Pions from and about Heavy Ions

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Introduction

When nuclei collide at energies in the center-of-mass system that exceed the rest mass energy of the pion of 140 MeV, it is energetically possible to produce a pion. I recall from a dozen years ago a flurry of excitement over the possibility that pions might be observable from heavy ion linear accelerators and cyclotrons at the modest heavy ion energies then available. I was working at the old Yale Heavy Ion Accelerator at the time, and on a visit to the University of Maryland it was suggested to me that we ought to look for pions with the 400 MeV argon ion beams at the Yale HIA. Though it sounded unlikely that all the nucleons could somehow concentrate their energy into producing a pion, it was argued that there was a coherence effect among the various nucleon-nucleon collisions that would greatly enhance pion production. The skeptics argued that if this coherence worked as advertised, the best way of making pions would be to drop an elephant into a pit—total energy well above threshold and a tremendous number of nucleons to act coherently.

We didn't actually get around to trying such long-shot experiments at Yale, either with argon ions or elephants, but ten years later I found myself very much immersed in this kind of experiment at the Berkeley BEVALAC. To this pre-history I should add that at Maryland they did succeed in observing neutral pions from ^3He on ^{12}C at the picobarn per steradian per MeV level¹. Work of Schimmerling *et al.*,² at the old Princeton-Penn accelerator measured pion production by nitrogen-14 ions at 520 A MeV. That the early ideas were not so preposterous was brought home to us in a MSU-Berkeley-Tokyo collaboration when we found³ we could still observe pions when the energy of our ^{20}Ne beam was lowered to around 100 A MeV. More recently at the CERN synchrocyclotron π^+ spectra have been measured from 86 A MeV C^{12} ions.⁴ Even more surprising are the CERN experiments⁵ that have measured pion production with the fused nucleus going to ground and first excited states, namely the reaction $^3\text{He} + ^3\text{He} = ^6\text{Li} + \pi^+$. That this unlikely process goes with a substantial (i.e. 111 ± 11 nb)³ cross section seems amazing and certainly provides some challenge to theorists.

1. Pion Thermometry

The various particles from high energy heavy ion collisions exhibit spectra that asymptotically approach an exponential fall-off with energy suggesting a thermal distribution. There are two main problems, one, the slopes depend on angle, and two, the apparent temperatures indicated by the different particles are not the same. We restrict ourselves here to the case of symmetric systems, where target and projectile are nearly the same charge and mass. The slopes determined at the more forward angles are not reliable indicators of any thermal equilibrium situation. In the case of protons there is a persistence of initial momentum through several collisions, since elastic scattering is forward-peaked at energies greater than 300 A MeV. In the case of pions there is a natural forward-backward peaking from the initial formation in a nucleon-nucleon collision, such distribution arising from the role of the Δ (1232) intermediate. For composite particles, like the deuteron, pick-up processes can give direct reaction components in the forward direction. Thus, we believe that the 90° c.m. spectra of various particles are the least admixed with particles from direct processes and may provide the best thermometry of the hot reaction region. A closely related method is applied by Kevin Wolf to streamer chamber π^- data, namely to use the Hagedorn

approach of plotting against $p_{\perp}$ the cross sections integrated for fixed $p_{\perp}$.

Fig. 1 is taken from Nagamiya *et al.*,⁶ and it shows proton and pion spectra from various heavy ion reactions at 0.8 A GeV. They are careful not to call the slope parameter E_0 the temperature, but it is closely related to a temperature. Fig. 1 is a plot of Lorentz-invariant cross sections, whereas to get a temperature from a Boltzmann distribution, one should plot just the differential cross section without multiplying by the relativistic total energy. The effective temperatures will be somewhat lower than Nagamiya's E_0 values.

