MASTER

SOURCE EFFECTS ON IMPURITY AND
HEAT TRANSPORT IN A TOKAMAK

by

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Abstract

A recently developed generalization of neoclassical theory is extended here to study heat flux contributions to impurity transport, as well as the heat fluxes themselves. The theory accounts for the first four source moments, with external drags, which has been studied previously with either fewer moments or restricted to a collisional plasma. Conditions are established for which a momentum source may be used to modify the particle and heat transport. In the course of this work, the particle and heat transport is evaluated for a two species plasma with arbitrary plasma geometry, beta, and collisionality.

I. Introduction

The neoclassical description of transport phenomena in a tokamak in the presence of external sources and drags is extended here to include the first four moments of the Boltzmann equation. This follows the work of Stacey and Sigmar,^{1,2} who carried the development through the first two moments and corresponding sources. Burrell³ has recently carried the development through the first four moments but only for a plasma in the collisional regime, while this work which utilizes Hirshman's general formalism,^{4,5} is applicable to all collisionality regimes. The resulting formalism is specialized to a two species plasma in which the main ion-impurity collisions are dominant. This report is a summary of a more detailed description of the work given elsewhere.⁶

II. Derivation of Transport Equations

A. First Order Flows

The starting point in determining the particle and heat flows is the moments of the Boltzmann equation. The first four moments are used in this study and, since each moments equation includes elements of higher moments, assumptions must be made to close the system.

The even moments for a species i ,

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i \bar{\mathbf{v}}_i = S_i^0 \quad (1)$$

and

$$\begin{aligned} \frac{3}{2} \frac{\partial p_i}{\partial t} + \bar{\mathbf{v}}_i \cdot \nabla \bar{\mathbf{v}}_i + \frac{3}{2} \nabla \cdot (\bar{\mathbf{v}}_i p_i) + \nabla \cdot \bar{\mathbf{q}}_i + p_i \nabla \cdot \bar{\mathbf{v}}_i \\ = R_i^2 + S_i^2 - \bar{\mathbf{v}}_i \cdot (\bar{\mathbf{R}}_i^1 + \bar{\mathbf{S}}_i^1) + \frac{1}{2} m_i (\bar{\mathbf{v}}_i \cdot \bar{\mathbf{v}}_i) S_i^0 \end{aligned} \quad (2)$$

express the conservation of particles and energy, respectively, while the odd moments,

$$\begin{aligned} m_i n_i \frac{\partial \bar{V}_i}{\partial t} + m_i n_i (\bar{V}_i \cdot \nabla) \bar{V}_i + \nabla p_i + \nabla \cdot \pi_i \\ = n_i e_i (\bar{E}_i + \bar{V}_i \times \bar{B}) + \bar{R}_i + (\bar{S}_i^1 - m_i \bar{V}_i S_i^0) \end{aligned} \quad (3)$$

and

$$\begin{aligned} \frac{\partial}{\partial t} (\bar{Q}_i - \frac{5}{2} n_i T_i \bar{V}_i) + \frac{5}{2} n_i \bar{V}_i \frac{\partial T_i}{\partial t} + \nabla \cdot \bar{\theta}_i \\ - \frac{5}{2} \frac{T_i}{m_i} \nabla \cdot \bar{M}_i - \frac{e_i}{m_i} (\bar{Q}_i - \frac{5}{2} n_i T_i \bar{V}_i) \times \bar{B} \\ - \frac{e_i}{m_i} \bar{E} \cdot [\bar{M}_i - \frac{1}{3} \text{Tr}(\bar{M}_i) \bar{I}] = \left[(\bar{R}_i^3 - \frac{5}{2} \frac{T_i}{m_i} \bar{R}_i^1) \right] + \left[(\bar{S}_i^3 - \frac{5}{2} \frac{T_i}{m_i} \bar{S}_i^1) \right] \end{aligned} \quad (4)$$

describe the momentum and energy flux balances. The scalar terms involved are m_i , e_i , n_i , T_i , and p_i ; the mass, charge, number density, temperature, and pressure, respectively, for species i . The vectors $\bar{E}$ and $\bar{B}$ are the electric and magnetic fields, and the vectors $\bar{V}_i$, $\bar{q}_i$ and $\bar{Q}_i$ represent the net velocity, heat flux, and heat flow. Tensors describing the anisotropic shear stress, energy flux, total pressure, and momentum stress are $\bar{\pi}_i$, $\bar{\theta}_i$, $\bar{P}_i$ and $\bar{M}_i$. Finally, $\bar{R}_i^n$ and $\bar{S}_i^n$ are the collisional friction and source moment of order n for species i . Previous authors have included the first four moments in multispecies transport studies, but have not included sources^{4,5,8} or were restricted to a collisionality regime.^{3,7,8}

Using gyroradius ordering (δ), the first-order perpendicular flows are found from the vector product of the magnetic field on the even moment equations,

$$\begin{aligned} n_i \bar{V}_{i\perp} &= \frac{1}{e_i B_i} \hat{n} \times (\nabla p_i + n_i e_i \nabla \phi) \\ &= \frac{RB}{e_i B} (p_i' + n_i e_i \phi') \frac{\hat{n} \times \nabla \psi}{|\nabla \psi|} \equiv \frac{B}{B_\phi} \frac{G_i(\psi)}{B} \hat{n}_\perp \end{aligned} \quad (5)$$

and

$$\begin{aligned}\bar{q}_{i\perp} &= \Omega^{-1} \hat{n} \times \left[v \frac{5}{2} \frac{p_i T_i}{m_i} - \frac{5}{2} \frac{T_i}{m_i} v p_i \right] \\ &= \frac{5}{2} \frac{1}{e_i B} R B_p p_i T_i \hat{n}_\perp \equiv \frac{B_p}{B B_\phi} D_i(\psi) \hat{n}_\perp\end{aligned}\quad (6)$$

where the prime denotes a derivative in the ψ (radial) direction. Both of these results agree with the subject reviews of Braginskii⁹ and Hinton and Hazeltine,¹⁰ so the effects of external sources and drags do not enter the theory at this level.

