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HIGH GRADIENT ACCELERATORS FOR LINEAR LIGHT SOURCES

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Ultra-high gradient radio frequency linacs powered by relativistic klystrons appear to be able to provide compact sources of radiation at XUV and soft x-ray wavelengths with a duration of 1 picosecond or less. This paper provides a tutorial review of the physics applicable to scaling the present experience of the accelerator community to the regime applicable to compact linear light sources.

1. INTRODUCTION

Linear light sources capable of producing picosecond bursts of XUV and soft x radiation have gained ever increasing attention with the rapid advances both in free electron laser (FEL) physics and in the technology of ultra-high gradient linacs capable of producing high brightness electron beams at high energy. Compact accelerators for linear light sources are based upon conventional radio frequency structures. Desired average accelerating gradients (≈ 200 MeV/m or more) are higher than those achievable in present-day accelerators (e.g., 17 MeV/m at SLAC). This goal is, however, being pursued actively at Lawrence Livermore National Laboratory (LLNL) in collaboration with the Stanford Linear Accelerator Center (SLAC) and the Lawrence Berkeley Laboratory (LBL) toward the design of a TeV electron-positron collider. A great deal is known about the basic scaling laws governing the operation of rf accelerators. They are presented here as a tutorial guide to those considering the design of compact linear light sources (or flavor factories).

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A critical beam characteristic with respect to the performance of an x-ray FEL (XFEL) is the emittance of the beam, because it strongly affects the gain of the laser. As discussed in Sec. 2, design goals of $\epsilon_n \approx 10$ mm-mrad and $N_b \approx 5 \times 10^{10}$ are consistent with the performance of existing electron sources. Constraints on beam characteristics imposed by cavity fill times, focusing, beam loading, wakefield control, and gas matching (in the case of plasma assisted wigglers) are presented in Secs. 3-6.

A major technological hurdle to be overcome in realizing high gradient rf accelerators is the gigawatt rf power source. Section 7 discusses the inductor-linac-powered relativistic klystron and the possibility of deploying branched magnetics in order to provide macro-pulses as long as $1 \mu\text{s}$ duration should the community of linear light source users require such a characteristic. Finally, Sec. 8 presents the various elements of a cost estimation algorithm that can be used by the designer of a linear light source (or flavor factory) to access the financial resources needed to build a high gradient linac powered by relativistic klystrons.

1.1. Application-Derived Constraints

Because of the diverse radiation output characteristics desired by the potential user communities, no specific facility or economic constraints have been formulated for linear light sources. It is useful, nonetheless, for the accelerator designer to adopt some rough guidelines as goals:

$$\text{Length/energy} < 5 \text{ m/GeV},$$

$$\text{Cost/energy/pulse duration} < 10 \text{ M\$/GeV}/\mu\text{s},$$

$$\text{Repetition rate} > 200 \text{ Hz}.$$

Cast in this form the cost and size goals for the light source designer are found to be roughly equivalent to those of the designer of the next generation of linear colliders at TeV energies. Exploring design concepts consistent with these goals has

been the object of active and steadily increasing efforts by the high energy physics research community throughout the world. Consequently, a substantial literature exists concerning the physics of accelerators relevant to compact linear light sources. Recent work by Wilson,^{1,2} and Palmer,³ and Amaldi⁴ form the basis of the analysis of Secs. 2-6. A spread-sheet program including these analyses, those concerning cost, and those of FEL physics is now in active use at LLNL with a goal of system optimization.

1.2. Energy Constraints

In addition to setting size and cost constraints, the physics of free electron laser operation set several constraints on the characteristics of the beam such as the beam energy, E (or relativistic factor, γ), the normalized emittance, ϵ_n , and the energy spread, $\Delta E/E$ (or momentum spread, $\Delta p/p$). For an FEL with a planar wiggler the well-known design equations linking the beam characteristics with FEL performance are specified in terms of the wiggler field, B_0 , the wiggler wavelength, λ_w , and the radiation wavelength, λ_s . The wiggler characteristics can be combined in terms of a dimensionless vector potential

$$K = e\lambda_w(B_0)/2\sqrt{2\pi} m_e c^2 \approx 0.66 B_0(T)\lambda_w(cm) \quad . \quad (1a)$$

The wiggler and radiation wavelengths are connected to the beam energy by the resonance condition

$$\lambda_s = \left(\lambda_w / 2\gamma^2 \right) \left(1 + K^2 \right) \quad . \quad (1b)$$

1.3. Mono-Chromaticity and Emittance Requirements

FEL physics place tight bounds on an important characteristic of the accelerator subsystem, namely, the allowable momentum spread, $\Delta p/p$, within the pulse or from bunch to bunch within a macro-pulse. The momentum spread has several sources, the largest of which originates from the duration of each pulse relative to

that of the rf cycle. This spread, plus longitudinal wakefield effects, will make it difficult to produce high energy beams with $\Delta p/p < \pm 0.1\%$.

The performance of the FEL scales⁵ with a single electron beam parameter, ρ :

$$\rho = \left(\frac{K \omega_p}{8 \omega_w} \right)^{2/3}, \quad (2)$$

where ω_p ($\sim \gamma^{-3} N_b^{1/2}$) is the relativistic plasma frequency of the electron beam. After passing through a wiggler of length $L \sim \gamma_s/\rho$, the output from the FEL will saturate with the power in the radiation being ρ times the electron beam power. The constraints on energy spread in the beam and the normalized beam emittance are

$$\frac{\Delta E}{E} \leq \frac{\rho}{4}, \quad (3a)$$

and

$$\varepsilon_n \leq \lambda_s \gamma. \quad (3b)$$

The final constraint on the desing of the XRFEL comes from requiring that the gain length, L_g , of the laser be shorter than the Rayleigh range, Z_R . The gain length can be expressed in terms of the FEL parameter; namely,

$$L_g = \frac{\lambda_w}{4\pi\rho}; \quad (4a)$$

the Rayleigh range is given by

$$Z_R = \frac{\pi a^2}{\lambda_s}. \quad (4b)$$

Hence,

$$a^2 > \frac{\lambda_s \lambda_w}{r \pi^2 \rho}, \quad (4c)$$

where a is the size of the beam in the wiggler. If the wiggler employs natural focusing, the betatron wavelength in the wiggler and the beam size are related by

$$\lambda_\beta = \frac{\lambda_w \gamma \sqrt{2}}{K}. \quad (5a)$$

The beam size and betatron wavelength are, in turn, related to the emittance by

$$\epsilon_n = \frac{2\pi\gamma a^2}{\gamma\beta} \quad (5b)$$

If the XRFEL employs a plasma-assisted wiggler⁶ in which the beam assumes its self-focused radius in the presence of an ion channel (or if the beam is to be focused to an extremely small radius as in a flavor factory), a magnet subsystem must match the beam size in the accelerator ($\sim 100 \mu\text{m}$) to the self-focused size in the plasma ($\sim 1 \mu\text{m}$). Relatively compact focal arrays with such capabilities have received considerable attention from the designers of linear colliders. Most notably, the Stanford Linear Collider (SLC) has such a final focus system in operation. For the SLC design, the size of the focal spot is related to the momentum spread by

$$a_{\text{focus}} \propto \left(\frac{\Delta p}{p} \right)^2$$

A focal system with quadrupoles with field strengths of less than 2 T and apertures of 1 cm will require that $\Delta p/p < \pm 0.5\%$.

2. CHARACTERISTICS OF EXISTING ELECTRON BEAM SOURCES

This section reviews the present state of the art concerning bright, high current sources of electron beams. These sources, which include electron injectors, synchrotron radiation sources, and linear colliders, form the data base for scaling accelerator design to the regime applicable to linear light sources and flavor factories.

2.1. High Brightness Sources

As the beam emittance plays a critical role in determining electron density and thereby the gain of a free electron laser, the examination of the scaling basis for accelerators for linear light sources begins with a discussion of the present status of high brightness electron sources. Over the past decade the accelerator community has expended considerable effort in the design of ever lower emittance sources of

electron beams with large peak currents. This work has been motivated by three applications: high energy linear colliders, high average power FELs operating at visible or shorter wavelengths, and effective sources of incoherent synchrotron radiation. During this period the beam brightness (current density divided by angular divergence) has been increased by at least three orders of magnitude, with presently achieved performances still a considerable distance from fundamental limits.

