Analysis of Production Decline in Geothermal Reservoirs

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I. INTRODUCTION AND CONCLUSIONS

Objectives and Rationale

The major objectives of the Decline Curve project were to

1. Test the decline analysis methods used in the petroleum industry on geothermal production data,

2. Examine and/or develop new analysis methods,

3. Develop a standard operating procedure for analyzing geothermal production data.

Various analysis methods have long been available but they have not been tested on geothermal data because of the lack of publicly available data. The recent release to publication of substantial data sets from Wairakei, New Zealand, Cerro Prieto, Mexico and The Geysers, U.S.A. has made this study possible. Geothermal reservoirs are quite different from petroleum reservoirs in many ways so the analysis methods must be tested using geothermal data.

Data and Analysis Methods

Data and analysis methods were gathered from the petroleum, geothermal, and hydrological literature. The data sets examined include

1. Wairakei, New Zealand - 141 wells
2. Cerro Prieto, Mexico - 18 wells
3. The Geysers, U.S.A. - 27 wells
4. Larderello, Italy - 9 wells and groups
5. Matsukawa and Otake, Japan - 8 wells
6. Olkaria, Kenya - 1 well

The analysis methods tested were

1. Arps's equations
2. Fetkovich type curves
3. Slider's method for Arps
4. Gentry's method for Arps
5. Gentry's & McCray's method
6. Other type curves
7. P/z vs. Q method
8. Coats' influence function method
9. Bodvarsson's Linearized Free Surface Green's Function method

Conclusions

The conclusions are

1. The exponential equation fit is satisfactory for geothermal data.
2. The hyperbolic equation should be used only if the data fit well on a hyperbolic type curve.
3. The type curve methods are useful if the data are not too scattered. They work well for vapor dominated systems and poorly for liquid dominated systems.
4. Coats' influence function method can be used even with very scattered data.
5. Bodvarsson's method is still experimental but it shows much promise as a useful tool.
II. THEORY OF RESERVOIR DECLINE MODELS

(1) Decline mechanism

A geothermal reservoir has essentially three capacitances, (1) fluid/rock compressibility, (2) free liquid surface mobility and (3) reservoir liquid vaporization. In essence, item (3) is also a compressibility effect similar to (1). In this section, we will very briefly review in a semi-quantitative manner, the relative magnitudes of the effects listed above.

Consider a reservoir consisting of a slab of thickness H and of a large horizontal extent. The porosity/permeability can be of the fracture or intergranular type but is assumed to be sufficiently homogeneous that an average porosity \( \phi \) and capacity \( s \) (storage coefficient) can be defined.

On these premises, we find that lowering the pressure by \( \Delta p \) in a vertical column of unit area, releases because of compressibility a total liquid mass of

\[
\Delta q_c = \rho s H \Delta p
\]

(1)

where \( \rho \) is the density of the liquid. We can then define a specific release per unit area of

\[
dq_c/dp = \phi s H
\]

(2)

Let \( g \) be the acceleration of gravity. Lowering the pressure by \( \Delta p \) corresponds to a lowering of the free liquid surface by \( \Delta p/\rho g \). Hence, for the same \( \Delta p \), the free surface releases a total of

\[
\Delta q_f = \phi \Delta p/\rho g = \phi \Delta p/g,
\]

(3)

and then the specific release

\[
dq_f/dp = \phi /g
\]

(4)

Finally, we consider the effect of intergranular vaporization. Let \( \rho_s \) be the density of the vapor, \( L \) the latent heat of vaporization of the liquid and \( T \) the temperature in kelvins. The Clausius-Clapeyron equation for the liquid is then approximately

\[
(dp_s/dT)_\nu = \rho_s L/T
\]

(5)

where \( \rho_s \) is the vapor pressure along the saturation line that is denoted by the subscript \( \nu \). Hence, assuming saturation conditions, the lowering of the pressure by \( \Delta p \) lowers \( T \) by

\[
\Delta T = T \Delta p/\rho_s L
\]

(6)

and the release of heat per unit volume of the wet formation is

\[
\Delta h = \rho CT \Delta p/\rho_s L
\]

(7)

where \( \rho \) is the density and \( C \) is the heat capacity of the wet formation. The release of vapor is then

\[
\Delta q_v = \Delta h/L = \rho_c CT \Delta p/\rho_s L^2,
\]

(8)

and we can thus define a specific rate per unit area of a slab of thickness \( H \)

\[
dq_v/dp = \rho_c CT H/\rho_s L^2.
\]

(9)

The ratio of free surface to compressibility effect follows from (2) and (4)

\[
(dq_f/dp)/(d_q_c/dp) = \phi/\rho s H
\]

(10)

Considering porosities in the range \( \phi = 0.01 \) to 0.2, a thickness of \( H = 10^3 \) m and taking that \( \phi = 2 \times 10^{-11} \) Pa$^{-1}$, we find that the ratio given in (10) varies from 50 to 100. Thus, at normal reservoir conditions the free surface lowering releases a much larger amount of reservoir liquid mass per unit pressure decline than the compressibility.

Along similar lines we obtain the ratio of the vaporization to the compressibility effect on the basis of (2) and (9)

\[
(dq_v/dp)/(d_q_c/dp) = \rho_c CT/\rho s L^2.
\]

(11)

Considering the case of \( T = 200°C = 473 K \) and using standard values \( \rho_c = 2500 \) kg/m$^3$, \( C = 10^3 \) J/kg*K, \( \phi = 2 \times 10^{-11} \) Pa$^{-1}$, \( \rho_s = 7 \) kg/m$^3$ and \( L = 2 \times 10^6 \) J/kg, we find a ratio of about 2 x 10$^3$. Since \( \rho_s \) is the main variable in (11) this ratio will decrease with increasing temperature.

Summing up the results of the present section, we conclude that in the case of liquid dominated reservoirs with common porosities and where no vaporization takes place, the free surface effect is larger than the compressibility effect by a factor of $10^2$-$10^3$. In such cases, the reservoir response to long-term production will be dominated by the free surface effect.

The situation is more complex when vaporization takes place. Theoretically, this effect can release approximately as much fluid mass as the free surface effect. However, in most practical cases where production is initiated at liquid dominated conditions, the vaporization is more or less confined to the local volumes around the boreholes and the ratio in (11) has then to be reduced by a volume factor that may very roughly be of the order of 0.1 or less. The free surface effect would also then dominate the global reservoir response to long term production.

Vapor dominated reservoirs have, as a matter of course, different characteristics. There is no near-surface free liquid surface and \( \rho_s \) in equation (2) has then to be replaced by the product \( \gamma \) where \( \gamma \) is the steam compressibility. Usually, there is a vaporization at a deep liquid surface and this effect dominates the long term reservoir behavior.
(2) Pressure-flow fields in slightly compressible formations with Darcy type flow

(2.1) Diffusion equation. Let \( p(t,P) \) be the pressure field at time \( t \) and at the point \( P \) in a Darcy type domain \( B \) with the stationary boundary surface \( \Sigma \). Consider a general setting where the permeability \( k \) is a linear matrix operator and the kinematic viscosity of the fluid \( v \) is also taken to be variable. It is convenient to introduce the fluid conductivity operator \( c = k/v \) and express Darcy's law

\[
\dot{q} = -cv \mathbf{p}
\]

where \( q \) is the mass flow density. Moreover, let \( \mathbf{p} \) be the fluid density, \( s \) the capacitance or storage coefficient of the formation and \( f \) be a source density. Combining (12) with the equation for the conservation of mass,

\[
\nabla \cdot q = -c\mathbf{s} q + f
\]

we obtain the diffusion equation for the pressure field

\[
p \mathbf{s} q + \Pi(c) p = f
\]

where \( \Pi(c) = -\nabla (c \nabla) \) is the generalized Laplacian operator. Appropriate boundary conditions that may be of the Dirichlet, Neumann, mixed or more complex convolution type, have to be adjoined to equation (14). The case of a homogeneous/isotropic/isothermal formation results in the simplification \( \Pi(c) = c I = -c \nabla^2 \) where \( c \) is a constant. Moreover, stationary pressure fields satisfy the potential equation

\[
\Pi(c) p = f.
\]

(2.11) Eigenfunctions of the Laplacian. The eigenfunctions \( u_n(P) \) of \( \Pi(c) \) in \( B \) associated with (14) satisfy the equations

\[
\Pi(c) u_n = \lambda_n u_n, \quad n = 1, 2, \ldots
\]

where the constants \( \lambda \) are the eigenvalues and the boundary conditions on \( \Sigma \) are homogeneous of the same type as those satisfied by \( p(t,P) \) in (14) and (15).

(2.111) Types of solutions. The key to solving equation (14) is the causal impulse response or Green's function \( G(P,Q,t) \) which represents the pressure response of the causal system to an instantaneous injection of an unit mass of fluid at \( t = 0^+ \) at the source point \( Q \). This function satisfies the same boundary conditions as the eigenfunctions \( u_n(P) \). Solutions to (14) in the case of a general source density \( f(t,P) \), non-causal initial values and general boundary conditions can then be expressed in terms of integrals over the Green's function (Duff and Naylor, 1966).

Two fundamental types of expressions for the Green's function are available. First, in the case of simple layered domains \( B \) with a boundary \( \Sigma \) composed of a few plane faces, \( G(P,Q,t) \) can be expressed as a sum (or integral) over the fundamental whole space source function

\[
G_0(P,Q,t) = (8ps)^{-1/2}(\pi t)^{-3/2}
\]

\[
\exp(-r^2/pq/4at)U(t)
\]

and its images. The symbol \( U(t) \) is the causal unit step function, \( a = c/ps \) the diffusivity, and \( r_{pq} \) is the distance from \( Q \) to \( P \). Whenever applicable, sums of this type represent the most elementary local and/or global expressions for \( G(P,Q,t) \).

Second, the Green's function can be expanded in a series or integral over the eigenfunctions of \( \Pi(c) \). If \( p \) and \( s \) are constants, then

\[
G(P,Q,t) = (1/ps) \int u_n(P)u_n(Q)\exp(-\lambda_n t/ps). \quad (18)
\]

The series expansion (18) is of a more general applicability than solutions of the type based on the fundamental source function (17).

The formal link between the two types (17) and (18) is provided by the Poisson summation formula (Stakgold, 1967). It is important to underline that all solutions of the type (17) can be expressed in the form (18).

From the numerical point of view, the form given by (17) is more convenient for the computation of relatively short term field responses, in particular, in the case of layered half-spaces. However, long-term responses in bounded domains are more effectively computed on the basis of (18). This expression is a sum over exponentials where the convergence improves with time.

A different type of solution of (14) that is of interest in the present context can be obtained by operational methods. Limiting ourselves to the pure initial value problem with \( p(0,P) = p_0(P) \) in the case of an infinite domain, we can, since \( p \), \( s \) and \( \Pi(c) \) are independent of \( t \), formally express the solution of the homogeneous form of (14) as

\[
p = \exp[-t\Pi(c)/ps]p_0 \quad (19)
\]

where the exponential operator is to be interpreted as a Taylor series in the operator \( \Pi(c) \)

\[
\exp[-t\Pi(c)/ps] = 1 - [t\Pi(c)/ps] + \{t\Pi(c)/ps\}^2/2 \ldots (20)
\]

The series represents an iteration process where the convergence is limited to (properly defined) small values of \( t \). The practical applicability is therefore fundamentally different from (18). Moreover, it is of considerable interest that rather general situations with regard to \( \Pi(c) \) can be admitted in (19) and (20).

A number of other analytical and/or numerical techniques are available for solving (14). These include the path-integral technique of the Feynman-Kac type (Simon, 1979), compartmentalization or lumping and, as a matter of course, a series of numerical techniques.
Nonstationary boundaries: effects of a free liquid surface

The presence of a free liquid surface in a reservoir requires the introduction of a rather complex non-stationary surface boundary condition. Let \( C \) now represent the free liquid surface at equilibrium and \( S_2 \) be the free surface in a perturbed state. The boundary \( \Sigma \) is a surface of constant pressure which without loss of generality can be taken to vanish. The free surface condition (Lamb, 1932) is then expressed

\[
\frac{Dp}{Dt}_{|_{p=0}} = 0
\]

where \( D/Dt \) is the material derivative. This is an essentially non-linear condition which leads to a much more complex problem setting. Losing the principle of superposition the construction of solutions to the forward problem becomes a difficult task.

Bodvarsson (1977) has shown that when \( \Sigma \) deviates only little from \( \Sigma \), \( \Sigma \) can be simplified and linearized. For this purpose, we place a rectangular coordinate system with the \( z \)-axis vertically down such that the \( (x,y) \) plane coincides with \( \Sigma \). Moreover, let the amplitude of \( \Sigma \) relative to \( \Sigma \) be \( u \) and the scale of the undulation of \( \Sigma \) be \( L \). Then provided \( |u/L| < 1 \), the condition (21) can be replaced by the approximation

\[
\frac{1}{\omega} \partial_z p - \partial_z^2 p = 0
\]

where \( \omega = c_g/\rho \) is a new parameter, namely, the free sinking velocity of the pore liquid under gravity (\( g \) = acceleration of gravity). Under these circumstances, the solution of the forward problem is obtained by constructing a solution to (14) which satisfies (22) at the free surface and appropriate conditions at other sections of the reservoir boundary.

The presence of a first order derivative with respect to time in the free-surface condition (21) obviously leads to an additional relaxation process analogous to the purely diffusive phenomena associated with the first order time derivative in the basic equation (14). As we shall conclude below, the individual time scales of the two phenomena are, however, different.

For the sake of brevity, we shall limit the present discussion to the simplest but practically quite relevant case of the semi-infinite liquid saturated homogeneous, isotropic and isothermal half-space. To consider the pure free-surface related phenomena, we eliminate pressure field diffusion by neglecting the compressibility of the liquid/rock system. As shown in section (1) above, the long term dynamics of liquid reservoirs is dominated by the free surface phenomena. In this setting we can combine the potential equation (15) and the surface condition (22) in one single equation confined to the \( \Sigma \) plane (Bodvarsson, 1978a), which expressed in terms of the fluid surface amplitude \( u(x,y) = p/\rho g \) takes the form

\[
\frac{1}{\omega} \partial_z u + \kappa^2 u = f/\rho g c
\]

where \( \kappa^2 = (-\partial_{xx} - \partial_{yy})^{1/2} \) is the square root of the two-dimensional Laplacian and \( f \) is an appropriately defined surface source density. To obtain the pressure field in the space \( z > 0 \), the boundary values derived from (23) have to be continued into the lower half-space on the basis of standard potential theoretical methods. The fractional order of the Laplacian in (23) is quite unusual, but the operator is well defined and poses no mathematical problems.

