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Plasma Effects in High Frequency
Radiative Transfer

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PLASMA EFFECTS IN HIGH FREQUENCY RADIATIVE TRANSFER

Introduction

This paper is intended as a survey of collective plasma processes which can affect the transfer of high frequency radiation in a hot dense plasma. We are rapidly approaching an era when this subject will become important in the laboratory. For pedagogical reasons we have chosen to examine plasma processes by relating them to a particular "reference" plasma which will consist of fully ionized carbon at a temperature $kT=1 \text{ KeV}$ (10^7 K) and an electron density $N = 3 \times 10^{23} \text{ cm}^{-3}$, (which corresponds to a mass density $\rho = 1 \text{ gm/cm}^3$ and an ion density $N_i = 5 \times 10^{22} \text{ cm}^{-3}$). We will consider the transport in such a plasma of photons ranging from 1 eV to 1 KeV in energy. Such photons will probably be frequently used as diagnostic probes of hot dense laboratory plasmas.

In this survey paper we have treated this problem classically, and in the continuum limit, but we do so with some trepidation. Atomic processes will not be considered here. Some of the phenomena to be discussed lie only marginally in the classical domain, and quantal effects may be important. For very small wavelengths the medium becomes discontinuous and Maxwell's equations are inappropriate. Our upper photon energy of 1 KeV reaches this discontinuous limit, and so the conclusions we make must be tempered by the realization that, for the higher-frequency photons, the problem becomes extremely complex because it involves the exact detailed structure of the plasma.

We use the term "plasma" loosely; basically we mean that in our medium the photon frequency is much higher than the electron-ion collision frequency, and also that the medium is of high enough temperature and low enough density that characteristic lengths in the medium are much larger than the inter-particle spacing. This latter criterion allows us to analyze the medium in terms of large-scale collective behavior rather than individual particle behavior.

One measure of "classicality" is to compare the photon wavelength with the inter-particle spacing in the plasma, for example with the Debye length, given by

$$D = \sqrt{\frac{kT}{8\pi N e^2}} = 4.9 \sqrt{\frac{T(\text{deg})}{N(\text{cm}^{-3})}} \text{ cm.} \quad (1)$$

D is the characteristic distance over which charges in a plasma screen the ionic fields, and thus it is a measure of the distance over which the charged particle motion is correlated. For our reference plasma $D = 3 \times 10^{-8}$ cm. The wavelength of a 1 KeV photon is 12.4×10^{-8} cm, so this photon is beginning to interact with ions in a quantum mechanical manner. Dispersion theory requires that there be many electrons per wavelength; at $h\nu = 1$ KeV we have only a few electrons per wavelength in our reference plasma, so we have also reached the limits of continuity.

For a fully ionized plasma the predominant interaction, however, will be via free-free (inverse Bremsstrahlung) absorption in which photons are absorbed by free electrons in the presence of ions. In the absence of collisions the classicality limit is given by $h\nu = mc^2 = 511$ keV. Classical theory is applicable to Bremsstrahlung if the photon energy is much less than the kinetic energy of the electron, i.e. if

$$h\nu \ll kT \quad (2)$$

In many cases, especially those involving equilibrium radiation, this criterion is not met, and the conductivity σ and permittivity ϵ should be calculated by quantum methods. We therefore have occasionally substituted quantum-mechanical absorption coefficients into otherwise classical plasma expressions in this paper.

Another criterion for classicality is that the plasma not be degenerate. The degeneracy temperature is given by

$$k T_0 = \frac{(\hbar c)^2}{m c^2} N^{2/3} \quad (3)$$

Below this temperature the free electrons will not obey classical Boltzmann statistics. For our reference plasma $k T_0 = 3.4$ eV. Thus we need temperatures $T > 10$ eV in order to apply classical plasma theory to the dense plasmas we are considering.

Another general question concerns the limits of applicability of kinetic theory. This poses an interesting and often overlooked restriction, since kinetic theory is valid only for photons of sufficiently low frequency. That is, derivations of the Boltzmann equation assume that the electron-electron or electron-ion collision time Δt is much less than the photon period $2\pi/\omega$. In other words, the Boltzmann equation derivations employed in plasma physics assume that the photon electromagnetic field is constant during e-e collision times. The usual criterion for kinetic theory to be valid is

$$\sqrt{\frac{4\pi N e^2}{m \omega^2}} = \frac{\omega_p}{\omega} \gg 1 \quad (4)$$

where ω_p is the natural plasma frequency and ω is the photon frequency. For our reference plasma $\omega_p = 3 \times 10^{16} \text{ sec}^{-1}$. Thus strictly speaking we can only use kinetic plasma theory to describe interactions with photons of frequency less than 10^{16} sec^{-1} (i.e. energy less than 10 eV). In practice, however, the kinetic expressions are usually used even when the above inequality is not upheld, and we will do the same having established this warning.

In summary the problem we have posed sometimes requires quantum mechanical cross sections but the plasma processes can be treated classically and continuously for temperatures above about 10 eV and photon frequencies below 1 KeV. For lower temperatures or higher frequencies the problem is very complex and we will not consider such ranges here.

In the following sections we explore certain aspects of plasma phenomena which can affect photon transport. No doubt other important aspects have been omitted. The discussion is only on a general level, with the intent of opening up certain questions to exposure and further research if warranted. Throughout the discussion we will estimate certain parameters relevant to the reference plasma described above. These numbers are accumulated in Table I.

We have relied heavily on the text of Ginzburg throughout this analysis. Some of the work in our present paper is original but most of these processes were examined years ago by Ginzburg, Dawson, Zel'dovich, and others in the context of low-frequency radiation in tenuous plasmas such as the ionosphere or laser-generated plasmas [1], [2], [3], [4]. In this paper we take a different approach, inspecting the potential effects on short wavelength radiation when the plasma density is very high.

A. High-Frequency Photon Propagation In Hot Dense Plasmas

A.a. Index of Refraction

Our main question here concerns indices of refraction for high frequency photons in hot dense plasmas. First we will treat simple homogeneous plasmas and then in subsequent sections we will discuss various complications that can arise.

The dense plasmas that we are considering are collision-dominated. Both electron-electron and electron-ion collisions are important. The plasma-photon interaction is complex because the interparticle spacing, the Debye length, and the photon wavelength are all comparable, and also photon absorption coefficients are not always negligible.

One way in which a plasma can interact with and affect a photon field is through the generation of longitudinal plasma waves. The plasma has its own natural oscillatory frequency, given by

$$\omega_p = \sqrt{\frac{4\pi N e^2}{m}} \quad (5)$$

which is $3 \times 10^{16} \text{ sec}^{-1}$ for our reference plasma ($h\omega_p = 20 \text{ eV}$). When this "plasmon field" resonates with the photon field, the radiation energy is converted to plasma wave (plasmon) energy. The plasma waves are damped by collisions, eventually returning the energy to the thermal pool. Thus there is a photon cutoff frequency, which occurs at 20 eV for our reference plasma, below which a radiation pulse is blocked because of plasma oscillations or the inability of the plasma electrons to radiate. In thermal equilibrium the radiation loss due to generation of longitudinal plasma oscillations must be balanced by the emission of transverse waves by the longitudinal oscillations.

The photon-plasma interaction described above is manifested in the index of refraction of the plasma. In what follows we will substitute for the photon a plane transverse electromagnetic wave, recognizing that, in the linear regime at least, the photon wave packet can be considered to be a superposition of plane waves. Following Ginzburg,^[1] the electromagnetic plane wave is described by

$$\vec{E} = \vec{E}_0 \exp i(\omega t - \vec{k} \cdot \vec{r}) \quad (6)$$

Application of the Maxwell relation $\nabla \cdot \vec{E} = 0$ gives the dispersive wave equation

$$\Delta \vec{E} + \frac{\omega^2}{c^2} \epsilon'(\omega) \vec{E} = 0 \quad (7)$$

where the permittivity $\epsilon(\omega)$ and the conductivity $\sigma(\omega)$ are defined by

$$\vec{D} = \epsilon'(\omega) \vec{E} \quad (8)$$

$$\vec{j} = \sigma(\omega) \vec{E} \quad (9)$$

Here $\vec{D}$ is the displacement and $\vec{j}$ the current. All the properties of the plasma are contained in the complex permittivity tensor $\epsilon_{ij}(\omega, \vec{k})$, which is usually written as

$$\epsilon'_{ij}(\omega, \vec{k}) = \epsilon_{ij}(\omega, \vec{k}) - i \frac{4\pi}{\omega} \sigma_{ij}(\omega, \vec{k}) \quad (10)$$

Equations (6) and (7) yield the complex dispersion relation

$$k^2 = \frac{\omega^2}{c^2} \epsilon'(\omega) = \frac{\omega^2}{c^2} \left(\epsilon - i \frac{4\pi\sigma}{\omega} \right) \quad (11)$$

which relates the magnitude of the wave vector k to the frequency ω and the permittivity ϵ . This important relation ties the photon field to the plasma properties. The first term represents the displacement current and the second represents the imaginary conduction current. In general when ω is real the wave vector k is complex:

$$\bar{k} = \frac{\omega}{c} (n - i\kappa) \frac{\bar{k}}{k} \quad (12)$$

where n is the index of refraction and κ is the index of absorption.