The correction to E_0 to get a temperature value is rather easy to make. Fig. 2 from Nagamiya shows π^- invariant cross sections $E \frac{d^3\sigma}{dp^3}$ vs. pion kinetic energy for Ne + NaF at several different bombarding energies. Note that the following simple relations connect T and E_0 :

$$\sigma_{INV} = E \frac{d^3\sigma}{dp^3},$$

where E is the relativistic total energy of the pion. Taking the natural logarithm of both sides and differentiating with respect to E gives us

$$-\frac{1}{E_0} = \frac{1}{E_x} - \frac{1}{T},$$

where E_x is the total energy at which the slopes are to be determined. Now we see that the effective temperature T will not be so well defined as is E_0 , for a plot of $\frac{d^3\sigma}{dp^3}$ against pion energy will be concave downward rather than the straight line forms of Nagamiya. Such behavior is physically reasonable for systems with a relatively small number of particles. We know especially from lower energy nuclear physics that the particle evaporation spectra depart from exponential behavior at high energies and indeed go to zero where all the available energy of the finite system is carried off. If we choose to evaluate the effective temperature at a kinetic energy around twice the E_0 value (total E three times greater), then the above equations tell us that T, the effective temperature is 0.75 times E_0 . With this in mind we can determine temperatures from Fig. 3 of Nagamiya.

The pion probe indicates temperatures smaller than given by the simple fireball model in which all the available kinetic energy of the geometrically overlapping matter (participants) is assumed to be thermalized. For example, at $E_{lab} = 0.18$ A GeV there is 45 MeV kinetic energy per nucleon available for a fireball model temperature of 2/3 of this, or 30 MeV. This is to be compared with a pion temperature of 0.75 times 25 Mev, or 18 MeV. Note also that the temperature indicated by the pions for a given system is always less than that indicated by the protons. Similarly, the deuterons and tritons indicate progressively higher temperatures according to Nagamiya.

In an attempt to resolve the discrepancy between proton and pion temperatures Phil Siemens and I some years ago proposed^{6a} a blast-wave model. We derived expressions for the spectra from a system in which the available energy was divided between thermal and ordered motion of a spherically expanding blast wave. If the energy were about equally distributed between these forms, the difference in apparent temperatures was resolved. Today I would not want to defend the literal blast wave but rather point out that any ordered motion of the thermal sources in the perpendicular direction can give systematic differences in the limiting slopes of the various mass particles boiling off. Arguments have been given that a "bounce-off" effect occurs⁷, and this effect provides an ordered sideways component in the thermal sources. Furthermore, cascade-code calculations of Gyulassy and Frankel⁸ show a fluctuation in sideways momentum, even of head-on central collisions. Nagamiya in ref. 9 presents a kaon temperature which does not fit into my general theme that it is ordered perpendicular motion that causes the difference in slopes. The temperature inferred from the kaon (K^+) production is even higher than that of the proton, rather than lying between proton and pion temperatures. I would merely point out that the kaons are not likely to be very good thermal probes, since their mean free paths in nuclear matter are longer than the other particles. Since they require the greatest energy for production, they may also probe just the most violent initial parts of the collision, before any thermal equilibration has set in.

Then what are these slopes and pion and proton temperatures telling us generally about the heavy ion collision processes? The simple fireball model of complete thermalization, as has certainly been pointed out before, cannot be used literally. At least for systems up to mass 40 on mass 40 at BEVALAC energies the forward momentum of nucleons is not totally degraded. One way of dealing with this incomplete degradation was outlined by C. Y. Wong¹⁰. He analyzes 800 MeV per nucleon $^{40}\text{Ar} + \text{KCl}$ results with an ellipsoidal Gaussian distribution for the proton spectra. He states the following:

We observe that the longitudinal momentum distribution has a larger width as compared to that of the transverse distribution, as one expects from the cascade model^{11,12}. Clearly there are substantial nucleon-nucleon collisions which transform the two-center momentum distribution into an ellipsoidal Gaussian distribution by filling in the space in between. The collisions are not numerous enough to erase the initial preference of the longitudinal momentum direction. The anisotropy in the momentum distribution indicates that complete thermal equilibrium of the whole system is not yet achieved.

