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EFFECTIVE CHARGES, THE VALENCE p-n INTERACTION, AND THE IBM

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INTRODUCTION

There are three recent themes in nuclear structure that come together in an interesting and useful way via the concept of effective charges and the framework of the IBM. These three concepts are the importance of dynamical symmetries in describing nuclear structure and the benefits that accrue from their exploitation, secondly, the critical role of the p-n interaction in the onset and development of collectivity in nuclei, and, thirdly, the importance of the valence nucleons in determining structure and its evolution.

We will illustrate this by showing that the interpretation of measured $B(E2)$ values in the context of the dynamical symmetries of the IBM leads to new insights into the meaning of effective charges and offers new avenues to understand the role of the proton-neutron (p-n) interaction in modulating the nature of the valence space and the growth of collectivity. In particular, we will show that effective charges in *valence* models, such as the IBM, can be interpreted in terms of *derivatives of the collectivity of the low lying levels*, that is, as measures of the rate of change of collectivity as the proton and neutron numbers vary. This paper is based on recent work¹ by the authors.

EFFECTIVE CHARGES IN VALENCE MODELS

In most nuclear models, effective charges are simply multiplicative factors on calculated transition rates. They are employed to ensure agreement with experiment and to compensate for simplifications or approximations in the model such as truncations of the basis space (in which case the effective charges would mock up effects such as core polarization), simplified forms of the transition operator, or the use of effective interactions amongst the nucleons. [Of course, some microscopic models can estimate the values of these effects, in

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which case the effective charges may not be free parameters or may be partially constrained. Multi- $\hbar\omega$ symplectic models² are a case in point.]

In valence models, however, that is, in models that consider only the valence nucleons in the determination of structure, effective charges take on an additional, and more physically intuitive, role. To see this, we exploit the simplicities of the Interacting Boson Model (IBM)³.

Let us first consider the three limiting symmetries of this model, the U(5), SU(3), and O(6) limits. We can then write expressions⁴ for the key observable $M(E2:2_1^+ \rightarrow 0_1^+)$, where $M(E2)$ is just the square root of the corresponding $B(E2)$ value, as follows:

$$U(5) \quad M(E2:2_1^+ \rightarrow 0_1^+) = \sqrt{1/N} (e_\pi N_\pi + e_\nu N_\nu) \quad (1)$$

$$O(6) \quad M(E2:2_1^+ \rightarrow 0_1^+) = \left(\frac{N+4}{5N}\right)^{1/2} (e_\pi N_\pi + e_\nu N_\nu) \quad (2)$$

$$SU(3) \quad M(E2:2_1^+ \rightarrow 0_1^+) = \left(\frac{2N+3}{5N}\right)^{1/2} (e_\pi N_\pi + e_\nu N_\nu) \quad (3)$$

Note that all three expressions have the same form, namely, a function of the total boson number N and a separate factor of the form $(e_\pi N_\pi + e_\nu N_\nu)$, incorporating the boson effective charges e_π, e_ν and the proton and neutron boson numbers N_π, N_ν .

Most nuclei, of course, do not satisfy the exact strictures of a specific symmetry, but are intermediate in structure. Exact analytic expressions cannot be written down for such cases, but very good approximate expressions are available: these incorporate parameters from the IBM-1 Hamiltonian. For example, for nuclei near U(5), Ginocchio and Van Isacker⁴ have developed the expression:

$$M(E2:2_1^+ \rightarrow 0_1^+) = \sqrt{\frac{1}{N}} \left(1 - \frac{\kappa(N-1)}{\epsilon}\right)^2 (e_\pi N_\pi + e_\nu N_\nu) \quad (4)$$

where ϵ and κ are the coefficients of the $\hat{n}_d$ and $Q \cdot Q$ terms in the IBM Hamiltonian, respectively, and where the latter gives the scale of U(5) symmetry breaking. For nuclei between O(6) and SU(3) [this category includes the vast majority of deformed nuclei such as the rare earths or the actinides], the following expression has been developed⁵

$$M(E2:2_1^+ \rightarrow 0_1^+) = 0.5 \left(\frac{N+1}{N}\right) (1-0.1\chi) (e_\pi N_\pi + e_\nu N_\nu) \quad (5)$$