The plane wave equation (6) becomes

$$E = E_0 \exp \mp \left(\frac{\omega \kappa z}{c} \right) \exp i \left(\omega t \mp \frac{\omega n z}{c} \right) \quad (13)$$

and the photon wavelength in the material is seen to be

$$\lambda = \frac{\lambda_0}{n} \quad \text{where} \quad \lambda_0 = \frac{2\pi c}{\omega} \quad (14)$$

The phase velocity is then

$$v_p = \frac{c}{n} \quad (15)$$

From (11 and 12),

$$(n - i\kappa)^2 = \epsilon' = \epsilon - i \frac{4\pi\sigma}{\omega} \quad (16)$$

The index of absorption κ is simply related to the usual absorption coefficient μ by

$$\mu = \frac{2\omega\kappa}{c} = \frac{4\pi\sigma}{cn} \quad (17)$$

where the photon flux changes by a factor e over a distance $1/\mu$, i.e.

$$I = I_0 e^{-\mu x} \quad (18)$$

Solving these equations gives the following relations:

$$\epsilon = n^2 - \kappa^2 \quad (19)$$

$$n^2 = \frac{\epsilon}{2} + \left[\left(\frac{\epsilon}{2} \right)^2 + \left(\frac{2\pi\sigma}{\omega} \right)^2 \right]^{1/2} \quad (20)$$

$$\kappa = \frac{2\pi\sigma}{\omega n} \quad (21)$$

Thus we need to know the plasma permittivity ϵ and the conductivity σ in order to estimate the indices of refraction and absorption.

In a collision-dominated plasma the permittivity for an electromagnetic wave of frequency ω is given by

$$\epsilon = 1 - \frac{4\pi e^2}{m} \frac{N}{\omega^2 + \nu_{eff}^2} - \frac{4\pi e^2}{M} \frac{Z N_i}{\omega^2 + \nu_{ii}^2} \quad (22)$$

where ν_{eff} is the effective electron-ion collision frequency and ν_{ii} is the effective ion-ion collision frequency. Similarly, the conductivity is given by

$$\sigma = \frac{e^2}{m} \nu_{eff} \frac{N}{\omega^2 + \nu_{eff}^2} + \frac{e^2 Z}{M} \nu_{ii} \frac{N_i}{\omega^2 + \nu_{ii}^2} \quad (23)$$

in the limit that ion-ion collisions are unimportant, and

$$\omega \gg \nu_{eff} \quad (24)$$

these expressions simplify to

$$\epsilon = 1 - \frac{4\pi N e^2}{m \omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \quad (25)$$

and

$$\sigma = \frac{4\pi N e^2}{m} \frac{\nu_{eff}}{4\pi} \frac{1}{\omega^2} = \frac{\nu_{eff}}{4\pi} \frac{\omega_p^2}{\omega^2} \quad (26)$$

This gives for the index of absorption

$$\kappa = \frac{\nu_{eff}}{2n} \frac{\omega_p^2}{\omega^3} \quad (27)$$

and for the absorption coefficient

$$\mu = \frac{\nu_{eff}}{hc} \frac{\omega_p^2}{\omega^2} \quad (23)$$

Note that the ω^2 -dependence of this collective classical absorption coefficient differs from the ω^3 -dependence of the high-temperature quantum mechanical expression for free-free absorption. This is because stimulated emission supplies an additional factor of $(h\omega/kT)^{-1}$.

A common expression for ν_{eff} , derived from kinetic theory is

$$\nu_{eff} = \frac{5.5 N_i Z^2}{T_e^{3/2}} \left[\ln \frac{280 T_e}{Z N_i^{1/3}} + \frac{1}{3} \ln \frac{T_i}{T_e} \right] \quad (29)$$

where T_e is the electron temperature (in degrees Kelvin) and T_i is the ion temperature. Putting all this information together, we calculate for a 1 keV photon in our reference plasma ($kT=1$ keV) the following set of numbers:

$$\begin{aligned} \omega &= 1.5 \times 10^{18} \text{ sec}^{-1} \quad (1 \text{ keV}) \\ \omega_p &= 3 \times 10^{16} \text{ sec}^{-1} \quad (20 \text{ eV}) \\ \nu_{eff} &= 3 \times 10^{15} \text{ sec}^{-1} \\ \sigma &= 1 \times 10^{11} \text{ sec}^{-1} \\ \epsilon &= 0.999576 \\ \kappa &= 4.2 \times 10^{-7} \\ \mu &= 42 \text{ cm}^{-1} \quad (\text{classical}) \\ n &= 0.999788 \end{aligned}$$

These estimates are included among the reference plasma parameters listed in Table I.

Thus the index of refraction of a 1 keV photon in a simple 1 keV homogeneous plasma is unity. For this photon frequency the plasma is a poor conductor since

$$\frac{4\pi\sigma}{\omega} \ll 1 \quad (30)$$

In the other extreme, when $\omega \approx \omega_p$ (20 eV photon), then $\epsilon \rightarrow 0$ and $n \rightarrow 0$. Photons of this frequency are totally blocked by the plasma. Figure 1 shows a plot of n and n^2 versus ω for a 1 keV temperature plasma. The radiation field intensity in a dispersive medium goes as n^2 , the plasma correction to the opacity goes as n^{-1} , and the group velocity goes as n .

Figure 1

In summary, we have found (see Figure 1) that in our reference plasma the index of refraction starts falling at temperatures near 100 eV. These curves are essentially the same for $kT=1000$ or $kT=100$ eV. (Note that ω_p does not depend on temperature. In reality it does if at the lower temperatures the plasma is not fully ionized.) Neglecting bound-free processes we found that in plasmas of this character the classical absorption has a negligible effect on the index of refraction; that is, the plasma is not strongly absorbing. Finally we have shown that for the frequency range 10-100 eV the effect of collective plasma oscillations on photon transport is significant.

A.b. Pulse Propagation Velocities:

The index of refraction of a real material is a strong function of frequency. If the absorption varies strongly with frequency the effects of gradients $dn/d\omega$ can be important. In this section we investigate the effect of this frequency dependence on photon pulse propagation. We might, for example, consider a Gaussian pulse propagating through a medium. It was suggested by Jeff Paisner^[5] that even in cases where the index of refraction n is unity, parts of a pulse can be delayed if the frequency derivatives $dn/d\omega$ are large.

The philosophical questions concerning the relation between group velocity and speed of energy transport are deep, especially in plasmas where there is dissipation and dispersion. Interested readers are referred to the monograph of Brillouin on the subject^[6]. Here we shall assume, as is usually done, that the group velocity is indeed a measure of photon pulse propagation velocity. We are not describing a situation of diffusional absorption and re-radiation here; rather the propagation of a photon in a slightly absorptive medium dominated by free-free interactions.

Electromagnetic energy density, even with absorption, is given by

$$W = \frac{E^2}{8\pi} \frac{d(\omega\epsilon)}{d\omega} \quad (31)$$

This translates into a general pulse group velocity given by

$$v_g = \frac{c}{\frac{d(\omega n)}{d\omega}} \quad (32)$$

It is this expression whose implications we shall explore.

In the simplest case of a constant index of refraction, the pulse phase and group velocities are given by

$$v_p = \frac{c}{n} \quad (33)$$

$$v_g = cn \quad (34)$$

leading to the well-known relation

$$v_g v_p = c^2 \quad (35)$$

We will now show that for a homogeneous plasma with no absorption the above relation (34) still holds. In that case the index of refraction is given by

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} \quad (36)$$

The phase velocity by definition is

$$v_p = \frac{c}{n(\omega)} \quad (37)$$

while the group velocity is given by (32). Applying the derivative at the peak of the pulse gives

$$n(\omega_0) + \omega_0 \left(\frac{dn}{d\omega} \right)_{\omega_0} = n(\omega_0) + \frac{\omega_p^2}{\omega_0^2} \frac{1}{n(\omega_0)} = \frac{1}{n(\omega_0)} \quad (38)$$

Thus

$$v_g = cn(\omega_0) \quad (39)$$

and relation (34) still holds, even though the frequency derivatives have been taken into account. Therefore if the index of refraction of the pulse is unity the pulse moves with the velocity of light in a simple homogeneous plasma with no absorption.

According to the estimates in the preceding section for our reference plasma, ignoring absorption and spatial dispersion, a photon pulse will not be slowed down until the centroid frequency ω_0 is in the vicinity of the plasma frequency (20-100 eV). Below 20 eV it cannot propagate at all.

