The wave function amplitudes from our computer program are per single electron.

The uncertainty in this comparison is that of the degree of cancellation of residual small cross terms in the nuclear matrix element sums omitted from numerator and denominator. I assume here that the cancellation is perfect, (this approximation was previously discussed); thus the numerical results should agree perfectly with those of the other two methods, in which the (virtual) transitions compared to the bound 1s beta decay are in the same $Z +$

Z+1 sense, as also are the 1s beta decay and neutrino captures. Equation (3) might not accurately give the experimental rate ratio if the cancellation is imperfect.

The compensating virtues of this method are its simplicity and symmetry. Apart from the statistical factor of 2 it is very difficult to justify insertion of any other factors into either numerator or denominator alone that should not apply to the other. This will appear importantly later. All questions of units vanish (this will be seen as not trivial later) and it is dimensionally clear.

Nevertheless, for consistency with later application I use natural units, in which $\hbar = m = c = 1$. As some of the input parameters are given in cgs or in atomic units ($\hbar = m = e = 1$), the transformation factors will be indicated, and the final transformation to decay rates in seconds also.

In equation (3) and in the two other calculations the parameters are:

$E_{\bar{\nu}} = 40.0 \pm 1$ keV, the antineutrino energy, already includes the appropriate binding energy term, 101.4 keV, as in equation (1). $W_{\bar{\nu}} = E_{\bar{\nu}}/mc^2 = 0.07828$ in natural units.

$E_{\nu} = 44 \pm 1 - 15.3$ keV = 28.7 ± 1 keV, where the 2s binding energy in

Tl = 15.3 keV. $W_{\nu} = E_{\nu}/mc^2 = 0.05616$.

All wave functions used here are evaluated using the multiconfiguration relativistic Dirac-Fock computer code of the Oxford group⁷, whose output is given in atomic units. A uniformly charged nuclear model was used, with radius $R = 1.342 \times 10^{-4}$ atomic units which corresponds to 7.10 fermis or $1.20 \times 10^{-13} A^{1/3}$ cm for the $A=207.19$ value used for Pb. The cases calculated were either neutral Pb atom initial state from which 2s electron capture occurs, or stripped final state Pb^{81+} ion with one 1s electron. The program gave both large and small components of the radial wave functions as a function of

radius R , on a fine grid. The radial wave function is defined as $\Psi(R)/R$, such that $|\Psi(R)/R|^2$ is the probability density for the electron at the nuclear surface, i.e., it has dimensions of L^{-3} . The large component is designated P , the small component, which we use throughout, Q , so $|\Psi(R)/R|^2 = |Q(R)/R|^2$.

A completely independent Dirac-Fock code of Desclaux¹¹ at NBS was used to generate the same set of wave functions for Pb (neutral atom). The agreement with the Oxford code results was within 0.5%. The Oxford code gave:

$[Q_{1s}^{81+}(R)/R] = 0.1129/1.342 \times 10^{-4}$ atomic units; $[Q(R)/R]^2/137.036^3 = 0.2750$ in natural units.

$[Q_{2s}^0(R)/R] = 0.04405/1.342 \times 10^{-4}$ atomic units; $[Q(R)/R]^2/137.036^3 = 0.04187$ in natural units.

The overlap and exchange factors B_{1s} and B_{2s} originate in the calculations of Bahcall¹² and Vatai¹³. Modernized versions of these factors are presented in Tables XI and XII of reference 2, from which $B_{2s} = 1.026$. For the decay into the vacant $1s$ orbital neither overlap nor exchange transitions are relevant; $B_{1s} = 1.00$.

