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SOME ESTIMATES OF MIRROR PLASMA STARTUP BY NEUTRAL
BEAM HEATING OF PELLET AND GAS CLOUD TARGETS

James W. Shearer and Peter A. Willmann

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SOME ESTIMATES OF MIRROR PLASMA STARTUP BY
NEUTRAL BEAM HEATING OF PELLETS AND GAS CLOUD TARGETS

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ABSTRACT

Hot plasma buildup by neutral beam injection into an initially cold solid or gaseous target is found to be conceivable in large mirror machine experiments such as 2XIIB or MFTF. A simple analysis shows that existing neutral beam intensities are sufficient to ablate suitable targets to form a gas or vapor cloud. An approximate rate equation model is used to follow the subsequent processes of ionization, heating, and hot plasma formation. Solutions of these rate equations are obtained by means of the "GEAR" techniques for solving "stiff" systems of differential equations. These solutions are in rough agreement with the 2XIIB stream plasma buildup experiment. They also predict that buildup on a suitable nitrogen-like target will occur in the MFTF geometry. In 2XIIB the solutions are marginal; buildup may be possible, but is not certain.

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1. Introduction

In recent years 20-40 kV beams of neutralized hydrogen and deuterium atoms have been created at equivalent current levels of 20-50 amperes(1). The twelve neutral beams that are used on the 2XIIB experiment(2) are so intense that it was found possible to build up a hot plasma by direct injection of 310 amperes (total equivalent current) into a warm streaming plasma(3).

These results raise the question whether hot plasma buildup may be possible by direct neutral beam injection into a neutral gas or pellet target. Previous unpublished assessments of this possibility were not hopeful;(4) however, the estimates to be described here are more encouraging. If these results are confirmed by experiment, then considerable simplification of mirror plasma experimental hardware is possible. No laser system would be needed, with its demanding focusing and alignment requirements.(5) The required number of streaming guns would be reduced to the smaller number needed for plasma stabilization.(2)

The production of the mirror plasma from a pellet or gas target is modeled in three successive stages. In the first stage the solid pellet target is vaporized by the beam to form a gaseous cloud. (Alternatively, a gas cloud could be directly injected.) Secondly, the gaseous vapor must be sufficiently well ionized by the neutral beams so that an appreciable fraction is trapped in the magnetic mirror field before it escapes to the walls or the vacuum pumps. Finally, the hot mirror plasma must be built up by trapping from the ionized target in the same way as the streaming plasma experiment.(3)

The following pages describe the model and the results in detail. The estimates are admittedly crude, and could be improved in the future, if desired.

2. Ablation of Plastic Pellet

As a "first cut", the hydrogen, carbon, and oxygen content of a common plastic or/ammonia pellet is modeled by assuming that it is equivalent to a nitrogen sphere of diameter D(cm):

$$D = (6NA M_H / \pi \rho)^{1/3}$$

where ρ is the density (gm/cc), M_H is the mass of hydrogen, A is the atomic weight, and N is the total number of atoms in the target. Nitrogen is

chosen because its beam interaction cross-sections are similar to those of carbon and oxygen but are better known.⁽⁷⁾ The hydrogen content of the target is neglected in comparison to the heavier components because its beam interaction cross-sections are smaller.⁽⁷⁾

The size of the pellet is estimated by assuming that the number of target atoms N should be of the order of magnitude of the number of final hot plasma atoms, N_h . For the MFTF facility one estimates:

$$N_h = n_h V_h \approx 10^{14} \times 2.5 \times 10^5 = 2.5 \times 10^{19} \quad (2)$$

where n_h is the hot atom density and V_h is the hot plasma volume. No optimization studies of the target atom quantity N have yet been done. Assuming that N is somewhat larger ($N = 4 \times 10^{19}$), equation (1) is used to find:

$$D = 1.2/\rho^{1/3} \quad (3)$$

for a single pellet in the MFTF roughly corresponding to a one millimeter diameter plastic sphere.

The pellet target does not have to be made of hydrogen because the hot plasma is formed from the neutral beams, as described in the introduction. The heavier target atoms ought to stream out of the mirrors at later times, thus purifying the resultant plasma.

The areas of each neutral beam at the position of the MFTF target plasma is approximately $S = 1000 \text{ cm}^2$,⁽⁶⁾ corresponding to an average beam intensity I_B for each module of:

$$I_B = E_V i / S \quad (4)$$

where E_V is the voltage and i is the current. The MFTF proposal lists 24 modules at $E_V = 20 \text{ kV}$, $i = 50 \text{ amp}$, plus another 24 modules at $E_V = 80 \text{ kV}$, $i = 32 \text{ amp}$.⁽⁶⁾ Assuming that 20 of each type are in operation and averaging over the spherical pellet surface area, one obtains an averaged intensity I at the surface of:

$$\left. \begin{aligned} I &= \left(\frac{\pi}{4\pi} \right) \frac{20}{1000} \left[(2 \times 10^4)(50) + (8 \times 10^4)(32) \right] \text{ w/cm}^2 \\ &= 18. \text{ kw/cm}^2 \end{aligned} \right\} \quad (5)$$

This energy flux is equivalent to a black body temperature of approximately 8,000°K; it will melt and ablate the surface of the pellet. The solid radius r will shrink at the rate:

$$\frac{dr}{dt} = - \bar{I} / \rho H_V \approx \frac{4.3}{\rho} \frac{\text{cm}}{\text{sec.}} \quad (6)$$

where H_V is the vaporization energy ($H_V \approx 1$ kcal/gm for plastics or ammonia). These results are intuitively reasonable when one recalls that individual neutral beam dumps have to be built of water-cooled high conductivity copper to avoid unwanted melting and vaporization phenomena.