Here, χ is an internal parameter in the IBM quadrupole operator, Q . χ varies from 0 [for O(6)] to -1.32 [for SU(3)] and is typically near -0.5 for deformed nuclei. [Equation 5, it will be noted, has only a weak dependence on χ .] A more general (and more complicated) formula exists⁶ which spans the entire IBM symmetry "triangle." It need not be given here. These approximate formulas, of course, differ in detail from those applicable to the limiting symmetries (eqs. 1-3), but they again share the factorizable form of an expression in N and one in the effective charges.

Thus, rather generally, in the IBM, we can write:

$$M(E2:2_1^+ \rightarrow 0_1^+) = f(N) (e_\pi N_\pi + e_\nu N_\nu) \quad (6)$$

Similar expressions may apply to other valence models.

There are two interesting aspects of eq. 6. One is that it allows the easy extraction of the effective charges from sets of $B(E2)$ values for a sequence of nuclei (e.g., an isotopic series). The second is that it reveals a new interpretation of the effective charges.

We can see the first point simply by re-writing eq. 6 as

$$\frac{M(E2:2^+_1 \rightarrow 0^+_1)}{f(N) N_v} = \left(e_v + e_\pi \frac{N_\pi}{N_v} \right) \quad (7)$$

The left hand side (lhs) is linear in the ratio N_π/N_v with slope e_π and intercept e_v . Since N_π and N_v are known from counting the numbers of valence nucleons (more on this later), $f(N)$ is known from the structure in a given region, and the $M(E2)$ values are measured quantities, it is possible to plot the lhs of eq. 7 against N_π/N_v for a series of nuclei and, if a linear behavior results, to extract the boson effective charges. (We will illustrate this process momentarily.) It is important to stress here that the effective charges thus extracted refer to, and indeed are only defined in the context of, a *series* of nuclei, not an individual nuclide.

The second point is evident if one inspects the forms of eqs. 1-5 and notes that $f(N)$ is nearly always more slowly varying than $(e_\pi N_\pi + e_v N_v)$. If we therefore make the approximation that $f(N)$ is constant, and differentiate eq. 6, we obtain the fascinating results that

$$\frac{\partial M(E2)}{\partial N_v} \approx f(N) e_v \quad \text{and} \quad \frac{\partial M(E2)}{\partial N_\pi} \approx f(N) e_\pi \quad (8)$$

These equations have the immediate physical interpretation that e_π and e_v are proportional to the rate of change of collectivity (as reflected in the $M(E2:2^+_1 \rightarrow 0^+_1)$ values), with respect to N_π and N_v , respectively. Thus, extracting these effective charges by means of eq. 7 immediately discloses the evolution of collective structure for a series of nuclei.

Of course, a key to this procedure is the assumption that $f(N)$ varies slowly enough that it is reasonable to treat it as constant for a set of nuclides. Except for U(5), $f(N)$ typically changes nearly an order of magnitude slower than $(e_\pi N_\pi + e_v N_v)$, especially for large N_π and N_v . This is clear, for example, in eqs. 2 and 3, where the $f(N)$ functions $((N+4/5N))^{1/2}$ and $(2N+3/5N)^{1/2}$ are asymptotically constant for large N . Even in the case of U(5) the leading order term in the derivative of eq. 1 is independent of $f(N)$. A more rigorous derivation of this result is given in the second article of ref. 7.

