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PASSIVE SEISMIC TECHNIQUES FOR GEOTHERMAL EXPLORATION

**QUARTERLY TECHNICAL PROGRESS REPORT
FOR PERIOD MARCH 1, 1977 - MAY 31, 1977**

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I. PROGRAM SUMMARY

The overall objective of the subject program is to develop geophysical methods into useful cost-effective tools for geothermal reservoir delineation and evaluation. More specifically, this program involves the development of forward and generalized inverse modeling techniques and the application of these techniques to teleseismic and gravity data for obtaining the three-dimensional velocity and density structure of a geothermal region. Our modeling capabilities are being proof-tested on data that were compiled for the Imperial Valley, California.

During the third three-month period of this program, our efforts concentrated on developing and applying various processing procedures to the seismic and gravity data and to the results of the forward modeling calculations. In parallel with the forward modeling efforts, we have formulated and developed an inversion scheme for final modeling of the processed data sets for the Imperial Valley.

A computer program was written to generate a regular grid of interpolated values of the gravity field based on a least square fit to a general quadratic surface of irregularly spaced observations. The resulting data set was subjected to extensive filtering and processing to produce a gravity data vector for input to the inversion scheme. Particular attention was paid to the baseline considerations encountered in the treatment of the gravity data and the impact of these considerations on the complexity of the inversion scheme.

The forward modeling of the near-surface sedimentary layers was completed. For consistency these data were subjected to the same processing used for the observed data base. The two data sets were then combined to produce a data vector from which we have removed the approximate effects

due to the sediments. At intermediate steps in the processing the result has been contoured and analyzed for accuracy and content which is relevant to the inversion method.

The particular inversion scheme that we have developed was formulated in such a way as to be compatible with the methods used for processing both the teleseismic and gravity data and, furthermore, to take advantage of the fact that both data sets are primarily sensitive to lateral variations in velocity and density. The method adopted also permits the incorporation of a smoothness criterion for extracting the most geologically reasonable estimates for velocity and density. Computational techniques, centering around the method of singular value decomposition, for implementing the inversion technique in an efficient manner were developed and are being tested.

II. TECHNICAL DISCUSSION

2.1 INTRODUCTION

The approach that we have adopted in the conduct of this program is a parallel development of data processing, and both forward and inverse modeling techniques. Throughout the course of this work we have paid particular attention to developing these three major program areas to be compatible with each other. This section of the report describes the results obtained to date employing this approach.

2.2 ANALYSIS OF TELESEISMIC TRAVEL TIME DATA

In the previous quarterly report (Savino, et al., 1977) we described some preliminary results of an analysis of the travel time data obtained from the Imperial Valley Array (IVA) of seismic stations. As noted in that report, the data base was reexamined and increased to include travel-time observations for 182 worldwide events, consisting of both earthquakes and underground explosions. To date, as part of the processing prior to analysis, the data have been converted to relative travel-time residuals, using as a reference station one of the better recording stations in the IVA (SUP, CRR or SGL) that is located on basement outcrop. This convention will be continued here and, for the sake of discussion, we repeat the expression used for computing relative residuals.

Given that teleseismic P-waves from a large number (m) of earthquakes and explosions have been recorded by the n stations of the IVA. Let the observed time of the m^{th} P-wave arrival at the n^{th} station be T_{nm}^{OBS} . Then we define the relative residual as:

$$\delta T_{nm} = T_{nm}^{\text{OBS}} - T_{om}^{\text{OBS}} - (T_{nm}^{\text{CALC}} - T_{om}^{\text{CALC}}) + C_{nm} .$$

In this expression the zero subscript designates the reference station and C_{nm} represents a small grid and elevation correction term. T_{nm}^{CALC} and T_{om}^{CALC} are the predicted arrival times of P-waves from the m^{th} event to the n^{th} station and the reference station, respectively, and are calculated using the Herrin [1968] travel-time tables and the known event locations. The corrections embodied in C_{nm} have not been applied to the results which will be shown in the following.

Figures 1a and 1b are relative residual maps based on the entire teleseismic data base that we have acquired. The reference station in this particular case is SUP, located on Superstition Mountain on the western flank of the Imperial Valley. As noted by Steeples [1975] in a similar study of travel-times in this region, but with a much smaller data set, the particular choice of a reference station, provided it is a flank station located on basement outcrop, does not significantly influence the results. For instance, we tried CRR and SGL as reference stations and the resulting residual maps varied by no more than 0.1 to 0.2 seconds from Figure 1.

Analysis of the travel-time data pointed up a marked dependence of the relative residuals on event epicentral distance and azimuth of approach into the Imperial Valley. Thus, the results shown in Figure 1 are grouped according to event distance (angle of incidence) and azimuth. Figure 1a shows results for events with angles of incidence between 13 degrees and 20.6 degrees, corresponding to an epicentral distance range between 65 degrees and 95 degrees. Figure 1b is for shallower angles of incidence, 23 degrees to 33 degrees. A final point about the data is that the residuals plotted in Figure 1 are actually mean values based on as many as 25 events in several of the azimuthal ranges. As shown in the first quarterly report written on this contract (Savino, et al., 1976), three major source regions situated along the

Figure 1b. Map of raw mean relative residuals for events with incoming rays with shallower angles of incidence.

circum-Pacific seismic belt (i.e., Fiji-Tonga at a mean azimuth of 235 degrees) contributed the majority of events recorded by the IVA.

Two observations that can be made based on the data shown in Figure 1 are the following.

1. There is a general trend in the residuals from fairly large positive (slow) values for stations near the center of the Imperial Valley to small positive or negative (fast) values for stations on the flanks of the valley.
2. At several of the stations, particularly those near the center and eastern side of the valley, there is a variation with azimuth in the mean relative residuals, or delays, of as much as 0.8 seconds.

With respect to the first point, the thick pile of low-velocity (relative to basement) sediments, with maximum thickness near the center and southern end of the valley (e.g., in the vicinity of BON in Figure 1), can account for much of the observed trend in the residuals. How much is the subject of the small grid calculation wherein we will incorporate all available information concerning the sediments in the Imperial Valley and correct the travel-time delay vector for these near surface effects. The details of how we have modeled the sediment pile and will use this model to correct the observed data are described in a later section of this report.