The pellet is ablated layer by layer because both the thermal conduction depth and the neutral beam particle range are small compared to the dimensions. The thermal diffusivity D at the pellet surface is:

$$D \equiv \frac{\partial T}{\partial t} / \frac{\partial^2 T}{\partial x^2} = \frac{k}{C_H \rho} \approx \delta_R \left| \frac{dr}{dt} \right| \quad (7)$$

where k is the thermal conductivity, C_H is the specific heat, and δ_R is an approximate measure of the depth of heating by thermal conduction. For a plastic, suitable values are $k \approx 3 \times 10^{-4}$ cal/cm - °k and $C_H = 0.3$ cal/gm - °k. Thus, from equation (6)-(7) one finds $\delta = 2.3 \times 10^{-4}$ cm, which is much smaller than the initial pellet radius. The range R_a of the neutral particles is of a similar order of magnitude; from well-known tabulations(7):

$$R_a = (1.2 - 2.8) \times 10^{-4} / \rho \quad (8)$$

for 20-80 keV deuterons in carbon. Similar results would be obtained for ammonia.

Next one finds the time τ_A for the $N = 10^{19}$ pellet to completely vaporize by this ablation process in the MFTF neutral beams.

$$\tau_A = \left(\frac{N}{2} \right) / \left| \frac{dr}{dt} \right| = .0088 \rho^{2/3} \text{ sec.} \quad (9)$$

where equations (3) and (6) have been used. Thus, this single solid pellet would take almost 10 milliseconds to vaporize. This might be too long a time, as will become more obvious later in this report. However, there are several possible ways to speed up the ablation process. The first (already implied in equation (9)) is to use a low density, foamed plastic target.

For example, if $\rho = .125$ g/cc, then $\tau_A \approx 2.2$ ms and the diameter is about 1.5 mm. Secondly, non-spherical shapes, which have a greater surface/volume ratio, can be used, such as threads or foils. Thirdly, a cloud of smaller pellets would not only speed up the ablation process but distribute the resultant gaseous vapor more uniformly in the plasma volume.

Although there are obvious analogies with the laser-pellet injection method(5), one important difference should be kept firmly in mind. The "focal volume" of the neutral beam is orders of magnitude larger than the pellet target and this has important advantages. Pellet projection and alignment apparatus can be much simpler and cheaper. The expanding vapor cloud intercepts more and more energy because the neutral beam energy flux is approximately constant and isotropic; therefore, the beams continue to heat the target over a wide range of densities.

The subsequent history of the ablated vapor cloud is discussed in the following sections.

3. Heating and Ionization of Gas or Vapor Cloud

As the vapor cloud expands away from the pellet, it intercepts more and more of the neutral beam flux, which heats it and eventually ionizes it. Important length parameters for the MFTF case are plotted versus target density in figure 1, which has been calculated for a nitrogen vapor cloud containing 4×10^{19} atoms expanding in a 20 kG magnetic field while being irradiated with 20 keV deuterons.

The diameter is estimated assuming a uniform density cloud; from equation (1):

$$D = \left(\frac{6N}{\pi n} \right)^{1/3} = 4.25 \times 10^6 / n^{1/3} \quad (10)$$

The range R_a for a 20 keV deuteron in nitrogen is found from equation (8):

$$R_a = 5 \times 10^{18} / n \quad (11)$$

From figure 1 one finds that for high densities ($n > 10^{18}$), the range is shorter than the cloud diameter. At lower densities the 20 keV beam range is greater than the cloud diameter; if there were no magnetic field, the beam would penetrate the cloud and converge on the opposite side. The incident beam is a neutral particle (D^0) beam, but inside the target cloud it is an equilibrium mixture of neutrals D^0 and ions D^+ , just as it is when it emerges from the neutralizer of a neutral beam source module.(1)

Figure 1

Various lengths versus target cloud density for a typical case.

The equilibrium fractions are determined from the reaction cross-section ratios.⁽⁷⁾ The mean free path (MFP) for the neutral deuterons incident on the cold nitrogen atoms is much smaller than the range, is also plotted in figure 1:

$$MFP = 1/\sigma n \quad (12b)$$

Another important parameter is the gyroradius ρ_i of the 20 keV deuteron in the 20 kG magnetic field:

$$\rho_i = \frac{c}{eB} (2A M_H E_B)^{1/2} = 1.4 \text{ cm} \quad (13)$$

At medium densities ($n < 10^{19}$), the charged D^+ particles will move in curved orbits; at lower densities they will become trapped. This will lengthen the effective range of the neutral beam in the target.

The last parameter shown in figure 1 is the mean free path for beam ionization on singly-charged nitrogen target atoms.

This is a larger cross-section and shorter mean free path than for the neutral nitrogen target. It will become important when the target cloud heats up sufficiently to become ionized. In addition to direct beam-induced ionization, thermal ionization can be important. The ions so produced will be contained by the mirror magnetic field for a longer time than the neutrals. Thus, one finds that target cloud ionization is desirable both for higher beam interaction cross-sections and for better containment.

There seems to be no way to extend this simple analysis beyond this point. The complexity of the interactions among neutral beam atoms, hot ions, target atoms, target ions, and electrons requires the development of a calculational model, which will be described in the next sections. The overall question which the model seeks to answer is: will a hot plasma be formed in time -- before the target comes apart?

4. Rate Equation Numerical Model

The numerical scheme is a "zero-dimensional" code that seeks to calculate the time evolution of seven quantities: densities of beam neutrals, hot ions, target neutrals, and target ions; temperatures of electrons and target ions (plus neutrals); and target cloud diameter. As many physical effects as possible were modeled, even though crudely approximated. The chief defect of the model is probably the neglect of multiple ionization states of the target atoms; this will be mentioned again later.

