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HEAT CONDUCTION IN PARTIAL VACUUM

Final Technical Progress Report

By
James R. Thomas

September 1980

Work Performed Under Contract No. AC04-78CS15367

Virginia Polytechnic Institute & State University
Blacksburg, Virginia

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FINAL TECHNICAL PROGRESS REPORT

JAMES R. THOMAS
SEPTEMBER 1980

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Abstract

Methods developed for computing conduction heat losses from evacuated solar collectors are reported. Results of such calculations are given, including the minimum vacuum necessary to effectively eliminate conduction. Experiments performed at Owens-Illinois, Inc. to assess helium penetration rates into evacuated collectors are analyzed, and estimates are given as to the likely penetration rate of atmospheric helium. Conclusions are drawn as to the probable effect of helium penetration on the lifetimes of evacuated solar collectors.

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I. Introduction

Evacuated solar collectors have been manufactured and utilized for a number of years, and their advantages are well known. Because of the insulating effect of the vacuum surrounding the collector surface, these solar collectors are quite insensitive to low ambient temperatures and high winds. Evacuated collectors are relatively expensive, however, and there is some concern about their long-term reliability. The study reported here had two objectives, both related to these concerns. The first objective was to develop a rigorous theoretical analysis of heat conduction in partial vacuum in order to determine the minimum vacuum necessary to effectively eliminate conduction losses. Hopefully, developing a minimum evacuation procedure will help reduce manufacturing costs. The second objective addresses one of the long-term reliability questions, that of helium penetration. Helium is present in atmospheric air at a partial pressure of 4×10^{-3} Torr, and will permeate through glass. Thus if the penetration rate of helium through solar collector covers is sufficiently fast, it is possible that the useful lifetime of an evacuated collector could be seriously reduced. The present investigation considered both the rate of helium permeation through glasses of interest in solar collector design and the effects of helium on the conduction heat losses from the collector.

The basic theory of heat conduction in rarefied gases, as well as the fundamentals of helium permeation are given in a previous report¹ and will not be repeated here. The present report is limited to developments since publication of Ref. 1, including more accurate theoretical methods for predicting heat conduction in low-density gases, comparisons of predictions of these methods with experiment, comparison of helium penetration calculations with data on actual solar collectors, and evaluation of data on heat conduction through helium introduced at various pressures into evacuated collectors. Finally, conclusions and recommendations for evacuated collector design are given.

II. Methods Development

Reference 1 gives the basic theory of heat conduction in rarefied gases. There it is pointed out that different methods are required in

each of four basic density regimes delineated by values of the Knudsen number, Kn , which is the ratio of the molecular mean-free-path to a characteristic dimension of the system. Evacuated solar collectors with vacuum levels of 10^{-4} Torr or less will operate in the free-molecular regime of large Knudsen numbers ($Kn > 10$), in which molecules collide much more with the walls of the system than with each other. Relatively simple formulas describe heat conduction in this regime, and they are summarized in Ref. 1.

As the pressure is increased, the molecular mean-free-path decreases and the point is reached at which intermolecular collisions become as important in determining the state of the gas as molecule-boundary collisions. This is called the transition regime, and is assumed to be in the range $10 > Kn > 0.1$. Beyond this lie the temperature-jump ($0.1 > Kn > 0.01$) and continuum ($Kn < 0.01$) regimes, which are of no concern in evacuated solar collector work. However, for a complete analysis of evacuated collectors, including the problem of helium penetration, it is necessary to have available methods to predict heat conduction in the transition regime. Existing methods were studied, and some new ones were developed during the course of the present work.

Heat conduction in the transition regime must be based on the Boltzmann equation for the molecular distribution function $f(\underline{r}, \underline{c})$, which represents the density of molecules at location $\underline{r}$ having velocity $\underline{c}$. This equation may be written²

$$\underline{c} \cdot \nabla f(\underline{r}, \underline{c}) = \iiint_{-\infty}^{\infty} \int_{dP_c} [f(\underline{r}, \underline{c}')f(\underline{r}, \underline{\xi}') - f(\underline{r}, \underline{c})f(\underline{r}, \underline{\xi})] |\underline{\xi} - \underline{c}| dP_c d^3\xi. \quad (1)$$

Once $f(\underline{r}, \underline{c})$ is known, the heat flux may be determined from

$$q_i = \frac{m}{2} \iiint_{-\infty}^{\infty} c_i c^2 f(\underline{r}, \underline{c}) d^3c. \quad (2)$$

In Eq. (1), dP_c is the molecular collision cross section, and in Eq. (2), q_i is the i^{th} component of the heat flux while m is the molecular mass. The formidable part of the problem, of course, is solving Eq. (1); approximations must always be used.

The two most useful approximations for the present purpose are the use of model equations and of moment methods. It suffices here to summarize the approximate results which are of importance in solar collector application. Results for both parallel-plate and concentric tubular geometry will be discussed. It is recognized that evacuated collectors are not likely to be constructed in flat plate geometry, however it is useful to discuss this case because of the availability of experimental data with which to compare. Such comparisons give guidance in judging accuracy of similar methods in cylindrical geometry.

A. Heat Conduction Between Parallel Plates in the Transition Regime

In Ref. (1), two equations were given for the heat conduction in a rarefied gas between parallel plates, one based on the linear BGK model^{3,4}

$$q = \frac{Q}{Q_{fm}} = \left[1.176 + \frac{4}{5\sqrt{\pi}} \left(\frac{\ell}{\lambda} \right) \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2} \right]^{-1}, \quad (3)$$

and the other derived from a linearization of the four moment method of Liu and Lees⁵ by Springer⁶:

$$q = \frac{Q}{Q_{fm}} = \left[1 + \frac{4}{15} \left(\frac{\ell}{\lambda} \right) \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2} \right]^{-1}. \quad (4)$$

In Eqs. (3) and (4) Q is the heat flow rate per unit area while Q_{fm} is the free molecular heat flow for the same conditions computed from Eqs. (3-6) of Ref. 1. Also ℓ is the spacing between the plates, λ is the molecular mean free path, and α_1 and α_2 are the thermal accommodation coefficients, defined in Refs. 1 and 7, for plates 1 and 2, respectively.

Eq. (3) is an approximation deduced from an exact analysis of the BGK equation for the case $\alpha_1 = \alpha_2 = \alpha$, since at the time of that report an exact solution for different values of α on each plate was not available. Since that time, however, such an exact analysis has been developed by the present author⁸ (included as Appendix I). From this exact solution (which requires iteratively solving coupled Fredholm integral equations), a more accurate approximation can be shown to be

$$q = \frac{Q}{Q_{fm}} = \left[1 + \frac{1}{\sqrt{\pi}} \left(\frac{\ell}{\lambda} \right) \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2} \right]^{-1} \quad (5)$$

This result agrees with the exact analysis within about 10%. However, it is more revealing to compare the predictions of Eqs. (4) and (5) and the exact analysis of Ref. 8 with experimental data.