Some solutions of equations (23) of practical interest have been obtained by Bodvarsson (1977). Confining ourselves first to the simple semi-infinite half-space, some important results are given below.

### (3.1) The source-free case

In a source-free case where \( f = 0 \), the homogeneous equation (23) is most easily solved by solving

\[
-\kappa^2 p = 0, \quad z \geq 0
\]

with the boundary condition (22) combined with a given initial condition which takes the form

\[
p = \rho g h_o, \quad t = 0, \quad z = 0
\]

where \( h_o(S) \) is a given initial free-surface amplitude.

This solution is obtained immediately by observing that a pressure function of the form

\[
p = p(x,y,z + wt)
\]

satisfies the boundary condition (22) at all times. Consequently, introducing the Dirichlet type Green's function for the half-space \( z > 0 \) (Duff and Naylor, 1966, page 276) which gives the pressure \( p(P) \) in \( z > 0 \) for a pressure \( p_o(S) \) on \( \Sigma \)

\[
p(P) = (z/2\pi) \int_{\Sigma} \frac{(1/\kappa^2) p_o(U) dU}{r^2} \quad z \geq 0
\]

where \( U = (x', y'), \quad dU = dx'dy' \) and

\[
r = [(x-x')^2 + (y-y')^2 + z^2]^{1/2}
\]

the solution to the present problem is

\[
p(P,t) = [\rho g(z+wt)/2\pi] \int_{\Sigma} (1/\kappa^2) h_o(U) dU, \quad t > 0, \quad z > 0
\]

where

\[
r_{PU} = [(x-x')^2 + (y-y')^2 + (z + wt)^2]^{1/2}
\]

The motion of the fluid surface is obtained by letting \( z = 0 \) in (13) and hence,

\[
h(S,t) = (wt/2\pi) \int_{\Sigma} (1/\kappa^2) h_o(U) dU, \quad t > 0
\]

where now

\[
r_{SU} = [(x-x')^2 + (y-y')^2 + (wt)^2]^{1/2}
\]

### (3.11) Flow fields with sources

To select a relevant and important case of flow fields with
sources, we will consider the following situation. Let the fluid at \( t = 0 \) be in static equilibrium and the fluid surface at \( t = 0 \) therefore coincide with \( \Sigma \). Consider a concentrated sink of strength unity at the point \( Q = (0,0,d) \) which at \( t = 0^+ \) starts withdrawing fluid mass at a constant rate equal to unity. In this case we have to solve

\[
\Pi_p = -\nabla^2 p = (-1/c)\delta(P-Q)U_+(t) \tag{33}
\]

where \( U_+(t) \) is the causal unit step function for which \( U_+(0) = 0 \). The boundary condition on \( \Sigma \) is again given by (22) and the initial condition is \( p = 0 \) at \( t = 0 \).

A simple method of solving this problem has been given by Bodvarsson (1977). In the present context, it is of some interest to present a different approach via the combined Laplace-Hankel transform method.

Let \( p(P,s) \) be the Laplace-transform of \( p(P,t) \). The transform of (33) and (22) are then

\[
\Pi_p = -(1/(cs))\delta(P-Q) \tag{34}
\]

and

\[
s\phi - w_0 \phi = 0, \quad z = 0 \tag{35}
\]

Moreover, let \( \tilde{p}(k,z,s) \) be the two-dimensional Hankel-transform of \( p(P,s) \) and \( D = d/dz \). The transform of (34) is then

\[
k^2\tilde{p} - D^2\tilde{p} = -(1/(2\pi cs))\delta(z-d) \tag{36}
\]

and (35) takes the form

\[
s\tilde{p} - w_0 \tilde{p} = 0, \quad z = 0 \tag{37}
\]

The solutions of (36) for \( z \geq z' \) are of the form \( \exp(\pm kz) \) and we thus obtain

\[
\tilde{p} = a\exp(kz) + b\exp(-kz), \quad 0 < z < z', \tag{38}
\]

and

\[
\tilde{p} = c\exp(-kz), \quad z > z', \tag{39}
\]

where \( a, b \) and \( c \) are integration constants (with respect to \( z \)). From (37) we obtain the relation

\[
A(s-wk) + B(s+wk) = 0, \tag{40}
\]

and our solution has to be continuous at \( z = d \) such that

\[
A\exp(kd) + B\exp(-kd) = C\exp(-kd), \tag{41}
\]

Finally, integrating (36) with respect to \( z \) from \( d \) to \( d^+ \), we obtain the necessary third condition

\[
\frac{d\tilde{p}}{dz}\bigg|_{z=d} = -(1/2\pi cs) \tag{42}
\]

which yields the relation

\[
-k\exp(-kd) - k[A\exp(kd) - B\exp(-kd)] = -(1/2\pi cs) \tag{43}
\]

Solving (40), (41) and (43) for \( A, B \) and \( C \) and inserting in (38) leads to

\[
p = -(1/(4\pi cs))\exp[-k(d-z)]\{[A\exp(-kd)] + [a\exp(kd)]\} \tag{44}
\]

which holds for \( z > 0 \). Using the identity

\[
(a-wk)/(s+wk) = [2/(s+wk)] - (1/n), \tag{45}
\]

(44) is easily Hankel-Laplace inverted into \((P,t)\) space (tables in Duff and Naylor, 1966) and the result is

\[
p(P,t) = -(1/(4\pi cs))\left[1/(r_{PQ}) + (1/r_{PQ''} + (1/(2\pi cs) \{[a\exp(kd)] + [a\exp(-kd)]\} \tag{46}
\]

where

\[
r_{PQ} = [(x-x')^2 + (y-y')^2 + (z-d)^2]^{1/2}, \tag{47}
\]

\[
r_{PQ''} = [(x-x')^2 + (y-y')^2 + (z+d)^2]^{1/2}, \tag{48}
\]

\[
r_{PQ''t} = [(x-x')^2 + (y-y')^2 + (z+w-t+d)^2]^{1/2} \tag{49}
\]

The surface elevation \( h = p/\rho g \) is

\[
h(S,t) = -(1/2\rho g c)[(1/r_{SQ}) - (1/r_{SQ''t})] \tag{50}
\]

where \( S = (x,y) \) and

\[
r_{SQ} = [(x-x')^2 + (y-y')^2 + d^2]^{1/2}, \tag{51}
\]

\[
r_{SQ''t} = [(x-x')^2 + (y-y')^2 + (w-t+d)^2]^{1/2} \tag{52}
\]

It is of particular interest to note that the Hankel-inversion leading to the last term in (46) follows upon a Laplace-inversion on the basis of the Sommerfeld integral

\[
\int_0^\infty \exp[-(z+w-t+d)k]J_0(kd)dk = 1/r_{PQ''t}, \tag{53}
\]

which we rewrite

\[
\int_0^\infty \exp[-(z+d)k]J_0(kr)dk = 1/r_{PQ''t}, \tag{54}
\]

where

\[
E(r,s,k) = \exp[-(z+d)k]J_0(r,k), \tag{55}
\]

Equation (46) reveals that the effect of the free fluid surface on the pressure drawdown due to the concentrated sink of strength unity starting at time \( t = 0 \) can be represented by the pressure field due to a stationary image sink of strength unity located at \( Q' = (x',y',-d) \) and a moving image source of strength 2 located at \( Q'' = (x',y',-w(t+d)) \). At time \( t = 0^+ \) the image sink and 1/2 of the image source cancel resulting in an initial pressure field of

\[
p(P,0^+) = -(1/(4\pi cs))\left[1/(r_{PQ}) + (1/r_{PQ''})\right]. \tag{56}
\]

At very large times, that is at \( t >> d/w \), when the image source has retreated far into the negative half space, the third term in (46) becomes negligible and the pressure field reaches its stationary value \( p_s \) given by

\[
p(P)\bigg|_{t=\infty} = -(1/(4\pi cs))[1/(r_{PQ}) + (1/r_{PQ''})], \tag{57}
\]
The source-sink situation is illustrated in Figure 1.

It is appropriate to reiterate that the above free surface results have been obtained by neglecting the rock/liquid compressibility.

(3.11) Flow in slab with a free surface. The results for the half-space set forth in the previous section are easily generalized to the model of a slab of thickness \( H \) and of infinite horizontal extent. As given in equation (46) and shown in Figure 1, the free surface dynamics reduces at any fixed time to a source-sink situation. Applying well known results of elementary potential theory, we can extend equation (46) to the case of the slab by adding an infinite sequence of source-sink images that is obtained by reflecting the source and the two sinks in Fig. 1 at the bottom and the equilibrium surface boundaries. Appropriate reflection coefficients have to be applied in this process. We will refrain from entering into details of the procedure. The practically most important case is obtained when the basement is impermeable and the reflection coefficient at the boundary is equal to unity. As shown above, the equilibrium surface has also a reflection coefficient of unity but on any reflection, we have to observe the splitting of an image source into a stationary image and a double moving image with an opposite sign. The picture is therefore a little more complex than in the usual cases involving single images.

(3.1v) Discussion. Equations (18) and (53) above show that both compressibility and free surface effects lead to decline functions that are sums or integrals over exponentials of negative time. In essence, therefore, the decline processes are governed by very simple functional relationships. Moreover, the analysis in section (I) indicates that from the quantitative point of view, the free surface effect dominates in all liquid reservoirs.

The decline or relaxation time is another parameter of major interest. By definition this is the time \( t_r \) during which the amplitude of a stationary wave of wavelength \( L \) decreases to \((1/e) \) of its initial value. Inserting a waveform \( \exp[-(t/t_r) + ikx] \) where \( k = (2\pi/L) \) is the wavenumber, into equation (14) gives for compressibility the time \( t_r = (1/ak^2) \). Similarly, we find on the basis of (33) for the free surface a value \( t_r = (1/\omega_k) \). At the same \( L \), the ratio of the free surface to compressibility time is \((ak/\omega) = (k/\rho g) \). Inserting values of interest for long-term reservoir behavior such as, for example, \( \phi = 0.1, L = 6 \) km, \( s = 3 \times 10^{-11} \) \( \text{Pa}^{-1} \) we find values of this ratio of about 300. This indicates quite clearly that the compressibility phenomena are on a much shorter time scale and smaller magnitude than the free surface phenomena. Our approach of neglecting compressibility in the above analysis is, therefore, well justified.

---

**FIG. 1a.** Infinite half space of linearized free surface method.

**FIG. 1b.** Reservoir half space for linearized free surface method with bottom layer.
Reservoir simulation by lumping

The fact that the principal decline functions for liquid reservoirs are of the negative time exponential type suggests the use of lumping as a method of reservoir simulation. Below, we will briefly look onto this possibility.

Consider a liquid geothermal reservoir that is producing a constant mass flow $q$ from a number of wells. We assume that the reservoir pressure is being monitored at a fixed point where a decreasing reference pressure function $p(t)$ is being observed. Moreover, it is being assumed that production, started at time $t = 0$ from equilibrium conditions where we can take that the reference pressure $p(0) = 0$. The producing holes have a bottom-hole pressure of $p_w(t)$ that is also taken from an appropriate reference point as $p(t)$ and therefore $p_w(0) = 0$.

The simplest lumped model to simulate this system is shown in Fig. 2 below.

![Diagram of a lumped parameter model of simulated reservoir.](image)

The model consists of a liquid capacitor or container (I) with vertical walls having an area $A$. The production $q$ is being extracted from this capacitor over a conductor that has a conductance $c_1$. This element represents the contact resistance of the producing holes. Recharge to the container is obtained from a capacitor (II) of an infinite area over another conductor that has a conductance $c_2$. Reference pressure in the large capacitor is taken to constant and equal to zero. The liquid level in (I) is measured by the pressure $p_1(t)$. In accordance with Darcy type flow conditions we assume that both conductors are linear and hence that the following system equations hold

$$q = c_1(p_1-p_w)$$

$$(A/g)Dp_1 = c_2(0-p_1)-q$$

where $g$ is the acceleration of gravity and $D = d/dt$. Since we don't observe $p_w$, equation (58) is irrelevant and does not enter into the discussion below. The principal parameters of the simulation system are thus the capacitor area $A$ and the conductance $c_2$. Given $q(t)$ and $p_1(t)$ for some fixed time interval starting at $t = 0$, we are now interested in deriving values of $A$ and $c_2$ such that the model simulates the given reservoir in the optimal way during at least a part of the production time. A convenient way of obtaining these values in the following.

Since we have assumed that $q$ is constant, the present decline function $p_1(t)$ is characterized by a smooth negative time exponential behavior. We can then expand the known $p_1(t)$ into a Taylor series in $t$ starting at $t = 0$ and that is truncated at the second order term,

$$p_1(t) = p_1(0) + tDp_1(0) + (t^2/2)D^2p_1(0),$$

where we have abbreviated $Dp_1|_{t=0} = Dp_1(0)$ and $D^2p_1|_{t=0} = D^2p_1(0)$.

Since $p_1(0) = 0$ this series reduces to

$$p_1(t) = tDp_1(0) + (t^2/2)D^2p_1(0),$$

Inserting this expression in equation (59) results in

$$(A/g)[Dp_1(0)+tD^2p_1(0)] = c_2[-tDp_1(0)-$$

$$(t^2/2)D^2p_1(0)]-q,$$

On a second order approximation, we obtain from the terms in $t^0$ and $t$ the parameter relations

$$A = -g[q/Dp_1(0)]$$

and

$$c_2 = -(A/g)D^2p_1(0)/Dp_1(0) =$$

$$(A^2/g^2)(D^2p_1(0)).$$

Since $p_1(t)$ is a known function, the derivatives at $t = 0$ are also known and we can thus derive $A$ and $c_2$ from (63) and (64) above.

On the basis of the known parameters, we can then solve equation (59) for a given variable input $q(t)$ and obtain

$$p_1(t) = (g/A)\int_0^t \exp[-gc_2(t-t)/A]q(t)dt$$

as a procedure to predict or extend $p_1(t)$ in time.

An analysis of the above type can be carried out on any field decline functions that have been obtained with sufficient accuracy to derive the derivatives. In most practical cases the mass production function will be a variable $q(t)$ and the input function $p_1(t)$ for the above analysis will then have to be obtained by a deconvolution, that is by solving an equation

$$p_F(t) = \int_0^t p_1(t-t)Dq(t)dt,$$

where $p_F(t)$ is the reference pressure that is the output to $q(t)$. There are no problems in solving (66).

