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ATOMIC PHENOMENA IN DENSE PLASMAS

BY

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ATOMIC PHENOMENA IN DENSE PLASMAS

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I. INTRODUCTION

Our understanding of atomic processes important in plasmas has benefitted considerably from the efforts of physicists and astronomers to analyze and relate laboratory and cosmic plasma radiation arising under similar conditions. Particle densities in conventional laboratory plasmas typically range from about 10^7 cm^{-3} to about 10^{18} cm^{-3} . As is illustrated in Figure 1, this range of densities is bracketed by gaseous nebulae and stellar atmospheres, and it has been extensively studied for several decades.

During the past few years, research directed toward controlled fusion by means of inertial confinement (laser or particle beams) has resulted in very hot laboratory plasmas at solid and even greater than solid densities, $N > 10^{22} / \text{cm}^3$. It is, of course, no surprise that inertial confinement fusion (ICF) experiments seek to generate plasmas having densities and temperatures like those occurring within stars. However, in contrast to stellar interiors, which--except for neutrino flux measurements--are not directly accessible to observation, the ICF plasmas can and are being studied in great detail. Besides the spectral data provided by conventional means, spatial and temporal resolution of ICF plasmas is provided by x-ray photographs from pinhole and streak cameras (Yaakobi and Goldman, 1976; Attwood et al., 1977; Lee and Rosen, 1979; Key et al., 1979).

The long timescales characteristic of stellar evolution insure that the dense gas inside stars almost always is in local thermodynamic equilibrium (LTE), but the nanosecond timescales characteristic of ICF plasmas generally preclude populations of atomic states from attaining LTE values. This dense, $10^{18} < N (\text{cm}^{-3}) < 10^{25}$, non-equilibrium plasma regime is the primary setting for the present discussion of atomic phenomena. In this environment, the dimensions of ionic orbitals can be comparable to distances between ions,

causing quantal states to be strongly influenced by surrounding plasma.

In addition to stellar physics and ICF research, high density plasmas are encountered in solid state science and in the study of thermonuclear detonations. However, collective phenomena are of primary interest in cold metal and semiconductor plasmas, and much of the physics of thermonuclear explosions is classified defense research. Therefore, the relevance of the present subject to these areas will not be considered here. The design and development of short-wavelength lasers is another active area of investigation often involving dense plasmas. (Because of their very brief radiative lifetimes, the lasing atoms require very rapid pumping, and for many lasing materials this is best done at very high densities; see, e.g., Chapline and Wood, 1975.) Specific laser schemes will not be described in this chapter, but many phenomena discussed here can affect the extent of stimulated emission in a high-density lasing medium.

Several other chapters collected in the present work provide up-to-date discussions of related subjects. Some recent monographs on associated topics are those by Hughes (1975), Bekefi (1976), Motz (1979), and McDowell and Ferendeci (1980). These references discuss either relevant atomic processes, but as they occur in low-density plasmas, or the laser/plasma aspects of fusion. The text by Cox and Giuli (1968) surveys atomic physics in stellar interiors.

II. The Plasma Environment

A convenient, one-dimensional characterization of a plasma is provided by the Coulomb parameter (DeWitt, et al. 1973)

$$\Gamma = \langle Z_i e \rangle^2 / R_i \theta$$

$$= \langle Z_i \rangle^{5/3} (N_e / 8.00 \times 10^{19} \text{ cm}^{-3})^{1/3} / \theta \text{ (eV)}, \quad (1)$$

where $\langle Z_i \rangle$ is the mean charge of all plasma ions, $\theta = kT$ is the plasma temperature* in energy units, and where the ion-sphere radius R_i approximates the ion-ion spacing. It is defined in terms of the electron concentration N_e by the equation

$$R_i = \{3\langle Z_i \rangle / 4\pi N_e\}^{1/3} \quad (2)$$

$$= (1.61 \times 10^{-24} \langle Z_i \rangle \text{ cm}^{-3} / N_e)^{1/3} a_0,$$

with $a_0 = 0.529 \times 10^{-8}$ cm being the Bohr radius. When $\Gamma \gg 1$ Coulomb interactions control particle motions, but when $\Gamma \ll 1$ thermal motions predominate. Two lines of constant Γ are shown on the $N_e - \theta$ diagram in Figure 1, for a hydrogen plasma with $\langle Z_i \rangle = 1$. It is evident that values $\Gamma \gtrsim 10^{-2} \langle Z_i \rangle^{5/3}$ are of interest here.

Another important parameter is the plasma electron frequency ω_e , which is the oscillation frequency for space-charge disturbances. At high densities, the energy of collective oscillations (plasmons),

$$\begin{aligned} \hbar \omega_e &= \hbar (4\pi e^2 N_e / m_e)^{1/2} \\ &= (N_e / 7.21 \times 10^{20} \text{ cm}^{-3})^{1/2} \text{ eV}, \end{aligned} \quad (3)$$

*Throughout this chapter, it is assumed that an equilibrium temperature can be ascribed to each charge species. The parameter Γ is useful when all species have the same temperature and, unless noted otherwise, this additional assumption is also made.

can be comparable with atomic transition energies, and electric fields associated with these oscillations can strongly perturb plasma ions.

A. Electrostatic Potentials of Ions In Plasmas

The long-range nature of the Coulomb force causes plasmas to be electrically neutral on a macroscopic scale. However, small departures from charge neutrality do occur, and their scale can be estimated from the range of the electrostatic potential $\phi(r)$ of a bare test charge placed in a uniform plasma. This potential also serves, in a first approximation, as a measure of the plasma's influence on a nucleus with some bound electrons, if one makes the simplifying assumption that the many-body wave function representing the background plasma and the ion of interest is separable. (The separation is analogous to that effected in the Born-Oppenheimer approximation for molecular structure calculations.)

In response to a test charge Ze , whose position is taken as the origin, the concentration of each (non-degenerate) particle species p departs from its uniform value N_p according to Boltzmann's equation,

$$\tilde{N}_p(r) = N_p g_p(r) = N_p \exp \left[-Z e \phi(r) / \theta_p \right], \quad (4)$$

such that ϕ satisfies the Poisson equation. The exponential quantity g is known as the radial distribution function. In the limit of high temperatures θ_p (i.e., $\Gamma \ll 1$), a linear approximation of $g_p(r)$ leads to the familiar result

$$\phi = \phi_{DH}(r) = (Ze/r) \exp(-r/D), \quad (5)$$

where D is the Debye-Hückel length,

$$D = \left[4\pi e^2 \sum_p \frac{Z_p^2 N_p}{O_p \Gamma} \right]^{-1/2}$$

$$= \left[\sum_p \frac{Z_p^2 N_p}{[1.97 \times 10^{22} \text{ cm}^{-3} O_p (\text{eV})]^{-1/2} a_0} \right]^{-1/2} \quad (6)$$

This statistical description of the plasma screening is valid only if there are many shielding particles (usually, electrons) within a "Debye sphere," i.e., if $(4/3)\pi D^3 N \gg 1$.

A complementary picture can be obtained in the low temperature limit $\Gamma \gg 1$. Under the assumptions (1) that the nuclei are fixed and (2) that each nucleus of charge Ze is completely shielded from its neighbors by a uniform cloud of Z electrons, Poisson's equation can be solved to give

$$\phi(r < R_i) = \phi_{IS}(r) = Ze \left[1/r - (1/2R_i) (3 - r^2/R_i^2) \right], \quad (7)$$

where R_i is the ion-sphere radius of Eq. (2). For $r > R_i$, ϕ_{IS} is defined to be zero.

Sufficiently far from a test charge, the ratio $e\phi/O_p \ll 1$ regardless of the temperature, so the Debye potential represents the asymptotic ($r \rightarrow \infty$) form of plasma screening; conversely, when r is very small, the ion-sphere potential is appropriate at all temperatures. When all particle species have the same temperature, the relationship between the two screening lengths is

$$(R_i/D) = [3\Gamma(\langle Z_i^2 \rangle + \langle Z_i \rangle)]^{1/2} / \langle Z_i \rangle. \quad (8)$$

The relationship between these two model potentials has been studied in detail

by Stewart and Pyatt (1966), also for the case of a one-temperature plasma. These authors give a simple interpolation formula that is valid when $\langle Z_1^2 \rangle \gg \langle Z_1 \rangle$.

Thus far, no mention has been made of the influence of bound electrons on $\phi(r)$. The Thomas-Fermi statistical model (cf. Bethe and Jackiw, 1973) does not consider bound states explicitly, but it does yield an electrostatic potential ϕ_{TF} that is self-consistent with a degenerate ($0_e = 0$) Fermi distribution of free electrons near the nucleus. For an ion of nuclear charge $Z_n e$ this potential satisfies the differential equation

$$x^{1/2} U''(x) = U^{3/2}(x), \quad (9)$$

in which

$$U = r(e\phi_{TF} + \zeta)/Z_n e^2, \quad (10)$$

where ζ is the chemical potential of the electron gas, $x = r/\xi$, and the length

$$\xi = \left[(3\pi/4)^{2/3} a_0 \right] / 2Z_n^{1/3}. \quad (11)$$

At the origin, $U(0) = 1$; different choices for the second boundary condition yield potentials for confined and isolated systems.

The Thomas-Fermi model has been extended to include electron exchange interactions (Dirac, 1930) and finite electron temperatures (Feynman, Metropolis and Teller, 1949; Cowan and Ashkin, 1957). Moreover, the effects of atomic shell structure on ϕ_{TF} have been treated approximately by Zink (1968) and by Carson, Mayers and Stibbs (1968). Plasma screening in the

regime of intermediate degeneracy--between the Debye-Hückel and Thomas-Fermi limits--recently has been investigated by Gupta and Rajagopal (1979).

The most elaborate computations for ions in hot, dense matter have been performed by Rozsnyai (1972). He used a relativistic Thomas-Fermi-Dirac calculation to describe the free-electron distribution and to provide the trial potential for a relativistic Hartree-Fock-Slater calculation of the bound-electron distribution. The result is a statistical, or average, atom (AA) with non-integral occupation numbers (Chandrasekhar, 1939). The total potential ϕ_{AA} due to the nucleus and its neutralizing cloud of bound and free electrons, all isolated in an ion sphere, is self-consistent with the total charge density of these same particles.

Thomas-Fermi and Hartree-Fock potentials cannot be expressed in closed form, even for an isolated atom. In order to illustrate the differences among the various model potentials, numerical results are shown in Figure 2 for neon ions in an LTE neon plasma of temperature $0 - 100$ eV and nucleon number density $N_{Ne} = 10^{21} \text{ cm}^{-3}$; in this plasma, the predominant charge state of neon ions is Ne^{+8} and the Coulomb parameter is $r = 1.36$. The plotted quantity is the effective nuclear charge seen by one of the bound electrons, $Z_{eff}(r) = -(r/e)\phi(r)$. For the ion-sphere, Thomas-Fermi and average-atom potentials, Z_{eff} was computed by reducing the ten, neutralizing electrons' contribution to ϕ by the factor $9/10$. For the Debye-Hückel potential, Z_{eff} was computed from Eq. (5) for an ion of net charge $+8$; this introduces a significant difference at small r -values due to the complete screening accorded the other bound electron. In the region of large r -values the discrepancies are due to the fact that partial screening by other ions and their electrons is not included in the ion-sphere, Thomas-Fermi or average-atom potential calculations. (All of these potentials correspond to a model of each plasma ion being confined in

a finite volume.)

On the scale of Figure 2, the Thomas-Fermi and average-atom effective charges cannot be distinguished. Although the agreement is not always as good as it is for this plasma of moderate temperature and density, in general these two schemes yield quite similar results (Rozsnyai, private communication).

In addition to these various potentials, some other models recently have been proposed for calculating macroscopic (equation-of-state) properties of matter at high densities (cf. Liberman, 1979, More, 1979). As yet, however, these models have not been applied to atomic problems.