Another way of dealing with the incomplete thermalization in terms of a simple model is the two-fireball model of Das Gupta¹³ and Das Gupta and Lam¹⁴. The overlapping matter distributions in the collisions pass through one another, depositing part of their translational kinetic energy into thermal energy, so one has two hot sources of pions, protons, etc. instead of the single source of the simple fireball model. Radi *et al.*, adapted this model to describe the pion source function in Monte Carlo trajectory studies of pion Coulomb effects¹⁵. Fig. 4 is from this work, and it shows final fireball velocities and temperatures as a function of impact parameter of the collision for a $^{20}\text{Ne} + ^{20}\text{Ne}$ system. Our beam energy was E/A of 655 MeV, whereas the dashed comparison from Das Gupta was for 800 MeV. Our calculations are done for two cases, for a longitudinal momentum decay length of 2.6 fm, the theoretical value given by Sobel *et al.*,¹⁶ and for double that value, which we believe to be more realistic and in line with various other evidence that collision mean free paths of nucleons in nuclei are longer than the simple first theories give. (cf. refs. 17, 18). The one-fireball temperature shown in Fig. 4 is, of course, independent of impact parameter. The two-fireball model, as we used it with a longitudinal momentum decay length or friction coefficient, qualitatively accounts for the variation of E_0 values with the mass of the system, as shown in Fig. 1.

Knoll¹⁹ has taken the two-fireball model a step forward in sophistication by allowing for sideways momentum of the fireballs arising from processes, such as, the "bounce-off" effect of hydrodynamics. He calls this a two-source model, and he gives the following equation (non-relativistic) for the particle distribution:

$$P(\vec{v}) \approx \exp \left[- \frac{(\vec{v} \pm \vec{v}_{\text{coll}})^2}{2 \langle v^2_{\text{thermal}} \rangle} \right]$$

Let us now recap this brief discussion of pion thermometry in heavy-ion collisions. (1) The asymptotic slope of the pion spectra at 90° c.m. gives probably the best simple measurement of temperature of the hot reaction region, since the 90° direction gives least contamination from direct unscattered pions and since the pion's low rest mass makes it less sensitive than protons to sideways collective motions of whatever sort in the sources. (2) The temperatures so determined, as well as other evidence, suggest that for mass 20-40 symmetric collision systems at BEVALAC energies about half the available energy in the c.m. system is thermalized, with the remainder in ordered motion along the beam direction and perpendicular to it. Global analysis of streamer-chamber events offers the hope of delineating details.

2. Hills and Valleys in Pion Spectra

In the preceding section we discussed information to be gained from the smooth exponentially-falling region of highest energy pions. At lower pion energies a number of special features have been found in the spectra. I discussed these at some length in the Relativistic Heavy Ion Winter School in Banff last February, and the Proceedings can be consulted for details²⁰. These features I referred to there as "funny hills" of the first, second, and third kind.

The first kind is a peaking that occurs at low $P_{\perp}$. This peaking is seen also in free nucleon-nucleon production of pions, as seen in Fig. 5a, and is to be associated with Δ decay. These hills of the first kind are clearly seen in Ne + NaF at E/A of 0.8 GeV (Fig. 5c) and of 0.4 GeV (Fig. 5b) but are pretty well washed out in Ar + Ca at 1.05 GeV. Contour plots of the Ar + Ca data are shown in Fig. 6, where the lower plot shows data of ref. 25 alone, and the upper plot combines it with our data²⁶. The Ar + Ca system is apparently sufficiently large to be effective at scattering and thermalizing the nascent pions from nucleon-nucleon collisions.

The hills (and valleys) of the third kind are clearly Coulomb effects of the spectator fragment charges. The hills, often quite sharp and pronounced, appear in π^- spectra at or just below beam velocity. They were first reported by our MSU-Berkeley-Tokyo collaboration²¹, and reports from more comprehensive studies are more recently published by Sullivan *et al.*,²² Fig. 7 from ref. 20 shows from work of Murphy *et al.*,²³ a nice illustration of the beam-velocity π^- peak. Radi *et al.*,²⁴ have made a theoretical analysis of some of these π^- peaks and conclude that the Coulomb focussing is sensitive to the primary distribution of projectile fragments before proton or alpha evaporation. Furthermore, they extract a velocity dispersion for the primary fragments that is somewhat lower than the measured velocity dispersion of the final fragments after nucleon boil-off. Corresponding to the π^- peaks at beam velocity there are holes in the π^+ spectra, caused by Coulomb defocussing by the charge of projectile fragments.