To illustrate the above procedure we will carry out the lumping of the free surface dynamics model leading to equation (46). We obtain then the time derivatives $Dp_1(0)$ and $D^2p_1(0)$ from equation (46) and use (53) and (64) to derive the lumped system parameters. To simplify the procedure,
we consider only the case \( q = \text{constant} = \text{unity} \). Omitting elementary details and irrelevant factors, the method, in essence, consists in approximating the function

\[
f_1(t) = -[1 - (R_o/R_1)]
\]

by the function

\[
f_2(t) = - [(z+d)^2/R_2^2][1 - \exp(-wtR_2^2/R_0(z+d))],
\]

where

\[
R_o = [r^2 + (z+d)^2]^{\frac{1}{2}}
\]

The results of a numerical evaluation are illustrated in Fig. 3. It is evident that the lumped approximation holds quite well until the factor \( wt \) is of the order of a few depths \( d \). Quite often \( w \) is of the order of \( 10^{-5} \text{ m/s} \) and \( d \) about \( 10^3 \text{ m} \). In this case the lumped approximation will give good results for a period of a few years.

\[
R_1 = [r^2 + (z+wc+d)^2]^{\frac{1}{2}}
\]

\[
R_2 = [2(z+d)^2 - r^2]^{\frac{1}{2}}
\]

with \( r^2 < 2(z+d)^2 \)

FIGS. 3a-3f. Comparisons between lumped parameter approximation and exact solution.
 III. REVIEW OF METHODS

**Petroleum Reservoirs**

Production decline methods are probably the most commonly used tool of the reservoir engineer because production data are always recorded and filed whereas temperature and pressure records are far less common. The uses of these methods are at least two fold. First, they are used to predict future production and second, they can provide insight into reservoir mechanisms and geology.

Production data for fields and individual wells are usually plotted on a monthly basis so a year's worth of data might be enough to use with the standard methods. When fields have been produced for a number of years, e.g., 10, production data are plotted on an annual basis and fitted. In the petroleum industry great care must be taken in trying to extrapolate past trends because conditions can change. For example, the reservoir pressure might pass through a bubble point causing dissolved gas to outgas thereby drastically changing flow conditions. The best discussion of and warning about decline methods is in Brosn (1962).

Reserve estimates are calculated from predicted future production. If the predictions are bad, the estimates are bad. Brosn shows an example using production from two wells, each with constant but different percentage decline rates. When their productions are added together and fitted with a hyperbolic eqn (the best fit) we get a very different reserve estimate from the one obtained by looking at each well separately. As always, the reservoir analyst must supply a great deal of insight.

Decline methods are not directly applicable to new fields except that if the new field appears to be similar to a previously studied field we might make some intelligent guesses about its production characteristics.

Decline methods are used to determine when additional wells should be drilled and when wells should be worked over. Production in individual wells can decrease in a steady regular manner from sand plugging the formation. This can be seen on a production vs. time graph.

**Geothermal Reservoir**

The decline methods developed for analyzing oil and gas wells can be used for geothermal wells but we must recognize that petroleum and geothermal reservoirs are very different from each other. These differences can cause production mechanisms to be drastically different in the two cases. Some of the more important differences and their consequences are as follows:

1. Petroleum reservoirs are usually sedimentary formations. Geothermal reservoirs are usually fractured igneous or metamorphic formations. Darcy flow holds in the first case and fracture flow in the second.

b. Temperature is relatively unimportant in petroleum production. It is critical in geothermal production. High temperatures stress tubing and cement in the wellbore.

c. Geothermal well flow volumes are often 1 to 2 orders of magnitude greater than petroleum volumes.

d. Precipitation is much more serious in geothermal wells than in petroleum wells.

e. Petroleum is a complex mixture with volatile components. Geothermal water is essentially one species.

Fracture size, quantity and distribution are drastically affected by precipitation, changes in temperature and seismic activity. Geothermal reservoirs are much more complex than petroleum reservoirs so methods taken into geothermal work must be examined carefully. We have done this with the data and methods available but more work must be done as we produce more geothermal fields over time.

1. **Arps**

Arps (1945, 1956) work forms the basis for all the decline curve methods currently in use. He brought together and codified work on oil reserve estimation that had been done as early as 1908. The commonest methods were graphical in which production \( q \) or cumulative production \( Q \) were plotted vs. time \( t \). See Fig. 4 from Arps (1956). Examinations of production data showed that data with constant first differences fit an exponential equation while data with constant second differences fit a hyperbolic or harmonic equation. All three equations can be expressed as

\[
\frac{a}{q} = Kt \quad \frac{b}{q} = -\frac{dq}{dt} \quad \text{(72)}
\]

where

- \( a = \) fractional decline
- \( b = \) fractional decline
- \( K = \) constant
- \( q = \) production rate of time \( t \)

The solutions to equation (72) are shown in Table 1.

Guerrero (1961) gives a good "cookbook" approach to analyzing data using these methods. See Table 2 for problems worked out by Guerrero. Arps's equations were considered to be strictly empirical until 1973 when Fetkovich proposed some theoretical basis for the exponential equation (see below). The hyperbolic equation is still considered to be empirical.

2. **Fetkovich**

Fetkovich (1973) showed that log-log type curves can be used to analyze production data in an analogous manner to analyzing pressure data. He presented log-log plots of dimensionless flow rate, \( q^2/\psi = q(t)/q_1 \) vs. dimensionless time, \( t^2/\tau = t/\tau_1 \), for \( \psi < 1 \) and \( \psi = 1 \) (see Fig. 5).

\( b = 0 \) is the exponential solution while \( b = 1 \) is the harmonic solution.
The exponential curve is given by
\[ q_{Dd} = \exp(-D_\text{d}t), \quad D_\text{d} = 1 \tag{73} \]
while the hyperbolic curves are given by
\[ q_{Dd} = (1+bD_\text{d}t)^{-1/b} \quad \text{for} \ 0<b<1. \tag{74} \]

Using an overlay technique as shown very clearly in Earlougher (1977), (see Fig. 6), production data can be plotted over the curves and a decline exponent can be picked. For \( t_\text{pd}<0.3 \) all the curves are coincident.

Fetkovich showed that the exponential decline has a fundamental base by deriving it as a solution to the constant well pressure case. The equation for dimensionless flow rate is
\[ q_{Dd} = q(t)/q_1 = \exp[-(q_1)_{\text{max}} t/N_{pl}] \tag{75} \]

This equation can be related to (73) by setting
\[ D_\text{d} = (q_1)_{\text{max}}/N_{pl} \tag{76} \]
then
\[ t_{Dd} = (q_1)_{\text{max}} t/N_{pl} \tag{77} \]

We define
\[ N_{pl} = \Pi(r_e^2 - r_w^2)\phi_h c_h p_1 \]
\[ (q_1)_{\text{max}} = \frac{k h p_1}{141.3 \mu B\left[\ln\left(r_e/r_w\right) - \frac{1}{2}\right]} \tag{78} \]
\[ q_{Dd} = \frac{q(t)}{kh(p_e - p_{wf})} \tag{80} \]

Fetkovich showed that production decline curve data could be used to derive values for permeability thickness \( kh \) which is usually obtained from pressure data. (see Fig. 7a and 7b). Compare \( kh \) calculations from rate-time data and pressure time data.

3. Slider's Method

Slider (1968) proposed a simple method of curve matching to obtain the hyperbolic exponent
### Table 1. Classification of Production Decline Curves

<table>
<thead>
<tr>
<th>DECLINE TYPE</th>
<th>I. CONSTANT-PERCENTAGE DECLINE</th>
<th>II. HYPERBOLIC DECLINE</th>
<th>III. HARMONIC DECLINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC CHARACTERISTIC</td>
<td>DECLINE IS CONSTANT $n=0$</td>
<td>DECLINE IS PROPORTIONAL TO A FRACTIONAL POWER $k$ OF THE PRODUCTION RATE $q &lt; n &lt; 1$</td>
<td>DECLINE IS PROPORTIONAL TO PRODUCTION RATE $n = 1$</td>
</tr>
<tr>
<td>$D<em>K</em> q^o - q^t = $</td>
<td>$D<em>K</em> q^o - q^t = $</td>
<td>$D<em>K</em> q^o - q^t = $</td>
<td></td>
</tr>
<tr>
<td>$\int_0^t dt = \int \frac{dq}{q}$</td>
<td>$\int_0^t \frac{dt}{q}$</td>
<td>$\int_0^t \frac{dt}{q}$</td>
<td></td>
</tr>
<tr>
<td>$-dt = \log_q \left( \frac{q}{q_0} \right)$</td>
<td>$-dt = \log_q \left( \frac{q}{q_1} \right)$</td>
<td>$-dt = \log_q \left( \frac{q}{q_1} \right)$</td>
<td></td>
</tr>
</tbody>
</table>

**RATE - TIME RELATIONSHIP**

- $q_1 = q_1 - e^{-Dt}$
- $q_2 = q_2 - e^{-Dt}$
- $q_3 = q_3 - e^{-Dt}$

**RATE - CUMULATIVE RELATIONSHIP**

- $Q_1 = \frac{q_1 - q_1}{D}$
- $Q_2 = \frac{q_2 - q_1}{D}$
- $Q_3 = \frac{q_3 - q_1}{D}$

| $D*$ Decline as a fraction of production rate | $q_1*$ Production rate at time $t$ | $Q_1*$ Cumulative oil production at time $t$ |
| $D_1*$ Initial decline | $q_1*$ Initial production rate | $K*$ Constant |
| $q_1*$ Time | $n*$ Exponent |

### Table 2. Example of Use of Exponential Equation

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Time, years</th>
<th>Ave. Oil prod. rate bbl./yr.</th>
<th>$\frac{q_0^{(2)}}{q_0^{(4)}}$</th>
<th>$\frac{(4)^{n-1}-(4)^n}{(5)^{(5)}(6)^{(6)}}$</th>
<th>$q_{avg}^{(6)}$</th>
<th>$\frac{D=}{\frac{\Delta q/dt}{N_p}}$</th>
<th>$\frac{N_p=}{Cum.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947</td>
<td>1</td>
<td>0.5</td>
<td>99,200</td>
<td>10,990</td>
<td>93,210</td>
<td>0.13</td>
<td>687,410</td>
</tr>
<tr>
<td>1948</td>
<td>2</td>
<td>1.5</td>
<td>88,210</td>
<td>14,970</td>
<td>80,725</td>
<td>0.18</td>
<td>260,650</td>
</tr>
<tr>
<td>1949</td>
<td>3</td>
<td>2.5</td>
<td>73,240</td>
<td>9,250</td>
<td>68,615</td>
<td>0.13</td>
<td>324,640</td>
</tr>
<tr>
<td>1950</td>
<td>4</td>
<td>3.5</td>
<td>63,990</td>
<td>9,250</td>
<td>59,450</td>
<td>0.13</td>
<td>379,550</td>
</tr>
<tr>
<td>1951</td>
<td>5</td>
<td>4.5</td>
<td>54,910</td>
<td>7,510</td>
<td>51,155</td>
<td>0.14</td>
<td>426,950</td>
</tr>
<tr>
<td>1952</td>
<td>6</td>
<td>5.5</td>
<td>47,430</td>
<td>5,820</td>
<td>44,490</td>
<td>0.13</td>
<td>468,530</td>
</tr>
<tr>
<td>1953</td>
<td>7</td>
<td>6.5</td>
<td>41,580</td>
<td>5,820</td>
<td>36,770</td>
<td>0.14</td>
<td>504,490</td>
</tr>
<tr>
<td>1954</td>
<td>8</td>
<td>0.5</td>
<td>35,960</td>
<td>5,620</td>
<td>33,350</td>
<td>0.14</td>
<td>535,569</td>
</tr>
<tr>
<td>1955</td>
<td>9</td>
<td>1.5</td>
<td>31,099</td>
<td>4,861</td>
<td>28,297</td>
<td>0.14</td>
<td>586,744</td>
</tr>
<tr>
<td>1956</td>
<td>10</td>
<td>2.5</td>
<td>26,895</td>
<td>4,204</td>
<td>25,079</td>
<td>0.14</td>
<td>630,302</td>
</tr>
<tr>
<td>1957</td>
<td>11</td>
<td>3.5</td>
<td>23,260</td>
<td>3,635</td>
<td>21,688</td>
<td>0.14</td>
<td>685,860</td>
</tr>
<tr>
<td>1958</td>
<td>12</td>
<td>4.5</td>
<td>20,116</td>
<td>3,144</td>
<td>18,757</td>
<td>0.14</td>
<td>632,257</td>
</tr>
<tr>
<td>1959</td>
<td>13</td>
<td>5.5</td>
<td>17,397</td>
<td>2,719</td>
<td>16,221</td>
<td>0.14</td>
<td>636,302</td>
</tr>
<tr>
<td>1960</td>
<td>14</td>
<td>6.5</td>
<td>15,045</td>
<td>2,352</td>
<td>14,029</td>
<td>0.14</td>
<td>631,313</td>
</tr>
<tr>
<td>1961</td>
<td>15</td>
<td>7.5</td>
<td>13,011</td>
<td>2,034</td>
<td>12,132</td>
<td>0.14</td>
<td>662,565</td>
</tr>
<tr>
<td>1962</td>
<td>16</td>
<td>8.5</td>
<td>11,252</td>
<td>1,759</td>
<td>10,492</td>
<td>0.14</td>
<td>682,296</td>
</tr>
<tr>
<td>1963</td>
<td>17</td>
<td>9.5</td>
<td>9,731</td>
<td>1,521</td>
<td>9,074</td>
<td>0.14</td>
<td>680,712</td>
</tr>
<tr>
<td>1964</td>
<td>18</td>
<td>10</td>
<td>8,416</td>
<td>1,315</td>
<td>8,043</td>
<td>0.14</td>
<td>680,712</td>
</tr>
</tbody>
</table>

Future Performance

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Time, years</th>
<th>Ave. Oil prod. rate bbl./yr.</th>
<th>$\frac{q_0^{(2)}}{q_0^{(4)}}$</th>
<th>$\frac{(4)^{n-1}-(4)^n}{(5)^{(5)}(6)^{(6)}}$</th>
<th>$q_{avg}^{(6)}$</th>
<th>$\frac{D=}{\frac{\Delta q/dt}{N_p}}$</th>
<th>$\frac{N_p=}{Cum.}$</th>
</tr>
</thead>
</table>

$D_{avg} = 0.868 t + 0.145$
FIG. 5. Log-log type-curve of dimensionless flow rate vs dimensionless time (after Fetkovich, 1973).

b and the initial decline rate q_1. To use the method one needs to construct a set of curves of q/q_1 vs. log time for various values of a_1 and b using Arps's hyperbolic equation. Production data can then be plotted on the curves by using a transparent overlay. The overlay can be moved around until the best fit is found thus giving b and a_1. From equations or from a second set of curves, future production rates q and future cumulative production Q can easily be estimated. This method is easy to apply but it requires a separate set of curves for each possible value of b. Later methods eliminate this shortcoming.