On time scales corresponding to the time of buildup of particles and heat due to an external source, the flux surface average of the odd moment equations become

$$\frac{\partial n_i}{\partial t_i} = \langle S_i^0 \rangle + O(\delta^2) \quad (7)$$

and

$$\frac{3}{2} \frac{\partial p_i}{\partial t_i} = \langle S_i^2 \rangle + O(\delta^2) \quad (8)$$

With this, the lowest order portions of the odd moment equations are solved, using the perpendicular flows above, for the parallel flows

$$\begin{aligned}n_i v_{i\parallel} &= -\frac{G_i}{B} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) + B(I_i^0 - \langle B^2 I_i \rangle / \langle B^2 \rangle) \\ &+ \langle B n_i v_{i\parallel} \rangle B / \langle B^2 \rangle \hat{n}\end{aligned}\quad (9)$$

and

$$\begin{aligned}\bar{q}_{i\parallel} &= - \left[\frac{D_i}{B} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) + B(I_i^2 - \langle B^2 I_i^2 \rangle / \langle B^2 \rangle) \right. \\ &\left. - \frac{5}{2} T_i R(I_i^0 - \langle R^2 I_i^0 \rangle / \langle R^2 \rangle) + B \langle B q_{i\parallel} \rangle / \langle B^2 \rangle \right] \hat{n}\end{aligned}\quad (10)$$

The terms $I_i^{0,2}$ are introduced for the asymmetric sources and defined as

$$I_i^{0,2} \equiv \int_0^{\ell_p} \frac{(S_i^{0,2} - \langle S_i^{0,2} \rangle)}{B_p} d\ell_p, \quad (11)$$

which vanishes if the particle or energy source is constant* over the flux surface. This leaves only the flux surface averaged parallel flows, the constants $\langle B n_i V_{i\parallel} \rangle$ and $\langle B q_{i\parallel} \rangle$, undetermined.

The parallel and perpendicular flows are combined to obtain

$$n_i \bar{V}_i = \left(\frac{\bar{B}_p \cdot n_i \bar{V}_i}{B_p^2} \right) \bar{B} - \frac{R}{e_i} (p_i + n_i e_i \phi') R \nabla \phi \quad (12)$$

and

$$\bar{q}_i = \left(\frac{\bar{B}_p \cdot \bar{q}_i}{B_p^2} \right) \bar{B} - \frac{R}{e_i} \left(\frac{5}{2} \frac{B_\phi^2}{B^2} p_i T_i' \right) R \nabla \phi. \quad (13)$$

The last term in each equation represents the rigid body rotation (of either particle or heat flux) in the toroidal ($\hat{\phi}$) direction. The remaining flows in the direction of the magnetic field are expressed

$$\begin{aligned} \Gamma_{ip} &\equiv \left(\frac{\bar{B}_p \cdot n_i \bar{V}_i}{B_p^2} \right) = \frac{F}{e_i \langle B^2 \rangle} (p_i + n_i e_i \phi') \\ &+ (I_i^0 - \langle B^2 I_i^0 \rangle / \langle B^2 \rangle) + \langle B n_i V_{i\parallel} \rangle / \langle B^2 \rangle, \end{aligned} \quad (14)$$

and

$$\begin{aligned} q_{ip} &\equiv \left(\frac{\bar{B}_p \cdot \bar{q}_i}{B_p^2} \right) = \frac{F}{e_i \langle B^2 \rangle} \left(\frac{5}{2} \frac{B_\phi^2}{B^2} p_i T_i' \right) \\ &+ (I_i^2 - \langle B^2 I_i^2 \rangle / \langle B^2 \rangle) - \frac{5}{2} T_i (I_i^0 - \langle B^2 I_i^0 \rangle / \langle B^2 \rangle) + \langle B q_{i\parallel} \rangle / \langle B^2 \rangle. \end{aligned} \quad (15)$$

where $F = R B_\phi = f(\psi)$. Both of these flows lie strictly on the flux surface $[f(\psi)]$ if the source terms cancel.

B. Parallel Flow Evaluation

In order to close the system of equations and evaluate the constants, the form for viscous shears given by Hirshman,^{4,5}

*constant including toroidal effects.

$$\langle \bar{\mathbf{B}} \cdot \nabla \cdot \vec{\pi}_i \rangle = 3\mu_{ii} \langle (\hat{\mathbf{n}} \cdot \nabla \mathbf{B})^2 \rangle_{ip} + 3\mu_{zi} \langle (\hat{\mathbf{n}} \cdot \nabla \mathbf{B})^2 \rangle_{ip} , \quad (16)$$

and

$$\left\langle \bar{\mathbf{B}} \cdot \nabla \cdot \left(\vec{\pi} - \frac{5}{2} \frac{T_i}{e_i} \vec{M}_i \right) \right\rangle = 3 \langle (\hat{\mathbf{n}} \cdot \nabla \mathbf{B})^2 \rangle \frac{T_i}{m_i} (\mu_{zi} r_{ip} + \mu_{zi} q_{ip}) , \quad (17)$$

are substituted into the parallel components of the flux-surface averages of the second and fourth moment equations,

$$\langle \bar{\mathbf{B}} \cdot \nabla \cdot \vec{\pi}_i \rangle = \langle \bar{\mathbf{B}} \cdot \vec{R}_i^1 \rangle + \langle \bar{\mathbf{B}} \cdot \vec{S}_i^1 \rangle \quad (18)$$

and

$$\left\langle \bar{\mathbf{B}} \cdot \nabla \cdot \left(\vec{\pi} - \frac{5}{2} \frac{T_i}{e_i} \vec{M}_i \right) \right\rangle = \frac{T_i}{m_i} \langle \bar{\mathbf{B}} \cdot \vec{R}_i^3 \rangle + \langle \bar{\mathbf{B}} \cdot \vec{S}_i^3 \rangle - \frac{5}{2} \frac{T_i}{m_i} \langle \bar{\mathbf{B}} \cdot \vec{S}_i^1 \rangle . \quad (19)$$

