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SSS-R-77-3165

PASSIVE SEISMIC TECHNIQUES FOR GEOTHERMAL EXPLORATION

Quarterly Technical Progress Report
For Period December 1, 1976 - February 28, 1977

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March 1977

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PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

Under Contract No. E(04-3)-1313

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I. PROGRAM SUMMARY

The overall objective of the subject program is to develop geophysical methods into useful cost-effective tools for geothermal reservoir delineation and evaluation. More specifically, the program involves the development of generalized inversion techniques and their application to combined seismic and gravity data bases for obtaining the three-dimensional velocity and density structure of a geothermal region. The inversion methods and data processing procedures will be proof-tested using data that have been compiled for the Imperial Valley, California.

During the second three-month period of this program, our efforts concentrated on the development and verification of forward modeling techniques and the processing and analysis of the teleseismic and gravity data bases for the Imperial Valley. Our present capabilities include the forward modeling of surface observations of teleseismic travel-time residuals and gravity anomalies associated with three-dimensional variations in near-surface seismic velocities and densities. Considerable time was spent devising test cases that both verify the modeling programs and provide insight for the discretization scheme to be used for modeling the Imperial Valley.

The teleseismic travel-time data base originally acquired during the first quarter of this project was subjected to various statistical tests for error analysis. As a result of these tests, all of the original 176 events were reanalyzed for accuracy of time picks and correlation of event waveforms across the Imperial Valley Array (IVA). Travel-time residuals based on the corrected data base, now increased to 182 events, were computed using several different stations within the IVA as reference stations. The resulting residuals are being analyzed as a function of azimuth and angle of incidence

(epicentral distance) for data consistency and baseline considerations, prior to calculating the residual data vector to be used in the inversion routines.

A very large data set of Bouguer gravity anomalies, covering the region bounded by 32°N - 34°N , 114°W - 117°W , which includes the Imperial Valley, was obtained from Professor Shawn Biehler of the University of California at Riverside. The data were provided in two formats corresponding to observations at the original irregularly spaced station locations and interpolated values on regular grids at different map scales (1:48,000; 1:62,500 and 1:96,000). Contour routines and smoothing programs have been written and will be used to process the data base prior to generation of the gravity anomaly data vector.

II. TECHNICAL DISCUSSION

2.1 INTRODUCTION

The objective of this research program is to develop generalized inversion techniques and apply them to surface observations of teleseismic travel times and gravity anomalies for a determination of the three dimensional variations in near-surface seismic velocities and densities. The approach that we have adopted toward a solution to this problem is a parallel development of the observational data bases and modeling procedures. This section of the report describes the results obtained to date employing this approach.

2.2 FORWARD MODELING PROGRAM DEVELOPMENT

The system of linear simultaneous equations that govern the forward and inverse modeling phases of both the travel-time and gravity portions of this study may be written in matrix form as follows:

$$\underline{\underline{M}}^{TT} \cdot \underline{\underline{\delta a}} = \underline{\underline{\delta T}} \quad (1)$$

$$\underline{\underline{M}}^G \cdot \underline{\underline{\delta \rho}} = \underline{\underline{\delta g}} \quad (2)$$

In these equations the elements of the column vectors $\underline{\underline{\delta T}}$ and $\underline{\underline{\delta g}}$ are based on observations of travel-time and gravity anomalies, respectively. The elements of the vectors $\underline{\underline{\delta a}}$ and $\underline{\underline{\delta \rho}}$ are comprised of perturbations to the model parameters slowness, $a = 1/v$ where v is the compressional wave velocity, and density, respectively. The elements of the matrices $\underline{\underline{M}}^{TT}$ and $\underline{\underline{M}}^G$ are partial derivatives or kernels relating first order variations in the model parameters to variations in the observed data. A detailed description of the generation of the vectors and matrices in Eqs. (1) and (2), and their application to test problems, is given in the following sections of this report.

2.2.1 Overall Program Structure

As a result of our code development efforts during the past three months, it was decided that a further subdivision of the main program TGVERT, described in the previous quarterly report [Savino, et al., 1976], was necessary. Thus a separate program capable of providing the near-surface correction for the travel-time and gravity vectors was developed. This program incorporates subroutines "MATGEN", "TKERN", "GKERN", "INDATA", "BIRCH", "DISAZ", "LOCATE" and "GRID", which were discussed in the previous report. In addition, a new routine "DOTPR" was developed to accomplish the vector product required for the forward problem.

The new main program "SGRID" which replaces the "SURCOR" portion of "TGVERT" is designed to do the complete forward problem. A detailed model of the structure is input in the form of a slowness perturbation vector with an entry for each element in the grid. From this a density perturbation vector can be derived for the gravity calculations via the relation [Press and Biehler, 1964]

$$\rho = C_1 V + C_2$$

which leads to

$$\delta\rho = - C_1 \frac{\delta a}{a^2}$$

where C_1 is a constant, chosen as 0.3, and δa is an element of the slowness perturbation vector.