Seven first order non-linear equations are written down. They are then time-integrated by means of the LLL "GEAR" package,⁽⁸⁾ which is a calculation technique especially developed for "stiff" systems of differential equations. In the following seven subsections each differential equation will be described in turn.

a) Beam Neutrals

$$\frac{dn_B}{dt} = \alpha_J - \alpha_{BL} - \alpha_{BH} \quad (a1)$$

where n_B is the neutral beam density (units of 10^{15} cm^{-3}) and t is the time (sec) α_J is the input rate from the source modules:

$$\alpha_J = \frac{J}{e} \frac{A'}{V} \quad (a2)$$

where e is the electron charge, J is the neutral beam "current" density (in amperes of an equivalent flux of singly charged ions), A' is the projected area of the gas target and V is its volume. For a sphere of diameter D (cm):

$$\frac{A'}{V} = \frac{\pi D^2}{4} \frac{6}{\pi D^3} = \frac{3}{2D} \quad (a3)$$

This must be corrected for the situation at late times when the sphere diameter D has expanded beyond the envelope of the neutral beams. This is done by introducing an input constant D_C which is a measure of the plasma diameter in a given mirror machine. Then the effective diameter D_L is approximated by:

$$\frac{1}{D_L} = \frac{1}{D} + \frac{1}{D_C} \quad (a4)$$

The current density J is also an input constant, based on the average current density per beam module multiplied by the number of modules. If J is in amperes/cm², then equation (a2) becomes:

$$\alpha_J = 7968 (J/D_L) \quad (a5)$$

for the corrected value of α_J .

α_{BL} is the escape rate of neutral beam particles from the target cloud. It is similar in structure to the beam input term α_J because the neutrals travel in straight lines. Even those neutrals which are created by charge-exchange from the hot ions are still assumed to have no velocity component parallel to the magnetic field. For a cylinder:

$$\frac{A'}{V} = ZD \frac{4}{\pi D^2 L} = \frac{4}{\pi D} \quad (a6)$$

Thus then the loss rate of neutrals, α_{BL} is:

$$\alpha_{BL} = v_B n_B (A'/V) = 4 v_B n_B / \pi D_L \quad (a7)$$

The difference between the cylindrical model used here and the spherical one used above (equation a3) is an uncorrected discrepancy of the code; the error is small compared to the overall crudeness of the computation.

The neutral atom velocity v_B (cm/sec) is:

$$v_B = \sqrt{2E_B/M_B} = 1.38 \times 10^6 \sqrt{E_B/A_B} \quad (a8)$$

where E_B is the average energy (ev) of neutral atoms and hot ions, and A_B is their mass number. These are input constants; E_B is usually estimated to be half of the maximum voltage E_V of the neutral beam modules, as a rough measure of the effects of the fractional energy components of the beam, electron drag, charge exchange effects, ion-ion scattering, and plasma instabilities. One can also estimate it from experiment; for example, $E_B \approx 10^4$ eV in 2XIIIB.

Finally the parameter α_{BH} is the net rate at which neutrals are converted to hot ions. The equation for α_{BH} is:

$$\begin{aligned}
 \alpha_{BH} = & + v_B n_B n_O \sigma_{BO} \\
 & + v_B n_B n_i (\sigma_{Bi} + \sigma_{XB i}) \\
 & + v_B n_B n_H \sigma_{BH} \\
 & - v_B n_H n_O \sigma_{XHO}
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} \alpha_{BH} = \\ + v_B n_B n_O \sigma_{BO} \\ + v_B n_B n_i (\sigma_{Bi} + \sigma_{XB i}) \\ + v_B n_B n_H \sigma_{BH} \\ - v_B n_H n_O \sigma_{XHO} \end{aligned}} \right\} \quad (a9)$$

where n_h is the hot ion density, n_o is the target atom neutral density, and n_i is the target atom ion density (all densities in units of 10^{15} cm^{-3}). The reaction cross-sections (units of 10^{-15} cm^2) are:

$$\sigma_{BO}: \quad D^0 + N^0 \rightarrow D^+ + N^- + e^- \quad (a10)$$

$$\sigma_{Bi}: \quad D^0 + N^+ \rightarrow D^+ + N^+ + e^- \quad (a11)$$

$$\sigma_{XB i}: \quad D^0 + N^+ \rightarrow D^+ + N^0 \quad (a12)$$

$$\sigma_{BH}: \quad D^0 + D^+ \rightarrow 2D^+ + e^- \quad (a13)$$

$$\sigma_{XHO}: \quad D^+ + N^0 \rightarrow D^0 + N^+ \quad (a14)$$

where deuterium (D) is used as an example for the neutral beams and hot ions, and where nitrogen (N) is used as an example for the target atoms and ions. The hot ion charge exchange reaction:

$$\sigma_{XBH}: \quad D^+ + D^0 \rightarrow D^0 + D^+ \quad (a15)$$

is not included because it does not change the inventory of n_b or n_h , and because we are assuming the same average energy E_B for both populations. The energy consequences of this reaction are considered later.

It should be pointed out that the net value of α_{BH} may be much smaller than some of the individual terms, especially for a neutral beam traversing a dense target plasma. This makes the system of equations

"stiff", and is the reason that the "GEAR" package⁽⁸⁾ is required for stable calculation of the solution.

b) Hot Ions

$$\frac{dn_H}{dt} = -\alpha_{HE} - \alpha_{HL} + \alpha_{BH} \quad (b1)$$

where n_H is the hot ion density (in units of 10^{15} cm^{-3}). The first term α_{HE} of this rate equation is the drag term:

$$\alpha_{HE} = \alpha_{HD} e^{-\alpha_{QX}/\alpha_{HD}} \quad (b2)$$

where the drag rate α_{HD} is given by:

$$\alpha_{HD} = n_H \left(\frac{1}{\tau_{HE}} + \frac{1}{\tau_{Hi}} \right) \quad (b3)$$

where τ_{HE} and τ_{Hi} are the respective time constants (in sec) for electron drag and cold ion drag on the hot ions. The exponential factor in equation (b2) is described later in this section.