Finally, $\lambda_{2s,ec}$ is evaluated from the assumed $\log ft = 5.90$ governing all transitions between the $^{205}\text{Tl}_{1/2+}$ and $^{205}\text{Pb}_{1/2-}$ states. For this $2s$ electron capture,²

$$f = (\pi/2) W_V^2 [Q_{2s}(R)/R]^2 B_{2s} = 2.128 \times 10^{-4}.$$

By assumption $ft = 10^{5.90} = 7.943 \times 10^5$ sec.

whence $t = 3.73 \times 10^9$ seconds, the partial half life, or

$$\lambda_{2s,ec} = \ln 2/t = 1.86 \times 10^{-10} \text{ sec}^{-1}.$$

(With a ratio of (total/2s) electron capture = 1.77 the total electron capture half life from the $1/2^-$ state is 66.6 years if $\log ft = 5.90$).

From equation (3) one has the bound state β -decay rate

$$\lambda_{\beta^-,1s} = 1.155 \times 10^{-9} \text{ sec}^{-1}. \quad (4)$$

Including the stated ± 1 keV uncertainty in the experimental value⁴ for the electron capture transition energy leads to an uncertainty (1 σ) in $\lambda_{\beta^-,1s}$ of $\pm 11\%$. Using $Q_{\beta^-,1s,W} = 28.0 \pm 1.6$ keV based on Wapstra's mass evaluation⁵, $Q_{ec,W} = 56 \pm 1.6$ keV gives

$$\lambda_{\beta^-,1s,W} = 5.66 \pm .66 \times 10^{-10} \text{ sec}^{-1},$$

a 12% uncertainty and a factor of 2 below the value above. Thus the measurement of the $1s$ bound state decay rate will be completely inconclusive with respect to the desired neutrino capture cross section unless the gross disparity with the Wapstra⁵ mass difference, 56 keV, is essentially eliminated or resolved, and the error reduced to at most 1 keV. I comment on this problem in the Appendix.

Method 2. Ratio of bound-state to continuum-beta-minus decays

A recent set of papers by Harston and Pyper^{14,15} presented the ratio of bound to continuum rates for allowed and first forbidden non unique transitions, respectively, as an extension of the earlier paper of Bahcall¹⁶ on allowed bound state beta decay. They specifically state that their equations are given in atomic units, but this turns out to be misleading; reasonable numerical results emerge only if they are calculated in natural

units, whose combination appearing in the equation differs from atomic units by $(\alpha = 137.036^{-1})^{-9} = 1.7 \times 10^{19}$

Their rate ratio in natural units is

$$\frac{\lambda_B}{\lambda_c} = \frac{\pi c^2 \frac{W^2}{v} \{ |Q_{1s}(R)/R|^2 / 137.036^3 \}}{4(2J_1 + 1)f} \frac{|P(R)/R|^2}{|Q(R)/R|^2} \times B_{1s} \quad (5)$$

where I have contracted their product of atomic wave function overlap integrals into the factor B_{1s} relevant to bound state beta decay into the 1s orbit of a stripped nucleus; $B_{1s} = 1.00$. For the virtual continuum beta minus decay to the totally stripped $^{205}\text{Pb}^{82+}$ nucleus, for which the transition energy $Q = -61$ keV, f represents the conventional integral-over-phase-space factor; note that this transition is in the same $Z \rightarrow Z+1$ sense as the bound state beta decay, so perfect cancellation of the nuclear matrix element sums in numerator and denominator follows.

I evaluate λ_c from the general ft relationship¹⁷

$$\lambda_c = \frac{g_v^2 m^5 c^4}{2\pi^3 \hbar^7} |M|^2 f = 1.123 \times 10^{-4} |M|^2 f \text{ sec}^{-1} \quad (6)$$

where $|M|^2$ is the nuclear matrix element sum the experiment is measuring, and $g_v = 1.4122 \times 10^{-49} \text{ erg cm}^3$ is the Fermi weak interaction coupling constant.¹⁸ In these calculations I assume a value for $|M|^2$ consistent with the assumed value of $\log ft = 5.90$, by correlation to the known value of $|M|^2 = 2$ for superallowed $0^+ \rightarrow 0^+$ transitions for which the measured value¹⁸ of $ft = 3085 \text{ sec}$. For the assumed $\log ft = 5.90$, $ft = 7.943 \times 10^5$ whence the corresponding value of $|M|^2 = (3085/7.943 \times 10^5) \times 2 = 7.77 \times 10^{-3}$. Hence,

$$\lambda_c = 8.723 \times 10^{-7} f \text{ sec}^{-1} \quad (7)$$

Inserting (7) in (5)

$$\lambda_B = \pi c^2 \tilde{W}_V^2 \{ |Q_{1s}(R)/R|^2 / 137.036^3 \} \times 8.723 \times 10^{-7} B_{1s}/4 \quad (8)$$