The form of the shears are consistent with those of Hirshman¹¹ and verified in its extension by Stacey and Sigmar.¹ The shears being set to zero apply to a collisional regime, which was examined by Burrell.³

The form for the "friction" terms are also taken from Hirshman.⁵ It is at this point that the development is specialized to a two species plasma. Denoting the species by i and z, the friction terms are

$$R_i^1 = -C_1^*(Z_i^*) m_i n_i v_{iz} (\bar{V}_i - \bar{V}_z) + \frac{2}{5} C_2^*(Z_i^*) m_i n_i v_{iz} \left(\frac{\bar{q}_{ip}}{p_i} - \frac{\bar{q}_{zp}}{p_z} \right) = -R_z^1 \quad (20)$$

and

$$\begin{aligned} \bar{R}_i^3 &= \bar{R}_i^3 - \frac{5}{2} \frac{T_i}{m_i} \bar{R}_i^1 = C_2^*(Z_i^*) m_i n_i v_{iz} (\bar{V}_i - \bar{V}_z) \\ &- \frac{2}{5} C_3^*(Z_i^*) m_i n_i v_{iz} \left(\frac{\bar{q}_{ip}}{p_i} - \frac{\bar{q}_{zp}}{p_z} \right) = -\bar{R}_z^3 , \end{aligned} \quad (21)$$

where C_n^* are the constants dependent on the effective charge for scattering, $Z_i^* = \sum_{j>i} v_{ij}/v_{ii}$. These constants are fit approximately

by

$$C_1^*(\alpha) \approx .48 + \frac{.31}{.59 + \alpha} \quad (22a)$$

$$C_2^*(\alpha) \approx .30 + \frac{.41}{.59 + \alpha} \quad (22b)$$

and

$$C_3^*(\alpha) \approx 1.13 + .50\alpha + \frac{.53\alpha}{.59 + \alpha} \quad (22c)$$

as given by Hirshman.⁷ Similar constraints were used by Rutherford¹² with the inter-relationships

$$C_1^* = C_1 + \frac{C_2^2}{C_3} \quad (23a)$$

$$C_2^* = \frac{5}{2} \frac{C_2}{C_3} \quad (23b)$$

and

$$C_3^* = \left(\frac{25}{4C_3} \right) \alpha \quad (23c)$$

The asterisk will be dropped in all further equations.

The source moments are defined here following Stacey and Sigmar² and are

$$S_i^0 = S_i \quad (24a)$$

$$\bar{S}_i^1 = \bar{K}_i^1 - m_i n_i v_{iz} \beta_i \bar{V}_i \quad (24b)$$

$$S_i^2 = K_i^2 - \frac{1}{2} m_i n_i v_{iz} \beta_i V^2 \quad (24c)$$

$$\bar{S}_i^3 = \bar{K}_i^3 - \frac{1}{2} m_i n_i v_{iz} \beta_i V^2 \bar{V}_i = \bar{K}_i^3 - v_{iz} \beta_i \bar{Q}_i \quad (24d)$$

where $K_i^{1,3}$ are the directed inputs of the momentum and heat flux and S_i^1 and K_i^2 are the sources of particles and heat. β_i is defined as the ratio of the external drag to the collision frequency, $\beta_i = v_{di}/v_{iz}$.

These equations are combined and solved simultaneously for two species. The averaged parallel heat flux is found to be

$$\frac{\langle Bq_{i\parallel} \rangle}{p_i} = [(a_z + C_s)M_i + C_s M_z] / m_i n_i v_{iz} D_{zi} \quad (25)$$

where

$$a_z = \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{3z}}{\mu_{1z}} + \frac{5}{2} \beta_z \quad (26a)$$

$$D_{zi} = (a_z + C_s)a_i + C_s a_z = D_{iz} \quad (26b)$$

$$\tilde{\mu}_i \equiv 3\mu_{1i} \langle (\hat{n} \cdot \nabla B)^2 \rangle / n_i \langle B^2 \rangle \quad (26c)$$

$$M_i = \frac{5}{2} \left\{ Q_i + m_i n_i v_{iz} \left[C_z \left(\frac{\langle B n_i V_{i\parallel} \rangle}{n_i} - \frac{\langle B n_z V_{z\parallel} \rangle}{n_z} \right) \right] - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \frac{\langle B n_i V_{i\parallel} \rangle}{n_i} \right\} \quad (26d)$$

and

$$Q_i = \frac{m_i}{T_i} \langle BK_{i\parallel}^3 \rangle - n_i \tilde{\mu}_i \frac{\mu_{2i}}{\mu_{1i}} \left[F \left(\frac{p_i}{n_i e_i} + \phi \right) + \frac{\langle \Delta_i^0 \rangle}{n_i} \right] - \frac{5}{2} \langle BK_{i\parallel}^1 \rangle - n_i \tilde{\mu}_i \frac{\mu_{3i}}{\mu_{1i}} \left[F \left(\frac{T_i}{e_i} \right) + \frac{\langle \Delta_i^2 \rangle}{p_i} \right] \quad (26e)$$