Braun and Frohn⁹ measured heat conduction between parallel plates in helium at a temperature difference of 100°C, and in Argon at temperature differences of 200 and 300°C. The data for helium are plotted in Fig. 1 along with the predictions of Eqs. (4) and (5), and the exact analysis of Ref. 8. It is clear that of the three, Eq. (4) is the more accurate over the full range of $Kn^{-1} = \ell/\lambda$. Equation (5) fits the data better for $Kn^{-1} < 3$ - i.e. as the free molecule limit is approached. This wider range of accuracy of Eq. (4) is probably due to the fact that it is derived from the full Boltzmann equation by taking moments, rather than from a model of the Boltzmann equation such as that of Bhatnagar, et al.³

None of these formulations give satisfactory results for the larger ΔT 's of 200 and 300°C. This is not too surprising since they are all based on a linearized theory derived by assuming $T_2/T_1 - 1 \ll 1$. In fact, it is surprising that these results are as accurate as they are for $\Delta T = 100^\circ$ since for this case

$$\frac{T_2}{T_1} - 1 = \frac{373}{273} - 1 \approx 0.4,$$

which is certainly not "much less" than unity. For larger temperature differences, however, a nonlinear method is required. The four-moment method of Liu and Lees⁵ has been extended to the case of arbitrary accommodation coefficient on either surface by Braun and Frohn^{10,11} for a binary mixture. Their methods can, of course, be simplified to a single gas, and the resulting solution shows excellent agreement with experimental data¹⁰. Unfortunately, this method requires the numerical solution of a system of nonlinear differential equations too lengthy to repeat here.

Figure 1. Comparison of Methods for Heat Conduction Between Parallel Plates

In summary, for heat conduction between parallel plates in the range $0.5 \leq \text{Kn}^{-1} \leq 10$, Eq. (4) is recommended up to temperature differences of 100°C ; for larger differences one must use the nonlinear moment method as described by Braun and Frohn in Ref. 10.

B. Heat Conduction Between Concentric Cylinders in the Transition Regime

Analytical solutions of model equations are not available in cylindrical geometry. However, the moment method has been formulated and solved for $\alpha = 1$ on both surfaces by Lees and Liu.¹² This method has recently been extended to arbitrary accommodation on either surface by Wideman and Thomas¹³ (see Appendix II). The nonlinear equations are solved numerically by a method described in detail in Ref. 14, while a linearized method yields an analytical solution which can be written

$$q = \frac{Q}{Q_{fm}} = \frac{\frac{R_1}{R_2} \left(\frac{1}{\alpha_2} - 1 \right) + \frac{1}{\alpha_1}}{\frac{R_1}{R_2} \left(\frac{1}{\alpha_2} - 1 \right) + \frac{1}{\alpha_1} + \frac{4}{15} \frac{R_1}{\lambda_1} \ln \left(\frac{R_2}{R_1} \right)} \quad (6)$$

This formula is the cylindrical geometry analogue of Eq. (4) and should be similarly accurate. Unfortunately there is no experimental data available for comparison for non-unity accommodation coefficients on both surfaces. Experiments in cylindrical geometry always utilize a fine wire surrounded by a cylinder with which heat is exchanged. Since the gas molecules have so many more collisions with the outer cylinder they are essentially completely accommodated to it. Yeh and Frohn¹⁵ have measured heat conduction in this manner and compared the predictions of Eq. (6) with $\alpha_2 = 1$ to their data. The agreement is excellent for small temperature differences. For large temperature differences the linear formula (6) differs significantly from the nonlinear, and thus it should not be used when T_1/T_2 exceeds about 1.5. Since there is no experimental data for comparison at large temperature differences, we assume that the nonlinear method described in Refs. 13 and 14 is about as accurate as the analogous method for parallel plates described above.

III. Application to Solar Collectors

A. Accommodation Coefficients

Before the method described above can be applied to evacuated solar collectors, values are needed for the accommodation coefficients of air and helium on the inside surfaces of the collector. The cover tube is, of course, glass, while the collector tube is generally glass with a selective coating of metal a few hundred angstroms in thickness.

Accommodation coefficients for such materials are difficult to find in the literature. Most experimenters measure accommodation coefficients for purposes of comparison with theoretical models of gas-surface interaction. Since the simplest system possible is desirable for this purpose, emphasis is on attaining ultra-clean surfaces by baking the sample at high temperatures for several hours. Unfortunately, such experimental surfaces do not approximate those of solar collectors under actual operating conditions. The presence of an adsorbed surface layer of gas apparently has a significant effect on measured accommodation coefficients¹⁹, and is likely to be present in a solar collector. This is particularly true in the case of helium which has permeated a glass cover or collector tube by dissolving into the glass matrix and migrating through the thickness.

For the ultra-clean surfaces obtained in laboratory experiments, very low values for helium accommodation are usually reported¹⁸, often in the range 0.01 to 0.1. For normal surface conditions, values for helium on glass as high as 0.35 have been reported^{18,19} and as high as 0.53 for helium on tungsten. For air on metal surfaces, values in the range 0.85 to 0.95 are common. A search of the literature has revealed no data for air on glass.

It was hoped that the Owens-Illinois experiments (to be discussed later) would yield some indication of accommodation coefficients for helium, but unfortunately this did not prove possible, as we shall see. For conservatism then, it would seem prudent to use values of about 0.40 and 0.6 for helium on glass and metal, respectively, and 0.95 for air on all surfaces. We must keep in mind that such values will probably yield conservatively large predictions of heat conduction through the respective gases.

B. Minimum Partial Vacuum

Reference 1 gave curves of conduction heat flux through air and helium as a function of pressure. These curves were based on free molecule conduction in plane geometry. With the methods described above it is now possible to calculate heat conduction through air and helium in a tubular evacuated collector at all pressure levels of interest. Fig. 2 is such a plot constructed for a tubular collector of the dimensions of the Owens-Illinois SUNPAKTM, assuming conservative values for the accommodation coefficients as described above. The radiation heat loss is also shown for comparison. Note that conduction losses are less than 10% of the radiation loss in air at pressures less than about 6×10^{-4} Torr, and in helium, for pressures less than about 1.5×10^{-3} Torr. This helium pressure would be reached after a time period of about 0.5τ , where τ is the time constant for helium penetration discussed in Section IV. Note also that at equilibrium pressure¹ conduction in helium would amount to about 25% of the radiation loss. For comparison, air at atmospheric pressure would produce a conduction loss of approximately 1100 W/m^2 for the conditions of Fig. 2.