Here the Fermi functions have cancelled, $(2J_i+1) = 1$ as the initial spin of the initially vacant $1s$ orbital equals 0, and the factor $[P(R)/R]^2 / [Q(R)R]^2$ introduced by Harston and Pyper¹⁵ to correct the allowed Fermi function f to a first forbidden form (correcting the continuum Coulomb wave function evaluated at the nuclear radius) was dropped because λ_c (and thereby the Fermi function appropriate to this first forbidden transition) was evaluated here by assumption.

Evaluating equation (8) in natural units, but noting that the factor λ_c is already in cgs units gives

$$\lambda_{B,1s} = 1.155 \times 10^{-9} \text{ sec}^{-1} \quad (10)$$

in agreement with equation (4).

Method 3. Rate of bound state beta-minus decay

Bahcall¹⁶ derived the rate of bound state beta decay for allowed transitions, and applied it to the simplified situation of decay to inner orbitals in stripped nuclei as in stellar interiors. I use his formulation, given in natural units, modified only for first forbidden non unique transitions by using the small components of the relativistic wave functions, following Harston and Pyper¹⁵

Simple algebraic manipulation of Bahcall's formulae (12) and (16) gives

$$\lambda_{B,1s} = g_v^2 m^5 c^4 (\alpha Z)^3 (a_0/2)^3 \frac{(W_0 + b - mc^2)^2}{(mc^2)^2} (Q_{1s}^{81+}(R)/R)^2 B_{1s} |M|^2 / 2\pi \hbar^7 \quad (11)$$

Here $\alpha = 137.036^{-1}$; $a_0 = 137.036$ is the Bohr radius in natural units;
 $W_0 = (-61 \text{ keV}/mc^2) + 1$ is the nuclear mass difference of ^{205}Tl and ^{205}Pb plus one electron and $b/mc^2 = 101.4 \text{ keV}/mc^2$ is the 1s hydrogenic binding energy in Pb^{81+} so $[(W_0 + b - mc^2)/mc^2]^2 = (40/511)^2 = 6.127 \times 10^{-3}$; $g_v = 1.4122 \times 10^{-49} \text{ erg cm}^3 \times (8.187 \times 10^{-7})^{-1} mc^2 \text{ erg}^{-1} \times (3.862 \times 10^{-11})^{-3} (\hbar/mc/\text{cm})^{-3} = 2.995 \times 10^{-12}$ natural units and the remaining parameters have been defined and evaluated for this case previously.

Evaluating equation (11)

$$\begin{aligned} \lambda_{B,1s} &= 1.869 \times 10^{-29} mc^2/\hbar \div 1.2881 \times 10^{-21} \text{ sec}/(mc^2/\hbar)^{-1} \quad (12) \\ &= 1.451 \times 10^{-8} \text{ sec}^{-1} \end{aligned}$$

which is exactly $4\pi \times \lambda_{B,1s}$ obtained by both methods 1 and 2.

In search of this factor of 4π disparity. I note that Bahcall has integrated over a solid angle in his equation (8) to generate an extra factor of 4π appearing in his equation (12) which, he states applies to "the total decay rate of initially unoriented nuclei if no attempt is made to measure the direction of the nuclear recoil", the circumstances of our present interest. However, I am unable to account for the $(2\pi)^3$ factor in his equation (8) so I cannot confirm the validity of his equation (12) in all respects, though the origin of the above referred 4π factor seems logical.