The solution for $\langle Bq_{z\parallel} \rangle$ is found by interchanging i and z subscripts. The asymmetric source terms are defined,

$$\langle \Delta_i^0 \rangle \equiv \langle B^2 \rangle I_i^0 - \langle B^2 I_i^0 \rangle \quad (27a)$$

and

$$\langle \Delta_i^2 \rangle \equiv \langle B^2 \rangle I_i^2 - \langle B^2 I_i^2 \rangle - \frac{5}{2} \frac{T_i}{m_i} \langle \Delta_i^0 \rangle \quad (27b)$$

Similarly, the averaged parallel particle flows are solved for two species and found to be

$$\frac{\langle B n_i V_{i\parallel} \rangle}{n_i} = [(\xi_z + \lambda)N_i + \lambda N_z] / m_i n_i v_{iz} d_{zi} \quad (28)$$

where

$$N_i = \langle BK_{i||} \rangle - n_i \tilde{\mu}_i \left[F \left(\frac{p_i'}{n_i e_i} + \phi \right) + \frac{\langle \Delta_i^0 \rangle}{n_i} + \frac{\mu_{2i}}{\mu_{1i}} \left(\frac{F}{e_i} T_i' + \frac{2}{5} \frac{\langle \Delta_i^2 \rangle}{p_i} \right) \right] \\ + C_2 (a_z Q_i - a_i Q_z) / D_{zi} - n_i \tilde{\mu}_i \frac{\mu_{2i}}{\mu_{1i}} [(a_z + C_s) Q_i + C_s Q_z] / m_i n_i v_{iz} D_{zi} \quad (29a)$$

$$\lambda = C_1 - C_2^2 (a_i + a_z) / D_{zi} + C_2 a_i \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} / D_{zi} \\ + \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \left[a_z C_2 + C_s \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} \right] / D_{zi} \quad (29b)$$

$$\xi_i = \beta_i + \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} + C_2 a_z \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} / D_{zi} - C_2 a_i \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} / D_{zi} \\ - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \left[(a_z + C_s) \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} + C_s \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} \right] / D_{zi} \quad (29c)$$

and

$$d_{zi} = d_{iz} = (\xi_i + \lambda) \xi_z + \lambda \xi_i .$$

$\langle B n_z V_{z||} \rangle$ is found by interchanging subscripts and, with back substitution to the averaged heat flows, the first order flows are known in terms of gradients, sources, and electrostatic potential gradients.

The potential gradient was determined from momentum balance in Appendix A of Ref. 6, where ambipolarity of the resultant particle fluxes was proven. The potential gradient is given by

$$- \left\{ \left\langle 1 - \frac{\langle B^2 \rangle}{B_\phi^2} \right\rangle + \frac{1}{m_i n_i v_{iz} (\beta_i + \beta_z) d_{zi}} \left[\gamma_z \frac{(N_z)_{\phi'}}{F \phi'} + \gamma_i \frac{(N_i)_{\phi'}}{F \phi'} \right] \right\} \phi' \\ - \frac{\langle R^2 \nabla \phi \cdot (\bar{K}_i^1 + \bar{K}_z^1) \rangle}{m_i n_i v_{iz} (\beta_i + \beta_z)} \frac{\langle B^2 \rangle}{F^2} + \left\langle 1 - \frac{\langle B^2 \rangle}{B_\phi^2} \right\rangle \left[\frac{\beta_i}{\beta_i + \beta_z} \frac{p_i'}{n_i e_i} + \frac{\beta_z}{\beta_i + \beta_z} \frac{p_z'}{n_z e_z} \right] \\ + \frac{1}{F (\beta_i + \beta_z)} \left[\beta_i \frac{\langle \Delta_i^0 \rangle}{n_i} + \beta_z \frac{\langle \Delta_z^0 \rangle}{n_z} \right] \\ + \frac{1}{m_i n_i v_{iz} (\beta_i + \beta_z) F d_{zi}} \left[\gamma_i (N_i)_{-\phi'} + \gamma_z (N_z)_{-\phi'} \right] \quad (30)$$

where

$$\gamma_z \equiv \lambda(\beta_i + \beta_z) + \beta_z \xi_i \quad (31)$$

and $(N_i)_\phi$ signifies ϕ terms of N_i and $(N_i)_{-\phi}$ signifies all non- ϕ terms.

C. Transport Fluxes

The particle and heat fluxes across flux surfaces are obtained from the toroidal component of the odd moment equations at steady state, and found to be, in Ref. 6,

$$\Gamma_i = \langle \nabla \psi \cdot \bar{n}_i \bar{V}_i \rangle = n_i \langle R^2 \nabla \phi \cdot \bar{V}_i \times \bar{B} \rangle = -\frac{1}{e_i} (\langle R^2 \nabla \phi \cdot \bar{R}_i^1 \rangle + \langle R^2 \nabla \phi \cdot \bar{S}_i^1 \rangle) \quad (32)$$

and

$$q_{\psi i} = \langle \nabla \psi \cdot \bar{q}_i \rangle = \langle R^2 \nabla \phi \cdot \bar{q}_i \times \bar{B} \rangle = -\frac{m_i}{e_i} \left[\langle R^2 \nabla \phi \cdot \bar{R}_i^3 \rangle + \langle R^2 \nabla \phi \cdot (\bar{S}_i^3 - \frac{5}{2} \frac{T_i}{m_i} \bar{S}_i^1) \rangle \right] \quad (33)$$