The azimuthally dependent variation in the residuals shown in Figure 1 are thought to reflect variations in the deep structure beneath the sediments. Steeples [1975] pointed

out that near-surface delays at a station are relatively independent of approach azimuth because of the steep angles of incidence as near-surface low velocity material is encountered. Thus, our conclusion that much of the 0.8 second azimuthally dependent delay must come from deeper sources. A quantitative interpretation of these data will be the product of the inversion scheme described in Section 2.5 of this report.

2.3 PROCESSING AND ANALYSIS OF GRAVITY DATA

As discussed in previous reports (Savino, et al., 1977) we obtained a large gravity anomaly data base from Professor Shawn Biehler of the University of California at Riverside. It consists of approximately 14,000 gravity observations taken at random locations in the Imperial Valley and surrounding area. Each observation is recorded on a card image which contains the latitude and longitude of the observation, its universal transverse mercator (UTM) northing and easting, elevation, terrain correction, free air anomaly, and complete Bouguer anomaly type one.

2.3.1 Reduction of Gravity Observations to a Uniform Grid

To integrate the gravity data into the inversion process it was first necessary to sort the observations in order of ascending UTM easting with UTM northing as a secondary sort key. Thus organized we were then able to calculate grid point gravity values (GPGV) on a regular rectangular grid of points. This was done for a large area including the region to be modeled by the inversion plus wide surrounding bands of data. The surrounding data are required to facilitate application of various filtering techniques which will be discussed later.

To calculate the GPGV a computer program (GRLSF) has been developed. The method used involves obtaining a least squares fit to a general quadratic surface using weighted gravity observations in a specified domain about the point to be calculated. An explanation of the mathematical method follows.

Assume the gravity field over the domain D can be approximated by the general quadratic surface:

$$\begin{aligned} g \approx f(x,y) &= A_0 x^2 + A_1 xy + A_2 y^2 + A_3 x + A_4 y + A_5 \\ &= \sum_{K=0}^5 A_K d_K \end{aligned} \quad (1)$$

For a grid point located at the origin of this coordinate system the value of this function is given by:

$$g \approx f(0,0) = A_5 \quad (2)$$

Using the procedure outlined by Hildebrand [1956] we specify the domain D as the set of all gravity observations which lie within a circle of radius R about the grid point. Since it is desired that the GPGV be influenced more strongly by nearby points and to a lesser degree by more distant observations a weighting function of the form given in Equation (3) was introduced.

$$\begin{aligned} W_i (X_i, Y_i) &= 0 & r_i &> R \\ W_i (X_i, Y_i) &= \left[\frac{R - r_i}{r_i + \epsilon} \right]^2 & r_i &\leq R \end{aligned} \quad (3)$$

where

$$r_i = (x_i^2 + y_i^2)^{1/2}$$

R = The domain radius

ϵ = Station location error $\ll r_i$

In practice the purpose of ϵ in the equation is to assure that W_i is defined throughout the domain including an observation which may fall exactly at the origin. This function is seen to decrease smoothly from its maximum value at the grid point to zero at the domain radius R. The degree of smoothing obtained in this procedure is governed by the exponent in the expression for W_i . Smaller exponents produce a high degree of smoothing while larger ones preserve the higher spatial frequency in the data.

If we define a residual $\gamma_i (X_i, Y_i)$ by:

$$\gamma_i (X_i, Y_i) = g_i - f(X_i, Y_i) \quad . \quad (4)$$

We obtain the best least squares approximation to the gravity surface by determining the coefficients A_k in Equation (1) such that the sum of $W_i \gamma_i^2$ is minimized. This requires then that

$$\frac{\partial}{\partial A_j} \sum_{i=1}^N W_i \gamma_i^2 = 0 \quad j = 0, 1, \dots, 5 \quad .$$

Substituting from Equation (1) we have:

$$\frac{\partial}{\partial A_j} \sum_{i=1}^N W_i \left(g_i - \sum_{k=0}^5 A_k d_k \right)^2 = 0$$

$$\sum_{i=1}^N W_i d_j \left(g_i - \sum_{k=0}^5 A_k d_k \right) = 0$$

$$\sum_{k=0}^5 A_k \sum_{i=1}^N w_i d_j d_k = \sum_{i=1}^N w_i d_j g_i \quad (5)$$

$$j = 0, 1, \dots, 5$$

Equation (5) is a system of six equations in the six unknowns A_k . These may be written out as:

$$\begin{aligned} A_0 \sum WX^4 + A_1 \sum WX^3 Y + A_2 \sum WX^2 Y^2 + A_3 \sum WX^3 + A_4 \sum WX^2 Y + A_5 \sum WX^2 &= \sum WgX^2 \\ A_0 \sum WX^3 Y + A_1 \sum WX^2 Y^2 + A_2 \sum WXY^3 + A_3 \sum WS^2 Y + A_4 \sum WXY^2 + A_5 \sum WXY &= \sum WgXY \\ A_0 \sum WX^2 Y^2 + A_1 \sum WXY^3 + A_2 \sum WY^4 + A_3 \sum WXY^2 + A_4 \sum WY^3 + A_5 \sum WY^2 &= \sum WgY^2 \\ A_0 \sum WX^3 + A_1 \sum WX^2 Y + A_2 \sum WXY^2 + A_3 \sum WX^2 + A_4 \sum WXY + A_5 \sum WX &= \sum WgX \\ A_0 \sum WX^2 Y + A_1 \sum WXY^2 + A_2 \sum WY^3 + A_3 \sum WXY + A_4 \sum WY^2 + A_5 \sum WY &= \sum WgY \\ A_0 \sum WX^2 + A_1 \sum WXY + A_2 \sum WY^2 + A_3 \sum WX + A_4 \sum Y + A_5 \sum W &= \sum Wg \end{aligned} \quad (6)$$

Where all sums Σ are over the domain D of N observations lying within the radius R about the grid point to be calculated.

A solution to the system of Equations (6) can now be found and the GPGV is thus obtained from Equation (2). To accomplish this a standard library mathematical routine was employed which uses the Gauss-Jordan reduction method for the solution of simultaneous equations. Certain error checking criteria were applied both to the set of N observations and the resulting solution. Input parameters for this scheme are as follows:

RZERO: The initial domain radius
RMIN: The minimum allowable radius
RMAX: The maximum allowable radius
DELR: An increment by which to either increase
or decrease the radius if necessary.