On the other hand, the numerical results show such a factor is not included even implicitly in my equation (5) copied from Harston and Pyper's¹⁵ equation (1), but no conclusion can follow since they do not exhibit their

derivation, saying only that it "is an extension of one derived previously by Bahcall".

The strongest argument I can bring to bear on this question is the aforementioned symmetry of my equation (3) for the ratio of two single channel bound state rates. It seems very difficult to justify the insertion of a factor of 4π in the right hand numerator, needed to bring the $\lambda_{B,1s}$ value into agreement with that from Bahcall's formulation, without including 4π also in the denominator, the electron capture channel. Thus it appears that Bahcall's final result may include an unjustified factor of 4π . I have as of this writing not received information from either authors on this question. The question can be settled experimentally by measuring the bound state beta decay rate of a heavy stripped ion for which the ft value of its continuum beta decay is known, as a calibration of the experimental method and I propose such a calibration procedure in Section V.

In the sequel I assume that the value of $\lambda_{B,1s}$ for $^{205}\text{Tl}^{81+}$ decay consistent with the value of $\log ft = 5.90$ or $|M|^2 = 7.77 \times 10^{-3}$ is that from methods 1 and 2,

$$\lambda_{B,1s} = 1.155 \times 10^{-9} \text{ sec}^{-1} .$$

If Bahcall's derivation turns out to be correct the rate of decay to produce $^{205}\text{Pb}^{81+}$ will be more favorable by 4π .

IV. ESTIMATION OF THE PRODUCTION RATE OF $^{205}\text{Pb}^{81+}$ IN A BEAM OF $^{205}\text{Tl}^{81+}$

To strip Tl ions completely the beam energy should be, say, 0.5 GeV/nucleon, which corresponds to relative velocity $v/c = 0.758$. With the beam coasting at this speed the relativistic time dilatation factor

$[1-(v^2/c^2)]^{1/2} = 0.652$ reduces the decay rate as measured in the laboratory by this factor, so

$$(\lambda_{B,1s})_{v/c} = 0.652 \lambda_{B,1s} = 7.53 \times 10^{-10} \text{ sec}^{-1} .$$

As example, consider a beam of enriched $^{205}\text{Tl}^{81+}$ ions with a charge current of 81 mamp. (1 mamp. nuclear "current"), coasting at 0.5 GeV/ μ in a storage ring of 100 meter circumference. (These parameter values are well within the anticipated capacities of the GSI 98 m circumference storage ring, in which 1 amp. of stripped U^{92+} ions at 1 GeV/ μ is described.) The beam contains $6.24 \times 10^{15} \times 100/0.758 \times 3 \times 10^8 = 2.74 \times 10^9$ ions = N. In this beam the above calculations predict a rate of production of ^{205}Pb by bound state 1s decay of

$$D = N(\lambda_{B,1s})_{v/c} = 2.06 \text{ sec}^{-1} = 7430 \text{ hr}^{-1} .$$

No allowance is made for the uncertainty in the estimate of the decay constant's dependence on transition energy, nor for the errors in measuring the beam current, beam velocity, the accumulation (decay) time, or the contributions of background reactions producing ^{205}Pb . The errors in the calculated wave functions and binding energies are comparatively insignificant. The error contribution of the transition energy, shown above, probably will dominate the measured value of the decay constant, with the error in measuring the number of Pb ions produced making the next most significant contribution. If the Q_{ec} value ultimately is determined to be 56 keV, the value of the bound 1s decay constant consistent with $\log ft = 5.90$ is

$$\lambda_{B,1s,W} = 5.66 \times 10^{-10} \text{ sec}^{-1}$$

and the ^{205}Pb production rate D in this example,

$$D = 3640 \text{ hr}^{-1},$$

an easily measured rate.

