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Boundary Conditions for OH, L, and H-mode Simulations

C. E. Singer*, G. Bateman, and D. P. Stotler

Plasma Physics Laboratory

Princeton University

Princeton, New Jersey 08543

PPPL--2527

DE88 012993

ABSTRACT

A method for prescribing appropriate boundary conditions for predictive simulations using flux-surface-averaged plasma transport codes is described. The model makes use of the present theoretical understanding of L and H-mode transport mechanisms and is constrained to be consistent with trends in existing data. It is calibrated against an ASDEX experiment and used to predict the edge behavior in CIT.

*Fusion Studies Laboratory, University of Illinois, Urbana, Illinois 61801

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1. Introduction

This note outlines a method for prescribing appropriate boundary conditions for predictive simulations using flux-surface-averaged plasma transport codes. The aim is to obtain a model which (1) gives a reasonable description of existing data on plasma edge parameters, (2) is compatible with present theoretical understanding of the transport mechanisms in L and H-mode plasmas, and (3) can be conveniently applied to predictive simulations using a variety of core-plasma transport models.

The primary criterion for the model described here is that it be compatible with trends in existing data. Under this point of view the details of how the model is derived are not critical; but they are interesting. Thus, we (1) give the derivation of the boundary condition model, (2) show its calibration against an ASDEX experiment, and (3) show predictions for CIT.

2. Derivation

The first question that arises is where to set the boundary condition. The present model allows the boundary condition to be set anywhere near the separatrix. The recommended choice is a location one scrape-off density scale-height thickness, λ , inside the separatrix¹ (i.e., at a minor radius $r_\lambda = a - \lambda$). The simplest approximation for this distance is

$$\lambda = 0.05/B \quad (1)$$

The constant 0.05 meter-Tesla in this formula is the median value of λB of the 13 values listed by Zweben and Taylor [1] and should be accurate to within about a factor of 2. (We use SI units with temperatures in keV throughout, except where otherwise indicated by an appropriate subscript.) This typically corresponds to a couple of poloidal ion gyroradii at a temperature of 0.1 keV, which is a natural theoretical width for the scrape-off. It also corresponds to a couple of banana-widths at the edge of an H-mode plasma. These observations imply that a proper treatment of plasma transport outside r_λ requires at least a two-dimensional analysis. Therefore, r_λ would seem to be an appropriate place to apply boundary conditions. Only for special heuristic

¹By "separatrix" we mean the first flux surface which intercepts a material boundary.

studies of the scrape-off would there seem to be much point in setting the boundary outside r_λ when using a flux-surface-average model.

Given the limitations just mentioned and present paucity of complete theories for the plasma boundary, there seems little to be gained in routine simulation work by using any but the simplest, analytically integrable, models for the transport outside r_λ (unless a full multidimensional code simulation is available). To this end, we integrate the equations

$$-\frac{1}{r} \frac{d}{dr} (r \kappa \frac{dT}{dr}) = H \quad (2)$$

$$-\frac{1}{r} \frac{d}{dr} (r D \frac{dn}{dr}) = S \quad (3)$$

$$\frac{1}{r} \frac{d}{dr} (r \Gamma_N) = S_N \quad (4)$$

One set of boundary conditions is applied near $r = 0$. These are zero energy flux and a specified electron density:

$$-\kappa \frac{dT}{dr} = -D \frac{dn}{dr} = 0 \quad (5)$$

$$n = n_0 \quad (6)$$

At the separatrix we choose a fixed temperature and assume equal and opposite ion and neutral atom fluxes:

$$T = T_s \quad (7)$$

$$\Gamma_N - D \frac{dn}{dr} = 0 \quad (8)$$

The sixth boundary condition on the above sixth-order set of equations is that the electron density should vanish beyond the separatrix:

$$\lim_{r \rightarrow \infty} n = 0 \quad (9)$$

In the above formulas, T is the electron-ion average temperature, n is the electron density, and Γ_N is the neutral flux. By setting the sum of the neutral and ion fluxes to zero at the separatrix, we have ignored ionization of neutrals in the scrape-off. This will be consistent with our assumed temperature, T_s ,

at the scrape-off provided T_e is on the steeply rising portion of $\xi_i(T)$, where ξ_i is the electron impact ionization rate coefficient. It is assumed here that atomic physics processes and transport along open field lines will regulate the separatrix temperature so that $T_e < 0.1$ keV, which is the temperature at which $\xi_i(T)$ has its maximum.