The first criterion to satisfy is a requirement for at least six observations within the domain radius in order to solve the system. The domain radius is increased from its initial value of RZERO until this requirement is met. If the radius is increased to RMAX and six observations have still not been encountered the GPGV is flagged as erroneous and calculation proceeds to the next point.

The routine which solves the system of equations provides alternate returns to the main program should a matrix singularity or an overflow condition occur. In the case of singularity, which can arise if all observations within the domain radius lie along a straight line the domain radius is increased and the calculation attempted again. If, upon reaching RMAX, singularity still occurs, the GPGV is flagged and we proceed to the next point.

To avoid overflow conditions the domain radius can be decreased in increments of DELR and the calculation attempted again. This continues until RMIN is reached at which time the GPGV is again flagged.

In some cases the matrix used in the Gauss-Jordan routine may be nearly singular, however due to round off error the routine will not take the error exit. The calculated value for GPGV will still be erroneous. This occurs most often when all observations within the domain radius lie on one side of the grid point and some considerable distance away. The resulting erroneous GPGV are usually well out of the range of the typical gravity observations. To combat this problem we simply require

that the calculated GPGV fall in a range between the minimum gravity observation minus ten milligals and the maximum observation plus ten milligals. If this criterion is not met the domain radius is increased incrementally until a suitable solution is obtained or RMAX is reached. Again the GPGV is flagged as erroneous if we are unable to obtain a solution before reaching RMAX.

The program (GRLSF) is designed to produce a regular grid of GPGV at a specified map scale. The following formula gives the grid spacing for a desired scale:

$$\Delta X = \text{Scale}/39370.0$$

Accordingly a map scale of 1:192,000 results in

$$\Delta X = 192000/39370 \approx 4.88 \text{ km}$$

A facility for producing blocks of GPGV and then merging them into a large data base was also developed. Using these techniques a grid of approximately 50 x 50 GPGV have been calculated at the 1:192000 scale. These 2500 data may now be subjected to filtering and decimation to create the final data base to be inverted.

2.3.2 Techniques for Filtering the Gravity Data

Various filtering procedures have been investigated to produce data appropriate for the discretized model which will be used. Components in the data arising from effects outside the model or whose spatial variations are greater in extent than can be accommodated must be removed. To do this a regional gravity field for a large area surrounding the model is calculated and subtracted from the data base.

All ways of calculating a regional field involve some form of filtering applied to the original data. The objectives are not only to produce an extremely smooth version of that data but also to account for structure in the region surrounding the model which yields unwanted effects in the data. Thus calculating a regional for one area may involve a much different technique than that appropriate for another.

One approach which has been presented (Griffin, 1949) is to use a simple average of all points which lie on the circumference of a circle centered on the desired point. A slight variation averages all points within an annulus of finite width small compared to its mean radius. This filter has a fairly complicated form in the frequency domain but it does have a general smoothing effect which increases as the radius increases. It also can be successful in distinguishing other regional effects which a simpler low pass filter would miss.

One of these simpler low pass filters is a simple or a weighted average of points on a disk of radius R. Depending on the weighting function used, this filter can have a well behaved form in the frequency plane with a tapered effect and no side lobes. In this form however it may not be as useful for removing other effects associated with the regional field.

To date we have examined both of the above mentioned filters particularly from the standpoint of relating the desired degree of smoothing to the radius of the filter. Since anomalies at depth appear as much broader features on the surface an important consideration is to preserve our capability to resolve features in the deepest portion of the model (i.e. approximately 50 km). It can be shown that the anomaly due to a point source at depth Z has a half amplitude width given by (Telford, et al., 1976):

$$\Delta X_{1/2} \approx 1.3 Z$$

It is seen then that a filter with a radius in excess of 50 km would be needed to preserve this resolving capability.

Applying a filter this large is undesirable for two reasons. The first of these arises from the sparceness of gravity observations in some areas, particularly at the southern edge where the filter apron would have to extend well down into Mexico. Where it was possible to calculate GPGV for these regions they were rarely comprised of enough observations surrounding the grid point to render them really reliable measures of the gravity field. To allow these data to contaminate the well defined data points 50 km or more to the north in the region we are modeling may introduce errors larger than those we hope to remove in the regional field.

The second is a practical consideration. In order to properly apply such a filter it must be done consistently for all components of the resulting data base. This means that the output from the forward modeling technique used to calculate the effects of the layered sediments in the valley (this phase of the program will be discussed in detail in later sections of this report) will have to be filtered in an identical manner. To do this would require producing a grid of calculated gravity points for a region more than four times as large, in terms of numbers of data, than the region to be modeled by the final inversion. Along with the vast numbers of GPGV also required the investment in labor and computer time to produce this type of regional outways its usefulness in correcting the data base.

At present it appears that the most advantageous approach to deregionalizing the gravity data may be to simply remove the mean value from all GPGV. In addition it may be desirable to compute a linear trend across the region and remove that as well. In view of the large overlap of the filters mentioned above into poorly recorded regions of data the difference between applying those filters and a simple

demeaning operation may be insignificant. More likely the data from the demeaning operation may be more reliable particularly at the southern extreme of the model.

Some significant added advantages to this simple deregionalizing are also apparent. For the final inversion a variance must be assigned to each datum according to an estimate of its reliability and its ability to contribute information about the region to be modeled. This variance ultimately will take the form of an operator applied to the matrix of differential kernels in the inversion. For the demeaning operation this operator has a well defined and understandable form. However, for the other techniques of deregionalizing the form becomes much more complex and difficult to reliably compute. This aspect of the procedure will be discussed in later sections on the development of the inversion methods we will employ.

Once the regional has been removed one more filtering operation is still required. This is a low pass process to remove effects of anomalies smaller in scale than can be resolved by the grid. If it could be shown that the only contribution of these effects to the data came in the form of random noise this procedure would perhaps not be necessary. It can be shown however that anomalies very small in extent which are near to a point of observation can contribute large values to the gravity field. Any possible coherence or systematic distribution of these small features could be an unacceptable source of error when inverting. Since no a priori information is known about their distribution (and this is what distinguishes this filtering from the deregionalizing) the approach to minimizing their effect is purely a frequency plane filtering or low band pass operation.