B. Electric Microfield Distributions

Because plasmas are good conductors, there are no steady-state electric fields within them. However, transient fields $\vec{E}(t)$ are produced by motions of incompletely screened charges. Perturbations of atoms and ions by passing electrons usually are brief, and are treated as collisions. On the other hand, the fluctuation times $\sim |\vec{E}|/|\dot{\vec{E}}|$ associated with the slower moving ions are, in many important instances, long compared with the lifetimes of atomic states, and therefore these so-called ion microfields can be treated quasi-statically (cf. Griem, 1975, Sec. II.2).

1. One-Component Plasmas

The problem of calculating the net electric field $\vec{E} = \sum_j \vec{E}_j$ of an ensemble of stationary, uncorrelated point charges Ze was first considered by Holtmark (1919). He found that the distribution of reduced field strengths $\beta = |\vec{E}|/F_0$, where

$$F_0 = \sqrt{8\pi/25} (Ze/R_1^2) \quad (12)$$

$$= Z^{1/3} (N_e/4.35 \times 10^9 \text{ cm}^{-3})^{2/3} \text{ V/cm},$$

is given by the integral

$$W_H(\beta) = (2\beta/\pi) \int_0^\infty \exp\{-x^{3/2}\} \sin(\beta x) x \, dx, \quad (13)$$

the normalization being $\int_0^\infty W_H(\beta) \, d\beta = 1$. This distribution, which has been tabulated by, for instance, Baranger and Mozer (1959), is plotted in Figure 3. For comparison, the distribution of field strengths due to the single nearest neighbor (1NN) ion in a uniform plasma (Margenau and Lewis, 1959),

$$W_{1NN}(\beta) = (3/2\beta^{5/2}) \exp\{-1/\beta^{3/2}\}, \quad (14)$$

also is plotted. The two field strength distributions are essentially the same when $\beta \gg 1$, but elsewhere the 1NN formula is a poor approximation of the Holtsmark distribution.

At finite temperatures, Coulomb interactions give rise to correlations between ions, and electron screening reduces the individual ionic fields $\vec{E}_j$. Two quite different techniques have been used to investigate how finite temperatures alter microfield distributions. The first (Baranger and Mozer, 1959, 1960; Hooper, 1966, 1968) is analogous to Holtsmark's approach, and it involves the use of collective coordinates and particle correlation functions. The published calculations of this sort all have used a linearized Debye-Huckel approximation of the correlation between ion pairs. In other words, the radial distribution of ions at a distance $r = xR_1$ from a given ion has been taken to be (See Eq. (4))

$$g(x) \rightarrow g_{DH}(x) = \exp \left[\frac{-Z_e^2}{r_0} \exp \left(\frac{-r}{D_e} \right) \right]$$

$$= 1 - \frac{\Gamma}{x} \exp(-\sqrt{3\Gamma} x) \quad , \quad (15)$$

where $D_e = (4\pi e^2 N_e / \epsilon_0 \epsilon_e)^{-1/2}$ is the Debye length that includes just the electrons' screening of the ion-ion repulsion (Baranger and Mozer, 1959). The linearization restricts Baranger and Mozer's approach to plasmas having low Γ -values. However Hooper's method, which separates the many-body interaction in order to treat differently its long- and short-range parts, can be used to determine microfields in plasmas with $\Gamma \sim 1$.

The second approach, which is not limited to low Γ -values, involves lengthy numerical (i.e., Monte Carlo or molecular dynamics) simulations of the plasma as a classical n -body system (Brush et al., 1966; Hansen, 1973). There exist few direct comparisons of microfield distributions, but it is generally believed that when $\Gamma < 0.2$ these different schemes yield distributions that are almost identical (Hooper, 1966). More extensive comparisons have been made of the radial distribution function $g(x)$. Brush et al., (1966) determined g from their Monte Carlo experiments for plasmas with $0.05 < \Gamma < 100$, and found that even the full Debye-Huckel expression g_{DH} is inaccurate when Γ exceeds a few tenths. (Polynomial fits to the computed values of g have been discussed by DeWitt et al., 1973, and Itoh et al., 1977.)

Two microfield distributions are shown in Figure 3 for single-component plasmas with Γ -values in the range of interest here. The likelihood of a strong field occurring in a dense plasma clearly is much less than that predicted by Holtsmark's expression for the plasma with $\Gamma = 0$, but, even when $\Gamma > 1$, the characteristic ion microfield strength is of order F_0 . It is interesting to compare this field strength with that associated with longitudinal plasma waves at the frequency ω_e . Their rms strength F_w (Griem,

1975, p 22) in fact is proportional to F_0 ,

$$(F_w/F_0) = \left(\frac{25}{8\pi^2 Z}\right) \left(\frac{3\Gamma}{Z}\right)^{1/2} = 0.551 Z^{-3/2} \Gamma^{1/2}. \quad (16)$$

From Figure 1 it can be determined that $F_w < F_0$ in most cases of interest here.

2. Multi-Component Plasmas

Holtmark's analysis can be extended to the case of a plasma with ions of different charge. According to Chandrasekhar and Von Neumann (1942), it is only necessary to re-define the normal field as $F_0 = \sqrt{8\pi/25} \langle Z_i^{3/2} \rangle^{2/3} e/R_i^2$. Formulae that generalize the nearest-neighbor (1NN) approximation--including two-particle correlations--have been given for ion mixtures by Weisheit and Pollock (1980).

Hooper and his collaborators (O'Brien and Hooper, 1972; Tighe and Hooper, 1976) have extended his original formalism to treat multi-component plasmas by means of collective coordinates. Weisheit and Pollock (1980) have also published some molecular dynamic calculations of microfield distributions for hydrogen plasmas with high-Z impurities. These computations for multi-component plasmas having $\Gamma > 0$ are very time-consuming. Therefore, reasonably accurate approximations are needed to describe microfields in the variety of plasmas now being generated. To this end, a two-nearest-neighbor scheme has been proposed by Weisheit and Pollock (1980); it yields accurate microfield distributions for plasmas with $\Gamma < 2$, and requires as input just radial distribution functions for mixtures (cf. Itoh et al., 1979). It should be noted that the Coulomb parameter as defined in Eq. (1) differs slightly from the expression proposed by Itoh et al. (1979). Their particular definition facilitates the analysis of numerical many-body calculations, but

for most other purposes Eq. (1) is more convenient and so it is used throughout this chapter.

III. PERTURBATIONS OF ATOMIC STRUCTURE

Modification of the Coulomb interaction has significant effects on atomic structure, and these, in turn, have important consequences for the statistical mechanical properties of plasmas (Theimer and Kepple, 1970; Rogers and DeWitt, 1973). Because of the finite range of screened electrostatic potentials, there is only a finite number of bound states for any ion in a plasma. Moreover, surrounding plasma lifts the orbital angular momentum degeneracy of hydrogenic states, and changes to some extent eigenenergies and eigenfunctions of all atomic systems. Observed shifts and intensity alterations of spectral lines, which are evidence of these perturbations, can serve as a check on the model potentials.

A. Continuum Lowering and Level Shifts

1. Theory

The Schrodinger equation for an electron bound by a Debye-Hückel potential to a nucleus of charge $Z_n e = (Z+1)e$ cannot be solved analytically.* However, from first-order perturbation theory one finds that the energy of the ionization continuum relative to the ground state is decreased by an amount

$$\Delta E_{DH}(n\ell) = - (Z+1)e^2 \langle n\ell | (e^{-r/D} - 1)/r | n\ell \rangle, \quad (17)$$

* This choice of nuclear charge is consistent with the definitions and formulae of Section II, wherein Z_e represents the net ionic charge.

and that the principal quantum number of the uppermost bound state of the system is roughly $[(Z+1)D/a_0]^{1/2}$. This continuum lowering modifies equilibrium populations of ionic states and brings about a higher degree of ionization in the plasma. Accurate energy levels $E_{DH}(n\ell)$ of a Debye-screened hydrogenic ion have been computed by Rogers et al. (1970) and by Roussel and O'Connell (1974) for a wide range of screening lengths $1 \leq (Z+1)D/a_0 \leq 10^3$. The number of bound states with orbital angular momentum $\ell = 0$ was found to be $1.126 [(Z+1)D/a_0]^{1/2}$, and the smallest screening length for which a bound state exists was determined to be $\hat{D} = (0.8399 a_0)/(Z+1)$. Critical screening lengths for the existence of the 45 lowest orbitals, as computed by Rogers et al., (1970), are reproduced in Table I.

The ion-sphere potential also does not admit an analytic solution of the one-electron Schrodinger equation, but perturbation theory again can be used to estimate the extent of continuum lowering. To first order, all one-electron levels in an ion of nuclear charge $(Z+1)e$ are shifted upward by an amount

$$\Delta E_{IS}(n\ell) = 3Ze^2/2R_i - (Ze^2/2R_i^3) \langle n\ell | r^2 | n\ell \rangle. \quad (18)$$

Variational calculations of the energies and wavefunctions for the $n=1$ and 2 states in an ion-sphere potential were performed by Zirin (1954), who tabulated his results as a function of the parameter $(Z+1)R_i$. Because it represents a more severe screening picture, for a specified screening length R_i or D , ϕ_{IS} has less tightly bound levels than does ϕ_{DH} .

The relationship between the Debye-Huckel, ion-sphere and Thomas-Fermi predictions of continuum lowering was explored by Stewart and Pyatt (1966), whose results are presented in Figure 4. As expected, in the

limits of large and small Γ -values, the ion-sphere and Debye-Huckel depressions ΔE are obtained. At intermediate Γ -values, Thomas-Fermi theory yields a continuum lowering ΔE_{TF} that always is less than ΔE_{DH} ; thus, the Debye-Huckel potential supports fewer bound states than does the Thomas-Fermi potential. This conclusion of Steward and Pyatt (1966) is consistent with the recent analysis of Gupta and Rajagopal (1979).

Latter (1955) numerically solved the Schrodinger equation with the ($C_e = 0$) Thomas-Fermi-Dirac potential, and obtained orbital eigenvalues for isolated neutral atoms that agree with experimental ionization energies almost as well as Hartree-Fock results do. Shalitin (1965, 1967) proposed a variant of the Thomas-Fermi potential that accounts for the self-interaction of the electrons. He then extended Latter's tables to include this modification; some positive ions were also considered. Although no extensive eigenvalue tabulations comparable to Latter's and Shalitin's exist for confined ions in plasmas, Zink (1968) described a model potential with properties like ϕ_{TF} that can be used to determine shell structure in dense, partially ionized matter.

Rozsnyai (1972, 1975) published average-atom eigenvalues for a few different ions. At low matter densities and temperatures his results agree with standard Hartree-Fock calculations, and his AA eigenvalues for ions in dense, hot plasmas generally are in accord with Thomas-Fermi results for confined ions.

There also exist more detailed investigations for the special case of a partially ionized hydrogen plasma. Theimer and Kepple (1970) treated unbound plasma particles classically, but corrected the Debye-Huckel model for the fact that the free electron distribution in the vicinity of a neutral atom is not the same as it is near a point charge. Recently, Dharna-wardana et al.

(1980) treated quantum mechanically both the bound and the free particles in the hydrogen plasma, and then determined the spectrum of elementary (one-particle) excitations of this many-body system. In both of these works, neutral atom level shifts much smaller than those computed for the Debye-Hückel potential were obtained.

2. Experiment

In principle, the perturbation of an individual level with respect to the continuum is observable in the displacement of the threshold for recombination continuum radiation, but in practice the Doppler effect may obliterate these shifts: It is easy to show for hydrogenic systems that, at a given plasma temperature θ , the Doppler width $\Delta E_{\text{Dopp}}(n)$ of the n^{th} level's recombination radiation edge and the Debye-Hückel shift (to lowest order) of the same level have the ratio

$$\frac{\Delta E_{\text{Dopp}}(n)}{\Delta E_{\text{DH}}(n)} \approx \frac{\theta(\text{eV})}{n^2} \left(\frac{2.16 \times 10^{13} \text{ cm}^{-3}}{N_e} \right)^{1/2}, \quad (19)$$

independent of the ionic charge. Thermal motions may not pose a problem even for the ground and low-lying excited states, but for any state the true continuum depression may also be obscured by the merging of emission lines near the series limit--the so-called Inglis-Teller effect (Inglis and Teller, 1939)-- unless there are only a few, well separated lines remaining.