These results are in close agreement with those of Ref. 1 for hypothetical flat plate evacuated collectors. Thus those results can be considered correct up to pressures of about 5×10^{-3} Torr.

IV. Analysis of Experimental Data

Under DOE sponsorship, Owens-Illinois, Inc. has been investigating the effects of helium on their SUNPAKTM evacuated tube collector, using experimental methods. Two of their experiments are pertinent to the present study. Several tubes were maintained at elevated temperature ($650\text{--}800^\circ\text{F}$) for an extended period of time and their performance was periodically checked for degradation due to helium penetration. Six of these tubes were subsequently sent to an independent laboratory for residual gas analysis. In addition, a tube was backfilled with helium at various pressures and its performance was checked to determine the effect of the helium on conduction heat losses. From the data of these

Figure 2. Heat Conduction in Air and Helium in an Evacuated Tube Collector $T_1 = 240^\circ F$, $T_2 = 68^\circ F$, $\epsilon_1 = 0.06$, $\epsilon_2 = 0.90$. For air, $\alpha_1 = \alpha_2 = 0.95$; for helium, $\alpha_1 = 0.60$, $\alpha_2 = 0.40$.

two experiments it should be possible to (1) estimate the time constant for helium penetration, and (2) estimate a mean thermal accommodation coefficient for helium molecules with the collector and cover surface, as discussed above.

A. Helium Penetration Rates

Ref. 1 shows that the helium partial pressure in the evacuated region of a collector tube varies with time as

$$P(t) = P_0 (1 - e^{-t/\tau}) \quad (7)$$

where P_0 is the helium partial pressure in the atmosphere (4×10^{-3} Torr at standard conditions) and τ is the time constant, given by

$$\tau = \frac{V}{S} \left(\frac{N_0}{RT} \right) \frac{\Delta x}{K} \quad (8)$$

Here, V/S is the volume-to-surface ratio of the tube, N_0 is Avagadro's number, R is the gas constant, T is the temperature, Δx the tube wall thickness, and K the permeation rate constant, which is strongly temperature dependent. From Eq. (7) it is clear that given the length of time t necessary to reach a given pressure $P(t)$, it is possible to determine the time constant τ . Table 1 gives the data from the residual gas analysis, along with the deduced time constants. In Table 1, the second column gives the time for which the tube was maintained at the corresponding temperature shown in the second column. Column four gives the helium pressure as determined by mass spectroscopy, and the last column displays the time constant. (In some cases, two determinations of pressure were made for the same tube). The uncertainty shown is based on $\pm 10\%$ uncertainty in the measured helium pressures¹⁶. The mean value at 700°F is:

$$\tau = 1.0 \pm 0.4 \text{ yrs.}$$

Using the values at 650, 700, and 800°F, it is possible to deduce an approximate temperature dependence for τ . This cannot be done with much

<u>Tube No.</u>	<u>t</u>	<u>T</u>	<u>P</u>	<u>ϵ'</u>	<u>τ</u>
X302	4145 hrs.	700°F	1.14×10^{-3} Torr	0.16	1.41 ± 0.17 yrs
X310	1670	700	5.02×10^{-4}	0.09	1.42 ± 0.15
X319	3565	650	1.40×10^{-3}	0.25	0.94 ± 0.12
X323	1795	800	1.24×10^{-3} 1.43×10^{-3}	0.19	0.55 ± 0.06 0.46 ± 0.06
X327	2420	700	1.31×10^{-3} 1.18×10^{-3}	0.13	0.70 ± 0.09 0.79 ± 0.09
X330	2400	700	1.50×10^{-3} 1.34×10^{-3}	0.13	0.58 ± 0.08 0.67 ± 0.08

Table 1. Data and Calculated Results from Owens-Illinois Life Tests

confidence, however, since there are so few data points. It is known^{1,17} that the temperature dependence should have the functional form

$$\tau = \frac{C_1}{T} \exp (C_2/T). \quad (9)$$

For the most conservative estimate (shortest time constant) we may use the values for tubes X323 (800°F) and X330 (700°F) to determine C_1 and C_2 . This yields

$$\tau = \frac{136}{T} \exp (1940/T) \quad (10)$$

for τ in years with T in °R. This is the curve sketched in Fig. 3 along with the experimental data. It is clear that many other curves of the form (9) could be fit to the data in Fig. 3, and the curve shown may be unduly pessimistic. In the absence of more complete data, however, it seems prudent to use Eq. (10). Note that at a normal operating temperature of 240°F, Eq. (10) predicts a time constant of only 3.1 years for an air heating collector. Clearly, more complete experimental results are highly desirable.

It should be pointed out that these data predict time constants for helium permeation at high temperatures which do not agree with those in Ref. 1 based on the data of Norton¹⁷. Predictions for the time constant based on the data of Table 1 are consistently larger - i.e. it appears that helium will penetrate the collectors more slowly (above 400°F) than would be expected from Norton's data¹. A likely explanation for this is provided by the selective coating applied to the absorber tube. This coating consists of 800 Å of aluminum covered with a similar thickness of black chrome. It is known that metals are extremely resistant to gas permeation¹⁷ and thus this behavior would be expected. The cover glass, of course, is not coated, but since it remains essentially at ambient temperature it will not admit helium very rapidly.

Figure 3. Temperature Dependence of Time Constant for Helium Permeation

B. Conduction in Helium

In order to explore the effect of helium on collector tube performance, OI tested a tube backfilled with helium at various pressures. The tube was heated by a solar simulator until a steady temperature was reached, and then the simulator was turned off and the collector tube temperature recorded as a function of time during the ensuing cooldown. The rate of decrease of temperature should reflect conduction losses through the helium. By doing this at several different helium pressures it was expected that a better understanding of the effect of helium on collector performance would be obtained. By comparing measurement with calculation it was hoped that values for the accommodation coefficient of helium on the glass surfaces of the collector tube could be determined.

At Owens-Illinois the data were correlated in terms of an "effective emissivity", ϵ' , the idea being that the higher heat losses with helium present could be expressed as an equivalent radiation loss from a surface with a higher emissivity than the actual collector surface.* This apparent emissivity was found²⁰ to range from 0.06 for helium pressure of 2.6×10^{-5} Torr, up to 0.75 at 1.2×10^{-3} Torr. These values certainly indicated that helium has a large effect - too large, in fact, since they are inconsistent with the data from the life tests and with calculations based on methods described earlier in this report.