I have saved until now the funny hills of the second kind. They appear as hills or ridges in the cross sections at very low energy in the c.m., around 15 MeV or momentum of 0.4-0.5 $m_{\pi}c$. They can be seen in Figs. 5c and 5f and perhaps in 5e; also Fig. 6 shows them. On the basis of the first experimental evidence from π^+ in symmetric systems^{25,26} Libbrecht and Koonin^{26a} made a Monte Carlo trajectory study and suggested that these bumps were a Coulomb effect. Subsequent studies²⁷ revealed (see Fig. 8) similar low energy bumps for π^- as well as π^+ , and π^+ studies showed the bump did not move out for high atomic number systems²⁸. On the basis of these studies it seems that the funny hills of the second kind cannot be explained solely as some trivial Coulomb effects. We should point out that they are not seen in the $p + p \rightarrow \pi^+ + X$ reaction, nor are they seen at E/A of 400 MeV. It has been suggested that they may be indicating some side-splash or collective flow effects or that spectator-shadowing could play a role. However, I currently favor the idea that these low-energy pion bumps are giving us a momentum-space snapshot of some sort of pion orbiting of nuclear dimensions.

2.1. Pionic Orbits of Nuclear Size?

What is striking about the mid-rapidity bumps is that a simple application of the uncertainty relation gives a distance about equal to the nuclear radius. That is, $\hbar$ over the bump momentum of 0.5 $m_{\pi}c$ gives a size parameter of 2 pion Compton wave lengths, or about 2.8 fm. Of course, the bump may be shifted somewhat by Coulomb effects, so we will need in the future to accumulate data for both π^+ and π^- for systems of widely differing mass to see if the mid-rapidity bumps move inward with increasing mass, as the uncertainty principle argument would predict. Ideally we should have data somewhat sorted between peripheral and central collisions to extract the most information. We could then take Fourier transforms of the momentum spectrum to infer the spatial wave functions of the pions.

It is worthwhile to trace some of the roots of these notions about possible pion orbiting. Kitazoe and Sano²⁹ solved equations for thermal and chemical equilibrium among particles in the hot dense matter of heavy ion collisions, and they drew attention to a "zero-energy" component of the pion spectra. Likewise, Zimanyi, Fai, and Jakobsson considered such a component, which they gave a finite width according to the nuclear size and uncertainty principle.³⁰ They claim that this component is a boson-condensed component analogous to superfluid liquid ^4He and a distinctly separate phenomenon to the virtual pions of the Migdal-Sawyer pion condensation.^{31,32}

Now it is fair to ask if we get boson condensation among identical pions in a box of nuclear size, what confines the pions to the nuclear volume? For guidance we look to the work of Ericson and Myhrer,³³ who pointed out the possibility of hadronically-bound pion orbitals within the nuclear volume. They approximated the interaction by a pion-nuclear optical potential with a Kisslinger-

type velocity-dependent attractive potential and a static repulsive potential. Parameters were taken from fits to shifts and widths of pionic atom s - and p -state levels. They called attention to a peculiar feature for π^- in neutron-rich matter that the effective mass could go through a singularity and become negative, inverting the order of binding of states in a well. Mandelzweig, Gal, and Friedman³⁴ also studied the problem and clarified the behavior of the wave function near the singularity. They show that the singularity can divide solutions into the classes of inner and outer solutions. For the inner solution where the effective mass of the pion is negative the pion energy is lower than the potential energy, essentially a negative kinetic energy. This "Kisslinger" catastrophe with its infinity of bound states needs some qualifying remarks. The Kisslinger term goes as ∇^2 , just as the kinetic energy in the Schrodinger equation, and the Kisslinger term can take on a greater magnitude than kinetic energy for attainable neutron-richness. However, the Kisslinger term is only part of an expansion of a non-local potential and should probably be regarded as insufficient for higher kinetic energies comparable to the 130 MeV pions from decay of Δ s.