On the other hand, relative shifts between pairs of levels can be determined by measuring the displacement $\Delta\lambda$ of spectral lines with respect to their vacuum wavelengths λ_0 . For most ions, broadening of transition lines is accompanied by a shift that cannot be computed with great accuracy. However, lines of hydrogenic ions are not Stark-shifted by uniform plasma

fields. Berg et al., (1962) first measured the shift of the $\text{He}^+(4686\text{\AA})$ line, and they attributed it to the polarization of the plasma near each radiating ion. The basic idea is that, for an electron bound in an orbital of principal quantum number n , free electrons shield the nucleus to the extent

$$e\Delta Z(n) = -4\pi e \int_0^{r_n} \tilde{N}_e(r) r^2 dr. \quad (20)$$

The resulting energy shift is $\Delta E(n) = -(Z+1)e^2\Delta Z(n)/n^2a_0$, and so the plasma polarization shift (PPS) in wavelength of a photon emitted in the transition $n \rightarrow m$ is*

$$\Delta\lambda_{\text{PPS}} = -\frac{2\lambda_0}{(Z+1)} \left(\frac{\Delta Z(n)}{n^2} - \frac{\Delta Z(m)}{m^2} \right) / \left(\frac{1}{n^2} - \frac{1}{m^2} \right). \quad (21)$$

No unique prescription exists to determine in Eq. (20) the free electron concentration within the ion or the effective orbital radius r_n . In a first approximation (Grieg et al., 1970), the effective radius may be equated to $\langle n|r|n \rangle = n^2a_0/(Z+1)$, and the influence of the bound electron on the free electron radial distribution may be accounted for by the replacement $\tilde{N}_e(r) = N_e g_e(r) + N_e \exp(V_n/\theta)$, where $V_n > 0$ is a constant representing the average interaction energy of the plasma and the radiating ion. This leads to the result

$$\Delta\lambda_{\text{PPS}} = -\frac{8\pi\lambda_0 N_e a_0^3}{3(Z+1)^4} \frac{[n^4 \exp(V_n/\theta) - m^4 \exp(V_m/\theta)]}{(1/m^2 - 1/n^2)}. \quad (22)$$

*The formal differences between "level shifts" and "line shifts", which have been noted by Dharma-wardana et al., (1980), are not considered here.

If $V_n = V_{\infty} = V$, the plasma polarization shifts are negative (i.e., blue shifts) and are proportional to $m^2 n^2 (n^2 + 1) N_e$. In contrast, the Debye-Hückel and ion-sphere potentials always predict red shifts of transition wavelengths; to lowest order both of these shifts also are proportional to N_e . The Hartree-Pock average-atom theory predicts red shifts for some spectral lines and blue shifts for others; moreover, the sign of the shift for some transitions changes as the plasma conditions vary (Rozsnyai, 1975).

Since the original experiment of Berg *et al.* (1962) there have been several other measurements confirming this plasma induced shift (Burgess and Peacock, 1971; Gabriel and Volante, 1973; Goto and Burgess, 1974; Neiger and Griem, 1976; Van Zandt *et al.*, 1976). Almost all of the FPS measurements have been of the He^+ principal series, and the published data are collected in Figure 5. All the measured wavelength displacements have been scaled linearly with N_e to the density $1 \times 10^{18} \text{ cm}^{-3}$. There is consensus regarding the sign of the shift, namely $\Delta\lambda (np + 1s) < 0$; however, where there is more than one measurement, the agreement among different values is poor.

Evidently, the Debye-Hückel and ion-sphere models are too crude to describe ion-plasma interactions at this level of detail. For the specified experimental conditions, $\theta = 3.5 \text{ eV}$ and $N_e = 1 \times 10^{18} \text{ cm}^{-3}$, the predicted values for the $9p + 1s$ transition are $\Delta\lambda_{\text{DH}} (9p + 1s) = + 0.70 \text{ \AA}$ and $\Delta\lambda_{\text{IS}} (9p + 1s) = + 0.14 \text{ \AA}$. Besides having the wrong sign, the magnitudes of these shifts are considerably larger than the measured value. Several alternatives have been proposed for the interaction energy V_n (c.f., Neiger and Griem, 1976), but all were chosen *a posteriori* to reproduce certain data. As one example, the simple expression $V = Ze^2/R_1$, suggested by Burgess and Peacock (1971), was used to obtain the curve shown in the figure.

Burgess (1972) emphasized that the plasma polarization shift not only is intrinsically interesting as a many-body phenomenon, it also is important as an (as yet unknown) restriction on the limit of accuracy of wavelength determinations from plasma spectra. What are needed in order to better understand this effect are data for a wide range of plasma temperatures, to check for the postulated exponential dependence of $\Delta\lambda_{\text{PPS}}$ on Ω , and data for different spectral series, to clarify the dependence of ν on the state of the radiating ion. At present, there are only a few such measurements (Burgess and Peacock, 1971; Van Zandt *et al.*, 1976; Pittman and Kelleher, 1980), and it is not yet certain if even the form of Eq. (22) is correct.

Finally, Skupsky (1980) has argued that PPS measurements of lines from high-Z impurities in a hydrogen plasma can be a useful density diagnostic for ICF experiments: At densities $N_e \gtrsim 10^{24} \text{ cm}^{-3}$, the ion-sphere model is accurate and a red shift $\Delta\lambda_{\text{PPS}} \propto N_e$ is expected (cf, Eq. 18).

B. Radiative Transition Strengths

Because screening affects the eigenfunctions as well as the eigenenergies of ions, strengths of radiative transitions in plasmas are changed from their vacuum values. The discussion here is limited to allowed transitions, so only matrix elements of the electric dipole operator $\vec{d}$ are needed.

1. Bound-Bound Transitions

The absorption oscillator strength $f(b \leftarrow a)$ and the spontaneous emission coefficient $A(a \rightarrow b)$ for transition between atomic states a and b , whose statistical weights are $w(a)$ and $w(b)$ and whose substates are specified by the

sets of quantum numbers q_a and q_b , both are proportional to the transition strength

$$S(b,a) = S(a,b) = \sum_{q_a q_b} |\langle a q_a | \hat{d} | b q_b \rangle|^2, \quad (23)$$

the familiar relations being

$$f(b \rightarrow a) = \frac{2\pi}{3h^2 e^2} \frac{[E(a) - E(b)]}{w(b)} S(a,b), \quad (24)$$

$$A(a \rightarrow b) = \frac{4[E(a) - E(b)]^3}{3h^4 c^2 w(a)} S(a,b). \quad (25)$$

The smallness of the plasma polarization shift suggests that any significant modification of f - and A - values in plasmas will be due to variation of the transition strengths, not the transition energies $[E(a) - E(b)]$.

Several authors (Herrman and Coulaud, 1970; Roussel and O'Connell, 1974; Weisheit and Shore, 1974) have investigated dipole transitions in a Debye-Hückel-screened hydrogenic system. The general result is that transition strengths begin to decrease from their vacuum values when the screening length D gets as small as a few times the critical screening length (Table 1) of the upper level, and then they drop precipitously as the critical screening length is approached. This trend, which is illustrated in Figure 6 for the hydrogenic ion resonance series, has two important consequences (Weisheit and Shore, 1974). First, there is less absorption of radiation by closely spaced lines near a series limit, and this can affect opacity calculations (cf. Mihalas, 1978, Sec. 7.2). Second, higher lines of a spectral emission series become progressively harder to detect, and this can affect the Inglis-Teller scheme for plasma density determinations (Roussel and

O'Connell, 1975).

No transition strength calculations involving the ion-sphere or Thomas-Fermi potentials have been published, but the same qualitative behavior displayed in Figure 6 is evident in the Hartree-Fock-Slater average-atom oscillator strengths reported by Rozsnyai (1975).

2. Bound-Free Transitions

The ever greater diminution of oscillator strengths with respect to their vacuum values as the limit of a spectral series $n\ell \rightarrow n'\ell'$ is approached is formally connected to the low-energy ($E \rightarrow 0$) behavior of the photoionization cross section $\sigma(n\ell \rightarrow E\ell')$, where E is the energy of the ejected electron (c.f., Shore, 1975). The photoionization cross section vanishes at threshold because of the finite range of the interaction between the ejected electron and its screened parent ion (Wigner, 1948). Just above threshold, Wigner showed that $\sigma \propto E^{\ell'+1/2}$. This behavior was reproduced by the cross section calculations of Weisheit and Shore (1974), who also found that well above threshold the cross sections regained their hydrogenic form. In fact, their numerical results can be approximated accurately by matching, at the energy $E = \hbar^2/2m_e D^2$, the low- and high-energy forms.

The work of Rozsnyai (1975) includes some average-atom cross sections for photoionization of ions in dense, hot plasmas, but does not discuss these results in particular. Similarly, the stellar opacity calculations of Carson, et. al., (1968) employ Thomas-Fermi-Dirac potentials to compute radiative transition strengths, but no specific results are given.

3. Free-Free Transitions

In the non-relativistic limit, the differential cross section

for bremsstrahlung, or free-free emission, in the frequency interval $(\omega, \omega + d\omega)$ as an electron of initial momentum $\hbar k_1$ scatters from a fixed point charge Ze is (Bethe and Salpeter, 1957, Sec. 77)

$$\frac{d\sigma(\omega)}{d\omega} = G \frac{d\sigma_K(\omega)}{d\omega} = G \frac{16\pi \alpha^3 Z^2}{3\sqrt{3} \omega k_1^2}, \quad (26)$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant, $d\sigma_K(\omega)/d\omega$ is the Kramers formula and G is the Gaunt factor, a complicated function of the electron's initial and final momenta, $\hbar k_1$ and $\hbar k_2$. There exist simple approximations to G in various limits (see Sobelman, 1979, Sec. 9.5.7) and extensive tables of numerical values (Karzas and Latter, 1961). Except very near threshold ($\omega=0$), $G \approx 1$ when the inequalities $Z/k_1 a_0 \gg 1$ and $(Z/k_2 a_0 - Z/k_1 a_0) \ll 1$, are satisfied. In the other limit, when $Z/k_1 a_0 \ll 1$, the Born-Elwert approximation of G is accurate,

$$G_{BE} = \frac{\sqrt{3}}{\pi} \left\{ \frac{k_1}{k_2} \frac{1 - e^{-2\pi Z/k_1 a_0}}{1 - e^{-2\pi Z/k_2 a_0}} \right\} \ln \left(\frac{k_1 + k_2}{k_1 - k_2} \right). \quad (27)$$

The Born approximation is valid when k_1 and k_2 both are much greater than Z/a_0 . In this case, the bracketed term in the preceding equation approaches unity and the usual Born result for bremsstrahlung emission in a Coulomb field is obtained.

In the (non-relativistic) Born approximation, the general expression for G is given in terms of an integral involving the Fourier transform of the electron-ion interaction ϕ_x (Grant, 1968),

$$G_{xB}(k_1, k_2) = \frac{\sqrt{3}}{\pi} k_1 k_2 \int_{-1}^{+1} dy \left| \int_0^\infty dr \left[\frac{r \phi_x(r)}{Z e} \right] \sin Kr \right|^2, \quad (28)$$

where $\hbar^2 K^2 = \hbar^2 |\vec{k}_1 - \vec{k}_2|^2 = \hbar^2 (k_1^2 + k_2^2 - 2k_1 k_2 \gamma)$ is the square of the momentum lost by the scattered electron, and $Z_n e$ again is the nuclear charge of the target ion. Thus, in the high-energy regime it is straightforward to investigate the effect of screening by bound or free electrons.