The desired characteristics for the filter consist mainly then of a well behaved form in the frequency plane. The filter we have employed is a Gaussian, given by Equation (7).

$$W_i = e^{-r_i^2/2\lambda^2} \quad (7)$$

where

2λ = The approximate cutoff

r_i = Radius of GPGV from point to be calculated

$$r_i = \left(x_i^2 + y_i^2 \right)^{1/2}$$

GPGV at radii large enough so that W_i is small compared to its central value of one can be ignored. This results in a disk shaped domain of radius R where R is chosen such that the volume under the filter beyond R is negligible. The formula for computing R such that W_i has dropped to some small value W_{sm} is:

$$R = \sqrt{-2\lambda^2 \ln W_{sm}}$$

For $\lambda = 5$ km and $W_{sm} = 0.01$ this radius is approximately 15 km. The volume under the filter beyond this radius is 0.01 of the total volume.

In the frequency plane this filter is also a Gaussian as shown by Equation (8).

$$F\{W_i\} = e^{-k^2 \lambda^2/2} \quad (8)$$

These functions are plotted in Figure 2.

Applying this filter will smear out the unwanted high frequency components but will also correlate their effects to some degree. The final step is to decimate the resulting data to the scale length indicated by the filter cutoff. These two steps produce data where the effects of high frequency localized

Figure 2. Form taken by the low-pass filter in both the spatial and frequency domains.

information have been reduced to low level random noise in the data. This noise will appear as uncorrelated scatter in the data about the surface which the inversion will actually be able to fit.

Some results of the data processing effort described above are presented in Figures 3 through 5. Figure 3 is a contour plot of GPGV for the region to be modeled. Figure 4 is the low pass filtered version of this data. Figure 5 is the high frequency information which was removed. This was calculated by simply subtracting the low pass from the original GPGV. These same operations were performed on the synthetic data produced in the small grid forward modeling which will be discussed in the next section. Similar figures for these data will appear there.

2.4 MODELING OF THE NEAR SURFACE STRUCTURE - SMALL GRID CALCULATION

The near-surface low velocity sediments in the Imperial Valley have a major effect on both the teleseismic travel-times and the gravity field observed there. In Section 2.2 above, we noted the large travel-time delays for valley stations relative to flank stations. In the case of the gravity field, Biehler [1964] showed that because of the increase of density with depth typical of this type of structure most of the observed gravity anomaly in a thick sedimentary basin, such as the Imperial Valley, arises from the upper few thousand feet of sediments. Thus, we must in effect strip off the sediments to uncover details of the deeper structure.

2.4.1 Depth-to-Basement Contouring and Density Model

Figure 6 is a depth-to-basement map of the Imperial Valley taken from Rex [1970]. Various kinds of geophysical information were used to make up this map ranging from surface

Figure 3. Complete Bouguer anomalies interpolated to a regular grid (GPGV) before filtering. Contour levels are in milligals and the data plotted is the anomaly plus 1000 milligals.

LO PASSED BOUGER ANOMALIES 1:192000 SCALE 4 MG LEVELS W-LENGTH 5KM

A= 9.2000+02B= 9.2400+02C= 9.2800+02D= 9.3200+02E= 9.3600+02F= 9.4000+02G= 9.4400+02H= 9.4800+02
I= 9.5200+02J= 9.5600+02K= 9.6000+02L= 9.6400+02M= 9.6800+02N= 9.7200+02O= 9.7600+02P= 9.8000+02
Q= 9.8400+02R= 9.8800+02S= 9.9200+02T= 9.9600+02

Figure 4. Result of the low-pass filtering operation on the observed gravity field. Levels depict actual plus 1000 milligals.

HI PASSED BOUGER ANOMALIES 1:192000 SCALE 4 MG LEVELS W-LENGTH 5KM

A=-2.4000+01B=-2.0000+01C=-1.6000+01D=-1.2000+01E=-8.0000+00F=-4.0000+00G= 0.0000 H= 4.0000+00

I= 2.0000+00J= 1.2000+01K= 1.6000+01L= 2.0000+01M= 2.4000+01

Figure 5. High frequency perturbations about zero which were removed from the complete Bouguer anomalies (GPGV) in the low-pass filtering operation. These are obtained by a point by point subtraction of the low-passed field from the original.

Figure 6. Map of the thickness of water filled sediments in the Imperial Valley. This may also be considered a depth to basement map with contours at 5000 foot increments.

mapping of major fault systems to refraction profiles and data from deep wells. Obviously, however, the basement depths in many locations are based on best estimates.

A digitized version of Figure 6 was obtained from Professor Shawn Biehler. The initial digitizing was performed on a regular 2 km x 2 km grid. In view of the scale lengths of the features of interest in this study we smoothed the digitized model to a 6 km x 6 km grid. This smoothing was accomplished by picking east-west slices through the model spaced every 6 kms going from north to south. For each slice, or cross-section, the complete depth to basement data were plotted and four major layers, 1.5 km thick, were then defined at increments of 6 kms. The top layer was further subdivided into three 0.5 km layers. Figure 7 shows three slices (cross-sections) that span the Imperial Valley, proceeding from the north (near the Salton Sea) to just south of the Mexican border (top to bottom of Figure 7, respectively). The motivation for a finer subdivision of the top layer was a result of the study by Biehler [1964] which showed the strong effect of the top most sedimentary layers to the observed gravity anomalies.

Twenty east-west slices were defined over the region of interest from which layer interfaces between slices are found by linear interpolation. Figure 8 shows the resulting depth-to-basement map that comprises the small grid model which was subsequently employed in a forward gravity calculation. The density contrast, relative to basement, model used for this calculation was taken from Biehler [1964] and is given in Table 1.

Figure 7. Cross-section view of three east-west slices through the Imperial Valley sediments. The slice at the top of this figure is representative of the sediments near the Salton Sea. The next two slices represent the sediments further to the south, with Slice 1 appropriate to sediments 20 to 30 km south of the Mexican border.

Figure 8. Smoothed version of depth to basement map used for the small grid calculation of the effects of the sedimentary layers.

TABLE 1
DENSITY MODEL FOR SMALL GRID

Depth (km)	Density Contrast (gm/cm ³)
0	
0.5	0.43
1.0	0.34
1.5	0.29
3.0	0.20
4.5	0.07
6.0	0.03

2.4.2 Calculated Gravity Field

Using the forward modeling program which was outlined in the last quarterly report (Savino, et. al., 1977) a 644 element gravity anomaly vector for the model described above was calculated on a 23 x 28 regular grid of points. This grid was designed to exactly coincide with a 23 x 28 subset of the GPGV previously discussed and just fits inside the region specified by the slice model.