For the impurity particle flux in the two-species plasma case this becomes

$$\begin{aligned} \Gamma_z = & -\frac{\langle R^2 \nabla \phi \cdot \bar{K}_z^1 \rangle}{e_z} - \frac{m_i n_i v_{iz}}{e_z \langle B^2 \rangle} \left\{ C_1 \left[\frac{p_i'}{n_i e_i} - \frac{p_z'}{n_z e_z} \right] - C_2 \left[\frac{T_i'}{e_i} - \frac{T_z'}{e_z} \right] \right. \\ & - \beta_z \left[\frac{p_z'}{n_z e_z} + \phi' \right] \left. \right\} \langle R^2 (B_a^2 - \langle B^2 \rangle) \rangle - \frac{m_i n_i v_{iz}}{e_z \langle B^2 \rangle} F \left\{ C_1 \left[\frac{\langle \Delta_i^0 \rangle}{n_i} - \frac{\langle \Delta_z^0 \rangle}{n_z} \right] \right. \\ & - \frac{2}{5} C_2 \left[\frac{\langle \Delta_i^2 \rangle}{p_i} - \frac{\langle \Delta_z^2 \rangle}{p_z} \right] - \beta_z \frac{\langle \Delta_z^0 \rangle}{n_z} \left. \right\} - \frac{F}{e_z \langle B^2 \rangle} \left\{ -C_2 [a_z Q_i - a_i Q_z] / D_{zi} \right. \\ & + \left. \left[(C_1 \xi_z - \theta_i - \beta_z \lambda) N_i - (C_1 \xi_i - \theta_z + \beta_z (\xi_i + \lambda)) N_z \right] / d_{zi} \right\}, \quad (34) \end{aligned}$$

where $\langle R^2 (B_\phi^2 - \langle B^2 \rangle) \rangle$ are the Pfirsch-Shüster terms, and

$$\begin{aligned} \theta_i = & \left[C_2 (a_i + a_z) \xi_z - C_2 a_z \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \xi_z \right] \\ & - \left[C_2 a_z \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} - C_2 a_i \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} \right] / D_{zi} \quad (35) \end{aligned}$$

The ion conduction heat flux is found to be

$$\begin{aligned}
q_{\psi i} = & - \frac{m_i n_i v_{iz} T_i}{e_i} \left[C_2 \left(\frac{p'_i}{n_i e_i} - \frac{p'_z}{n_z e_z} \right) - C_3 \left(\frac{T'_i}{e_i} - \frac{T'_z}{e_z} \right) - \frac{5}{2} \beta_i \frac{T'_i}{e_i} \right] \frac{\langle R^2 (B_\phi^2 - \langle B^2 \rangle) \rangle}{\langle B^2 \rangle} \\
& - \frac{m_i n_i v_{iz} T_i}{e_i} F \left[C_2 \left(\frac{\langle \Delta_1^2 \rangle}{n_i} - \frac{\langle \Delta_2^2 \rangle}{n_z} \right) - \frac{2}{5} C_3 \left(\frac{\langle \Delta_1^2 \rangle}{p_i} - \frac{\langle \Delta_2^2 \rangle}{p_z} \right) - \beta_i \frac{\langle \Delta_1^2 \rangle}{p_i} \right] \\
& - \frac{T_i F}{e_i \langle B^2 \rangle} \left\{ \left[D_{zi} - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} (a_z + C_3) / D_{zi} \right] Q_i + \left[C_3 \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} / D_{zi} \right] Q_z \right. \\
& + \left[\left(C_2 \xi_z - \frac{C_3}{C_2} \theta_i - \frac{5}{2} \beta_i A_i / D_{zi} \right) N_i \right. \\
& \left. \left. - \left(C_3 \xi_i - \frac{C_3}{C_2} \theta_z - \frac{5}{2} \beta_i B_z / D_{zi} \right) N_z \right] / D_{zi} \right\} - \frac{m_i}{e_i} \langle R^2 \nabla \phi \cdot \left(\bar{K}_i^3 - \frac{5}{2} \frac{T_i}{m_i} \bar{K}_i^1 \right) \rangle
\end{aligned} \tag{36}$$

where

$$A_i = C_2 a_z \xi_z - (a_z + C_3) \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \right) (\xi_z + \lambda) - C_3 \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} \right) \lambda \tag{37a}$$

and

$$B_z = C_2 a_z \xi_i + (a_z + C_3) \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}} \right) \lambda + C_3 \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{\mu_{2z}}{\mu_{1z}} \right) (\xi_z + \lambda) \tag{37b}$$

III. Reduction to Collisionality Regime

A. Regime Structure

The plasma transport is dependent largely on the frequency and step size of a collisional scattering. The relative frequency of interspecies collisions is measured in terms of the collisionality parameter, $v_{*i} = (v_{ii}/\omega_{ti}) \epsilon^{-3/2}$, which is the ratio of the interspecies collision frequency to the banana orbit frequency, multiplied by the aspect ratio to the 3/2 power. If particles do not in general complete a banana orbit, $(v_{ii} > \omega_{ti})$, the plasma is said to be collisional and is marked by $v_{*i} > 1$. The species is collisionless if the orbits are completed and larger scattering steps are seen, with $v_{*i} < 1$.

The plasma viscosity, $\tilde{\mu}_i$, is dependent on the collisionality parameter such that it is finite in a collisionless plasma, and no viscous effects are seen in a collisional plasma, ($\tilde{\mu}_i = 0$). The first state reached

in a plasma is the collisional regime. In a two-species plasma, the higher main ions will become collisionless before the heavier impurities, and will be called the mixed regime here. Similarly, the collisionless regime is when both or all species reach the collisionless state.