The time constants τ_{HE} and τ_{Hi} are obtained from the "equilibration time" τ_{eg} given on page 135 of ref. 9. In this code they are:

$$\tau_{He} = \frac{3.16}{10^8} \frac{A_B}{n_e} \left(T_e + \frac{E_B}{2754 A_B} \right)^{3/2} \quad (b4)$$

$$\tau_{Hi} = \frac{3.16}{10^8} \frac{A_T}{n_i} \left(\frac{T_i}{1836 A_T} + \frac{E_B}{2754 A_B} \right)^{3/2} \quad (b5)$$

where A_T is the atomic number of the target atoms, T_e is the electron temperature (eV), T_i is the target ion temperature (eV), and where n_e is the electron density (units of 10^{15} cm^{-3}):

$$n_e = n_H + n_i \quad (b6)$$

Strictly speaking, these drag time constants are energy exchange parameters rather than particle loss constants. They are being used here as an approximate measure of the particle loss time.

An important correction to this loss rate is the exponential factor in equation (b2). Physically, it represents the reduction in the drag constant due to charge exchange, where a lower energy hot ion is replaced by a new,

full energy beam ion.(3) The charge exchange rate α_{QX} is:

$$\alpha_{QX} = v_B n_B n_H \sigma_{XBH} \quad (b7)$$

where σ_{XBH} corresponds to the charge exchange reaction of equation (a15).

The second term of the hot ion rate equation (b1) is the escape rate α_{HL} of hot ions from the target cloud.

$$\alpha_{HL} = \left(\frac{4}{\pi} \frac{v_B n_H}{D_L} \right) \left(e^{-(D_L/\rho_H)^2} \right) \left(\frac{1}{1-\beta^2} \right) \quad (b8)$$

where the first parenthesis is obtained from the geometry in the same manner as equation (a7). The second parenthesis is an ad hoc correction which reduces this loss rate in the important case where the cloud diameter D_L (equation a4) is larger than the hot ion gyroradius ρ_H . Thus, magnetic containment of the hot ions is approximately accounted for.

The gyroradius ρ_H (cm) is an input constant; the magnetic field is not explicitly given. This field, of course, is well-determined and can be computed from the input constants E_B and ρ_H , if desired. In this model there is no finite beta correction to ρ_H .

The third term of equation (b8) is an ad hoc "beta limit", which prevents the code from generating a total plasma pressure which exceeds the magnetic pressure

$$\beta \equiv 8\pi(\sum n k T)/B^2 \quad (b9)$$

As $\beta \rightarrow 1$, the hot ion loss rate becomes very large. For the parameters used in this code, one has:

$$\beta = \frac{1.28\pi}{E_b} \left(\frac{\rho_H}{E_B} \right)^2 \left[(2/3)n_H E_B + n_i T_i + n_e T_e \right] \quad (b10)$$

The third term in equation (b1) is the net increase of hot ions from the beam; it was previously discussed in equations (a9 - a14).

c) Target Neutral Atoms

$$\frac{dn_0}{dt} = -\alpha_{0i} - \alpha_{0D} + \alpha_{0G} \quad (c1)$$

where n_0 is the target neutral atom density (in units of 10^{15} cm^{-3}).

The first rate term, α_{0i} , is the net production rate of target ions from target neutrals; it is the sum of several terms due to various effects:

$$\begin{aligned}
 \alpha_{0i} = & \alpha_X - \alpha_R - \alpha_Z \\
 & - \nu_B n_B n_i \sigma_{XB i} \\
 & + \nu_B n_H n_O \sigma_{XHO} \\
 & + \nu_B n_B n_O \sigma_{TB} \\
 & + \nu_B n_H n_O \sigma_{TH}
 \end{aligned} \tag{c2}$$

The last four terms are beam reactions: the charge exchange reactions (a12) and (a14), and the following target ionization reactions:

$$\alpha_{TB}: \quad D^0 + N^0 \rightarrow N^+ + D^0 + e^- \tag{c3}$$

$$\alpha_{TH}: \quad D^+ + N^0 \rightarrow N^+ + D^+ + e^- \tag{c4}$$

where the notation is similar to equation (a10) - (a15).

The first three terms of equation (c2) are target reactions which are independent of the beam. Approximate expressions are used, taken from reference 10, and adapted to the units of this code. The thermal ionization rate α_X is:

$$\alpha_X = 2.5 \times 10^9 \left(\frac{n_e n_0}{T_e^{3/2}} \right) \left(\frac{n e^{-X}}{X(X+1)} \right) \tag{c5}$$

where n is an input constant designating the number of electrons in the outermost electron shell of the target neutral atom, and where the parameter X is:

$$X = X/T_e \tag{c6}$$

where X is the first ionization potential (eV) of the target neutral atom.

The radiative recombination term α_R is:

$$\alpha_R = 52 n_e n_i x^{1/2} \left[.429 + \frac{.469}{x^{1/3}} + \frac{1}{2} \ln x \right] \quad (c7)$$

and the three-body recombination term α_Z is:

$$\alpha_Z = .14 n_e^2 n_i x^2 e^{x/4} \quad (c8)$$

The approximation of equations (c5)-(c8) are only applicable to neutrals and singly-ionized species. Multiple ionization of the target cloud is not considered.