The same type of measurement was made for the six tubes subjected to life test, as described earlier. The "apparent emissivity" for these tubes is also given in Table 1. The highest value is 0.25 at a measured helium pressure of 1.4×10^{-3} Torr. This is certainly not consistent with the values given above for the helium backfilling tests and would suggest a systematic error in one or the other experiments.

A mathematical model for the backfilling experiments can be constructed as follows. Assuming the cover tube remains near ambient temperature and is opaque to long-wavelength radiation emitted by the absorber tube, the rate of change of absorber tube temperature, T_1 , can be expressed as

*In the opinion of the present author, such a concept is not very useful, since equating conduction heat flow to radiation yields an ϵ' which varies as T_1^3 .

$$C \frac{dT_1}{dt} = -Q_{\text{rad}} - Q_{\text{cond}}, \quad (11)$$

where Q_{rad} represents the radiation loss

$$Q_{\text{rad}} = \frac{\pi d_1 L \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{d_1}{d_2} \left(\frac{1}{\epsilon_2} - 1 \right)} \quad (12)$$

and Q_{cond} the conduction loss, which can be computed by the methods described in Section II. Here C is the heat capacity of the absorber tube, d_1 and d_2 the appropriate tube diameters, L the tube length, T_2 the temperature of the cover, and σ is the Stefan - Boltzmann constant. Equation (11) is a nonlinear differential equation which can be solved straightforwardly by a numerical method to simulate the cooldown of the absorber tube after the heat source is removed. This calculation was performed for each of the eight different pressures used in the experiments, ranging from 2.6×10^{-5} to 1.2×10^{-3} Torr. Excellent agreement was obtained at the lower pressures where radiation dominates, the calculated results generally being within $\pm 1^\circ\text{F}$ of the data through the first forty minutes of the transient. At higher pressures, where conduction is significant, the calculations did not agree with the experimental data at all. For all pressures greater than about 5×10^{-5} Torr, the tube cools much faster than predicted by the theory. This is especially puzzling since the methods described above for transition regime heat conduction in cylinders are known to agree well with experiments and, if anything, to conservatively overpredict the heat conduction. This reinforces the suspicion of a systematic error in the experiments, especially since the "apparent emissivities" reported by the experimenters for the backfilled tubes is in such complete disagreement with those for the life-tested tubes. In fact, if the reported pressures for the backfilled tubes are artificially increased by a factor of about 10, consistent results are obtained for the tube cooldown rates, and the apparent emissivities also appear to be much more reasonable. This would suggest the possibility of a systematic error in the pressure measurement. Whatever the reason, there is no possibility of utilizing these experiments to determine the accommodation coefficients for helium on the inside surfaces of the collector tubes.

In conclusion, analysis of the data from experiments performed at Owens-Illinois, Inc. with their SUNPAKTM solar collector tube provides the very conservative and tentative equation (10) for the time constant for helium penetration, but no information about helium accommodation coefficients.

V. Summary and Conclusions

Methods have been developed for the calculation of conduction heat losses from evacuated solar collectors in either flat plate or tubular geometry. The methods and equations recommended in Section II should be correct to within 10% over the full range of Knudsen number applicable to solar collectors, provided the accommodation coefficients are known to that degree of accuracy.

The minimum partial vacuum necessary to limit heat conduction losses to no more than 10% of radiation losses is about 5×10^{-4} Torr for either evacuated tube or flat plate collectors. The radiation loss is about 5% of the conduction loss which would be experienced without evacuation at operating temperatures and thus evacuation to 5×10^{-4} Torr reduces the total loss to essentially this value.

Helium can be expected to penetrate the evacuated space of borosilicate glass collectors, though the rate of penetration is still quite uncertain. Based on the conservative estimate of Eq. (10), we can say that an air-heating collector of the Owens-Illinois SUNPAKTM design and construction, operating at 240 °F collector surface temperature for an average of 8 hours a day, would have helium at 3×10^{-3} Torr in about 4.5 years. However, at this pressure total heat losses would only increase by about 20%, which is roughly equivalent to a 20% increase in the emittance of the selective surface. Thus it would appear that helium penetration is not likely to produce a serious reduction in the lifetime of an evacuated tube collector.

Reference 1 gives strong evidence that collectors constructed of ordinary soda-lime glass are much less vulnerable to helium penetration than are those constructed of borosilicate.

It would be comforting to have reliable data on which to base these estimates. However, on a cost/benefit basis it is difficult to justify another expensive experimental program. The theoretical results of the present study do not suggest a sufficiently large risk of collector failure as a result of helium penetration.

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Appendix I

Transition Regime Heat Conduction: Influence of Unequal Surface Accommodation

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I. Introduction

The problem of heat conduction between parallel plates is high on the list of the most-studied problems of kinetic theory. It is not the purpose of the present paper, however, to review the literature connected with this problem. It suffices to point out that all previous studies of heat conduction in the transition regime, to this author's knowledge, have assumed a single value for the accommodation coefficient α at both plates. However, several applications (e.g. evacuated solar collectors), and some recent data [1] require a theory which includes the effects of different values of α on each plate. Since such a theory of heat conduction in the transition regime is significantly more complicated than that for a single value due to the lack of symmetry, an accurate approximate value for a mean $\alpha = \bar{\alpha}$ would be quite useful. In the free molecule regime such a mean value is readily available from the classical Knudsen formula [2] as follows:

$$\frac{\bar{\alpha}}{2 - \bar{\alpha}} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2} \quad (1)$$

The purpose of the work reported here was to construct an exact solution of the linear BGK equation [3] for this problem using analysis previously developed by the present author and coworkers [4], for a single α to test the applicability of the approximation (1), and to develop better approximations if necessary.

In section II, the problem is formulated in terms of the linearized BGK model equation with appropriate boundary conditions. Since the solution parallels that of Ref. [4], only a brief outline is given. Section III presents results for the heat flux and temperature profiles, and compares values for the heat flux computed using the mean $\alpha = \bar{\alpha}$ from Eq. (1) with that computed using the two different values. In Section IV conclusions are drawn as to the accuracy of Eq. (1) and some interesting results concerning the temperature profile are pointed out.