B. Collisional Regime

The lowest temperature regime in which both the impurities and ions are collisional is examined in this section. The parallel viscosity factors, $\tilde{\mu}_i$, $\tilde{\mu}_z$ are set to zero in the limit, and in the large aspect ratio, low beta limit, the fluxes reduce to

$$\begin{aligned} \Gamma_z = & + \frac{2\epsilon^2 m_i n_i v_{iz}}{e_z} \frac{R_o}{B_{po}} \left[\left(C_1 + \frac{\beta_i \beta_z}{\beta_i + \beta_z} \right) \left(\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right) \right. \\ & - C_2 \left(\frac{1}{e_i} \frac{\partial T_i}{\partial r} - \frac{1}{e_z} \frac{\partial T_z}{\partial r} \right) \Big] \\ & + \frac{\epsilon^2 R_o B_o}{e_z B_{po}} m_i n_i v_{iz} \left[\left(C_1 + C_2 + \frac{\beta_i \beta_z}{\beta_i + \beta_z} \right) \left(\frac{\delta_z^0}{n_z} - \frac{\delta_i^0}{n_i} \right) \right. \\ & - \frac{2}{5} C_2 \left(\frac{\delta_z^2}{p_z} - \frac{\delta_i^2}{p_i} \right) \Big] - \epsilon^2 \frac{R_o}{e_z (\beta_i + \beta_z)} (\beta_i K_{\phi z}^1 - \beta_z K_{\phi i}^1) \end{aligned} \quad (38)$$

and

$$\begin{aligned} q_{\phi i} = & \frac{2\delta^2 m_i n_i v_{iz} T_i}{e_i} \frac{R_o}{B_{po}} \left[C_2 \left(\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right) \right. \\ & - C_3 \left(\frac{1}{e_i} \frac{\partial T_i}{\partial r} - \frac{1}{e_z} \frac{\partial T_z}{\partial r} \right) - \frac{5}{2} \beta_i \frac{1}{e_i} \frac{\partial T_i}{\partial r} \Big] \\ & - \frac{m_i}{e_i} \epsilon^2 R_o \left(K_{\phi i}^3 - \frac{5}{2} \frac{T_i}{m_i} K_{\phi i}^1 \right) - \frac{\epsilon^2 R_o B_o}{B_{po}} \frac{m_i n_i v_{iz} T_i}{e_i} \\ & \left[(C_2 + C_3) \left(\frac{\delta_i^0}{n_i} - \frac{\delta_z^0}{n_z} \right) - \frac{2}{5} C_3 \left(\frac{\delta_i^0}{p_i} - \frac{\delta_z^0}{p_z} \right) - \beta_i \left(\frac{\delta_i^0}{n_i} - \frac{5}{2} \frac{\delta_i^2}{p_i} \right) \right], \end{aligned} \quad (39)$$

where

$$\langle \Delta_i^0 \rangle \approx \frac{\epsilon^2 R_o B_o^2}{B_{po}} \delta_i^0 \quad (40a)$$

and

$$\langle \Delta_i^2 \rangle \approx \frac{\epsilon^2 R_O B_O}{B_{po}} \left(\delta_i^2 - \frac{5}{2} \frac{T_i}{m_i} \delta_i^0 \right) \quad (40b)$$

are the low beta limits of the asymmetric source terms. The particle results agree with those of Stacey and Sigmar² to within constant factors attributable to different forms taken for the friction terms, but now include temperature gradient effects and the asymmetric heat sources. Both fluxes agree with those of Burrell,³ but now include impurity temperature gradients, asymmetric impurity heat sources, and a directed heat flux in the heat flux equation.

No major differences with the results of previous authors were found in this limit, the usual negative ion pressure gradient drives impurities inward and heat outward, and coinjection to the main ions impedes the inward flow of impurities and enhances the outward flow of heat. It is noted here that, since $C_1 > C_2$, the additional temperature gradient shown here in the impurity flux will not reverse these results.

C. Mixed Regime

In the next collisionality regime reached the ions have become collisionless, showing parallel viscous effects, and the impurities are still collisional, $\tilde{u}_z = 0$. This limit leaves the impurity particle flux (as well as the ion flux because of ambipolarity) of order ϵ^2 , but the ion conducted heat flux is near first order. The impurity flux is

$$\begin{aligned} \Gamma_z = & \frac{2\epsilon^2 R_O}{e_z B_{po}} m_i n_i v_{iz} \left\{ (C_1 + \beta_z) \left(\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right) \right. \\ & \left[- C_2 - \beta_z \left(\frac{\mu_{zi}}{\mu_{zi}} - \frac{\gamma_i \mu_{zi}}{D_\phi} \left(1 - \frac{\mu_{zi} \mu_{ii}}{\mu_{zi}^2} \right) \right) \right] \frac{1}{e_i} \frac{\partial T_i}{\partial r} - \frac{C_2}{e_z} \frac{\partial T_z}{\partial r} \left. \right\} \\ & + \epsilon^2 \frac{2m_i n_i v_{iz}}{e_z} \frac{R_O^2 B_O}{B_{po}} \left[(C_1 + C_2 + \beta_z) \frac{\delta_z^0}{n_z} - (C_1 + C_2) \frac{\delta_i^0}{n_i} - \frac{2}{5} C_2 \left(\frac{\delta_z^2}{p_z} - \frac{\delta_i^2}{p_i} \right) \right] \\ & - \epsilon^2 \frac{R_O}{e_z} \left\{ \left[1 - \frac{2\beta_z (d_{zi} - \gamma_i)}{n_i \tilde{u}_i / m_i n_i v_{iz} D_\phi} \right] K_{\phi z}^1 - \frac{2\beta_z (d_{zi} - \gamma_i)}{n_i \tilde{u}_i / m_i n_i v_{iz} D_\phi} K_{\phi i}^1 \right. \\ & + \beta_z \frac{C_2 (\gamma_i - \gamma_z)}{n_i \tilde{u}_i / m_i n_i v_{iz} D_\phi D_{zi}} \left[\frac{5}{2} \beta_z \left(\frac{m_i}{T_i} K_{\phi i}^3 - \frac{5}{2} K_{\phi i}^1 \right) - a_i \left(\frac{m_z}{T_z} K_{\phi z}^3 - \frac{5}{2} K_{\phi z}^1 \right) \right] \\ & \left. - \beta_z \frac{\gamma_z}{D_{zi} D_\phi} \frac{\mu_{zi}}{\mu_{ii}} \left[\left(\frac{5}{2} \beta_z + C_3 \right) \left(\frac{m_i}{T_i} K_{\phi i}^3 - \frac{5}{2} K_{\phi i}^1 \right) + C_3 \left(\frac{m_z}{T_z} K_{\phi z}^3 - \frac{5}{2} K_{\phi z}^1 \right) \right] \right\} \end{aligned} \quad (41)$$