I can make little contribution to the experimental plan other than to record that minimizing beam scattering at high energy to reduce Pb-producing nuclear reaction rates is vital. The charge to mass ratios of $^{205}\text{Tl}^{81+}$ and $^{205}\text{Pb}^{81+}$ are practically indistinguishable, $\Delta M/M = 2.1 \times 10^{-8}$, but if Pb^{81+} can be stripped to Pb^{82+} without generating ^{205}Pb from ^{205}Tl , they are easily separated by magnetic analysis; if not, accelerator mass spectrometry techniques on electromagnetically slowed beams must be used.

From the experimental value of the production rate D_{exp} , normalized per $^{205}\text{Tl}^{81+}$ ion in the beam and per second, also corrected for (difference from .652 of) the actual time dilatation factor used, one obtains the measured value of $|M|^2$ or ft as

$$|M|_{\text{exp}}^2 = D_{\text{exp}} \times 7.77 \times 10^{-3} / 2.06 \text{ sec}^{-1} = 6170 / (ft)_{\text{exp}}. \quad (12)$$

V. EXPERIMENTAL CALIBRATION OF THE METHOD

As noted, both the theory and the experimental technique can and should be verified by measuring the rate of bound state inner orbital decay in a stored stripped ion beam as here described, in a nuclear decay system for which the $\log ft$ value is known experimentally. No complete analogue to the ^{205}Tl - ^{205}Pb system exists, but the pair ^{241}Pu - ^{241}Am is a very good stand-in. In this case the neutral atom ^{241}Pu decays by very low energy beta-minus emission $E_0 = 20.5 \text{ keV}$,¹⁹ to the ground state of ^{241}Am ; $\log ft = 5.77$; $t =$

14.4 y. The electron binding energy difference between Am and Pu, $\Delta B.E. = 22.4 \text{ keV}$; ¹⁵ thus it contributes the entire $Q_{\beta-}$ of the beta decay, such that a stripped $^{241}\text{Pu}^{94+}$ nucleus is about 2 keV lighter than a stripped $^{241}\text{Am}^{95+}$ nucleus, and cannot decay by continuum beta emission, just as in the case of stripped $^{205}\text{Tl}^{81+}$. It can decay by bound state beta decay to inner ns or $np_{1/2}$ orbitals ($n \leq 6$) in $^{241}\text{Am}^{94+}$ (only these have significant amplitudes at the nuclear surface and satisfy the selection rules) at a predictable rate that can be checked experimentally. The $^{241}_{95}\text{Am}^{94+}$ ion cannot decay back to ^{241}Pu by electron capture since it lacks the weakly bound outer electrons which alone can be captured. Like the $^{205}\text{Tl} \rightarrow ^{205}\text{Pb}$ case the ^{241}Pu decay is a first forbidden non unique transition from $5/2+ \rightarrow 5/2-$.

This case is one of the rare occurrences in which the normal continuum beta decay is completely turned off when the parent ion is stripped, so the bound state decay product ions are not overwhelmed by the continuum decay daughters. Moreover, its normal continuum beta decay and the bound state decay of stripped $^{241}\text{Pu}^{94+}$ are in the same $Z \rightarrow Z+1$ sense so thus do not involve any question of possible imperfect cancellation of the nuclear matrix element sums as would be the case in comparing normal electron capture rates with bound state beta decay, as in, e.g., ^{193}Pt or ^{163}Ho low energy electron capture decays.

The Oxford program can yield the hydrogenic ns and $np_{1/2}$ small and large components of the wave functions, respectively, and their individual binding energies in $^{241}\text{Am}^{94+}$, needed for determining the neutrino momenta. Equation (5) (or perhaps equation (11)) gives the individual $\lambda_{B,ns}$ or $np_{1/2}$, the partial decay rates; which sum agrees with the measured ^{241}Am production rate selects among equations (5) and (11).

VI. CALCULATION OF THE NEUTRINO REACTION CROSS SECTION

Given the composite solar neutrino spectrum, $\lambda_\nu(q_\nu) dq_\nu$, and the experimental ft_{exp} value from equation (12), one obtains the neutrino absorption cross section of ^{205}Tl to the $1/2^-$ state of ^{205}Pb from the equations of Bahcall²⁰ for allowed neutrino absorption. These can be applied without change to first forbidden non unique absorptions provided that the experimental ft value used is obtained by measuring a beta-minus decay, as in this experiment, rather than from an electron capture or positron decay.