Now consider the same problem with weak classical absorption^{[7], [8]}. (If the absorption is strong the concept of group velocity becomes meaningless). In this case

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} - \left(\frac{2\pi\sigma}{\omega n} \right)^2 \quad (40)$$

assuming $\nu_{eff} < \omega$. Using (26) this may be rewritten

$$(n\omega)^4 = (n\omega)^2(\omega^2 - \omega_p^2) - \frac{\nu_{eff}^2}{4} \frac{\omega_p^4}{\omega^2} \quad (41)$$

Completing the square and applying (32) gives

$$\frac{v_g}{c} = \frac{4f \left[\frac{1}{2}(\omega^2 - \omega_p^2) + f \right]^{1/2}}{2\omega f + \omega(\omega^2 - \omega_p^2) + \frac{\nu_{eff}^2}{2} \frac{\omega_p^4}{\omega^3}} \quad (42)$$

where

$$f \equiv \frac{1}{2} \left[(\omega^2 - \omega_p^2)^2 - \nu_{eff}^2 \frac{\omega_p^4}{\omega^2} \right]^{1/2} \quad (43)$$

In the limit $\nu = 0$ this gives (34). Taking the limit $\omega \rightarrow \omega_p$ results in imaginary group velocities. The group velocity analytically goes to zero at the real roots of the cubic (for $f = 0$)

$$\omega^3 - \omega \omega_p^2 - \nu_{eff}^2 \omega_p^2 = 0 \quad (44)$$

In the regime where $\omega_p \gg \nu_{eff}$ this root gives the collisionless plasma result $\omega = \omega_p$ ($v_g = 0$). For cases where $\nu_{eff} < \omega_p$, such as in our reference plasma, there are three real roots near ω_p .

In the parameter ranges we are considering, the electron-ion collision frequency ν_{eff} is smaller than the photon frequency. Thus the collisions can be treated as independent. (Otherwise, correlations would produce complex interference phenomena.) In the above treatment we considered the classical conductivity model of free-free absorption, which assumes instantaneous interaction times. This model applies to a

weakly ionized gas and results in the $1/\omega^2$ dependence of the absorption as exhibited in equation (28). In a fully ionized plasma with finite collision times and a Maxwellian velocity distribution the inverse Bremsstrahlung absorption coefficient is given by^[3]

$$\mu_{ff} = \sqrt{\frac{2\pi}{3mkT}} \frac{4}{3} N_i \frac{Z^2 e^6 N_e}{hc n \omega^3} g \left(1 - e^{-h\nu/kT}\right) \quad (45)$$

where g is the Gaunt factor which folds in quantum mechanical effects. Note this absorption has a $1/\omega^3$ dependence because of the contribution from stimulated emission. Including the index of refraction correction, we can re-write equation (45) as

$$\mu_{ff} = \alpha \frac{\omega_p^2}{n\omega^3} \quad (46)$$

where

$$\alpha = \sqrt{\frac{2\pi}{3mkT}} \frac{8}{3} \frac{N_i Z^2 e^4 \pi^2}{hc} g \left(1 - e^{-h\nu/kT}\right) \quad (47)$$

which gives

$$\mu_{ff} \approx 10 \frac{Z^2 N_i}{T^{1/2}} \frac{\omega_p^2}{n\omega^3} \text{ cm}^{-1} \quad (48)$$

where T is in $^{\circ}\text{K}$.

For our reference plasma this "free-free" absorption coefficient is 1.4 cm^{-1} , which is smaller than the classical value of 42 cm^{-1} . Note that the analysis leading to the above absorption coefficient assumes a thermal ion distribution. In case of a non-thermal ion density

Correlation the low-frequency absorption coefficient can be much stronger^[2]. We shall re-define the conductivity in terms of the above high-frequency free-free absorption coefficient, keeping the weak absorption expression (40) for the index of refraction. Then

$$\mu_{ff} = \frac{4\pi\sigma}{cn} \quad (48)$$

defines σ and we keep

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} - \left(\frac{2\pi\sigma}{\omega n} \right)^2 \quad (49)$$

or,

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\beta^2 \omega_p^4}{n^2 \omega^3} \quad (50)$$

where

$$\beta = \frac{c\alpha}{2} \approx 5 \times 10^{11} \frac{z^2 N_i}{(T \text{ deg})^{3/2}} \quad (51)$$

For our reference plasma $\beta = 3 \times 10^{32} \text{ sec}^{-1}$, which is of the order of v_p^2 . The solution for the group velocity is then

$$\frac{v_g}{c} = \frac{4f \left[\frac{1}{2}(\omega^2 - \omega_p^2) + f \right]^{1/2}}{2\omega f + \omega(\omega^2 - \omega_p^2) + 4 \frac{\beta^2 \omega_p^2}{\omega^3}} \quad (52)$$

where

$$\xi \equiv \frac{1}{2} \left[(\omega^2 - \omega_p^2)^2 - \frac{\beta^2 \omega_p^4}{\omega^4} \right]^{1/2} \quad (53)$$

This group velocity goes to zero at the real roots of the quartic

$$\omega^4 - \omega^2 \omega_p^2 - \beta \omega_p^2 = 0 \quad (54)$$

These roots are given by

$$\omega_c^2 = \frac{1}{2} \left[\omega_p^2 + \omega_p (\omega_p^2 + 4\beta)^{1/2} \right] \quad (55)$$

For our base 1 KeV plasma this gives a cutoff frequency only slightly above the plasma frequency ($\omega_p = 20$ eV). That is, the free-free absorption in our plasma is not quite strong enough to affect the cutoff frequency. At lower temperatures it will, however, soon begin to raise the cutoff frequency. For example the cutoff frequency for photons entering a 10 eV plasma ($\beta = 3 \times 10^{33}$) is approximately 70 eV (ignoring bound-free effects on ω_p).

4. c. Plasma Effects in Front Propagation

We finish our study of photon propagation in dense homogeneous plasmas by commenting on the effect of collective plasma effects on front propagation. We begin with Dawson's expression for blackbody radiation inside a weakly absorbing plasma, [2], [9].

$$P_{\omega} = 2 \frac{(\hbar\omega)^3}{(\hbar c)^2} \frac{1}{\exp(\hbar\omega/kT-1)} \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad (56)$$

This is the specific intensity inside a thin plasma sample surrounded by Planckian radiation. Figure 2 shows the low energy end of the Planckian intensity B_{ν} compared with the intensity P_{ν} in our reference plasma at a temperature of 1 KeV.

Figure 2

The total radiation energy in the Planckian is

$$W = \frac{4\pi}{c} \int_0^\infty B_\nu d\nu = a T^4 \quad (57)$$

with equal partition between the electric and magnetic modes. Inside the plasma, on the other hand, some of the energy is removed from the radiation field and given to collective electron kinetic energy (e.g., plasma waves). The partition becomes

$$W_E/W_T = 1/2$$

$$W_M/W_T = 1/2 (1 - v_p^2/c^2)$$

$$W_e/W_T = 1/2 v_p^2/c^2$$

Consider next a slab of material on which impinges at $t=0$ a constant beam of high-frequency photons. The photons are absorbed in the medium, raising its temperature. At some future time a radiation front has moved through the material, heating up the material behind the front. Suppose that the material behind the front is composed of our base carbon plasma at $kT=1$ KeV. The temperature distribution looks like the illustration on the next page (Figure 3).

Figure 3

The material at the leading edge of the front is at temperatures of 10-100 eV. Ignoring bound-free effects, note that the plasma frequency ω_p is independent of temperature, so the cutoff frequency for the material at the leading edge is 20 eV. In the absence of plasma effects the thermal re-radiation of this cooler material is shown by the solid line in Figure 4; the blocked-out region shows the amount of energy that actually gets deposited in collective electron motion. This is roughly 25% of the re-radiation energy for a leading edge at a temperature of 10 eV.

Thus as the front sweeps through the cold material, there is constant generation of plasma waves at the leading edge, where the thermal re-radiation encounters strong refractive effects. The high frequency photons coming from behind the front continue to heat the material at the leading edge because they are not inhibited by plasma effects.

At any instant of time the amount of energy bound in collective motion at the leading edge itself is very small compared to the total energy behind the front. However if the material were infinitely long (many high-frequency photon mean-free-paths) and the lifetime of the plasmons were infinite, eventually most of the radiation energy could be converted to plasma oscillations, which are steadily absorbing some of the radiation energy at the leading edge, thus lowering the material temperature.

In reality the lifetime of the plasma waves is not infinite. We can estimate the time it takes plasma waves to damp out through electron-ion collisions thus returning their collective energy to the thermal pool. The characteristic time for this energy transfer is

$$\Delta T = \frac{M}{m} \frac{1}{\nu_{ei}} \quad (58)$$

since in one elastic collision the electron loses a fraction of its energy proportional to the masses. For our reference plasma this gives $\Delta T \approx 10^{-11}$ sec. Thus the energy is retained in electron motion only for this short amount of time before it is returned to the equilibrated thermal pool.