2.1. Temperature

We begin the integration of these equations by specifying profiles for thermal conductivity and the heating power density. A moment's reflection shows that the steep temperature gradient found at the edge of H-mode plasmas is a natural result of standard core-plasma transport models [2,3] which have a thermal conductivity which decreases with decreasing temperature. (That this occurs in nature is well illustrated by the sharp transition region between the hot solar corona and cold chromosphere.) The absence of such steep edge gradients in L-mode plasmas suggests the existence of another energy loss mechanism. Support for such a mechanism is found in a number of theories (cf. Hahm and Diamond [4], Hinton [5], and Bishop [6]). These theories suggest the existence of a second contribution to the heat flux, which gets larger as the temperature decreases. Thus, we add two terms to get

$$\kappa = \kappa_0 \left[\left(\frac{T}{T_*} \right)^t + \frac{s-1}{t+1} \left(\frac{T}{T_*} \right)^{-s} f_s \right] f(\rho) \quad (10)$$

Here T_* is the temperature at which the two contributions to κ are comparable. The factor $(s-1)/(t+1)$ could have been absorbed in the definition of T_* , but the final result turns out simpler using the formula for κ given here [cf. Eq. (13), below]. All of the theories of interest here give results qualitatively similar to the above formula with $t, s > 0$. However, they make varying predictions about the value of T_* and the dependence of κ_0 on other plasma parameters. We shall, therefore, determine T_* empirically. Since both Hinton [5] and Hahm and Diamond [4] that the $s > 1$ type of contribution will be larger far away from diverted field lines than near diverted field lines, we set the factor f_s equal to one when a poloidal X-point lies near the separatrix and set $f_s > 1$ otherwise. Since the various theories make various contradictory predictions about the dependence on edge-plasma transport on other parameters, f_s will be taken here to be only a function of the magnetic geometry. While this is obviously rather simplistic, an examination of

the transport formulas given in the papers referred to above suggests that it should capture their most important features. The radial variation $\kappa \propto f(\rho)$ (where $\rho \equiv r/a$) will play no significant role in the present application of this model, as we will assume that $(d \ln f / d \ln \rho)|_{\rho=1} \ll a/\lambda$. (A factor $f(\rho) = \rho^u$ is included for future use of the model for comparison with global energy confinement scalings.) The radial scaling of heating profiles also plays no significant role in the edge but is also included for completeness in the assumed heating profile,

$$H = \frac{2+\gamma}{2} P_V \rho^\gamma \quad (11)$$

where P_V is the volume-average heating power density.

The solution of the energy transport equation under these assumptions is

$$\left[\left(\frac{T}{T_*} \right)^{1+t} - \left(\frac{T}{T_*} \right)^{1-s} f_s \right]_{T_*}^{T_\lambda} = \frac{h}{\gamma+2-u} (1 - \rho^{2+\gamma-u}) \quad (12)$$

$$\approx h \frac{\lambda}{a} \quad (13)$$

where

$$h = \frac{(1+t)a^2}{2\kappa_* T_*} P_V \quad (14)$$

is a dimensionless measure of the heating power. The approximation in Eq. (14) is valid at $r = a - \lambda$, since we shall always have $\lambda/a \ll 1$. The only positive integer values of t and s for which this gives a conveniently solvable expression for $T_\lambda \equiv T(r_\lambda)$ are $t = 1$ and $s = 2$. Since these are reasonably close to theoretical expectations, we shall limit ourselves to these values. This gives the final result

$$\left(\frac{T_\lambda}{T_*} \right)^2 - \frac{T_*}{T_\lambda} f_s = h \frac{\lambda}{a} + \left(\frac{T_s}{T_*} \right)^2 - \frac{T_*}{T_s} f_s \quad (15)$$

This equation for $T_\lambda \equiv T|_{r=a-\lambda}$ has been calibrated to the OH and H-mode temperatures observed on an ASDEX case for various values of T_* , with the results shown in Fig. 1. To do this, we adjust $T_s = 0.07$ keV and $h\lambda/a = 4.66$ to match the observed [7] values of T_λ for the ohmic and H-mode phases of the ASDEX discharge described in Table 1.

To complete the calibration of this model, it is interesting to note that the function $T(P_{MW})$ at fixed $a-r$ is equivalent to $T(a-r)$ at fixed heating

Table 1. Parameters for ASDEX Calibration Case

Parameter	Symbol (Units)	Value			
Toroidal Field	B (T)	2.2			
Major Radius	R (m)	1.64			
Minor Radius	a (m)	0.40			
Plasma Current	I_{MA} (MA)	0.375			
Boundary Thickness	$\lambda = 0.05/B$ (m)	0.023			
Safety Factor	q_{cyl}	2.86			
			<u>OH</u>	<u>L</u>	<u>H</u>
Heating Power	P_{MW} (MW)	0.35	2.2	2.9	
Temperature at r_λ	T_λ (keV)				
	observed	0.08	0.21	0.34	
	predicted	*	0.24	*	
Line-average Density	$\bar{n}_{20}$ (10^{20} m^{-3})	0.41	0.40	0.43	
Density at r_λ	$n_{\lambda,20}$ (10^{20} m^{-3})				
	observed	0.17	0.20	0.25	
	predicted	0.20	*	0.22	
"Bolometric" Correction	f_{Mu}	0.75	0.76	0.73	