The second term of equation (c1) is the geometric loss term α_{OD} :

$$\alpha_{OD} = 2 v_A n_0 \left(\frac{1}{Z_L} + \frac{2}{D_L} \right) \quad (c9)$$

where D_L is given by equation (a4) and where the length parameter Z_L is:

$$\frac{1}{Z_L} = \frac{1}{D} + \frac{1}{Z_C} \quad (c10)$$

where the length Z_L is an input constant which is characteristic of the plasma length in a given mirror machine. This geometric loss term differs from the previous ones (eq. a7 and b8) because the velocity distribution of the target neutral atoms is isotropic and independent of the magnetic field. Thus the loss is directly proportional to the acoustic velocity v_A :

$$v_A = \left(\frac{1.26 \times 10^6}{\Lambda_T^{1/2}} \right) \left(T_i + \frac{n_e T_e}{(n_0 + n_i)(1 + f)} \right)^{1/2} \quad (c11)$$

where the temperature T_i is assumed to be equal for both target ions and target neutral atoms. The electron temperature contribution to the acoustic velocity is important only when the parameter f is small (collisional case)

$$\tau_{ee} \omega_e \ll f = \frac{.0584 A_B E_B^{1/2} T_e^{3/2}}{\rho_H n_e} \quad (c12)$$

where τ_{ee} is the electron self-collision time, and where ω_e is the electron gyrofrequency. Thus, when f is large, the electrons are assumed to be tied to the field lines, so they would not contribute to the target plasma pressure or the acoustic velocity.

The third term of equation (c1) is a target gas input term, intended to mock up the addition of neutral target gases from such sources as the "gas box". Its equation is:

$$\alpha_{0G} = 2v_G n_G / Z_L \quad (c13)$$

where the gas velocity v_G is:

$$v_G = 1.2 \times 10^6 (T_G / A_T)^{1/2} \quad (c14)$$

The temperature T_G and the density n_G of the incoming gas are input constants. The geometry factor $2/Z_L$ is similar to loss factors such as equation (c9).

d) Target Ions

$$\frac{dn_i}{dt} = \alpha_{0i} - \alpha_{iD} - \alpha_{iE} + \alpha_{is} \quad (d1)$$

where n_i is the target ion density (in units of 10^{15} cm^{-3}). The first term, α_{0i} , was described in equations (c2)-(c8); it is the net rate of production of target ions from target neutrals.

The second term, α_{iD} , is the geometric loss term in the radial direction for target ions:

$$\alpha_{iD} = \frac{4v_A n_i}{D(1-\beta^2)} \quad (d2)$$

This loss is most important for dense targets; it is taken approximately into account by using the uncorrected target diameter D (equation a3) and the "beta factor" from equations (b9)-(b10) in the denominator. α_{iD} is only important when these terms are small - that is, when the target is dense or when the overall plasma pressure is large.

The third term, α_{iE} , is the geometric loss term in the axial direction for target ions.

$$\alpha_{iE} = \frac{2v_A n_i}{Z_L(1-\beta^2)} \quad (d3)$$

This may be an overestimate because it neglects the containment effect of the magnetic mirror; however, the ambipolar potential is probably more important than the magnetic mirror for the low energy target ions, so the geometric loss is used.

The fourth term, α_{is} is a target plasma input term. If the target plasma is specified to be deuterium, then this term mocks up the effects of adding a plasma stream. Its equation is:

$$\alpha_{is} = 2 v_s n_s / Z_L \quad (d4)$$

where the stream velocity v_s is:

$$v_s = 1.2 \times 10^6 (T_s / \Lambda_T)^{1/2} \quad (d5)$$

The temperature T_s and the density n_s of the incoming stream are input constants. The treatment of this stream input (equations (d4)-(d5)) parallels the equations used for the gas input (equations (c13)-(c14)).

e. Cloud Diameter

$$\frac{dD}{dt} = 2 v_A \quad (e1)$$

where D is the diameter of the target cloud and v_A is the acoustic velocity (equation (c11)). In this equation the cloud is assumed to expand unimpeded by magnetic fields or walls. Corrections for the finite size of the experiment are made, where necessary, by means of the parameters of equations (a4) and (c10). These are all admittedly crude approximations to the real magnetohydrodynamic problem.

f. Target Ion Temperature

$$\frac{dT_i}{dt} = \beta_{ie} - \beta_{iL} + \beta_{is} + \beta_{i0} \quad (f1)$$

where T_i is the temperature of the target ions in electron-volts (eV). The target neutral atom temperature is also assumed to be equal to the ion temperature T_i .

The first term of equation (f1), represents the heating of the target ions by Coulomb collisions with the electrons and the hot ions:

$$\beta_{ie} = \left(\frac{1}{n_0 + n_i} \right) \left[\frac{n_e (T_e - T_i)}{T_{ie}} + \frac{n_h (T_h - T_i)}{T_{hi}} \right] \quad (f2)$$

where τ_{Hi} is given by equation (b5), and where the electron-target ion time constant τ_{ie} is a similar equation:

$$\tau_{ie} = \frac{3.16}{10^8} \frac{A_T}{n_e} \left(T_e + \frac{T_i}{1836 A_T} \right)^{3/2} \quad (f3)$$

The second term of equation (f1) is the cooling term due to adiabatic expansion of the target cloud:

$$\beta_{iE} = \frac{2}{3} \frac{T_i}{n_0 n_i} (\alpha_{0D} + \alpha_{iD} + \alpha_{iE}) \quad (f4)$$

where the expansion rates are found from equations (c9), (d2), and (d3).

The third term of equation (f1) represents heating (or cooling) by the plasma stream:

$$\beta_{is} = \left(\frac{T_s - T_i}{n_0 + n_i} \right) \alpha_{is} \quad (f5)$$

where α_{is} is given by equation (d4). Similarly, the last term of equation (f1) mocks up the heating (or cooling) by the "gas box" input term:

$$\beta_{0G} = \left(\frac{T_G - T_i}{n_0 + n_i} \right) \alpha_{0G} \quad (f6)$$

where α_{0G} is given by equation (c13).

