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MOLD HEATING AND COOLING SYSTEMS

By R. H. Floersch

Published September 1978
Interim Report

Prepared for the United States Department of Energy
Under Contract Number EY-76-C-04-0613.

**Kansas City
Division**

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By R. H. Floersch

Published September 1978

Interim Report
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SUMMARY

This project was initiated in October, 1968, to study the dynamics of control systems required to effectively control mold time/temperature profiles. The study was initially directed toward solutions to problems which had been encountered in the existing analog control system. The preliminary work led to consideration of a computerized control system and to procurement of a direct digital control process computer which will be used to provide simultaneous control of all Bendix filled-elastomers and plastics molding operations.

The most outstanding feature of the proposed DDC system is the convenience and time saving in setting up new processes or making process changes. Changes are required only in the computer software -- no change in hardware is necessary.

Adoption of the DDC system will circumvent the problems which have been encountered in use of the present analog systems. Significant cost savings will result from reduction in manpower requirements and elimination of new equipment costs associated with process changes and production change-over. Down-time resulting from process changes and equipment disorders will be reduced significantly, thereby favoring production schedules and advancing delivery dates.

Because of the short time required to make the programming changes necessary for new parts or new processes, the system will be of great value in process development and mold design work.

Work has advanced well beyond the point of proving the desirability of the system and the feasibility of adapting the DDC concept to Bendix applications. Extensive basic analytical work has been done, mathematical models of oil-loop and mold temperature responses have been developed, and computer programs have been written to lessen the necessary task of calculating the heat-transfer fluid and mold response properties which are required to write the DDC computer software for each process.

Much additional work will be required to perform closed-loop analyses of oil-loop and mold responses, construct mathematical models, and write the software for each of seventeen filled-elastomers and five plastics molding operations which are anticipated in the predictable future.

Section 1

HISTORY

Development was initiated in October, 1968, to determine the dynamics of control systems required to effectively control mold time/temperature profiles. Time/temperature cycling is required to produce many high-quality precision parts.

Some of the molding variations which had been encountered were believed to have been caused by long time constants and excessive dead-time, both of which appeared to have been due to inadequacies in the heat-transfer characteristics of the mold and the control system.

Three major alterations had been made in the control systems during the preceding two years, and the system was not yet fully perfected. It was apparent that the manufacturer of the system did not completely understand the complexities of the time/temperature cycling required to conform to Bendix specifications and the control problems which are frequently encountered in Bendix molding operations.

Since fifteen new analog pumping stations were being acquired for control of additional filled-elastomers molding facilities, and other new applications were being planned, further investigation in advance of such expansion was warranted.

PURPOSE

The initial intent of the development program was to obtain additional knowledge by study of mold designs, system dynamics, and heat-transfer mechanisms, thereby alleviating the problems which had been encountered, and were again expected in the proposed new filled-elastomer systems. As the program progressed, however, it became apparent that the ultimate solution to the control problem might lie in use of a direct digital control (DDC) computer system for simultaneous control of several types of molding operations. Therefore, for reasons discussed in this report, the program was expanded to include development of a real-time, on-line digital computer system to be used for time-sharing control of seventeen filled-elastomer and five plastics molding substations, with capacity to accommodate possible future additions.

APPROACH

Success of the effort was contingent upon optimum mold designs and a thorough knowledge of oil-loop dynamics, heat-transfer characteristics, reduction of dead-time and the time constant of the molds, and a complete understanding of the analog control system in use at the outset of the program.

A thorough knowledge of the third-modification analog control system was acquired. Research relative to use of a real-time, on-line digital computer system for control of molding time and temperature was being conducted concurrently with production molding operations.

Sufficient knowledge was gained through the research and experience with the existing analog control system to indicate that adoption of a DDC system was both feasible and highly desirable.

When problems were encountered in procuring the fifteen additional analog control systems for the filled-elastomers facility, the resulting delay provided an opportunity to re-evaluate the control requirements. As a result of a committee effort, a direct digital control scheme was suggested at that time as a possible means of consolidating the time/temperature controls of all plastics molding operations into a central computer control system. An internal memorandum which summarizes the conclusions of that committee is given in Appendix A to this report. As an outcome of the preliminary research and the work of the committee, a proposal to adopt the DDC system was submitted to the AEC in September, 1969, and was approved by the AEC a short time thereafter.

Heavy production schedules demanded that an effective control scheme be placed in service as quickly as possible. Therefore, work was immediately begun to develop the DDC system and the custom software necessary for the required control functions.

Section 2

THE DDC DEVELOPMENT

Since cyclic temperature control of plastics molding operations at Bendix is frequently more exacting than that commonly used in the industry as a whole, purchased systems usually require some modification or supporting developmental work to adapt them to specific Bendix requirements. Therefore, the most reliable source of information required to establish new molding processes or to up-date existing facilities often lies within the company. Although the computer and some of the system elements were purchased items which were designed to a similar concept, it was necessary to incorporate them into a system arranged to Bendix needs. For that reason, an appreciable amount of developmental and investigative work was required.

Any closed-loop system which is continuously self-correcting to some predetermined level requires a signal derived from sensing one or more of the variables which are to be controlled. Accuracy of control depends upon the sensitivity of the system and the rate at which corrections can be made, with minimal time-lag and overshoot.

To obtain the required accuracy of control, it is logical to control both the oil temperature and the temperature of the mold. Therefore, thorough analyses of oil-loop and mold dynamics will be necessary for development of each of the control algorithms, which require accurate mathematical models of both the oil-loop and the mold responses. For that reason, extensive future experimental development work will be required in that area.

The computerized heating and cooling system is basically composed of three subsystems: (1) the central hot and cold oil supplies and the oil return system; (2) a substation at each press location; and (3) a centrally located computer, which will control all of the substations, thus controlling several different types of molding operations at the same time.

A basic block diagram of the hot and cold oil supply and return system is shown in Figure 1. The hot supply employs two gas-fired heaters, each with a capacity of 7,000,000 Btu per hour. The heaters are capable of maintaining the temperature of the oil at 600°F under 60 percent of maximum load. The cold supply is similar to the hot portion of the system, except that the heaters are replaced by heat exchangers which can maintain the temperature in the cold supply loop at 90°F under 60 percent of maximum load. Pumps, controlled by central pressure regulators, move the oil continuously to maintain a constant pressure in both hot and cold portions of the central system, regardless of the demands of the substations which are tapped into the supply lines.

Figure 1. The Hot and Cold Oil Supply System

Figure 2 illustrates one loop of a local substation. Hot or cold oil enters the recirculating loop through the computer-controlled pneumatically actuated valves to maintain the temperature of the oil flowing into the mold at a value determined by the setpoint temperature programmed into the computer. When oil is allowed to flow from either the hot or cold supply into the local loop, an equal amount flows through the return valves to the respective hot or cold supply reservoirs, thus maintaining a constant volume of oil in all parts of the local substation and the central supply system.

The central computer controls the oil temperature at the inlet of the mold as follows: At two-second intervals, the computer samples the output from the thermocouple in the line at the inlet of the mold (single asterisk in Figure 2) and compares it to the setpoint data of the software. The difference is the error signal (ϵ), which is used by the P+I+D (Proportional + Integral + Derivative) algorithm to compute an output signal (m) which is applied to the input of a QSE* incremental amplifier where it is converted to a pneumatic pressure signal which opens or closes the hot or cold supply and return valves in proper proportion to restore the temperature of the oil to programmed value. (See Figure 3.)

The initial work principally consisted of a heat-transfer analysis, molding experiments, and attendance at a two-week training program. Theoretical analyses of the step-forcing and ramp-forcing functions were developed later. Research into the necessary modes of control continued, including oil temperature feedback, combining oil temperature and mold temperature feedbacks (cascade), dual mode control, dead-time compensation, adaptive gain changing, and gain changing based upon absolute error. Descriptions of those activities follow.

HEAT TRANSFER

An analysis of the heat transfer characteristics of the mold heating and cooling fluid was conducted to determine the film coefficient, which defines the ability of the fluid to absorb or relinquish heat.