Table 2. CIT Parameters

Parameter	Symbol (Units)	Value			
Toroidal Field	B (T)	10			
Major Radius	R (m)	1.75			
Minor Radius	a (m)	0.55			
Plasma Current	I_{MA} (MA)	9			
Boundary Thickness	$\lambda = 0.05/B$ (m)	0.005			
Elongation	K	2			
Safety Factor	q_{cyl}	2.40			
		<u>OH-lim</u>	<u>OH-div</u>	<u>RF</u>	<u>α</u>
Heating Power	P_{MW} (MW)	6	6	17	34
Temperature at r_λ	T_λ (keV)	0.11	0.61	1.1	1.7
Line-average Density	$\bar{n}_{20}$ (10^{20} m^{-3})	3.3	3.3	4.5	4.5
Density at r_λ	$n_{\lambda,20}$ (10^{20} m^{-3})	2.2	2.2	3.3	3.3
"Bolometric" Correction	f_{Mu}	0.70	0.70	0.55	0.55

power for $(a - r) \ll a$ under the assumptions made here. The observation that $T = 0.8$ keV at $r = a - 3\lambda$ for the H-mode phase of the calibration case described in Table 1 leads to a choice of $T_* = 0.3$ keV (cf. Fig. 1). The variation of edge temperature with power in similar limiter plasmas² in ASDEX [8] show that $f_s = 5$ is an appropriate choice to complete the overall scaling, at least for a comparison of ASDEX divertor and limiter geometries.

The results of applying this model to compact ignition tokamak (CIT) design parameters from Table 2 are shown in Fig. 2. The edge temperature scales as $T_\lambda \propto P_{MW}^{1/2}$ for $T_\lambda \gg T_* = 0.3$ keV. Within the confines of the present model, this scaling in the high power limit should be the same as the scaling of the volume-average temperature.

The results presented so far are based on the assumption that the plasma

²For example, $T_e = 0.17$ keV at $r = 0.31$ m for $\bar{n} = 5 \times 10^{19} \text{ m}^{-3}$ with other conditions similar to the case described in Table 1.

is near thermal equilibrium. A correction for time-varying heating is discussed below.

2.2. Density

Analytic solution for the plasma density at r_λ requires specifying the particle sources and transport coefficients. The plasma source is dominated by ionization of atoms incident from outside the plasma boundary, except for transients just after pellet injection. It is, therefore, usually appropriate to model the fuelling source as $S = nN\xi_i$, where N is the neutral atom density found by computing the transport of neutrals inward from the boundary. (Molecules are assumed in the present treatment to be ionized so far out in the scrape-off *is* to have negligible impact on the present calculation. The neutral atoms which significantly affect plasma fuelling are implicitly assumed to have energies of at least 0.003 keV.) Except for cases with very intense injection of high energy neutrals, the maximum penetration length for most of the incident neutrals is about

$$L = \frac{2}{n_e \Sigma} \quad (16)$$

where n_e is the central electron density and $\Sigma \sim (3 \pm 2) \times 10^{-19} \text{ m}^2$ is the effective cross section for diffusion and attenuation of neutrals with energy in the range $0.003 \rightarrow 2 \text{ keV}$ [9]. We will restrict ourselves to plasmas with a scrape-off thickness $\lambda \ll L$ and a penetration depth $L \ll a$. In this approximation, ionization can be neglected outside r_λ . For the plasma, we also include a term of the form $-n/\tau_\parallel$ outside $r = a$ to account for losses along open field lines. The plasma and neutral sources are then

$$S = \begin{cases} nN\xi_i & r < r_\lambda \\ 0 & r_\lambda < r < a \\ -n/\tau_\parallel & a < r \end{cases} \quad (17)$$

$$S_N = \begin{cases} -nN\xi_i & r < r_\lambda \\ 0 & r_\lambda < a \end{cases} \quad (18)$$

Radial variation of the ionized particle diffusion coefficient has relatively little impact on the boundary density with the sources just defined. This can

be illustrated by setting

$$D = \begin{cases} D_0 & r < r_\lambda \\ D_1 & r_\lambda < r < a \\ D_2 & a < r \end{cases} \quad (19)$$

where D_0 , D_1 , and D_2 are constants. When combined with a diffusive approximation for the neutral transport,

$$\Gamma_N = -\frac{\xi_i n}{\Sigma^2} \frac{dN}{dr} \quad (20)$$

and the boundary conditions listed above, this gives the following solution to the particle transport equations to lowest order with $\lambda \ll L \ll a$:

$$n = \begin{cases} n_0 \tanh[(r_* - r)/L] & r < r_\lambda \\ \frac{2D_0 n_0 (a-r)}{D_2 L} (1 - \frac{n^2}{n_0^2}) & r_\lambda < r < a \\ n_s \exp[-(r-a)/\lambda] & a < r \end{cases} \quad (21)$$

where $r_* = r_\lambda + L \operatorname{arctanh}(n_\lambda/n_0)$. Matching the fluxes and densities at the boundaries of these three regions gives the following equation for the desired boundary density, $n_\lambda = n(r_\lambda)$:

$$\frac{n_\lambda}{n_B B} = -1 + \sqrt{1 + (\frac{\bar{n}}{n_B B})^2} \quad (22)$$

Here

$$n_B = \frac{D_2}{2D_0 B \lambda \Sigma} \quad (23)$$

is independent of D_1 ! The parameter n_0 has been replaced by the line-average density, $\bar{n}$, because we have flat density profiles over almost all of the plasma in the present approximation.