4. Gentry's Method

Gentry (1972) developed curves which are much easier to use than Slider's because only one set is needed for all values of b between 0 and 1. (see Figs. 8a and 8b) We can find b from a plot of Q/tq1 vs. log q/q_1. With this b we go to a plot of q/t vs. log q/q_1 and find a_1. This gives us all the factors we need for a reserve analysis.

5. Gentry and McCray

Reservoir analysts have usually assumed that 0<b<1 in the solution of Arps's equations. See Higgins and Lechtenberg (1970) for exceptions. There is no mathematical basis for this restriction. b = 0 and b = 1 are special cases, the exponential and harmonic, respectively, but this does not restrict b from being larger than 1. Gentry and McCray (1978) investigated decline curve methods using semi-log plots of q/q_1 vs. Q/q_1t, cartesian plots of q/q_1 vs. Q, and semi-log plots of q/q_1 vs. a_1t. See Figures 9a, 9b, and 9c. Some of their conclusions are (N_p = Q)

1. The dimensionless curves N_p/q_1t vs. q_1/q a_1t vs. q_1/q for a particular fluid-permeability system are not affected by the absolute permeability or size of the reservoir. The behavior of these plots is determined by (1) the characteristics of the contained fluid, (2) the relative permeability characteristics of the reservoir rock, (3) the reservoir drive mechanism, (4) reservoir heterogeneity, and (5) manual manipulation of production.

2. Reservoir heterogeneity tends to increase the magnitude of b as the degree of heterogeneity is increased. It is also apparent that b for a heterogeneous system
(a) Choose a type curve.

(b) Overlay with tracing paper.

(c) Trace major grid lines.

(d) Label axes.

(e) Plot observed data using type-curve grid.

(f) Slide tracing paper to match a type curve.

(g) Trace the matched curve.

(h) Pick a match point.

FIG. 7a. Type-curve matching example for calculating Kh using decline curve data (after Fetkovich, 1973).

\[ kh = \frac{q(t) 141.3 (\ln \frac{r_w}{r_i} - .5)B}{q_{dD} (P_1 - P_w)} = \frac{1000 \text{ BOPM} 141.3(\ln 50 - .5)}{30.4 \text{ D/B} (.054) (7259)} = 40.5 \]

FIG. 7b. Type-curve matching example for calculating Kh from pressure buildup (after Fetkovich, 1973).

\[ P_D = \frac{141.3q^2B}{kh} \]

\[ P_{ws} - P_{wf} \text{ psi} \]

\[ t_DL = \frac{0.00634 kn}{\phi \mu_c L^2} \]

\[ kh = 47.5 \text{ md-ft} \]
FIG. 8a. Decline curve analysis chart relating production rate to time (after Gentry, 1972).

FIG. 8b. Decline curve analysis chart relating production rate to cumulative production (after Gentry, 1972).

FIG. 9a. Q/(q,t) vs q/ q (after Gentry & McCray, 1978).
FIG. 9b. Plots of cumulative production vs $q/q_i$ for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME

FIG. 9c. Dimensionless $a_t$ curve histories for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME
will increase to a maximum value and then as the ratio \( q_1/q \) becomes large, \( b \) will decrease and approach its homogeneous value.

3. Reservoir heterogeneity can and does cause \( b \) values to be greater than 1.0.

4. Manual manipulation of production can and does cause \( b \) values to be greater than 1.0.

5. The dimensionless plots for heterogeneous systems of 1 and 3 md, 3 and 9 md, and 5 and 15 md all plotted the same curve. This indicates that heterogeneous systems in the ratio of 1:3 will plot congruous dimensionless curves.

6. It appears that the relative-permeability characteristics of the reservoir have the greater effect on the decline exponent \( b \), while the fluid characteristics have a greater influence on the constants \( a_1 \) and \( q_1 \).

7. The equation \( N_p/(q_1 t) = (q/q_1)^b \)

may better define certain decline curves than do the Arps equations.

8. The plotting of production data on the \( N_p/(q_1 t) \) vs. \( q_1/q \) curve can be a helpful diagnostic tool for evaluating the production history of a well or lease.

6. Other Type Curves

Fetkovich developed log-log type curves using dimensionless production vs. dimensionless time, \( Q_p \) vs. \( t_p \), but other variables can be used. We tried plots of dimensionless cumulative production vs. dimensionless time and dimensionless production vs. dimensionless cumulative production, \( q_0 \) vs. \( Q_p \). The plots were made by using the exponential equation and the hyperbolic equation for several values of \( b \). See Figs. 10a and 10b. We had the same data scatter problem with these type curves as we did with Fetkovich’s. A few data sets plotted very nicely on a particular curve, but most sets plotted very ambiguously.

7. \( p/z \) vs. \( Q \)

The natural gas industry has long used decline curves in which pressure divided by gas deviation factor, \( p/z \), is plotted against cumulative production, \( Q \) (Katz, 1959). The straight line can be extrapolated to the economic limit of producing pressure quite easily. Brigham and Morrow (1974) have proposed adapting this method to steam fields. In plotting computer generated data they found that curve shape was strongly influenced by porosity. Also, the presence of a boiling interface is critical. "If the wells are completed in the vapor zone it would be natural to graph \( p/z \) vs. production, as though this were a gas reservoir, and use an extrapolation of the best straight line as a predictive method to calculate reserves. The efficiency of this technique will be strongly dependent on the porosity if the actual reservoir contains boiling liquid." (see Fig. 11.)

Pruess et al. (1979a, 1979b) have used the simulator SHAFT78 to test the use of \( p/z \) vs. \( Q \) plots for geothermal reservoirs. They conclude that "... the standard technique of estimating reserves by extrapolating a plot of \( p/z \) vs. cumulative production is not applicable to two-phase geothermal reservoirs." and "... in many cases pressure will be a linear function of cumulative production, with the slope allowing an estimate of reservoir volume. Reserve assessment requires knowledge of average porosity and vapor saturation, which cannot be obtained from pressure decline curves."

Brigham (1979) applied \( p/z \) techniques to a study of depletion in the Cabbro zone at Larderello, but he stated that the linearity of \( p/z \) with cumulative production doesn’t hold for the entire life of a reservoir with a boiling interface. He claims that linearity is a good approximation for the first one-third to one-half of the reservoir’s life.
8. Influence Functions

Unsteady state isothermal flow of slightly compressible liquid through a porous medium can be described using the diffusivity equation

$$\frac{\partial p}{\partial t} = \frac{\kappa A \partial^2 p}{\mu}$$

The equation can be solved using a Green's function approach to derive a "response", "resistance", "memory", or "influence" function. Katz and Coats (1969) in describing water movement in aquifers defined two influence functions: 1) $P(t) = \text{the "rate case" influence function which is defined as the pressure drop at the reservoir boundary (a function of time) corresponding to a unit rate (e.g. 1 cu. ft./day) of water influx.}$. For a constant flow rate $q$ we get $P(t) = Q(t)$, the constant terminal rate case equation. 2) $Q(t) = \text{the "pressure case" influence functions since a constant pressure $p_b$ is specified at the outer boundary.}$ The constant terminal pressure equation is $q(t) = (p_0 - p_b)Q(t)$.

$P(t)$ and $Q(t)$ can be calculated either for idealized models or from field data. Let $F(t) = Q(t)$ or $P(t)$. For an idealized $F(t)$ we must specify "1) model geometry, 2) exterior boundary conditions (e.g. infinite, closed or constant pressure), and 3) model parameters." The specification of reservoir parameters and geometry is particularly difficult in geothermal reservoirs so the calculation of $F(t)$ from field data is more attractive and easier than trying to devise a thoroughly specified model. The advantages of the field method are "1) none of the above choices are required, and 2) an influence function which reflects unknown (and practically speaking, indeterminate) aquifer properties, as reflected by actual field performance, is determined. Disadvantages are 1) the resemblance of the backed out $F(t)$ to the true function is proportional to the accuracy of field data, and 2) the influence function is obtained only up to the time of last available field data; extrapolation is required for purposes of predicting future water movement."

Coats et al. (1964) recommended the use of field influence functions for oil fields with adjacent aquifers. The method is directly applicable to geothermal fields. The influence function $F$ can easily be generated as a function of pressure $p$ and flow rate $q$ using the following equations:

Integral form

$$\Delta p = \int_0^t \frac{dq(t-\tau)}{dt} F(t) d\tau$$

Discrete form

$$\Delta p_1 = p_0 - p_1 = \sum_{j=1}^{\infty} (q_{i-1} - q_{i-1})F_{i-1} - F_i$$

Boðvarsson (1980, personal communication) has shown how the influence function problem can be formulated in a slightly different manner. The function $F$ defined by Coats is a unit step response function. Instead of the unit step response, we can use the impulse response $h$, where $h = \frac{dF}{dt}$. The equation to be solved is then

Integral form $\Delta p = \int_0^t \frac{dq(t)}{dt} F(t) d\tau = \int_0^t q(t) \frac{dF(t-\tau)}{dt} d\tau$

Discrete form $\Delta p_1 = p_0 - p_1 = \sum_{j=1}^{\infty} q_{i-1}F_{i-1} - F_i$

The first derivative of the curve from the $F$ formulation should be identical to the curve derived from the $h$ formulation. $F$ can be calculated by hand (Jargon and van Pooten, 1965) and Hutchinson and Sikora, 1959) but we recommend against it. An $F$ can be calculated which fits the data well, but which has no physical meaning.

Katz and Coats cite an example in Katz et al. (1963) in which an influence function is calculated by direct methods which exactly reproduces past performance but which cannot be extrapolated. The smoothness constraints below assure a physically meaningful solution and they can be arried at both intuitively and analytically. From Katz and Coats "if water is injected into an aquifer at a constant rate through some fixed inner aquifer boundary (surface), then intuitively the pressure change at that boundary must always be positive. In addition, the pressure should always increase and the rate of increase should continually decrease with time." The analytical proof for the constraints is given in Coats et al.
Linear programming methods such as the package MPOS should be used with the smoothness constraints on $F$ or $h$:

$$F > 0, t > 0 \quad h > 0, t > 0$$

If the data are not "smooth and regular enough" the use of simple hand calculations can lead to results shown in Fig. 12. The $F$ function will reproduce the pressure very well, but the function cannot be extrapolated and is physically meaningless. The $F$ calculated by the linear program is shown on the same figure for comparison.

Hutchinson and Sikora and Coats et al. discuss the effects of field geometry on the behavior of the pressure drop and the influence function. As production time increases, the rate of pressure change decreases: If the reservoir outcrops, both the pressure drop and the influence function become constant. This is an effect we will look for in geothermal areas which we know have fluid recharge. If the reservoir is infinite-acting or bounded the influence function and the pressure drop will increase monotonically for all time greater than 0.

Hutchinson and Sikora show how to extrapolate calculated influence functions. If a definite straight line has developed from the field data it may be extrapolated and the field assumed to be bounded. If no definite straight line has developed, the last 3 or 4 values of $F$ should be examined. If the average $\Delta F$ for these times gives a good match to past performance the curve may be extrapolated using the slope of the average $F$. The extreme extrapolation assumes an infinite aquifer. In this case

$$\Delta F_{n+1} = \Delta F_n \frac{\log((n+1)/n)}{\log(n/(n-1))}$$

All these extrapolations are included in our MPOS program.

9. **Linearized Free Surface-Green's Function**

One of the main virtues of the influence function method is described above is that it can be used to predict reservoir behavior without specifying a physical model for the reservoir. Long-time behavior of the influence function can tell something about the boundaries. If the reservoir has a free liquid surface and is assumed to be a porous half-space, Fig. 1a, a simple, distributed parameter model can be posited. Bodvarsson (1977) linearized the free surface condition and derived the following equations for pressure

$$p(P,t) = \frac{-d}{4c} \left( \frac{1}{r_{PQ}^2} + \frac{1}{r_{PQ'}^2} - \frac{2}{r_{PQ't}} \right)$$

where

- $p$ = pressure, meters of head
- $q$ = flow rate, kg/s (constant)
- $r_{PQ} = ((x-x')^2 + (y-y')^2 + (w+d)^2)\frac{1}{2}$
- $r_{PQ'} = ((x-x')^2 + (y-y')^2 + (z+d)^2)\frac{1}{2}$
- $r_{PQ't} = ((x-x')^2 + (y-y')^2 + (z+wt+d)^2)\frac{1}{2}$

Bodvarsson (1978) also showed that the impulse response of a linearized free surface can be expressed as

$$G(t,S,Q) = \frac{1}{2\pi\rho}(w+d)(x^2 + y^2 + (wt+d)^2)^{-3/2}U_+(t)$$

where

- $d$ = depth from free surface to sink
- $G$ = Green's function
- $S$ = $(x,y)$ a point on the free surface
- $\phi$ = porosity, fraction
- $\rho$ = density, kg/m$^3$
- $w$ = kg/(v$\phi$) = sinking velocity, m/s
- $v$ = kinematic viscosity, m$^2$/s
- $g = 9.8$ m/s$^2$
- $k$ = permeability, m$^2$
\( U_+(t) = \text{unit impulse function} \)
\( r = (x^2+y^2)^{\frac{1}{2}}, \text{radial distance from sink}. \)

He obtained the following expression for drawdown in meters for \( P \) at a distance \( r \) from the sink (wellbore)

\[
h(t) = \int_0^t G(t-\tau)q(\tau)d\tau
\]

(86)

\[
\approx \sum_{\tau=0.5}^{t-0.5} G(t-\tau)q(\tau)\Delta \tau
\]

(87)

See Chpt. II for the derivation of these equations.