The cross section is given by

$$\sigma_a = [\ln 2 (h/mc)^3 (2\pi^2)/(mc^3) ft_{\text{exp}}] \int_0^{q_\nu^{\text{max}}} (p_e/mc) \frac{(q_\nu - Q_{ec} + .511)}{.511} F(Z, p_e) \lambda(q_\nu) dq_\nu / \int_0^{q_\nu^{\text{max}}} \lambda_\nu(q_\nu) dq_\nu$$

$$= [5.14 \times 10^{-41} \text{ cm}^2 / ft_{\text{exp}}] \times \int_0^{q_\nu^{\text{max}}} (p_e/mc) \frac{(q_\nu - Q_{ec} + .511)}{.511} F(Z, p_e) \lambda(q_\nu) dq_\nu / \int_0^{q_\nu^{\text{max}}} \lambda_\nu(q_\nu) dq_\nu$$

where p_e is the resulting electron momentum (in mc units) $= [(q_\nu - Q_{ec} + 1)^2 - 1]^{1/2}$, q_ν and Q_{ec} in MeV, and $F(Z, p_e)$ is the Fermi function accounting for the screened Coulomb field of the Pb nucleus in its influence on the amplitude of the continuum electron wave function at the nucleus.

For the contributions of the sharp neutrino peaks from electron capture decays in the solar spectrum the integrals are replaced by δ functions in momentum of p_e , q_ν and F . The small contributions of higher energy neutrino absorption to other final excited states of ^{205}Pb must be otherwise evaluated.²¹

APPENDIX

THE ELECTRON CAPTURE DECAY ENERGY Q_{EC} OF ^{205}Pb GROUND STATE*

The only direct determination of this transition energy was by a careful measurement⁴ of the intensity ratio of M-to-L x rays in the decay of the long lived ground state ^{205}Pb . The activity was incorporated in volatile tetramethyl lead as a minor gas constituent in a proportional counter surrounded by a windowless anticoincidence shield. All data corrections and calibrations were well assayed. In my judgement it would be very difficult to achieve a significant improvement in the measurement of this ratio with available technique. The ratio is quite sensitive to the transition energy in the region of 50 keV. Comparison to the theoretical prediction of the ratio as a function of energy gave $Q_{EC} = 41.4 \pm 0.4$ keV.

The 1983 atomic mass evaluation⁵ rejected this measurement, described as "well documented data", for its inconsistency with the $^{205}\text{Pb} - ^{205}\text{Tl}$ mass difference of 53.5 ± 1.6 keV determined from the closure energies of three reaction-plus-beta-decay energy loops.

In search of this 12 keV disparity I recalculated the theoretical prediction for the M/L x ray intensity ratio using the same set of relativistic electron wave functions (Table IX, ref. 2) used in ref. 4, duplicating the author's results. Repeating the calculation with the most modern Dirac-Fock wave functions of Desclaux¹¹ yielded a still lower value for $Q_{EC} = 38.3$ keV.

However, as was pointed out by the authors⁴, the overlap-plus-exchange factors, B_M and B_L of equation (2) were not included in either calculation. In this first forbidden unique decay the captures are principally from L3 and

* Q_{EC} denotes the transition energy of the ground state of ^{205}Pb . The electron capture energy of the 2.3 keV, 1/2-level is $Q_{ec} = Q_{EC} + 2.3$ keV.