In early 1987 Brookhaven National Laboratory hosted a workshop⁷ concerning the production of low emittance beams for both collider and FEL applications. The most stressing goal, that for XRFELs as set for the participants by Pellegrini, was a factor of a few more demanding than that for linear colliders; namely,

Beam Energy, E	1 GeV,
Normalized horizontal emittance area, $\epsilon_{nx} = \gamma\epsilon_x$	$1 \times 10^{-6} \pi \text{ m} - \text{rad}$,
Normalized vertical emittance area, $\epsilon_{ny} = \gamma\epsilon_y$	$1 \times 10^{-8} \pi \text{ m} - \text{rad}$,
Longitudinal brilliance, $B_L = I_{\text{beam}}/\gamma$	200 A.

Working groups studying beam injectors, damping rings, and the problems of emittance preservation during the acceleration process concluded that the injectors would have to be improved beyond the present state of the art to meet the stated goals for an FEL. They found no mechanism incompatible with this goal and expected that such a beam could be accelerated from a few tens of MeV to a few GeV without degradation.

The best performance in building high current, low emittance guns using photocathodes has been achieved by Fraser, Sheffield et al.^{8,9} at Los Alamos. They have produced a beam at 1.1 MeV with a peak current of 130 A and a normalized rms emittance of $5 \times 10^{-6} \pi \text{ m-rad}$. This value is approximately twenty times smaller than that of the SLAC gun,¹⁰ which produces a beam of $\approx 1 \text{ kA}$ at an emittance of $2 \times 10^{-4} \pi \text{ m-rad}$. At SLAC this beam is accelerated to $\approx 1 \text{ GeV}$ for insertion

into a damping ring. Upon exiting the damping ring the beam emittance has been reduced to $1 \times 10^{-6} \pi$ m-rad.

Low emittance beams have also been produced with thermionic cathodes in rf guns, albeit at somewhat lower peak currents. For example, the Mark III used for FEL experiments at Stanford's High Energy Physics Laboratory delivers a peak current of 30 A at 44 MeV with normalized horizontal and vertical emittances of $\epsilon_{nx} = 3.6 \times 10^{-6} \pi$ m-rad and $\epsilon_{ny} = 1.8 \times 10^{-6} \pi$ m-rad, respectively.

2.2 Existing and Planned Synchrotron Beams

Existing and planned synchrotron sources¹¹ in the USA are (in alphabetical order) Argonne 6 GeV Ring, Berkeley 1-2 GeV Ring, Cornell High Energy Synchrotron Source (CHESS), National Synchrotron Light Source (NSLS) (Brookhaven), Stanford Synchrotron Radiation Laboratory (SSRL), Stanford Photon Research Laboratory (SPRL), Synchrotron Radiation Center (SRC) (Stoughton), and Synchrotron Ultraviolet Radiation Facility (SURF) (Gaithersburg). Their properties are detailed in Table 1.; note that the emittances given are unnormalized, i.e., $\epsilon = \epsilon_n/\gamma$.

2.3. The Stanford Linear Collider (SLC)

The most relevant data base for scaling the design of rf linac technology to the parameter regime suitable for linear light sources (and for linear collider flavor factories) is the 50 GeV SLC at the Stanford Linear Accelerator Center (SLAC). The SLC linac consists of a 3-km-long assembly of sections of S-band (2.87 GHz) disk-loaded waveguide (traveling wave). Each linac section, consisting of nearly 100 cavities with a length one-third the rf wavelength, is fed by a high peak power klystron. Although other types of rf structures (e.g., standing wave) may also be suitable, the disk-loaded structure is amenable to analytical calculation and has received considerable study. Consequently, scaling the SLC design is a convenient way to scope the characteristics of the accelerator. The following analysis requires

Table 1. Existing and Planned Storage Rings

Status*	Name	Energy (GeV)	ϵ_{horiz} ($\pi \times 10^{-9}$ m rad)	ϵ_{vert}
o	ALADDIN (Stoughton)	0.75	63	110
o	TANTULUS (Stoughton)	0.24	230	4000
p	ALS (LBL)	1.3	6.8	4.0
p	ANL (Argonne)	7.0	7.0	0.2
o	CESR (Cornell)	5.5	200	6.6
o	NSLS I(BNL)	0.75	130	230
o	NSLS II(BNL)	2.5	80	13
o	PEP (SLAC)	15	150	0.67
	" "	8.0	10	0.16
o	SPEAR (SLAC)	3.0	450	50
o	SURF II(Gaithersburg)	0.28	270	3400
c	SPRL (SLAC)	1.0	10	10

* o=operational, c=in construction, p=planned

the definition of several quantities listed in Table 2. along with their respective values for the SLC. Units are mks unless noted.

3. SCALING BASIS FOR ACCELERATORS FOR LINEAR LIGHT SOURCES

For use in a linear light source, a compact accelerator should both be highly efficient in transferring rf energy from the source to the high energy beam and highly conservative in use of physical space. As shown in the next section, these constraints push one toward high gradients, high frequencies, and short fill times.

Table 2. SLC values of scaling variables

Beam energy	E	50 GeV
RF frequency	f	2.87 GHz
RF wavelength	λ	10^5 mm
Iris aperture (radius)	a	11 mm
Cavity Q	Q	$1.39 \cdot 10^4$
RF Group velocity	v_g	$0.011 c$
Fill time	t_f	921 ns
Length per feed	L_f	2.97 m
Attenuation time	T_o	1.4 μ s
Fill length / attenuation	t	0.6
Average gradient	E_a	17 MeV/m
Peak field in cavity	$E_{\max}$	41.3 MeV/m
Loading	h_b	3.12%
Peak RF power	P_{rf}	9.8 MW/m
Particles per bunch	N_b	$\leq 5 \times 10^{10}$
Bunch length (Gaussian σ)	s_z	1 mm (3.43°)
Momentum spread ($\Delta p/p$)	σ_p	0.39%
Beam size in linac	$\sigma_{x,y}$	77 μ m

3.1. High Voltage Breakdown

The physical phenomenon which forms the basis for scaling rf linacs from the SLC value of 17 MeV/m to the required > 200 MeV/m is the observation¹² that the peak electric field that can be sustained without breakdown rises with increasing rf frequency and with shortening duration of the rf power (see Fig. 1).

Not all of the potential for increased fields can be realized in practical high

Figure 1. The peak electric field that can be sustained in rf structures without breakdown. Generally, only approximately half this value can actually be used.

energy accelerators. At field values somewhat below breakdown values, small currents will flow near the cavity walls. These currents give rise to copious x radiation and, more significantly, to magnetic fields that exert random transverse forces on the beam, thereby exciting wakefield instabilities. To avoid this difficulty the peak field should be reduced by a safety factor, F_s , with $0.5 \leq F_s \leq 0.75$. The interdependence model uses a value $F_s = 0.66$. Commonly, in the discussion of linear colliders, the scaling of E_{pk} with frequency is given as $E_{pk} \propto f^{7/8}$. This scaling takes advantage of the increase of sustainable peak field with decreasing rf pulse

duration:

$$E_{pk} = 120 \text{ MV/m} \left(\frac{F_s}{0.66} \right) \left(\frac{f}{2.87 \text{ GHz}} \right)^{7/8} . \quad (6)$$

Of course, this value can only be achieved if sufficient power is supplied to the structure; it is not in the case of SLC. Should the application require an output period of $\sim 1 \mu s$ independent of frequency, the exponent in Eq. 6 should be reduced to $1/2$.

3.2. Cavity and rf Characteristics

The scaling characteristics of $2\pi/3$ disk-loaded structures as described in Refs. 1-3, 13, and 14 are based on computer calculations with Farkas's code TWAP. The disk-loaded structure has a phase velocity equal to c , the speed of light, and a group velocity that is a function of the rf wavelength and the aperture of the iris. Palmer's fits³ to the TWAP calculations¹³ as shown in Fig. 2(b) give

$$\beta_g = \frac{v_g}{c} \approx \exp \left[3.1 - 2.4 \left(\frac{\lambda}{a} \right)^{1/2} - 0.9 \left(\frac{a}{\lambda} \right) \right] . \quad (7)$$

With the group velocity specified the remainder of the cavity characteristics can be calculated. Initially assume that no rf energy is lost as the wave travels through the structure. In that case the energy per unit length, w_a (J/m), required

Figure 2. Palmer's³ fits to Farkas's TWAP¹³ calculations of SLAC-like accelerating structures. (a) iris radius a divided by wavelength; (b) normalized corrected elastance S_{at} ; (c) attenuation time constant T_0 in μ sec; (d) ratio of peak and average accelerating fields; (e) ratio of outer cavity and iris radii; (f) the relative peak rf power.