The program employs "MATGEN" to calculate a matrix of travel time and gravity kernels provided by "TKERN" and "GKERN". This matrix is then multiplied in "DOTPR" with the appropriate model vector. The results are a travel-time residual vector and a gravity residual vector which can then

be used to provide near surface corrections to the vectors produced by "TPREP" and "GPREP".

2.2.2 Matrix Generation

The rows of the matrix fall in two categories. Those containing travel-time kernels are calculated first, followed by the rows of gravity kernels. For each datum in the travel-time data vector, a row is calculated by a single call to subroutine "TKERN". Similarly "GKERN" calculates an entire row for each entry in the gravity data vector.

For a travel-time row, "TKERN" calculates an element, that is a travel-time kernel, for each of the cells in the three dimensional grid. A row of the matrix results from tracing a single ray backward from the recording station to the point where it exits the grid. Along the ray path the kernel for each cell is the path length traversed in that cell. This is later used in Eq. (1) to calculate a contribution to the travel-time residual vector. Since but a few cells are traversed by any given ray, the travel-time rows of the matrix are very sparse. Figure 1 shows how "TKERN" begins with the cell which lies directly below the recording station. The indexing follows the ray into each of the cells it traverses and the points of entry and exit are found. When the exit point is found to be at a cell boundary which coincides with an outer boundary of the grid the calculation of kernels for this row is complete. Kernels for all cells not traversed by the ray are zero. At each x-y plane in the grid the ray may be refracted by entering a layer of differing slowness from that above it. Thus, each time the exit point is on the cell floor and a new layer is entered a new angle of incidence is calculated from the ray parameter and the slowness.

When all entries in the travel-time data vector have been computed, "MATGEN" goes on to form the rows of gravity kernels. Unlike the travel-time rows this portion of the

Find indices of cell containing recording station

Loop over all X-Y planes of the grid

Calculate the tangent of the angle of incidence from the ray parameter (p) and the slowness (a)

Distance from entry point to each possible exit point for this cell

Three possible projections in the X-Y plane where θ is the back azimuth for the ray path

Choose the shortest projection

Calculate kernel having excited bottom of cell within grid

Check to see if ray has left the grid at the bottom

Figure 1. Logic flow diagram for subroutine TKERN.

matrix is densely packed with an entry for every cell of the grid in each row. Each row and its corresponding entry in the gravity data vector are associated with a unique point of observation on the surface of the grid. The gravity kernel which "GKERN" calculates is a measure of the cell's ability to contribute to the gravity field at this point of observation based on the volume of the cell and its distance from the point. Like travel-time kernels the gravity kernels have units of length (note that each element of the density vector is multiplied by the universal constant of gravitation, G). A contribution to the gravity residual vector is later found by application of Eq. (2). Figure 2 shows the indexing convention used in applying Nagy's formula [1966, 1967] for right rectangular prisms. The distance to cell vertices from the point of observation are used in evaluating the following:

$$K_G = X \ln (Y+R) + Y \ln (X+R) - Z \sin^{-1} \left[\frac{Y^2+Z^2+YR}{(Y+R) \sqrt{Y^2+Z^2}} \right] \begin{matrix} Z_2 & Y_2 & X_2 \\ \left| \right. & \left| \right. & \left| \right. \\ Z_1 & Y_1 & X_1 \end{matrix} \quad (3)$$

where

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

The full form of Eq. (3) is worked out for the case where an elemental prism is completely contained within any one of the four xy quadrants. The form given below is a corrected version of the one proposed by Nagy [1966]

Figure 2. Indexing convention used to calculate the gravity kernel for a typical cell in the three-dimensional grid.