To our knowledge, processes such as this have not been considered in the literature. In most cases the net effect of collective plasma oscillations on front propagation is so small that it is entirely negligible, both because of the strong damping of collective electron motion through electron-ion collisions and also because the amount of radiation energy in the leading edge is very small compared to the total. It is possible, however, that complicating features such as those described in the rest of this paper, particularly in the nonlinear range, can enhance the degradation of a moving radiation front to the point where the energy balance is noticeably affected.

B. Complicating Features

In the preceeding section we made many simplifications and thus ignored certain complicating features which are often present in a complex plasma. In the rest of the paper we shall briefly describe some of these complications and try to estimate their importance. We shall consecutively discuss the effects of spatial dispersion, inhomogeneities, fluctuations, and nonlinear phenomena.

C. a. Spatial Dispersion and Absorption

By concentrating on high-frequency photons and not dense plasmas, we are inviting spatial dispersion. "Spatial dispersion" for a homogeneous medium technically means that the permittivity ϵ is a function of the photon wave number k as well as the photon frequency ω . (The dependence of ϵ on ω , which we have already treated in Section A is called "frequency dispersion", and this is what occurs in what is usually referred to as a "dispersive medium".) In a spatially dispersive medium the permittivity ϵ is generally a tensor, as indicated in equation (10), and its components are intimately tied to the detailed structure of the medium. In a homogeneous medium k and ω are related by the "dispersion relation" (for example equation (11) in Section A). In fact, if ω is independent of k the plasma waves will transfer no energy (the group velocity is zero). To find the correct function for $\omega(k)$ we must take spatial dispersion into account.

Physically, the process leading to spatial dispersion occurs when a free electron is not interacting with a uniform field. Thus spatial dispersion can result from thermal motion at high temperature, from strong absorption when the collision frequency is comparable to the

electron oscillation frequency, or from high-frequency photon fields whose wavelength is of the order of the interparticle spacing. Another source of spatial dispersion is magnetic fields. The process leading to spatial dispersion occurs when the inertial time scales of the electron processes are of the same order of magnitude as the time scales for the electromagnetic field to change; then the polarization and displacement of the medium at some time are not simply determined by the value of the field at that instant. All of these processes can be important in the types of plasma we are contemplating^[1].

We have already treated some aspects of weak spatial dispersion in Section A by including weak absorption in the conductivity terms. For cases of stronger spatial dispersion and absorption the characteristic plasma frequency will not be given simply by ω_p (Eq. (5)) and the concept of group velocity becomes meaningless. Absorption can be considered to be strong when the collision frequency is of the same order of magnitude as the plasma oscillation frequency (i.e. $\nu_{eff} \approx \omega_p$). For our reference plasma, $\nu_{eff} = 10^{15} \text{ sec}^{-1}$ and $\omega_p = 10^{16} \text{ sec}^{-1}$, so absorption would appear to be weak but not negligible. However these classical estimates are crude, and in fact "absorption" covers a variety of complex processes which can feed back and change the actual plasma frequency. For example if the absorption coefficient is strongly frequency-dependent (e.g. spectral lines or edges) then it can of course lead to spatial dispersion.

If the collision frequency is small, then another measure of spatial dispersion is that $kD \ll 1$, where D is the Debye length given by Eq. (1) and k is the wave number of the photon. This criterion compares the photon wavelength λ to the characteristic interparticle distance. If $kD \ll 1$ then $D \gg \lambda$ and the electron is free to respond instantaneously to the photon field without interference from other particles. The Debye

length folds in thermal velocity effects. For our reference plasma and a 1 keV photon, $kD \approx 1$ ($k = 0.5 \times 10^{-8}$ cm and $D = 3.05 \times 10^{-8}$ cm).

Therefore spatial dispersion can be an important problem, even neglecting collisions, just because of the density of the plasma.

One important aspect of spatial dispersion is that even when slight it can bring in new effects. Thus we saw in Section A that weak dispersion can lead to longitudinal plasma waves with finite group velocities (which can therefore carry energy). The interesting effects associated with generation of polarisation are numerous, as are the effects introduced by a magnetic field. We shall discuss some of these processes briefly later. Generally the greater the spatial dispersion the more energy that can be invested in these phenomena.

The above arguments apply to the plasma wave as well as to the photon. That is, the plasma wave has its own wave number k_p (which for our plasma is 1.06×10^6 cm⁻¹, so $k_p D = 0.03$, which is weakly dispersive). We can define an index of refraction $n_{\perp}$ for the transverse wave (the photon) and another index of refraction $n_{\parallel}$ for the longitudinal wave (the plasma wave).

In the case of no absorption and weak spatial dispersion one can form the tensor^[1]

$$\epsilon_{ij}(\omega, \vec{k}) = \epsilon_{\perp}(\omega, k^2) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + \epsilon_{\parallel}(\omega, k^2) \frac{k_i k_j}{k^2} \quad (59)$$

with the approximations

$$\epsilon_{\perp} = \epsilon' - \int_{\perp} k^2 \quad (60)$$

$$\epsilon_{\parallel} = \epsilon' - \int_{\parallel} k^2 \quad (61)$$

which leads to the following dispersion relations:

$$k^2 = \frac{\omega^2}{c^2} (\epsilon - \delta_{\perp} k^2) \quad (62)$$

$$\epsilon - \delta_{\parallel} k^2 = 0 \quad (63)$$

Thus we get the transverse and longitudinal indices of refraction

$$n_{\perp}^2 \equiv \frac{c^2 k^2}{\omega^2} = \frac{\epsilon}{1 + \omega^2 \delta_{\perp} / c^2} \quad (64)$$

$$n_{\parallel}^2 \equiv \frac{c^2 k^2}{\omega^2} = \frac{\epsilon}{\omega^2 \delta_{\parallel} / c^2} \quad (65)$$

From these we see that when $\delta_{\perp}$ is small, the transverse wave (photon) is not affected by spatial dispersion. But the plasma wave is always affected by spatial dispersion through $\delta_{\parallel}$. Therefore, in plasmas where plasma waves are carrying some of the energy, the spatial dispersion, even neglecting absorption, is very important. From previous arguments, the coefficients $\delta_{\perp}$ and $\delta_{\parallel}$ should be compared with D^2 ; we have estimated that $\delta_{\perp} D^2 = 1$ and $\delta_{\parallel} D^2 = 10^{-2}$. Therefore both are probably important for our reference plasma.

Obviously the possibility of strong spatial dispersion, which is quite real for our reference plasma, brings with it a whole set of complicated phenomena which we cannot calculate, nor can we adequately describe them in a survey paper like this. One such phenomenon that deserves mention is Cherenkov emission of the plasma waves. If $kD \approx 1$ then it is possible to satisfy $\omega = \vec{k} \cdot \vec{v}$ for the longitudinal waves and thus produce Cherenkov radiation (and therefore absorption) from plasma waves. This in fact is the process through which plasma waves are damped in a collisionless plasma. (For transverse waves in a plasma Cherenkov emission is not possible.)

So far we have not dealt with strong absorption, and we cannot do so quantitatively here because of the complexity of the problem. Obviously strong absorption, which is necessarily coupled with strong spatial dispersion, poses a formidable calculational problem. Yet it can be a very real problem for hot dense plasmas. It would take an intense calculational and experimental effort to evaluate the effects of strong spatial dispersion combined with strong absorption on the total energy balance of a hot plasma.

B.b. Inhomogeneous Plasmas

This presents a vast and complex subject, and we shall but briefly investigate three common inhomogeneities found in hot dense plasmas, and their effect on photon transport. They include nonuniformities in the density distribution, in the material velocity field, and in an imposed or self-generated magnetic field.

B.b.1. Density Gradients

The condition for geometric optics to be valid in a weakly absorbing medium is that^[1]

$$\frac{\lambda \left| \frac{dn}{dz} \right|}{2\pi n^2} \ll 1 \quad (66)$$

It is possible that n can be close to unity but that its derivative dn/dz is very large. For example consider a density distribution $N(z)$ (which could include some sharp changes). Then the index of refraction is given by

$$n^2(z) = 1 - \frac{\omega_p^2(z)}{\omega^2} = 1 - \frac{4\pi e^2 N(z)}{m \omega^2} \quad (67)$$

The derivative dn/dz is proportional to the density gradient dN/dz . When dn/dz becomes large enough that equation (66) is no longer satisfied, reflection off steep density gradients can occur. The coefficient of reflection will be large only if the distance over which n changes is of the order of $\lambda/2\pi$ or less, where λ is the wavelength of the photon. Thus for our reference plasma we would need n to change over a distance of the order of 10^{-7} cm for a 1 keV photon in order to observe reflection from such a density gradient. Such a change is not likely for the high frequency photons under consideration since λ is close to atomic dimensions. Therefore we do not expect that high-frequency photon transmission will not be affected by large density gradients when absorption is absent. One possible exception to this conclusion is the shock front discussed in the last section.