to produce a given acceleration of the beam is related to the average accelerating gradient by a monotonically increasing function of the group velocity, namely, the corrected elastance, s_t :

$$s_t = \frac{E_a^2}{w_a} = 5.7 \cdot 10^{14} \text{ V/mC} \left(\frac{10 \text{ mm}}{a} \right)^2 \left(\frac{v_g}{c} \right)^{0.4} . \quad (8a)$$

Frequently accelerator designers prefer to quote a "normalized" elastance that is independent of the iris aperture, i.e.,

$$s_{at} = s_t a^2 . \quad (8b)$$

Because both the beam and the rf move through the structure with a finite velocity, the length of the accelerating pulse can be less than that of the section. Therefore, the amount of energy, w_f , needed to fill the structure is greater than w_a by an amount

$$w_f = \frac{w_a}{(1 - \beta_g)} . \quad (9a)$$

Similarly, the uncorrected elastance is related to s_t by

$$s = (1 - \beta_g) s_t . \quad (9b)$$

In actual structures the rf wave is attenuated as it travels down the section. The characteristic attenuation time,

$$T_o = 1.45 \mu s (1 + 1.29 \beta_g^{1.5}) \left(\frac{\lambda}{105 \text{ mm}} \right)^{1.5} , \quad (10)$$

is related to the Q of the cavities by

$$Q = \pi f_{rf} T_o . \quad (11)$$

The fill time of the section is

$$t_f = \frac{L_f}{v_g} . \quad (12)$$

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In terms of the ratio of the fill time to attenuation time,

$$t = \frac{t_f}{T_0} , \quad (13)$$

the rf energy needed for the same acceleration in a constant impedance structure is

$$w_{rf} = w_a \left[\frac{\tau^2}{(1 - e^{-\tau})^2} \right] = \frac{w_a}{h} , \quad (14)$$

where h is defined as the structure efficiency. Note that as the length of the section is decreased the fill time goes to zero, but $w_{rf} \rightarrow w_a$. Therefore, the required power increases without bound:

$$P_{rf} = w_{rf}/t_f . \quad (15)$$

Combining Eq. (7) with eq. (15), one can express the rf power per unit length as

$$P_{rf} = \frac{E_a^2 a^2}{h T_0 \tau s_{at}} . \quad (16)$$

The maximum gradient is determined from the peak permissible value of the electric field:

$$E_a = \frac{E_{pk}}{2 + 6\beta_g} , \quad (17)$$

where E_{pk} is given by Eq. (6). Hence,

$$E_a = 1.2 \cdot 10^8 \text{ V/m} (2 + 6\beta_g)^{-1} \left(\frac{F_s}{0.66} \right) \left(\frac{f}{2.87 \text{ GHz}} \right)^{1/2} . \quad (18)$$

In designing a linear light source of minimum length for a given beam energy, one can choose only the frequency, iris aperture, and the attenuation parameter; all the other cavity characteristics are determined. Completing the cavity parameter descriptions is

$$\frac{b_c}{a} \approx 1.04 - 0.29 \ln \beta_g + 0.068 \ln^2 \beta_g , \quad (19)$$

where b_c is the inner radius of the accelerating cavity.

4. FOCUSING IN THE LINAC

The design of the focusing system for a high energy linac has two impacts. The first and least important of these is to increase the overall length of the accelerator. In a transport design scaled from the SLC FODO lattice this consideration can be quantified by a single number, F_q , the fraction of the linac occupied by the quadrupoles (typically <10%). The second implication is much more important. The strong focusing needed to control wake field instabilities can lead to severe tolerance constraints on the linac design. The beam transport in the linac is described by the beta function of the focusing lattice. The average strength of the focusing in a FODO structure consisting of quadrupoles with field strength B_q and aperture a_q is

$$\langle \beta_y \rangle = \left(\frac{\sin \mu}{\mu^2} \frac{E}{ec} \frac{2a_q}{B_q F_q} \right)^{1/2}, \quad (20)$$

where 2μ is the phase advance per cell. Following the design of the SLC linac, one chooses

$$\mu = 45^\circ, \quad (21a)$$

$$B_q = 1.5 \text{ T}, \quad (21b)$$

$$a_q = 1.2 \text{ a}. \quad (21c)$$

With these choices one can rewrite Eq. (20) as

$$\langle \beta_y \rangle = 2.49 \text{ m} \left(\frac{E}{10 \text{ GeV}} \frac{a}{10 \text{ mm}} \frac{1.5 \text{ T}}{B_q} \frac{0.1}{F_q} \right)^{1/2}. \quad (22)$$

Of course, the local value of the focusing strength will vary throughout the accelerator as the beam gains energy. Using ζ to denote distance along the accelerator, one can compute the number of quadrupoles in the accelerator:

$$N_q = \int_0^L \frac{d\zeta}{\beta(\zeta)\mu}, \quad (23)$$

where L is the length of the accelerator. The beam size produced by this lattice is

$$\sigma_y(\zeta) = \left(\frac{\beta_y(\zeta)\epsilon_n}{\gamma(\zeta)} \right)^{1/2} . \quad (24)$$

Palmer³ shows that the alignment tolerances along the linac can be kept constant, if the strength of the quadrupoles is varied weakly ($\approx \gamma^{1/3}$) along the linac; whence,

$$\beta(\zeta) \propto \gamma^{1/3} \propto \zeta^{1/3} . \quad (25)$$

Then, Eq. (23) gives

$$N_q = 1.5 \left(\frac{L}{\beta(L)\mu} \right) \quad (26)$$

Calculations¹⁵ of the ratio of beam displacement to beam radius at the end of the linac indicate that in order to preserve the beam emittance, the quadrupoles must be aligned with an rms accuracy of

$$\langle dy \rangle = \frac{\sigma_y}{\mu\sigma_p} \left(\frac{2}{N_q} \right)^{1/2} , \quad (27)$$

where σ_p is the momentum spread, $(\Delta p/p)$. Probably the largest source of momentum spread in the main body of multi-GeV linacs will be a head-to-tail energy variation which is deliberately introduced and maintained throughout the accelerator in order to damp transverse wakefield effects. This technique is often erroneously called Landau damping, which refers to an instantaneous spread in betatron frequencies. The head-to-tail spread with the $4k_\beta$ increasing at the tail is an idea of Balakin, Novokhatskii, and Smirnov and should be called BNS damping. The linear component of the head-to-tail spread can be removed in a small accelerating section at the end of the accelerator that operates at a phase advance of 90° . This section has a length

$$\begin{aligned} l_{\text{corr}} &= \frac{\sigma_p p \lambda}{2\pi E_a \sigma_z} \\ &= 0.81 \text{ m} \left(\frac{E}{10 \text{ GeV}} \right) \left(\frac{\lambda}{100 \text{ mm}} \right) \left(\frac{E_a}{200 \text{ MeV/m}} \right)^{-1} \left(\frac{\sigma_z}{10 \text{ mm}} \right)^{-1} , \end{aligned} \quad (28)$$

where σ_z is the rms bunch length.

Wilson² has showed that the tolerance on alignment of the focusing system in large linacs is generally more severe than the tolerance on alignment of the accelerator sections. A prudent approach, however, may be to adopt Eq. (27) as establishing the tolerance requirements for all beamline components. Similarly the position of the beam in the linac must be controlled to this level. For linacs of applicability to linear light sources and flavor factories, the alignment tolerances should not be worse than $\approx 100 \mu\text{m}$. The difficulty may be compounded by the fact that vibrational tolerances in such linacs tend to be an order of magnitude tighter than the static alignment tolerances. Tolerance requirements may be relaxed if alternatives to BNS damping such as rf focusing with elliptical irises prove practical; however, such means of controlling wake fields in high gradient linacs must still be considered to be untested at this time.

5. WAKEFIELDS AND BEAM LOADING

In passing through the accelerating structure each beam pulse will both remove energy from the fundamental accelerating mode of the cavity and excite higher order modes in the cavity. Via the excitation of these higher order modes (wakefields), the head of a beam pulse can affect the energy of (longitudinal wakes) and can deflect (transverse modes) the tail of the pulse. The results will be an energy spread in the beam and a head-to-tail growth of emittance as the beam travels through the accelerator.