Calibrating this model to the L-mode phase of the ASDEX discharge described in Table 1 gives (for $n_\lambda = 2.0 \times 10^{19} \text{ m}^{-3}$, $\bar{n} = 4.0 \times 10^{19} \text{ m}^{-3}$, and $B = 2.2 \text{ T}$)

$$n_B = 1.4 \times 10^{19} \text{ m}^{-3} \text{ T}^{-1} \quad (24)$$

It should be noted that the simpler approximation $n_\lambda \approx \bar{n}/2$ would also fit the ASDEX calibration case; and it would give almost as good a fit to the

density rise observed in the PDX case also described by Singer *et al.* [7]. But this simpler approximation would not be compatible with the scaling of the separatrix density, $n_s \propto (\bar{n})^2$ reported for relatively low-density T-10 [9] and TEXT [10] cases, so we recommend use of the quadratic equation for n_s shown above.

It should be kept in mind that the present model will overestimate n_s when the edge plasma is too cold to ionize neutrals or there is a strong inward plasma pinch. The effects of lowering n_s by a factor of two or so should, therefore, always be examined when extrapolating to large values of $\bar{n}/(n_B B)$.

3. Extrapolation

Suppose we wish to apply the above formulas to a simulation where data on the boundary conditions is incomplete or absent. To do this, we need only to know how the normalized heating power density, h , scales from the calibration case to the machine conditions we wish to simulate. The energy confinement scaling calibrated by Sugihara [11] gives a particularly convenient method for doing this. This confinement scaling is based on the assumption that the global energy confinement time in the absence of nonconductive losses scales as

$$\tau_E = \tau_{SN} f_{profile} \quad (25)$$

Here Singer's modification of the Neoalcatraz scaling (extrapolated to on-axis heating without sawteeth) is

$$\tau_{SN} = 9.9 \times 10^{-22} I_{MA}^{0.8} R^{3.15} a^{-0.2} K^{0.6} Z_{eff}^{0.4} \bar{n} / \langle T \rangle_n \quad (26)$$

where I_{MA} is the toroidal plasma current in MA, R is the major radius, K the elongation at the plasma boundary, Z_{eff} is the effective charge, and $\langle T \rangle_n$ the density-weighted volume average temperature in keV. For the type of heating profiles considered here, this scaling should be multiplied by a heating efficiency factor, $f_{profile} \approx 0.4$ (cf. Appendix).

This confinement scaling was based on the usual assumption that $\tau_E \propto a^2 K / (\kappa/n)$ for any given heating profile.³ Assuming the $\kappa \propto \kappa_0 T$ term in our

³A typographical error omits a factor of n where this assumption is quoted in Sugihara and Singer [11].

above formulation dominates the L-mode scaling, it is appropriate to make the identifications $a^2/\kappa_o \rightarrow \tau_{SN} < T >_n / \pi$ and $(2 + \gamma)^{-1} \propto f_{profile}$ from this model when extrapolating the above formula for the dimensionless heating power density, h .

For a more accurate extrapolation, we should also account for losses due to radiation and charge exchange. In Sugihara's calibration of the above model to gas-fuelled discharges, this was accomplished by calibrating a factor

$$f_{Mu} = [1 + (\pi/n_{MS})^2]^{-1} \quad (27)$$

which reduces the global confinement time as the density increases. Here n_{Mu} is the modified Murakami Scaling limit

$$n_{Mu} = \begin{cases} 1.5 \times 10^{20} B / (Rq_{cyl}) & \text{with gas-puffing} \\ 2.1 \times 10^{20} B / (Rq_{cyl}) & \text{with pellets} \end{cases} \quad (28)$$

For the H-mode phase of the ASDEX calibration case described in Table 1, this factor would be $f_{Mu}^{ASDEX} = 0.73$. The introduction of such a correction factor into transport simulations would be redundant if the corresponding losses were already computed during the simulation. For this reason, we omit the f_{Mu} from our basic prescription of boundary conditions for predictive simulations. The applied power in the formula we derive here should, therefore, be interpreted as the net power reaching the boundary at $r = r_A$ after subtracting out the losses which would be observed by a bolometer array looking inside $r = r_A$. When experimental or predictive estimates of these "bolometric" losses are unavailable or inconvenient, then the total power should be reduced by the factor f_{Mu} . (Note that a higher density limit may be appropriate for computing f_{Mu} for elongated plasmas, cf. [11,12].)