M3 orbitals (over 95%) for which orbitals no B_{L3} or B_{M3} factors have been calculated. The few calculations of $X^{M/L} = B_M/B_L$ at high Z were for allowed transitions dominated by s orbital capture; these gave $X^{M1/L1} = 1.03-1.07$. Using the latter for $X^{M3/L3} = 1.07$ raised the value to only $Q_{EC} = 42$ keV, still far from agreement with the Wapstra calculation.⁵ To attain agreement using the data of ref. 4 requires that $X^{M3/L3} = 1.17$. It would aid in resolving this important problem if a calculation of this correction factor could be made; Dr. M. H. Chen² is considering this problem currently. Moreover, there is an unsettled dispute concerning the best approximations to be made in these calculations, different Anzätze^{12,13} yielding a few percent variance in $X^{M/L}$ at high Z. Unfortunately no other cases exist of first forbidden unique electron capture transitions at high Z and known low Q_{EC} value, for which the M/L x ray intensity ratio could be measured to "calibrate" a nearby $X^{M/L}$ value.

A reexamination of all the unadjusted input data given in ref. 5, pages 289-290, pertinent to the Q_{EC} value, indicated three reaction-plus-beta-decay energy loops involving independent sets of measurements, shown on Fig. 2. The closure energies $Q_{EC}(^{205}\text{Pb}-^{205}\text{Tl})$ of these loops are as follows:

- (1) a. $^{205}\text{Tl}(n,\gamma)^{206}\text{Tl}$; 6503.4 ± 0.4 keV
- b. $^{206}\text{Tl}(\beta^-)^{206}\text{Pb}$; 1527 ± 4 keV
- c. $^{206}\text{Pb}(\gamma,n)^{205}\text{Pb}$; -8087 ± 3 keV

- d. $^{205}\text{Pb}(\text{EC})^{205}\text{Tl}$; 56.6 ± 5 keV;

- (2) a. $^{204}\text{Pb}(n,\gamma)^{205}\text{Pb}$; $6731.6 \pm .15 \text{ keV}$
 b. $^{204}\text{Tl}(\beta^-)^{204}\text{Pb}$; $763.4 \pm .2 \text{ keV}$
 c. $^{205}\text{Tl}(\gamma,n)^{204}\text{Tl}$; $-7548 \pm 3 \text{ keV}$

 d. $^{205}\text{Pb}(\text{EC})^{205}\text{Tl}$; $53 \pm 3 \text{ keV}$; and
- (3) a. ^{205}Tl (mass doublet) ^{207}Pb ; $14779.6 \pm 2.6 \text{ keV}$
 b. $^{206}\text{Pb}(n,\gamma)^{207}\text{Pb}$; $6737.8 \pm .2 \text{ keV}$
 c. $^{206}\text{Pb}(\gamma,n)^{205}\text{Pb}$; $-8087 \pm 3 \text{ keV}$

 d. $^{205}\text{Pb}(\text{EC})^{205}\text{Tl}$; $45.2 \pm 4 \text{ keV}$.

In addition to these eight input data [(1)c=(3)c], there are eight other reaction and decay data which interact with the data above, in two cases significantly influencing their adjusted values. The energy sum

- (4) a. $^{203}\text{Tl}(n,\gamma)^{204}\text{Tl}$; $6655.8 \pm .8 \text{ keV}$
 b. $^{205}\text{Tl}(\gamma,n)^{204}\text{Tl}$; $-7548 \pm 3 \text{ keV}$

 c. $^{203}\text{Tl}(2n)^{205}\text{Tl}$; $14,203.8 \pm 3 \text{ keV}$ matches the observed mass doublet difference
 d. $^{203}\text{Tl} \rightarrow ^{205}\text{Tl}$; $14,203.8 \pm 0.9 \text{ keV}$, thus leading to a reduced adjusted error on the former, and hence on (2)c, thus on (2)d, to $\pm 1.3 \text{ keV}$.

Similarly the energy sum

- (5) a. $^{206}\text{Pb}(\gamma,n)^{205}\text{Pb}$; $-8087 \pm 3 \text{ keV}$
 b. $^{204}\text{Pb}(n,\gamma)^{205}\text{Pb}$; $6731.6 \pm 0.15 \text{ keV}$

 c. $^{204}\text{Pb}(2n)^{206}\text{Pb}$; $14818.6 \text{ keV} \pm 3 \text{ keV}$ compares to the observed mass

doublet difference

- d. $^{204}\text{Pb} \rightarrow ^{206}\text{Pb}$; 14819.5 ± 1.1 keV, which implies that
 c. $^{206}\text{Pb}(\gamma, n)^{205}\text{Pb}$ should be increased to -8088 ± 1.0 keV, thus
 increasing (1)d. to 57.6 ± 4 keV and (3)d. to 46.2 ± 3 keV.