The basic equation for forced convection in tubes and ducts illustrates the importance of the film coefficient:

$$Q = h(\hat{T})A[T_o(t) - T_m(t)] = \text{Btu/hr} \quad (1)$$

Where: $h(\hat{T})$ = Temperature-dependent film coefficient

A = Heat-transfer surface area

$T_o(t)$ = Average oil temperature as a function of time

$T_m(t)$ = Metal temperature as a function of time

*Quasi-Special Equipment

Figure 2. An Individual Pumping Substation for One Loop

Figure 3. Schematic Diagram of the On-Line Computer

The derivation of the film coefficient for forced convection in tubes and ducts (for turbulent flow only) is shown in Appendix B. The film coefficient (h) equation from that derivation is:

$$h = 0.023 \cdot \text{Re}^{0.8} \cdot \text{Pr}^{0.33} \cdot k/d \quad (2)$$

Where: h = Film coefficient
 Re = Reynold's Number
 Pr = Prandtl's Number
 k = Fluid conductivity
 d = Duct diameter

Equation 2 therefore yields the film coefficient as a function of Reynold's Number, Prandtl's Number, conductivity, and duct diameter.

$$\text{Reynold's No.} = \frac{\text{Velocity} \cdot \text{Diameter} \cdot \text{Density}}{\text{Viscosity } (\mu_m)} \quad (3)$$

$$\text{Prandtl's No.} = \frac{\text{Specific heat} \cdot \text{Viscosity } (\mu_i)}{\text{Conductivity}} \quad (4)$$

Equations 3 and 4 (Reynold's and Prandtl's Numbers) are functions of flow rate, duct diameter, and thermal characteristics of the fluid; therefore, if the heat transfer fluid is constant, the film coefficient can be increased only by larger values of fluid flow or duct diameter. For that reason, those two variables are important factors in designing the system and the mold.

Equations 2, 3, and 4 show that the film coefficient changes with every change in the temperature dependent thermo-properties of the oil (viscosity, conductivity, specific heat, etc.) as well as with changes in duct diameter. Since the oil temperature must be ramped or step-forced to obtain the mold-temperature responses required for molding various types of parts, calculation of film coefficient for each set of molding conditions would be very time-consuming. For that reason, two programs were developed for the computer to derive the film coefficient for a specified temperature, flow rate, and duct diameter.

The first two of the special programs calculates the Reynold's Number for each temperature for a specific flow rate and duct diameter. A second-order least-squares curve fit was then developed to obtain the Reynold's Number as a function of temperature for that set of conditions.

The second program applies the Reynold's Number equation obtained from the first program to calculate the film coefficient.

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If the average oil temperature and the film coefficient as a function of temperature were known, Equation 1 might be solved to determine the heat-load requirements for a desired mold response. Since those factors are not known at this point in the calculations, however, the heat-load required to raise the mold temperature a given amount in a given period of time must be determined by solving the internal energy equation:

$$U = M \cdot C_p \cdot dT_m(t)/dt \quad (5)$$

Where: U = Internal energy

M = Metal mass

C_p = Specific heat of the metal

$dT_m(t)/dt$ = Temperature rise rate of the metal

The solutions to Equations 1 and 5 can be compared to determine if the oil has the ability to supply enough heat to obtain the desired mold response.

The only drawback to the previous analysis is that the outlet oil temperature must be known to determine the average oil temperature.

The outlet oil temperature is dependent upon the type of forcing, the mass of the metal in the mold, and the heat-transfer area. According to Kreith,* the outlet oil temperature must be known to evaluate the film coefficient at the film temperature.

The film temperature (T_f) is determined as follows:

$$T_f = [T_o(t) + T_m(t)]/2, \text{ and } T_o(t) = (T_{in} + T_{out})/2$$

Where: T_f = Film temperature

$T_o(t)$ = Time dependent average oil temperature

$T_m(t)$ = Time dependent mold temperature

T_{in} = Inlet oil temperature at time (t)

T_{out} = Outlet oil temperature at time (t)

Since outlet oil temperature must be known to determine both film temperature (T_f) and loop gain, it was necessary to devise a method of determining the temperature at the outlet from the mold. Such a method was derived, based upon an earlier experiment. That experiment is described in the paragraphs immediately following:

*Frank Kreith, Principles of Heat Transfer

THE EXPERIMENT

During a nine-month interval when the principal oil supply system was out of operation, three heaters and a series of solenoid valves were used to heat the heat-transfer fluid and regulate the flow of the fluid to the mold. The experimental data from this work could not be correlated to the dynamic mold responses expected of the computer-controlled system because of inconsistencies in flow rate, transfer time, and the different transfer fluid used. However, the nature of the mold response could be formulated. A plot of the mold response exhibited momentum, and the curve resembled a semi-second-order response (semi-second-order meaning that it can be approximated by a first-order plus a dead-time response).

The order of the response is important for two reasons: (1) The first-order response is easier to model mathematically, thus a given control policy can be formulated more conveniently; and (2), it is much easier to develop simulation programs to predict the oil forcing required to achieve a given mold response with a first-order response pattern.

A method was developed to determine the outlet oil temperature, approximate the time constant of the mold, and calculate the open-loop gain of the oil loop for an oil temperature step-forcing function. That derivation is shown in Appendix C.

The basic approach was to take an enthalpy balance across the mold. The basic equation, neglecting losses and assuming a lumped system with no kinetic or potential differences, is shown below:

$$\dot{m}[h_{in}(t) - h_{out}(t)] = [MC_p \cdot dT_m(t)/dt] \quad (6)$$

Where: $\dot{m}$ = Mass flow rate of the oil

$h_{in}(t)$ = Inlet oil enthalpy as a function of time

$h_{out}(t)$ = Outlet oil enthalpy as a function of time

M = Mass of the metal of the mold

C_p = Specific heat of the metal

$dT_m(t)/dt$ = Time rate of change of the metal temperature

If a first-order step response is used, the mold metal response as a function of time is:

$$T_m(t) = A(1 - e^{-t/\tau}) + T_{mi}e^{-t/\tau} \quad (7)$$

Where: $T_m(t)$ = Metal temperature as a function of time
 A = Magnitude of the oil temperature
 τ = Time constant
 T_{mi} = Initial metal temperature

(See Appendix C for the derivation of Equation 7)

For a step increase, the inlet oil temperature is constant; therefore, the inlet enthalpy is constant. By taking the derivative of Equation 7 and substituting in Equation 6, the outlet oil enthalpy can be determined.

If the outlet enthalpy is known, the outlet oil temperature may be determined by means of Equation 8. That equation, as derived in Appendix C, expresses outlet temperature as a function of outlet enthalpy, as shown below:

$$T_{out}(t) = -1276 + [\sqrt{0.0692 + 0.000412 (8.686 + h_{out}(t))} / 0.000206] \quad (8)$$

The minimum possible time constant is:

$$\tau = MC_p (A - T_{mi}) / \dot{m} [h_{in} - h_{out}(0)] \quad (9)$$

When outlet oil temperature, T_o , has been determined, open loop gain may be calculated by means of Equation 10, as derived in Appendix C.

$$\text{Open Loop Gain} = [T_h - T_o(t)] / \dot{m} \quad (10)$$

Where: T_h = Central hot supply temperature
 $T_o(t)$ = Oil outlet temperature as a function of time
 $\dot{m}$ = Total mass flow rate in the loop

The loop gain is important in determining controller gain and in analyzing the stability problems that might result from large gain variations. The overall gain is important in determining the need for adaptive gain changing.

The foregoing mathematical solutions are essential to ensuring that the mold will respond and in determining the control schemes necessary to produce the desired response. For that reason, the writer developed a computer program to predict the required responses, thus virtually eliminating the necessity for time-consuming manual calculations. That program is shown in Appendix D.

The findings indicated that, when fast response is required, some form of gain changing must be employed to maintain a stable control system. The program will be useful in mold design work, also, if the process requirements are known. If the time constant is known, the heat transfer surface area may be designed to match the heat transfer characteristics. The time constant and the heat transfer area may be varied to obtain the required mold response. The mold characteristics can thus be optimized in the design stage.

A third derivation, shown in Appendix E, established a means of determining outlet oil temperature and open-loop gain for a given ramp forcing function. That derivation requires that the time constant, as determined by the preceding program (Appendix D), be known. The computer program shown in Appendix F was subsequently developed from the third derivation outlet oil temperature and open-loop gain calculations.

STATUS AT CLOSE OF QUARTER (FY70-3)

Use of the three computer programs which were developed enables virtually all heat control requirements and mold responses to be predicted before the molds are placed in the machines. Those programs, however, were all open loop configurations, and were not yet adapted to the closed loop configurations necessary to determine the controller P+I+D constants required for adequate control.