$$\begin{aligned}
K_G = & x_1 \ln \left(y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2} \right) - x_1 \ln \left(y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2} \right) \\
& + x_2 \ln \left(y_2 + \sqrt{x_2^2 + y_2^2 + z_1^2} \right) - x_2 \ln \left(y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2} \right) \\
& + x_1 \ln \left(y_2 + \sqrt{x_1^2 + y_2^2 + z_2^2} \right) - x_2 \ln \left(y_2 + \sqrt{x_2^2 + y_2^2 + z_2^2} \right) \\
& + x_2 \ln \left(y_1 + \sqrt{x_2^2 + y_1^2 + z_2^2} \right) - x_1 \ln \left(y_1 + \sqrt{x_1^2 + y_1^2 + z_2^2} \right) \\
& + y_1 \ln \left(x_1 + \sqrt{x_1^2 + y_1^2 + z_1^2} \right) - y_1 \ln \left(x_2 + \sqrt{x_2^2 + y_1^2 + z_1^2} \right) \\
& + y_2 \ln \left(x_2 + \sqrt{x_2^2 + y_2^2 + z_1^2} \right) - y_2 \ln \left(x_1 + \sqrt{x_1^2 + y_2^2 + z_1^2} \right) \\
& + y_1 \ln \left(x_2 + \sqrt{x_2^2 + y_1^2 + z_2^2} \right) - y_1 \ln \left(x_1 + \sqrt{x_1^2 + y_1^2 + z_2^2} \right) \\
& + y_2 \ln \left(x_1 + \sqrt{x_1^2 + y_2^2 + z_2^2} \right) - y_2 \ln \left(x_2 + \sqrt{x_2^2 + y_2^2 + z_2^2} \right) \\
& - z_1 \sin^{-1} \frac{z_1^2 + y_1^2 + y_1 \sqrt{x_1^2 + y_1^2 + z_1^2}}{\left(y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2} \right) \sqrt{z_1^2 + y_1^2}} \\
& + z_1 \sin^{-1} \frac{z_1^2 + y_2^2 + y_2 \sqrt{x_1^2 + y_2^2 + z_1^2}}{\left(y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2} \right) \sqrt{z_1^2 + y_2^2}} \\
& - z_1 \sin^{-1} \frac{z_1^2 + y_2^2 + y_2 \sqrt{x_2^2 + y_2^2 + z_1^2}}{\left(y_1 + \sqrt{x_2^2 + y_2^2 + z_1^2} \right) \sqrt{z_1^2 + y_2^2}} \\
& + z_1 \sin^{-1} \frac{z_1^2 + y_1^2 + y_1 \sqrt{x_2^2 + y_1^2 + z_1^2}}{\left(y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2} \right) \sqrt{z_1^2 + y_1^2}}
\end{aligned}$$

$$\begin{aligned}
& - z_2 \sin^{-1} \frac{z_2^2 + y_2^2 + y_2 \sqrt{x_1^2 + y_2^2 + z_2^2}}{(y_2^2 + \sqrt{x_1^2 + y_2^2 + z_2^2}) \sqrt{z_2^2 + y_2^2}} \\
& + z_2 \sin^{-1} \frac{z_2^2 + y_1^2 + y_1 \sqrt{x_1^2 + y_1^2 + z_2^2}}{(y_1^2 + \sqrt{x_1^2 + y_1^2 + z_2^2}) \sqrt{z_2^2 + y_1^2}} \\
& - z_2 \sin^{-1} \frac{z_2^2 + y_1^2 + y_1 \sqrt{x_2^2 + y_1^2 + z_2^2}}{(y_1^2 + \sqrt{x_2^2 + y_1^2 + z_2^2}) \sqrt{z_2^2 + y_1^2}} \\
& + z_2 \sin^{-1} \frac{z_2^2 + y_2^2 + y_2 \sqrt{x_2^2 + y_2^2 + z_2^2}}{(y_2^2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) \sqrt{z_2^2 + y_2^2}}
\end{aligned}$$

A systematic and efficient scheme for evaluating these kernels is set forth in subroutine "GKERN". Figure 3 is used to illustrate the procedure. First a loop over all cell vertices in the grid is executed to evaluate Eq. (3) at each vertex and store the result in a triply dimensioned array. An economy in storage can be realized by overlaying this array with the buffer which will ultimately contain the matrix row. This is possible since the final evaluation for each cell is a sum of only terms with the same indices as the cell and those have indices which are one greater. Next a loop over all cells in the grid is performed to provide the cyclic permutation of indices needed to combine terms for the evaluation of Eq. (3). Since the loop cycles through indices from smallest to largest in the grid rather than outward in all directions from the point of observation, some care must be taken in handling the result. First, the terms for all cells below and to the right, and above and to the left, of the point of observation (second and fourth quadrants in a

GKERN

Loop over cell vertices in
X-direction

Loop in Y-direction

Loop in Z-direction

Find X-Y-Z coordinates relative
to the point of observation

Store the square to be used many
times in evaluating the expres-
sion (3)

Calculate intermediate
quantities

Store the result for
each vertex

Figure 3. Logic flow diagram for subroutine GKERN.

Loop over all X-Y planes of grid

Loop over all cells in column containing point of observation

Save terms needed for on-axis correction (i_p is index of this column)

Intermediate quantities for $X=0$ correction terms

Calculate $X=0$ correction terms

Loop over row of on-axis cells

Save correction terms (j_p is index of this row)

Intermediate calculations

$Y=0$ correction terms

Figure 3 (Continued)

Loop in both directions over
all cells in this layer
(X-Y plane)

Combine terms for each
of the eight vertices of
the cell whose indices
are i j k

$$\begin{aligned}
 A_{ijk} = & A_{i+1 \ j+1 \ k} \\
 & - A_{i \ j+1 \ k} + A_{i \ j \ k} \\
 & - A_{i+1 \ j \ k} + A_{i \ j+1 \ k+1} \\
 & - A_{i+1 \ j+1 \ k+1} + A_{i+1 \ j \ k+1} \\
 & - A_{i \ j \ k+1}
 \end{aligned}$$