5.1. Transverse Wakefields

The dipole (deflecting) modes have an E_z component that varies linearly with distance from the cavity axis and with the cosine of the azimuthal angle, f . The n th traveling wave modes generated by a charge q traveling parallel to the cavity

axis at a radius r_q have an electric field at a distance $\zeta = ct$ behind q of

$$E_{zn} = -2k_n q \left(\frac{r}{a}\right)^{m-1} \left(\frac{r_q}{a}\right)^m \cos m\phi \cos \omega_n \tau, \quad (29)$$

where k_n is Wilson's¹ loss parameter which relates the field intensity to the stored energy for the n th mode (E_n^2/W_n or one-fourth the elastance for the mode). The interaction of the beam with these modes can be characterized by a transverse shunt impedance $r_\perp$. For typical disk-loaded structures

$$\frac{r_\perp}{Q} \approx \frac{100}{\lambda} \text{ ohms} \quad (30)$$

gives a rough estimate of the transverse shunt impedance. According to Wilson,¹ the lowest order (beam break-up) mode has a frequency $\approx 40\%$ higher than the fundamental accelerating mode. Therefore, one expects the Q of the dipole mode to be only slightly lower than that of the accelerating mode. Consequently, without special precautions the transverse wakefield of a pulse will persist for many tens of nanoseconds. Via these long-range wakes, both the transverse motion and the emittance can increase from one micro-pulse to another.

A theorem by Panofsky and Wenzel shows that the total deflecting kick on the particle at ζ can be expressed solely in terms of the E_{zn} (or the corresponding potentials). The difficulty however, in computing the transverse deflection of the particle is that one must include as many modes as possible. The values of ω_n and k_n in Eq. (29) are obtained by solving the boundary value problem for a charge-free cavity or structure with a computer code such as SUPERFISH.¹⁶ Figure 3 shows the results of such calculations of the dipole wake per unit accelerating cell for the SLAC disk-loaded structure for three different time durations. In particular, Fig. 3(a) indicates the size of the contribution of the higher modes to the total dipole wake.

For the purposes of estimating the characteristics of high gradient linacs for linear colliders, Palmer³ has fitted such computer calculations of the initial linear

Figure 3. (a)-(c): Dipole wake per cell for the SLAC disk-loaded structures over various time intervals; (d) longitudinal wake per cell with a cell length of 3.5 cm and beam aperture radius of 1.163 cm.

rise, ${}_1W_t(\zeta)$, and the maximum of the wakefield, ${}_2W_t$, to obtain the scaling laws (in mks units):

$${}_1W_t(\zeta) = 6.64 \cdot 10^{10} \text{ V}/(\text{C} - \text{m}^2) \zeta a^{-7/2} \lambda^{-1/2} \quad (31)$$

for $\zeta \ll a$ and

$${}_2W_t(\zeta) = 3.28 \cdot 10^{10} \text{ V}/(C - m^2) a^{-2.2} \lambda^{-0.8} , \quad (32)$$

which occurs at $\zeta \approx a$. For the SLAC geometry a reasonable fit to W_t at intermediate values of $\zeta < a$ is obtained from

$$W_t(\zeta) = [{}_1W_t^{-2}(\zeta) + {}_2W_t^{-2}(\zeta)]^{-1/2} . \quad (33)$$

As one can see from Fig. 3(c), the dipole fields persist long after each beam pulse. Using an asymptotic approximation valid for long pulse trains, Yokoya¹⁷ has computed the value of the the dipole wakefield for multi-bunch macro-pulses in long linacs. He finds that the multi-bunch wake field has the same form as the single bunch wake with the growth determined by the average current during the macro-pulse. For the case of no spread in betatron frequency and strong focusing ($k_\beta = 1 \text{ m}^{-1} [E/2 \text{ GeV}]^{-2/3}$), a fit to Yokoya's calculations of the wake of the j th pulse on the k th rf cycle yields

$$W(j) = 0.36 {}_2W_t j^{-0.8} \exp \left[\frac{-k}{Q^*} + 1.38 j^{1/2} \right] , \quad (34)$$

where ${}_2W_t$ is given by Eq. (32) and Q^* is the effective quality factor of the dipole modes. This expression is overly pessimistic for short ($\approx 1 \text{ GeV}$) linacs designed to accelerate ~ 10 pulses in a macro-bunch.

As argued above, Q^* is approximately 70% of the value for the fundamental ($\approx 10^4$); hence, the decay term is of little help in preventing disastrous growth of the dipole mode. However, the frequency difference between the deflecting modes and the fundamental accelerating mode can, in principle, be used to out-couple the energy of these modes. Such a procedure is used in low-frequency cw linacs and has been cold-tested for disk-loaded structures by Palmer and his collaborators at Brookhaven and SLAC. They have been able to design structures with a Q of 20 for

the dipole modes. In assessing system requirements for a XRFEL accelerator, one can characterize the requisite level of energy out-coupling by a dequeing parameter, D , where

$$Q^* = 0.7 \frac{Q}{D} \quad (35)$$

This requirement is easily satisfied by the slotted structures cold-tested at BNL.

5.1.1. Damping transverse motion. The transverse wakes from a single pulse can be effectively controlled by BNS damping the transverse motion through the introduction of a spread in betatron frequency increasing from head-to-tail within the beam. This spread can be obtained by appropriately centering the pulse in phase with respect to the rf so as to maintain an energy spread between the front and back of the bunch. If the bunch is approximated as two point charges separated by $2\sigma_z$, the energy spread required is

$$\Delta E \approx 2E\sigma_{p,BNS} \approx \frac{eN_b}{4} (2\sigma_z) W_t \beta_y^2, \quad (36)$$

where ΔE and the betatron function are specified at the full energy of the linac. Solving for σ_p and substituting in Eqs. (27) and (28) yields the alignment tolerance requirements and the length of the correction section, respectively. If one uses Eq. (34) in Eq. (36), it is apparent that BNS damping will be insufficient to control multi-bunch transverse wakes in multi-GeV flavor factories. This technique may, however, be sufficient for several pulses in short (≈ 1 -GeV) linacs suitable for early linear light sources.

5.2. Longitudinal Wakes and Beam Loading

As the beam particles move through the waveguide structure they will lose energy into the structure. For the disk-loaded structure this loss can be considered a diffraction loss by a plane wave with the same power spectrum and Poynting vector at the iris radius as the actual field due to the charge. The wake potential due to

this process can be calculated using the same procedures as are used to compute transverse wakes; again as many modes as possible should be included. Fig. (3b)¹ shows the results of such a calculation for the SLAC structure. The importance of the higher order modes can be seen by the comparison of the total wake with that due to the excitation of the fundamental accelerating mode. (Note that a positive potential is decelerating.)

Figure 4. Beam loading voltage within a Gaussian bunch for the SLAC structure for three values of bunch length (adopted from Ref. 1).

The beam loading potential is the convolution integral of the wake potential with the beam current. The value of the beam loading voltage, $E_b(t)$, within a Gaussian bunch (centered at $t = 0$) is shown in Fig. 4 for bunches of three different lengths. The total energy gain of a particle at time t in the bunch is the sum of the accelerating and beam loading voltages:

$$E(t) = E_a \cos(\omega t - \theta) - E_b(t) \quad , \quad (37)$$

where θ is the phase angle by which the center of the bunch leads the crest of the rf accelerating wave. Both components of the energy gain give rise to an energy spread within a bunch of finite length. To some extent this spread can be reduced

Figure 5. (a) Relationship between single-bunch beam loading $E_b(t)$, rf accelerating wave E_{rf} , and net gradient $E(t)$; (b) energy spread and average gain per particle for the SLAC structure with $E_a = 17$ MV/m, $N_b = 5 \times 10^{10}$, and $\sigma_z = 1.0$ mm (adopted from Ref. 1).

by choosing the q so that the negative-going beam loading balances the increasing accelerating voltage. This effect is illustrated schematically in Fig. 5(a) and quantified in Fig. 5(b). The penalty for reducing the energy spread in this manner is reducing the average value of the energy gain of the pulse, $\langle E \rangle$, by a small amount.