When ϕ is the Debye-Hückel interaction, the Born approximation for G takes the form (Grant, 1958)

$$G_{\text{DHB}}(k_1, k_2, D) = \frac{\sqrt{3}}{\pi} \left\{ \ln \left[\frac{1 + (k_1 + k_2)^2 D^2}{1 + (k_1 - k_2)^2 D^2} \right]^{1/2} + \frac{1}{2[1 + (k_1 + k_2)^2 D^2]} - \frac{1}{2[1 + (k_1 - k_2)^2 D^2]} \right\}. \quad (29)$$

Examination of Eq. (29) shows that a finite screening length tends to reduce G and, hence, the free-free cross section at all emission frequencies ω . In the limit of negligible screening, $D \rightarrow \infty$, the (Coulomb) Born formula is regained. Grant (1958) and Rozsnyai (1979) modeled the actual potential in the vicinity of a partially stripped ion by a linear combination of Debye-Hückel potentials having different screening lengths, and then used a generalization of Eq. (29) to compute the bremsstrahlung emission from electron-ion collisions.

In plasmas with $\Gamma > 1$, the Born approximation to the Gaunt factor can be computed for the ion-sphere potential from the integral expression

$$G_{\text{ISB}}(k_1, k_2, R_i) = \frac{\sqrt{3}}{\pi} \int_{(k_1 - k_2)R_i}^{(k_1 + k_2)R_i} \left[1 + \frac{3}{2} \left(\cos x - \frac{\sin x}{x} \right) \right]^2 \frac{dx}{x}. \quad (30)$$

Various Born Gaunt factors are compared in Figure 7 for an electron of initial momentum $\hbar k_1 = 5\hbar/a_0$ (initial energy = 340 eV) scattering in a hydrogen plasma with $N_e = 3.4 \times 10^{24}$ and $\phi = 340$ eV. The screening lengths in this plasma are $D = 1a_0$ and $R_1 = 0.78 a_0$, and the Coulomb parameter is $\Gamma = 0.10$.

It should be mentioned that Zirin (1954) has published a few tables of Gaunt factors for free-free emission in an ion-sphere potential; these calculations employed crude (one-term) variational wavefunctions for the scattering electron. Also, Jackson (1975) has presented a classical electrodynamic description of free-free emission when screening is important, and Pratt and his collaborators (cf. Tseng and Pratt, 1979) have extensively studied relativistic effects in electron bremsstrahlung. These effects become significant when $k_1 \gtrsim 60/a_0$, i.e., when the incident energy of the electron exceeds about 50 keV.

Cross sections for free-free emission and free-free absorption (inverse bremsstrahlung) are related through the principle of detailed balance. Therefore, the absorption process, which plays an important role in the heating of plasmas by laser irradiation, also is affected by plasma screening of the ionic charges. Recently, Lima *et al.* (1979) investigated this topic and concluded that the rate at which free electrons gain energy via inverse bremsstrahlung is significantly reduced unless the laser frequency $\omega \approx \omega_e$, the plasma frequency; in that resonant situation the absorption is enhanced.

In addition to work, already cited, involving one or another screened potential, there is the paper by Watson (1970), who investigated the effect on the Gaunt factor of ion-ion correlations in dense plasmas. He found that, in degenerate plasmas characteristic of some stellar interiors, the

total radiative opacity typically decreased by less than 10 percent due to this effect, while in non-degenerate plasmas the effect is even smaller.

For many applications to laboratory or astrophysical plasmas, what is needed is a Gaunt factor averaged with respect to the distribution of incident electron momenta. Such averages for the Maxwellian distribution can be found in the works of Karsas and Latter (1961), Green (1974) and Grant (1958), Zirin (1954), Grant (1958), and Kozsnyai (1979) for the pure Coulomb, Debye-Hückel, ion-sphere, Thomas-Fermi, and average-atom potentials, respectively.

IV. PERTURBATIONS OF ATOMIC COLLISIONS

It is well known that the infinite range of the Coulomb interaction causes the total cross section for elastic scattering of two point charges to diverge at all collision energies, and the total cross section for excitation of ions by charged particles to be finite at threshold. Besides modifying the eigenstates of scattering systems, plasma screening changes these general features of atomic collisions.

Let $\vec{v}_*$ be the constant velocity of a charge Z_*e moving in an otherwise uniform plasma, and let $\vec{s}_*(t)$ be its position at time t . In this time-dependent situation, the test-particle concept (Rostoker and Rosenbluth, 1960) provides a quantitative description of the screening in terms of the plasma dielectric function $D(\vec{k}, \omega)$ (cf. Krall and Trivelpiece, 1973). At a given point $\vec{s}$,

$$\phi_*(r) = \frac{4\pi Z_*e}{(2\pi)^3} \iiint_{-\infty}^{+\infty} \frac{e^{i\vec{k} \cdot [\vec{s} - \vec{s}_*(t)]}}{k^2 D(\vec{k}, \vec{k} \cdot \vec{v}_*)} d^3\vec{k} \quad (31)$$

where $r \approx |\vec{s} - \vec{s}_*(t)|$ is the instantaneous distance between the test charge and the point in question. This concept, which is based on the Debye-Hückel model of plasma screening and consequently is valid only if there are many screening particles within a Debye sphere (i.e., if $\Gamma \ll \langle Z_i \rangle$), leads to the following results:

i) When v_* is much smaller than the mean thermal speeds of all particle species, then $D \rightarrow 1-D$ and the potential of the test charge is

$$\phi_*(r) \rightarrow \frac{Z_* e}{r} \exp(-r/D). \quad (32)$$

ii) When v_* is much larger than the mean thermal speeds of all ion species but much smaller than the mean thermal electron speed, then $D \rightarrow 1 - D_e$ and ϕ_* has the same form as in Eq. (32), but with D replaced by D_e .

iii) When v_* is much larger than the mean thermal speeds of all particle species, then $D \rightarrow 1$ and the potential of the test charge is unshielded.

In other words, only plasma particles with speeds at least comparable to v_* are polarized on a timescale short enough that they can contribute effectively to the screening of the moving test charge's potential.

For the ion-sphere model, there is no analog of the test-particle scheme, so most of the present discussion of collisional processes is limited to plasmas that satisfy the inequality $\Gamma \ll \langle Z_i \rangle$, and to systems that interact via Debye-Hückel potentials. There is an extensive literature on the formal aspects of this scattering topic (cf. Joachain, 1975; Semon and Taylor, 1976; and references cited therein), in part because the exponentially screened $1/r$ potential also occurs in the study of nuclear phenomena.

A. Elastic Scattering

In general, the elastic scattering of a charge Z_*e by a central potential $\phi(r)$ cannot be computed easily except at high energies, where the (first) Born approximation is valid. Born's formula for the differential cross section for scattering through an angle θ from a fixed center is

$$\left(\frac{d\sigma}{d\Omega}\right)_B = \left(\frac{2m_*Z_*e}{\hbar^2}\right)^2 \left| \int_0^\infty r^2 \phi(r) \left(\frac{\sin K_* r}{K_* r}\right) dr \right|^2, \quad (33)$$

where m_* is the mass of the scattered particle, $\hbar k_*$ is its momentum, and $K_* = 2k_* \sin(\theta/2)$. For a binary collision, m_* and $\hbar k_*$ are replaced by the reduced mass μ of the two particles and the relative momentum $\hbar k$, and θ becomes the scattering angle in the center-of-mass coordinate system.

According to the test-particle picture, the differential cross section for the elastic scattering of a fast (unscreened) charge Z_*e by a screened charge Ze in a plasma follows directly from Eqs. (32) and (33),

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{DHB}} = \left[\frac{2\mu Z_* Z e^2 D^2 / \hbar^2}{1 + (2KD)^2} \right]^2, \quad (34)$$

where D is the relevant Debye-Huckel screening length and $K = 2k \sin(\theta/2)$. This expression smoothly approaches the Rutherford cross section in the limit $D \rightarrow \infty$, and upon integration with respect to solid angle it yields a finite total scattering cross section,

$$\sigma_{\text{DHB}} = \frac{\pi (2\mu Z_* Z e^2 D^2 / \hbar^2)^2}{(1 + 4k^2 D^2)}. \quad (35)$$

Comparison of the Born formula (35) and the cross section σ_{DH} calculated by a full quantal treatment shows that the Born approximation is accurate to within 10 percent even when $kD \sim 1$ (Joachain, 1975).

The energy lost per unit path length by a charge Z_*e as it moves through a one-component plasma having, per unit volume, N μ screened ions of charge Ze and mass M is (Mott and Massey, 1965, p626)

$$-\left(\frac{dE_*}{ds}\right)_C = \frac{\pi N}{M} \left(\frac{2\mu Z Z_* e^2}{\hbar k}\right)^2 \ln \left(\frac{\hbar^2 k^2 \rho_{\max}}{\mu e^2 |Z Z_*|}\right), \quad (36)$$

where $\mu = (M^{-1} + m_*^{-1})^{-1}$ and $\rho_{\max}$ is a somewhat arbitrary maximum impact parameter for which there is any significant energy transfer in individual Coulomb collisions. When the particle slowing down is a charge selected from a thermal distribution in the plasma the approximation $\hbar^2 k^2 = 3\mu\theta$ in the logarithmic term, together with the substitution $\rho_{\max} \rightarrow D_e$, yields the familiar Coulomb logarithm (Spitzer, 1962),

$$\ln \Lambda = \ln \left(\frac{3\theta D_e}{|Z Z_*| e^2}\right) = \ln \left(\frac{3Z^3}{Z_*^2 \Gamma^3}\right)^{1/2}. \quad (37)$$

Spitzer (1962, Table 5.1) has published a convenient table of values of $\ln \Lambda$ for a wide range of plasma conditions. In the regime explored by ICF experiments, $\ln \Lambda \sim 3$ to 10 for hydrogen plasmas.

Because of its particularly simple form, Eq. (34) can be used to obtain a Debye-Hückel (Born) formula analogous to Eq. (36),

$$-\left(\frac{dE_*}{ds}\right)_{DHB} = \frac{\pi N}{M} \left(\frac{2\mu Z_* Z e^2}{\hbar k}\right)^2 L_{DHB}(kD), \quad (38)$$

where

$$L_{DHB}(x) = \ln \sqrt{1 + 4x^2} - 2x^2/(1 + 4x^2) . \quad (39)$$

If L_{DHB} also is evaluated at the mean thermal energy of the plasma particles, then for electron screening of the plasma ions

$$\begin{aligned} L_{DHB}(kD_e) + \ln nT &\approx \ln (2D_e \sqrt{3\mu_0/\hbar}) \\ &= \ln [ZZ_* | \Lambda \sqrt{(\mu/m_e)(13.3 \text{ eV}/\theta)}] , \end{aligned} \quad (40)$$

since the quantity $(2D_e \sqrt{3\mu_0/\hbar})$ generally is much greater than unity in hot, dense plasmas. Note that, unlike the Coulomb logarithm, this quantity depends upon the masses of the scattering particles.

The transport properties of an ionized gas are regulated by small-angle deflections in charged-particle encounters. For scattering in a Coulomb field, the rate of increase of the mean square angular deflection $\langle(\Delta\theta)^2\rangle$ of a test particle Z_*e from its original trajectory satisfies

$$\left(\frac{d\langle \tan^2(\Delta\theta/2) \rangle}{ds} \right)_C = 2\pi N \left(\frac{\mu ZZ_* e^2}{\hbar k} \right)^2 \ln \Lambda . \quad (41)$$

For the Debye-Huckel potential, Eqs. (34) and (40) yield

$$\left(\frac{d\langle \sin^2(\Delta\theta/2) \rangle}{ds} \right)_{DHB} = 2\pi N \left(\frac{\mu ZZ_* e^2}{\hbar k} \right)^2 \ln T . \quad (42)$$

The deflection in any given collision usually is quite small, so $\sin^2(\Delta\theta/2) \approx \tan^2(\Delta\theta/2) \approx (\Delta\theta/2)^2$ and therefore the derivatives in Eqs. (41) and (42) have

the ratio $\ln\Lambda/\ln T$, just as do the energy loss rates calculated for these two potentials.

A specific numerical example of results of the preceding paragraphs is given by a particle with $|Z_*| = 1$ scattering in the hydrogen plasma described in connection with Fig. 7; for that plasma, $\Gamma = 0.10$ and $\ln\Lambda = 4.0$. It follows from Eq. (40) that $\ln T = 2.4$ when $m_* = m_e$, and that $\ln T = 5.8$ when $m_* = (1836 m_e)$.