I should say that the states calculated in refs. 33 and 34 for the most part had very large widths. That is, the true absorption of pions by pairs of nucleons is so great that these broad states may be of only academic interest for pionic states of stable nuclei or mildly neutron-rich beta-unstable species. However, in heavy-ion reactions there are two possible new species of interest. First, there are the fireballs of double or higher nuclear density, where even broad states of pion orbiting might give rise to measurable bumps in the spectra. Second, there are highly neutron-rich spectator fragments about which π^- orbiting could occur. This latter situation provides the candidate for Bill McHarris' and my explanation³⁵ of the anomalous puzzle, to which I return shortly.

2.1.1. Pion Confinement in the Fireball

To get some orientation on pion orbiting conditions I have developed a computer code to solve for eigenvalues of pions in a nuclear optical potential. The methods of Ericson and Myhrer³³ have been employed except that we obtain eigenvalues first in the real part of the potential, using the WKB approximation and applying Bohr-Sommerfeld quantization conditions, with modifications of ref. 34 where there is a singularity in the effective mass. The widths of the states are calculated perturbatively from the imaginary parts of the potential; the states of greatest interest to us will have rather small widths and thus be adequately estimated by the perturbative treatment of the imaginary phase. I have taken potential-parameter starting values from the recent work of Carr, McManus, and Stricker-Bauer,³⁶ though they are not very different from the pionic atom values used by Ericson and Myhrer³³ except in the isospin dependence of the static repulsive potential. We shall refer to the 25 MeV pion potential (set F) of ref. 36 as the CMS potential.

The optical potential in the notation of ref. 36 is as follows:

$$2\omega U = -4\pi \left[b + B - \nabla^2 \frac{L}{1 + \frac{4}{3}\lambda L} \nabla^2 \right]$$

plus the Coulomb term. Here

$$b = \bar{b}_0 \rho - \epsilon_s b_1 \delta \rho$$

$$L = c + C$$

$$c = c_0 \rho - \epsilon_s c_1 \delta \rho$$

$$B = B_0 \rho^2 - \epsilon_s B_1 \rho \delta \rho$$

and

$$C = C_0 \rho^2 - \epsilon_s C_1 \rho \delta \rho$$

where ρ is the nucleon density and $\delta \rho$ is the neutron density minus the proton density. Capitalized parameters B and C are coefficients of terms arising from true pion absorption and are dependent on the square of the nuclear density. The lower case parameters b and c denote terms arising from single nucleon scattering and are dependent linearly on the density. Isoscalar and isovector terms are

distinguished by the subscripts zero and one, respectively. The Lorenz-Lorentz-Ericson-Ericson (LLEE) parameter for polarization of the medium is denoted by λ . In writing the above equations I have suppressed the radial argument r , which affects all the densities and density dependent variables. Furthermore, I have set the kinematic factors of ref. 36 to unity for simplicity, since they differ from unity only by terms of the order of the pion-nucleon mass ratio. The quantity ω is the relativistic total energy of the pion at infinity. The quantity ϵ_π is $+1$ for π^+ and -1 for π^- .

First, with the nuclear potential turned off I have verified that the code gives about the right Bohr value for the pionic binding energy in lowest s- and p-states. Then the first calculations with the double density of a fireball made clear that the central part of the nucleus will have a negative effective pion mass for both π^+ and π^- under a broad range of neutron-to-proton ratios. The mass-singularity near the surface thus provides always a confining boundary for inner solutions. In general they are deeply bound states, but they show large widths.