As the acceptable energy spread of the beam is restricted (Sec. 1.) by FEL physics constraints (or luminosity constraints and energy resolution in flavor factories, one must be able to estimate how the wakefields affect ΔE . One begins with an estimate³ of the longitudinal wake that depends on rf wavelength and iris aperture, a . For very short bunches, that is, for $\zeta \ll a$,

$${}_1W_1(\zeta) = 1.78 \cdot 10^{10} \frac{V}{(C - m)} a^{-2} \quad (38)$$

For $\zeta \approx a$,

$${}_2W_1(\zeta) = 1.25 \cdot 10^{10} \frac{V}{(C - m)} \zeta^{-1/2} a^{-1} \lambda^{-1/2} \quad (39)$$

For intermediate values of ζ ,

$$W_1(\zeta) = [{}_1W_1^{-3}(\zeta) + {}_2W_1^{-3}(\zeta)]^{-1/3} . \quad (40)$$

Next, following Palmer, one notes four distinct contributions to σ_p : (1) the average energy of the bunch is reduced (zeroth order); (2) the tail of the pulse has a greater loss than the head of the pulse (first order); (3) long pulses have a significant second order term with the rate of change of momentum falling at the tail; (4) Gaussian bunches have a significant third order term. To account for each of these contributions in computing the beam loading, Palmer divides the bunch into four equal sub-bunches: two at $\sim 0.2\sigma_z$ and two at $\sim 1.4\sigma_z$. The energy losses of the four bunches are

$$V_1 = MW_i(0) , \quad (41a)$$

$$V_2 = M[W_i(0) + W_i(1.2\sigma_z)] , \quad (41b)$$

$$V_3 = M[W_i(0) + W_i(0.4\sigma_z) + W(1.6\sigma_z)] , \quad (41c)$$

$$V_4 = M[W_i(0) + W_i(1.2\sigma_z) + W(1.6\sigma_z) + W_i(2.8\sigma_z)] , \quad (41d)$$

where

$$M = \frac{eN_b}{4E_a} . \quad (42)$$

From these beam loading voltages, Palmer then computes the first, second, and third order momentum spreads in the four-bunch approximation:

$${}_1\sigma_p = 0.05(V_3 - V_2) + 0.35(V_4 - V_1) \quad (43a)$$

$${}_2\sigma_p = 0.25(V_3 + V_2) + 0.25(V_4 + V_1) \quad (43b)$$

$${}_3\sigma_p = 0.22(V_3 - V_2) + 0.033(V_4 - V_1) \quad (43c)$$

A continuous pulse convolution integral gives more accurate results, but the four-bunch approximation should be sufficient for parameter surveys.

As noted above, the first order effect can be canceled by the rf with an appropriate choice of θ . Since some of this momentum spread is required for BNS damping of the transverse wake, one should choose

$$\tan \theta = \frac{1}{2\pi} \frac{\lambda}{\sigma_z} ({}_1\sigma_p - \sigma_{p,\text{BNS}}) \quad , \quad (45)$$

where $\sigma_{p,\text{BNS}}$ is given by Eq. (36). Recall that $\sigma_{p,\text{BNS}}$ is removed at the end of the linac. Including the effects of the rf accelerating field, the total second order spread is

$${}_2\sigma_{p,\text{tot}} = {}_2\sigma_p - 0.5 \left(\frac{2\pi\sigma_z}{\lambda} \right)^2 \quad . \quad (46)$$

For the purpose of designing the beam matching sections into the wiggler, one can use as the final momentum spread

$$\sigma_p = ({}_2\sigma_{p,\text{tot}}^2 + {}_3\sigma_{p,\text{tot}}^2)^{1/2} \quad . \quad (47)$$

5.2.1. Beam Efficiency. A quantity of considerable interest to the designer of linear colliders is the efficiency, η_b , with which the beam extracts energy from the rf structure:

$$\eta_b = esN_b \frac{\langle E \rangle}{E_a^2} \approx \frac{esN_b}{E_a} \propto \frac{N_b}{\lambda^2} \quad , \quad (48)$$

where the uncorrected elastance s is given by Eq. (2.9b). From Eq. (37) $\langle E \rangle$ differs from E_a by the beam loading voltage, E_b . An approximation to Eq. (48) for structures with the ratio a/λ roughly the same as the SLAC structure is

$$\eta_b \approx 0.13\% \left(\frac{N_b}{10^{10}} \right) \left(\frac{100 \text{ mm}}{\lambda} \right)^2 \left(\frac{100 \text{ MeV/m}}{E_a} \right) \quad . \quad (49)$$

The efficiency of transferring energy from the rf source to the beam is the product of the beam efficiency and the structure efficiency, i.e., $h_s \eta_b$.

A XRFEL or collider macro-pulse will contain B bunches of equal charge. In order that each bunch extract the same fraction of the the stored energy from the

Figure 6. Efficient energy extraction from a travelling wave linac section for short pulse trains

structure, the beam loading must be compensated in one of several possible ways (Fig. 6). For example, the first pulse can be injected before the section is completely filled, and the bunch-to-bunch spacing and power flow can be adjusted so that the energy added between bunches just compensates for the pulse-to-pulse sag.

An alternative procedure suitable for use with pulse trains of duration much greater than T_{fill} is to recirculate the rf power that exits from a section by adding it back at the input of the same section. A steady-state condition can be achieved after $\sim 3 - 5 T_{fill}$, at which point the beam is injected.

Although both schemes of compensating for beam loading will have the side effect of reducing the gradient of the linac, they allow one to raise the overall

electrical efficiency of the accelerator to

$$\eta_{\text{tot}} = h_s \eta_b \eta_{rf} \left(\frac{\beta T_{\text{fill}}}{T_{rf}} \right) , \quad (50)$$

where η_{rf} is the efficiency of converting electrical power to rf and T_{rf} is the total period during which rf is supplied to the high gradient structure. With both efficient rf power sources and beam-loading compensation it should be practical to obtain $\eta_{\text{tot}} \approx 10\%$.

From the analysis of the previous section one should note that increasing N_b to increase the beam efficiency is limited by wake field effects. Via longitudinal wakefields raising N_b will lead to the simultaneous introduction of an irreducible, nonlinear momentum spread in the beam. Raising the beam loading has the additional drawback of increasing the amount of energy spread needed to BNS damp the single bunch transverse wakefield. The length of the section designed to remove this energy spread will, therefore, increase with increased beam loading. Fortunately, however, in accelerators for linear light sources this increase in overall linac length is a 1-2% effect.

6. PULSE MANIPULATION

Linear light sources will probably have several output beamlines. In addition to directing the beam to the appropriate beamline, the final beam manipulation subsystem may be required to perform other functions:

- 1) Stretch the length of the bunch from its value in the high frequency, high gradient accelerator to a much longer pulse matched to user requirements.
- 2) Focus the beam from its $\approx 100 \mu\text{m}$ size in the accelerator to the $\approx 1 \mu\text{m}$ size appropriate to self-focused plasma wigglers or other plasma experiments.
- 3) Provide for a vacuum-to-gas transition in the case of plasma devices.

6.1. Pulse Stretching

The stretching of the pulse can be accomplished by taking advantage of the irreducible momentum spread acquired during the acceleration process. Although many forms of dispersive transport may be used as a debuncher, for scaling purposes it is convenient to choose the four dipole chicane arrangement (Fig. 7), typically used as a debuncher in conjunction with damping rings.

Figure 7. Four dipole pulse stretcher in a chicane arrangement

Dipoles 1 and 2 and dipoles 3 and 4 are separated by L_d meters; the distance between dipoles 2 and 3 is J_d meters. With four dipoles of equal strength and with the beam trajectory normal to the entrance and exit edges of the dipoles, the chicane can be made doubly achromatic to second order. The effect of the chicane is described¹⁸ by a transfer matrix, M , with unit diagonal elements and two nonzero off-diagonal terms. The dispersion of the chicane is described by a debunching parameter (in units of cm / % $\Delta p/p$):

$$M_{34} = 2L \tan^2 \alpha + 4\rho \tan \alpha + 4\rho \tan \alpha - 8\rho \sin \alpha \quad , \quad (51)$$

where α is the angle of bend and ρ is the radius of curvature in the magnetic field. The only other nonzero matrix relates the entrance position to the exit angle; that is

$$M_{12} = 4\rho \tan \alpha + \frac{2L}{\cos^2 \alpha} + J_d \quad . \quad (52)$$

The magnetic field and the radius of curvature are related by

$$B\rho = 33 \text{ T-m} \left(\frac{E}{10 \text{ GeV}} \right) \quad (53)$$

Precision, small aperture (4 to 5 cm) dipoles are practical with a field strength of 6.5 T. With such magnets, pulse stretching to ≈ 1 cm with $\Delta p/p = \pm 1\%$ is achievable in a chicane of overall length less than 7 m. Thus, compact linear light sources can tailor the duration of the pulse up to that available with circular synchrotron sources.

The drawback to stretching the pulse by more than a factor of several is that the peak current in the linac is raised to more than 1 kA. Although increasing the peak current does not lead to a critical constraint in the linac proper, it does make the task of designing the low emittance injector ever more difficult. In tradeoff studies it is probably unwise to drive peak current from the injector much above 1 kA.