Loop again over column of
on-axis cells

Apply on-axis correction
to results of loop 10

$$\begin{aligned}
 A_{ip \ j \ k} = & A_{ip \ j \ k} + 2 W_{jk} \\
 & - 2 W_{j+1 \ k} + 2 W_{j+1 \ k+1} \\
 & - 2 W_{j \ k+1} + 2 C_{j+1 \ k} \\
 & - 2 C_{j \ k} + 2 C_{j \ k+1} - 2 C_{j+1 \ k+1}
 \end{aligned}$$

Loop over row of on-
axis cells

Apply correction to
terms produced by
loop 10

$$\begin{aligned}
 A_{i \ jp \ k} = & A_{i \ jp \ k} + 2 V_{ik} \\
 & - 2 V_{i+1 \ k} + 2 V_{i+1 \ k+1} \\
 & - 2 V_{i \ k+1} + 2 B_{i+1 \ k} \\
 & - 2 B_{i \ k} + 2 B_{i \ k+1} - 2 B_{i+1 \ k+1}
 \end{aligned}$$

Figure 3 (Continued)

Correct cell directly
below point of
observation

Loop over all k (all
layers) for i and j
in second quadrant

Negate these terms

Also negate all terms
in the fourth
quadrant

Figure 3 (Concluded)

map view) will have acquired the wrong sign. This is pictured in Figure 4 which shows how the indices are cycled.

For proper evaluation of Eq. (3), the X_i , Y_j and Z_k terms must always be those closest to the point of observation while the X_{i+1} , Y_{j+1} , Z_{k+1} terms must be the furthest. This rule is violated in quadrants two, three and four. However, since both indices are backward in quadrant three, the sign will be correct. This problem is easily handled by negating the result for all cells in quadrants two and four just prior to exiting the routine. Secondly, the results for all cells which are in line with the point of observation (cross-hatched in Figure 4) will have been miscalculated. To properly evaluate the expression for the cell labeled "A" in the figure, terms for points ① through ⑥ must be combined. The cell should have been divided into two parts and a contribution for points ②, ③, ⑤ and ⑥ added to that for points ①, ④, ⑤ and ⑥. This can be accomplished by saving all points in line with ⑦ and ④ on the first pass through the grid. Points ⑤ and ⑥ are then calculated and combined with these. The result is doubled and applied as a correction to the original value calculated for the cell and the correct number is obtained.

Final adjustment is needed for a cell which lies directly below the point of observation (P). This cell should be calculated from the four rectangular blocks surrounding the point of observation. Part of the desired correction was already supplied in the two loops which correct on axis cells. However, in the process the terms for points P, ⑦, ⑧ and ⑨ were added both times with the wrong sign. This can be corrected by adding four times the contribution from these points. All information needed for this correction, save the evaluation of point P, is still present in the temporary arrays used for an on-axis correction. Since expression (3) takes on

Figure 4. Map view of a typical layer in the grid showing indices used in the initial evaluation of Eq. (3). The numbered vertices denote terms which must be used in the on-axis correction and the correction for the cell directly below the point of observation.

a particularly simple form when both X and Y are zero

$$(A = - Z \pi/2)$$

the task is very easy indeed and performed just before proceeding to the next layer in the grid. When all layers in the grid have been exhausted, the matrix row is complete and "GKERN" returns control to "MATGEN"; to be called again for each remaining row.

The rows of the matrix are written to mass storage as they are calculated. A double buffering technique is used to improve execution speed by allowing one row to be calculated at the same time the preceeding row is being transmitted to the peripheral device.

Now it is time to perform the vector products indicated by Eqs. (1) and (2). For this purpose, rows of the matrix are read back in using an identical double buffering scheme to that with which they were written. As each row is made available, subroutine "DOTPR" is called to multiply its elements by the corresponding element in the model vector (δa and $\delta \rho$). The sum of these terms for a travel time row becomes an element (δt) of the travel time residual vector. Elements of the gravity residual vector (δg) are obtained in the same fashion from the gravity rows.

This process completes the calculations for the entire forward problem. The residual vectors can be examined for correlation with measured gravity and travel time data. They may alternatively be used as near surface corrections to residual vectors derived from recorded data.

2.3 PROGRAM TESTING PROCEDURE AND RESULTS

Since the new main program SGRID and the main inversion program use several identical subprograms, and the importance of this phase of the operation to the final result was apparent,

it was decided to undertake a rigorous testing and evaluation phase. To facilitate this, a convenient method for generating the initial model vector was desired. It should have the ability to create model vectors for both idealized geometrically shaped anomalies and "real world" anomalies of arbitrary dimensions.

To accomplish this task an interactive program "MODGEN" was developed. The basic technique for both types of model generation is identical. It consists of assigning a value of the model parameter to each cell in the grid based on whether that cell is fully contained, partially contained, or fully excluded by the anomalous region.