The test-particle concept is not applicable to collisions in plasmas having large Γ -values. However, if the collision involves a fast particle and a slowly moving (i.e. thermal) ion, the elastic scattering may be described reasonably well by Eq. (33), with $\phi = \phi_{IS}$ or ϕ_{TF} or ϕ_{AA} . There are no simple expressions for ϕ_{AA} or ϕ_{TF} at a finite temperature and density, but substitution of ϕ_{IS} into the Born formula, followed by some algebra, leads to the equation

$$\left(\frac{d\sigma}{d\Omega}\right)_{ISB} = \left(\frac{2\mu Z_* Z_n e^2 R_i^2}{\hbar^2}\right)^2 \left| \frac{1}{6} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell (2+\ell) (KR_i)^{2\ell}}{(2\ell+5)!} \right|^2, \quad (43)$$

$Z_n e$ being the nuclear charge of the screened target ion. Integration over solid angle yields the total elastic scattering cross section for the ion-sphere potential,

$$\sigma_{ISB} = \pi \left(2\mu Z_* Z_n e^2 R_i^2 / \hbar^2 \right)^2 Y(2kR_i), \quad (44)$$

where the function $Y(x)$, which is evaluated in Table II, decreases slowly from $Y(0) = 1/25$.

A general formula for the differential (Born) cross section for elastic scattering in a Thomas-Fermi potential can be written in terms of the

dimensionless variables U and ξ of Eqs. (10) and (11),

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{TFB}} = \left| \frac{2uZ_*Z_n e^2 \xi}{n^2 K} \int_0^\infty U(x) \sin(K\xi x) dx \right|^2. \quad (45)$$

For neutral atoms, the differential cross section is proportional to the square of a universal function of the product $K\xi$, which has been tabulated by Mott and Massey (1965, p. 462). Although it has not been done, it should be possible to treat elastic scattering from ions in plasmas by using in Eq. (45) the potential function U computed for finite matter temperatures and densities. Unfortunately, it seems that Eqs. (43) and (45) will not yield simple expressions for the rates of energy loss and deflection.

All the results of this section pertain to a non-degenerate plasma. Energy loss by charged particles moving in a Fermi gas has been studied extensively (e.g., Fermi and Teller, 1947; Ritchie, 1959; Peres and Ron, 1976), and will not be treated here. It is interesting to note, though, that in these very dense environments the logarithmic divergence of dE/ds due to small transfers in individual collisions is removed not by plasma screening (i.e., $\rho_{\text{max}} \rightarrow D$ in Eq. (36)), but by the fact that there is a minimum energy which a degenerate plasma can absorb, namely, the smaller of the energy needed to raise a scattering electron to an unoccupied portion of velocity space (on average, $1/3$ the Fermi energy), and the energy $\hbar\omega_i = (Z_i m_e / M_i)^{1/2} \hbar\omega_e$ of collective ion motions.

B. Inelastic Scattering

Very little is known about a plasma's influence on inelastic processes such as excitation, ionization, and rearrangement. However, for excitation collisions involving slow (i.e., thermal) ions, the qualitative

effects of screening by plasma electrons can be determined.

Consider the excitation of two isoelectronic systems, one neutral and the other charged, upon collision with an electron. A comparison of cross sections $\sigma(b,a)$ for a particular transition $b \rightarrow a$ shows how the scattering event is affected when the long-range Coulomb interaction is eliminated. This is done in Fig. 8a for the transition $(2p^3)^4S + (2p^3)^2P$ in $O^+ - e^-$ and $N - e^-$ collisions; to facilitate the comparison, the theoretical curves are plotted as a function of impact energy measured in units of the threshold energy ΔE for each system. The finite range of the electron-atom interaction causes the $N - e^-$ excitation cross section to vanish at threshold, and plasma screening will cause this same behavior in electron-ion excitation cross sections. In fact, the model potential study by Hatton *et al.* (1991) shows that even the magnitude of the effect is similar to that displayed in Fig. 8a. A few of the results of these authors, who used the first Born approximation to compute various excitation cross sections for electron scattering from a Debye-Hückel-shielded one-electron ion, are drawn in Fig. 8b.

Now, a particular stage of ionization of an element usually is prevalent when the plasma temperature is a few tenths of its ionization potential. A given excitation process $b \rightarrow a$ therefore is most likely to be important in a plasma when $0 \ll \Delta E(a,b)$. Under this condition, the thermal rate of excitation,

$$N_e C(b,a) \approx N_e (80/\pi n_e)^{1/2} \int_{\chi_0}^{\infty} \chi^{-1}(\chi) e^{-\chi} d\chi, \quad (46)$$

with $\chi = E/Q$ and $\chi_0 = \Delta E(a,b)/Q$, is very sensitive to changes in the cross section near threshold, and the effect of plasma screening is enhanced.

Burgess et al. (1978) devised an experiment to measure inelastic electron-atom collision rates in a hydrogen plasma. From an analysis of Balmer emission line data they concluded that inelastic collisions with excited ($n=2$) H atoms in their laser-pumped plasma had rates lower by a factor of at least 5 than predicted by calculations (for unscreened atoms). Detailed computations will be required to determine to what extent this surprising result can be attributed to the modest screening expected for neutral atoms in their plasma, wherein $D_e \approx 500 a_0$. In contrast to this result, Kunze et al., (1968) inferred, from C^{+4} spectral line intensities in a low density γ -pinch plasma ($N_e \sim 10^{15} \text{ cm}^{-3}$), carbon excitation rates in good agreement with values computed for unscreened interactions.

A comparison of collisional ionization cross sections for neutral and ionized, isoelectronic targets would not adequately illustrate the effects of plasma screening because in both cases the ejected electron interacts via a Coulomb potential with its parent ion and with the scattered charged particle.

V. FORMATION OF SPECTRAL LINES

Line radiation from dense plasmas is strongly affected by Stark broadening, i.e. pressure broadening by charged perturbers. Intensities of lines arising in dense plasmas also may be influenced by radiative transfer effects due to large optical depths. Moreover, the electric microfields in plasmas mix atomic states of different parity and thereby give rise to "forbidden components" of allowed transition lines. Before the present, brief discussions of these topics are given, it is helpful to review other broadening mechanisms relevant to the analysis of dense-plasma spectra. These

are Doppler and natural broadening, which occur in gases of any density and temperature.

Let $Q(\omega)d\omega$ be the power per unit volume emitted by a plasma in the (angular) frequency interval $d\omega$ about ω . In the vicinity of a spectral feature due to the atomic transition $a \rightarrow b$, that is, ω near $\omega_0 = \Delta E(a,b)/h$, it is*

$$Q(\omega)d\omega = \frac{4\omega^4}{3c} N_a I_{ab}(\omega)d\omega, \quad (47)$$

where N_a is the number density of radiating atoms, and where $I(\omega)$, the distribution of radiation intensity within the line, is called the line shape. $I(\omega)$ is proportional to the transition strength $S(a,b)$ (cf. Eq. 23). The Lorentz formula,

$$I_{ab}^{(L)}(\omega) = \frac{\gamma_L S(a,b)}{\pi \omega(a) [(\omega - \omega_0)^2 + \gamma_L^2]}, \quad (48)$$

describes natural broadening when $(2\gamma_L)^{-1}$ is the harmonic mean of the radiative lifetimes of states a and b ; note that $2\gamma_L$ also is the full width at half-maximum (FWHM) of this profile. For a Maxwellian distribution of ion speeds, the Doppler profile is

$$I_{ab}^{(D)}(\omega) = \frac{S(a,b)}{\gamma_D \omega(a) \sqrt{\pi}} \exp \left[- \left(\frac{\omega - \omega_0}{\gamma_D} \right)^2 \right], \quad (49)$$

*In the general case of overlapping lines, Eq. [47] includes a summation with respect to all relevant transitions; and if there is an underlying emission continuum, its contribution to $Q(\omega)d\omega$ needs to be added to Eq. [47].

where $\gamma_D = \omega_0 \sqrt{2Q_i/M_i c^2}$ is $(2\sqrt{2}n_2)^{-1}$ times the Doppler FWHM. As the plasma temperature increases, Doppler broadening becomes more and more important. This trend also is exhibited by natural broadening of lines from the higher charge states that occur in hot plasmas: the scaling is roughly $\gamma_L/\omega_0 \sim Z^2$.

Convolution of $I^{(L)}$ and $I^{(D)}$ yields the well known Voigt profile that describes a line shape influenced by both Gaussian and Lorentzian mechanisms. Properties of the Voigt function have been discussed by Armstrong (1967), and a useful estimate of the Voigt FWHM is (Allen, 1975, p.85)

$$2\gamma_V = \gamma_L + \sqrt{\gamma_L^2 + (4\sqrt{2}n_2)\gamma_D^2} \quad (50)$$

It is clear from the discussion of Sec. III. B.1 that the perturbation of an atom's eigenstates by surrounding plasma decreases radiative transition strengths and, hence, results in somewhat lower intensities I . At the same time, plasma screening decreases natural line widths. Doppler widths for radiating ions also can be reduced by a dense plasma environment: If the radiator's collision mean free path between large-angle deflections is much shorter than the wavelength of the emitted photon, Doppler narrowing occurs because some of the radiator's momentum can be collisionally transferred to other particles during the emission process (Dicke, 1953, Burgess *et al.*, 1979). In fact, the narrowing can exceed that estimated by Burgess *et al.* Their calculation employed a Coulomb mean free path, which is proportional to $(2n\lambda)^{-1}$. But, if a Debye-Hückel potential is used, the ions mean free path is smaller, being proportional instead to $(2n\lambda)^{-1}$ (cf. Eqs. 41 and 42). As yet, none of these plasma screening effects have been incorporated into calculations of spectral line shapes.

A. Stark Broadening

The current theory of Stark broadening was developed independently by Baranger (1958) and by Kolb and Griem (1958), and is treated in detail in the texts by Sobel'man (1972, chapt. 10) and by Griem (1974) and in the review article by Baranger (1962).

1. Basic Concepts

Stark broadening can be treated as a perturbation problem in quantum mechanics. The plasma surrounding a given radiating atom (or ion) introduces a time-varying interaction $V(t)$ into the atomic Hamiltonian. In terms of the eigenfunctions $\psi_i(t)$ of this Hamiltonian the definition of the line shape is (Sobelman, 1972, p. 402)

$$\begin{aligned}
 I_{ab}(\omega) &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} dt e^{i\omega t} \langle \psi_b(t) | \hat{d} | \psi_a(t) \rangle \right|^2 \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \phi(\tau), \quad (51)
 \end{aligned}$$

where $\hat{d}$ is the instantaneous electric dipole moment of the radiator, and where $\phi(\tau)$, Fourier transform of $I(\omega)$, is also the autocorrelation function of the dipole transition moment,

$$\phi(\tau) = \{ \langle \psi_a(0) | \hat{d} | \psi_b(0) \rangle \cdot \langle \psi_b(\tau) | \hat{d} | \psi_a(\tau) \rangle \}_{AV}. \quad (52)$$

In Eq. (52), which follows from the ergodic hypothesis for stationary random processes, $\{ \}_{AV}$ denotes an average with respect to the ensemble of plasma configurations. If state a or b is degenerate, or is one of several closely spaced states, then Eq. (52) becomes a statistically weighted sum of terms.