6.2. Beam Matching

The matching section for a flavor factory or a plasma-focused wiggler XRFEL system is analogous to the final focus of a linear collider flavor factory and scales accordingly. In the final focus the beam radius must be reduced by a factor ≈ 100 in as short a distance as possible; i.e., the design must have the minimum achievable β^* . Let β_o^* be the minimum that can be achieved for a given momentum spread by a focal array with no chromatic correction. Chromatic correction elements can reduce this value by a factor S which scales inversely with σ_p . Thus,

$$\beta^* \geq \frac{\beta_o^*}{S} = \frac{\beta_o^* \sigma_p}{S_o} = 25 \beta_o^* \sigma_p \quad (54)$$

Unlike the case of the linear collider, the focal spot for a plasma focused XRFEL may not be highly elliptical; therefore, quadrupole doublet designs are ruled out.

An appropriate design for scaling is a quadrupole triplet with a layout (Fig. 8) based on the SLC final focus, which brings the beam to a circular focus. One scales such a design by modifying all lengths by one factor and all transverse dimensions by another. The scaling factors are respectively f^* , the “ideal” focal length, and a^* , the aperture radius of the first quadrupole, which has pole tips of field strength B^* . The “ideal” focal length is

$$f^* = \left[\left(\frac{a^*}{B^*} \right) (B\rho) \right]^{1/2}, \quad (55)$$

where $B\rho$ is given by Eq. (53). Palmer³ expresses the performance of the focal system in terms of f^* , a^* and constants that depend on details of the magnet system: T , A_x , A_y and Λ .

Figure 8. Quadrupole triplet array for beam matching based on the SLC design. All quadrupoles have a field gradient of 50 T/m. Distances are given in meters.

In terms of the scaling parameters, the β^* of the focal array is

$$\beta_o^* = 2\sigma_p f^* T. \quad (56)$$

The maximum angular acceptance of the focal array and the free space before the first quadrupole are

$$\Theta_{x,y} = \frac{a^*}{A_{x,y} f^*} \quad (57)$$

and

$$l_1 = \Lambda f^*, \quad (58)$$

respectively. Combining Eqs. (54), (55) and (56), one obtains

$$\beta^* = \left(\frac{T}{S_o} \right) \beta \rho \left(\frac{a^*}{f^* B^*} \right) \sigma_p^2 \quad (59)$$

For this design, $T = 2.96$, $A_x = 4.3$, $A_y = 3.2$, $\Lambda = 1.36$, and $S_o = 0.04$.

In order to focus the beam to the very small radii, the pole tip fields must either be very strong or the aperture must be very small. Electro-mechanical engineering considerations are likely to limit the minimum aperture of superconducting quadrupoles to 2–3 cm. In that case the field strength will have to be raised to > 6 T. The alternative is to choose a conventional magnet design with iron pole pieces. For this case the pole tip fields will be limited to ≈ 1.5 T. More than compensating for this limitation is the fact that the field can be led into a very small region of space (apertures ≈ 1 –2 mm) at the expense of magnet efficiency. Therefore, the system designer should opt to use conventional magnets, even though cryogenic systems will be almost mandatory in the design of a pulse stretcher (if that is desired).

6.3. Gas Matching for Self-Focused XRFELs

If the size of the focal spot is not equal to the self-focused radius of the beam in gas, the equilibrium beam size will differ from the size of the focal spot. This consideration is especially important in the case that the vacuum-air transition is a very thin foil rather than a differentially pumped transition. Foils may be useful in bringing the accelerator to early operational status at low repetition rates.

If the focal system produces a spot of radius a_f , then in the gas the beam will suffer radial oscillations, eventually settling to an equilibrium radius given by

$$a_{eq}^2 = a_f^2 \exp \left(\theta_{tot}^2 \frac{\gamma I_o}{I_b} - 1 \right) \quad (60)$$

where $I_o = 17$ kA. The mean betatron angle, θ_{tot} , is determined by a combination of the emittance and the foil induced scattering:

$$\theta_{tot}^2 = \left(\frac{\varepsilon_n}{\gamma a_f} \right)^2 + \theta_{scat}^2 \quad (61)$$

For very thin foils of thickness X , the Moliere-corrected rms scattering angle is

$$\theta_{\text{scat}} = \left(\frac{28}{\gamma}\right) \left(\frac{X}{X_{\text{rad}}}\right)^{1/2} \left[1 + 0.111 \log_{10} \left(\frac{X}{X_{\text{rad}}}\right)\right] , \quad (62)$$

where X_{rad} is the radiation length of the foil material or of the gas in the transition region in the case of a plasma assisted XRFEL.

7. CHOICE OF FREQUENCY AND RF POWER SOURCES

The generation of bright beams at multi-GeV energies requires the accelerator designer to consider new classes of machines that can operate with gradients exceeding 200 MeV/m in structures suitable for high-repetition rate-operation. The approach must be highly reliable and energy efficient, if the electrical power demands of accelerator operation are to remain within currently accepted bounds for most facilities. As discussed in the Sec. 3, the accelerating gradient can be increased and the energy required per meter of accelerator structure reduced by scaling the (effective) rf structure of the accelerator to high frequencies (≥ 10 GHz.) The practical limits of this approach for conventional rf cavities are set by the electron induced breakdown limit and the surface heating limit, with little gain in gradient being achievable for frequencies exceeding 30 GHz. Considering that both the cost of fabricating the rf structure and the difficulty of maintaining the alignment of the structure is likely to increase once the cavity is miniaturized beyond a scale size of ≈ 2 cm, one can select 10–20 GHz as the best frequency range for a compact accelerator. If standing-wave linacs with geometries more complex than that of the disk-loaded waveguide are used to accelerate long pulse trains, ease of fabrication will tip the balance even further in the direction of choosing longer rf wavelengths.

The peak rf power needed to drive an accelerator based on a generic $2\pi/3$ disk-loaded waveguide can be determined by substituting Eqs. (6)–(13) into Eq. (16). A useful, though less exact, estimate can be obtained by ignoring the dependence

of the elastance on the group velocity. Scaling the SLC linac design at constant β_g —that is, constant a/λ —one can rewrite Eq. (16) as

$$P_{rf} = 74 \text{ MW/m} \left(\frac{E_a}{100 \text{ MeV/m}} \right)^2 (h\tau)^{-1} \left(\frac{\lambda}{105 \text{ mm}} \right)^{1/2}. \quad (63)$$

For the SLC design $h\tau$, the product of the structure efficiency and attenuation parameter is ≈ 0.3 ; halving the fill time will reduce $h\tau$ to ≈ 0.2 . Thus, a rough estimate of the total peak power needed to drive a accelerator is

$$P_{rf} \approx 60 \text{ GW} \left(\frac{E}{10 \text{ GeV}} \right) \left(\frac{\lambda}{105 \text{ mm}} \right)^{1/2}. \quad (64)$$

One concludes that a new class of rf power sources will be needed to drive compact accelerators of beams for compact linear light sources and flavor factories.

7.1. Relativistic Klystrons

The high energy accelerator community has considered many approaches to providing the large amounts of rf power at X-band and higher frequencies needed to achieve ultra-high accelerating gradients. Recently a consensus¹⁹ has emerged in the U.S., Western Europe, and the USSR that the range from 10 to 30 GHz holds the most promise for the next generation TeV linear collider. The same considerations apply to the accelerator for a compact linear light source. At the low end of this frequency range one can perhaps combine many conventional klystrons with an advanced rf pulse-compression network to power the accelerator. More novel approaches, however, seem to offer the required performance levels at substantially lower cost and complexity. One of these approaches, the relativistic klystron, offers particular promise, especially if high repetition rates are desired as in the case of flavor factories. The relativistic klystron²⁰ (Fig. 9) is a concept for modulating a multi-kA, multi-MeV beam produced with an linear induction accelerator (LIA) at the desired rf frequency (10 to 15 GHz) and using the modulated high current beam to excite an rf generating transfer structure. The high peak power rf is then fed via

Figure 9. Conceptual design of the front end of a relativistic klystron powered by a multi-kA, multi-MeV beam from an induction linac.

waveguides to the miniaturized rf cavities of the high gradient (200–400 MeV/m) accelerator of the high energy beam.

Three basic variants of the relativistic klystron concept are now under active investigation by a SLAC/LLNL/LBL collaboration. These are:

- 1) Extrapolation of conventional (velocity modulation), high gain klystron design to 1–3 MV operation (≈ 1 GW).
- 2) Adiabatic deceleration of an energy modulated, bunched, 5–10 MeV beam with multiple klystron output cavities (≈ 10 GW).
- 3) Alternating multiple cavity deceleration with induction re-acceleration of the high current beam to form a single power “tube” (≈ 100 GW).