II. Analysis

We consider a gas confined between two parallel plates spaced a distance d apart and maintained at temperatures T_1 and T_2 , respectively. Assuming

$T_1 < T_2$ and

$$1 - \frac{T_1}{T_2} \ll 1 \quad (2)$$

we may treat the problem on the basis of the BGK model equation [3, 5]

$$\left[\frac{\partial}{\partial t} + \underline{u} \cdot \nabla \right] f(\underline{z}, \underline{u}, t) = \nu [\hat{f}(\underline{z}, \underline{u}, t) - f(\underline{z}, \underline{u}, t)], \quad (3)$$

where $f(\underline{z}, \underline{u}, t)$ is the particle distribution function, $\underline{z}$ is the position vector, $\underline{u} = (u_1, u_2, u_3)$ the velocity, ν , collision frequency, and t the time. Here $\hat{f}(\underline{z}, \underline{u}, t)$ represents a local Maxwellian distribution. Assuming steady conditions and linearizing about an absolute Maxwellian $f_1(\underline{u})$ based on the temperature at plate 1, we may write the equation as [6]

$$(\underline{c} \cdot \nabla + 1) \bar{h}(\underline{x}, \underline{c}) = \int \bar{h}(\underline{x}, \underline{c}') K(\underline{c}', \underline{c}) e^{-c'^2} d^3 c', \quad (4)$$

with

$$K(\underline{c}', \underline{c}) = \pi^{-3/2} \left[1 + 2 \underline{c} \cdot \underline{c}' + \frac{2}{3} (c^2 - \frac{3}{2}) (c'^2 - \frac{3}{2}) \right], \quad (5)$$

and where

$$\underline{x} = \nu \left(\frac{m}{2kT_0} \right)^{1/2} \underline{z}, \quad (6)$$

$$\underline{c} = \left(\frac{m}{2kT_0} \right)^{1/2} \underline{u}, \quad (7)$$

and

$$f(\underline{x}, \underline{c}) = f_1(\underline{c}) [1 + \bar{h}(\underline{x}, \underline{c})]. \quad (8)$$

Since the present problem is one dimensional in space, we replace $\underline{x}$ with x , measured from the left-hand plate. Using Maxwell's diffuse/specular model for the interaction of the molecules with the walls, we write

$$f(0, \underline{c}) = (1 - \alpha_1) f(0, \underline{c}^*) + \alpha_1 f_1(\underline{c}), \quad c_1 > 0, \quad (9a)$$

and

$$f(\delta, \underline{c}) = (1 - \alpha_2)f(\delta, \underline{c}^*) + \alpha_2 f_2(\underline{c}), \quad c_1 > 0, \quad (9b)$$

where

$$\underline{c}^* = (-c_1, c_2, c_3),$$

$$\delta = v \left(\frac{m}{2kT_o} \right)^{1/2} d,$$

is the inverse Knudsen number, α_1 and α_2 are the accommodation coefficients at plates 1 and 2 respectively, and $f_2(\underline{c})$ is an absolute Maxwellian based on temperature T_2 . Linearizing Eqs. (9) about the same absolute Maxwellian $f_1(\underline{c})$ gives the boundary conditions for Eq. (4) as

$$h(0, \underline{c}) = (1 - \alpha_1)h(0, \underline{c}^*), \quad c_1 > 0,$$

and

$$h(\delta, \underline{c}^*) = (1 - \alpha_2)h(\delta, \underline{c}) + \alpha_2 \left(c^2 + A - \frac{3}{2} \right), \quad (10)$$

where

$$h(x, \underline{c}) = \frac{T_1}{\Delta T} \bar{h}(x, \underline{c}),$$

$$\Delta T = T_2 - T_1,$$

and A is a constant to be determined by the condition of mass conservation at either plate.

Next, following Cercignani [5] we project $h(x, \underline{c})$ onto a basis in the Hilbert space B as follows:

$$h(x, \underline{c}) = \sum_{\ell=1}^4 \psi_{\ell}(x, c_1) g_{\ell}(c_2, c_3) + \psi_5(x, \underline{c}), \quad (11)$$

where

$$\begin{aligned} g_1(c_2, c_3) &= \pi^{-1/2} \\ g_2(c_2, c_3) &= \pi^{-1/2} (c_2^2 + c_3^2 - 1), \end{aligned} \quad (12)$$

$$\begin{aligned} g_3(c_2, c_3) &= \left(\frac{\pi}{2} \right)^{-1/2} c_2, \\ \text{and} \quad g_4(c_2, c_3) &= \left(\frac{\pi}{2} \right)^{-1/2} c_3. \end{aligned}$$

In Eq. (5), $\psi_5(x, c)$ is the component of h which is orthogonal to the space B . The g_i form an orthogonal set with respect to the inner product

$$(g_\ell, g_m)_B = \int_{-\infty}^{\infty} dc_2 \int_{-\infty}^{\infty} dc_3 g_\ell(c_2, c_3) g_m(c_2, c_3) \exp(-c_2^2 - c_3^2). \quad (13)$$

It has been shown [5] that only ψ_1 and ψ_2 relate to temperature-density variations. The decomposition of Eq. (4) according to (11) leads to a pair of coupled equations for $\psi_1(x, c_1)$ and $\psi_2(x_1, c_1)$ which may be written [6]

$$u \frac{\partial}{\partial x} \psi(x, u) + \psi(x, u) = \pi^{-1/2} Q(u) \int_{-\infty}^{\infty} Q^T(u') \psi(x, u') e^{-u'^2} du', \quad (14)$$

where

$$\psi(x, u) = \begin{bmatrix} \psi_1(x, u) \\ \psi_2(x, u) \end{bmatrix},$$

$$Q(u) = \begin{bmatrix} \frac{2}{3} (u^2 - \frac{1}{2}) & 1 \\ \frac{2}{3} & 0 \end{bmatrix},$$

and the superscript T denotes the transpose operation. When the boundary conditions are similarly decomposed we find

$$\psi(0, u) = (1 - \alpha_1) \psi(0, -u), \quad (15)$$

and

$$\psi(\delta, -u) = (1 - \alpha_2) \psi(\delta, u) + \alpha_2 \sqrt{\pi} \begin{bmatrix} u^2 + A \frac{1}{2} \\ 1 \end{bmatrix}. \quad (16)$$

From this point the solution closely follows that of Thomas, Chang, and Siewert [4]. The general solution of Eq. (14) is expressed as an expansion in the elementary solutions (singular eigenfunctions) developed by Kriese, Chang, and Siewert [6], and constrained to satisfy the boundary conditions (15) and (16). This leads to a set of four coupled singular integral equations for the coefficients in the elementary solution expansion. These are converted

to regular Fredholm equations by application of the orthogonality relations of Kriese, Chang, and Siewert [6]. Only two equations were obtained in Ref. [4] since the symmetry allowed certain simplifications in that case. Nevertheless, rapidly convergent iterative numerical solutions to the four Fredholm equations are straightforwardly constructed; since the heat flux and temperature and density profiles are directly related to the expansion coefficients, these quantities are readily computed.