At each cell in the three-dimensional grid a gravity kernel is calculated by use of Nagy's [1966, 1967] formula for the gravitational attraction of a right rectangular prism. When this is complete the dot product of the rows of kernels with the model parameter vector is performed. The model parameter in this case is the density perturbation in a cell relative to a basement rock density of 2.7 gm/cc. The result is a vector containing a value for the gravity anomaly at each of the 644 points of observation. A contour map of this data is presented in Figure 9.

IMPERIAL VALLEY MODEL 1:192000 SCALE

A= 9.5000+02B= 9.5400+02C= 9.5800+02D= 9.6200+02E= 9.6600+02F= 9.7000+02G= 9.7400+02H= 9.7800+02
I= 9.8200+02J= 9.8600+02K= 9.9000+02L= 9.9400+02M= 9.9800+02

Figure 9. Contour map of gravity data vector produced in forward modeling of near surface sedimentary layers in the Imperial Valley. Levels shown reflect the actual data plus 1000 milligals.

In Figure 9 we notice that the contours start to close toward the southern edge of the grid. This is an artificial effect produced by the absence of information outside of the finite grid which was employed. Some waveyness in the contours results from the discretization introduced by assignment of a constant value of the model parameter throughout each cell.

Next the filtering techniques described earlier in this report, which were applied to the observed data, were also applied to this vector. A contour map of this result appears in Figure 10. Notice that the waveyness has been removed and the contours have opened somewhat to the south.

The next step toward producing data suitable for inversion is to combine this small grid data with the observed gravity data. The low passed version of both was chosen and a point by point subtraction of the small grid from the observed was performed. This produces a low passed data vector from which the approximate effects of the sedimentary layers have been removed. This was again contour plotted and appears as Figure 11. Remaining steps include any possible demeaning, detrending or other deregionalizing procedures necessary before introduction of the data into the inversion scheme.

2.5 INVERSION SCHEME DEVELOPMENT

The development of a travel time-gravity inversion scheme involves three steps. The first is to cast the governing equations, which relate the data base to subsurface density and slowness, into a standard form suitable to the application of linear inversion. A convenient, general form is (Jordan, 1972)

$$\underline{y} = A \underline{x} + \underline{n}, E(\underline{n}) = 0, \text{Var}(\underline{n}) = I, \quad (9)$$

LO PASSED SMALL GRID 1:192000 SCALE 4 MG LEVELS W-LENGTH 5KM

A= 9.2000+02B= 9.2400+02C= 9.2800+02D= 9.3200+02E= 9.3600+02F= 9.4000+02G= 9.4400+02H= 9.4800+02

I= 9.5200+02J= 9.5600+02K= 9.6000+02L= 9.6400+02M= 9.6800+02N= 9.7200+02O= 9.7600+02P= 9.8000+02

Q= 9.8400+02R= 9.8800+02S= 9.9200+02T= 9.9600+02U= 1.0000+03

Figure 10. Result of the small grid forward modeling effort after completion of the low-pass filtering operation. Actual data plus 1000 milligals is displayed.

BOUSER ANOMALIES - SMALL GRID CORRECTION 4MG LEVELS 5KM FILTER

A=-8.0000+01B=-7.6000+01C=-7.2000+01D=-6.8000+01E=-6.4000+01F=-6.0000+01G=-5.6000+01H=-5.2000+01
I=-4.8000+01J=-4.4000+01K=-4.0000+01L=-3.6000+01M=-3.2000+01N=-2.8000+01O=-2.4000+01P=-2.0000+01
Q=-1.6000+01R=-1.2000+01S=-0.8000+01T=-0.4000+01U= 0.0000 V= 4.0000+00W= 8.0000+00X= 1.2000+01

Figure 11. Filtered gravity observations with the approximate effect of sedimentary layers removed. The result of the subtraction process is contoured in milligals.

where $\underline{y}$ is a vector of M observed data, $\underline{x}$ is a vector of N unknown parameters that describe a model of the earth's density and slowness, $\underline{n}$ is a vector of M observational errors, and the M by N kernel matrix A establishes the physical relationship between $\underline{y}$ and $\underline{x}$. "E" denotes "expected value," "Var" denotes variance, and I is the unit matrix. Therefore, (9) gives a stochastic interpretation to the data and observational errors and states that $\underline{y}$ is an unbiased estimate for $A\underline{x}$ with unit variance.

The second step is to define a criterion for finding an optimal estimate, $\hat{\underline{x}}$, for $\underline{x}$. This criterion determines an N by M "inverse" matrix, A^+ , for A such that

$$\hat{\underline{x}} = A^+ \underline{y} . \quad (10)$$

As shown below, this criterion is translated into an algorithm for finding $\hat{\underline{x}}$ that gives an optimal compromise between fitting the observed data and being physically tenable.

The third step is the development of numerical techniques for calculating $\hat{\underline{x}}$ and the quantities we need to interpret its accuracy and uniqueness.

The remainder of this section is divided into four parts. The first gives a brief summary of inversion theory. The last three outline the progress made on the three steps described.

2.5.1 Linear Inverse Theory

Equations (9) and (10) imply

$$\hat{\underline{x}} = A^+ A \underline{x} + A^+ \underline{n} . \quad (11)$$

Equations (9) and (11) imply that the expected value and variance of $\hat{\underline{x}}$ are given by

$$E(\hat{\underline{x}}) = A^+ A \underline{x} \equiv R \underline{x} \quad (12)$$

$$\text{Var}(\underline{x}) = A^+ (A^+)^T \equiv P$$

Equation (12) states that the solution $\hat{\underline{x}}$ is an unbiased estimate for $R\underline{x}$ with variance P . If P were zero and R were the unit matrix, we could say $\hat{\underline{x}}$ was a unique solution and in fact was $\underline{x}$ itself. In general, though, the size of the diagonal components of P and the difference between R and the unit matrix tell us how nonunique $\hat{\underline{x}}$ is. The rows of R , called "averaging kernels," indicate which weighted averages of the parameters the data really determine. These weighted averages can be interpreted as spatial averages of density and slowness. P indicates how accurately the spatial averages are determined.