Early experiments²¹ at LLNL have already yielded record power levels (≈ 80 MW) for X-band klystrons at high conversion efficiency ($\approx 55\%$). Over 200 MW have been produced at higher frequency (11.4 GHz) although at lower efficiency. This power was sufficient to yield an accelerating field of 130 MeV/m in a small section of high gradient structure. The most convincing demonstration of the practicality of the relativistic klystron approach would be the construction and operation at the earliest possible date of an ultra-compact (5-m), 1-GeV linac powered by the beam from the 7-MeV, 3-kA ETA-II at LLNL.

Before such an ambitious goal can become a reality, the collaboration must address several fundamental physics issues relating to (1) rf generation and power handling and (2) stability of the drive beam in the rf generating structures. Still, for the purposes of scoping studies, the design space for the induction linac beam source can already be roughly delimited:

- 1) Both the acceleration and deceleration rates for the high current beam can exceed 1 MeV/m.
- 2) Using present design techniques, the maximum rf current that can be handled efficiently with standing wave klystron cavities is 3–4 kA.
- 3) The minimum voltage beam that can be efficiently decelerated in a multi-cavity transfer structure is 2–3 MeV.
- 4) Maximum beam-to-rf conversion efficiencies of 50–70% seem possible.
- 5) The efficiency of converting electrical energy into usable beam energy with induction linacs is $\approx 50\%$.

Thus, a flavor factory using the relativistic klystron approach would require ≈ 60 MeV of LIA cells operating with a 2-kA beam to provide sufficient power to drive two 6-GeV, compact rf linacs operating at ≈ 11 GHz.

The large power flow carried by LIA beams is not the sole reason for their interest to the high energy accelerator designer. Because the magnitude of the transverse wake varies as a^{-3} , one is unlikely to scale the disk-loaded structure to X-band or higher frequencies while maintaining $A_l \equiv a/\lambda$ constant. Rather one should increase A_l by a factor of 2 to 3 to control wakefields. Increasing A_l has several additional effects. First, by Eq. (7), raising A_l will increase the rf group velocity thereby shortening the fill time of the structure. The rf attenuation time of the structure also varies as $\lambda^{3/2}$. Therefore, scaling with constant attenuation parameter t implies shortening the fill time from the $\approx 1 \mu\text{s}$ at SLC to $\approx 50 \text{ ns}$ for an X-band linac.

These short fill times match the pulse durations of modern induction linacs. The second effect of increasing A_l is to raise the maximum surface field that is associated with a given accelerating field. As long as sufficient rf power is available, one will probably choose to operate the linac at as close to the maximum surface field as is practical. Therefore, shortening the fill time by opening the iris aperture comes at the expense of reducing gradient.

With respect to the design of its rf power source, a linear light source could have one significant difference from the linear collider. The collider's power source must deliver rf for approximately one fill time. In contrast, some applications of compact linacs may require hundreds of bunches in a macro pulse. In that case, the rf power must be supplied for a time, T_{rf} , several times longer than the fill time; i.e., $\approx 300\text{--}1000 \text{ ns}$.

Modern induction linacs with ferrite cores produce pulses with a duration of several tens of nanoseconds. From the design principles described in Ref. 22, one sees that scaling ferrite loaded LIAs to produce continuous pulses much longer than $\approx 100 \text{ ns}$ is probably impractical. By changing the ferromagnetic material to a

dielectric coated metallic glass, the pulse duration may be extendible to ≈ 200 -300 ns. However, the beam break-up effects of this change are not known. The first induction linacs such as the Astron with soft iron tape cores produced 300 ns pulses. Indeed, a $1\text{-}\mu\text{s}$ pulse core was built at NBS in the early 1970s. At $1\text{ }\mu\text{s}$ such cores are hardly small or inexpensive. Moreover, eddy current losses in the metallic cores will be high. The alternative is to recycle the induction cores at multi-MHz rates with branched magnetic drives.²³

7.1.1. Branched Magnetic Drives. The magnetic pulse compressors that supply pulsed power in modern LIAs consist of a repetitive pulse source followed by several serial stages of pulse compression. These networks can drive the LIA at any repetition rate up to the maximum firing rate of the primary commutator. In the ETA-II, for example, this rate is set by the recovery time of the thyatron switches. If the application were to require a fixed number (3-10) of output pulses from the LIA in a burst ($1\text{-}2\text{ }\mu\text{s}$), a new scheme would become feasible. The technique, called branched magnetics, involves deploying several capacitive stores and switches in parallel that are fired in a sequence at any repetition rate permitted by the load. One such circuit was tested with an eight-pulse burst (Fig. 10). This proof-of-principle experiment demonstrated pulse generation into a resistive load at 16 MHz.

In the application of this approach to synthesize microsecond bursts of LIA beam power, the firing rate will be limited by the time required to reset the induction cores in the linac. Unless the reset times are at least twice as long as the original pulse time of the core, the amount of power consumed by the reset circuits will be comparable to that of the beam drive circuits. Hence, the highest practical duty factor of the LIA during the macro-pulse is $\approx 30\%$. Therefore, even with branched magnetics, supplying rf power for $\approx 1\text{ }\mu\text{s}$ will either require multiple LIAs or an innovation in rf power handling. One such idea is recycling the rf power that leaves

Figure 10. Simplified circuit diagram of the burst mode generator designed to provide any desired repetition rate for a fixed number of pulses (in this case, eight). The numbers indicate the order in which the capacitors are discharged.

one linac section into another section. Maintaining adequate rf phase control and stability at high power is a critical physics issue for this latter approach.

From the point of view of making the LIA beam source as simple and economical as possible, the accelerator designer should endeavour to make T_{rf} as small as possible. In particular, the fill time must be kept below 100 ns without making L_f so short that both the peak power and number of rf feeds are raised excessively. Therefore, even if the size (gradient) constraints were eased considerably, the accelerator designer would be strongly motivated to build the linac at X-band or higher frequency.

8. COST AND SIZE ESTIMATES

The primary capital cost element of the accelerator will be the LIA driven, relativistic klystron power sources for the rf linac. The considerations that govern the installed capital cost of such rf sources is discussed at length in Ref. 22 for the case of single branch magnetic drives. It remains only to indicate how to modify those scaling relations to the case of an induction drive accelerator with branched magnetics.

One begins by determining the total active length of high gradient accelerator, including the section to correct energy spread:

$$L_{act} = 50 \text{ m} \left(\frac{E}{10 \text{ GeV}} \right) \left(\frac{E_a}{200 \text{ MeV/m}} \right) + L_{corr} \quad (65)$$

The overall length of the linac is then

$$L_{acc} = (1 + F_q) L_{act} \quad (66)$$

Using Eq. (16), one then calculates the total voltage to be supplied by the induction cells as

$$V_{ind} = \frac{\Pi_{rf}}{I_{ind} \eta_{rf}} = 10^{-3} P_{rf} L_{acc} \left(\frac{2 \text{ kA}}{I_{ind}} \right) \left(\frac{0.5}{\eta_{rf}} \right) \quad (67)$$

where current in the LIA is I_{ind} . The LIA pulse length, T_{ind} , the number of LIAs, N_{ind} , and the total number of magnetic drive branches in all LIAs, N_{br} , depends on the maximum pulse length, T_{imax} , assumed for the induction cores and on the induction reset duty factor during the macro-pulse, Δ_{ind} . In terms of the standard number theoretic functions,

$$N_{ind} = \text{Min} \left[\left(\text{Int} \left(\frac{T_{rf}}{T_{imax}} \right) + 1 \right), \left(\text{Int} (\Delta_{ind}^{-1}) + 1 \right) \right] \quad ; \quad (68)$$

$$N_{br} = \text{Int} \left(\frac{T_{rf}}{T_{imax}} \right) + 1 \quad ; \quad (69)$$

$$T_{ind} = \frac{T_{rf}}{N_{br}} \quad (70)$$

Once these quantities are determined the cost scaling algorithm can be applied with the modifications indicated in the Appendix. For quick estimates of the cost of powering the one can use the graph of rf cost vs total “tube” power given in Fig. 11.

Figure 11. Rf cost vs. “tube” power for various LIA pulse lengths

With respect to scaling the cost per active meter of the disk-loaded waveguide, actual data at many frequencies are scarce. The following heuristic equation seems, however, to summarize the experience of the accelerator community:

$$C_{\text{wave}} = 45 \text{ k\$/m} \left(\frac{\lambda}{500 \text{ mm}} \right)^{1/3} + 2.5 \text{ k\$/m} \left(\frac{50 \text{ mm}}{\lambda} \right)^{5/2} + 5 \text{ k\$/m} . \quad (71)$$

The first term, representing a cost of materials and machining, scales as the waveguide volume. The second term is an alignment cost, which scales as the transverse wake. The final term is a fixed handling cost.