The impulse response, \( h \), is the drawdown in meters at the point, \( P \), caused by the instantaneous withdrawal of one unit fluid mass at point \( Q \). At a continuous withdrawal the total drawdown at \( P \) would be a summation over all fluid sinks. The equation is

\[
h_{\text{total}}(P) = \sum_{n=1}^{N} G_n(t-\tau)q_n(\tau)d\tau
\]

(87)

This equation for drawdown can be compared directly with the \( h \) function formulation of the influence function as described above.

If the reservoir has a relatively impermeable zone below the producing zone the half space assumption can be modified. An image source term or terms as necessary can be added to the equation for \( G \). See Fig. 1b. With one image term the expression is

\[
G = \frac{-1}{2\pi \phi \rho} \left[ \frac{wt+d}{(x^2+y^2+(wt+d)^2)^{3/2}} + \frac{2H-d+wt}{(x^2+y^2+(2H-d+wt)^2)^{3/2}} \right]
\]

(88)

More terms can be added as necessary.

The distributed model described above can be approximated by a lumped parameter model as described more fully in Chpt. II.
IV. DATA PROCESSING

Data Sets

The most complete data set is from Wairakei, New Zealand by Pritchett et al. (1978) published by Systems, Science and Software for Lawrence Berkeley Laboratory. Individual well monthly heat and mass flow rates are given from 1953 through 1976 for 141 wells. Furthermore, a fair amount of pressure and temperature data are presented.

The authors presented a substantial amount of data on the geology and subsidence problems at Wairakei. In addition to this report we received from Malcolm Grant, DSIR, a set of annotated individual well production graphs which indicated when wells were shut in and which steam lines the wells were connected to. See Fig. 13a for map.

The data set for Cerro Prieto by Bermejo et al. (1979) published by Lawrence Berkeley Laboratory included graphical production histories of most of the wells from 1973 to 1978. These graphs were digitized for analysis. The production was broken down into liquid and vapor production. In addition, we received data from Marcelo Lippman, LBL, which showed individual well total mass flow rates. The two data sets were treated separately and then compared. A theoretical pressure drawdown curve was taken from Sanchez and de la Palma (1979). See Fig. 13b for map.

The last large data set is from The Geysers, California, courtesy of the California Dept. of Conservation, Div. of Oil and Gas. The data include production injection and pressure data from 27 wells from March 1971 through December 1979. Additional pressure data are from Lipman, Strobel, and Gulati (1977). See Fig. 13c for map.

The Larderello data were taken from Sestini (1970). The sparse data for other fields were from various sources. See Fig. 13d for map.

Graphical Treatment of Data

The first step in the analysis was graphing all the available production data on cartesian paper using SPSS. These graphs allowed us to eliminate from further consideration wells with severely irregular production such as Bore 11.

Arps (1945, 1956) pointed out that the exponential equation would graph as a straight line on semilog paper. We tried plotting the data for several wells at Wairakei but found that production decline was insufficient to make the semilog plots look very different from the cartesian plots. The log-log plots, however, were significantly different from the cartesian plots so most of the data were plotted on log-log plots. We tried matching the log-log plots against Fetkovich's type curves. For the most part the data scatter rendered the method useless. We were also hindered by the fact that dimensionless time for almost all our wells was fairly short.

FIG. 13a. Bore locations in main bore field (from Pritchett et al., 1978).
about 1.0. The exponential and hyperbolic curves only start diverging at about $t_{pd} = 0.2$, so with rough data we would like the last point to have $t_{pd} = 2.0$, at least. We could not reproduce the fits reported by Rivera-R. (1977, 1978) using Cerro Prieto data. None of the data from liquid-dominated fields fit very well, but this is probably much more a function of data scatter than of the efficacy of the methods. See below for a discussion of data scatter. The only fair fits were for several wells from Larderello. Successful use of type curves with rough data may require a great deal of insight on the part of the analyst.

We tried two other kinds of type curves, Figs. 10a and 10b, with no more success than with Fetkovich's curves. Scatter and small dimensionless time caused problems again.

Gentry and McCray (1978) proposed the use of several different graphs, Figs. 9a, 9b, and 9c, for decline curve analysis. We had difficulty with the plots involving $a_1$ because the data scatter gave $a_1$ a very large uncertainty. We plotted $N_i/q_1 t$ vs. $q_i/q$ for several wells and got very peculiar results which were of no use. Again, the data are far more problematical than the models.

We tried plotting $p/z$ vs. $Q$ for The Geysers data using pressure from Cobb Mountain #1 well and yearly total production data from Finn (1975) and from the California Dept. of Oil and Gas (see Figs. 14a and b). Brigham (1979) analyzed some Larderello data using $p/z$ vs. $Q$, but as mentioned above he cautions against expecting linearity after one-third to one-half of the fluid has been produced (see Figs. 15a and 15b for plots of the data).
Statistical Treatment of Data

Most of the data sets had so much scatter that statistical treatment was the only reasonable approach. We used SPSS (Statistical Package for the Social Sciences) to reduce the data. See Appendix for discussion of SPSS and for the programs we used. SPSS is available at many computer centers and it requires a minimum of data handling.

We used SPSSPLOT to generate cartesian plots of the q vs. t data for all the wells. From the plots we chose wells to analyze further. Some of the wells had drastic rate changes in their histories so only selected parts of their histories were analyzed. We used a non-linear least squares regression subroutine to analyze the exponential equation

$$q(t) = q_1 e^{-at}$$

The program requires initial estimates of $q_1$ and $a$, and it returns $\hat{q}(t)$, the predicted value of $q$, and best estimates $\hat{a}$ and $\hat{q}_1$ for the fractional decline and the initial flow rate. A fit to the linear equation

$$q(t) = q_1 + kt$$

is also generated. The primary statistic generated is $R^2$ defined as

$$R^2 = \frac{SSR}{SSTO} = \frac{\text{regression sum of squares}}{\text{total sum of squares}}$$

$$1 - \frac{SSE}{SSTO} = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$

$$SSTO = \sum_{i=1}^{n} (q_i - \bar{q})^2$$

$$SSR = \sum_{i=1}^{n} (\hat{q}_i - \bar{q})^2$$

FIG. 13c. The Geysers, California (from California Dept. of Conservation, Division of Oil & Gas, 1978).
FIG. 13d. Larderello geothermal area (from Sestini, 1970).


FIG. 15a. Gabro Zone pressure—production history match, lag time=36 months (from Brigham, 1979).

FIG. 15b. p/z vs Q—Larderello data (from Brigham & Neri, 1979).

\[ \text{SSE} = \text{SSTO} - \text{SSR} \]

Values of \( R^2 \) greater than 0.65 indicate a good fit which can be extrapolated with some confidence. The value 0.65 is arbitrary, but is generally considered to be a good fit for raw data.

For the influence function method, we developed a fitness measure, \( \rho \), which is the average fractional deviation of computed pressure differences, \( \Delta p \), from observed pressure difference, \( \Delta p = p_1 - p(t) \). For example, if \( \rho = 0.1 \) and \( \Delta p = 100 \), the true value is between 90.91 and 111.11 because \( \Delta p = \Delta p/(1 + \rho) \).
Discussion of Data Scatter

Field data often have a great deal of scatter in them which can cause difficulties in analyzing them. The scatter can be of two general types, reservoir related and operations related. Reservoir related scatter can be caused by

1) rainfall
2) recharge
3) earthquakes
4) subsidence.

Production related scatter can be caused by

1) changes in production schedules
2) bad well completions
3) workovers
4) poor calibration techniques
5) poor data gathering techniques

Little can be done to prevent reservoir related scatter, but operations related scatter can always be reduced. Methods for reducing the chance of scatter are discussed in the Standard Operating Procedure section. Scattered data can be analyzed with the following techniques:

1) averaging the data
2) least squares fitting
3) subtracting our known effects and trends
4) using insight and experience.

We tried averaging data from several Wairakei and Cerro Prieto wells to see whether we could use Guerrero's method for Arps's equations. We could not get reasonable values for the decline exponent. See Fig. 16 for a graph of six month average production vs. time for Bore 18.

![Graph of six month average production, Bore 18, Wairakei, New Zealand.](image)

The easiest known effects to take into account are periodic shut downs. The annotated production graphs from Wairakei showed that many of the wells were shut in for periods of about 1 month every 1-2 years. The monthly production during these shut-in months is obviously much lower than the preceding and following month's production. If the other data are on a smooth trend, the low values can effectively be ignored in fitting an equation to the trend line. Since these points represent production, however, they should be included in any calculation involving the cumulative production, Q.
V. RESULTS

Arps's Equations

We tested Arps's exponential equation (73) on all individual well data, total field data and on several groups of wells. The results are summarized in Table 3 with complete results in the Appendix. \( D \) is the average calculated monthly fractional decline. \( D \) based on total field production from Wairakei, The Geysers, and Cerro Prieto ranges from 0.003 for Wairakei to 0.0115 for The Geysers. This converts to yearly declines of 3.6\% and 13.8\%, respectively. \( R^2 \) for individual wells ranges from 0.0004 for a well at Otake to 0.9712 for a well at Larderello. Eight of the ten wells and groups at Larderello had \( R^2 \)'s greater than 0.87, indicating a very good fit to the equation. Also, all three wells at Matsukawa had \( R^2 \)'s greater than 0.76. The wells from The Geysers did not fit as well as the wells from Larderello and Matsukawa, so we cannot draw definite conclusions about vapor-dominated fields and the exponential equation.

Cerro Prieto and Wairakei are both liquid-dominated fields, and their data did not fit the exponential equation quite as well as the vapor-dominated fields. However, for all the fields the equation fit at least several of the well's data quite well. See Figures 17a-g for a fit of the exponential equation to total Wairakei production and to several individual wells.

We only tested a few wells using the hyperbolic equation and got \( R^2 \)'s greater than 0.989 in all cases. This indicates that the equation is either nearly perfect or that it is a very poor model and will fit virtually any data set. We hold the latter view. Because the equation has no physical basis, we recommend against using it. However, if a particular data set fits a hyperbolic type curve well over a long stretch of dimensionless time, the curve can be used to extrapolate production.

Type-Curve Methods

None of the data sets fit any of the type curves well. The scatter is so high that no value of \( b \) can be picked with confidence. Some of the Larderello data fit Fetkovich's exponential curve for up to 80 months but then develop constant production which takes them off the curve. See Figure 18a. Figure 18b shows typical Cerro Prieto data plotted on the same curve. No value of \( b \) can reasonably chosen.

Coats' Influence Function Method

Coats' method can be used with any but the most bizarre data because the derivative constraints imposed on the solution method guarantee that either a meaningful solution or no solution is generated. The fitness measure tells how useful

<table>
<thead>
<tr>
<th>Field</th>
<th>( D )</th>
<th>( R^2 )</th>
<th># of Wells</th>
<th>Range on Individual R(^2)s</th>
<th># of Wells at R(^2)&gt;0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wairakei; NZ total prod.</td>
<td>0.0030</td>
<td>0.7847</td>
<td>36</td>
<td>0.0016-0.9049</td>
<td>11</td>
</tr>
<tr>
<td>Cerro Prieto</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.C., Mexico Liquid prod.</td>
<td></td>
<td></td>
<td>17</td>
<td>0.0066-0.8524</td>
<td>6</td>
</tr>
<tr>
<td>Total prod.</td>
<td></td>
<td></td>
<td>19</td>
<td>0.0405-0.9409</td>
<td>8</td>
</tr>
<tr>
<td>The Geysers, CA., USA</td>
<td>0.01151</td>
<td>0.8103</td>
<td>26</td>
<td>0.0126-0.8127</td>
<td>6</td>
</tr>
<tr>
<td>Larderello, Italy</td>
<td></td>
<td></td>
<td>10</td>
<td>0.0416-0.9712</td>
<td>8</td>
</tr>
<tr>
<td>Matsukawa, Japan</td>
<td></td>
<td></td>
<td>3</td>
<td>0.7609-0.8633</td>
<td>3</td>
</tr>
<tr>
<td>Otake, Japan Liquid</td>
<td></td>
<td></td>
<td>4</td>
<td>0.1528-0.7043</td>
<td>1</td>
</tr>
<tr>
<td>Vapor</td>
<td></td>
<td></td>
<td>4</td>
<td>0.0004-0.7981</td>
<td>1</td>
</tr>
</tbody>
</table>

FIG. 17b. Pozo 15, Cerro Prieto, Mexico.

FIG. 17c. Pozo 25, Cerro Prieto, Mexico.

FIG. 17d. Bore 72, Wairakei, New Zealand.

FIG. 17e. Bore 66, Wairakei, New Zealand.

FIG. 17f. Bore 18, Wairakei, New Zealand.
FIG. 17g. Larderello, Italy—Nella Sasso Rosso Nr. 82.

the solution is. We tried the method on Wairakei and Cerro Prieto total production and on Travale 22 from Larderello. The fitness measure, \( p \) for Travale 22 was 0.038 indicating a very good fit (see Fig. 19). \( p \) was 0.1001 and 0.3366 for Cerro Prieto liquid production and Wairakei total production, respectively. We tested the predictive value of the method by fitting Wairakei data from 1955-62 and then extrapolating. The pressures obtained using both the infinite and bounded aquifer approximations are shown in Table 4 along with the actual pressures. The pressure drop is calculated as

\[
\Delta p = \frac{QF}{t}
\]

Where \( Q \) is cumulative production, \( F \) is the influence function and \( t \) is time. Figure 20 shows the calculated influence functions for Wairakei 1955-1962. The high fitness measure indicates very rough data. The observed pressures fall within the fitness measure for the infinite aquifer case.

We tried the method on Wairakei for the rate of pressure decline decreases with time as is the case for Wairakei.

**Sodvarseon's Linearized Free Surface Method**

We had enough data to try the linearized free surface method only with Wairakei. We divided the field into six regions and then assumed that the total production from each region was coming from a virtual well in the "center" of the region. The production depth for each virtual well was the production weighted average center of open zone for the wells in the region. A centroid was chosen for the entire field and then the pressure drawdown at the centroid was calculated for each virtual well and summed to get total drawdown. The drawdown curve obtained is shown in Figure 21 as Curve #2 with the actual drawdown as Curve #1 for comparison. By adding the term for a bottom as described in Chapter III and by adjusting the porosity, \( \phi \), permeability, \( k \), and depth, \( h \), we obtained Curve #3,

FIG. 18b. Pozo 25, Cerro Prieto, fit to Fetkovich type-curve.

In an infinite aquifer the rate of pressure decline decreases with time as is the case for Wairakei.

FIG. 18a. Larderello 82 (Nella Sasso Rosso Nr. 82) fit to Fetkovich type-curve.