where the electrostatic potential denominator is

$$D_{\phi} = \gamma_i \left[1 + \frac{5}{2} \beta_z \frac{C_2}{D_{zi}} \frac{\mu_{zi}}{\mu_{ii}} - \left(\frac{\mu_{zi}}{\mu_{ii}} \right)^2 \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\left(\frac{5}{2} \beta_z + C_3 \right)}{D_{zi}} \right] + \gamma_z \frac{5}{2} \beta_z \frac{C_2}{D_{zi}} \frac{\mu_{zi}}{\mu_{ii}} \quad (42)$$

The effects of the directed heat fluxes on the particle transport is first observed here, and it is seen that the impurity flux follows the same pattern as it did in the collisional regime with modified coefficients on the gradients. For an effective charge for scattering of unity, $\alpha = Z_1^* = 1$, the numerical values of $C_1 = .67$, $C_2 = .56$, and $C_3 = 2$ are found. For the numerical fit to the parallel viscosity coefficients of Hirshman¹³ (also found in Appendix D of Ref. 6), numerical values for ratios of viscosity coefficients are found, $\mu_{zi}/\mu_{ii} \sim 10$ and $\mu_{zi}/\mu_{ii} \sim -1.5$ for a typical collisionless plasma with $v_{*i} \sim 10^{-1}$. Since $\tilde{\mu}_i \sim \sqrt{\epsilon} m_i v_{ii}$, the major contributing terms to the impurity flux are found

$$\begin{aligned} \Gamma_z \approx & \frac{2\epsilon^2 R_0}{e_z B_{po}} m_i n_i v_{iz} \left[(C_1 + \beta_z) \left(\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right) \right. \\ & \left. - C_2 \left(\frac{1}{e_i} \frac{\partial T_i}{\partial r} - \frac{1}{e_z} \frac{\partial T_z}{\partial r} \right) - \beta_z \frac{1}{e_i} \frac{\partial T_i}{\partial r} \right] \\ & + \epsilon^2 \frac{m_i n_i v_{iz}}{e_z} \frac{R_0^2 B_0}{B_{po}} \left[(C_1 + C_2 + \beta_z) \frac{\delta_z^0}{n_z} - (C_1 + C_2) \frac{\delta_i^0}{n_i} - \frac{2}{5} C_2 \left(\frac{\delta_z^2}{p_z} - \frac{\delta_i^2}{p_i} \right) \right] \\ & - \frac{\epsilon^2 R_0}{e_z} \left\{ \frac{[C_1(\beta_i - \beta_z) + \beta_i \beta_z] K_{\phi z}^1 - 2\beta_z (C_1 + \beta_z) K_{\phi i}^1}{[C_1(\beta_z + \beta_i) + \beta_i \beta_z]} \right\} \quad (43) \end{aligned}$$

The effect of the directed heat source is seen to be minor, and the reduced results compare closely with those of Stacey and Sigmar⁴, but now with inclusion of the temperature gradients.

No previous studies including more than momentum sources and gradients have been done on the mixed regime heat flux, the most significant terms being given here,

$$\begin{aligned}
q_{\psi i} = & - \frac{m_i n_i v_{iz}}{e_i} \frac{T_i R_o}{B_{po}} \left[\frac{D_{zi} - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} (a_z + C_3)}{D_{zi}} \right] \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} \frac{1}{e_i} \frac{\partial T_i}{\partial r} \\
& - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} \frac{(a_z + C_3)}{D_{zi}} \frac{m_i}{e_i} R_o \left(K_{\phi i}^3 - \frac{5}{2} \frac{T_i}{m_i} K_{\phi i}^1 \right) \\
& - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} \frac{C_3}{D_{zi}} \left(\frac{m_z}{T_z} \right) R_o \left(K_{\phi z}^3 - \frac{5}{2} \frac{T_z}{m_z} K_{\phi z}^1 \right). \quad (44)
\end{aligned}$$

The quantity in brackets is positive and less than one, so that the usual temperature gradients will drive heat outward. Also, coinjection of momentum to either species will enhance the outward heat flux.