3. Anomalons

From research on relativistic heavy ions has come a remarkable puzzle with which the name anomalon has been associated. When heavy ions in the GeV/nucleon energy range pass through photographic emulsions or other matter, they often undergo peripheral fragmentation reactions from which a fragment of somewhat reduced charge continues on with nearly the beam velocity. Fig. 9 is a photomicrograph of a multiple fragmentation of a 1.88 GeV/nucleon ^{56}Fe ion from the Bevalac. Three fragmentation processes, giving successively charges of 24, 20, and 11, are seen before the chain is terminated by a violent central collision.³⁷

The puzzle is that some of the secondary, tertiary, etc. fragments appear to exhibit anomalously short mean free paths for further reactions. Primary beam fragments exhibit normal mean free paths that can be well represented by the expression

$$\lambda_z^* = \Delta^* Z^{-1}.$$

The coefficient Δ^* is called the "charge independent" mean free path, and for beam particles it takes the value of 30 cm with parameter b about 0.4. Now this reduced mean free path for other-than-primary fragments is plotted in ref. 37 and in Gyulassy's talks in this school. The plot is against path length for three different experimental groups, combining experiment^{38,39} with Bevalac beams of ^{56}Fe , ^{16}O , and ^{40}Ar and with cosmic rays.⁴⁰ The reduced mean free path dips to 80% of its normal value and returns to normal over a characteristic length of 2.5 cm. No special characteristics have been identified that would distinguish the reactions in the early part of the path from those further along the path or those of primary beams. That is, if the anomalons have a decay mode, it must be by neutrons or gamma rays, though possibly decay by a near-minimum-ionizing singly charged particle could be missed in emulsions. The anomalous behavior seems to go with all charges ≥ 3 , with the behavior of charge 1 and 2 perhaps, but not clearly, normal. What must we assume to explain these deviations? The solid curve on Fig. 10 is based on the model that 6% of all projectile fragments are anomalons with a MFP of 2.5 cm. Such a short MFP for a light nucleus, such as, ^{12}C would mean it has a reaction cross section nearly ten times geometric. Nor does it help to assume that some unobserved decay process is involved, since even if 100% of the fragments were assumed anomalous with decay lengths of a few centimeters, one still would need cross sections around twice those of normal nuclei. Though this behavior is by no means rare, it is hard to design counter experiments to check it. Plastic track detector experiments with automated microscopes to eliminate scanning bias are being analyzed now by Prof. Price's group at Berkeley, but they have not yet presented results.

While anomalons are not universally accepted, it is incumbent on theorists to try to find possible explanations. I shall not try to catalog the various theories, but shall mention some of the exotic possibilities and then conclude with a theory McHarris and I are pursuing that is closer to conventional nuclear physics.

I quote from Harry Heckman's brief article³⁷ of June, 1982:

Theories that suggest nuclear collisions can alter the quark structure in nuclei to produce color polarization inside the nucleus, thereby giving rise to larger nuclear collision cross

sections, have been put forward by Y. Karant (LBL)⁴¹, W. Romo, and P. Watson (Carleton University, Ottawa)⁴² and S. Fredriksson and M. Jandel (Stockholm)⁴³. J. Boguta (LBL)⁴⁴ has pursued a Lagrangian field theoretic approach in nuclear theory and has found "hadroid" solutions that exhibit the appropriately "long-range" forces required to explain the large interaction cross sections of anomalous. So far it is beyond the grasp of any theoretical concept to explain both the enhancement of the cross section and the remarkably long lifetime of anomalous, estimated to be in excess of 10^{-10} sec.

McHarris and I were intrigued by the speculations of van Dantzig and van der Velden⁴⁵ that a π^- and a few neutrons might form a bound system, a "pinut." The normal strong absorption channel for pionic atoms would not be present, for charge conservation requires that "true absorption" occur on a pn pair or a pp pair. We reasoned that at least 6% of projectile fragments might be quite neutron-rich, thus have neutron haloes to which π^- could bind, with quite small absorption if only the pionic wave function were small enough in the nuclear interior, where the proton density is significant.

4. Schrodinger Equation Solutions for Pionic Atoms

It is not easy to make quantitative the above speculations about exotic pionic atoms, but we have made a start along the lines pioneered by Ericson and Myhrer³³. We have started with the pion optical potential outlined above, using parameters of ref. 36. Their parameters have never been tested on very neutron-rich species, so the isovector terms are uncertain.