Since the basic principles discussed and the control schemes that are yet to be developed involve only computer software, and do not require different or altered hardware items, control algorithms can be readily modified, as required, for a variety of molded parts or different molding parameters. The proposed computerized control system will enable down-time during change-over, repair, or process changes to be greatly reduced, and will virtually eliminate change-over equipment costs. Change-over for new part production can be accomplished in a much shorter time than that required with the present control methods, thereby improving production and delivery schedules. The resultant savings in manpower and overall costs will be significant. Furthermore, the ease of change in the control programs makes the system ideally suited to development work.

Section 3

FUTURE PLANNING

Work during the Fourth Quarter of FY70 will consist of investigation and simulation of modes of control which employ a closed-loop supply and control system.

A major portion of Fiscal Year 1971 will be devoted to experimentation to determine the oil-loop and mold dynamics necessary for development of the required mathematical models and to development of actual control software for the new system. The oil feedback mode of control provided by the computer will be studied in depth. During the latter part of FY71-1 and the early part of FY71-2, the vendor will complete adaptation of the system to Bendix requirements at the Bendix Kansas City facility.

A block diagram mathematical model of the oil temperature feedback mode of mold temperature control is shown in Figure 4. The system operates as follows: The setpoint (R) is compared to the process variable (feedback oil temperature). The difference or error signal (ϵ) is applied to the P+I+D algorithm, which computes an output to actuate the oil flow valves. Opening or closing the hot or cold supply line valves in the proper proportions restores the temperature of the oil at the mold to the predetermined setpoint value, at which time the error signal is reduced to zero and no further corrective action occurs. Although this is an over-simplified description, it explains the basic principles of the system.

When the central control computer is added to the system, the outputs from the thermocouples in the substation system will be sampled at two-second intervals. The computer will perform the functions described in the preceding paragraph, thereby providing signals which will operate the valves in the oil lines to control the temperature of the mold. Since oil-loop dynamics is the most important factor in attaining effective control of mold temperature, accurate mathematical modeling of the oil-loop parameters is necessary to program the computer and to predict the temperature response characteristics of the mold.

The major disadvantage of the oil-feedback control mode shown in Figure 4 is that mold response is dependent upon the temperature of the oil in the loop, but has no part in control of that temperature. The major advantage is the rapid temperature response of the oil, as compared to the slower response of the mold. A long dead-time creates a large phase shift, and when combined with a slow mold response, causes the system to become uncontrollable. Because of the inherent slow mold response, mold temperature feedback will not suffice as a lone mode of control.

* PV = (PROCESS VARIABLE) OIL TEMPERATURE - SINGLE ASTERISK IN FIGURE 2.

Figure 4. Schematic Diagram of Oil-Loop Feedback

By use of both hot and cold oil supplies and suitable modes of control, the temperature of the oil at the inlet to the mold can be made to change very rapidly; therefore, oil-temperature feedback is the most effective single mode of control. For that reason, it is the basic control concept from which all of the more complex control schemes will evolve.

Figure 5 is a block diagram of an oil-temperature feedback system of control which incorporates adaptive gain changing. The necessity for some form of gain changing was first mentioned in a preceding section.

Adaptive gain changing consists of computing a short-interval process gain and using the result to change the gain of the P+I+D control element, so that the overall loop gain remains essentially constant. Operation is essentially the same as that of the oil-feedback system shown in Figure 4, except that the P+I+D controller gain (P) is variable, and is dependent upon the closed-loop gain.

Figure 6 illustrates the cascade control mode, wherein mold-temperature feedback is used to regulate the setpoint of the oil loop, thereby providing compensation for changes in the temperature of the mold. It thus permits rapid correction of mold temperature variations which result from process disturbances. The cascade mode is ideally suited for dwells and slow ramp rates, but the dual mode scheme is more suitable for fast rise rate applications.

The cascade control system (Figure 6) functions as follows: The temperature of the mold is compared to the setpoint temperature and the resulting error signal is utilized by the P+I control algorithm to calculate a compensated setpoint for the minor oil loop. The oil-temperature variable at the inlet to the mold is compared to the compensated minor-loop setpoint, and a second error signal is generated. That signal is utilized by the P+I+D algorithm to calculate the valve responses required to correct the overall temperature variable. Cascade control combined with adaptive gain changing is shown in Figure 7.

A dual mode control scheme is normally required in time-optimum processes and those that require fast ramp or rise rates. The first part of the dual mode consists of an ON-OFF action to obtain fast response. When the process levels out, control is switched to a second mode, oil-feedback or cascade, with the P+I+D algorithm in the loop.

Dead-time is another important factor that must be considered. Excessive dead-times create phase shifts that tend to make the system unstable. Dead-time compensation requires modification of the P+I+D algorithm to offset the effects of the time lapse between the controller output and the start of the process response. A solution to that problem requires use of a process model to predict the response of the system before it occurs.

Figure 5. Schematic Diagram of Adaptive Gain Oil Feedback

Figure 6. Schematic Diagram of Cascade Control

Figure 7. Schematic Diagram of Adaptive Gain - Cascade Control

The predictive control scheme provides better control of processes which exhibit extremely fast responses. The prediction is based upon an analysis of preceding and present levels of the process. The process variable is predicted one-half time period ahead of the need for correction, thereby the corrective action can be taken in time to prevent the variation that would have occurred if the action had not been taken.

Fourth Quarter FY70 activities will include some dynamic testing on the existing analog system with available equipment and molds, further theoretical study of control loop simulations, development of simulation programs to establish loop tuning parameters (times, magnitudes, rates, etc.), development of a second-order response model, and continued study of custom software.

First Quarter FY71 efforts will be almost entirely devoted to experimental work with the new system and with new test equipment which is now being constructed. That work will principally consist of determination of major and minor oil-loop dynamics for each of the subsystems.

Dynamics studies of each of the minor and major oil loops will determine oil-loop time constants, dead-times, flow-rate variations, mixing-rates, and loop gains. Oil-loop responses to different forcings will be determined, and the effects thereof will be examined. The experimental results which are obtained will be compared to theoretical values.

Study of the new systems will continue into the Second Quarter of FY71, and will probably require most of that quarter. More accurate mathematical models and additional computer programs will be developed for use in future, more complex, computer control schemes.

During the ensuing quarters, actual control techniques will be incorporated into systems to achieve a higher degree of control of mold temperature (the principal process variable). Since more sophisticated control schemes require more accurate and more complex mathematical modeling, refinement of test procedures will be continued to achieve the required accuracies.

During the last two quarters of FY71 and the first two quarters of FY72, the control schemes outlined in this report will be implemented, and will be modified and updated to reflect the latest knowledge. Additional schemes will be devised to establish control criteria by insertion of an ideal temperature cycle, thus eliminating the necessity for tuning the loop. Another subroutine will be developed to analyze each cycle automatically, and to update the following cycle to compensate for the discrepancies found in the preceding one.

Section 4

CONCLUSION

The work outlined in this report and that planned for future quarters reflects the importance of accurate mathematical modeling and a thorough understanding of oil-loop and mold responses to the final attainment of adequate control of mold temperature. It is also evident that the control techniques are quite complex, and that incorporation of the necessary data into the computer software will require a comprehensive knowledge of the language of the control computer and the necessary custom software.

Although the computerized time/temperature control concept and the capability of formulating computer programs have been firmly established, much additional work will be required to develop mathematical models and control computer programs for each of the several present and projected types of Bendix molding processes.

When the hardware is delivered, the vendor will require a considerable period of time at this facility to complete the adaptation of the equipment to Bendix applications.

Future developments will continue to be reported on a quarterly basis, and a final report will be prepared upon completion of the work. Because of the length of this interim report, however, the entire content will not be repeated, but may be referenced frequently. Therefore, because of the importance of the basic knowledge contained herein to an understanding of the overall final system, this publication should be retained by those who have an interest in the final outcome of the work.