With the substitutions just described and assuming $P_V \propto P_{MW} / (Ra^2 K)$, this gives $h \propto R^{2.15} a^{-2.2} K^{-0.4} I_{MA}^{0.8} P_{MW}$. Here we have suppressed the scaling with Z_{eff} , since this scaling is weak and was not actually calibrated by Sugihara. Dropping a factor $R^{-0.05} \approx 1$ and normalizing this to the ASDEX H-mode conditions given in Table 1 (with $P_{MW} = 2.9 f_{Mu}$), this gives

$$h = 3.8 (R/a)^{2.2} K^{-0.4} I_{MA}^{0.8} P_{MW} \quad (29)$$

The reason for choosing the Sugihara calibration for scaling h should be apparent from this relatively simple result. Use of the similar Uckan-Houlberg

modification of the Kaye-Goldston scaling (which has been used for various machine design studies [12]) would give a much messier formula with no expected improvement in accuracy.

A qualitative comparison of H-mode and limiter L-mode scalings for global energy confinement is evident from the above formula for h if one identifies the H-mode with a regime where auxiliary heating is strong enough to give $T_\lambda \sim T_*$. Namely, the H-mode confinement should be largest at high aspect ratio, retain strong scaling with plasma current, and have less degradation of confinement with heating power. These predictions seem roughly consistent with observations.

Inserting this formula for h in the above equation for T_λ gives an *a priori* determined value for the boundary condition on a flux-surface-averaged transport simulation. The results are plotted in Fig. 2 for the projected compact ignition tokamak (CIT) parameters listed in Table 2.

4. Simulations

Here we discuss how to handle the transients in predictive simulations and show some sample applications. The reason there is a problem with transients is that the above analysis was based on analysis of steady-state energy and particle balances. The direct application of these results to simulations with pellet injection or discontinuous heating would result in inappropriate and inconvenient discontinuities in the boundary conditions. One could use the actual net power outflux computed by the simulation code to determine T_λ , but this might be inconvenient. A more convenient method is to gradually scale up and down the boundary conditions as the global plasma parameters evolve. A general method of doing this is to compute the value of h using the transformed heating power

$$P_t = e^{-s}[P(0) + \int_0^s e^s P(s) ds] \quad (30)$$

where

$$s = t/\tau \quad (31)$$

This result follows from assuming the plasma stored energy evolves according to $dE/dt = P - E/\tau$ and defining $P_t = E/\tau$ where τ is taken to be a constant. To the accuracy to which the overall model has been calibrated, it should be

adequate determine τ for this formula from a simple scaling. Here we use Gruber's [13], with a factor $A_i^{1/2}$ omitted:

$$\tau = 0.065 I_{MA} R \quad (32)$$

This gives confinement times intermediate between L- and H-mode for most cases. We also use fixed values of I_{MA} , B , and ohmic heating when computing the evolution of the boundary values. A similar transform of the boundary density is used when modelling cases with pellet injection. For the step-function or linear ramps typically used in predictive transport simulations, these transforms are easy to compute.

While the above prescriptions should be adequate for most purposes, it may occasionally be desirable to illustrate an abrupt transition to higher boundary values and global energy confinement. Since sawteeth usually trigger the H-mode transition, it would seem appropriate to use a relatively abrupt increase in boundary temperature after a sawtooth crash to do this. A simple method is to apply the boundary temperature increase given by the above transform as a constant between sawteeth. This would only be strictly valid in the limit where the heating takes place inside the mixing radius and the sawtooth period is short compared to the energy confinement time, but it serves here to illustrate how sawteeth might trigger an abrupt increase in the temperature of the edge plasma.

Simulations of ASDEX and CIT using the boundary conditions described here with the Singer-Ku " $t = 1$ " transport model [12] are done as follows. In the integrand of Eq. (28), these simulations use an abrupt increase in heating power to the L- value quoted in Tables 1 and 2, followed by a linear rise to the value listed for the maximum heating power. For the CIT case, the value of f_s is rapidly dropped from 5 to 1 at time $t = 3.0s$, when the elongation reaches its final value of $K = 2$. The line-average density used for computing the CIT boundary density was transformed as in Eq. (28) to give a slow rise after each pellet injection. The boundary conditions computed and applied to the ASDEX are shown in Fig. 3.

The results of ASDEX simulations with fixed and time-varying boundary conditions are shown in Figs. 4 and 5. The transport model used here is identical to that given by Singer, Ku, and Bateman [12]. Other simulation parameters are as in Singer, Redi, et al. [7] except that (a) sawtooth mixing events exchange plasma outside and inside the inversion radius instead of

flattening the profiles, and (b) the neutral influx energy is fixed at 3 eV. Given that the transport model was taken without modification from one based on a calibration against various types of ohmic and L-mode data for five different tokamaks, the fits to the data shown in Figs. 4 and 5 are at least as good as would be expected from such a simple transport model. The differences in the plasma core between simulations with fixed and time-variable temperature boundary conditions are rather small, as shown in Fig. 4. The main effect of applying the complete new model is a better fit to the wing on the density profiles, as shown in Fig. 5.