Finally one has the (nearly) independent Q_{EC} values;

- (1)d. 57.6 ± 4 keV
 (2)d. 53 ± 1.3 keV
 (3)d. 46.2 ± 3 keV, and from the M/L x ray measurement,
 (4) 38.3 ± 1.4 KeV, but to be changed by $\chi^{M/L} = ?$.

Clearly the external errors of their weighted ($1/\sigma^2$) average, including (4),

$$Q_{\text{EC}} = 46.8 \pm 0.9 \text{ keV},$$

are inconsistent with normally distributed errors; $\chi^2 = 4.9$ for 2 degrees of freedom. The only reason to selectively reject the outlying 38.3 keV result while retaining the 57.6 keV value which is based on the same types of measurement as are the 53. and 46.2 keV data, as was done in ref. 5, is the uncertain influence of the omitted $\chi^{M/L}$ factor. Omitting this result one has

$$Q_{\text{EC}} = 52.4 \pm 1.2 \text{ keV}$$

for which $\chi^2 = 1.65$ for one degree of freedom. Wapstra and Audi⁵ gave $Q_{\text{EC}} = 53.5 \pm 1.6$ keV, based on their use of their adjusted input data. Subjectively this appears to be an example in which the formalities of statistical treatment of data give a misleadingly optimistic assessment of errors as a result of the somewhat arbitrary omission of a datum. In my judgement it will be necessary not merely to improve on the consistency among the three values

(1)d, (2)d and (3)d, but also to reconcile the value (4) from the x ray measurement with the others for the purposes of the neutrino flux measurement.

The most fruitful studies to reduce the experimental uncertainties in Q_{EC} are seen to be the following: (1)b, $^{206}\text{Tl}(\beta^-)^{206}\text{Pb}$; (3)a, ^{205}Tl - ^{207}Pb mass doublet; and the theoretical evaluation of $X^{M/L}$ for the decay $^{205}\text{Pb}(\text{EC})^{205}\text{Tl}$. There are attractive new direct experimental approaches: the mass spectro-metric difference ^{205}Pb -- ^{205}Tl viewed via two resolvable doublet pairs: $^{205}\text{Tl}^{35}\text{Cl}$ -- $^{203}\text{Tl}^{37}\text{Cl}$, and $^{203}\text{Tl}^{37}\text{Cl}$ -- $^{205}\text{Pb}^{35}\text{Cl}$; the direct pair $^{205}\text{Pb}^{35}\text{Cl}$ -- $^{205}\text{Tl}^{35}\text{Cl}$ peaks are separated by only 1/4 of a typical 1 ppm line width, thus not resolvable, with 1 keV precision requiring centroid location to 1/400 of the line width. This experiment is being discussed with mass spectroscopists. Two other direct experiments are: $^{205}\text{Tl}(\text{p},\text{n})^{205}\text{Pb}$ using time of flight spectroscopy on the neutrons; or $^{205}\text{Tl}(^3\text{He},^3\text{H})^{205}\text{Pb}$ which has the advantage of magnetic analysis on the ^3H product. Whether either experiment can possibly achieve the desired precision of 1 keV is being explored.

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FIG. 1

ENERGETICS OF $^{205}\text{Tl} - ^{205}\text{Pb}$

FIG. 2.

ENERGIES (ERROR) IN KEV.

LESS ACCURATE TRANSITIONS (NOT USED) SHOWN - - - - -

REACTION ENERGIES CONVERTED TO NEUTRON SEPARATION ENERGIES
VALUES ARE UNADJUSTED INPUT DATA.