We parametrize the nuclear density as the usual Woods-Saxon form but also give the proton-to-nucleon ratio a Woods-Saxon form with separately adjusted radius, so as to set up a neutron skin for neutron-rich species. One may note that the true absorption should greatly decrease for the very deeply bound pion internal solutions of the type proposed by Ericson and Myhrer. This decrease results from the fact that the number of channels open for the dominant absorption process of ejection of two neutrons into the continuum will strongly decrease. True absorption of a negative pion converts a proton into a neutron and thus drives in the opposite direction to negative beta decay, which may be exocergic by more than 10 MeV in the neutron-rich light nuclei.

As pointed out in ref. 34, the radius at which the effective mass singularity occurs divides the pionic solutions into essentially internal and external classes. Mandelzweig *et al.* give the Bohr-Sommerfeld phase values appropriate for solutions bounded by the mass singularity, and we use their prescriptions.

Use of the CMS pionic atom potential did not give us a mass singularity, even for the neutron-rich ³⁴Na test nucleus. Thus, we first softened the LEE effect by reducing the lambda parameter from 1.4 to 1.0, certainly within the range of values used in the pion optical parameters. Secondly, we reasoned that the neutron-rich nuclei would have a large neutron-density polarizability which would result in the local nuclear density, especially of neutrons, in the vicinity of an orbiting pion to be larger than normal. With these modifications the effective mass became negative throughout the nuclear interior.

We have not attempted yet to calculate the degree of excess nuclear density induced by pions. It will not be easy to calculate, but it is analogous to the effective charge problem in the nuclear shell model. Rather have we simply made calculations for three different degrees of neutron-density enhancement about the pion. Our calculations are summarized in Table I, where the first column is the orbital angular momentum L, the second column is the fractional excess neutron density assumed (proton density was unaltered). The third column gives the location of the mass singularity in units of the Woods-Saxon half-density radius. The fourth column gives the negative eigenvalues (binding energies); for the system as a whole the binding energy should be reduced by the energy cost of local neutron density enhancement, which we have not attempted to calculate. The last column gives the average orbital radius of the solution in units of the half-density nuclear radius.

The last two solutions, L=4 and 5 for intermediate neutron density polarizability, are so deeply bound that all or most channels of true pion absorption may be closed altogether, and the lifetime could lengthen to that required for anomalous. Let us ask why the reaction cross sections of

these internal pion species should be so much larger than normal nuclei. The long-range part of the nuclear force is generally assumed to come from virtual pion exchange. That is, the intermediate state is classically energy-forbidden by the rest mass energy of 140 MeV, so the characteristic range is the pion Compton wave length. Contrast this with the range of chemical binding forces in, say, the simplest molecule, the hydrogen molecule ion. The electron, real, not virtual, is the exchanged particle, and the characteristic force range is then the tunneling length of an electron with the Rydberg binding of 13.5 eV. By the same token, the Ericson-type deeply bound pions are real, not virtual, so their tunneling range outside the nucleus will be longer than the pion Compton wave length. Indeed, the more lightly bound the pion, the longer the characteristic distance it can tunnel. We are now faced with a trade-off, though. The deeply bound pion states may have the longest lifetimes, but the largest nuclear interaction cross sections will go with the more lightly bound solutions. It remains to try to calculate estimates of these properties. One also is faced with the problem of explaining the sheer abundance of anomalous production. To the extent that there is any thermal equilibration in the production of reaction fragments, we note that the masses of the deeply bound pionic species are not much greater than the corresponding ordinary nuclei.