We might also ask whether a density gradient could affect the group velocity of a pulse, which will be given by

$$v_g(z) = \frac{c}{d\{\omega n(z)\}/d\omega} \quad (68)$$

We shall answer this question in the next section, where we treat density and velocity gradients together.

B.b.2. Velocity Fields

Some plasmas of astrophysical significance can possess extremely high velocity fields. Lerche has studied the effects of such fields in a dispersive medium^[10]. The moving medium can act as a prism, displaying anomalous effects when the index of refraction is less than unity^[101,VI].

Furthermore, even when v/c is so small as to be ordinarily negligible, if it multiplies terms containing large derivatives, those terms cannot always be neglected. We shall now investigate such terms.

Consider an inhomogeneous medium of index of refraction $n(x, \omega, v, \dots)$ moving along the x -axis. (For simplicity we will hold this discussion to one dimension, ignoring some interesting angle-dependent features such as Snell's law for moving media.) The medium has a velocity distribution $v(x)$ and a density distribution $N(x)$ which determines $n(x)$ where n is the index of refraction. Suppose an external observer measures the phase and group velocities of a pulse that has entered a slab of length dx . This is the famous Fizeau experiment. The phase velocity with respect to the co-moving frame of the medium is $u' = c/n(x)$. The observer at rest, however, measures

$$\frac{u}{c} = \frac{1}{n} + \frac{v}{c} \left(1 - \frac{1}{n^2} \right) \quad (69)$$

We would like to investigate how this phase velocity can change when there are gradients over the slab. Differentiating by using the chain rule we get, to order v/c ,

$$\frac{n}{c} \frac{du}{dx} = -\frac{1}{n} \frac{dn}{dx} - \frac{v}{cn^2} \omega \frac{dn}{d\omega} + \frac{dv}{dx} \frac{n}{c} \left(1 - \frac{1}{n^2} \right) \quad (70)$$

From this we see how the various gradients affect the phase velocity. The group velocity, which for weak absorption equals the velocity of energy transfer, is given by

$$v_g = u + \omega \frac{du}{d\omega} \quad (71)$$

Thus, again applying the chain rule and dropping second derivatives,

$$\frac{dv_g}{dx} = \frac{du}{dx} + \frac{d\omega}{dx} \frac{\partial u}{\partial \omega} = \frac{du}{dx} + \frac{v\omega}{nc} \frac{\partial u}{\partial \omega} \quad (72)$$

Combining (70) and (72),

$$\frac{1}{c} \frac{dv_g}{dx} = -\frac{1}{n^2} \frac{dn}{dx} - \frac{v(x)}{c n^3} \omega \frac{dn}{d\omega} + \frac{1}{c} \frac{dv(x)}{dx} \left(1 - \frac{1}{n^2}\right) + \frac{v(x)\omega}{nc} \frac{dv/d\omega}{(73)}$$

The first term gives the effect of a density gradient, the second and fourth terms the effect of the frequency dispersion, and the third term describes the effect of the velocity gradient.

Let us estimate the strength of these terms for our reference plasma, assuming arbitrarily that $v/c = 10^{-2}$, and making some reasonable assumptions about the gradients. Suppose there is a density gradient over the scale of microns, leading to changes in n of the order of 10^{-3} . Then $dn/dx \approx 10 \text{ cm}^{-1}$, where n is near unity and

$$\Delta v_g \approx -c \Delta x \frac{dn}{dx} \approx 10^{-3} c \quad (74)$$

or in this case the group velocity, which for a homogeneous medium would equal c , has slowed down by one part in a thousand because of the density gradient. That is, it takes a little longer for energy to be transported through a density gradient.

Next consider a velocity gradient over the scale of microns, say $d(v/c)/dx \approx 100 \text{ cm}^{-1}$. If n is very close to unity then the factor $(1-1/n^2)$ in the third term is essentially zero, and the velocity gradient doesn't matter. Suppose $n = 0.9$; then $(1-1/n^2) = -0.2$ and $\Delta v_g = -\Delta x d(v/c)/dx = 2 \times 10^{-3}$. Again the pulse slows down about one part in a thousand.

Finally consider the dispersive gradients $dn/d\omega$. For example, consider the effects of an edge such that $\Delta(\hbar\omega) = 10 \text{ eV}$ and $\Delta n = 10^{-2}$, for $\hbar\omega = 1 \text{ keV}$. Then $\omega dn/d\omega \approx 1$ and the second term yields $\Delta v_g = c \Delta x (v/c) \approx 10^{-6} c$. This is a negligible change in the group velocity.

The above perturbations on a 1 KeV photon are fairly small. In the 10 - 100 eV frequency range however, where n changes rapidly, the density and velocity gradient contributions to the slowing down of energy transfer can be much higher than in the above estimates. For example, if $\Delta n / \Delta x = 0.1 / 10^{-6} = 10^5 \text{ cm}^{-1}$ and $n = 0.9$, then $\Delta v_g = - .12c$, and the pulse propagation is delayed by about 10%.

Another consequence of a velocity field is that electron and ion density correlations can greatly affect the free-free absorption cross section. It is possible that the absorption becomes enhanced by orders of magnitude in the presence of strong nonthermal velocity and/or density gradients.[2]

We have not discussed here such phenomena as the effects of nonuniform electric fields, nor have we discussed effects of temperature and pressure gradients. These will have to be reserved for future investigation, although we will allude to them again in the last section.

B.b. 3. Magnetic and Electric Fields

This is a vast subject and we shall only try to summarize some aspects of particular interest to the transport of high-frequency waves in a hot dense plasma^{[1], [4]}. The magnetic or electric field could be generated either externally or internally through fluid plasma flow. We will concentrate on the effect of such a field, however it was established, on the collective properties of the plasma with respect to photon transport.

Given a magnetic field of strength H , the parameter of interest is the gyration frequency of the electrons

$$\omega_H = \frac{eH}{mc} \quad (75)$$

In our reference plasma, for ω_H to equal the plasma frequency ω_0 would require a magnetic field of 2×10^9 gauss. Obviously this is a much higher magnetic field than is found in the laboratory, so we will only investigate what basic differences a small magnetic perturbation (such that $\omega \gg \omega_H$) could have on the complex permittivity tensor ϵ_{ij} .

Basically the important contribution of a weak magnetic field is to supply a direction for anisotropy in the plasma. A plasma in a very weak magnetic field can exhibit marked activity because ϵ_{ij} is not real. The index of refraction takes on two values, one for the ordinary wave and one for the extraordinary wave, such that, for example, when the electromagnetic wave travels across the magnetic field,

$$n_e^2 = 1 - \frac{\omega_p^2}{(\omega^2 - \omega \omega_H)} \quad \text{extraordinary wave} \quad (76)$$

$$n_o^2 = 1 - \frac{\omega_p^2}{(\omega^2 + \omega \omega_H)} \quad \text{ordinary wave} \quad (77)$$

Thus an incoming plane wave splits into two separate components which have separate velocities, absorptions, and polarizations. These waves in a plasma will be elliptically polarized. This is in contrast to the three normal waves which exist in an isotropic plasma (two transverse and one longitudinal). When the wave travels parallel to the magnetic field the plasma exhibits "stop bands" for right and left circularly polarized waves where electromagnetic waves in certain frequency ranges cannot propagate.

Suppose for example that our reference plasma is subjected to a field of 10 MG. Then $\omega_H = 1.76 \times 10^{14} \text{ sec}^{-1}$, which is bigger than the collision frequency. A 30 eV wave would be split into two components with indices of refraction $n_e^2 = 0.5482$ and $n_o^2 = 0.5517$. From this we conclude that for the photon frequencies of interest in our reference plasma the effect of reasonable magnetic fields will not be large.

Strong electric fields are not usually considered in fully ionized plasmas because of the strong forces to preserve macroscopic charge neutrality. Spitzer has estimated that on a scale of 10 Debye lengths the charge neutrality is good to 1%^[11]. For high frequency waves in dense plasmas such as we are considering, however, the local electric fields could be very high on the scale of a wavelength. Unfortunately we cannot deal with such a problem using continuous dispersion theory.

At the boundaries of a plasma (for example when the containing wall is a solid surface) a potential gradient will develop naturally in order to maintain charge neutrality in the interior, because the plasma electrons have higher thermal velocities than the ions. In this "wall sheath", whose thickness is approximately a Debye length, neutrality is not preserved. In fact the potential energy changes by an amount kT over a Debye length, leading to large field gradients. For our plasma this gradient would amount to

$$\frac{kT}{eD} = 3 \times 10^7 \frac{\text{V}}{\text{cm}} \quad (78)$$

This is also the order of magnitude of the local atomic electric fields for our plasma, where $\lambda \approx 0$ for a 1 keV photon.