For the geometric case the anomalous region can be buried sphere, a right circular cylinder oriented either vertically or horizontally, or a rectangular prism located anywhere within the grid. These were chosen for testing purposes since closed form gravity and travel-time solutions for these shapes [Telford, et al., 1976] are available for comparison.

The arbitrary shaped anomaly is generated by employing a slice model which allows specification of a few vertical slices through the grid at which the location of interfaces between regions are known. This would, for instance, allow generation of a model based on refraction profiles collected for a particular region of interest (i.e., the Imperial Valley). This slice model approach can be illustrated by Figure 5 which shows two vertical slices. The depth to the points denoted by dots on the diagram would be input for each slice along with the values of the model parameter above and below the interface. Linear interpolation is used between slices to determine surfaces delineated by the long dashed lines. These surfaces separate regions for model parameter assignment.

For both geometric and slice models, one of two approaches to model parameter assignment can be employed. These are

Figure 5. Depths to points on an interface specified as input for slice model generation.

volume weighting of the model parameter and all-or-nothing assignment. In the all-or-nothing approach a cell acquires the value of the model parameter associated with the region which contains at least half of the volume of the cell (Figure 6). Volume weighting assigns an intermediate value of the model parameter to a cell which is partially contained in an anomalous region by weighting the parameters by the volume subtended in each region. Figure 7 shows that cells fully contained (cross-hatched) in a region simply take on the value for that region while those partially contained (stippled) are volume weighted.

To accomplish these tasks two basic program units were developed. For any configuration a function subprogram "INFUNC" is employed. "INFUNC" is given the X-Y-Z coordinates of a point in the grid and returns a value of one if the point lies inside a chosen region and zero if outside. The main program first loops through all vertices in the grid calling "INFUNC" and storing the results in a triply dimensioned array ITEST (I,J,K). It then loops over all cells and determines for each cell if ITEST for all eight vertices is one or zero. If all eight have the same value, the assignment of the appropriate model parameter can be made immediately. For those cells with vertices in two regions, subroutine "SBDVD" is called to subdivide the cell and provide an approximation for the volume subtended by each region.

"SBDVD" does a simple three-dimensional numerical integration to obtain the volumes by dividing a cell into smaller blocks. The block dimensions are provided as input to the program. "INFUNC" is furnished the coordinates of the centroid of each block and the volume inside a region is calculated as the sum of volumes of blocks for which "INFUNC" returns to one. The finer the subdivision (smaller typical block dimension), the better the approximation to the volume. The volume returned by "SBDVD" is used either to determine which model

Figure 6. All-or-nothing model parameter assignment. All cells acquire either the model parameter associated with the cross-hatched region or that associated with the unshaded region.

Figure 7. Volume weighting of model parameter cross-hatched and unshaded cells acquire the parameter for the appropriate region. Stippled cells are assigned an intermediate value found by volume weighting.

parameter to assign for all-or-nothing weighting or to determine an intermediate value for volume weighting. Diagrams depicting implementation of these two routines are given in Figures 8 and 9. The "INFUNC" shown is for the simplest case of a sphere of radius R_s with center at coordinates X_s, Y_s, Z_s .

2.3.1 Test Case I. Buried Cube

For the first test case a cube, each edge of which is 15 km in length, is positioned with its center 15 km below the surface of the grid. A ten percent velocity contrast between material inside and outside the cube was assumed. The cells of the grid were of varying dimension but cell interfaces were arranged to fall on the boundaries of the cube. Thus, the cube should be exactly modeled by the blocks which contain it. The forward gravity problem was run with a regular 5×5 array of points of observation on the surface of the grid. It was then rerun with a 25×25 array so that better resolution could be obtained for contour plotting of the result.

It was found that the 5×5 spacing produced an exact subset of 25×25 array as expected. Contour plots of the result are presented in Figures 10 and 11. For this problem we can conclude that the distortion of the contours for the 5×5 case is due solely to the poor sampling provided by this coarser spacing since again the grid elements exactly model the desired anomaly.

2.3.2 Test Case II. Buried Sphere

The results presented for Case II are for a sphere 15 km in radius buried 25 km in the geometric center of the grid. Three grid configurations were used to approximate the sphere with both volume and all-or-nothing weighting. These are shown in Figures 12 through 14. Each map view shows the

Figure 8. Function subprogram INFUNC for the case of a sphere of radius R_s with its center at grid coordinates X_s, Y_s, Z_s .