EMPIRICAL EXTRACTION AND INTERPRETATION OF EFFECTIVE CHARGES

We now illustrate the extraction of the boson effective charges and their interpretation in light of eq. 8 and the discussion above. In fig. 1 we show plots of the lhs eq. 7 against N_π/N_v for the Pd and Sm isotopes. We note that the plots are indeed linear, in fact, often to higher accuracy than might be expected from the experimental uncertainties. In each case we show more than one fit, using different expressions of the type of eqs. 1-6 to illustrate that the extracted effective

Figure 1. Examples of the linear fits described in the text. $T=M(E2)/f(N)N_p$, where $\rho=v$ for Pd and π for Sm. Left: Fits, with different formulas, for Pd. Top right: two fits to Sm. (Note that $e_v > e_\pi$.) In the "perturbed" U(5) formula, κ and ϵ are IBM parameters. Bottom right: fit to Sm using effective N_π values, yielding "normal" e_π and e_v values. (See text.)

charges are not very sensitive to uncertainties in which forms of $f(N)$ are used. (We will discuss the lower right panel of fig. 1 later.)

We have carried out fits like those in fig. 1 for isotopic chains from Mo to the actinides. The extracted effective charges reveal some interesting features, especially in light of the natural expectation that e_v should be substantially less than e_π . We show plots of e_π , e_v and e_v/e_π against Z in fig. 2. The e_π values, with few disruptions, exhibit a nearly linear, monotonically increasing, behavior with Z , from values near 0.1 for $A \sim 100$ to 0.35 for the actinides. More striking, e_v shows two remarkable features, namely, first, a repeated undulation or oscillating behavior in each shell, with large values near magic numbers and in shape transitional regions and very small values near midshell, and, secondly, values in the former regions that can actually *exceed* e_π . This is highlighted in the ratio plot on the top of the figure which shows at least two regions where $e_v/e_\pi > 1$. (Incidentally, these plots dramatically point to the need for further data in the Pb region which has all the earmarks of another region where e_v may be greater than e_π .)

The "derivative" interpretation of these effective charges allows a simple physical interpretation of the large e_v values which have, heretofore, been extremely puzzling. At the most superficial level, large e_v values merely imply that collectivity $[M(E2)]$ changes faster with neutron than proton number in a given region. But the real issue is *why* should this be so? Disregarding minor orbit dependencies of the p-n interaction, the success of the $N_p N_n$ scheme⁷

Figure 2. Empirical e_π , e_v and e_v/e_π values for isotopic sequences of all non-magic even-even nuclei with $Z > 40$. The uncertainties reflect the fit quality and the variations in e_π and e_v with different analytical formulas. Open symbols for Mo and Ba give values for the rotational isotopes of these elements following the spherical-deformed phase transitional regions.

suggests that changes in collectivity should vary symmetrically with N_π and N_v . However, the regions where $e_v > e_\pi$ are exactly those where this is not the case because, here, the neutrons play a dual role^{8,9}. Of course, additional neutrons add to the total number of valence nucleons and to the $N_p N_n$ product, and thus increase the collectivity, primarily via the quadrupole component of the p-n interaction¹⁰. However, they *also* affect the single particle energies of the *protons* via the attractive monopole p-n interaction¹⁰. For example, in the $A = 100$ (150) region, the addition of neutrons in the $1g_{7/2}$ ($1h_{9/2}$) orbit, lowers the effective single particle energy of the proton $1g_{9/2}$ ($1h_{11/2}$) orbit. This effect can be large, can grow as a function of neutron number, and can actually alter (create or destroy) important sub-shell gaps in the proton energy levels. Again, in the $A = 100$ (150) regions, near $N = 60$ (90), the addition of neutrons obliterates the proton sub-shell gaps at $Z = 40$ (64). This in turn affects the way one counts the number of valence proton bosons, N_π , used in eqs. 1-7. However, since N_π is *defined* to be constant in applying these equations for an isotopic series, this effect shows up as an enhanced multiplier, e_v , of N_v . Thus, we see that the effective charges can, in a formalism of valence models, give important information on subtle yet critical aspects of the p-n interaction which are key to understanding the onset of collectivity and shape/phase transitions in nuclei.