2.5.2 Data Reduction Prior to Inversion

The processing of the gravity and travel time data, discussed in Sections 2.2 through 2.4, accomplishes the reduction of the data to standard form. This section discusses the data processing in the context of linear inversion.

Our model of the earth consists of a finitely thick slab composed of a three-dimensional grid of N rectangular cells. Each cell has uniform density and slowness, which can be taken to be the average of the earth's density and slowness over the volume of that cell. The density of all the cells define an N component vector $\underline{\rho}$. The slownesses define a vector $\underline{a}$.

We will treat the interpolated uniform grid of observed gravity values (GPGV) as a raw data vector, $\underline{g}'$. The relationship between $\underline{g}'$ and $\underline{\rho}$ is

$$\underline{g}' = G' \underline{\rho} + \underline{\gamma}' \quad , \quad (13)$$

where G' is the gravity kernel matrix and γ' is an error term that accounts for contributions to g' that cannot be explained by an N-cell model. There are four such contributions:

1. Reading, interpolation, and Bouguer correction errors.
2. High-frequency effects caused by density variations, particularly near the surface, on a scale smaller than our cell sizes. These effects can be very large as discussed in Section 2.3.
3. A special case of 2., the effect of sedimentary layer of the Imperial Valley.
4. A regional contribution, including an unknown d.c. component, due to density below our slab model.

To reduce (13) to standard form, we derive a transformation that reduces γ' to an error vector with zero mean and unit variance. This transformation corresponds to filtering and correcting the raw gravity data.

The effect of the sediment layer is approximately removed by the small-grid correction (g'_{SG}) discussed in an earlier section. The high-frequency effect is reduced by low-pass filtering and decimating the data. The result of these two operations produces a data vector $\underline{g}$ which contains errors $\underline{\gamma}$:

$$\underline{g} = G \underline{p} + \underline{\gamma} \quad (14)$$

where

$$\underline{g} = F_1 (\underline{g}' - \underline{g}'_{SG})$$

$$\underline{Y} = F_1 (\underline{Y}' - g' SG)$$

$$G = F_1 G'$$

and where the matrix F_1 represents the low pass-decimate filter. Filtering G' to produce G , which $F_1 G'$ implies, is not really necessary - a close approximation to this is to let G be the gravity kernels evaluated for the gravity stations that remain after decimation.

The resulting error term, $\underline{y}$ in (14), now has two primary contributions:

1. Reading, interpolation and Bouguer correction errors; small-grid correction errors; high-frequency components - all low-pass filtered and decimated.
2. Regional contribution - unaffected by low-pass filtering.

This is symbolized with

$$\underline{Y} = \underline{\varepsilon} + \underline{Y}_{reg}$$

We treat $\underline{\varepsilon}$ as a zero-mean error vector whose components have finite variance and are uncorrelated - namely

$$E(\underline{\varepsilon}) = 0, \text{Var}(\underline{\varepsilon}) = \Sigma_g^2,$$

where Σ_g is a diagonal matrix whose components are the standard deviations of the components of $\underline{\varepsilon}$. We will assign these standard deviations on the basis of a minimum error we believe to be in the data and on the proximity of the stations to the edge of the grid, where the errors are expected to be largest.

The components of γ_{reg} are highly correlated because a regional gravity field must be slowly varying. In addition, the components of γ_{reg} have an unknown d.c. component which effectively gives γ_{reg} an infinite variance. The next step is to derive a filter F_2 that makes $F_2\gamma$ an error term with finite variance and uncorrelated components.

The filter F_2 essentially removes a regional field from g . As with F_1 and (13), F_2 must be applied to each term in (14). Three approaches to deregionalization are to (1) remove the effect of a known regional density structure below our slab model, (2) high-pass filter the data, and (3) zero-mean and detrend the data. We do not have the necessary information to do the first approach. As discussed above, the lack of broad areal coverage of our data makes the second approach difficult to do without removing gravity effects we wish to invert. We adopt third approach which will be adequate if our slab model is thick enough so that the gravity field due to possible density variations in the earth below our slab is nearly linear.

In the zero-mean and detrend approach, we model γ_{reg} as a linear function of the station coordinates:

$$\gamma_{\text{reg}} = \gamma_1 \underline{1} + \gamma_x \underline{x} + \gamma_y \underline{y}$$

where $\underline{x}$ and $\underline{y}$ are vectors of station coordinates and $\underline{1}$ is a vector all of whose components are unity. The filter F_2 is derived in terms of variances which we assign to the scalars γ_1 , γ_x , and γ_y . We plan to assign a very large variance to γ_1 to express the fact that the baseline of our gravity data is unknown. The variances of γ_x and γ_y will be chosen in accordance with how steep a trend (in milligals/km) we estimate can be caused by density variations in the mantle.

The filter F_2 has the convenient form

$$F_2 = (I - \beta_1 \underline{1} \underline{1}^T - \beta_x \underline{x} \underline{x}^T - \beta_y \underline{y} \underline{y}^T) \Sigma_g^{-2}$$

where the constants β_1 , β_x , and β_y are derived in terms of the variances assigned to γ_1 , γ_x , and γ_y . Applying F_2 to Equation (14) gives

$$\begin{aligned}\Delta \underline{g} &= \Delta G \underline{\rho} + \Delta \underline{\gamma} , \\ E \Delta \underline{\gamma} &= 0, \text{ Var } \Delta \underline{\gamma} = \Sigma_g^2\end{aligned}\tag{15}$$

where

$$\Delta \underline{g} = F_2 \underline{g} ,$$

$$\Delta G = F_2 G ,$$

$$\Delta \underline{\gamma} = F_2 \underline{\gamma} .$$

Multiplication of (15) by the diagonal matrix Σ_g^{-1} completes the reduction to standard form.