SBDVD

Find dimensions of
cell to be subdivided

$$D_1 = X_{i+1} - X_i$$

$$D_2 = Y_{j+1} - Y_j$$

$$D_3 = Z_{R+1} - Z_k$$

Determine number of sub-
divisions required in
each direction using
typical subdivision
size s which is input

$$N_1 = D_1 / S$$

$$N_2 = D_2 / S$$

$$N_3 = D_3 / S$$

Determining actual sub-
division sizes which will
exactly divide the cell

$$S_1 = D_1 / N_1$$

$$S_2 = D_2 / N_2$$

$$S_3 = D_3 / N_3$$

Volume of each block

$$V_B = S_1 * S_2 * S_3$$

Coordinates of centroid
of first block

$$U = X_i + S_1 / 2$$

$$V = Y_j + S_2 / 2$$

$$W = Z_k + S_3 / 2$$

Initialize volume inside
region to 0

$$V_I = 0$$

A

Loop over block in
X-direction

In Y direction

In Z direction

Accumulate volume inside
region as sum of blocks
for which INFUNC = 1

Update coordinate of
centroid in X direction

and in Y direction

and in X direction

Figure 9. Logic flow diagram for subroutine SBDVD.

BURIED CUBE 10% VELOCITY CONTRAST 25X25 GRID

Figure 10. Calculated gravity residual due to a buried cube contoured for a regular 25×25 array of points of observation. Contour levels are labeled in milligals.

BURIED CUBE 10% VELOCITY CONTRAST 5X5 GRID

Figure 11. Contours of calculated gravity residual for buried cube using only a 5×5 regular array of points of observation. Contour levels are in milligals.

Figure 12. Regular grid configuration for buried sphere problem showing weighting factors applied to the model parameter inside the sphere.

Figure 13. Irregular grid in map view for buried sphere problem. The weighting factors for each layer are shown in the effected cells.

Figure 14. Irregular grid in all directions for buried sphere problem.

weighting factors applied to the model parameter in each of the five layers of the grid. In the case of all-or-nothing weighting, factors of either 0 or 1 are shown in the column on the right hand side, while those used in volume weighting are indicated on the left.

2.3.3 Case IIA. Buried Sphere in Regular Grid

Of the three configurations tested, the regular 10×10×10 km spacing of grid elements provided the worst approximation for the shape of the sphere. As in the other two cases yet to be discussed, a regular 23×23 array of points of observation on a 4 km spacing at the surface of the grid was calculated.

Figure 15 shows a comparison of values for the 23 positions which lie above a line through the center of the sphere. The solid line represents the profile of gravity anomalies computed with the analytic solution for the buried sphere [Telford, et al., 1976]. The expression used is

$$\delta g = \frac{4\pi G \delta \rho R_s^3}{3} \frac{z}{(x^2 + z^2)^{3/2}}$$

where z is the depth to the center of the sphere, G is the universal constant of gravitation and $\delta \rho$ is the density contrast. It is seen that the all-or-nothing weighting (short dashes) provides a gross over-estimate of the magnitude of the anomaly while volume weighting (long dashes) affords a solution which differs from the closed form by just over three percent at the peak.

2.3.4 Case IIB. Buried Sphere in Irregular Grid (Map View)

An enhancement to the modeling of the spherical shape afforded by the grid configuration was used in the second test

Figure 15. Comparison of results for all-or-nothing weighting (short dashes), volume weighting (long dashes) and the closed form expression (solid line) for a buried sphere. The results shown are for the regular grid case.

case. A larger block completely encompassed by the sphere was introduced along with narrower block sizes near the right edge. The depth of the blocks was as in the first test case; constant 10 km spacing.

This case is characterized by a marked improvement in the ability of the all-or-nothing scheme to model the anomaly. Figure 16 shows again the comparison with the closed form expression. It is still apparent, however, that the volume weighting scheme provided a more accurate solution preserving both the magnitude estimation of the anomaly and its symmetry across the array.

2.3.5 Case IIC. Buried Sphere in Irregular Grid (All Dimensions)

A further enhancement is provided by introducing variations in layer thickness which delineate the sphere boundaries more closely. The results are shown in Figure 17. Here the magnitude estimation afforded by the all-or-nothing weighting shows good agreement with the closed form. The ability of the technique to accurately locate an anomaly, however, is improved again by volume weighting as is seen by the preservation of symmetry in the solution.

Contour maps of the computed anomalies for the two extreme cases IIA and IIC are presented in Figures 18 and 19. These depict the result for volume weighting in each case. A map view of the grid is overlayed in dashed lines. Some distortion of the innermost contour due to the discrete sampling is apparent in both cases. A slight asymmetry in this distortion is present for the irregular grid but the overall concentricity of contour levels which lie near or beyond the radius of the sphere is excellent.

Some conclusions can be drawn from these three test cases. A judicious choice of cell boundaries produces a

Figure 16. Results for buried sphere problem with irregularities in the map view only. The legend for the long and short dashes, and the solid line is the same as Figure 15.

Figure 17. Result for buried sphere problem where cell dimensions were varied in all directions. Same legend for the curves as in Figure 15.