Finally, we mention one more interesting possibility and complication. If two π^- are associated with a single nuclear fragment, it may be that extra binding will occur by virtue of admixture of "di-deltas," strongly bound combinations of two delta particles in the isospin 3 and spin 0 state.^{46,47}

We must conclude that there is need for fundamental theoretical work to pin down the effects on the pion optical potential for multiple pions and the consequent di-delta admixtures. Pending such work we can only say that moderate alterations of the CMS potential of neutron-rich nuclei can give somewhat deeper bound pion s-states with lifetimes that can get into the anomalous range. The quark-bag rearrangement candidates for anomalous have at least as great difficulty explaining the properties of the anomalous. Whatever the outcome of anomalous physics it continues to excite the imagination and stimulate new experiments. In particular I am much intrigued by possibly learning something about pionic atoms with more than one pion, a field of study uniquely accessible by relativistic heavy ion research.

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Intranuclear Pionic States for Sodium-34				
L	$\Delta \frac{p_N}{p_N}$	$\frac{R_\infty}{R_N}$	$-E_\pi(\text{MeV})$	$\frac{\langle r \rangle_{\text{orbit}}}{R_N}$
0	.86	1.011	17.32	.50
1	.86	1.011	59.20	.67
2	.25	.863	24.08	.62
3	.25	.863	40.07	.65
0	.40	.932	9.72	.47
1	.40	.932	27.60	.62
2	.40	.932	53.12	.67
3	.40	.932	80.94	.71
4	.40	.932	110.7	.74
5	.40	.932	142.1	.76

Figure Legends

- Fig. 1. Semi-logarithmic plots of Lorentz-invariant production cross sections at 90° vs. particle kinetic energy. The left figure is for protons and the right for negative pions. Various projectile-target systems are shown, all at a beam energy of 0.8 A GeV. (from Nagamiya et al.⁶)
- Fig. 2. Same as Fig. 1, except for the Ne + NaF system and negative pions at various beam energies. The lowest energy data are compared with various theoretical predictions. (from Nagamiya et al.)
- Fig. 3. Summary of slope parameters E_0 for various beam energies. (from Nagamiya et al.)
- Fig. 4. Fireball velocities and temperatures in the two-fireball model for Ne + Ne at 655 A MeV. The calculations are plotted against impact parameter, and two different cases are shown. The dots represent the more realistic case in which the longitudinal-momentum mean free path is double the first-published estimate; the open triangles represent the case with the published longitudinal-momentum-mean free path of 2.6 fm. (from Radi et al.¹⁵)
- Fig. 5. Contour plots of Lorentz-invariant pi-plus cross sections in the plane of rapidity y and perpendicular momentum. The numbers on the contour lines are the Lorentz-invariant cross sections for pi-plus production in units of $\text{mb sr}^{-1}\text{GeV}^{-2}c$. The dots indicate points at which data are given. The arrows on the abscissas indicate target, nucleon-nucleon center-of-mass, and projectile rapidities. (from Nakai et al.²⁵)
- Fig. 6. Same as Fig. 5, except for the Ar + Ca system, and change of units to $\text{b sr}^{-1}\text{GeV}^{-2}$. The upper figure combines data of two groups, and the lower figure is for the data of one group, Wolf et al.²⁶
- Fig. 7. Isometric and contour plots of "sub-threshold" negative pion production data. Cross section units are those of Fig. 6. Note the pronounced peak at beam velocity, and the smooth exponentially-falling behavior elsewhere. The small ripples are probably statistical fluctuations.
- Fig. 8. Same as Fig. 7 except for the higher beam energy of $E/A = 655$ MeV.
- Fig. 9. Photomicrograph mosaic illustrating repeated projectile fragmentation of an incident ^{56}Fe nucleus at E/A of 1.88 GeV. The occurrence of abnormally short mean free paths of secondary, tertiary, etc. fragments in the first 2 or 3 cm of their path is called the anomalon effect.

XBL 8012-2516

Fig. 1.

XBL 8112-13235

Fig. 2.

Fig. 3.

XBL 788-1495B

Fig. 4.

XBL 808-11345

Fig. 5.

XBL 816 - 2375

Fig. 6.

$E/A = 138 \text{ MEV}$

$NE+NAF \rightarrow X+\pi^-$

XBL 823-8269

Fig. 7.

XBL 8112-12886

Fig. 8.

XBL 794-9277

Fig. 9.