The filter F_2 becomes a zero-meaning filter if the variance of γ_1 is made very large or infinite. A consequence of this is that the residual gravity data, $\Delta \underline{g}$, are sensitive only to lateral variations in density. Let $\bar{\rho}$ be the laterally uniform model whose value of density in each layer of the cell model is the average of ρ for all the cells in that layer. Further, let $\Delta \underline{\rho}$ be the difference between $\underline{\rho}$ and $\bar{\rho}$:

$$\Delta \underline{\rho} = \underline{\rho} - \bar{\rho} .\tag{16}$$

Then ΔG has the property that

$$\Delta G \bar{\rho} = 0 ,\tag{17}$$

so our gravity-density relation can be written also as

$$\underline{\Delta g} = \Delta G \underline{\Delta \rho} + \underline{\Delta \gamma} . \quad (18)$$

The reduction of the travel time data to standard form is analogous to the gravity data reduction - the errors in the data for one event are similar to the gravity errors. In this case, the "regional," correlated errors are due to travel time delays accumulated through the mantle below our slab model. These delays are at least partially reduced by referencing the travel times to a key station. We are contemplating a further reduction of these "regional" delays by applying a filter similar to F_2 for the purpose of zero-meaning or de-trending the data from each event.

The final travel time residuals for all the events can be expressed as

$$\underline{\Delta t} = \Delta T \underline{a} + \underline{\Delta \tau}$$

$$E(\underline{\Delta \tau}) = 0, \text{ Var } (\underline{\Delta \tau}) = \underline{\Sigma}_t^2 \quad (19)$$

where $\underline{\Delta \tau}$ is the vector of travel time errors. $\underline{\Sigma}_t$ is a diagonal matrix of standard deviations of the travel time residuals, assigned primarily on the basis of the correlation analysis done in the data preparation phase of the program. ΔT is the kernel matrix appropriate to relative travel-time residuals.

The travel time data $\underline{\Delta t}$ are almost totally dependent on lateral variations and only slightly dependent on depth variations of slowness. Letting $\bar{a}$ be the one-dimensional average of $\underline{a}$, we have

$$\Delta T \bar{a} = 0 . \quad (20)$$

Defining $\underline{\Delta a}$ as

$$\underline{\Delta a} = \underline{a} - \underline{\bar{a}}, \quad (21)$$

the travel time-slowness relation becomes

$$\underline{\Delta t} \doteq \Delta T \underline{\Delta a} + \underline{\Delta \tau}. \quad (22)$$

Without a physical relationship between $\underline{\rho}$ and $\underline{a}$, the inversion of $\underline{\Delta g}$ and $\underline{\Delta t}$ are separate problems. We will assume a linear relation of the form

$$\underline{\rho} = c_1 \underline{a} + c_2 \underline{1} \quad (23)$$

where $\underline{1}$ is a vector of ones and c_1 and c_2 are scalar constants. Any relation between density and slowness (or velocity) can be approximated as linear for geologically reasonable values of density and slowness.

From (23) it follows that

$$\underline{\Delta \rho} = c_1 \underline{\Delta a}. \quad (24)$$

Equations (24), (22) and (18) show that c_2 and the average one-dimensional models, $\underline{\bar{\rho}}$ and $\underline{\bar{a}}$, drop out of the problem. They are needed only if we want to know the depth variations of $\underline{a}$ and $\underline{\rho}$ or if we do not make the approximation in Equation (20).

Substituting (24) into (18) allows us to write

$$\begin{aligned} \underline{\Delta g} &= c_1 \Delta G \underline{\Delta a} + \underline{\Delta \gamma} \\ \underline{\Delta t} &= \Delta T \underline{\Delta a} + \underline{\Delta \tau} \end{aligned} \quad (25)$$

$$E(\underline{\Delta \gamma}) = E(\underline{\Delta \tau}) = 0$$

$$\text{Var } \underline{\Delta \gamma} = \Sigma_g^2, \text{ Var } \underline{\Delta \tau} = \Sigma_t^2.$$

Relating (25) to (9), we make the following associations:

$$\underline{y} = \begin{bmatrix} \Sigma_g^{-1} \underline{\Delta g} \\ \Sigma_t^{-1} \underline{\Delta t} \end{bmatrix}$$

$$\underline{x} = \underline{\Delta a}$$

$$A = \begin{bmatrix} c_1 \Sigma_g^{-1} \Delta G \\ \Sigma_t^{-1} \Delta T \end{bmatrix}$$

$$\underline{n} = \begin{bmatrix} \Sigma_g^{-1} \underline{\Delta y} \\ \Sigma_t^{-1} \underline{\Delta \tau} \end{bmatrix}$$

Based on (25), an estimate of $\underline{\Delta a}$, $\hat{\underline{\Delta a}}$, is obtained with linear inversion. An estimate for $\underline{\Delta \rho}$ is derived simply as $\hat{\underline{\Delta \rho}} = c_1 \hat{\underline{\Delta a}}$.

We are not restricted to assuming that one linear relation between $\underline{\rho}$ and $\underline{a}$ applies to the whole model. We can assume a different relationship for each layer in the model without altering the essential features of the inversion problem.

2.5.3 Inversion Algorithm

We return to the general notation of

$$\underline{y} = A\underline{x} + \underline{n}$$

Our criterion for defining a solution, $\hat{\underline{x}}$, is a double one. The first criterion is that $\hat{\underline{x}}$ give a close fit to the data. Namely, $\hat{\underline{x}}$ should make $\phi_1(\hat{\underline{x}})$ small, where

$$\phi_1(\hat{\underline{x}}) = |\underline{y} - A\hat{\underline{x}}|^2 = [\underline{y} - A\hat{\underline{x}}]^T [\underline{y} - A\hat{\underline{x}}]$$

The vector $\underline{y} - A\hat{\underline{x}}$ contains the residuals of fit between $\underline{y}$ and the data predicted by the model $\hat{\underline{x}}$. ϕ_1 is the sum of the

squares of these residuals. Since $\underline{y}$ was transformed to have unit variance, no weighting coefficients are needed in the definition of ϕ_1 .

The second criterion is that $\hat{\underline{x}}$ be as "smooth" as possible in some sense. Two interpretations of this are (1) that $\hat{\underline{x}}$ be as small as possible and (2) that $\hat{\underline{x}}$ have the smallest spatial variations possible. Both these, and other, interpretations of smoothness can be expressed as the requirement that, for some matrix B , $\phi_2(\hat{\underline{x}})$ be as small as possible, where

$$\phi_2(\hat{\underline{x}}) = |B\hat{\underline{x}}|^2 = \hat{\underline{x}}^T B^T B \hat{\underline{x}} .$$

Our criterion for obtaining a solution is that $\hat{\underline{x}}$ minimize both ϕ_1 and ϕ_2 . Unfortunately, though, no model makes ϕ_1 and ϕ_2 simultaneously absolutely minimum. However, if we minimize, for some value of α , the quantity

$$\phi = \phi_1 + \alpha \phi_2$$

$\hat{\underline{x}}$ has the following optimum property: ϕ_2 is locally minimized with respect to all the models that have the same value of ϕ_1 as $\hat{\underline{x}}$ does. For a given α , then, $\hat{\underline{x}}$ is the smoothest model of all the models that fit the data as well as $\hat{\underline{x}}$.