The boundary conditions computed for CIT simulation are shown in Fig. 6 (cf. Stotler and Bateman [14]). We used this *a priori* prescription of the boundary conditions as the first step in an iterative procedure. After running the code with these boundary conditions, the resulting power outflow (conductive and convective in ions and electrons) was used to compute the evolution of new boundary conditions, and additional simulations were run. The resulting evolution of the volume average plasma to toroidal magnetic pressure ratio, $\langle \beta \rangle$ for these boundary conditions is compared to results from a more traditional set of fixed boundary conditions in Fig. 7. This simulation is based on the baseline simulation of Singer et al. [12] using their " $t = 1$ " model except that (a) equal deuterium and tritium densities are used, (b) the final density is lower (cf. Table 2), and (c) energy transport is enhanced by a large factor whenever the computationally determined ideal ballooning pressure gradient threshold is exceeded. The plasma transport and sawtooth mixing models were identical to those used for the ASDEX simulations described above. The main difference between the old and new models occurs when the density boundary condition is changed. A pronounced difference between the results with old and new boundary conditions is the virtual disappearance of neutral recycling in the new model after the initial start-up. Variation between different fixed and time-varying temperature boundary conditions has relatively little effect on the achieved temperature. This is perhaps not surprising, as no attempt was made to adjust the pellet velocity to compensate for the higher edge temperature, and the central temperature is large enough for fusion even in the worst cases considered here. It would be interesting to investigate these effects using less favorable transport models (such as the " $t = 3/2$ " version of the model calibrated by Ku, cf. [12]).

It should be noted that the method used here to scale the edge temperature requires that the core-plasma transport model gives a reasonably accurate treatment inside r_λ . This should be adequate for transport models calibrated against L-mode plasma results if all of the alterations in the H-mode occur within a distance λ of the separatrix. This assumption is appropriate to the type of theory described by Hinton [5], where a "barrier" to radial heat flow is set up within a radial distance determined by particle orbits. We shall, therefore, refer to the standard type of extrapolation adopted here as the "barrier" method.

If the core-plasma transport model does not have appropriate behavior near the separatrix, the boundary condition for H-mode plasmas must be set to compensate for this. If the configuration of interest has similar magnetic geometry to the case for which the model was calibrated, it may be appropriate to set the temperature boundary condition at the same fractional minor radius in both cases. This can be accomplished by scaling λ and the net heating power by B/B_{cal} , where $B_{cal} = 2.2T$ for the calibration described here. (The density can be found for this new boundary location using the equations given above.) This method of extrapolation might be called the "geometric" method. For large heating power, the boundary temperature will be larger by a factor $(B/B_{cal})^{1/2}$ than for the barrier method. The results from the two methods differ by a factor of 2.1 for extrapolation from ASDEX to CIT. The choice of an appropriate method can only be resolved by better theories or by accurate edge-plasma measurements from H-mode plasmas with very different magnetic fields. Neither of these seems likely to be available in the near future. There are also uncertainties of about a factor of 1.5 or so in the choice of conduction coefficients, the measurement of calibration temperatures, etc. The effect of changing boundary values by a factor of 2 for either extrapolation method should, therefore, be investigated in any detailed study of sensitivity of the results to modelling assumptions.

Acknowledgments

One of us (CES) wishes to acknowledge the support of D. Post and the Princeton Plasma Physics Laboratory. This work was supported by U.S. DOE Contract No. DE-AC02-76-CHO-3073.

Appendix A. Confinement Scaling

Here we make two comments about the global energy confinement scaling used in the derivations above. First, the heating efficiency factor can be written as

$$f_{profile} = 1 - 1.2x_{saw} + 0.2x_{saw}^3 \quad (A1)$$

where

$$x_{saw} = 2^{1/2}/q_{cyl} \quad (A2)$$

with

$$q_{cyl} = \frac{5a^2 B}{RI_{MA}} \frac{1 + K^2}{2} \quad (A3)$$

for resonant or fusion-product heating concentrated inside the quasisteady sawtooth mixing radius at $r \sim ax_{saw}$. This gives $f_{profile} = 0.33$ for the CIT case discussed above. The actual heating efficiency should be somewhat larger since the current profile does not evolve to bring the sawteeth to their maximum radius in most CIT scenarios, so $f_{profile} = 0.4$ seems a reasonable approximation. When heating is dominated by neutral beams, $f_{profile}$ can be computed from Eqs. (5)-(10) of Sugihara and Singer [11]. For example, one obtains $f_{profile} = 0.6(1 - x_{saw}^3) \exp(-0.51\pi_{20}) \approx 0.4$ for the ASDEX case shown in Table 1, (where $\pi_{20} = 10^{-20}\pi$). The ohmic heating efficiency has been approximated as $f_{profile} = 0.3(1 + 0.7q_{cyl})^{1/2}(1 - x_{saw}^3)$, which also gives $f_{profile} \sim 0.4$ for the cases of interest here.