The target ion temperature parameter varied only rather slowly in all of the code runs, and not much attention has been paid to it.

g. Electron Temperature:

This important parameter is believed to be only a rough approximation, despite the large number of terms in its rate equation.

$$\frac{dT_e}{dt} = \beta_{eD} - \beta_{eX} - \beta_{eR} + \beta_{eZ} + \beta_{eB} - \beta_{ee} + \beta_{es} - \beta_{sec} \quad (g1)$$

The first term of equation (g1) is the electron heating from the drag on the ions - especially the hot ions.

$$\beta_{eD} = \frac{1}{n_e} \left[\frac{n_i(T_i - T_e)}{\tau_{ie}} + \frac{n_H(\frac{2}{3} E_B - T_e)}{\tau_{He}} \right] \quad (g2)$$

where the time constants τ_{ie} and τ_{He} are given by equations (f3) and (b4), respectively.

The second term of equation (g1) is the electron temperature loss due to the ionization process:

$$\beta_{eX} = \left(\frac{\chi}{n_e} \right) \alpha_X \quad (g3)$$

where α_X is given by equation (c5), and χ is defined after equation (c6). The third term of equation (g1) is the electron temperature loss due to radiative recombination:

$$\beta_{eR} = \left(\frac{T_e}{n_e} \right) \alpha_R \quad (g4)$$

where α_R is given by equation (c7). The fourth term of equation (g1) is the energy gain from three-body recombination, where the extra electron receives the energy of recombination:

$$\beta_{eZ} = \left(\frac{\chi + T_e}{n_e} \right) \alpha_Z \quad (g5)$$

where α_Z is given by equation (c8).

The fifth term of equation (g1) models the heating of the electrons in a neutral gas by the absorption of energy from fast neutrals and hot ions. It is written,

$$\beta_{eB} = \frac{36 - \chi}{n_e} \alpha_{Ti} \quad (g6)$$

where the rate parameter α_{Ti} is given by:

$$\alpha_{Ti} = v_b n_0 [n_B \sigma_{TB} + n_H \sigma_{TH}] \quad (g7)$$

where σ_{TB} and σ_{TH} are the target ionization cross-sections given by equations (c3) and (c4). The constant 36 eV used in equation (g6) is based on the known experimental fact that approximately 36 eV per ionization is the energy loss rate for high energy electron and ion beams in a wide variety of neutral gases. (11)

The sixth term of equation (g1) is the cooling term due to adiabatic expansion:

$$\beta_{eE} = \left(\frac{4}{3} \right) \left(\frac{v_a T_e}{D} \right) \quad (g8)$$

where v_A is the acoustic velocity (equation (c11)). This term is large only at early times when the cloud diameter D is small.

The seventh term of equation (g1) models the heating (or cooling) by the electrons of the plasma stream:

$$\beta_{es} = \left(\frac{T_s - T_e}{n_e} \right) \alpha_{is} \quad (g9)$$

where the plasma stream is assumed to be a singly ionized plasma, as before. No electron heating term is needed for the "gas box" flow because it is not ionized.

The eighth term of equation (g1) is an attempt to model electron energy losses to the ends of the mirror machine. It is based on the assumption that T_e is independent of geometry up to the plasma sheath at the end wall. A simple derivation(12) has been made to show that the energy flux E_W to the end wall is given by:

$$E_W = f_i T_e [\eta_{MB} + (1 + \gamma_i)/(1 - \gamma_e)] \quad (g10)$$

where f_i is the ion flux, and where η_{MB} is the ratio of ion energy to electron energy at the wall [$\eta > 1$, due to acceleration of the ion by the ambipolar potential]. The coefficients γ_i and γ_e are the secondary emission coefficients for the ions and electrons, respectively. For this model linear fits are made for these coefficients:

$$\gamma_i = \eta_{MB} T_e / V_{SIN} \quad (g11)$$

$$\gamma_e = T_e / V_{SEE} \quad (g12)$$

where η_{MB} , V_{SIN} , and V_{SEE} are input constants. Then the eighth term of equation (g1) is:

$$\beta_{SEC} = \left(\frac{T_e}{n_e} \right) (\alpha_{iE} + \alpha_{iD}) \left(\eta_{MB} + \frac{1 + \eta_{MB} T_e / V_{SIN}}{1 - T_e / V_{SEE}} \right) \quad (g13)$$

It might seem odd that the ion energy loss term η_{MB} should be included in this expression for electron energy loss. It should be remembered, however, that the ion energy at the end wall is (to a large extent) derived from

acceleration by the ambipolar potential, whose energy is supplied by the electrons.

5. Comparison of Rate Model with 2XIIB Buildup Experiment

The model just described in the previous section was coded in FORTRAN language and run on the MFE computer time-sharing system. The first three problems were intended to mock up the 2XIIB buildup experiment reported in reference (3). The input numbers are listed in Table I, and the code result for the total ion density ($n_i + n_h$) is plotted versus time in fig. 2. Fig. 2 also shows the experimental data, taken from line integral ($\int ndl$) measurements and renormalized to the estimated maximum ion density.

In comparing theory and experiment only the first two milliseconds are important, because the plasma streams were shut down at that time. It is seen that for both the "stream only" experiment and for the "stream plus beam" experiment, the rate code reproduces the experiment within a factor of two up to 2 msec. The details of the shape of the curve are different, but that is not surprising considering the crudeness of the rate equation model.

Although this comparison is not a definitive test of all of the terms of the code calculation, it is still gratifying to find that it can roughly match the data from the existing 2XIIB stream buildup experiment. One thus has somewhat greater confidence in the results calculated for other situations.

6. Rate Model Estimates for Startup of MFTF Plasmas on a Dense Target Cloud

Using the rate equation model described in section 4 and tested in section 5, we are now ready to continue the discussion of section 3 on the heating and ionization of the vapor cloud. The MFTF experiment(6)(13) was chosen because this startup technique could be very useful there. If successful, one could cut back or eliminate a considerable quantity of auxiliary apparatus such as stream guns (and their power supplies) and the gas box (plus its extra vacuum requirements).