The standard evaluation of $\phi(r)$ employs Dirac's variation-of-the-constants method in which each $\psi(t)$ is expressed in terms of unperturbed atomic eigenfunctions $|i\rangle$ and coupling matrix elements $V_{ij}(t) = \langle i|V(t)|j\rangle$. In the quasi-static limit, $V(t)$ is essentially constant during the radiation process, while in the impact limit $V(t)$ is presumed to vary significantly during this time because of collisions between the radiator and rapidly moving charges. In principle, the quasi-static approximation is valid over that portion of the profile for which the distance to the line center is much greater than the inverse of the timescale for microfield fluctuations, $\Delta\omega \gg |\dot{\vec{F}}|/|\vec{F}|$, an inequality that can be rewritten as

$$\frac{\Delta\omega}{\gamma_D} \gg \frac{\Gamma}{\langle Z \rangle^2 a} \frac{Q}{\hbar\omega_0} . \quad (53)$$

Likewise, the impact approximation is strictly valid only for the central part of the profile well within the inverse of the typical collision duration, $\Delta\omega \ll [\langle v^2 \rangle / \sigma_a(\text{total})]^{1/2}$, where $\sigma_a(\text{total})$ is the total scattering cross section of atoms in state a . In practice, however, most Stark broadening calculations employ -- over the whole profile -- the impact approximation to treat electrons and the quasi-static approximation to treat ions. The complete Stark profile then is gotten by convoluting these two results. Smith *et al.* (1969) described a unified theory that treats the electron broadening over the whole profile in a more satisfactory way. It reduces to the impact limit in the core of the line, and to the quasi-static limit in the wings of the line. This theory, which was improved upon by Greene *et al.* (1975) and applied by them to hydrogenic ion lines, unfortunately requires even more computations than the impact theory.

The quasi-static autocorrelation function for an isolated line (neither

upper nor lower state degenerate) follows directly from Dirac's perturbation formulae, and when substituted into Eq. (51) yields

$$I_{ab}(\omega) = S(a,b) P(\Delta\omega) \delta(\omega - \omega_0 - \Delta\omega) / w(a), \quad (54)$$

where $\hbar\omega = V_{aa} - V_{bb}$ is the change in the transition energy, and $P(\Delta\omega) d(\Delta\omega)$ is the probability of occurrence of plasma configurations that give rise to a frequency shift $\Delta\omega$. Accordingly, the calculation of $I(\omega)$ in the quasi-static approximation reduces to the determination of the plasma microfield distribution, which was discussed in Sec. II.B, and the matrix elements of V , which will be presented later in this section.

The classical impact approximation, in which every collision just changes the phase of the radiating atom, usually is inadequate. The quantal formalism involves a lengthy derivation, and only the final result is quoted here. For an isolated line (cf. any of the general references cited above),

$$I_{ab}(\omega) = \frac{1}{\pi} \frac{\gamma_{ab} S(a,b) / w(a)}{(\omega - \omega_0 - \delta_{ab})^2 + \gamma_{ab}^2}, \quad (55)$$

where the Lorentzian half-width γ and the shift from line center δ both are defined in terms of elements of the complex scattering matrix S :

$$\gamma_{ab} - i\delta_{ab} = \{ 1 - \langle a | S^* | a \rangle \langle b | S | b \rangle \}_{Av} \quad (56)$$

In this equation, the asterisk denotes complex conjugation, and for impact broadening $\{ \}_{Av}$ represents an average with respect to collision speeds v , having a distribution $f(v)$, and collision impact parameters ρ ,

$$\{[...]\}_{Av} \rightarrow 2\pi N \int dv \int (v) v \int dp \rho [...] \quad (57)$$

When perturbations of the lower state can be ignored, as is often the case, then the FWHM $2\gamma_{ab} \rightarrow N\langle\sigma_a(\text{total})v\rangle$, the total rate of scattering by atoms in state a . A more general form of Eqs. (54) and (55) is needed to describe lines from hydrogenic ions, because of their states' degeneracy. The line shape formulae for these ions involves the trace, over the degenerate atomic states involved, of a particular product of electric-dipole, S-matrix, and density-matrix operators.

Almost all impact broadening calculations are based on impact parameter formulae and, up to the present time, screening effects on electron impact widths and shifts have been treated by restricting the impact parameters to values less than the electron Debye length D_e . It would be very interesting to compute widths and shifts from screened-Coulomb scattering matrix elements.

2. Quasi-static Shifts for Ion Resonance Lines

In order to determine quasi-static profiles, diagonal matrix elements of the plasma-radiator interaction V must be calculated. Here, attention is restricted to resonance lines of one- and two-electron ions because they are most commonly observed (e.g. Boiko, et al., 1973).

Consider a perturbing ion of charge $Z_j e$ at a distance R_j from the nucleus of a radiating ion; its interaction V_j with the radiator, whose active electron is at $\vec{r}$, can be expanded in a series of Legendre polynomials, P_j . The quasi-static shift of the transition energy due to all perturbing ions is just

$$\hbar\Delta\omega = -\sum_{\text{ions}} \left[\frac{Z_j e^2}{R_j} - \sum_{v=1}^{\infty} \langle a | (r/R_j)^v P_v(\cos\theta_j) | a \rangle \right], \quad (58)$$

since the ground state of a H- or He-like ion is spherically symmetric

For hydrogenic radiators the leading term ($v=1$) is non-vanishing, and the use of parabolic coordinates leads to the linear Stark formula

$$\begin{aligned} \hbar\Delta\omega &= \frac{3}{2} F (n_a/Z_n) (n_1 - n_2) e a_0 \\ &= \beta \left(\frac{n_a}{Z_n} \right) (n_1 - n_2) \left(\frac{N_e \langle Z_i^{3/2} \rangle / \langle Z_i \rangle}{6.15 \times 10^{21} \text{ cm}^{-3}} \right)^{2/3} \text{ eV}, \end{aligned} \quad (59)$$

where $F = \beta F_0$ is the strength of the net ionic field, n_a is the principal quantum number of the upper state, $Z_n e$ is the nuclear charge of the radiator, and $(n_1 - n_2) = 0, \pm 1, \pm 2, \dots, \pm (n_a - 1)$ is the difference of two parabolic quantum numbers. Each of the $(2n_a - 1)$ Stark substates of $|a\rangle$ is perturbed by a different amount. The complete profile of the resonance line n_a+1 , which has an approximate FWHM of $2\gamma_S \approx [3F n_a (n_a - 1) e a_0 / Z_n]$, consists of overlapping components, symmetrical about $\Delta\omega = 0$.

This familiar picture becomes invalid in very dense plasmas: Plasma screening and, for high-Z radiators, fine-structure interactions lift the orbital degeneracy of the hydrogenic states; moreover, the ion-quadrupole interaction (the $v = 2$ term in Eq. 58) is no longer negligible. The complicated, asymmetrical Stark patterns that can result from all these simultaneous perturbations have been discussed by Weisheit and Rozsnyai (1976), and one of their examples is reproduced in Fig. 9.

Because there are only accidental degeneracies among the various atomic states of two-electron systems, their Stark profiles usually differ considerably from those of one-electron systems. First, the spectrum lines are isolated, and this means that the one-term formula (55) can be used for the impact broadening. Second (and unfortunately), the quasi-static shifts do not depend linearly on the ion microfield. Instead, the Stark shift has a first-order term proportional to the gradient $H = -1/2\hat{u} \cdot (\partial \vec{F} / \partial \mathbf{u})$, where $\hat{u}$ is a unit vector in the direction of $\vec{F}$, plus a second-order (quadratic-Stark) term proportional to F^2 ,

$$\hbar\Delta\omega = B_3(a) H + B_4(a) F^2. \quad (60)$$

(The coefficients B_3 and B_4 , which in the nearest neighbor approximation multiply terms proportional to R^{-3} and R^{-4} , have been given by Weisheit and Pollock, 1980). Even in this situation, when broadening of the lower state can be neglected, the quasi-static profile, Eq. (54), still depends upon the joint microfield distribution of F and H , a difficult quantity to determine (cf. Chandrasekhar and von Neumann, 1942; Weisheit and Pollock, 1980).

3. Measured and Computed Line Shapes

Griem, Kolb, and Shen (1959) first applied the quantal impact theory to the calculation of hydrogen line profiles. As extensive tabulation of H^0 and He^+ results, in which only the linear Stark term is considered, has been published by Griem (1974, Appendix I). Sholin (1969; see also Demura and Sholin, 1975) extended the Stark profile formulae to include the ion-quadrupole term of Eq. (58), and used field-strength distributions for independent particles to determine quasi-static profiles in hydrogen plasmas

with $N_e \gtrsim 10^{18} \text{ cm}^{-3}$. Bacon (1973, 1977) added the quadratic-Stark correction, and then calculated complete hydrogen Stark profiles. In the line wings, his results for Lyman- α are in accord with the measurements of Fussman (1975), but for the Lyman- β line the agreement is not as good. However, it does appear that the higher order Stark terms are responsible for the small, observed line asymmetries.

A more serious discrepancy between theory and experiment was uncovered by Grützmacher and Wende (1977). They measured the core of the Lyman- α profile in plasmas having $N_e \gtrsim 10^{17} \text{ cm}^{-3}$, $T \approx 1 \text{ eV}$ and found FWHM almost twice as large as predicted values. It has been argued that relative motion between the radiator and perturber ions (Voslamber, 1977) and/or fluctuations in the number of screening electrons within a given ion's Debye sphere (Griem, 1979) cause the disagreement, but as yet there are too few comparisons between measured and computed profiles to identify with certainty additional broadening mechanisms.

Recently, Griem, Blaha and Kepple (1979) computed a grid of linear Stark fractional widths (full-widths-at-1/n-maximum, with $n=2, 4$ and 8) for several one-electron ions in dense, hot plasmas. Also, numerous laboratories have published shapes of lines emitted by hydrogenic ions in laser-heated plasmas (cf. Yaakobi et al., 1977; Key et al., 1979), and then matched them to computed profiles in order to infer plasma densities. However, it is not known how accurate the theoretical profiles actually are because there were no independent means of determining N_e . Until such additional information is obtained, plasma densities inferred from Stark-broadened hydrogenic ion lines are uncertain, and may be accurate only to within a factor of 2 or so.

There exist just a few studies of line broadening for He-like ions in hot, dense plasmas, and much work remains to be done on this subject. Lee,

Bromage, and Richards (1979) performed calculations for two-electron silicon and argon resonance lines. They considered the ion-quadrupole interaction but not the quadratic Stark term, which usually is comparable at very high densities. Weisheit and Pollock (1980) computed quasi-static profiles for He-like neon and argon resonance lines that include both of these Stark interactions, but then convoluted the results with Voigt profiles in order to approximate complete line shapes. Boiko *et al.* (1977) measured the positions and intensities of intercombination lines of several highly-stripped, two-electron ions, but did not published individual line profiles.

B. Forbidden Components and Plasma Satellites

It frequently happens that excited atomic states of different parity but the same multiplicity lie close to one another. These states are mixed by the microscopic electric fields in the plasma. Suppose, as shown in Fig. 10A, that there are only two adjacent states, and that in vacuo the transition $a \rightarrow b$ is allowed but the transition $\tilde{a} \rightarrow b$ is (dipole) forbidden. In the limit of degenerate states a and $\tilde{a}$, or in the limit of strong electric fields, a hydrogenic profile would be observed. Under less extreme conditions, a forbidden component ($\tilde{a} \rightarrow b$) may be observed as a separate feature adjacent to the line $a \rightarrow b$ (cf. Fig. 10B).

Most of the early work on forbidden components related to closely spaced levels in the spectrum of neutral helium (see, e.g., Griem, 1968). However, the large microfields arising in dense plasmas can strongly couple states that are quite widely separated, and X-ray spectra may show forbidden components. It is straightforward to calculate mixing coefficients and, from them, separations and relative intensities of allowed and forbidden components in the presence of a quasi-static field $\tilde{E}$. For the two-state case, first-

order perturbation theory yields the formulae

$$(E_a^{(1)} - E_{\tilde{a}}^{(1)}) = [(E_a^{(0)} - E_{\tilde{a}}^{(0)})^2 + 4V_{a\tilde{a}}^2]^{1/2}, \quad (61)$$

$$\frac{I_{ab}^{\sim}}{I_{ab}} = \frac{\int I_{ab}^{\sim}(\omega) d\omega}{\int I_{ab}(\omega) d\omega} = \left(\frac{V_{a\tilde{a}}^{\sim}}{E_a^{(0)} - E_{\tilde{a}}^{(0)}} \right)^2, \quad (62)$$

where the $E^{(0)}$'s are unperturbed eigenenergies and where

$$V_{a\tilde{a}}^2 = |\langle a | \hat{d} \cdot \hat{E} | \tilde{a} \rangle|^2 = \frac{1}{3} F^2 S(a, \tilde{a}) / w(a) \quad (63)$$

is the square of the coupling matrix element. Since $F \sim N_e^{2/3}$, the lines' separation and the relative strength of the forbidden component both increase with density, and therefore may be useful as plasma diagnostics.