FIG. 19. Influence function—Travale 22.
a plausible fit. This method is difficult to use because the necessary geologic and production data are usually lacking or sparse at best.

Table 4. Check of Extrapolability of Coats' Method Using Wairakei Data

(Values of the influence function are from Fig. 20)

<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative Prod., Q</th>
<th>Producing Time, t, yrs.</th>
<th>( F_0 )</th>
<th>( \Delta p )</th>
<th>( P )</th>
<th>( F_{\text{bound}} )</th>
<th>( \Delta p )</th>
<th>( P )</th>
<th>( P_{\text{obs}} )</th>
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<td>1965</td>
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<tr>
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<td>292</td>
<td>.006126</td>
<td>670</td>
<td>81</td>
<td>405</td>
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</table>
VI. STANDARD OPERATING PROCEDURE FOR DATA GATHERING AND ANALYSIS

Data Gathering

The most important step for the analysis of data is the proper collection of it. The data must be as complete and clear as possible so that "bad" data can be eliminated as a possible cause of unusual results in the analysis. Some steps for ensuring good data gathering are:

1) Set up regular testing schedules and stick to them.
2) Set up calibration schedules for all instruments used such as pressure gauges and temperature bombs.
3) Keep an updated calibration log for each instrument.
4) Use clear standard forms for recording data.

A data chart for routine measurements should include at least the following information:

1) Well name and location
2) Date and time
3) Pressure—well head, tubing, bottom hole, meter run, etc. gauge or absolute
4) Temperature
5) Flow rate
6) Location of test points
7) Units for all measured quantities
8) Well status
9) Type of test being conducted — buildup, interference, etc.
10) Zone being tested
11) Instrument numbers
12) Name of tester

Data Analysis

The data can be analyzed by wells, by groups of wells and by fields.

Graphs

Graphing the data provides an easy way of examining the data for unusual behavior such as occasional high, low or erratic production. Such data sets can be flagged for special attention. The data should be plotted and analyzed according to Arps (1945, 1956) using cartesian semi-log, and log-log plots of production vs. time. However, this provides only a "quick look" and further analysis should be done. If the data are smooth enough, log-log type curves and Gentry's and McCray's curves can be used to fit current data and to extrapolate to future behavior. If the field is vapor dominated, p/z vs. Q plots can be used but only with great caution.

Least Squares Fits to Arps's Equations

Production data (q vs. t) should be fit to Arps's exponential equation using a non-linear least squares program. The program should calculate R^2 to indicate goodness of fit. A reasonably high value of R^2, e.g., greater than 0.65, allows extrapolation with some degree of confidence.

We recommend against using the computer to fit data to Arps's hyperbolic equation for the reasons described in Chapter V. However, if the data fall very well on a particular type curve then one may reasonably predict future production using that curve.

<table>
<thead>
<tr>
<th>Field</th>
<th>Well #</th>
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<th>Calculated Fractional Decline</th>
<th>R^2</th>
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<td></td>
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### Geysers Total Field

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</table>

### references

If adequate production and pressure data are available, they should be analyzed using Coats's influence function method and a computer program with the constraints described in Chapter III. Data preparation is straightforward and data handling is minimal. The first half of a data set can be modeled and extrapolated in several different ways. Comparing the extrapolation with the second half of the data can give insight to the placement of reservoir boundaries such as faults or outcrops.

Bodvarsson's linearized free surface method should be tried if the reservoir has a free liquid surface, and if enough data are available to estimate a sinking velocity.

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APPENDIX

Statistical Package for the Social Sciences SPSS

SPSS is a set of programs developed for general statistical analysis. We have used the two subprograms PLOT (Tuccy, 1977) and NON-LINEAR (Robinson, 1977) quite extensively. The listings for our SPSS main programs which used these subprograms are given below. SPSS will plot a set of data. SPSS4 will do a nonlinear least squares fit using the exponential equation. \( B(i) \) and \( B(j) \) are initial guesses for initial production, \( q_0 \), and monthly fractional decline, \( D \). The other program names are self-explanatory. A complete description of SPSS is given in Nie, 1975.

Multiple Purpose Optimization System MPOS

MPOS is a linear programming package designed to solve a wide variety of linear programming problems. Coats' influence method can be formulated as a linear programming problem as follows.

\[
\begin{align*}
\sum_{j=1}^{n} q_{i-j} x_j + u_i - v_i &= b_i \quad (1) \\
0 &= x_j - F_j - F_{j-1} \quad (2a) \\
x_i &\geq 0 \quad (2b) \\
x_{n-1} - x_n &\geq 0 \quad (2c) \\
x_{i+1} - 2x_i + x_{i-1} &\geq 0 \quad (2d)
\end{align*}
\]

Values for \( u_i \) and \( v_i \) are given in the output so that the fitness measures, \( \rho \), can be calculated directly as

\[
\rho = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{b_i} (u_i + v_i). 
\]

The listing for program INFUNC is given below.
SPSS PROGRAMS

==== SPSS2 =====

PAGESIZE NOEJECT
RUN NAME PLOT ONLY -- POZO 5
VARIABLE LIST MONTH53, MASS
INPUT FORMAT FIXED (24X,F3.0,27X,F7.0)
N OF CASES UNKNOWN
READ INPUT DATA
PLOT PLOTS=MASS(0,320) WITH MONTH53(0,72)
       TITLE=POZO 5/
       TITLEX=MONTH/
       TITLEY=MASS/
       SIZE=10.5,8.0/
       XDIV=12/
       YDIV=8/
       SYMBOLS=-7/
OPTIONS 1,10
FINISH

==== SPSS4 =====

PAGESIZE NOEJECT
RUN NAME NONLINEAR REGRESSION -- RORE 10A
VARIABLE LIST MONTH53, AMONTH, YEAR, MASS
INPUT FORMAT FIXED (9X,F5.0,5X,F5.0,9X,F7.0)
N OF CASES 159
COMPUTE PMASS=MASS
MISSING VALUES MASS(0)
IF (YEAR LT 1964) MASS=0
IF (YEAR EQ 1964 AND AMONTH LT 8) MASS=0
COMPUTE MONTH=MONTH53-140
REJECT IF (MASS EQ 0)
NONLINEAR VARIABLES=MASS WITH MONTHR, NB=2
MODEL YHAT=1000000*B(1)*EXP(B(2)*.00001*MONTHR)
DERIVATIVES G(1)=EXP(B(2)*.00001*MONTHR)*1000000
G(2)=10*MONTHR*9(1)*EXP(B(2)*.00001*MONTHR)
PARAMETERS B(1)=435.299562
B(2)=392.55
REGRESSION VARIABLES=MASS, MONTHR/
 PLOT REGRESSION=MASS WITH MONTHR/
 PLOTS=(MASS(0,800000000)) WITH MONTH53(0,26A1)
       TITLE=RORE 10A/
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       SIZE=10.5,8.0/
       XDIV=12/
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OPTIONS 1,10
FINISH
... ===== SPSS6 =====

PAGE SIZE: NO EjECt
RUN NAME: SEMILOG PLOT ONLY -- BORE 22
VARIABLE LIST: MONTH53, AMONTH, YEAR, MASS
INPUT FORMAT: FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES: UNKNOWN
COMPUTE: LOGMASS=LG10(MASS+1)
READ INPUT DATA:
PLOT: PLOTS=LOGMASS(0.9) WITH MONTH53(0,288)/
      TITLE=BORE 22/
      TITLX=MONTH/
      TITLH=LOG MASS/
      SIZE=10.5,8.0/
      XDIV=12/
      YDIV=9/
      SYMBOLS=7/
OPTIONS:
FINISH

1,10

... ===== SPSS6 =====

PAGE SIZE: NO EjECt
RUN NAME: LOG-LOG PLOT ONLY -- BORE 72
VARIABLE LIST: MONTH53, AMONTH, YEAR, MASS
INPUT FORMAT: FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES: UNKNOWN
COMPUTE: LOGMASS=LG10(MASS+1)
COMPUTE: LOGTYPE=LG10(MONTH53)
READ INPUT DATA:
PLOT: PLOTS=LOGMASS(4,9) WITH LOGT(0,3)/
      TITLE=BORE 72/
      TITLX=LOG TIME/
      TITLH=LOG MASS/
      SIZE=-9.0,-15.0/
      XDIV=12/
      YDIV=20/
      SYMBOLS=7/
OPTIONS:
FINISH

1,10

... ===== SPSSGEN =====

PAGE SIZE: NO EjECt
RUN NAME: GENTRY SEMILOG PLOT -- BORE 22
VARIABLE LIST: XFM, YPRD, XCM
INPUT FORMAT: FIXED (10X,F3.0,5X,F20.0,5X,F20.0)
N OF CASES: UNKNOWN
COMPUTE: QINIT=354000000
MISSING VALUES: YPRD(0)
COMPUTE: QI=LG10(QINIT/YPRD)
ASSIGN MISSING: QI(0.1)
COMPUTE: RQCDQIT=1-XCM/QI(QINIT*XFM)
READ INPUT DATA:
DESCRiPTiVE: QI, RQCDQIT
STATISTICS: ALL
PLOT: PLOTS=QI(0.2) WITH RQCDQIT(0.1)/
      TITLE=BORE 22/
      TITLX=BOR(E22.CUM. PROD. OVER INITIAL PROD.*TIME/
      TITLH=LOG(INITIAL PROD. OVER CURRENT PROD.)/
      SIZE=-15.10/
      XDIV=20/
      YDIV=15/
      SYMBOLS=7/
OPTIONS:
FINISH

10
==== SPGEN2 ====

PAGESIZE  NOEJECT
RUN NAME  GENTRY CARTERIAN FOR A-INITIAL - BORE 20
VARIABLE LIST  XTM, YPRD, XCM
INPUT FORMAT  FIXED(10X,F3.0,5X,F20.0,5X,F20.0)
N OF CASES  UNKNOWN
COMPUTE  QINIT=QOOOO0000
MISSING VALUES  YPRD(1)
COMPUTE  QODI=YPRD/QINIT
ASSIGN MISSING  QODI
READ INPUT DATA
CONDESCRIPTIVE  QODI
STATISTICS  ALL
PLOT  PLOTS=QODI(0.01) WITH XCM (0,1000000000)/
      TITLE=BORE 20/
      TITLEX=BORE 20 PROD RATIO VS CUM PROD/
      SIZE=15.10/
      XDIV=20/
      YDIV=12/
      SYMBOLS=-9/
OPTIONS  10
FINISH

==== SPSSCUM ====

PAGESIZE  NOEJECT
RUN NAME  CUMULATIVE VS CURRENT PRODUCTION, BORE 24
VARIABLE LIST  XTM, YPRD
INPUT FORMAT  FIXED(14X,F5,0,10X,F20.2)
N OF CASES  UNKNOWN
COMPUTE  XCM=ACCUM(YPRD)
WRITE CASES  (10X,F3,0.5X,F20.0,5X,F20.0)
XTM, YPRD, XCM
READ INPUT DATA
PLOT  PLOTS=YPRD(0.600000000) WITH XCM(0,100000000000)/
      TITLE=BORE 24/
      TITLEX=CUMULATIVE PRODUCTION/
      TITLEY=CURRENT PRODUCTION/
      SIZE=10.5.8.0/
      XDIV=10/
      YDIV=8/
      SYMBOLS=-7/
PLOT  PLOTS=XCM(0,100000000000) WITH XTM (0,288)/
      TITLE=BORE 24/
      TITLEX=TIME IN MONTHS/
      TITLEY=CUMULATIVE PRODUCTION/
      XDIV=12/
      YDIV=10/
      SYMBOLS=-7/
FINISH

==== SPSSAVG ====

PAGESIZE  NOEJECT
RUN NAME  OUTPUT SEMI-ANNUAL AVERAGES -- BORE 72
VARIABLE LIST  MONTH53, ANMONTH, YEAR, MASS
INPUT FORMAT  FIXED(9X,F5.0,5X,2F5.0,F20.2)
N OF CASES  UNKNOWN
MISSING VALUES  MASS(10)
IF (AMONTH LE 6) HAFYEAR=1
IF (AMONTH GE 7) HAFYEAR=2
READ INPUT DATA
AGGREGATE  GROUPVARS=YEAR, HAFYEAR/
           VARIABLES=MONTH53, MASS/
           AGGSTATS=MEAN/
OPTIONS  3
STATISTICS  3
FINISH
NONLINEAR REGRESSION -- B0KE 16

NONLINEAR

MODEL

DERIVAT

PARAMETERS

STATISTICS

FINISH

SPSS RELOADED

00561400 CM NEEDED FOR NONLINEAR

OPTION = 1

IGNORE MISSING VALUE INDICATORS
NONLINEAR REGRESSION -- BORE 18

79/07/25. 08:28:54. PAGE 3

FILE NAME (CREATION DATE = 79/07/25)

NONLINEAR PROBLEM SUMMARY

252 CASES
1 DEPENDENT VARIABLE(S)
2 PARAMETERS
50 ITERATION LIMIT

METHOD = MARQUARDT
TOL1 = 1.5968688E-08 REL. CHANGE IN A PARAMETER
TOL2 = 0 REL. CHANGE IN SUM OF SQUARES
TOL3 = 0 RATIO TO INITIAL SUM OF SQUARES
TOL4 = 1.0000000E-06 PIVOT TOLERANCE

PARAMETERS

NO. NAME INITIAL VALUE
1 81 1.056589E+02
2 82 -4.5719080E+02

0 DERIVATIVE ERRORS WERE DETECTED

SETUP TIME = 0.47 SECONDS

THE LAST ITERATION

ITERATION NO. 5 BASE POINT TEST POINT
SUM OF SQUARES 1.2714952E+17 1.2714952E+17
LAMBDA 0 0
GAMMA 1.0000000E+00 1.0000000E+00
ANGLE IN DEGREES 6.717b 6.7257
MAX. PIVOT REDUCTION 3.9411999E-01 3.9411999E-01
PAR. 1 81 1.9510662E+02 1.9510662E+02
2 82 -4.9533609E+02 -4.9533609E+02

CUMULATIVE NO. OF FUNCTION CALLS = 6
ITERATION TIME = 0.262 SECONDS
CUMULATIVE TIME = 1.1 SECONDS
ITERATION TERMINATES
MAX. RELATIVE CHANGE IN A PARAMETER v.t. TOL(I) = 1.5000000E-08

NONLINEAR REGRESSION - SORCE 10
FILE NONAME (CREATION DATE = 79/07/25.)