Rate code calculations were done with two target materials: deuterium and nitrogen. One looks at a deuterium target as a standard of comparison; in addition, it does not introduce an impurity into the hot plasma experiment. For the other target material nitrogen was chosen because its interaction cross-sections with hydrogen are well-known,⁽⁷⁾ and because it seems representative of its neighbor elements carbon and oxygen. Thus, it is a reasonable first approximation to a wide variety of common gases and vaporized plastics. All of these elements have larger cross-sections for interaction with the beam than a hydrogen target does; therefore, they are

Table 1Rate Code Input Parameters for 2XII8 Buildup Comparisons

<u>Beams:</u>	$A_B = 2$	$E_B = 10^4 \text{ eV}$	$\rho_H = 3.0 \text{ cm}$
$J(\text{amp/cm}^2)$	0.0 for "Stream Only" Case 0.5 for "Low Beam" Case 1.0 for "High Beam" Case		
<u>Target Stream:</u>	$A_T = 2.$	$\chi = 13.59$	$\eta = 1.0$
	$D_C = 20 \text{ cm}$	$Z_C = .40 \text{ cm}$	$n_S = .002 \times 10^{15} \text{ cm}^{-3}$
	$T_{Si} = T_{Se} = 50. \text{ eV}$		
<u>End Wall:</u>	$V_{SIN} = 3 \times 10^3 \text{ V}$	$V_{SEE} = 200 \text{ V}$	$\eta_{AMB} = 3$
<u>Cross-Sections:</u>	$\sigma_{BO} = \sigma_{TB} = .042$		
(Units of 10^{-15} cm^2)	$\sigma_{BI} = \sigma_{BH} = \sigma_{TH} = .040$		
	$\sigma_{XBI} = \sigma_{XHO} = \sigma_{XBH} = 1.10$		

Figure 2

Rate code comparison with 2XIIIB buildup experiment⁽³⁾:
total ion density versus time.

of interest for this application. It is true that carbon, nitrogen, and/or oxygen are contaminants, but it can be argued that the hot plasma will gradually purge itself of these elements through the loss cone after startup. Also, it is known that the present 2XIIB plasma experiments contain a significant percentage of oxygen contamination.(14)

Table II sets out the input parameters for the six rate code studies done; some of these input numbers should be explained. The average beam and hot ion energy E_R was set at 20 keV rather than 40 keV because both the 80 kV and 20 kV beams are turned on during the MFTF buildup phase.(6) The ion gyroradius ρ_H is low because the MFTF magnetic field is much higher than for the 2XIIB experiments. The average beam current density J can be estimated from the same figures that were used to estimate the intensity (fig. 4):

$$J \approx \frac{20}{1000} (50 + 32) = 1.64 \text{ amp/cm}^2 \quad (15)$$

This value was rounded off to 1.5 amp/cm^2 for two of the runs; the reasons for the other four values will become obvious shortly.

The initial conditions of the target gas n_0 and D were chosen as dense as possible within the constraints of the calculation scheme, which assumes that the range of the neutral beams is greater than the diameter of the cloud. Figure 1 shows that for 20 kV beams the initial density is too great to meet this criterion; however, the higher energy 80 kV beams would penetrate further. Although the accuracy is not good at such high densities, the cloud rapidly expands and the calculational assumption is well-met most of the time.

The small amount of flowing neutral gas n_g was found to be mathematically necessary to keep the calculations stable. Physically, it is reasonable to expect that the plasma will experience an influx of neutral gases from the walls and other sources.

Figure 3 presents some results for the three calculations with a deuterium cloud target. The hot ion density, n_H is seen to be a sensitive function of both the time and the average beam current density. Deuterium target buildup to high hot ion plasma densities is successful only for average current densities which are greater than $J = 1.5 \text{ amp/cm}^2$ estimated for the MFTF beams. The curves show a fast buildup at early times when the target cloud is still dense, followed by a slow buildup on the hot ions if the current density is high enough.

Table IIRate Code Input for MFTF Buildup Studies

		<u>D₂ Target</u>	<u>N₂ Target</u>
<u>Beams</u>	A _B	2	2
	E _B (eV)	2 x 10 ⁴	2 x 10 ⁴
	ρ _H (cm)	1.414	1.414
(6 Runs)	J(amp/cm ²)	1.5, 3.0, 6.0	.375, .75, 1.5
<u>Target Gas</u>	A _T	2	14
	χ(eV)	13.59	14.55
	η	1.	3.
	D _C (cm)	50.	50.
	Z _C (cm)	100.	100.
	n _G (10 ¹⁵ cm ⁻³)	.001	.001
	T _G (eV)	0.1	0.1
(Initial Values)	{ n _O (10 ¹⁵ cm ⁻³)	10 ⁴	10 ⁴
	{ n _I (10 ¹⁵ cm ⁻³)	.1	.1
	{ D(cm)	2.	2.
<u>End Wall</u>	See Table I for v _{SIN} , v _{SEE} and η _{AMB}		
<u>Cross-Sections</u>	σ _{BO}	.077	.314
	σ _{BI}	.090	.314
	σ _{BH}	.090	.090
	σ _{XBI}	.83	1.0
	σ _{XHO}	.83	1.1
	σ _{XBH}	.83	.83
	σ _{TB}	.077	.43
	σ _{TH}	.090	.54

Figure 3

Calculations of hot ion density versus time for D_2 cloud in MFTF for three current densities.

Figure 4 presents similar results with a nitrogen cloud target. Here the slow buildup to high hot ion plasma densities is successful for a lower range of current densities, including the estimated value of $J = 1.5 \text{ amp/cm}^2$. Thus, the rate code calculation predicts that hot ion buildup in the MFTF neutral beam environment will work for the nitrogen target, but not for the deuterium target. The qualitative shape of the buildup curves is similar for both targets.