Appendix A

Alternate Proposal for Mold Heating and Cooling

Internal Memorandum, August 18, 1969
R. H. Floersch to R. E. Holm

A report of a Meeting of Engineering Personnel
to Discuss Computer Control of Mold Temperature

ALTERNATE PROPOSAL FOR MOLD HEATING AND COOLING CONTROL

The following is a summation of the meeting held on August 12, 1969. The people in attendance were: Facilities - Pete Garcia, Fred Davis, and C. B. Welter; Computer group - Matt Grogger and Bill Brown; Special Projects - Dave Hahn; and from Processing - Bob Klee and Dick Floersch.

The subject of the meeting was concerned with mold heating and cooling systems for Departments 26 and 27. Discussions involved present needs, capabilities, and future needs in temperature control.

The state of the art in direct digital control (DDC) has come from virtual nonexistence a year ago to a highly developed and reliable control system today. This is due to the innovation of the necessary hardware and software. Because until recently DDC was not practical, the present analog control systems were implemented. However, now that DDC is a proven art, it must be examined for its pros and cons.

One DDC computer of average core size can handle up to 200 or more different control loops at one time. The control loop processes can be as diversified as need be and may be made as simple or complex as the process dictates. Therefore, one DDC computer can replace sixty or more analog systems. A big advantage of DDC is the fact that different levels of sophistication can be achieved by simply modifying the particular loop algorithm rather than adding or deleting analog control equipment. Not only the ease is an advantage, but the modification of the algorithm can be accomplished in a matter of minutes versus the time required to install additional analog equipment which could take months.

The DDC machine can control a process variable with more accuracy than an analog system by the simple fact that set point comparators, ramp generating devices, etc., would be eliminated. The same accuracy problems would exist for the sensing devices, however. Nonlinearities can be compensated for with DDC, which analog systems cannot do. Also, the complexity of the control problem may be such that an analog system could not readily control the process.

One DDC computer can handle diversified control problems, while an analog system is restricted to one control function defined by its design criteria. Thus, the DDC system will not become antiquated when the need for a particular type of control has become obsolete due to different process controls or innovation of improved processing.

The expansion capabilities of the DDC system would be limited only to the size of the hardware and the ability of the software to handle more sophisticated control problems. Thus, all of rubber and plastics heating cycles could be controlled

from one flexible DDC computer.

The speed and ease of changing a parameter, or a series of parameters, would be of immense value, as opposed to the tuning required for changing the same parameters on an analog system. Parameters of the temperature contour could be changed via an input console to the DDC computer, and the control program will automatically retune the set point values and break points to achieve the new contour. An excellent example of tuning analog systems is the Honeywell four-zone heating and cooling system in Department 26. It takes a well informed man five or more hours to make a major alteration to a cycle or to initially tune a new mold. There are fifteen different variables that must be juggled by trial and error. The fact that there are five interrelated variables to accomplish one objective makes the task more difficult. This work would be alleviated by DDC control.

The DDC system can, by use of a software program, be a self-tuning system. This would be accomplished by continually updating the proportional band, integral, and derivative time constants. These constants vary in magnitude with large variations in ambient conditions, with large range changes, and when new molds are incorporated for a particular cycle. It would also continually update the set points and switch points, automatically, by comparing the deviations of the last cycle to the ideal cycle.

A DDC capability would be invaluable in research and development in that it is quite flexible and new processes could be attempted without procuring special sophisticated and complex temperature control equipment. DDC is more flexible in that the inputs can be programmed to achieve a wide variety of desired mold cycle outputs. For injection molding alone, computer simulations of the part temperature versus the mold temperature throughout the cycle have suggested that there might be a different cycle that is better than the existing one. This cycle, however, is considerably different and the capability of pursuing it in actual molding tests is nonexistent, because of the lack of a temperature controller.

The system has control enforcement, in that a change is recorded at the time it was instigated. Thus, this system has the advantage of historically recording any changes, and makes cycle changes by nonexperienced personnel less likely to occur.

Use of the DDC system would only require the knowledge of the process computer and an in-depth knowledge of the process. The computer language could be taught with virtually no problems, and the engineer should know the cycle he desires ahead of time. This would be much simpler than acquiring the necessary knowledge to control each analog system and would aid in standardizing procedures.

There would be considerably fewer maintenance problems, since there would be only one system, versus the maintenance required for each individual analog system.

The one big disadvantage to a centralized computer control is computer failure. Since the computer hardware does need maintenance, there is a possibility of failure. However, a study of the most probable failure points could be made, and backup standby hardware procured for the weak points. This would mean a standby computer and related hardware to be absolutely sure of continual on-line control. This standby computer could be utilized as a research and development control system and for background operations and, should the main computer fail, this system would automatically transfer to the control of production processes. Also, to support the backup capability, the systems in use at the present time would be retained. It is doubtful that this would be required, because most failures on the system hardware can be located and repaired within an hour of failure.

In summary, in the group's opinion, it would be a mistake not to look in detail at a DDC system. It is more flexible. It will not be a here today - no longer needed system tomorrow because a particular type of temperature control has become superannuated. It will provide a capability for exotic research and development temperature control. It will provide a quick, easy, and accurate means of changing cyclic parameters, and the experience realized from implementation and operation of the system will aid in developing future process control applications as the need dictates.

Appendix B

Derivation of the Film Coefficient for
Turbulent Flow Inside Tubes and Ducts

By

R. H. Floersch

DERIVATION OF THE FILM COEFFICIENT FOR TURBULENT FLOW INSIDE TUBES AND DUCTS

Physical properties of the heat transfer fluid, as a function of temperature, that are required for this analysis are:

1. Conductivity (k) = Btu/ft/hr/°F
2. Specific Heat (Cp) = Btu/lbm/°F
3. Specific Gravity (SG) = dimensionless
4. Viscosity (μ) = centipoises
 - a. To convert centipoises to lbm/hr/ft, multiply centipoises by 2.42: $\mu \cdot 2.42 = \mu_i$.
 - b. To convert centipoises to slug/ft/sec, multiply centipoises by 0.0000209: $\mu \cdot 0.0000209 = \mu_m$.
5. Film Coefficient (hc) = Btu/ft²/hr/°F

Other properties required:

1. Velocity (V) = ft/sec
2. Area (A) = in²
3. Diameter (D) = ft
4. Density (ρ) = slug/ft³ = 1.935 (water)

Equations:

$$V = \frac{\text{gpm} \cdot 2.31}{A \cdot 60 \cdot 12} = (\text{ft/sec}) \quad (1)$$

$$\text{Reynold's Number (Re)} = \frac{V \cdot D \cdot \text{SG} \cdot 1.935}{\mu_m} \quad (2) *$$

$$\text{Prandtl's Number (Pr)} = \frac{C_p \cdot \mu_i}{k} \quad (3) *$$

$$\text{Stanton's Number (St)} = 0.023 \cdot \text{Re}^{-0.2} \cdot \text{Pr}^{-2/3} \quad (4) *$$

*Equations 2, 3, 4, 5, and 6 were taken from Principles of Heat Transfer, by Frank Kreith, 3rd Edition International Textbook Comp. Chapter 8, entitled "Forced Convection Inside Tubes and Ducts."

$$\text{Nusselt's Number } (N_u) = St \cdot Re \cdot Pr \quad (5)*$$

$$\frac{hc}{Cp \cdot SG \cdot 1.935 \cdot V} = 0.023 \cdot \left(\frac{\mu_m}{D \cdot SG \cdot 1.935 \cdot V} \right)^{.2} \cdot Pr^{-2/3} \quad (6)*$$

DERIVATION

Multiplying the right hand side and left hand side of Eq. 6 by $Cp \cdot SG \cdot 1.935 \cdot V$ results in:

$$hc = 0.023 \cdot \left(\frac{\mu_m}{D \cdot SG \cdot 1.935 \cdot V} \right)^{.2} \cdot Pr^{-2/3} \cdot Cp \cdot SG \cdot 1.935 \cdot V \quad (7)$$

Recognizing that $\frac{\mu_m}{D \cdot SG \cdot 1.935 \cdot V}$ is the inverse Reynold's Number, Eq. 7 becomes:

$$hc = 0.023 \cdot Re^{-0.2} \cdot Pr^{-2/3} \cdot Cp \cdot SG \cdot 1.935 \cdot V \quad (8)$$

Multiplying and dividing the right hand side of Eq. 8 by Prandtl's Number, the result is:

$$\begin{aligned} hc &= 0.023 \cdot Re^{-0.2} \cdot Pr^{1/3} \cdot \frac{Cp \cdot SG \cdot 1.935 \cdot V}{Cp \cdot \mu_{i/k}} \\ &= 0.023 \cdot Re^{-0.2} \cdot Pr^{1/3} \cdot \frac{k \cdot SG \cdot 1.935 \cdot V}{\mu_i} \end{aligned} \quad (9)$$

Multiply and dividing Eq. 9 by D and changing μ_i to μ_m the resulting equation is:

$$hc = 0.023 \cdot Re^{-0.2} \cdot Pr^{1/3} \cdot \frac{k}{D} \cdot \frac{V \cdot D \cdot SG \cdot 1.935}{\mu_m} \quad (10)$$

Recognizing that $\frac{V \cdot D \cdot SG \cdot 1.935}{\mu_m}$ is Reynold's Number; by substitution:

$$hc = 0.023 \cdot Re^{0.8} \cdot Pr^{0.33} \cdot k/D \quad (11)$$

*See note on previous page.