FINAL PARAMETER VALUES

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NONLINEAR REGRESSION - SORCE 10
FILE NONAME (CREATION DATE = 79/07/25.)

FINAL FUNCTION VALUES AND RESIDUALS

ROOT MEAN SQUARE RESIDUAL = 2.255211E+07

CASE NO.  | v. NO. | PREDICTION | OBSERVATION | RESIDUAL |
-----------|--------|------------|-------------|----------|
1          | 1      | 1.9518662E+02 | 1.8208000E+02 | -1.3166229E+07 |
2          | 1      | 1.3161023E+07 | 1.3161023E+07 | 0         |
3          | 1      | 1.139130E+07  | 1.139130E+07  | 0         |
4          | 1      | 1.0871200E+08 | 1.0871200E+08 | 0         |
5          | 1      | 1.033400E+08  | 1.033400E+08  | 0         |
6          | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
7          | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
8          | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
9          | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
10         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
11         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
12         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
13         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
14         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
15         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
16         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
17         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
18         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
19         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
20         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
21         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
22         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
23         | 1      | 1.000000E+08  | 1.000000E+08  | 0         |
28 23 30 31 32
33 34 35 36 37
38 39 40 41 42
43 44 45 46

1.7447691E+08 2.1200000E+08 3.7521088E+07
1.7363329E+08 2.1200000E+08 3.8566713E+07
1.7729176E+08 1.9200000E+08 1.920821E+07
1.7729176E+08 1.9200000E+08 1.920821E+07
1.7021578E+08 1.1000000E+08 -6.0291970E+07
1.6946624E+08 7.3000000E+07 6.0666238E+07
1.6866416E+08 1.9000000E+08 2.1355093E+07
1.6702276E+08 2.0170000E+08 3.3853113E+07
1.6702276E+08 2.0170000E+08 3.3853113E+07
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1.6599202E+08 2.7000000E+08 -1.3639903E+07
1.6379140E+08 2.0700000E+08 3.6010561E+07
1.6300549E+08 1.1630000E+08 2.2940623E+07
1.6216959E+08 1.7700000E+08 4.0465989E+07
1.6147750E+08 4.0100000E+07 -1.1042976E+08
1.6073738E+08 7.9000000E+06 3.6352624E+07
1.5966790E+08 1.3100000E+08 3.1131214E+07
1.5849375E+08 1.8600000E+08 3.0990378E+07
1.5702291E+08 1.3500000E+08 3.0677022E+07
1.5735598E+08 1.0500000E+08 2.7444104E+07
1.5679185E+08 1.0700000E+08 3.0208222E+07

NONLINEAR REGRESSION -- 30RE 13
FILE NO NAME (CREATION DATE = 79/07/25.)

FINAL FUNCTION VALUES AND RESIDUALS

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### FINAL FUNCTION VALUES AND RESIDUALS

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<td>-1.77621E+08</td>
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<tr>
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NONLINEAR REGRESSION -- 30 RE...
## NONLINEAR REGRESSION -- BORE 18

### FILE: HOMAME (GREATEN GATE) -- 79/07/25.

<table>
<thead>
<tr>
<th>CASE</th>
<th>VAR</th>
<th>PREDICTION</th>
<th>OBSERVATION</th>
<th>RESIDUAL</th>
<th>GRAPH OF RESIDUAL</th>
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### FINAL FUNCTION VALUES AND RESIDUALS
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<th>PREDICTION</th>
<th>OBSERVATION</th>
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<th>-1</th>
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<th>+1</th>
<th>+2</th>
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</tr>
<tr>
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<td>6.1223257E+07</td>
<td>-2.263869E+06</td>
<td>**</td>
<td>**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>5.9679296E+07</td>
<td>6.1223257E+07</td>
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<td>**</td>
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<td></td>
<td></td>
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<td>**</td>
<td>**</td>
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</table>

**TIME SINCE END OF THE LAST ITERATION** = 1.4311 SECONDS
**TOTAL TIME** = 3.250 SECONDS
The linear programming technique of Coats, Rapoport, McCord and Drews (JPT, Dec., 1964), is used to determine the aquifer influence functions from field data. Several programs have been written to take raw data and convert it into a tableau used by the M.P.O.S linear programming package, then extract the results and reformat it. The procedure is as follows:

1. INPUT DATA

The input data must follow the following arrangement:

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Header Card</td>
</tr>
<tr>
<td>2</td>
<td>Input Format. Fields are integer-real-real for</td>
</tr>
<tr>
<td></td>
<td>time-pressure-volume</td>
</tr>
<tr>
<td>3</td>
<td>N = number of data cards to follow</td>
</tr>
<tr>
<td>4</td>
<td>First Data Card (contains data for time, pressure,</td>
</tr>
<tr>
<td></td>
<td>and volume)</td>
</tr>
<tr>
<td>N+4</td>
<td>Last Data Card</td>
</tr>
<tr>
<td></td>
<td>repeat for more data sets</td>
</tr>
</tbody>
</table>

2. GENERATING THE INFLUENCE FUNCTION

The procedure to do this is simply:

GET, INFUNC. or GET, INFUNC/UN=BKO85C.
INFUNC, IN.

where IN is the file containing the input data as described above.

The results will be found in file OUT and are arranged as follows:

1) header card
2) card containing number of data points and fitness value RHO in (I5,5X,F17.0) field
3) N data cards in the format (I5,5X,4F17.7) in order of time, F, delta-F, P, Q.
4) E-O-F mark
5) repetition of 1) to 3) for each data set

These results should be saved for future plotting. There is also a file named RESULTS generated which contains output from the M.P.O.S. package which should be sent to the line printer with carriage control.
3. PLOTTING THE INFLUENCE FUNCTION

To plot the influence functions generated above, it is necessary to use the INFPLT program. The necessary libraries and object decks will have been gotten by INFUNC. All that is necessary is to enter

INFPLT,OUT

where OUT is the file generated in step 2) above. The program should be run from a graphics terminal. Output can be sent to the GERBER plotter or plotted on a 4662 plotter. The program will prompt the user for the necessary information.

March 13, 1980
Influence Function Programs

**PROC**, **INFUNC**, **IN**, **OUT**, **RESULTS**.

---

**THIS PROCEDURE WILL TAKE RAW TIME/PRESSURE/VOLUME DATA AND CALL THE PROPER PROGRAMS TO PRODUCE AN INFLUENCE FUNCTION USING THE LINEAR PROGRAMMING ALGORITHM OF OATS, ET AL. (1964).**

**CALLING SEQUENCE:** **BEGIN**, **INFUNC**, **JPROC**, **IN**, **OUT**, **RESULTS**.

**WHERE:**

**IN** - FILE WITH DATA SETS IN PROPER FORMAT.
**OUT** - FILE WHICH WILL CONTAIN TIME, INFLUENCE FUNCTION, DELTA-T, PRESSURE, AND VOLUME IN (12.55, 17.7) FORMAT.
**RESULT** - FILE WHICH WILL CONTAIN OUTPUT FROM N.P.O.S. LINEAR PROGRAMMING PACKAGE.

---

**COPY, INF4, OUTPUT.**
**REVERT.**
**IF** **FILE** (JLTB) **NOT AS** GET, JLI3/UN=AE150.
**LIBRARY** = SOLUB/JLI3/COMPLT.
**BEGIN** GET PREF, F.PROGS, JTAGEN, INF, OUT.
**IF** **FILE** (JPOS) **NOT AS** ATTACH, JPOS/UN=LIBRARY.
**REVERT.**
**RETURN**, ZTAB.
**JTAGEN**, IN, ZTAB.
**COPY, INF4.**
**MPOS**, ZTAB, RESULTS.
**BEGIN, REFORM**, JPROC, RESULTS, **IN**, **OUT**.
**COPY, INF4.**
**RETURN**, INF4, INF2, INF3.
**REVERT**, JPROC/INF4.
**DATA, INF.**
L.P. TABLEAU GENERATED IN LOCAL FILE <ZTAB>
**ENTERING** N.P.O.S.
**DATA, INF.**
N.P.O.S. RUN COMPLETE, RESULTS ARE IN LOCAL FILE <RESULTS>
**BEGIN DATA REFORMATTING.**
**DATA, INF.**
REFORMAT COMPLETE, INFLUENCE FUNCTION PLUS ORIGINAL DATA ARE NOW IN LOCAL FILE <OUT>. DATA SETS ARE SEPARATED BY E-5-F MARKS. USE <APPEND, JINFUC, OUT> TO ADD TO PREVIOUS RESULTS, OVERWRITE <SAVEOUT=NEWNAME> TO SAVE THE OUTPUT.
**DATA, INF.**
PROGRAMS ARE NOT WORKING TEMPORARILY. CALL JEFF AT 754-4515 OR 4599.
PROGRAM TABGEN(INPUT, OUTPUT, TAPE8=INPUT, TAPE6=OUTPUT)

DIMENSION P(300), Q(300), 9UFL(8), FORM(8)

DATA U(2000), W(1), WNEG(-1), WONE(2), WONE2, WONE3, WONE4, WONE5

THIS PROGRAM READS IN PRESSURE AND PRODUCTION DATA AND CREATES INPUT FOR USE IN LINEAR PROGRAMMING ROUTINE M.P.O.S.

READ HEADER AND INPUT AND OUTPUT FORMATS.

1 READ(5,101) HDR
   IF(.EOF(5),110,2)
2 READ(5,101) FORM
   101 FORM(1:8)

READ NUMBER OF DATA POINTS

READ SUMM VARIABLE IN TIME FIELD, READ PRESSURE AND PRODUCTION DATA

READ(5,*) N
   READ(5, FORM) IDUM, P(I), Q(I)
   5 CONTINUE

SETUP INDEXES

   N1 = N-1
   N2 = N1+1
   N3 = N1+2
   N4 = 2*N1-1

COMPUTE DELTA-P

   DO 10 I=2,N
       10 PI1 = ABS(P(I) - P(I-1))

PRINT M.P.O.S. CONTROL CARDS

WRITE(6,201)
   201 FORMAT(1X,TITLE1)
   WRITE(6,101) HDR

WRITE(6,202) 297
   202 FORMAT(3X,REVISIONS/ VARIABLES/)
   WRITE(6,102) FORM(1:8)
   WRITE(6,103) 'U' , N1, N2, N3, N4
   WRITE(6,104) EQ, N1, N2, N3, N4
   WRITE(6,105) (DASH, I=1,N1)

WRITE(6,203) 204
   203 FORMAT(1X,MINIMIZE/ CONSTRANTS/)
   WRITE(6,106) (EQ, I=1,N1)
   WRITE(6,107) (DASH, I=1,N1)

WRITE(6,204) 205
   204 FORMAT(1X,READ/)

WRITE(6,205) 206
   205 FORMAT(1X,WRITE/)

WRITE OBJECTIVE FUNCTION

   111 FORMAT(2I5,F20.5)
   WRITE(6,111) (ZERO, I=1,N1, ONE, I=1,N2)

WRITE SECOND ORDER CONSTRAINTS

   DO 20 J=1,N1
       WRITE(6,111) J, WONE, J, ONE
       WRITE(6,111) J, WONE2, J, ONE
       WRITE(6,111) J, WONE3, J, ONE
       WRITE(6,111) J, WONE4, J, ONE
       WRITE(6,111) J, WONE5, J, ONE
       20 CONTINUE

WRITE THIRD ORDER CONSTRAINTS

   DO 22 J=E3,N4
       WRITE(6,111) J, WONE, J, ONE
       WRITE(6,111) J, WONE2, J, ONE
       WRITE(6,111) J, WONE3, J, ONE
       WRITE(6,111) J, WONE4, J, ONE
       WRITE(6,111) J, WONE5, J, ONE
       22 CONTINUE
WRITE(6,111) ZERO,ZERO,ZERO
WRITE(6,225)
205 FORMAT(*CP1MIZE*)
GO TO 1
99 REWIND 6
STOP
END
*INFPL0

PROGRAM INFPL0(OUT,EXTRAP,INPPUT,OUTPUT,TAPE1=OUT,TAPE5=INPPUT,
     * TAPE6=OUT,TAPE7=EXTRAP,TAPE6=OUTPUT)

PLTS THE INFLUENCE FUNCTION FOR GEOThERMAL FIELD

TIME AND VALUE OF THE FUNCTION AS GENERATED BY PROGRAM <LP> ARE READ AND PLOTTED ON TEKTRONIX 4062 PLOTER

LOGICAL UNITS FOR INPUT/OUTPUT:

LUN 6 READS INFLUENCE FUNCTION AS GENERATED BY PROCEDURE <REFORM>
FORMAT IS ONE HEADER CARD FOLLOWED BY TIME AND FUNCTION VALUE CARDS
IN (F5.0,F15.6,F17.6) FORMAT. SEPARATE DECKS SEPERATED BY --EDF--
DEFAULT DATA FILE NAME IS <OUTPUT>

LUN 5 WRITE COMPUTED EXTRAPOLATIONS ON DEFAULT FILE <EXTRAP>
LUN 5 ASSIGNED TO FILE <INPUT> FOR RECEIVING OPERATOR INSTRUCTIONS

INTEGER EXLAB1,EXLAB2(3),EXLAB3(3),EXLAB4(4)
DIMENSION HOR(8),V(361),F(361),VLAB(8),BUFF(2),FOOT(8)
DATA YES,'YES'/
DATA NO,'NO'/
DATA EXLAB1,'PRESSURE RESPONSE PER UNIT VOLUME'/
DATA EXLAB2,'FIELD DATA'/
DATA EXLAB3,'ASSUMES BOUNDED AQIFERS'/
DATA EXLAB4,'ASSUMES INFINITE AQIFER'/
DATA FOOT,'FITNESS MEASURE ^Q<^Y = #/

INITIALIZATION -- DETERMINE WHICH, IF ANY, EXTRAPOLATIONS ARE REQUIRED

CALL ALPHA
PRINT 1,00 YOU WISH TO PLOT ON THE GERBER*
Integer 1, 0 = NO/ 1 = YES
PRINT 2, ENTER NUMBER OF TIME PERIODS FOR EXTRAPOLATION
READ (5, *) NEXT
IF (5.EQ.1) NEXT=0
IF (5.EQ.0) GO TO 1
WRITE(OUT,'(15,10.4)') EXLAB2
104 L2 = NOTES(5)
WRITE(L2,'(15,10.4)') EXLAB3
L3 = NOTES(5)
WRITE(L3,'(15,10.4)') EXLAB4
L4 = NOTES(5)