The 2^1S and 2^1P states of He-like ions, whose unperturbed energy separation is approximately $0.0019 Z_n^2 e^2 / a_0$, provide a numerical example of forbidden-component line strengths. For highly stripped ions in dense plasmas, the nuclear charge $Z_n e \approx (Z_n - 1)e$, the charge seen by the active electron, and also the microfield typically has a strength near the Holtsmark normal value, $F \approx F_0$ (cf. Fig. 3). Under these circumstances, if hydrogenic eigenfunctions are used to evaluate $V_{a\tilde{a}}^{\sim}$, the intensity ratio of the forbidden component and the allowed line is approximately (Weisheit et al., 1975)

$$\frac{I(2^1S \rightarrow 1^1S)}{I(2^1P \rightarrow 1^1S)} \approx \left(\frac{\langle Z_n \rangle}{Z_n} \right)^{2/3} \left(\frac{N_e}{6 \times 10^{19} Z_n^4 \text{ cm}^{-3}} \right)^{4/3}. \quad (64)$$

Two problems complicate the actual measurement of forbidden-component features in the spectra of hot, dense plasmas. First, significant Stark

broadening of an allowed line can cause it to overlap its forbidden components. Second, transition probabilities for intercombination lines, e.g., $2^3P - 1^1S$, increase rapidly with nuclear charge because of the breakdown of L-S coupling; at large Z_n - values these lines are strong (Boiko *et al.*, 1977) and they too may obscure forbidden components. (Indeed, there may even be forbidden components of the intercombination lines.) High resolution spectra from plasmas with $N_e > 10^{23} \text{ cm}^{-3}$ will be required to identify these various features due to radiating ions with $Z_n \sim 10$ to 20.

There is a bound-free radiative process analogous to the formation of forbidden components of bound-bound transitions. Some doubly-excited ionic states are unable to autoionize to a (degenerate) continuum state because of their parity; in vacuo such states therefore decay via spectral line emission. In certain instances these states can be efficiently mixed, by plasma microfields, with adjacent doubly-excited states that do autoionize rapidly. Davis and Jacobs (1975) first investigated this subject and they found, for example, that the He-like ion line arising from the transition $(2p^2)^3P \rightarrow (1s2p)^3P$ would be completely quenched by this mixing if the plasma density exceeded about 10^{20} cm^{-3} . This phenomenon can have a large effect on dielectronic recombination rate coefficients (see Sec. VI.).

As pointed out by Baranger and Mozer (1961), the interaction of an atomic dipole and plasma oscillations at a given frequency ω' can produce additional spectral features in the vicinity of an allowed line and a nearby forbidden line: A second order process involving virtual atomic states results in plasma satellite lines displaced by an amount $\pm \hbar\omega'$ from the energy of the forbidden line; this is illustrated in Figs. 10C and 10D, again for the simple case of two adjacent excited states. The relative strengths of the satellites in this instance are

$$\frac{I_{\pm}}{I_{ab}} = \frac{\langle F^2 \rangle S(a, \tilde{a})}{6w(a) [\Delta E(a, \tilde{a}) \pm \hbar\omega]^2}, \quad (65)$$

where $\Delta E(a, \tilde{a}) = E_a^{(0)} - E_{\tilde{a}}^{(0)}$. If the wave field F is due to thermal oscillations at the (electron) plasma frequency ω_e , then $\hbar\omega = \hbar\omega_e \ll \Delta E(a, b)$ for resonant X-ray transitions, even when $N_e \sim 10^{23} \text{ cm}^{-3}$ (cf. Eq. 3). In consequence, the satellite lines and the forbidden component have a characteristic intensity ratio given by Eqs. (62), (63), (65), and (16),

$$\frac{I_{\pm}}{I_{ab}} \approx \frac{F_w^2}{2 F_o^2} = \frac{0.15 \Gamma}{\langle Z_1 \rangle^3}. \quad (66)$$

Plasma satellites therefore are expected to be weak features in dense plasma X-ray spectra, unless there exist strong, non-thermal oscillations.

Lee (1979) has calculated shapes of satellite lines in non-equilibrium plasmas, and several experimental results have been summarized by Bekefi et al., (1976). Recently, Drawin and others (cf. Drawin and Ramette, 1979) have argued that most observed features which previously were identified as plasma satellites in fact are weak molecular transitions. This point remains to be resolved; however, new hydrogen and helium spectra obtained by Kunze and his collaborators (Finken et al., 1980; Hildebrandt and Kunze, 1980) exhibit features that probably are due to the interaction of radiating atoms and plasma oscillations.

C. Optical Depth Effects

All the spectral line formulae presented above were derived under the assumption that the emitting gas is optically thin to its own radiation. However, even though their dimension is small, $s \leq 10^{-2} \text{ cm}$, very dense

laboratory plasmas can have significant optical depths at resonance line frequencies. When this happens, spectral line radiation is strongly modified, a fact evidenced in several recent studies involving ICF plasmas (Irons, 1975; 1976; Weisheit et al., 1975; Key et al., 1979; Duston and Davis, 1980). The theory of radiation transfer in optically thick lines (and these lines' shapes) is complicated, and is beyond the scope of this chapter (see, e.g., Athay, 1972; Ivanov, 1973). On the other hand, one often is interested only in the total amount of radiant energy transported in a given line, or the net rate of radiative decay (emission minus resonance absorption), and in these instances there are simple but useful ideas concerning integrated line emission, $Q_{ab} = \int ds \int d\omega Q_{ab}(\omega)$, and effective transition probabilities; these will be described here.

An atom's cross section for absorption of radiation in the interval $(\omega, \omega + d\omega)$ near one of its transition frequencies $\omega_0 = \Delta E(a,b)/\hbar$ is

$$\begin{aligned} \sigma_{ba}(\omega) d\omega &= (2\pi\omega_0/3\hbar c) I_{ab}(\omega) d\omega \\ &= (\pi e^2 f_{ba}/m_e c) \psi_{ab}(\omega) d\omega, \end{aligned} \quad (67)$$

where, as before, f is the oscillator strength, and where

$$\psi_{ab}(\omega) = I_{ab}(\omega) / \int I_{ab}(\omega) d\omega \quad (68)$$

is the normalized emission-line profile function.* The corresponding optical

* Absorption and emission profiles are essentially the same when spectral line shapes are dominated by Stark broadening, because then the "complete frequency redistribution" approximation is valid (Edmonds, 1955; Hummer, 1962).

depth for a gas of thickness s , having N_b absorbing atoms per unit volume, is

$$\tau_{ba}(\omega) = N_b s \sigma_{ba}(\omega) . \quad (69)$$

Usually, it is convenient to characterize the plasma's absorptivity in the vicinity of a spectral line by the optical depth at that line's center, $\tau_0 \equiv \tau_{ba}(\omega = \omega_0)$.

Consider a transition line photon created at a particular depth τ_0 from the surface of a plasma. This photon may suffer several resonance scatterings (absorptions and subsequent re-emissions) before it escapes the plasma, or it may be "destroyed" during a particular scattering if the excited atom suffers an inelastic collision or emits a different frequency photon. Yet a third possibility is that the photon escapes without scattering at all. Clearly, this last contingency is likely only if the photon is created in the wings of the line, where $\tau_{ba}(\omega) \ll 1$. Therefore, for a given line shape ψ , a good estimate of the probability of direct escape is (Osterbrock, 1962)

$$P_{esc}(\tau) = \int_{x_1}^{\infty} [\psi(x) + \psi(-x)] dx, \quad (70)$$

where $x = \omega - \omega_0$ is the frequency displacement from line center, and where x_1 satisfies $\tau_0 \psi(x_1) = \psi(0)$. (For clarity, subscripts identifying the transition have been omitted.) A precise definition of P_{esc} involving exponential integral functions has been given by Hummer (1964).

Escape probabilities provide a means of approximating the effect of resonance absorption on atomic level populations in optically thick media (see, for example, McWhirter, 1965). Specifically, it has been shown that

when the upper and lower level populations can be assumed to be uniform throughout the plasma, the effective probability of radiative transitions between them is just the Einstein coefficient A_{ab} times the photon escape probability for that line, viz.

$$A_{ab}(\tau_0) = A_{ab} P_{esc}(\tau_0) . \quad (71)$$

Photon escape probability formulae for Doppler, Lorentz, and Voigt line shapes have been known for some time (cf. Athay, 1972, p.22). Recently, Weisheit (1979) used the nearest-neighbor approximation (Eq. 14) to derive an expression for the probability of photon escape in the wings of linearly-Stark-broadened resonance lines ($np+1s$),

$$P_{esc}(\tau_0) = 3n \left(\frac{2\psi(0)F_0}{3\tau_0 Z_n} \right)^{3/5} \Delta_n^{2/5} , \quad (72)$$

where F_0 is the normal ion microfield strength and $\Delta_n \approx 0.04n$ is a given function of parabolic quantum numbers. To date, there are no published escape probability formulae for Stark-broadened lines of non-hydrogenic ions; the Voigt result,

$$P_{esc}(\tau_0) = (2/\pi)^{3/4} (\gamma_L/\tau_0 \gamma_D)^{1/2} , \quad (73)$$

probably is best to use for these lines. (This equation is valid only if the ratio of half-widths $\gamma_L/\gamma_D < 1$.)

In order to estimate an optically thick plasma's line emission, let $\langle X(\tau_0) \rangle$ be the mean number of scatterings experienced by photons, in a particular transition line $a \rightarrow b$, that escape the plasma from their point of

creation at a depth τ_0 ; and let ϵ_{ab} be the probability per scattering that such photons are destroyed. In terms of NC_a^{tot} and A_a^{tot} , the total rates of inelastic collisional and radiative transitions from the excited state, ϵ_{ab} is defined as

$$\epsilon_{ab} = (N C_a^{\text{tot}} + A_a^{\text{tot}}) / A_{ab} - 1. \quad (74)$$

In the absence of any continuum opacity source, the probability that a given photon, once created, ultimately is emitted by the plasma is

$$P_{em}(\tau_0) = (1 - \epsilon_{ab})^{\langle X(\tau_0) \rangle} = \exp [-\epsilon_{ab} \langle X(\tau_0) \rangle], \quad (75)$$

the second expression being correct in the limits of large X -values and small ϵ -values. It follows directly from Eqs. (47) and (75) that the integrated line emission from the plasma surface (energy/area/time) is

$$Q_{ab} = \int ds(\tau_0) P_{em}(\tau_0) \int d\omega Q_{ab}(\omega, s), \quad (76)$$

with Q , the power per unit volume and frequency interval, depending upon depth through the local density of emitters $N_a(s)$ and perhaps also the intrinsic shape of the line. If these dependences are known a priori or can be estimated, then Q can be determined once $\langle X(\tau_0) \rangle$ is specified.

Hummer (1964) derived approximate expressions for the mean number of scatterings when the line has a Doppler, Lorentz, or Voigt profile. Hummer and Kunasz (1980) performed numerical calculations for lines with a Voigt profile; a slab geometry was adopted, and both mid-plane and uniformly distributed photon sources were considered. Some of their results have been

plotted in Fig. 11. Over a wide range of Voigt parameters, $10^{-5} \lesssim (\gamma_L/\gamma_D) \lesssim 1$, $\langle X(\tau_0) \rangle$ in both cases lies between 1/2 and 2 times the quotient $(\tau_D/\psi(0))$. Values of $\langle X(\tau_0) \rangle$ have not yet been determined for strongly Stark-broadened lines, but this problem is worth investigating.