SPHERE OF RADIUS 15 KM BURIED 25 KM .075 DENSITY CONTRAST

Figure 18. Contour map of gravity anomaly due to a buried sphere using regular 10×10×10 km grid elements. Contour levels are labeled in milligals.

SPHERE OF RADIUS 15 KM BURIED 25 KM .075 DENSITY CONTRAST

Figure 19. Contour map of buried sphere with grid elements of varying dimensions as shown by the dashed lines. Contour levels are in milligals.

marked improvement in the accuracy of the solution when a block model is employed. This has also been shown by Cordell and Henderson [1968] who devised an iterative scheme for optimizing the size of the modeling elements. This can be an expensive approach, however, requiring nine minutes of processor time on an IBM 360 for the sphere problem.

Whether or not it is convenient, or practical, to specify optimized cell boundaries to model a particular anomaly, however, volume weighting of the model parameter seems to improve the result in all cases. Further, this form of optimization is a one pass process which resulted in processing times on our Univac 1108 for the combined optimization and forward solution for a large number of points of observation of under three minutes. In addition, this technique requires no a priori knowledge of the shape of the anomaly for its use. Thus, even a "worst case" modeling with a regular grid and large block sizes can produce excellent agreement with the closed form solution where it exists.

2.3.6 Travel-Time Calculations

Travel-time residual vectors were also calculated for the sphere using the regular grid and an array of ten stations above the center line of the anomaly. The angle of incidence of rays impinging on each station was varied from vertical to 30° in 10° increments. For non-zero angles of incidence the azimuthal angle of the impinging ray was incremented in 30° steps from 0° to 360° . This yields 360 ray tracings or a total of 370 when combined with the 10 observations for vertical incidence.

Figure 20 illustrates some results for the vertically traveling rays. The horizontal distance in km from a point above the center of the sphere is given along with the calculated travel-time residual. Note that these observations

Figure 20. Variation of travel-time residual for vertically traveling rays with distance from the point above the center of the spherically shaped anomaly.

are taken at the centers of grid elements. For vertical incidence the points on cell boundaries produce discontinuities in the solution, the magnitude of which is related to the size of the modeling element.

Figure 21 shows the variation of calculated residual with angle of incidence for the station directly above the center of the sphere. Discontinuities are much less apparent when several AOI's are sampled since cell boundaries are traversed at an angle and more cells are sampled by each ray. Figure 22 shows that some azimuthal dependence is still introduced into the solution due to the block model used.

2.4 PROCESSING AND ANALYSIS OF DATA FOR THE IMPERIAL VALLEY

2.4.1 Teleseismic Travel-Times

As discussed in Section 2.2 of this report, the application of the general linear inverse modeling technique to the teleseismic problem requires the generation of a travel-time data vector. The elements of this data vector take the form of travel-time residuals based on measurements of the arrival times of body waves. In the case of the Imperial Valley, we are interested in travel-time observations based on P waves recorded at the 16 IVA stations from a large population of worldwide events (earthquakes and explosions).

We suppose that teleseismic P-waves from a number (M) of earthquakes and explosions are recorded by an array of N stations. Let the observed time of the mth P-wave arrival at the nth station be T_{nm}^{OBS} . We can define the relative residual as follows:

$$\delta T_{nm} = T_{nm}^{OBS} - T_{om}^{OBS} - (T_{nm}^{CALC} - T_{om}^{CALC}) - (T_{nm}^{SG} - T_{om}^{SG}) \quad (4)$$

In Eq. (4) the zero subscript designates the reference station relative to which all residuals are formed. The terms T_{nm}^{CALC}

Figure 21. Variation of travel-time residual with angle of incidence for rays which are recorded at a station directly above the center of the sphere.

Figure 22. Azimuthal dependence of calculated travel time residual due to a buried sphere for three AOIs.

and T_{om}^{CALC} are the expected arrival times of P waves from the m^{th} event to the n^{th} station and the reference station calculated using the Herrin [1968] travel-time tables and the known location of the events. The terms T_{nm}^{SG} and T_{om}^{SG} are travel-times of rays from the m^{th} event that traverses a small dimensioned grid immediately below the surface of the n^{th} and reference stations.

As explained in the previous quarterly report, several arrival time measurements were made from each waveform. The measurements were taken at consecutive quarter cycles. The mean difference between the different arrival time measurements at the n^{th} station and the corresponding measurements at the reference station were computed and subjected to various statistical checks to test for accuracy of the data base. As a result of these tests, all of the original 176 events were reanalyzed for measurement and transcription errors and correlation of event waveforms across the array.

Travel-time residuals, based on the corrected data base, were computed using several different stations within the IVA as reference stations. It should be noted that the data base has been increased to a total of 182 events recorded with sufficient signal-to-noise to be included in the study. Figure 23 shows the station locations within the Imperial Valley and new values for the percentage of events recorded at each of the IVA stations.