Equation 11 defines the film coefficient in terms of Reynold's Number, Prandtl's Number, conductivity, and the duct diameter.

The film coefficient may also be defined in terms of Nusselt's Number, conductivity, and duct diameter. It is obtained by recognizing that Eq. 11 may be rewritten as follows:

$$hc = (0.023 \cdot Re^{-0.2} \cdot Pr^{-2/3}) \cdot (Re \cdot Pr) \cdot (k/D) \quad (12)$$

Recognizing $0.023 \cdot Re^{-0.2} \cdot Pr^{-2/3}$ (Stanton's Number), Eq. 12 may be rewritten as follows:

$$hc = (St \cdot Re \cdot Pr) \cdot (k/D) \quad (13)$$

Since Nusselt's number = $St \cdot Re \cdot Pr$, Eq. 13 may be written as follows:

$$hc = N_u \cdot k/D \quad (14)$$

Equation 14 defines the film coefficient in terms of Nusselt's Number, conductivity, and duct diameter.

Appendix C
The Mold Heating and Cooling System
Analysis by
R. H. Floresch

THE MOLD HEATING AND COOLING SYSTEM

To determine the oil temperature outlet of a mold given the inlet temperature: The mold temperature response approximates a first-order lag plus dead time. Neglect dead time. The loop is a recirculating loop with one hot inlet, one cold inlet, one hot return, and one cold return. Assume a step increase in oil temperature.

Given:

Oil enthalpy Btu/lbm.

$$h = 0.000103 \cdot \text{Temp}^2 + 0.26287 \cdot \text{Temp} - 8.686$$

Specific heat oil (Sp) = Btu/lbm/°F

$$\text{Sp} = 0.000211 \cdot \text{Temp} + 0.262$$

$$\text{Density oil (lbm/gal)} = -0.00463 \cdot \text{Temp} + 11.9169$$

$$\text{Specific heat metal (Cp)} = -0.11 \text{ Btu/lbm/°F}$$

Solution

The equation for a first-order transfer stage is:

$$\tau \frac{dT_m(t)}{dt} + T_m(t) = X(t) \quad (1)$$

$X(t) = A$ for a step function

τ = Time constant

$T_m(t)$ = Mold metal temperature

A = Value of the step forcing function (Magnitude)

Initial conditions at $t = 0$: $T_m(0) = T_{mi}$

Rewriting Eq. 1:

$$\tau \dot{T}_m(t) + T_m(t) = A \quad (2)$$

Taking the Laplace Transform of Eq. 2 yields:

$$\tau [s \cdot T_m(s) - T_m(0)] + T_m(s) = A/s \quad (3)$$

$$\tau s \cdot T_m(s) + T_m(s) - \tau T_{mi} = A/s \quad (4)$$

$$T_m(s) = \frac{A/\tau}{s(s+1/\tau)} + \frac{T_{mi}}{(s+1/\tau)} \quad (5)$$

$$L^{-1} T_m(s) = L^{-1} \frac{A/\tau}{s(s+1/\tau)} + L^{-1} \frac{T_{mi}}{(s+1/\tau)} \quad (6)$$

$$T_m(t) = A(1 - e^{-t/\tau}) + T_{mi} e^{-t/\tau} \quad (7)$$

Equation 7 is the time response of the mold temperature to a step forcing with an initial mold temperature T_{mi} .

If A , T_{mi} , and τ are known in Eq. 7 the mold temperature as a function of time can be found. If A , T_{mi} , C_p , M , and $\dot{m}$ are known the time constant can be found as will be illustrated later.

Taking a heat balance across the mold:

The system is a filled system with constant flow, therefore, $\dot{m}_{in} = \dot{m}_o = \dot{m}_t$.

T_{in} and T_o = Oil temperature in and out

V_{in} and V_o = Oil velocity in and out

Assuming that the losses are negligible, the heat given up by the oil is equal to the heat absorbed by the mold:

$$\begin{aligned} \dot{m} \left[\left(h_o(t) + \frac{V_o^2}{2g_c} + z_o \right) - \left(h_{in}(t) + \frac{V_{in}^2}{2g_c} + z_{in} \right) \right] \\ = - MC_p \frac{dT_m(t)}{dt} \end{aligned} \quad (8)$$

Assuming no potential or kinetic differences, (8) reduces to:

$$\dot{m} [h_o(t) - h_{in}(t)] = - MC_p \frac{dT_m(t)}{dt} \quad (9)$$

If the inlet oil temperature is assumed constant, $h_{in}(t)$ = constant.

$$h_{in} = 0.000103 \cdot T_{in}^2 + 0.26287 \cdot T_{in} - 8.686 \text{ (Btu/lbm)} \quad (10)$$

Restating Eq. 7 and taking the derivative yields:

$$T_m(t) = A(1 - e^{-t/\tau}) + T_{mi}e^{-t/\tau}$$

$$\frac{dT_m(t)}{dt} = (Ae^{-t/\tau} - T_{mi}e^{-t/\tau}) / \tau \quad (11a)$$

$$\frac{dT_m(t)}{dt} = \frac{e^{-t/\tau}}{\tau} (A - T_{mi}) \quad (11b)$$

Inserting Eq. 11b in Eq. 9 yields:

$$\dot{m} [h_o(t) - h_{in}] = - \frac{MC_p e^{-t/\tau}}{\tau} (A - T_{mi}) \quad (12)$$

Solving Eq. 12 for $h_o(t)$ yields:

$$h_o(t) = h_{in} - \frac{MC_p e^{-t/\tau}}{\dot{m}\tau} (A - T_{mi}) \quad (13)$$

Since the output enthalpy is a function of temperature, the outlet temperature is as follows:

$$T_o(t) = -1276 + \left[\sqrt{0.0691 + [0.000412 \cdot (8.686 + h_o(t))]} \right] / 0.000206 \quad (14)$$

Equation 14 is the equation for the outlet oil temperature. If the time constant (τ) is not known, the minimum time constant can be calculated as follows:

Knowing that the outlet oil temperature at time = 0 can be no less than the initial metal temperature at $t = 0$, and if the outlet oil temperature is in fact equal to the initial mold temperature, a maximum number of Btu's are given up. Therefore, the time constant must be a minimum.

Assuming that at $t = 0$:

$$T_o(0) = T_m(0) = T_{mi}, \text{ then}$$

$$h_o(t) = 0.000103 \cdot T_{mi}^2 + 0.26287 \cdot T_{mi} - 8.686$$

Solving Eq. 13 for τ , yields:

$$\tau = \frac{MCp e^{-t/\tau(A-T_{mi})}}{\dot{m}[h_{in}-h_o(0)]} \quad (15)$$

At $t = 0$, Eq. 15 reduces to:

$$\tau = \frac{MCp \cdot (A-T_{mi})}{\dot{m}[h_{in}-h_o(0)]} \quad (16)$$

Since τ is in seconds, $\dot{m}$ must be lbm/sec.

Plugging τ back into Eq. 13 and solving for $h_o(t)$, and plugging $h_o(t)$ into Eq. 14 yields $T_o(t)$.