The integral equations were solved by iteration, with values computed at the nodes of a Gauss quadrature set. The number of quadrature points was successively increased until no further change in the results were noted to the total number of significant figures quoted. The final computation utilized 81 Gauss nodes and weights.

III. Results and Discussion

Figure 1 is a plot of the heat flux ratio $q = Q/Q_{fm}$, where Q_{fm} is the heat flux under free molecule conditions. This figure shows q as a function of α_1 for various values of α_2 . Note that the heat flux approaches free molecule conditions as either α_1 or α_2 approaches zero.

In Table 1, the heat flux ratio $q(\alpha_1, \alpha_2)$ is compared with that computed for a single mean accommodation coefficient, $q(\bar{\alpha})$, where $\bar{\alpha}$ is determined from Eq. (1). The two methods yield results which differ by less than 2% over the full range of α_1 and α_2 . Though all results in the table were computed for $\delta = 5.0$, similarly close agreement was obtained for $\delta = 10.0$. Since Eq. (1) would certainly be expected to hold as free molecule conditions are approached ($\delta < 1.0$), it would appear that this approximation holds throughout the free molecule and transition regimes.

Figure 2 shows the temperature profile in the gas for $\alpha_1 = 0.5$ and several values of α_2 . Note that for $\alpha_2 = \alpha_1 = 0.5$, the profile is symmetric about the mid-plane as would be expected, but for other values of α_2 the profile is quite asymmetric. In particular, it is interesting to note that the temperature jump at plate 1 increases with increasing values of α_2 . Apparently, more complete thermal accommodation at the hotter plate increases the temperature throughout the gas, not just near plate 2.

IV. Conclusions

An accurate analytical/numerical solution has been obtained for the problem of heat conduction in a rarefied gas between two parallel plates with different thermal accommodation coefficients on each plate. An approximate mean accommodation coefficient computed from

$$\frac{\bar{\alpha}}{2-\bar{\alpha}} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2} ,$$

when used in a similar analysis, was found to yield results for the heat flux within 2% of the exact values computed using different α_1 , α_2 over the full range of α_1 and α_2 .

In the transition regime, the degree of thermal accommodation on one plate strongly influences the temperature throughout the gas, not just near that plate.

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<u>α_1</u>	<u>α_2</u>	<u>$\bar{\alpha}$</u>	<u>$q(\alpha_1, \alpha_2)$</u>	<u>$q(\bar{\alpha})$</u>
0.7	0.9	0.78750	0.38479	0.38459
0.7	0.5	0.58333	0.49292	0.49263
0.7	0.3	0.42000	0.59906	0.59771
0.7	0.1	0.17500	0.79070	0.80172
0.5	0.3	0.37500	0.63083	0.63041
0.5	0.1	0.16667	0.81145	0.80996
1.0	0.1	0.18182	0.79920	0.79504

Table 1. Heat flux for "mean" α (Eq. 1) compared to exact heat flux $q(\alpha_1, \alpha_2)$, $\delta = 5.0$.

Figure 1. Influence of accommodation on heat transfer.

Figure 2. Influence of accommodation on temperature profiles.

Appendix II

Heat Conduction in Rarefied Gases

Between Concentric Cylinders:

Arbitrary Accommodation on Both Surfaces

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LIST OF SYMBOLS

a_1	inner accommodation coefficient
a_2	outer accommodation coefficient
d	parameter defined by Eq. (23)
f	molecular velocity distribution function
G	function of normalized radius, defined by Eq. (7b)
K	function of normalized radius, defined by Eq. (7a)
k	Boltzmann's constant
Kn	Knudsen number
m	mass of gas molecule, kg
n_1	number density associated with molecules moving from inner to outer cylinder, part./m ³
n_2	number density associated with molecules moving from outer to inner cylinder, part./m ³
q_R	radial heat flux, W/m ²
R	Radius, cm
R_1	radius of inner cylinder, cm
R_2	radius of outer cylinder, cm
T_1	temperature associated with molecules moving from inner to outer cylinder, °K
T_2	temperature associated with molecules moving from outer to inner cylinder, °K
T_I	temperature of molecules leaving inner cylinder, °K
T_{II}	temperature of molecules leaving outer cylinder, °K
T_{w1}	inner cylinder temperature, °K
T_{w2}	outer cylinder temperature, °K
α	angle shown in Fig. 1
β	integration constant
ϵ	temperature difference defined by Eq. (13)
ξ	molecular velocity
λ	mean free path, m

I. Introduction

Existing methods [1-5] for the calculation of heat conduction in low-density gases in cylindrical geometry are limited to the case of complete accommodation of the gas molecules on the outer cylindrical surface. This is appropriate for the classical experiment involving heat transfer from a fine wire to an outer cylinder of much larger radius, since the gas molecules undergo many more collisions with the outer cylinder than with the inner. However, many applications, e.g., evacuated tube solar collectors, involve heat conduction in rarefied gases between concentric cylinders of nearly equal radii and different materials. The analysis reported here extends the four moment method of Lees [4] and Hurlbut [5] to the case of arbitrary accommodation on both inner and outer cylindrical surfaces. Both linear and nonlinear methods are presented, and results computed from each are compared to determine the regime of applicability of the linear method.

II. Analysis

For purposes of the present analysis, the Knudsen number, Kn , is defined as the ratio of molecular mean free path to the spacing between inner and outer cylinders. In the transition regime, $10 > Kn > 0.1$, the state of the gas is described by the Boltzmann Equation [1] for the molecular distribution function. The moment method [4] consists of multiplying the Boltzmann equation by functions of velocity and integrating over all velocities; approximate distribution functions are inserted to yield equations for the macroscopic quantities density, temperature, heat flux, and momentum. For the distribution function, Lees and Liu [1] utilize a two-sided Maxwellian given by

$$f = f_1 = n_1 \left(\frac{m}{2\pi k T_1} \right)^{3/2} \exp \left[\frac{-m}{2k T_1} (\xi_p^2 + \xi_z^2) \right], \quad \alpha < \phi < \pi - \alpha, \quad (1)$$

for velocities in region 1 and

$$f = f_2 = n_2 \left(\frac{m}{2\pi k T_2} \right)^{3/2} \exp \left[\frac{-m}{2k T_2} (\xi_p^2 + \xi_z^2) \right], \quad \pi - \alpha < \phi < 2\pi + \alpha, \quad (2)$$

for velocities in region 2, where regions 1 and 2 are defined in Fig. 1. The resulting moment equations are [1]

$$\text{continuity: } n_1 T_1^{1/2} = n_2 T_2^{1/2}; \quad (3)$$

momentum:

$$(\sin 2\alpha - 2\alpha) \frac{d}{dR} (n_1 T_1 - n_2 T_2) + \pi \frac{d}{dR} (n_1 T_1 + n_2 T_2) = 0; \quad (4)$$

energy:

$$n_1 T_1^{3/2} - n_2 T_2^{3/2} = \beta; \quad (5)$$

and heat flux:

$$\begin{aligned} & (\sin 2\alpha - 2\alpha) \frac{d}{dR} (n_1 T_1^2 - n_2 T_2^2) + \pi \frac{d}{dR} (n_1 T_1^2 + n_2 T_2^2) \\ & + \frac{4}{15} \frac{R_1}{\lambda_1 \beta R} n_1 (\pi - 2\alpha) + n_2 (\pi + 2\alpha) = 0, \end{aligned} \quad (6)$$

where β is a constant of integration.