Second, the temperature pedestals derived above could be used with either the Sugihara calibration or other models such as those of Gruber or Kaye and Goldston to give a detailed comparison of confinement scalings for limiter and divertor plasmas. A systematic treatment of this problem is beyond the scope of the present work. If this is eventually done, it should be noted the formula given above for τ_{SN} in the Sugihara model correctly includes a factor of 10^{-20} which was omitted in the Eq.(1) of the Sugihara and Singer paper. With this correction, the formulas given in the present paper allow a complete, and probably the most accurate available, specification of global energy confinement formulas appropriate for extrapolation to CIT parameters.

Appendix B. Calibration and Extrapolation

This Appendix gives some details of the methods used for calibration and extrapolation of boundary conditions for the present model. BASIC coding (for the SHARP EL-5500II computer) is listed here for each step in these procedures. This coding should be easily recoverable, (e.g. for conversion to use on Macintosh computers) by extracting everything between the "verbatim" commands used to list it in the L^AT_EX source file for the present document.

The calibration was started by solving a linear equation for $h\lambda/a$:

```
10 DIM P(2)
20 DIM N(2)
30 DIM X(3)
40 RADIANT
110 TH=.34
120 TO=.08
130 AL=(.35/2.9)*(.75/.73)
140 INPUT "TN";TN
150 XN=((TH*TH-TO*TO)/TN^2 - TN*(1/TH - 1/TO))/(1-AL)
```

and then a cubic equation for the separatrix temperature, T_s .

```
160 AC=(XN - (TH/TN)^2 + TN/TH)*TN*TN
170 BC=-TN*TN*TN
180 CS=BC*BC/4 + AC*AC*AC/27
190 IF CS<0 GOTO 230
200 C=SQR(CS)
210 TS=CUR(C-BC/2) - CUR(C+BC/2)
220 GOTO 300
230 M=2*SQR(-AC/3)
240 TS=M*COS((1/3)*ACS((3*B)/(AC*M)))
```

This was done for various values of the parameter T_λ . The power P_{MW} required for various boundary temperatures, T_λ was computed for each T_λ for the ASDEX parameters listed in the top of Table 1 with a fixed density of $n_{20} = 0.43$:

```

300 INPUT "B"; B
310 IF B>0 GOTO 390
320 B=2.2
330 R=1.64
340 K=1
350 A=.4
360 IM=.375
370 CM=1.5
380 GOTO 450
390 INPUT "B"; B
400 INPUT "R"; R
410 INPUT "K"; K
420 INPUT "A"; A
430 INPUT "IM"; IM
440 INPUT "CM"; CM
450 QC=(5*A*A*B/(R*IM))*(1+K*K)/2
460 NM=CM*B/(R*QC)
470 L=.05/B
480 TL=.17
490 PL=2.0
500 PN=2.9
510 NL=.5
520 NN=.43
530 GL=1/(1+(NL/NM)^2)
540 GN=1/(1+(NN/NM)^2)
550 XL=(PL/PN)*(.09/L)*(GL/GN)*XN
560 FL=(XL-(TS*TS-TL*TL)/TN^2)/(TN*(1/TS - 1/TL))
570 USING "####.###"
580 PRINT "FL"; FL
600 INPUT "FS"; FS
610 XS=(TS/TN)^2 - FS*TN/TS
620 INPUT "T"; T
630 IF T<0 GOTO 730
640 PA=((T/TN)^2 - FS*TN/T - XS)*PN/XN
650 PRINT "PA"; PA
660 GOTO 610

```

The best value of T_e was chosen by examination of the resulting graphs of $T_e(P_{MW})$; cf. Fig. 1. All of this was done with the geometry parameter $f_s = 1$. This gave $h\lambda/a = 4.66$ for $T_e = 0.3$. The value of f_s required to give the limiter-plasma result quoted above was found by multiplying the net heating power for the limiter result quoted in footnote 3 by the ratio $(0.09/\lambda)(f_{M_u}^{lim}/f_{M_u}^{div})$ to the location ($a - r = 0.09$) and density at which it was measured.

Calibrating the scaling of the dimensionless heating power density was done by solving the equation $4.66 = c(1.64/0.4)^{2.2}(0.375)^{0.8}(2.9)(0.73)(0.05/0.4)/2.2$ to for ASDEX H-mode parameters to give $c = 3.8$. A cubic equation for T_e was then solved to obtain the "predicted" values shown in Tables 1 and 2.

```

730 INPUT "P(0)"; P(0)
740 INPUT "P(1)"; P(1)
750 INPUT "P(2)"; P(2)
760 INPUT "N(0)"; N(0)
770 INPUT "N(1)"; N(1)
780 INPUT "N(2)"; N(2)
790 FOR J=0 TO 2
800 FM=1/(1 + (N(J)/NM)^2)
810 X(J)=3.8*(R/A)^2.2*K^(-.4)*IM^-.8*P(J)*FM*L/A
820 TA=-1
830 IF TA>0 THEN LET X(J)=XT
840 AC=-(X(J)+XS)*TN*TN
850 BC=-FS*TN*TN*TN
860 CS=BC*BC/4 + AC*AC*AC/27
870 IF CS<0 GOTO 910
880 C=SQR(CS)
890 T=CUR(C-BC/2) - CUR(C+BC/2)
900 GOTO 930
910 M=2*SQR(-AC/3)
920 T=M*COS((1/3)*ACS((3*BC)/(AC*M)))
930 IF TA>0 GOTO 1130
940 PRINT "J"; J
950 PRINT "T"; T
960 NEXT J