Knowing $T_o(t)$, the loop gain can be calculated:

$$\dot{m}_p \cdot T_{in} = \dot{m}_h \cdot T_h + \dot{m}_t \cdot T_o(t) \quad \dot{m}_r = (\dot{m}_t - \dot{m}_h) \quad (17)$$

$$\dot{m}_h \cdot T_h + T_o(t) \cdot (\dot{m}_t - \dot{m}_h) = \dot{m}_t T_{in} \quad (18)$$

$$\text{Open loop gain} = \frac{dT_{in}}{d\dot{m}_h} \quad (19)$$

$$\dot{m}_h \cdot (T_h - T_o) + \dot{m}_p T_o(t) = T_{in} \cdot \dot{m}_t \quad (20)$$

$$\text{Therefore: } T_{in} = \left[\frac{\dot{m}_h}{\dot{m}_t} (T_h - T_o(t)) \right] + T_o(t) \quad (21)$$

$$\frac{dT_{in}}{d\dot{m}_h} = \frac{T_h - T_o(t)}{\dot{m}_t} \quad (22)$$

Equation 21 is the loop gain as a function of time.

Appendix D

Computer Program to Obtain Mold Time Constants for a
First Order Response With a Step Forcing Function

```

DIMENSION TM(950),H0(950),T0(950)
WRITE(1,('TI GPM A B WM M K AR TH'))
ACCEPT TI,GPM,A,B,WM,M,K,AR,TH
DEN=-.00463*TI+11.91688
AL=.000103*B**2+.26287*B-8.686
AM=.110
HI=.000103*TI**2+.26287*TI-8.686
OF=GPM*DEN/60.
BR=WM*AM/OF
STR=HI-AL
RET=A-B
T=BR*RET/STR
DO 20 IM=1,K
N=4.5*T
IF(N.GT.950)GO TO 4
GO TO 5
4 N=950
5 WRITE(1,(//))
WRITE(1,('THE INLET TEMP='F5.0,5X'TAU='F6.1'SEC'))TI,T
WRITE(1,('HI='F5.0,3X'METAL MASS='F9.3))HI,WM
WRITE(1,('FLOW LBM/SEC='F8.4,3X'GPM='F5.0))OF,GPM
WRITE(1,('HOT SUPPLY TEMPERATURE='F5.0))TH
WRITE(1,('HEAT TRANSFER AREA='F6.2'FT*FT'))AR
WRITE(1,('TIME'3X'TM'5X' H0'6X'T0'5X'GAIN'3X'FILM'4X'CFIL'))
WRITE(1,(55('*'))
DO 10 I=1,N,M
Q=1-1
R=Q/T
TM(I)=A*(1-EXP(-R))+B*EXP(-R)
SP=.110
RAT=WM*SP/OF
DTM=A*EXP(-R)-B*EXP(-R)
H0(I)=HI-(RAT*DTM/T)
DH=HI-H0(I)
AH=(.0691+(.000412*(8.686+H0(I))))
AT=SQRT(AH)
T0(I)=(-1276.+(AT/.000206))
T0H=(T0(I)+TI)/2
WE=WM*SP/AR
DTMT=DTM/T
DMT=1/(T0H-TM(I))
FIL=WE*DTMT*DMT*3600
IF(GPM.EQ.20)GO TO 7
GO TO 8
7 CFIL=-.00004297*T0H**2+.81904*T0H+9.198
8 IF(GPM.EQ.40) GO TO 9
GO TO 16
9 CFIL=-.0000763*T0H**2+1.426*T0H+16.26
16 IF(GPM.EQ.80)GO TO 12
GO TO 11
12 CFIL=-.0001324*T0H**2+2.1483*T0H+28.03
11 IF(GPM.EQ.160)GO TO 14
GO TO 13
14 CFIL=-.00022789*T0H**2+4.321*T0H+49.27
13 GA=(TH-T0(I))/OF

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(Continued on next page)

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WRITE(1,(I3,6(3XF5.0)))Q,TM(I),H0(I),T0(I),GA,FIL,CFIL
GPI=TI-T0(I)
IF(GPI.LE.0)GO TO 87
10 CONTINUE
87 WRITE(1,(55('*'))))
T=T+20
WRITE(1,(////))
20 CONTINUE
STOP
END
+