One consequence of a strong field gradient is that it introduces a nonlinearity into the problem, since the permittivity ϵ depends on the electric field through the resulting density gradient. The change in density depends on the wave field E , which in turn is determined by nonlinear equations. Such nonlinear processes are described in the next section.

B.c. Nonlinear Phenomena and Fluctuations

Many aspects of plasma dynamics are inherently nonlinear, including those involving unstable hydrodynamic motion. We shall discuss here only a small portion of the nonlinear problem, namely the effect of a strong field

$E = E_0 e^{i\omega t}$ on an isotropic plasma. The basic criterion for an electromagnetic process in a plasma to be non-linear is that^[1]

$$E_0^2 > E_p^2 \quad (79)$$

where E_0 is the amplitude of the electromagnetic field $E = E_0 e^{i\omega t}$ and E_p is the "plasma field" given by

$$E_p^2 = \frac{3m k T}{e^2} \delta (\omega^2 + \nu_{ee}^2) \quad (80)$$

(or, more simply, $E_p^2 \propto NT$). Here δ is the fraction of energy transferred from an electron to an ion in one collision; in the elastic limit $\delta = 2m/M$. We can re-write the above equation as

$$E_p^2 = 12 \pi k T \delta N \frac{\omega^2}{\omega_p^2} \quad (81)$$

Inserting the values for our reference plasma, we find that

$$E_p^2 = 4 \times 10^{23} \text{ keV/cm}^3 \text{ (1 keV photon)}$$

$$E_p^2 = 4 \times 10^{21} \text{ KeV/cm}^3 \text{ (100 keV photon).}$$

In the equilibrium limit, the field E_0 can be estimated from the Poynting vector, assuming photon superposition:

$$\frac{E_0^2}{8\pi} \approx \frac{c T^4}{c} \quad (82)$$

This gives for our plasma $E_0^2 = 5 \times 10^{23} \text{ keV/cm}^3$. From this rough estimate we conclude that in the limit of high densities and high temperatures, (such as are found in our reference plasma), electromagnetic processes begin to become nonlinear for photons of frequency less than about 1 keV. In the equilibrium limit

$$\frac{E_0^2}{E_p^2} \sim T^3 \quad (83)$$

so that as temperature increases the nonlinear problems become worse.

The nature of nonlinear phenomena depends on the collision rate. If the plasma is collisionless the nonlinearity manifests itself through wave-particle and wave-wave interactions. If the plasma is collision-dominated the nonlinearity appears in the form of runaway heating effects or filamentation.

Consider first a collisionless plasma. A high-frequency electromagnetic wave cannot directly produce plasma oscillations if $\omega \gg \omega_p$. In the nonlinear range, however, two high-frequency waves can beat to produce a wave with frequencies near ω_p , or a wave can self-interact and produce harmonics. Thus in the nonlinear range it is possible to invest much more energy in collective plasma oscillations because they can be indirectly generated by interacting high-frequency waves. In the nonlinear range the scattering and exchange of energy between the various types of wave (plasma waves, transverse waves, harmonics, etc.) is much more probable than in the linear range.

In a collisionless plasma the nonlinearity manifests itself through density changes, because the density N depends on the field E_0 through Maxwell's relation

$$\nabla \cdot \vec{E} = 4\pi e (N - N_0) \quad (84)$$

and therefore the permittivity ϵ becomes a function of the electromagnetic field (which is basically what we mean by nonlinearity). In this nonlinear situation superposition no longer holds.

In a homogeneous plasma transverse waves have $\nabla \cdot E = 0$, but this is not true for longitudinal waves. Hence the interaction of transverse waves with plasma waves is nonlinear. In an inhomogeneous plasma $\nabla \cdot (eE) = 0$ so one can have a density gradient in a constant field E if $\text{grad } \epsilon \neq 0$. The changes will be of order

$$\frac{\Delta N}{N} \sim \frac{\Delta \epsilon}{\epsilon} \sim \frac{\Delta E}{E} \quad (85)$$

They only exist during the short time that ΔN exists. Note that if the wave equation becomes nonlinear the possibility for soliton solutions exists.

In a plasma with collisions the nonlinearity can manifest itself through changes in the electron velocity as well as through density gradients. Changes in velocity are usually more important because they last longer. In the following we present Ginzburg's analysis of such a process^[1]. An intense field E affects the collision frequency ν_{eff} so that the effective electron temperature changes. Thus in a strong field T_e depends on E . In the simplest model of energy balance, the work the electric field does on the plasma electrons is equated to the energy they lose in collisions with the ions:

$$e N (\vec{r} \cdot \vec{E}) = \frac{d}{dt} \left(\frac{3}{2} N k T \right) - \frac{3}{2} \delta \nu_{\text{eff}}(T) N k (T - T_e) \quad (86)$$

with the equation of motion of the electron giving r :

$$m \frac{d \vec{r}}{dt} = e \vec{E} - m \nu_{\text{eff}}(T) \vec{r} \quad (87)$$

If the random velocity of the electron is much greater than the directed velocity $|\vec{r}|$, then these equations are solvable and in the high-frequency case ($\omega \gg \delta \nu_{\text{eff}}$)^[1].

$$k (T - T_e) = e^2 E_0^2 / 3 \delta m (\omega^2 + \nu_{\text{eff}}^2) \quad (88)$$

This is the mean value about which the temperature will fluctuate. To first order it is not time-dependent. The relaxation time for temperature is $1/\delta = 10^5$ times greater than the momentum relaxation time $1/\nu_{\text{eff}}$ (which is 10^{-15} sec for our reference plasma in

equilibrium). Therefore the temperature takes about 10^{-10} seconds to reach its adjusted value. If in addition the directed electron velocity is large, as in early times or in strong fluid flow, then δ and v_{eff} depend on r and the problem gets extremely complicated. The nonlinear effect of the field E is apparent if we write

$$\frac{T_e}{T} = 1 + \frac{E_0^2}{E_p^2} \quad (89)$$

thus when $E \ll E_p$ the plasma is only slightly perturbed and $T_e \approx T$.

The permittivity now becomes

$$\epsilon' = 1 - \frac{\omega_p^2}{\omega[\omega - i \nu_{eff}(T_e)]} \quad (90)$$

where $\nu_{eff}(T_e)$ is given by (29) as a consequence of kinetic theory.

Using the above expressions Ginzburg calculates the self-interacting field for a strong high frequency plane wave entering the ionosphere. He finds that the amplitude of the wave saturates at a value given by

$$E_0(Z) = 2E_p e^{-\kappa Z} \quad (91)$$

where κ is the absorption coefficient. This is shown in the diagram below.

Figure 5

The remaining energy has been converted to the thermal pool. it does not get transmitted as electromagnetic radiation.

Physically the wave heats up the initial layers of plasma through the nonlinear heating described above. The absorption coefficient κ decreases rapidly with T_e , so the self-interaction increases rapidly with $E_0(z)$. The wave eats into the plasma, where heating drives $E_0(z)$ to the "plasma field" value $2E_p$. It then cruises through the plasma with reduced absorption. Ginzburg plots the following values for $E_0(z)/E_p$ as a function of $E_0(0)/E_p$:

To summarize, the perturbations caused in a plasma by a wave with $E_0 > E_p$ affect its own propagation. In the ionosphere (where molecules are important) this causes the intensity to saturate at a value determined not by its initial amplitude but by E_p . It is possible that similar saturation effects could occur in a fully ionized plasma as well. We plan to investigate the temperature dependence of such effects. Perturbations caused by an intense wave can also affect the transmission of other, perhaps weaker, waves because the permittivity ϵ and conductivity σ have been changed. As was mentioned previously, cross-modulation between different waves is also possible. Thus some of the original electromagnetic wave energy can be converted to plasma wave energy.

Another related problem concerns the effects of thermal fluctuations in a hot dense plasma. Thermal density fluctuations go as $\sqrt{N}$; therefore in our reference plasma the density fluctuations δN are of the order of 10^{11} cm^{-3} . These fluctuations occur over a Debye length. Short wavelength photons and plasma waves can scatter from these microscopic inhomogeneities convert into one another. Basically this occurs because the density fluctuation can acquire some polarization. For example, if a density fluctuation is smaller than a wavelength, the local material can acquire an induced dipole moment. Such a dipole can emit both plasma and electromagnetic radiation. Scattering from fluctuations is a non-linear phenomenon. Such processes are not necessarily limited to the frequency range where $\omega = \omega_p$. They are more likely to occur in situations where the intensity of the electromagnetic pulse is high or where the material is undergoing complex hydrodynamic changes. A treatment of such cases is beyond the scope of this paper.

Thermal fluctuations can also render an electromagnetic pulse incoherent. For example, if a diagnostic electromagnetic pulse enters a dense plasma, it is possible that a detector on the other side of the plasma would receive no signal because the phases of the waves could randomize through scattering and cancel one another.