The scalar α is a "trade-off" parameter whose value regulates the relative importance of minimizing ϕ_1 compared to ϕ_2 . The minimization of ϕ defines a family of solutions, $\hat{\underline{x}}(\alpha)$, each giving its own optimal compromise between fit and smoothness. ϕ_1 and ϕ_2 are, respectively, monotone increasing and monotone decreasing functions of α .

For a given α , minimization of ϕ produces a solution of the form

$$\hat{\underline{x}} = A^+ (\alpha) \underline{y} .$$

$A^+(\alpha)$ determines a resolution matrix $R(\alpha)$ and a variance matrix $P(\alpha)$, defined in (12). Similar to the fit versus smoothness trade-off, α also regulates a trade-off between the size of P and the similarity of R to the unit matrix.

We are considering two possible definitions of smoothness, each defining a form for the matrix B . The first definition is

$$\phi_2 = \sum_i \hat{x}_i^2 w_i$$

where the weighting coefficients w_i depend on the dimensions of the model cells. This leads to a diagonal matrix for B .

The second definition is more strict and has the potential for producing a smoother solution. In this case, ϕ_2 is defined as a sum of the squared differences between the components of $\hat{x}$. This method is tantamount to minimizing spatial gradients in the estimates for slowness and density. The result is that B is nondiagonal, which will require more calculations in the inversion algorithm. However, the number of extra calculations will be minimal if we restrict this method to minimize only lateral and not vertical gradients.

A convenient and efficient procedure for computing $\hat{x}$, A^+ , R and P for one or more values of the trade-off parameter α makes use of singular value decomposition (SVD). If A is any real $M \times N$ matrix of rank K , it can be shown (Lanczos, 1961) that there exist orthogonal matrices S and T such that

$$\begin{matrix} A & = & S & \Lambda & T^T \\ M \times N & & M \times K & K \times K & K \times N \end{matrix}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_K)$ and the λ 's are the square roots of the nonzero eigenvalues of $A^T A$.

In terms of this decomposition,

$$A^+ = T \Lambda (\Lambda^2 + \alpha I)^{-1} S^T .$$

The K by K matrix product $\Lambda (\Lambda^2 + \alpha I)^{-1}$ is diagonal and easily evaluated for several values of α . Furthermore, singular values much smaller than $\sqrt{\alpha}$ contribute little to A^+ . Hence, singular values that are small compared to the smallest value of α we use need not be calculated. We estimate that only on the order of 50 singular values and vectors are needed. This permits the use of an algorithm we have developed for reducing the size of A before decomposing it and hence reducing the total number of calculations involved.

In terms of S, Λ , and T, the quantities $\hat{x}$, P, and R are efficiently calculated as

$$\hat{x} = T \Lambda (\Lambda^2 + \alpha I)^{-1} S^T y$$

$$P = T \Lambda^2 (\Lambda^2 + \alpha I)^{-2} T^T$$

$$R = T \Lambda^2 (\Lambda^2 + \alpha I)^{-1} T^T .$$

2.5.4 Computational Methods

The matrix A mentioned above will have dimensions on the order of 1000 by 1000. Two kinds of problems arise when such large matrices have to be processed, relating to accuracy and efficiency. In order to generate results that are not swamped by accumulated round-off error, it is necessary to avoid using procedures that exhibit any form of numerical instability. The efficiency requirement arises from economic considerations: the inversion procedure would be quite expensive even if 100 percent efficiency were attainable. The

difficulty in achieving high efficiency is caused by the fact that the matrices that we are working with are 10 to 20 times larger than the space available in the internal memory of the computer (UNIVAC 1108 with 192,000 words of memory, some of which is occupied by instructions, operating system, etc.). Consequently, the matrices must be held in bulk storage devices such as drums and magnetic tape. Under these circumstances, the cost of retrieving data from bulk storage can easily exceed the cost of numerically processing them once they are obtained.

By far the most frequently used operation involved in inversion is matrix-vector multiplication. A procedure was developed that allows a matrix which is stored on high-speed drum to be multiplied by a set of up to 20 vectors held in the internal memory. When the number of vectors is three or more, obtaining one row of the matrix can be accomplished during the time that the previous row is being processed so that there is no interruption in the operation of the computer's arithmetic unit. This procedure will also be used for matrix-matrix multiplication which is equivalent to the multiplication of a matrix by a series of vectors.

A complication that occurs when multiplying large matrices is that the left-hand factor must be stored in row-form and the right-hand factor in column-form. Therefore, if either matrix happens to be available in the wrong form, it must be transposed before it can be accessed. Transposition is a specialized form of sorting and an efficient transposition procedure has been developed. It should require less than five percent the time required for matrix multiplication. Tests are underway to determine whether this degree of efficiency has been achieved.

Finding eigenvalues and eigenvectors is also required for performing the singular value decomposition referred to in the

previous subsection. The most efficient method for eigenvalue/eigenvector determination is based on the QR algorithm (Francis, 1961). Unfortunately, a straightforward application of the QR algorithm requires that the entire matrix be held in the internal memory because row operation and column operations are applied alternately. We have developed a reduction procedure that operates first only on columns and then, after a single transposition, only on rows. This procedure is capable of recovering at least the top 200 eigenvalues and their vectors. The efficiency of the method will be well over 95 percent and so we expect to suffer very little from the restrictions imposed by the finite size of the internal memory of the computer. A topical report explaining the reduction procedure is currently in preparation.

III. FUTURE PLANS

All work under this project is proceeding according to schedule. During the final quarter period of the program we will concentrate on inverting the teleseismic and gravity data compiled for the Imperial Valley for estimates of the velocity and density structure. We also plan on examining test cases that will point the way to obtaining the maximum amount of information on the three-dimensional structure of a geothermal environment from more ideal seismic and gravity data sets.

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