```

```

+G
TI  GPM  A  B  WM  M  K  AR  TH
500 160 500 100 10450 30 2 54.71 575

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THE INLET TEMP= 500.    TAU= 138.3SEC
HI= 148.    METAL MASS=10450.000
FLOW LBM/SEC= 25.6050    GPM= 160.
HOT SUPPLY TEMPERATURE= 575.
HEAT TRANSFER AREA= .54.71FT*FT

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TIME	TM	H0	T0	GAIN	FILM	CFIL
0	100.	19.	100.	19.	1094.	1325.
30	178.	44.	187.	15.	1065.	1506.
60	241.	64.	253.	13.	1045.	1644.
90	291.	81.	304.	11.	1031.	1750.
120	332.	94.	344.	9.	1020.	1833.
150	365.	105.	376.	8.	1012.	1898.
180	391.	113.	401.	7.	1006.	1949.
210	412.	120.	421.	6.	1001.	1990.
240	429.	126.	436.	5.	997.	2022.
270	443.	130.	449.	5.	994.	2048.
300	454.	134.	459.	5.	991.	2069.
330	463.	137.	467.	4.	989.	2085.
360	470.	139.	474.	4.	987.	2099.
390	476.	141.	479.	4.	986.	2109.
420	481.	142.	483.	4.	984.	2118.
450	485.	143.	486.	3.	983.	2125.
480	488.	144.	489.	3.	981.	2130.
510	490.	145.	491.	3.	980.	2135.
540	492.	146.	493.	3.	978.	2138.
570	494.	146.	494.	3.	976.	2141.
600	495.	147.	495.	3.	974.	2143.

THE INLET TEMP= 500. TAU= 158.3SEC
 HI= 148. METAL MASS=10450.000
 FLOW LBM/SEC= 25.6050 GPM= 160.
 HOT SUPPLY TEMPERATURE= 575.
 HEAT TRANSFER AREA= 54.71FT*FT

TIME	TM	HO	TO	GAIN	FILM	CFIL
0	100.	35.	157.	16.	837.	1444.
30	169.	55.	222.	14.	825.	1579.
60	226.	71.	273.	12.	815.	1686.
90	273.	84.	315.	10.	808.	1772.
120	313.	95.	348.	9.	803.	1841.
150	345.	105.	375.	8.	799.	1897.
180	372.	112.	398.	7.	795.	1943.
210	394.	118.	416.	6.	792.	1980.
240	412.	124.	431.	6.	790.	2011.
270	427.	128.	443.	5.	788.	2036.
300	440.	131.	453.	5.	786.	2056.
330	450.	134.	461.	4.	785.	2073.
360	459.	137.	468.	4.	784.	2087.
390	466.	139.	473.	4.	783.	2098.
420	472.	141.	478.	4.	782.	2108.
450	477.	142.	482.	4.	781.	2116.
480	481.	143.	485.	4.	780.	2122.
510	484.	144.	488.	3.	780.	2128.
540	487.	145.	490.	3.	779.	2132.
570	489.	145.	492.	3.	778.	2136.
600	491.	146.	493.	3.	777.	2139.
630	493.	146.	494.	3.	776.	2141.
660	494.	147.	495.	3.	775.	2143.
690	495.	147.	496.	3.	773.	2145.

STOP
 (\$MAIN\$)20+1

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Appendix E
Problem Analysis
By R. H. Floersch

PROBLEM ANALYSIS

To determine the inlet oil temperature, the outlet oil temperature, the mold temperature response, the loop gain, and film coefficient, for a first order ramp forcing:

$$\text{Given: Enthalpy: (Btu/lbm)} = 0.000103 \cdot \text{Temp}^2 + 0.26287 \cdot \text{Temp} - 8.686$$

$$\text{Density: (lbm/gal)} = -0.00463 \cdot \text{Temp} + 11.917$$

Assume the time constant, flow rate (gpm), the pipe diameter, and heat transfer area are given.

SOLUTION:

The equation for a first order lag is:

$$\tau \frac{dT_m(t)}{dt} + T_m(t) = x(t) \quad (1)$$

$$x(t) = kt + T_i \quad (2)$$

$$\text{At } t = 0, T_m(t) = T_{mi} = T_m(0) \quad (3)$$

Equation 2 is the equation for a ramp forcing function at a rate (k) in °F/sec., and T_i is the initial oil temperature.

In Equation 1, τ is the mold time constant, and $T_m(t)$ is the mold time/temperature response.

Combining Equations 1 and 2 and taking the Laplace transform:

$$\tau \dot{T}_m(t) + T_m(t) = kt + T_i \quad (3)$$

$$\tau [s T_m(s) - T_m(0)] + T_m(s) = K/s^2 + T_i/s \quad (4)$$

$$T_m(s) \cdot (s\tau + 1) = k/s^2 + T_i/s + \tau T_{mi} \quad (5)$$

$$T_m(s) = \frac{k/\tau}{s^2 (s+1/\tau)} + \frac{T_i/\tau}{s (s+1/\tau)} + \frac{T_{mi}}{(s+1/\tau)} \quad (6)$$

Multiplying each term on the right hand side of Eq. 6 by unity to achieve a common denominator yields:

$$T_m(s) = \frac{k/\tau}{s^2 (s+1/\tau)} + \frac{s}{s} \cdot \left(\frac{T_i/\tau}{s (s+1/\tau)} \right) + \frac{s^2}{s^2} \cdot \left(\frac{T_{mi}}{s+1/\tau} \right) \quad (7)$$

Collecting terms and rewriting Eq. 7:

$$T_m(s) = \frac{k/\tau + sT_i/\tau + s^2T_{mi}}{s^2 (s+1/\tau)} \quad (8)$$

Utilizing partial fraction expansion of Eq. 8:

$$T_m(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1/\tau} \quad (9)$$

Combining Eq. 8 and 9:

$$T_m(s) = \frac{k/\tau + sT_i/\tau + s^2T_{mi}}{s^2 (s+1/\tau)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1/\tau)} \quad (10)$$

Multiplying through Eq. 10 by s^2 will isolate A:

$$s^2 \cdot T_m(s) = \frac{k/\tau + sT_i/\tau + s^2T_{mi}}{(s+1/\tau)} = A + sB + \frac{s^2C}{s+1/\tau} \quad (11)$$

Evaluating Eq. 11 for $s = 0$ will yield the value of A:

$$\begin{aligned} \left[s^2 \cdot T_m(s) \right]_{s=0} &= \left[\frac{k/\tau + sT_i/\tau + s^2T_{mi}}{s + 1/\tau} \right]_{s=0} \\ &= \frac{k/\tau}{1/\tau} = k = A \end{aligned} \quad (12)$$

The quickest way of isolating the constant B is to take the derivative of Eq. 11 with respect to s:

$$\frac{d}{ds} \cdot \left[s^2 T_m(s) \right]_{s=0} = B \Big|_{s=0} \quad (13)$$

Therefore:

$$B = \frac{d}{ds} \left[\frac{k/\tau + sT_i/\tau + s^2 T_{mi}}{s+1/\tau} \right]_{s=0}$$

$$= \left[\frac{(T_i/\tau + 2sT_{mi}) \cdot (s+1/\tau) - (k/\tau + sT_i/\tau + s^2 T_{mi})}{(s+1/\tau)^2} \right]_{s=0}$$

therefore:

$$B = \frac{(T_i/\tau)(1/\tau) - (K/\tau)}{(1/\tau)^2} = T_i - k.$$

The quickest way to isolate C is to multiply through Eq. 10 by $(s+1/\tau)$:

$$\left[(s + 1/\tau) \cdot T_m(s) \right] = \frac{k/\tau + sT_i/\tau + s^2 T_{mi}}{s^2} = \frac{s+1/\tau}{s^2}$$

$$A + \frac{s+1/\tau}{s} B + C \quad (14)$$

Evaluating Eq. 14 at $s = -1/\tau$ yields the value of the constant C:

$$C = \left[(s+1/\tau) \cdot T_m(s) \right]_{s=-1/\tau} = \frac{k/\tau + sT_i/\tau + s^2 T_{mi}}{s^2} \quad s = -1/\tau$$

$$= \frac{k/\tau - T_i/\tau^2 + T_{mi}/\tau^2}{(1/\tau)^2}$$

therefore,

$$C = k - T_i + T_{mi}$$

Substituting the constants A, B, and C back into the right hand side of Eq. 10:

$$T_m(s) = \frac{k}{s^2} + \frac{(T_i - k\tau)}{s} + \frac{(k\tau - T_i + T_{mi})}{(s+1/\tau)} \quad (15)$$

Taking the inverse Laplace transform of Eq. 15 yields the time solution:

$$T_m(t) = kt + T_i - k\tau + (k\tau - T_i + T_{mi}) \epsilon^{-t/\tau} \quad (16)$$

Collecting terms and rewriting:

$$T_m(t) = kt + T_i (1 - \epsilon^{-t/\tau}) + k\tau (\epsilon^{-t/\tau} - 1) + T_{mi} \epsilon^{-t/\tau} \quad (17)$$

Equation 17 is the mold time/temperature response to a first order ramp forcing function.

Taking a heat balance across the mold, the heat given up by the oil equals the heat absorbed by the mold, plus losses:

Assuming no kinetic or potential differences, i.e. $V_{in} = V_o$ and $Z_{in} = Z_o$, the enthalpy balance is:

$$\dot{m} \cdot [h_{in}(t) - h_o(t)] = MCp \frac{dT_m(t)}{dt} \quad (18)$$

$\dot{m}$ = mass flow rate (lbm/sec)

$h_{in}(t)$ = inlet oil enthalpy (Btu/lbm)

$h_o(t)$ = outlet oil enthalpy (Btu/lbm)

M = mass of the mold metal (lbm)

Cp - specific heat of metal (Btu/lbm/°F)

$dT_m(t)/dt$ = time derivative of the mold metal temperature response in °F/sec

The mass flow rate is calculated as follows:

Density: ρ , (lbm/gal) = $-0.00463 \cdot (T_{in}(t) + T_o(t)/2) + 11.917$

$\dot{m} = \text{gpm} \cdot \rho / 160$ (lbm/sec)

$C_p = 0.11$ for tool steel

$$h_{in}(t) = 0.000103 \cdot T_i(t)^2 + 0.26287 \cdot T_i(t) = 8.686$$

$$T_{in}(t) = kt + T_i$$

Solving Eq. 18 for the outlet enthalpy:

$$h_o(t) = h_{in}(t) - \frac{MC_p}{\dot{m}} \frac{dT_m(t)}{dt} \quad (19)$$

Restating Eq. 17 and taking the derivative:

$$T_m(t) = kt + T_i \cdot (1 - \epsilon^{-t/\tau}) + k\tau \cdot (\epsilon^{-t/\tau} - 1) + T_{mi} \epsilon^{-t/\tau} \quad (17)$$

$$\frac{dT_m(t)}{dt} = k + \left(\frac{T_i}{\tau} \right) \epsilon^{-t/\tau} - k \epsilon^{-t/\tau} - \left(\frac{T_{mi}}{\tau} \right) \epsilon^{-t/\tau} \quad (20)$$

Collecting terms:

$$\frac{dT_m(t)}{dt} = k(1 - \epsilon^{-t/\tau}) + \frac{\epsilon^{-t/\tau}}{\tau} (T_i - T_{mi}) \quad (21)$$

Substituting Eq. 21 into Eq. 19:

$$h_o(t) = h_{in}(t) - \frac{MC_p}{\dot{m}} \cdot \left[k(1 - \epsilon^{-t/\tau}) + \frac{\epsilon^{-t/\tau}}{\tau} \cdot (T_i - T_{mi}) \right] \quad (22)$$

Solving the given enthalpy equation vs. temperature yields the outlet temperature as a function of the outlet enthalpy:

$$T_o(t) = -1276 + \left[\frac{\sqrt{0.0691 + [0.000412(8.686 + h_o(t))]} }{0.000206} \right] \quad (23)$$

With the inlet and outlet temperatures known, and the mass flow rate known, the loop gain may be formulated as follows on the next page.

$$\dot{m}_h T_h + \dot{m}_r T_o = \dot{m}_t T_m \quad (24)$$

$$\dot{m}_h T_h + \dot{m}_t T_o - \dot{m}_h T_o = \dot{m}_t T_m \quad (25)$$

$$T_m = \frac{\dot{m}_h}{\dot{m}_t} (T_h - T_o) + T_o \quad (26)$$

$$\text{Loop Gain} = \frac{dT_m}{d\dot{m}_h} \quad (27)$$

$$\frac{dT_m}{d\dot{m}_h} = \frac{T_h - T_o}{\dot{m}_t} \quad (27)$$

Where: $\dot{m}_r = \dot{m}_t - \dot{m}_h$

$$T_m = kt + T_i = T_{in}(t).$$

$$T_h = \text{Constant}$$

$$T_o = f(h_o) = T_o(t)$$

$$\dot{m}_t = \text{Total Mass Flow Rate}$$

Calculating the film coefficient ($\bar{h}$)

$$\bar{h}A (\hat{T}_O - T_m) = MCp \frac{dT_m(t)}{dt} \quad (28)$$

$$\bar{h} = \frac{MCp}{A} \cdot \frac{1}{\hat{T}_O - T_m} \cdot \frac{dT_m(t)}{dt} \quad (29)$$

Where:

$$\hat{T}_O = [T_{in}(t) + T_O(t)]/2$$

Rewriting Eq. 29 and substituting in Eq. 20:

$$\bar{h} = \frac{MCp}{A} \cdot \frac{1}{\hat{T}_O - T_m} \cdot \left[k(1 - \epsilon^{-t/\tau}) + \frac{\epsilon^{-t/\tau}}{\tau} \cdot (T_i - T_{mi}) \right] \quad (30)$$

A = heat transfer area in square feet.

Appendix F

Computer Program to Obtain the Mold Temperature
Response to a Ramp Forcing (First Order Response)