A. Nonlinear Method

Following Lees and Liu [1], we define the two functions

$$K(R) = n_1 T_1 - n_2 T_2 \quad (7a)$$

and

$$G(R) = n_1 T_1 + n_2 T_2. \quad (7b)$$

When these functions are substituted into Eqs. (4) and (6), we find the following two nonlinear differential equations for K and G :

$$\frac{dG}{dR} = \frac{2\alpha - \sin 2\alpha}{\pi} \frac{dK}{dR}, \quad (8)$$

and

$$\frac{dK}{dR} = \frac{\frac{32}{15} \frac{R_1}{\beta^3 \lambda_1} \left(\frac{\pi G + 2\alpha K}{1/K^2 - 1/G^2} \right) \frac{1}{R}}{2(2\alpha - \sin 2\alpha) \left(\frac{1}{G^2} - \frac{1}{K^2} \right) + \pi \frac{G}{K^3} - \frac{K}{G^3} \cdot \frac{(\sin 2\alpha - 2\alpha)}{\pi}} \quad (9)$$

It should be noted that in Eq. (9), several mistakes have been corrected from the comparable equation of Lees and Liu. [1]

Once K and G are determined by solving Eqs. (8) and (9), n_1 , n_2 , T_1 , and T_2 can be recovered through Eqs. (3), (5), and (7). The radial heat flux, q_R , is proportional to β , and can also be determined by solving Eqs. (8) and (9). Boundary conditions for the temperature at the inner and outer cylinders are defined through the accommodation coefficients:

$$a_1 = \frac{T_I - T_2(R_1)}{T_{w1} - T_2(R_1)}, \quad (10)$$

and

$$a_2 = \frac{T_{II} - T_1(R_2)}{T_{w2} - T_1(R_2)}, \quad (11)$$

where T_{w1} and T_{w2} are the temperatures of the inner and outer surfaces, and

T_I and T_{II} are the temperatures of the molecules leaving the inner and outer surfaces, respectively. Using Eqs. (3) and (5) in Eq. (10) yields two coupled equations for $T_1(R_1)$ and $T_2(R_2)$; combining these, we find that $T_1(R_1)$ must satisfy

$$T_1(R_1)^3 - 2T_1(R_1)^2 + T_1(R_1) - \frac{\beta(1-a_1)^2}{a_1} = 0. \quad (12)$$

In general, Eq. (12) has 3 roots. However, two of these solutions can be eliminated on physical grounds. The remaining solution provides a boundary condi-

tion for Eqs. (8) and (9). Boundary conditions for n_1 , n_2 , T_1 , and T_2 are derived similarly. From these, boundary conditions on G and K are obtained, which are seen to depend upon β .

We have developed a numerical scheme to solve Eqs. (8) and (9) as follows. Choosing a value for β , starting values for K and G at $R = R_1$ are generated. Eqs. (8) and (9) are then integrated numerically to find K and G at $R = R_2$, and these values are compared to the correct values for the chosen accommodation coefficient, a_2 . If they do not match, another value for β is tried and the integration is carried out again. In this way, the correct β is converged upon iteratively, and the resulting value is used to compute the heat flux. These results are used as the standard of comparison for the linear method to be described.

B. Linear Method

First we summarize the methods and results of Lees and Lui. [1] Equations (3) - (6) are linearized under the assumption that

$$\epsilon = 1 - \frac{T_{II}}{T_1} \ll 1. \quad (13)$$

This allows us to write [1]

$$n_1 = 1 + N_1; \quad n_2 = 1 + N_2;$$

$$T_1 = 1 + t_1; \quad T_2 = 1 + t_2;$$

where N_1 , N_2 , t_1 , and $t_2 \ll 1$. When Eqs. (3) - (6) are linearized in this way, they can be solved explicitly to yield [1]

$$N_1 = \frac{4}{15} \frac{R_1}{\lambda_1} \beta \ln\left(\frac{R}{R_1}\right), \quad (14)$$

$$N_2 = \beta \left[\frac{1}{2} + \frac{4}{15} \frac{R_1}{\lambda_1} \ln\left(\frac{R}{R_1}\right) \right], \quad (15)$$

$$t_1 = -\frac{4}{15} \frac{R_1}{\lambda_1} \beta \ln\left(\frac{R}{R_1}\right), \quad (16)$$

and

$$t_2 = -\beta \left[1 + \frac{4}{15} \frac{R_1}{\lambda_1} \ln\left(\frac{R}{R_1}\right) \right], \quad (17)$$

where

$$\beta = \frac{\epsilon}{1 + \frac{4}{15} \frac{R_1}{\lambda_1} \ln\left(\frac{R_2}{R_1}\right)}. \quad (18)$$

This result has been extended to the case of arbitrary accommodation coefficient at the inner surface by Hurlbut. [5] We have extended it to the case of arbitrary accommodation at either surface as follows: If complete accommodation prevails, T_I and T_{II} in Eq. (13) are equal to the wall temperatures T_{w1} and T_{w2} respectively. For arbitrary accommodation, the definition of the accommodation coefficient gives

$$a_1 = \frac{T_I - T_2 (R_1)}{T_{w1} - T_2 (R_1)} \quad (19)$$

and

$$a_2 = \frac{T_{II} - T_1 (R_2)}{T_{w2} - T_1 (R_2)} \quad (20)$$

Using the linearized forms for T_1 and T_2 , and eliminating t_1 and t_2 with Eqs. (16) and (17), we find [8] expressions for T_I and T_{II} in terms of T_{w1} and T_{w2} :

$$T_I = \frac{(a_1 a_2 d - a_1 a_2 + a_1) T_{w1} + (a_2 - a_1 a_2) T_{w2}}{a_1 a_2 d - 2a_1 a_2 + a_1 + a_2}, \quad (21)$$

and

$$T_{II} = \frac{(a_1 a_2 d - a_1 a_2 + a_2) T_{w2} + (a_1 - a_1 a_2) T_{w1}}{a_1 a_2 d - 2a_1 a_2 + a_1 + a_2}, \quad (22)$$

where

$$d = 1 + \frac{4}{15} \frac{R_1}{\lambda_1} \ln \left(\frac{R_2}{R_1} \right). \quad (23)$$