Yet another consequence of nonlinearity is that the inverse Bremsstrahlung absorption coefficients can change dramatically when the electron motion becomes dominated by the radiation field amplitude.

Clearly nonlinear phenomena, including those associated with fundamental thermal fluctuations, cannot be ignored in a study of electromagnetic processes in hot dense plasmas.

C. Photon Transfer in Shocked Plasmas

One last subject with which we would like to deal in a qualitative manner is the question of what happens to an electromagnetic wave when it propagates through a shock. In section B.b.1 we pointed out that electromagnetic waves are reflected from density gradients if the gradients are sharp enough to satisfy equation (66). We shall now examine shocked plasmas to see if reflection of high-frequency photons off strong shocks is possible.

First we need to describe the detailed structure of a shock in a hot dense plasma. Many authors have investigated this structure, both theoretically and computationally as well as experimentally. Extensive references are given by Zel'dovich and Raizer, [3], [12], [13]

In the absence of magnetic fields, the detailed shock structure has the general features illustrated in Figure 7.

The shock contains a region of large gradients in density, velocity, and pressure. The spatial width of this region is of the order of the ion-ion collision mean free path, which for our reference plasma is 3×10^{-7} cm. or 10 D.

This gradient region is not electrically neutral, despite the strong forces of charge neutrality. The reason for this polarization is the greater mobility of the electrons compared to the ions, which results in charge separation over the gradient region. Ahead of the gradient region there is a thermal precursor dominated by electron thermal conduction and radiative transfer. Behind the gradient region is a broad transition region where thermal equilibration is taking place, mostly via electron-ion Coulomb interactions. If thermal radiation fields are important they tend to smooth out the discontinuities.

In the plasma we are considering, an interesting feature is that even when the index of refraction is essentially unity throughout the plasma (i.e., if the photon frequency is high) the derivatives dn/dz and d^2n/dz^2 can be large at the gradient region of the shock. This is illustrated in Figure 7h. For example, suppose the density discontinuity in the shock quadruples our reference plasma density over a Debye length. Then

$$\frac{dN}{dx} \sim \frac{12 \times 10^{23}}{3 \times 10^{-7}} \sim 10^{30} \text{ cm}^{-4} \quad (92)$$

The gradient of the index of refraction is

$$\frac{dn}{dx} = \frac{2\pi e^2}{mn\omega^2} \frac{dN}{dx} \sim 10^3 \text{ cm}^{-1} \quad (93)$$

for a 1 KeV photon and 10^5 cm^{-1} for a 100 eV photon even though the change in n itself is only one part in a thousand (0.999576 vs. 0.998304).

The problem then reduces to investigation of the probability of reflection from a discontinuity not in n but in the gradient dn/dz . This can be analyzed in the approximation of geometric optics. The dispersive wave equation in an inhomogeneous medium is

$$\frac{d^2 E}{dz^2} + \frac{\omega^2}{c^2} \epsilon(\omega, z) E = 0 \quad (94)$$

and the solution is

$$E(z) = \frac{C_{\pm}}{[e(z)]^{1/4}} \exp \left(\pm i \frac{\omega}{c} \int_{z_0}^z \sqrt{e'(z)} dz \right) \quad (95)$$

Neglecting absorption (which also has a discontinuity) this can be approximated by

$$E(z) = \frac{E_0}{\sqrt{n(z)}} \exp \left(-i \frac{\omega}{c} \int_{z_0}^z n(z) dz \right) + E_1(z) \exp \left(+i \frac{\omega}{c} \int_{z_0}^z n(z) dz \right) \quad (96)$$

The second term is the reflected wave ($E_1 \ll E_0$). Substituting this solution into the wave equation, and assuming n is constant over the shock region (so that we retain only terms in $d^2 n/dz^2$), we can extract the reflection coefficient

$$|R| = \frac{c}{4\omega} \frac{\left| \frac{dn}{dz} \right|_2 - \left| \frac{dn}{dz} \right|_1}{n^2} \quad (97)$$

In our case where $n \approx 1$ this gives

$$|R| \approx \frac{c}{2\omega} \left| \frac{dn}{dz} \right| \quad (98)$$

Thus for a 1 KeV photon the reflection coefficient is $R(1 \text{ KeV}) \approx 10^{-4}$ and for a 100 eV photon the reflection is $R(100 \text{ eV}) \approx 10^{-2}$.

We might have anticipated such low reflectivities because we have so far used the approximation of geometric optics, equation (66). When this is valid, Fresnel's reflectivity for a sharp boundary is

$$|R|^2 = \frac{(1-n)^2}{(1+n)^2} \quad (99)$$

In the limit of strong amplitudes, however, spatial dispersion becomes important and the above analysis is not applicable. The problem also becomes nonlinear.

The above simple analysis also ignored the effects of absorption and conductivity gradients, velocity gradients, and charge separation. The complex permittivity is

$$\epsilon'(z) = \epsilon(z) - i \frac{4\pi\sigma(z)}{\omega} = (n - i\kappa)^2 \quad (100)$$

which for free-free absorption in a fully ionized plasma is

$$\epsilon'(z) = 1 - \frac{4\pi e^2}{m\omega^2} N(z) - 4\pi i \frac{a}{\omega^3} \frac{N(z)}{\sqrt{A T(z)}} \quad (101)$$

where a incorporates all the constants of equation (45). Obviously in a shocked plasma with density and temperature discontinuities the effective reflection coefficient can be quite complicated and is possibly large even for high-frequency photons. This means that photon transport from behind the shock could be inhibited.

Another different effect may be possible if the shock is viewed as an ion density pulse which is highly correlated. Dawson has shown that ion density waves can increase the absorption coefficient by many orders of magnitude^[2]. By analogy perhaps the presence of a shock could greatly enhance the free-free absorption coefficient if the effective wavelength of the shock were of the right order of magnitude. The amount of absorbing material in the shock, however, is not large.

We intend to deal with such problems in detail in future papers. The subject of plasma effects in high-frequency photon transport through shocked plasmas appears to be rich and uninvestigated. We finish by speculating about possible consequences of charge separation.

First let us examine the electric field in the gradient region of a shock in a dense plasma. This field is caused by the potential difference resulting from the ion-electron separation, which is a consequence of the higher mobility of the electrons. Work is done by the ions on the electron gas such that the potential difference is maintained by the thermal motion of the electrons:

$$e d\phi = k T_e \ln(\rho_2/\rho_1) \quad (102)$$

This potential energy is extracted at the expense of the ion kinetic energy.

To estimate the electric field we need to know the distance over which the potential operates. The two relevant characteristic lengths of the plasma are the Debye length (3×10^{-8} cm for our reference plasma) and the ion-ion collision length (3×10^{-7} cm). Their ratio D/λ in fact determines the importance of charge separation effects. If $D > \lambda$ then the net charge separation tends to be small because plasma oscillations damp it out; if $D \lesssim \lambda$ then large separation can occur

(see Figure 7e-7g) and if $D \ll \lambda$ there is again a small effect^[12]. Therefore our reference plasma has characteristics that maximize the effects of charge separation in a shock.

The charge separation itself, being due to an electron-ion density perturbation, occurs over a distance of the order of the Debye length ($\delta n \sim D$). The density however is proportional to the ion-ion collision length λ . Thus

$$\frac{\delta n}{n} \sim \frac{D}{\lambda} \quad (103)$$

which is approximately 0.1 for our dense plasma. The potential gradient is given by

$$d\phi \sim \frac{kT}{e} \quad (104)$$

The electric field, which varies over the ion-ion collision length, is then

$$E = \frac{d\phi}{\lambda} = \frac{kT}{e\lambda} \sim \frac{1000 \text{ eV}}{3 \times 10^7 \text{ cm}} = \frac{3 \times 10^9 \text{ statvolt}}{\text{cm}} \quad (105)$$

This strong double electric layer moves through the plasma at the shock velocity, which for a strong shock is approximately the thermal ion velocity. As long as the shock moves in the same direction as the field it generates (which is of course the usual case), no magnetic field is generated since B is given by

$$\vec{B} = \frac{\vec{v}_s}{c} \times \vec{E} = \frac{\vec{v}_s}{c} \times \sqrt{\frac{kT}{mc^2}} \times \vec{E} \quad (106)$$

The energy density in the charge separation field is very high. For our reference plasma we estimate

$$\frac{E^2}{8\pi} \approx 4 \times 10^7 \frac{\text{statvolt}^2}{\text{cm}^2} = 4 \times 10^{17} \frac{\text{erg}}{\text{cm}^3} = 2 \times 10^{27} \frac{\text{keV}}{\text{cm}^3} \quad (107)$$

This is stronger than the microscopic "plasma field" energy density or the equilibrium thermal radiation energy density, both of which are of the order of 10^{23} keV/cm^3 . Denoting the charge separation field by E_c we then have the conditions

$$\begin{aligned} E_0 &\ll E_c \\ E_p &\ll E_c \\ E_0 &\approx E_p \end{aligned} \quad (108)$$

where E_0 is the electromagnetic wave amplitude and E_p is the plasma field. Thus the electromagnetic wave propagation is nonlinear in the plasma field but linear in the shock field.