Mean relative residuals were computed for each station-event pair in the corrected data base. Three of the 16 stations, which were among the highest reporting stations in the IVA (e.g., SUP, CRR and SGL), were used as reference stations. The residuals were sorted on event-station azimuth and examined for consistency and signal content.

Figure 24 shows mean relative residuals plotted as a function of epicentral distance, or angle of incidence, for

Figure 23. Station location map with percentage of teleseismic events recorded at each station in parenthesis. This figure gives an indication of the areal coverage obtained to date for the Imperial Valley.

Figure 24. Plot of mean relative residual versus epicentral distance (angle of incidence, AOI) for AMS relative to SUP and CRR for events in restricted azimuthal ranges.

the station AMS on the eastern flank of the Imperial Valley referenced to two stations, SUP and CRR, on the western flank of the valley. Residuals were computed using Eq. (4) without the small-grid correction. The results are shown for events in two station-event azimuthal ranges; 130° - 140° corresponding to Central and South American events, and 310° - 320° for events in the Kurile Islands. The bars attached to the symbols represent one standard deviation from the mean residual.

The most striking aspect of the results in Figure 24 is the magnitude of the signal present in the travel-time residuals. Note how AMS appears fast (negative residuals) relative to either SUP or CRR for propagation paths from Central and South America to the Imperial Valley. For paths that enter the Imperial Valley from the Kurile Islands, however, AMS is slow (positive residuals) relative to the two reference stations. The difference in the mean relative residuals between the two approach azimuths is approximately 0.6 to 0.8 seconds, depending upon the angle of incidence.

The trend of the mean residual signal is consistent with the gross structural features of the Imperial Valley region. For instance, for approach azimuths of 130° - 140° , the path to AMS is primarily restricted to the western flank of the Chocolate Mountains-Sand Hills region, whereas portions of the Imperial Valley proper are included for the paths of SUP and CRR. The opposite situation occurs for approach azimuths of 310° - 320° , where the path to AMS now includes portions of the Imperial Valley proper and the paths to SUP and CRR are restricted to the higher velocity mountainous regions west of the Valley.

The results shown in Figure 24 are further highlighted by comparing with the mean relative residuals plotted in Figure 25. In this case we are comparing SGL relative to SUP, two stations that are located in the western side of the

Figure 25. Plot of mean relative residual versus epicentral distance (angle of incidence, AOI) for AMS and SGL relative to SUP for events in same azimuthal ranges as in Figure 24.

Imperial Valley approximately 30 km apart. Note how the residuals (open and closed squares) tend to cluster about a value of 0.0 seconds for both event-station azimuthal ranges. The mean residuals for AMS relative to SUP (open and closed circles) are repeated for ease of comparison.

Figure 26 shows the azimuthal dependence of mean residuals (AMS-SUP, closed circles; AMS-CRR, X's) for a restricted range of angles of incidence, 18-20°. The important point to note here is the grouping of residuals into negative and positive values. While the restricted range of angles of incidence results in a fairly limited data set, the striking dependence of the residuals on angle of incidence demonstrated in Figure 24 precludes inclusion of all the events on a plot such as that in Figure 26.

These preliminary results are extremely encouraging and similar analysis of the full IVA is underway. Information obtained from this type of analysis will aid in the interpretation of the formal inversion results.

2.4.2 Gravity Data

Bouguer anomaly gravity data for the region bounded by 32°N-34°N, 114°W-117°W, which includes the Imperial Valley, were obtained from Professor Shawn Biehler of the University of California at Riverside. These data were supplied in two formats. Raw gravity anomalies with supporting station information were supplied on magnetic tape. The magnetic tape was converted to an S³ format and gravity anomalies and station information for a small region within the Imperial Valley were printed and checked for accuracy.

The second set of data consists of interpolated Bouguer anomalies at map scales of 1:48,000, 1:62,500 and 1:96,000 supplied on punched cards. These data provide regular grids of gravity anomalies for areas within the Imperial Valley.

Figure 26. Azimuthal plot of mean relative residual for AMS referenced to SUP (●) and CRR (X). The events are restricted to angles of incidence in the range 18° to 20° . The numbers within parentheses represent the number of events recorded in the particular azimuthal range.

Our initial processing tasks with the gravity data are concerned with expanding the regular grid to interpolated anomalies and contouring the results for comparison with maps supplied by Professor Biehler.

III. FUTURE PLANS

All work under this project is proceeding according to schedule. During the next quarter period our research efforts will focus more on developing and applying the inverse modeling procedures to the teleseismic and gravity data for the Imperial Valley. One of our first steps in this regard will be to assemble a starting model (velocity and density structure) for the region including the Imperial Valley and compare first order predictions from the forward modeling procedures described in this report with the observed data sets. At that point we will be ready to try the initial inversion runs. Several additional months of seismic data are available for the IVA at Caltech. We plan to make one more trip there to bring the teleseismic data base up to date.

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