```

The final boundary temperatures were then computed from the result of

an analytic integration using a trapezoid as the integrand in formula for the transform shown in the main text:

```

1000 TA=.065*IM*R
1010 INPUT "DT"; DT
1020 IF DT<0 GOTO 2000
1030 INPUT "TIME"; TI
1040 IF TI<0 GOTO 390
1050 DE=DT/TA
1060 XD=(X(2)-X(1))/DE
1070 S=TI/TA
1080 D=S
1090 IF D>DE THEN LET D=DE
1100 ES=EXP(-S)
1110 ED=EXP(D-S)
1120 XT=X(0)*ES + (X(1)-XD)*(ED-ES) + D*XD*ED + X(2)*(1-ED)
1130 GOTO 830
1140 PRINT "T"; T
1150 GOTO 1030

```

The cubic equations were solved using the methods given in the CRC Standard Math tables. This used cube root formulas when there was one real root and the first trigonometric formula given in the CRC Tables when there were three real roots. (The other two real roots were negative.)

The boundary density transform for the CIT pellet injection case was found as follows:

```

2000 NP=5
2010 DIM E(NP+1)
2020 DIM NE(NP)
2030 DIM NT(NP)
2040 DIM TP(NP+1)
2050 NB=.14
2060 B=10
2070 IM=9
2080 R=1.75
2090 TP(1)=1.4

```

```

2100 TP(2)=2.4
2110 TP(3)=3.6
2120 TP(4)=4.7
2130 TP(5)=5.6
2140 TP(6)=20
2150 NE(0)=.45
2160 NE(1)=1.0
2170 NE(2)=2.7
2180 NE(3)=2.3
2190 NE(4)=3.3
2200 NE(5)=4.5
2220 FOR J=0 TO NP
2230 NT(J)=NB*B*(-1+SQR(1+NE(J)/(NB*B))^2))
2240 PRINT "J";J
2250 PRINT "NT";NT(J)
2260 NEXT J
2270 INPUT TI
2280 IF TI<0 GOTO 2400
2290 TA=.065*IM*R
2300 FOR J=1 TO NP+1
2310 E(J)=EXP((TP(J)-TI/TA)
2320 IF E(J)>1 THEN LET E(J)=1
2330 NEXT J
2340 NL+NT(0)*E(1)
2350 FOR J=1 TO NP
2360 NL=NL+NT(J)*(E(J+1)-E(J))
2370 NEXT J
2380 PRINT "NL";NL
2390 GOTO 2270
2400 END

```

It was assumed that the line-average density was $\bar{n} = (4/3) \langle n \rangle$, where $\langle n \rangle$ was a volume average density program similar to that used the work of Singer, Ku, and Bateman [12].

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Figures

Fig. 1. Edge-plasma temperature *vs.* heating power applied to ASDEX for various values of the free parameters T_e and f_s .

Fig. 2. Temperature at $\lambda = 5$ mm inside the separatrix *vs.* net heating power for CIT parameters.

Fig. 3. Edge-plasma temperature *vs.* time computed for ASDEX without sawteeth (smooth curve) and as applied to ASDEX simulation with sawteeth (piecewise linear curve).

Fig. 4. Electron temperature Thomson scattering data (symbols) *vs.* simulations with fixed temperature boundary conditions (simulation bas01p, dashed curve) and the time-varying boundary temperature given by the piecewise linear curve in Fig. 3 and boundary density given by Eq. (22) (simulation bas01o, solid curve).

Fig. 5. Electron density profiles for ASDEX discharge and simulations of Fig. 4. The short-dashed curves show density profiles from a simulation with fixed edge density as well as fixed edge temperature.

Fig. 6. Edge-plasma temperature and density *vs.* time computed for CIT.

Fig. 7. Evolution of $\langle \beta \rangle$ for CIT using $T_\lambda = 0.1$ keV (simulation bit01x), limiter boundary conditions (from Eq. (15) with $f_s = 5$, simulation bit01w), transition to divertor boundary conditions after $t = 3$ s (from Eq. (15) with $f_s = 1$, simulation bit01v), and two times the T_λ of bit01v after $t = 3$ s (simulation bit01y). These simulations used a fixed edge electron density of $2 \times 10^{19} \text{ m}^{-3}$. Instead, bit01t uses n_λ from Fig. 6 (simulation bit01t, dashed curve). The power outflow used to compute T_λ for simulations bit01t, v, w, and y came from a simulation (bit01c, not shown) using both boundary conditions shown in Fig. 6.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

88P0085

FIG. 7

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