When an electromagnetic wave impinges on the charge separation region, it encounters a material with very different properties from the surrounding plasma.

The permittivity drops from the normal electron-dominated value

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad (109)$$

to the ionic value

$$\epsilon = 1 - \frac{4\pi ze^2 N_i}{M \omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{m}{M} = 1 - \frac{\omega_i^2}{\omega^2} \quad (110)$$

Thus the cutoff frequency drops to ω_i^2 , and the conductivity drops by the ratio m/M .

Therefore the shocked region produces large gradients in the index of refraction due to charge separation:

$$\frac{dn}{dz} = \frac{1}{2n} \frac{d\epsilon}{dz} \approx \frac{1}{2n} \frac{\omega_p^2}{\omega^2} \frac{1}{D} \left(1 - \frac{m}{M}\right) \quad (111)$$

which for our reference plasma gives $dn/dz \approx 10^4 \text{ cm}^{-1}$ for a 1 KeV photon and 10^6 cm^{-1} for a 100 eV photon. Thus the charge separation can produce reflectivity effects that could be an order of magnitude larger than the effects of the density gradient alluded to in Section B.1.1.

In summary the details of shock structure in a plasma produce strong gradients which complicate the transport of electromagnetic waves by means of several processes, including possible reflection of the wave off the shock. In addition some aspects of the processes are likely to be nonlinear. Exact solution of such problems will be difficult.

Conclusion

We have tried to make a general survey of plasma processes which affect the transfer of high-frequency radiation in a hot dense plasma. At first blush the index of refraction for such radiation is unity and the speed of propagation is c , the speed of light in vacuum. However certain complicating features can change the ability of the plasma to transmit electromagnetic signals, resulting in reflections or inhibited transport. Among such features are sharp gradients in density and temperature, strong velocity fields, charge separation, correlated ion density distributions, strong magnetic fields, spatial dispersion, and density fluctuations. These effects are especially pronounced at the lower frequencies. A notable process occurs in the nonlinear range, where a plasma cannot transmit a large-amplitude electromagnetic wave because the transmitted energy saturates.

In order to be at least somewhat quantitative we defined a reference plasma whose characteristics are listed in Table I. For this plasma we attempted to estimate the importance of the various complicating features listed above for transport of photons in the 10-1000 eV energy range. In some cases our estimates may very well be inaccurate and conservative, especially for phenomena like shocks where many of the above features operate simultaneously. What we have found (so far) is that reflectivities and other cutoff phenomena due to gradients, etc. are indeed small, but not very small. At the same time some other processes, such as ion density correlations, fluctuations, and spatial dispersion, could possibly change these results by orders of magnitude. Furthermore, nonlinear saturation effects may be very large - changing all of the above estimates.

Besides pointing out the potential complexity and pitfalls of high-frequency diagnosis of future dense laboratory plasmas, which will probably be afflicted by several of the above complications, we have also proposed two new phenomena for investigation. One concerns the importance of plasma effects at the leading edge of travelling pulse or front of radiation. Under certain conditions the collective plasma effects can drain energy from the pulse; for example, if plasma waves are generated and survive for a length of time important for the problem, or if radiation at the leading edge is reflected back into the pulse. The other concerns the interaction of an electromagnetic wave with a shock front in a plasma. In this case reflection from the shock front is possible, even for relatively high-frequency photons, and the shock may also exhibit enhanced absorption properties. Both of these effects are likely to be small in the linear regimes but large in the nonlinear regimes.

Due to the extreme complexity of most of these phenomena we made many simplifications and ignored several undoubtedly important aspects. One assumption in particular was that we were always dealing with a fully ionized plasma. In fact bound-free contributions may often be significant. We also ignored many time-dependent effects, and for convenience we used blackbody equilibrium to make estimates when in fact many laboratory plasmas will not attain such equilibrium.

We conclude from these survey studies that the transport of high-frequency photons in a spatially complex and dense plasma can in many cases be significantly affected by collective plasma processes, especially if nonlinear effects are important.

Table I. Plasma Parameters

A. Definition of Reference Plasma

Temperature $kT = 1 \text{ KeV}$ $T = 1.16 \times 10^7 \text{ }^\circ\text{K}$
 Density $\rho = 1 \text{ gm/cm}^3$ $N_e = 3 \times 10^{23} \text{ cm}^{-3}$ $N_i = 5 \times 10^{22} \text{ cm}^{-3}$
 Atomic Number $Z = 6$ $A = 12$

B. Diagnostic Photon Characteristics

Energy $h\nu = \hbar\omega = 1 \text{ KeV}$ ($\hbar\omega \approx kT$)
 Wavelength $\lambda = 1.24 \times 10^{-7} \text{ cm}$ (in vacuum)
 Frequency $\omega = 1.5 \times 10^{18} \text{ sec}^{-1}$
 $f = \omega/2\pi = 2.4 \times 10^{17} \text{ sec}^{-1}$

C. Thermal and Particle Properties of Plasma

Thermal electron velocity $v_T/c = .04$
 Thermal ion velocity $v_i/c = 3 \times 10^{-4}$
 Electron-ion collision frequency $\nu_{eff} = 3 \times 10^{15} \text{ sec}^{-1}$
 ($\nu_{eff} < \omega_p < \omega$)
 Ion-ion mean free path $\ell = 3 \times 10^{-7} \text{ cm}$
 Debye length $D = 3 \times 10^{-8} \text{ cm}$
 Interparticle spacing $d = 1.4 \times 10^{-8} \text{ cm}$
 Degeneracy temperature $T_0 = 3.4 \text{ eV}$

D. Collective Homogeneous Plasma Parameters

Plasma frequency $\omega_p = 3 \times 10^{16} \text{ sec}^{-1}$ ($\hbar\omega_p = 20 \text{ eV}$)
 Permittivity $\epsilon = 0.999576$
 Conductivity $\sigma = 10^{11} \text{ sec}^{-1}$
 Index of refraction $n = 0.999788$
 Index of absorption $\kappa = 4 \times 10^{-7}$
 Classical absorption coefficient $\mu = 42 \text{ cm}^{-1}$ ($\hbar\omega \ll kT$)
 Quantum absorption coefficient $\mu_{ff} = 1.4 \text{ cm}^{-1}$ ($\hbar\omega \approx kT$)

References

- [1] V. L. Ginzburg, "The Propagation of Electromagnetic Waves In Plasmas", Ed 2., Pergamon Press, Oxford. (1970)
- [2] J. Dawson, "Radiation from Plasmas", Advances in Plasma Physics, Vol. 1, Simon & Thompson, ed., Interscience, New York (1968)
- [3] Zel'dovich and Razier, "Physics of Shock Waves and High-Temperature Hydrodynamics Phenomena", Vol. II, Academic Press, New York (1966)
- [4] T. P. Hughes, "Plasma and Laser Light", John Wiley, New York (1975)
- [5] Jeff Paisner, LLNL, Private Communication
- [6] L. Brillouin, "Wave Propagation and Group Velocity", Academic Press, New York (1960)
- [7] J. Dawson and C. Oberman, "High-Frequency Conductivity and the Emission and Absorption Coefficients of a Fully Ionized Plasma" Phys. Fluids 5, 517 (1962)
- [8] J. Stack and O. Sessler, "Bremsstrahlung in a Dense Plasma", Phys. Fluids 6, 1193 (1963)
- [9] E. Garelis, Private Memo (1973)
- [10] I. Lerche, "On The Passage of Radiation Through Inhomogeneous Moving Media.
 - I. The Plane, Differentially Sheared Medium: Ap. J. 187, 589, (1978)
 - II. The Rotating, Differentially Sheared Medium, Ap. J. 187, 597 (1974)
 - III. The Steady-State Fields of Inertial Charge Distributions: Ap. J. 188, 627 (1974)
 - IV. Radiative Transfer under Single-Particle Compton Scattering Ap. J. 191, 191 (1974)
 - VI. Dispersion Effects on Phase and Ray Paths In a Plane, Differentially Shearing Medium, Ap. J. 191, 759, (1974)
 - VII Concerning the Validity of the Geometrical Optics Approximation, Ap. J. 191, 763 (1974)
 - XIII Causality, Analyticity, Group Speed, Transparency, and Allied Phenomena, Ap. J. 222, 1020 (1978)

- [11] L. Spitzer, "Physics of Fully Ionized Gases", Interscience
New York (1962)
- [12] M. Jaffrin and R. Probstein, "Structure of a Plasma Shock Wave",
Physics of Fluids, Vol. 7, p. 1658 (1964)
- [13] D. Holm, K. Longren and S. Johnson, "Structure of Shock Implosion
in a Plasma", LA-UR-80-3290 (LASL)