With this explanation in hand, one could, of course, turn the procedures we have used around by making a *model* of the changing proton shell structure (that is, a model for how N_π changes with neutron number) in some region, insert these N_π values in eq. 7, re-do the fits and the extraction of the effective charges

Figure 3. As in Fig. 1, except for the isotonic and iso- $N_p N_n$ sequences indicated.

and determine if "normal" (that is, $e_\nu < e_\pi$) values result as expected. The lower right panel in fig. 1, for Sm, illustrates this procedure (we took $N_\pi = 1, 2, 4$ for $^{144}, ^{148}, ^{150}\text{Sm}$, respectively) and shows that more traditional values of e_ν do result.

We noted before that the e_π and e_ν values extracted were inherently defined with respect to a specifically chosen series of nuclei. (In the cases above, these were isotopic chains.) They should not necessarily be expected to be identical to values extracted, for example, from several transitions in a single nucleus. More significantly, e_π and e_ν values for a nucleus in, say, an isotopic chain, should not be expected to coincide with those for the same nucleus extracted from a *different* (say, isotonic) sequence. For example, the e_π and e_ν values for ^{106}Pd obtained from the Pd isotopes in fig. 1 will not necessarily be the same as those extracted from the $N = 60$ isotonic series. This is evident from our derivative interpretations. Collectivity does not vary isotropically in the N - Z plane: since e_π and e_ν reflect these rates of change, they will, in general, be different for different "cuts" through the plane. Note also that one is not limited to isotopic or isotonic sequences. One can inspect isobaric sequences or more exotic sets such as iso- $N_p N_n$, iso- P , iso- F -spin, and so on. We illustrate these ideas in fig. 3 which shows an isotonic and an iso- $N_p N_n$ sequence. In isotopic (isotonic) series, if the $B(E2)$ values were constant, the e_ν (e_π) values extracted would be zero and, hence, values of e_π (e_ν) of the order of 0.1-0.3 eb [see plots in ref. 1 and, here, in figs. 1, 2 and 3 (left)] point to significant rates of change of collectivity with $N(Z)$ in these chains of nuclei. In contrast, for an iso- $N_p N_n$ sequence, inspection of the form of eq. 6 shows that constant $B(E2)$ values lead to equal (and non-zero) e_π and e_ν . For the iso- $N_p N_n$ nuclei in fig. 3 (right) a constant $B(E2)$ at the average value observed for these nuclei would give $e_\pi = e_\nu = 0.14$. It is then to this value that the extracted $e_\pi = 0.106$, $e_\nu = 0.170$ values should be compared: we see that the small deviations of about 0.03 eb for both e_π and e_ν from the ideal average value do indeed reflect the relative constancy of structure in this $N_p N_n$ multiplet.

Extension of these results to other nuclides is an on-going project. Clearly, with extensive mapping in this way, and with the availability of sufficient data, the full 2-dimensional surface of the derivatives of collectivity could be mapped out. Such a result could be highly interesting in itself and a challenge to both algebraic and microscopic models of nuclear structure.

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REFERENCES

1. R.F. Casten and A. Wolf, *Phys. Lett.* B280:1 (1992); A. Wolf and R.F. Casten, *Phys. Rev.*, to be published.
2. O. Castanos et al., *Nucl. Phys.* A524:469 (1991).
3. A. Arima and F. Iachello, *Phys. Ref. Lett.* 35:1069 (1975).
4. J.N. Ginocchio and P. Van Isacker, *Phys. Rev.* C33:365 (1986).
5. R.F. Casten and A. Wolf, *Phys. Rev.* C35:1156 (1987).
6. A. Wolf, O. Scholten and R.F. Casten, *Phys. Rev.* C43:2279, (1991).
7. R. F. Casten, *Phys. Rev. Lett.* 54:1991 (1985).
8. P. Federman and S. Pittel, *Phys. Lett.* B69:385 (1977); B77:29 (1977).
9. R.F. Casten et al., *Phys. Rev. Lett.* 47:1433 (1981).
10. K. Heyde et al., *Nucl. Phys.* A466:189 (1987).

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