VI. DIELECTRONIC RECOMBINATION

The comment was made in the Introduction that level populations of ions in dense laboratory plasmas may be far from their equilibrium (LTE) values. Due to the transient nature of these plasmas, ionization and recombination events, as well as radiative and collisional transitions between bound states of the same ionization stage, can strongly influence the densities $N_Z(a)$. Effects of the plasma environment on rates of various bound-bound transition processes already have been discussed. Also, it has been noted that there is no specific information on collisional ionization rates in dense plasmas, but for photoionization rates changes due to plasma screening are not expected to be large (cf. Section II. B.2). These two statements also are true, respectively, for the inverse processes, three-body recombination and radiative recombination. For the latter process, the comment follows in part from the fact that radiative recombination predominantly is to the ion's lower levels, which are its less perturbed ones.

In contrast, dielectronic recombination tends to populate highly excited ionic states (see, for example, Shore, 1969), and so rates for this process in dense plasmas can vary significantly from vacuum values. To see how dielectronic recombination of a $(Z+1)$ -times ionized atom, denoted E_{Z+1} , and a free electron of energy E_e is altered by a high density environment, recall that in the low-density limit it involves the following sequence of processes.

$$\text{Resonance capture: } \Xi_{Z+1}(b) + e^-(E_e) \rightarrow \Xi_Z(a^{**}); \quad (77)$$

$$\text{radiative stabilization: } \Xi_Z(a^{**}) \rightarrow \Xi_Z(a^*) + \hbar\omega; \quad (78)$$

$$\text{radiative cascade: } \Xi_Z(a^*) \rightarrow \Xi_Z(\tilde{a}^*) + \hbar\omega. \quad (79)$$

(For convenience, singly- and doubly-excited states have been identified in these formulae by one and two asterisks.) The rate coefficient for a dielectronic capture that leaves the recombined ion in the singly-excited state a^* (processes (77) plus (78)) is usually written in terms of the autoionization probability of state a^{**} , the inverse of process (77) (Burgess, 1964),

$$\kappa_{Z+1}(a^*) = \left(\frac{2\pi\hbar^2}{m_e \lambda} \right)^{3/2} \frac{w(a^{**})}{w(b)} \exp\left(\frac{-E_e}{\lambda}\right) \frac{A_{\text{auto}}(a^{**}) A_{\text{rad}}(a^{**})}{A_{\text{auto}}(a^{**}) + A_{\text{rad}}(a^{**})}. \quad (80)$$

Here, the w 's are the statistical weights of the initial and final states and A^{rad} is the sum of the probabilities of stabilizing radiative transitions. The total dielectronic rate coefficient κ_{Z+1} is obtained by summing Eq. (80) with respect to all single-excited states a^* of the recombined ion. Seaton and Storey (1976), and Dubau and Volante (1981) have summarized the information on dielectronic recombination at low densities.

Burgess and Summers (1969) first described how collisional processes at high densities can enhance the dielectronic process by de-exciting the states a^{**} and by changing these states' angular momentum quantum numbers so as to impede the autoionization rate; in addition, collisions can reduce dielectronic recombination by ionizing the states a^{**} . These effects were

investigated by Weisheit (1975), who pointed out that continuum lowering also reduces the dielectronic rate because it removes many excited states from the summation $\sum \kappa_{Z+1}(a^*)$. A sample of results obtained from the computer code described by Weisheit (1975), which uses unscreened radiative and collisional rate coefficients, are plotted in Fig. 12 for 100 eV neon plasmas of different densities. The kink in the curve for $N_e = 10^{21} \text{ cm}^{-3}$ is due to collisions which stabilize $N_e^{+8}(1s n \ell)$ ions with angular momentum $\ell \geq 8$ against autoionization. One important effect that was not included in these numerical calculations is the Stark mixing of autoionizing and non-autoionizing states by the quasi-static ion microfield. This process, which tends to increase the rate coefficient for dielectronic recombination, already has been mentioned in the discussion of forbidden components (Section V.B).

It has been known for some time (cf. Gabriel and Paget, 1972) that the stabilizing radiative transitions (78) are a major source of satellite lines situated on the long-wavelength side of X-ray resonance lines. Peacock et al. (1973) observed that in dense laser-plasmas the intensities of certain satellite lines increase rapidly with ion charge. This was explained by Weisheit (1975) as being due to the fact that collisional rates scale roughly as Z^{-3} while radiative rates scale as Z^{+4} , and therefore that the satellite lines from ions of lower net charge were more readily quenched at a given density. Other interesting effects of high density on X-ray satellite line intensities have been explored in some detail by Vinogradov et al., (1977) and by Jacobs and Blaha (1980). In particular, the possible uses of these lines' intensities as density diagnostics were considered (see also Seely, 1979). Recently, Duston and Davis (1980) studied optical depths effects relevant to the analysis of satellite X-ray line spectra.

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TABLE I.*

Critical screening lengths $(Z + 1) \hat{D}/a_0$ for one-electron eigenstates $|n\ell\rangle$

n/ℓ	0	1	2	3	4	5	6	7	8
1	0.8399								
2	3.223	4.541							
3	7.171	8.872	10.947						
4	12.687	14.731	17.210	20.068					
5	19.772	22.130	24.985	28.257	31.904				
6	28.423	31.079	34.285	37.950	42.018	46.458			
7	38.64	41.581	45.122	49.159	53.630	58.500	63.730		
8	50.44	53.641	57.501	61.894	66.752	72.028	77.691	83.720	
9	63.81	67.258	71.426	76.162	81.392	87.064	93.143	99.604	106.43

*From Rogers, Graboske, and Harwood (1970).

Table II.

The function $Y(x)$ for elastic scattering from an ion-sphere potential.

x	$Y(x)$	x	$Y(x)$
0.0	0.0400	7.5	0.0310
0.5	0.0393	10.0	0.0286
1.0	0.0386	12.5	0.0265
1.5	0.0379	15.0	0.0246
2.0	0.0373	17.5	0.0230
3.0	0.0360	20.0	0.0214
4.0	0.0348	22.5	0.0201
5.0	0.0336	25.0	0.0188

Figure Captions

Fig. 1 Density and temperature regimes of some natural and man-made plasmas. The two dashed lines show (N_e, θ) -values corresponding to different constant values of the Coulomb parameter Γ (Eq. 1) for a hydrogen plasma.

Fig. 2 The effective charge number Z_{eff} seen by a bound electron in a Ne^{+8} ion that is imbedded in a neon plasma having $\theta = 100$ eV and $N_{\text{Ne}} = 10^{21} \text{ cm}^{-3}$. The ion-sphere radius is $R_i = 11.8 a_0$ and the Debye length is $D = 5.24 a_0$. DH, IS, TF, and AA denote results for a Debye-Huckel, ion-sphere, Thomas-Fermi, and (Hartree-Fock-Slater) average-atom model potential.

Fig. 3 Distributions of the normalized electric microfield $\beta = F/F_0$ (Eq. 12). The solid and dashed curves corresponding to the case when the Coulomb parameter $\Gamma = 0$ are, respectively, the Holtsmark (1919) and (uniform-density) nearest-neighbor results. The curve marked $\Gamma = 0.2$ is from Hooper (1968), and that marked $\Gamma = 2.0$ is from Weisheit and Pollock (1980).

Fig. 4 The Thomas-Fermi continuum depression ΔE_{TF} for an ion of net charge Ze in a plasma with $\langle Z_i^2 \rangle / \langle Z_i \rangle = \tilde{Z}$. The energy unit is $\Delta E_{\text{SP}} = (3/2) \{ [(R_i/D)^3 + 1]^{2/3} - 1 \} (D/R_i)^3 (Ze^2/D_0)$ (From Stewart and Pyatt, 1966). When $(R_i/D) \ll 1$, $\Delta E_{\text{SP}} \rightarrow \Delta E_{\text{DH}}$; when $(R_i/D) \gg 1$, $\Delta E_{\text{SP}} \rightarrow \Delta E_{\text{IS}}$.

Fig. 5 Measured plasma polarization shifts $\Delta\lambda$ for the He^+ principal

series. All values have been scaled linearly with N_e to an electron density of 10^{18} cm^{-3} . (ϕ): Greig et al., 1970. ($\bar{\Pi}$): Gabriel and Volonte, 1973. (I) and ($\bar{\Gamma}$): Goto and Burgess, 1974. ($\clubsuit$): Neiger and Griem, 1976. The theoretical curve is the prediction of Burgess and Peacock (1971).

Fig. 6 The ratio S_{DH}/S of Debye-Huckel-screened to unscreened transition strengths for the principal series in a hydrogenic ion of nuclear charge $(Z + 1)e$. As an aid to the eye, curves have been drawn to connect values for a given screening length D .

Fig. 7 The Gaunt factor G for free-free emission by an electron of initial momentum $\hbar k_1 = 5\hbar/\alpha_0$ scattering in a hydrogen plasma with $\theta = 340$ eV and $N_e = 3.4 \times 10^{24} \text{ cm}^{-3}$; the electron's final momentum is $\hbar k_2$. B denotes the Born approximation to G for scattering in a Coulomb potential, and BE , the Born approximation with Elwert's correction. DHB and ISB denote, respectively, the Born approximations for scattering in Debye-Huckel and ion-sphere potentials. The two filled circles denote values of the (Coulomb) Gaunt factor, as computed by Karsas and Latter (1961).

Fig. 8 (a) The computed electron impact excitation cross section (Henry, et al.; 1969) for the transition $(2p^3)^4S \rightarrow (2p^3)^2P$ in N and O^+ . When the nitrogen cross section is scaled to agree at high energies with the oxygen cross section, the dashed curve shows its low energy behavior. For nitrogen, the threshold energy is $\Delta E = 3.58$ eV; for oxygen, it is $\Delta E = 5.02$ eV. (b) The computed electron impact excitation cross section for the transition $1s \rightarrow 2s$ in a one-electron ion of nuclear charge $(Z+1)e$. The

Debye-screened ion results are the Born calculations of Hatton et al., (1981), and the unscreened ion result ($D \rightarrow \infty$) is the Coulomb-Born calculation of Oh et al., (1978). The threshold energy is $\Delta E = 10.2 (Z + 1)^2$ eV.

Fig. 9 Lyman- α Stark patterns for neon (Ne^{+9}) ions in homogeneous neon plasmas of two different ion densities $N_1(\text{cm}^{-3})$; in each case, the ion field strength is taken to be F_0 . The half-width $\gamma_S(e^2/Q_0)$ of the linear Stark pattern is the unit of energy, and the crosses indicate the Lyman- α fine-structure components' positions and intensities in the low-density limit. Full and broken lines show results obtained with and without taking into account ion-sphere screening of the radiating ion. The zero-point of energy is the position of the unperturbed Lyman- α line, and the direction of increasing transition energy is to the right. (From Weisheit and Rozsnyai, 1976.)

Fig. 10 (A) Schematic energy level diagram of real states giving rise to a forbidden component ($\tilde{a} \rightarrow b$) of the allowed line ($a \rightarrow b$); (B) resulting spectrum. (C) Schematic energy level diagram of real and virtual ($-$, $+$) states giving rise to a forbidden component of the allowed line and to plasma satellite lines ($\{-\} \rightarrow b$ and $\{+\} \rightarrow b$); (D) resulting spectrum.

Fig. 11 Mean number of scatterings $\langle X \rangle$ of resonance line photons escaping from a plasma slab of total optical thickness $\tau_0 = T\psi(0)$. A Voigt profile ψ was used, with widths γ as shown. Full curves are results for a mid-plane source, and dashed curves, those for a uniformly distributed source. (Hummer and Kunasz, 1980).

Fig. 12 Rate coefficients $\langle \kappa_0(n) \rangle$ [$\text{cm}^3 \text{sec}^{-1}$] for dielectric recombination to excited states ($1s n\ell$) of Ne^{+8} in a 100 eV plasma. The logarithm of the plasma electron density N_e (cm^{-3}) is shown by each curve; at densities $N_e \geq 10^{20} \text{ cm}^{-3}$, continuum lowering removes all but the few states indicated.

Fig. 1. (GILLES 1979)

Fig. 1. (P-P1-802422)

FIG. 1. (PPPL-802421)

Fig. 4. (PEPL-892424)

Fig. 5. (UFL-40/417)

Fig. 6. (Hill 1964b)

Fig. 7. (PPPL-802455)

FIG. 2. (continued)

Fig. 9.

Fig. 10.

81T0029

Fig. 11.

#81T0028