```

DIMENSION TIN(950),TM(950)
WRITE(1,('TI IS THE INITIAL OIL TEMP**GPM IS GPM**WM IS THE METAL'))
WRITE(1,('MASS**M IS THE INCREMENT SIZE OF TIME**AR IS THE'))
WRITE(1,('HEAT TRANSFER AREA IN FT*FT**T IS THE TIME CONSTANT'))
WRITE(1,('SL IS THE SLOPE IN F/SEC**TMI IS THE INITIAL'))
WRITE(1,('MOLD TEMP**TU IS THE UPPER OIL LIMIT OF THE'))
WRITE(1,('OIL LOOP NOT THE MAX SUPPLY TEMP'))
1 WRITE(1,('TI GPM WM M AR T SL TMI TU'))
ACCEPT T1,GPM,WM,M,AR,T,SL,TMI,TU
OMP=0
SLM=SL*60
TC=600
U=M
AP=.11
N=9*T
IF(N.GT.950)GO TO 4
GO TO 5
4 N=950
5 WRITE(1,(//))
WRITE(1,('METAL MASS='F9.3,4X'TIME CONSTANT='F6.1))WM,T
WRITE(1,('HTRAN AREA(FT*FT)='F8.3,4X'GPM='F5.0))AR,GPM
WRITE(1,('CENTRAL HOT SUPPLY TEMP='F5.0))TC
WRITE(1,('THE UPPER TEMP LIMIT IN LOOP='F5.0))TU
WRITE(1,('THE RAMP RATE IN DEG F/SEC='F8.4))SL
WRITE(1,('THE RAMP RATE IN DEG F/MIN='F8.3//))SLM
WRITE(1,('TIME'4X'MOLD'4X'OTEMP'3X'OTEMP'3X'LOOP'3X'FILM'7X'MASS'))
WRITE(1,(' SEC'4X'TEMP'6X'IN'5X'OUT'4X'GAIN'3X'COEFF.'5X'FLOW'))
WRITE(1,(56('*'))
DO 10 I=1,N,M
Q=1-1
QRS=SL*Q+T1
R=Q/T
L=Q-M
IF(QRS .GE. TU)GO TO 21
GO TO 32
21 TIN(I)=TU
IF(OMP.LT.1)GO TO 44
GO TO 46
44 B=CPU
WRITE(1,('SWITCH POINT B='F5.0'*****TIME IS='I4'SEC'))B,Q
46 OMP=OMP+1
Y=U/T
TM(I)=TU*(1-EXP(-Y))+B*EXP(-Y)
DEN=-.00463*TIN(I)+11.917
OF=GPM*DEN/60
FAT=WM*AP/OF
HI=.000103*TIN(I)**2+.26287*TIN(I)-8.686
DTM=(TU*EXP(-Y)-B*EXP(-Y))/T
U=U+M
GO TO 26
32 TIN(I)=SL*Q+T1
TM(I)=SL*Q+T1*(1-EXP(-R))+SL*T*(EXP(-R)-1)+TMI*EXP(-R)
DTM=SL*(1-EXP(-R))+((T1-TMI)*(EXP(-R)/T))
DEN=-.00463*TIN(I)+11.917

```

(Continued on next page)

```

ØF=GPM*DEN/60
FAT=WM*AP/ØF
HI=.000103*TIN(1)**2+.26287*TIF(1)-8.686
CPU=TM(1)
26 HØ=HI-(FAT*DTM)
AH=(.0691+(.000412*(8.686+HØ)))
AT=SQRT(AH)
TØ=(-1276+(AT/.000206))
GP1=TU-TØ
IF(GP1.LE.0)GØ TØ 25
TØH=(TØ+TIN(1))/2
WE=WM*AP/AR
DMT=1/(TØH-TM(1))
FIL=WE*DTM*DMT*3600
GA=(TC-TØ)/ØF
WRITE(1,(I4,4(3XF5.0),3XF6.0,3XF7.3))Q,TM(1),TIN(1),TØ,GA,FIL,ØF
10 CONTINUE
25 WRITE(1,(56('*'),////////))
GØ TØ 1
STOP
END
*
*
*

```

+G
 TI IS THE INITIAL OIL TEMP**GPM IS GPM**WM IS THE METAL
 MASS**M IS THE INCREMENT SIZE OF TIME**AR IS THE
 HEAT TRANSFER AREA IN FT*FT***T IS THE TIME CONSTANT
 SL IS THE SLOPE IN F/SEC**TMI IS THE INITIAL
 MOLD TEMP**TU IS THE UPPER OIL LIMIT OF THE
 OIL LOOP NOT THE MAX SUPPLY TEMP
 TI GPM WM M AR T SL TMI TU
 100 160 10450 30 54.71 13.8 2 100 450

METAL MASS=10450.000 TIME CONSTANT= 138.0
 HTRAN AREA(FT*FT)= 54.710 GPM= 160.
 CENTRAL HOT SUPPLY TEMP= 600.
 THE UPPER TEMP LIMIT IN LOOP= 450.
 THE RAMP RATE IN DEG F/SEC= 2.0000
 THE RAMP RATE IN DEG F/MIN= 120.000

TIME SEC	MOLD TEMP	ØTEMP IN	ØTEMP OUT	LOOP GAIN	FILM COEFF.	MASS FLOW
0	100.	100.	100.	16.	0.	30.544
30	106.	160.	108.	17.	1055.	29.803
60	123.	220.	127.	16.	1053.	29.062
90	148.	280.	154.	16.	1050.	28.322
120	180.	340.	187.	15.	1046.	27.581
150	217.	400.	227.	14.	1042.	26.840

(Continued on next page)

SWITCH POINT B= 217.*****TIME IS= 180SEC

180	263.	450.	274.	12.	1035.	26.223
210	299.	450.	310.	11.	1025.	26.223
240	329.	450.	338.	10.	1018.	26.223
270	352.	450.	361.	9.	1012.	26.223
300	371.	450.	378.	8.	1007.	26.223
330	387.	450.	393.	8.	1003.	26.223
360	399.	450.	404.	7.	1000.	26.223
390	409.	450.	413.	7.	998.	26.223
420	417.	450.	420.	7.	996.	26.223
450	424.	450.	426.	7.	994.	26.223
480	429.	450.	431.	6.	992.	26.223
510	433.	450.	435.	6.	991.	26.223
540	436.	450.	438.	6.	989.	26.223
570	439.	450.	440.	6.	988.	26.223
600	441.	450.	442.	6.	986.	26.223
630	443.	450.	444.	6.	984.	26.223
660	444.	450.	445.	6.	982.	26.223
690	445.	450.	446.	6.	979.	26.223
720	446.	450.	447.	6.	976.	26.223
750	447.	450.	447.	6.	973.	26.223
780	448.	450.	448.	6.	968.	26.223
810	448.	450.	448.	6.	963.	26.223
840	448.	450.	449.	6.	956.	26.223
870	449.	450.	449.	6.	948.	26.223
900	449.	450.	449.	6.	938.	26.223
930	449.	450.	449.	6.	926.	26.223

TI GPM WM M AR T SL TMI TU
(SMAINS)1+1

+

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