When Eqs. (21) - (23) are used in Eq. (18), the resulting value for β yields the heat flux per unit area at radius R :

$$q_R = \left(\frac{2}{\pi m} \right)^{1/2} \frac{R_1}{R} k^{3/2} \beta, \quad (24)$$

where

$$\beta = \frac{1 - T_{w2}/T_{w1}}{\frac{R_1}{R_2} \left(\frac{1}{a_2} - 1 \right) + \frac{1}{a_1} + (d-1)} \quad (25)$$

A similar result has been obtained by Shih [6], but no numerical results were given for arbitrary accommodation at both surfaces.

III. Numerical Results

Numerical results have been computed by the methods described above to determine the accuracy of the simple linear equation relative to the more complicated nonlinear method. Some of the results of this numerical study are shown in the accompanying figures and tables in terms of the ratio of the actual heat flux to that which is predicted by free-molecule theory. [7]

Figure 2 shows the influence of the outer accommodation coefficient, a_2 , on the heat transfer ratio, and also compares the results of the linear and nonlinear formulations for two different Knudsen numbers. It can be seen that incomplete outer accommodation has more effect at the larger Kn, and also that the linear equation is within approximately 5% of the nonlinear over the full range at $Kn = 1.0$.

Figure 3 indicates how the nonlinear and linear methods converge to the same result as free molecule conditions are approached. Note also that the linear equation becomes increasingly in error as the Knudsen number tends toward smaller values (mean free path decreasing relative to tube dimensions) and as the temperature ratio deviates more from unity. Both of these trends are, of course, expected on physical grounds. Similar but more pronounced results are seen in Fig. 4 where the inner and outer accommodation coefficients are different from unity.

These results tend to mask the accuracy of the linear method in computing actual heat transfer results in practical cases, as shown by Fig. 5. Here the actual heat flux is computed for a temperature ratio of 0.5 and radius ratio of 2. Clearly, the linear formula gives very accurate results for $Kn \gtrsim 0.3$.

To establish confidence in our nonlinear results, we have compared with nonlinear results of Shih [6]. Shih uses two different nonlinear methods, the first consisting of a series expansion of four radial functions in terms of the normalized temperature difference. The results of this method are given in Table 1 for a temperature ratio of 0.75. The second method uses a bimodal two-stream distribution function to iterate on a constraint equation derived from the boundary conditions. The results of this method are shown in Table 2 for a temperature ratio of 0.5. The second method should be the more accurate of the two. Both methods assume complete accommodation at each surface. At radius ratios up to 10, the present results match Shih's results very closely. At higher radius ratios, Shih's heat transfer ratio becomes somewhat higher, between 15 and 20 percent higher at a ratio of 100. Both of Shih's methods yield larger values than the present nonlinear method, but the results of Shih's more accurate method are closer to the results of the method derived in this analysis. This would seem to indicate that the nonlinear approach derived here is at least as accurate as Shih's expansion method. The obvious advantage of the present method is the ability to include arbitrary accommodation at each surface.

IV. Conclusions

The linear description of heat transfer in a low density gas between concentric cylinders with arbitrary accommodation at both surfaces, Eqs. (24) and (25), gives good results for $Kn \gtrsim 3.0$ and temperature ratios $T_{w_2}/T_{w_1} \gtrsim 0.75$.

Outside this range, the nonlinear formulation described in section II should be used. The linear formulae always overpredict the heat transfer, but the error decreases as accommodation coefficients decrease.

Figure 1. Problem Geometry

Figure 2. Influence of a_2 on Heat Transfer Ratio, $a_1 = 1.0$.

Figure 3. Effect of Knudsen Number on the Heat Transfer Ratio, $a_1 = a_2 = 1.0$

Figure 4. Effect of Knudsen Number on Heat Transfer Ratio, $a_1 = 0.3$, $a_2 = 0.9$.

Figure 5. Effect of Knudsen Number
on Heat Flux, $T_{w2}/T_{w1} =$
 0.5 , $R_2/R_1 = 2.0$.

Table 1.

NON-LINEAR HEAT TRANSFER RATIO COMPARISON

WITH SHIH⁽⁶⁾, $T_{w2}/T_{w1} = 0.75$ $K_n = 1.0$ $K_n = 0.1$

R_{II}	<u>Present Method</u>	<u>Shih</u>	<u>Present Method</u>	<u>Shih</u>
1.005	0.9996	1.0	0.9926	1.0
1.5	0.9433	0.94	0.6223	0.62
2.0	0.9053	0.91	0.4875	0.49
2.5	0.8771	0.88	0.4163	0.42
3.75	0.8291	0.83	0.3277	0.34
5.0	0.7075	0.80	0.2837	0.28
7.0	0.7617	0.77	0.2443	0.25
10	0.7260	0.73	0.2116	0.22
20	0.6586	0.67	0.1642	0.18
35	0.6029	0.64	0.1342	0.15
50	0.5645	0.61	0.1172	0.14
75	0.5168	0.58	0.0990	0.13
100	0.4799	0.56	0.0867	0.12

Table 2.

NON-LINEAR HEAT TRANSFER RATIO COMPARISON

WITH SHIH⁽⁶⁾, $T_{w2}/T_{w1} = 0.5$

$\bar{R}_{II}$	$K_n = 1.0$		$K_n = 0.1$	
	Present Method	Shih	Present Method	Shih
1.005	0.9990	1.0	0.9923	1.0
1.5	0.9357	0.93	0.5900	0.57
2.0	0.8924	0.88	0.4522	0.44
2.5	0.8601	0.86	0.3819	0.37
3.75	0.8057	0.80	0.2967	0.29
5.0	0.7699	0.77	0.2555	0.24
7.0	0.7307	0.74	0.2190	0.21
10	0.6920	0.70	0.1892	0.18
20	0.6217	0.64	0.1467	0.14
35	0.5666	0.60	0.1206	0.13
50	0.5301	0.57	0.1059	0.12
75	0.4865	0.55	0.0906	0.11
100	0.4535	0.53	0.0803	0.10

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