Klystron transfer cavities and power feeds to the high gradient structure are estimated to cost

$$C_{\text{cav}} = 20 \text{ k\$/m} . \quad (72)$$

The final beam manipulation beamline is estimated to cost

$$C_{\text{bmln}} = 200 \text{ k\$/in} \quad . \quad (73)$$

As is done for the cost of the rf power, the costs of Eqs. (71), (72), and (73) should be multiplied by ≈ 1.35 to account for the associated manpower needed for installment, procurement, and project management.

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Appendix A. COST SCALING ALGORITHM FOR BRANCHED DRIVES

Variables used in the cost scaling Eqs. (22) for the induction driver with branched magnetics include repetition frequency f in Hz, the pulse length (full-width half maximum), the total accelerating voltage V in MV, the total volt-seconds of ferrimagnetic core $W = VT_{\text{ind}}$, the peak gap stress E_g in kV/cm, the single pulse energy $E = VI_{\text{beam}}T_{\text{ind}}$ in joules, the average power $P = fE$ in MW, the effective gradient G in MV/m, and the inner radius of the ferrites in the induction cells, r_i , in cm which is taken as the beam pipe radius plus 2 cm to account for the thickness of the focusing solenoids. In the scaling equation (A1) italicized quantities refer to injector voltage, power, pulse energy, etc. Costs are specified in constant FY88 k\$. The branched magnetic drives are composed of N_{br} branches and N_{ind} induction linacs.

A.1. Injector Subsystem

The scaling equation for the injector is divided into five separate components:

$$C_{\text{inj}} = \left\{ 545 \left(\frac{W}{0.225} \right)_{\text{cells}} (I/3000) + 320 \left(\frac{E}{630} \right)_{\text{mag}} + 450 \left(\frac{P}{2.25} \right)_{\text{isps}} \text{Max} \left(1, \frac{f}{500} \right) \right. \\ \left. + 165 \left(\frac{V}{3} \right)_{\text{vac}}^3 + \left(60 \right)_{\text{fixtures}} + 60 \left(\frac{I}{3000} \right)_{\text{cathode}}^{1/2} \right\}. \quad (\text{A1})$$

To provide a stiff voltage source for the injector the magnetic compressors provide 50% additional energy into resistive loads. As branched drives may comprise as many as three separate induction linacs, the injector cost should be multiplied by N_{ind} in the estimate of total hardware cost.

A.2 Beam Transport Subsystems

For solenoidal transport, the cost has three separate components:

$$C_{\text{sol}} = \left\{ \left[\left(57 + 40 \left(\frac{r_i}{7.5} \right) \left(\frac{I}{3000} \right) \right) \left(\frac{V}{1.5} \right) \left(\frac{I}{3000} \right) \left(\frac{r_i}{7.5} \right) \left(\frac{0.75}{G} \right) \right]_{\text{focus}} + \left[42 \left(\frac{V}{1.5} \right) \left(\frac{r_i}{7.5} \right) \left(\frac{0.75}{G} \right) \right]_{\text{steer}} + \left[78 \right]_{\text{match}} \right\}. \quad (A2)$$

For the alternative laser guiding scheme the scaling equation is

$$C_{\text{laser}} = \left(\frac{7.5V}{G} \right) N_{\text{ind}}, \quad (A3)$$

which includes laser, gas handling, and matching magnet costs for $f \leq 250$ Hz.

A.3. Accelerator Cell Subsystems

The cell block cost is

$$C_{\text{block}} = \left[150 \left(\frac{V}{1.5} \right) \left(\frac{r_o}{20} \right)^2 \left(\frac{0.75}{G} \right) \left(\frac{175}{E_g} \right) \right] N_{\text{ind}}, \quad (A4)$$

where r_o is related to r_i by

$$r_o = r_i + \left(\frac{GT_{\text{ind}}}{p} \right) \Delta B. \quad (A5)$$

Here p is the ferrite packing fraction ~ 0.8 and ΔB is the total flux swing ~ 0.6 Wb/m² ($= 180$ MV/m), and effective gradients range from 0.5 to 1.5 MeV/m.

The ferrite cost is given by

$$C_{\text{ferrite}} = \left[140 \left(\frac{W}{0.225} \right) \left(\frac{r_o + r_i}{27.5} \right) \right] N_{\text{ind}}, \quad (A6)$$

where we have used the relation between the ferrite area A and volume $\pi A(r_o + r_i)$.

The cost of intermediate stores and power supplies scales as

$$C_{\text{isps}} = \left[678 \left(\frac{P}{3.2} \right) \text{Max}(1, 500/f) + 5 \left(\frac{E}{630} \right) \right] N_{\text{br}}. \quad (A7)$$

If the repetition rate of the system is less than 600 Hz, the power in Eqs. (A1) and (A7) should be replaced by a value corresponding to 600 Hz operation. The scaling for the magnetic pulse compressors is

$$C_{\text{mag}} = \left[241 \left(\frac{E}{630} \right) \right] N_{\text{ind}} \quad . \quad (\text{A8})$$

For power systems delivering pulses at repetition rates ≈ 100 Hz the magnetic modulators and intermediate stores can be re-engineered to reduce costs by nearly factor of two. The cost of the strongback alignment structure is

$$C_{\text{strong}} = \left[450 \left(\frac{V}{12.5} \right) \left(\frac{r_o}{20} \right)^2 \left(\frac{0.75}{G} \right) K \right] N_{\text{ind}} \quad . \quad (\text{A9})$$

where the constant K depends on the focusing scheme; namely,

$$\begin{aligned} K &= 0.6 \quad \text{for solenoids,} \\ &= 0.5 \quad \text{for laser guiding.} \end{aligned}$$

A.4. Ancillary Subsystems

The cost for low and extremely low conductivity water will scale as

$$C_{\text{lcw}} = 90 \left(\frac{P}{13.1} \right) N_{br} \quad , \quad (\text{A10})$$

for $f > 100$ Hz. For $f < 100$ Hz one should use the value of C_{lcw} at $f = 100$ Hz. The vacuum system is scaled as if it were pumping speed limited (valid for $r_i < 2.5\text{cm}$):

$$C_{\text{vac}} = 660 \left(\frac{V}{12.5} \right) \left(\frac{r_i}{7.5} \right)^2 \left(\frac{0.75}{G} \right) K N_{\text{ind}} \quad , \quad (\text{A11})$$

where K is the same as in Eq. (A9). The cost of electrical fluids is proportional to average beam power;

$$C_{\text{fluid}} = 542 \left(\frac{P}{13.1} \right) N_{br} \quad . \quad (\text{A12})$$

The scaling of dump costs is similar to that for reactors; i.e., \$1 per watt of time average beam power into the dump, P_d . For the relativistic klystron assume that

the average beam voltage at the dump, V_d , is 5 MeV; hence, $P_d = V_d T_{ind} I_{beam} f$. The cost of the dump is, therefore,

$$C_{dump} = 1000 P_d N_{ind} \quad . \quad (A13)$$

The cost of fixtures is proportional to the length of the induction linac:

$$C_{fixture} = 20 \left(\frac{V}{12.5} \right) \left(\frac{0.75}{G} \right) N_{ind} \quad . \quad (A14)$$

The cost of the instruments and controls scales as a fixed (buy-in) value plus a percentage of the cost of the hardware to be monitored and controlled:

$$C_{i\&c} = 1500 + 0.04 (C_{inj} + C_{cell} + C_{focus}) \quad . \quad (A15)$$

Summing the cost equations yields the total hardware cost for the induction linac driver.

A.5. Installation and Engineering Support

The installation costs for the base design were estimated for each component; for estimating purposes the installation costs can be taken as a fixed percentage of the total hardware costs adjusted to the fully loaded labor rate, R , in k\$/man-month.

$$C_{install} = 0.09 C_{hardware} \left(\frac{R}{10} \right) \quad . \quad (A16)$$

Similarly, a cost for engineering management and support is estimated as a percentage of the hardware costs; i.e.,

$$C_{engin} = 0.125 C_{hardware} \quad . \quad (A17)$$

Supplies and equipment used for engineering and installation increase the total cost by 10% ;

$$C_{s\&e} = 0.1 C_{hardware} \quad . \quad (A18)$$

Adding these installation costs to the hardware cost yields a total cost which includes $\sim 10\%$ contingency distributed (unevenly) among the various cost centers.

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