Further details of the rate calculation can be seen by examining one of these calculations in greater detail. The nitrogen target in the MFTF neutral beam flux of $J = 1.5 \text{ amp/cm}^2$ is a good example; figure 5 displays all seven variables for this case.

At about 50 μsec the first major change in the calculation takes place. The expansion of the target cloud has lowered its density and increased its overall exposure to the neutral beams. The beam heating raises the electron temperature until thermal ionization becomes important. Suddenly the target becomes ionized, thus increasing its interaction cross-sections, and trapping more beam. The rise in electron temperature also decreases the drag, so that the trapped ions have a longer lifetime. Thus, the hot ion density soon exceeds 10^{13} cm^{-3} .

The subsequent rise in hot ion density is much slower as the target cloud continues to expand. Thereafter, further buildup depends on the hot ions being a sufficiently efficient target to trap more neutral beam at a faster rate than the hot ion loss rate. That is the case for this particular run. Figures 3 and 4 each include one run with an insufficient hot ion trapping rate to build the density up beyond the initial level created from the target cloud.

The target ion density $n_i \approx 10^{11}$ at late times in figure 5 is caused by the small "gas box" source used in this run (see n_0 in Table II). As explained earlier, this was necessary to keep the problem stable.

The electron temperature approaches its upper limit of 200 eV for the last 9 milliseconds of the calculation. This limit is set by the end wall loss term (equation (g13)) and the input value $V_{\text{SEE}} = 200$. One of the principal weaknesses of this particular set of rate equations is that it seems unlikely that this is a realistic portrayal of the physical effects which determine this parameter. As mentioned earlier, all multiple ionization events have been neglected. These would absorb additional energy and lower the electron temperature T_e . However, the multiply charged target ions (N^{+2} , N^{+3} , etc.) might have larger interaction cross-

Figure 4

Calculations of hot ion density versus time for H_2 cloud in MFTF for three current densities.

Figure 5

All seven variables plotted versus time for MFTF N_2 calculation at $J = 1.5 \text{ amp/cm}^2$.

sections with the beam, and thus lead to more trapped hot ions and greater heating. Inclusion of these additional effects would be a sizeable research task involving additional differential equations, estimation of unknown cross-sections, et al, and it has not been done.

Another effect that is not included is ionization of the neutral beam by the plasma electrons. Omission of this effect is pessimistic; it causes the rate code to underestimate the beam trapping.

In summary, the present rate code predicts that in MFTF the presently planned neutral beams are sufficiently powerful to build up a hot plasma from a nitrogen-like gas cloud, but insufficient to build up from a deuterium to build up from a deuterium gas cloud.

7. Rate Model Estimates for 2XIIB Plasma Startup on a Dense Nitrogen-like Cloud

It would be interesting to make an experimental test of these plasma buildup ideas in the near future. For this reason a number of calculations were made to model the 2XIIB experiment; three of them are presented here (See Table III and figure 6). The 2XIIB system was found to be more marginal than the MFTF system; consequently, only the nitrogen target calculations are presented because the deuterium results did not show significant buildup.

Two methods of making a gas target were modeled: a "gas cloud" similar to the MFTF runs, and a continuous gas source such as might be provided by the "gas box" facility in 2XIIB. The most successful run was the isolated gas cloud; the "gas box" runs were less successful, probably because the constant influx of cold gas keeps the temperature low and the electron drag term high.

The lower current density and the smaller plasma size also served to lower the resultant 2XIIB densities relative to the MFTF experiment. Nevertheless the results are sufficiently tantalizing to encourage an experimental test.

8. Concluding Remarks

If experiment confirms the predictions of these calculations, this buildup method would simplify many mirror plasma experiments on MFTF-size machines. The stream guns would be needed only for stabilization; fewer would be required. If a "gas box" were used for stabilization, then no stream guns would be needed at all.

There are many possible methods for injection of a suitable target; as mentioned earlier such solids as foils, threads, foams, or multiple

Table III

Rate Code Input for 2XIIB Buildup Studies

Beams: See Table I "High Beam" $J = 1.0 \text{ amp/cm}^2$

<u>Target Gas:</u>	A_T	14.
(common input)	$\chi \text{ (er)}$	14.55
	r_i	3.
	$D_C \text{ (cm)}$	20.
	$Z_C \text{ (cm)}$	40.

<u>Target Gas</u>	1.	2.	3.
(options)	"Gas Box"	Both	"Cloud"
$n_G (10^{15} \text{ cm}^{-3})$	.05	.05	10^{-4}
$T_G \text{ (eV)}$	.1	.1	.1
(Initial Values)	$n_o (10^{15} \text{ cm}^{-3})$	10^4	10^4
	$n_i (10^{15} \text{ cm}^{-3})$	.1	.1
	$D \text{ (cm)}$	2.	2.

End Wall See Table I for V_{SIN} , V_{SEE} , and n_{AMB}

<u>Cross-Sections</u>	$\sigma_{\text{BO}} = .19$	$\sigma_{\text{XHO}} = 1.2$
(all problems)	$\sigma_{\text{BI}} = .19$	$\sigma_{\text{XBH}} = 1.1$
(10^{-15} cm^2)	$\sigma_{\text{BH}} = .04$	$\sigma_{\text{TB}} = .29$
	$\sigma_{\text{XBH}} = .85$	$\sigma_{\text{TH}} = .23$

Figure 6

Calculations of hot ion density and temperature versus time for three 2X113 examples.

particles are all possible. In addition, direct injection of a gas cloud is conceivable.

In April, 1978, a first attempt of this method was made on 2XIIB. It was unsuccessful, but subsequent measurements showed that the graphite fiber bundle target had been misaligned so that it was not near the focus of the neutral beams. It is hoped to repeat the test in the near future with kevlar fibers, urethan foam, and better alignment.

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