D. Collisionless Regime

In a high temperature plasma, both ion species may be collisionless. At this point, the particle transport does not reduce to the ϵ^2 ordering of the previous collisionality regimes. To lowest order in ϵ , the most significant terms are

$$\begin{aligned}
\Gamma_z = & \frac{R_o}{e_z B_{po}} m_i n_i v_{iz} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \left[C_1 \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \right) + \gamma_z \frac{n_i \tilde{\mu}_i \gamma_i}{D_{\phi i}} \right] \\
& \left[\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right] / d_{zi} \\
& + \frac{R_o}{e_z} C_1 \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \right) \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \frac{\beta_i n_i \tilde{\mu}_i - \beta_z n_z \tilde{\mu}_z}{D_{\phi i} d_{zi}} (K_{\phi i} + K_{\phi z}) \\
& + \frac{R_o}{e_z} \left\{ \frac{n_z \tilde{\mu}_z \gamma_z}{D_{\phi i}} \left[\frac{\xi_z}{d_{zi}} \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \right) \right] - \frac{5}{2} \frac{\mu_{2i}}{\mu_{1i}} \frac{a_z}{D_{zi}} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \right\} K_{\phi i}^1 \\
& - \frac{R_o}{e_z} \left[\left(\frac{n_i \tilde{\mu}_i \gamma_i}{D_{\phi i}} \right) \left(\frac{\xi_i}{d_{zi}} \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) - \frac{5}{2} \frac{\mu_{2z}}{\mu_{1z}} \frac{a_i}{D_{zi}} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \right] K_{\phi z}^1 \quad (45)
\end{aligned}$$

where

$$D_{\phi i} = \left[\gamma_z N_z \Big|_{\phi} + \gamma_i N_i \Big|_{\phi} \right] / F_{\phi} \quad (46)$$

The gradient part reduced directly to the results of Stacey and Sigmar.² The first momentum input term is small, so that coinjection to the main ions drives impurities outwards if

$$\left[\left(\frac{n_z \tilde{\mu}_z \gamma_z}{D_{\phi i}} \right) \left(\frac{\xi_z}{d_{zi}} \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \right) - \frac{5}{2} \frac{\mu_{zi}}{\mu_{1i}} \frac{a_z}{D_{zi}} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \right] K_{\phi i}^1$$

$$> \left[\frac{n_i \tilde{\mu}_i \gamma_i}{D_{\phi i}} \left(\frac{\xi_i}{d_{zi}} \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) - \frac{5}{2} \frac{\mu_{zi}}{\mu_{1z}} \frac{a_i}{D_{zi}} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \right] K_{\phi z}^1 \quad (47)$$

Here, the coefficients of $K_{\phi i}^1$ and $K_{\phi z}^1$ are strictly positive. The most significant contribution of the directed heat flux source ($K_{\phi x}^3$) is

$$\Gamma_z \Big|_{K_{\phi}^3} = \frac{R_o}{e_z} \left(\frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \right) \frac{\mu_{zz}}{\mu_{1z}} \left[C_1 \left(\frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \right) + \gamma_z \frac{n_i \tilde{\mu}_i \gamma_i}{D_{\phi i}} \right] / d_{zi}$$

$$\left[a_z \frac{m_i}{T_i} K_{\phi i}^3 - a_i \frac{m_z}{T_z} K_{\phi z}^3 \right] / D_{zi} \quad (48)$$

and coinjection helps to drive the impurities outward if

$$a_z \frac{m_i}{T_i} K_{\phi i}^3 > a_i \frac{m_z}{T_z} K_{\phi z}^3$$

The ion conducted heat flux is less affected by the impurities becoming collisionless, and much of it is unmodified from the mixed regime, and is

$$q_{\psi i} = q_{\psi i} \Big|_{\text{mixed}} - \frac{m_i n_i v_{iz}}{e_i} \frac{T_i R_o}{B_{po}} \left[\frac{C_3}{D_{zo}} \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} \right] \frac{n_z \tilde{\mu}_z}{m_i n_i v_{iz}} \frac{1}{e_z} \frac{\partial T_z}{\partial r}$$

$$- \frac{m_i n_i v_{iz}}{e_i} \frac{T_i R_o}{B_{po}} \left[\frac{D_{zi} - \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{3i}}{\mu_{1i}} (a_z + C_3)}{D_{zi}} \right] \frac{n_i \tilde{\mu}_i}{m_i n_i v_{iz}} \frac{\mu_{2i}}{\mu_{1i}}$$

$$\left(- \frac{n_z \tilde{\mu}_z \gamma_z}{D_{\phi i}} \right) \left(\frac{1}{n_i e_i} \frac{\partial p_i}{\partial r} - \frac{1}{n_z e_z} \frac{\partial p_z}{\partial r} \right) \quad (49)$$

Only the general dependence of the heat flux on the temperature gradients has been shown before in this regime. All else is first seen here. The impurity temperature gradient also drives heat outward, opposed by the pressure gradient differential (μ_{21}/μ_{11} is negative).

IV. Summary

Neoclassical transport theory in a tokamak plasma has been extended here to the description of heat fluxes and includes external sources of particles, momentum, heat, heat flux, and drags. This was developed for arbitrary plasma shape, beta, and collisionality, and has been specialized to a two-species ion-impurity plasma.

The scaling of the particle transport was found to follow the scaling of the collisionality regime of the heavier impurity ions. This is as the inverse aspect ratio squared when the impurities are collisional, and the square root of the inverse aspect ratio when the impurities are collisionless. The collisional regimes also are effected to like order by asymmetric source of particles, and the mixed regime (impurities collisional, ions collisionless) particle flux for the first time was found to be effected by asymmetric heat sources. The observation was made that in none of the collisionality regimes could the usual negative temperature gradient cancel the inward impurity flux driven by the density gradient. The heat flux source was examined and found to contribute nothing to the collisional particle transport, as seen previously, and that it contributes to the other two regimes only with small coefficients. The prime driving force in the totally collisionless regime for particle transport was seen to be the difference in the pressure gradients and momentum sources, and similar to the results of previous authors.

Unlike the particle transport, the heat transport was found to follow collisional scaling only in the totally collisional regime. Beyond that, heat flux scaling is dominated by collisionless effects. Hence, asymmetric sources are important only in the collisional regime. It was also found here that a coinjected momentum source drives the heat flux outward not only in the collisional regime, as seen before, but also in the mixed regime. The effects of these sources were also first reported here for the collisionless regime, and conditions are established for which coinjection in this regime retards the outward heat flux.

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