READ HEADER FOR X-LABEL

1 READ (8,101) HDR
   IF (EOF(8)) 59,10
   99 READ (8,107) RHO
   107 FORMAT(15X,F17.4)
   IF (RHO>0.0) GO TO 15
   CALL TEKPAUS
   101 FORMAT(5A18)

READ DATA

15 N = N + 1
READ (8,102) T(N), F(N)
102 FORMAT(5X,6.5X,F17.6)
   IF (N.GE.301) GO TO 20
   CALL EOFP(8) 20,12
20 N = N - 1

PLOTTING SECTION

PHYSICAL PROPERTIES SETUP
IF(MARK.EQ.1) GO TO 30
MARK=1
WIDTH=72
CALL FLOTYP(LITE)
CALL TTYPE(4,662)
CALL AUDI(1,293)
CALL SIZE(WIDTH+1.0,HEIGHT+1.0)

RANGING

38 NMENTS=4
FINDS=F(N)-F(N-1)
FINDS(F(N)+F(N-1))/FINDS
CALL RANGE(T,F(N)+NEXT*FINDS,NINT1,YMIN,YMAX,YTIC)
CALL RANGE(T,F(N)+NEXT*FINDS,6,XMIN,XMAX,XTIC)
IF(XTIC.LT.0.0) XTIC=1.0

SCALING (INCHES/USER UNIT)

XBIAS=2.0
YBIAS=1.5
XDEL1 = XMAX - XMIN
YFACT=(HEIGHT+8)*XBIAS/XDEL1
CALL SCALEIT(XFACT,YFACT,XBIAS,YBIAS,XMIN,YMIN)

GRID GENERATION AND DIMENSIONING

CALL ERASE
CALL ADDPHAS

GOEB

PRINT 17,*XMIN,F,XMIN,F,XMAX,F,XMAX,F,XTIC,F,XTIC,F,XFACT,F,XFACT,F
CALL AXES(XMIN,XMAX,XMIN,XMAX,YMIN,YMAX,YMIN,YMAX,XTIC,XTIC,8,9)
CALL AXLABL
* (*XMIN,XMAX,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1,YDISP)
* (*XMIN,XMAX,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1,XDISP)

WRITE AXIS LABELS

CSIZE = 0.1
NCHARS=31
Y = YMIN + Y/2.
CALL SYMBEL(Y,-8.0/YFACT,0.0,CSIZE,NCHARS,HDR)

CALL FNUM(HDR,0.00001,BUFF,NCHAR)
CALL MOVEST(BUFF,1,FOOT,2,HCHAR)
CAL XDELT = SYMWIDTH/FOOT/2
X = XDEL1 - SYMWIDTH/8,FOOT/XFACT
CALL SYMBEL(X,0.98/YFACT,0.0,CSIZE,80,FOOT)

NCHARS=33
Y = YMAX - SYMWIDTH/16/FOOT/YFACT
Y = Y/2.
X = TAFCT + XMIN
CALL SYMBEL(X,7.90,CSIZE,NCHARS,YLAB)

CALL LINE(T,F,17,N)

CHECK IF EXTENSION WAS REQUESTED BY USER

IF(NEXT.LE.0) GO TO 1
IF(N.LT.5) GO TO 1
CALL DASHSZ(05,03,07,03)
71 IF(L3.EQ.0) GO TO 72.
   X = WIDTH/3
   Y = HEIGHT/5 - CSIZE*MM
   MN = MN + 2
   CALL WMARK(X,Y,CSIZE,271)
   CALL SYMBOL(X,Y,0,CSIZE,24,EXLAB33)
   CALL DASHES
72 IF(L4.EQ.0) GO TO 1
   X = WIDTH/3
   Y = HEIGHT/5 - CSIZE*MN
   CALL WMARK(X,Y,CSIZE,211)
   CALL SYMBOL(X,Y,0,CSIZE,35,EXLAB84)
   GO TO 1
END

99 CALL PLOTENC
999 STOP #END INFLOT#
THIS PROCEDURE WILL REFORMAT THE OUTPUT FROM M P.O.S. LINEAR PROGRAMMING AND MERGE IT WITH THE TIME VALUES FROM THE ORIGINAL DATA. THE FINAL FORM IS DESIGNED TO BE USED WITH THE PLOTTING PROGRAM <INFPLT>.

FILES:

DATA CONTAINS OUTPUT FROM M P.O.S.
OUT FILE WHICH WILL CONTAIN THE OUTPUT USED TO CALL <INFPLT>

RETURN,OUT,EDUMP,NUL,Z1232.

EDIT: IN=EDIT1,L=EDUMP,NH.
FILE(Z1232,NUL,AS),B.
END:

EDIT: NUL,IN=EDIT2,L=EDUMP,NH.
EDIT(KIFH,NUL,AS)BEGIN,GETPROG.P,,LPM.
END:

RETURN,EDIT,EDIT2,BADAT.
REVERT: PROC/REFORM, <OUT> CONVERTED.
ENDIF:BADAT.
RETURN,BADAT,EDIT1,EDIT2.
REVERT: PROC/REFORM, ERROR FOUND.

=DATA EDIT
=DATA EDIT

VIL/NO FEAS/IL/NO FEAS/BIGCOPY Z1232 11,STOP
STOP

=DATA EDIT

D/SLACK/

=DATA BADAT

RUN ABORTED.

FILE EDIT2 CONTAINS A DATA SET FOR WHICH NO FEASIBLE SOLUTION CAN BE FOUND BY THE LINEAR PROGRAMMING PACKAGE.
PLEASE EXAMINE FILE <IN> TO DETERMINE WHICH SET IS NOT FEASIBLE.
### Example Calculations Using INFUNC

#### Table 1

**INFLUENCE FUNCTION** -- 30E28 -- 1956 to 1975

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>NSD</th>
<th>SDF</th>
<th>DSDF</th>
<th>DS2DF</th>
<th>D3SDF</th>
<th>D3S2DF</th>
<th>D4SDF</th>
<th>D4S2DF</th>
<th>D5SDF</th>
<th>D5S2DF</th>
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<tr>
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<tr>
<td>V5</td>
<td>74</td>
<td>2</td>
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</table>

**PARAMETERS**

- SCALE = 1.000E+01

**Problem Summary**

- **Constraints**: 19
- **Variables**: 85
- **Non-Zeros**: 74
- **Real**: 13
- **Integer**: 3

**Input Translation Time**: 7570 SECONDS
# Summary of Results

## Variable Status

<table>
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<tr>
<th>Var</th>
<th>Var</th>
<th>Cons Status</th>
<th>Activity</th>
<th>Opportunity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
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<tr>
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<td>0.000000</td>
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<td>0.000000</td>
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<tr>
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<td>0.000000</td>
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</table>

**Minimum Value of the Objective Function:** 216.067277

**Calculation Time:** 4.1527 seconds for 93 iterations.

**Data Storage Memory:** 904564000 bytes. Total memory: 94500380 bytes.
Linearized Free Surface Programs

LFS
  PROGRAM LFS(INPUT=65/153,OUTPUT=65/153,LFSQUT,TAPES=INPUT,
  TAPES=OUTPUT,TAPES,TAPES=LFSQUT)

FIELD DRAWDOWN AND PRESSURE WITH A LINEARIZED FREE SURFACE CONDITION

PROGRAM BY OREGON SYSTEMS ANALYSTS
6568 RESERVOIR ROAD
GALLIS, OREGON 97330
PHONE 503/975-4522

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THIS PROGRAM CALCULATES PRESSURE AT A FIELD POINT USING A DESCRIBED
ESTIMATE OF THE GREENS FUNCTION. THE FIELD POINT LIES ON THE
STATIC EQUILIBRIUM FREE SURFACE AT SOME RADIAL DISTANCE FROM
THE BOREHOLE.

THE PROGRAM READS IN ONE DATA SET AT A TIME AND WILL CALCULATE THE
FIELD PRESSURE FOR UP TO SIX RADIAL DISTANCES FROM THE BORE.
DATA SETS MAY BE SEPARATED BY AN END-OF-FILE OR NOT BY AN END-OF-FILE.
A SINGLE DATA SET CONSISTS OF CONTROL CARDS, PARAMETERS, AND
THE TIME OR FLOW DATA FOR THE REAL OR VIRTUAL WELL BEING ANALYZED. THIS TIME-FLOW
DATA MAY BE READ EITHER IN FREE FORMAT FROM THE SAME INPUT STREAM
AS THE CONTROL CARDS OR IT MAY BE READ FROM A DESIGNATED
PERMANENT INDIRECT ACCESS FILE.

A SINGLE DATA SET CONSISTS OF CONTROL CARDS AND DATA AS DESCRIBED
BELOW. ORDER OF INPUT IS UNIMPORTANT EXCEPT THAT THE BEGIN CARD MUST COME FIRST. THE END CARD MUST COME LAST.

1) A CARD WITH THE WORD BEGIN IN COLS 1-5,
   FOLLOWED BY A TITLE CARD OF UP TO 80 CHARACTERS
1) A CARD WITH THE WORD PARAMETER IN COLS 1-10 WITH THE
   FOLLOWING PARAMETERS ENTERED IN FREE FORMAT (BLANK SEPARATED)
   AND IN ORDER:
   ONE OR MORE LINES (CARDS) MAY BE USED.
   A) THE TIME INCREMENT IN SECONDS (TSTEP)
   B) THE WELL DEPTH IN METERS (I)
   C) THE POROSITY (PHI) WHICH IS DIMENSIONLESS
   D) THE DENSITY (RHOS) IN KILOGRAMS/CUBIC METER
   E) THE PERMEABILITY (k) IN SQUARE METERS
   F) THE DYNAMIC VISCOITY (MU) IN KILOGRAMS/SECOND-METERS
   G) THE STICKINESS (M) IN KILOGRAMS/SECOND
   H) THE NUMBER OF RADII TO BE USED IN THE CALCULATIONS (MAXIMUM OF EIGHT)
   I) THE VALUES OF THE RADII (IN METERS)

**** NOTE ****

THE P-Z THICKNESS, ITEM G, IS USED IN THE SECOND TERM OF THE EXPANSION
IN THE FREE SURFACE FUNCTIONS. IF CHOOSE THE SECOND TERM IS NOT
INCLUDED IN THE GREENS FUNCTION CALCULATION.

2) A <DATA> CARD WHICH SPECIFIES LOCATION OF THE TIME AND FLOW DATA
   THERE ARE TWO FORMS OF THE DATA CARD DEPENDING ON HOW THE INPUT
   IS TO BE READ.

   A) FREE FORMAT -- FIRST READ IN A <DATA N> CARD WITH N EQUAL TO THE NUMBER OF DATA POINTS. E.G. <DATA 27>
      FOLLOWED BY <N> DATA POINTS FOR TIME, THEN READ IN <N> DATA POINTS FOR FLOW.

   B) STORED ON FILE -- READ IN A <DATA N FILE> CARD WHERE
      <FILE> IS THE FILE NAME, <FORM> IS THE INPUT FORMAT,
      <S> IS AND OPTINAL INTEGER FROM 1 TO 9 WHICH TELLS THE
      PROGRAM TO SKIP THAT MANY CARDS BEFORE READING THE DATA.
      THE FILE MUST BE AN INDIRECT ACCESS PERMANENT FILE.
      ANY LOCAL FILE WITH THE SAME NAME WILL BE IRRETRIEVABLY LOST.
      DATA IS TYPICALLY STORED IN FREE FORMAT AND MUST BE READ IN INTEGER-REAL FORMAT. AN EXAMPLE IS:
         <DATA 27 LFSQUT (110,15X,F15.5) 3>

         WHICH READS 27 OBSERVATIONS OFF THE PERMANENT FILE NAMED
         LFSQUT USING THE SPECIFIED FORMAT AND SKIPPING THE
         FIRST 3 LINES OF THE FILE/
4) An optional card which has the word <GREEN> in columns 1-5. This will print the values of the <GREEN> function for values of real time and all radii.

5) An optional <LINES N Card> where N is the maximum number of output lines on a page. The default is 66 lines. If <N> is not specified, the value will be 66. The multiplier for the first radius must have the range 10-99. This card need only be entered once per run.

6) An optional <MULT> card which indicates the program is to start summing pressure drops for a central point which is at a distance equal to the first specified radius for all wells at each time period. This summation will continue until a <MULTEND> card is found in a data set or until the end of file is reached. Additional <MULT> cards encountered between the first one and the <MULTEND> or E-O-F are extraneous and are ignored.

7) An optional <CONVERT CODE> card which tells the program to convert the flow data from the units given to KG/SEC. Valid arguments for <CODE> are:

- CODE: CONVERSION FROM:
- A TON/HOUR
- B TON/HR
- C TON/MONTH
- D TON/yr
- E LBS/HR
- F LBS/DDAY
- G LBS/DDAY EXTERNAL SUPPLIED MULTIPlicative FACTOR GIVEN AS REAL NUMBER FOLLOWING CODE LETTER <Z>.

8) An optional <FIELD P> card which specifies an initial pressure P in pounds per square foot for all. If this card is supplied then a table giving field pressure over time is produced from the calculated drawdowns. If <P> is zero an error message is given and the option is skipped.

9) An end card which has the word <END> in columns 1-3.

Input and output logical units:

- UNIT 5: Data set and control card input
- UNIT 6: Output
- UNIT 7: Optional unit assigned to file specified in <FILE> card
- UNIT 8: Temporary file <LFSDUM> assigned to store headers
READ A CONTROL CARD, DETERMINE WHICH TYPE IT IS AND GO TO PROPER SECTION

1 IF (EOF(1)) GO TO 990
   CALL FINDC(CARDS+9, IEST, I, IFIND, IGF, IRET)
   WRITE(6,105) IEST
   IF (IEST .NE. 1) GO TO 1
   IF (IEST .EQ. 6 AND IGF .GT. 2) GO TO 1
   IBEG = LOGC(BLANK, IEST, 6, 80)
   GO TO (801, 802, 803, 804, 805, 806, 807, 808, 809), ISF

CHECK FOR INACTIVE STATUS, THEN

601 IF (IEST .EQ. 1) GO TO 997
   READ(10) HDR
   IF (EOF(6), IEST, 1) GO TO 995
   WRITE(6,138)

138 FORMAT(1H1)
   CALL TIME(TM)
   CALL DATE(DAT)
   PTFEG0 = 0.8
   PTFEG1 = 0.1
   ISIG = 0
   GO TO 1

READ PARAMETERS IN FREE FORMAT

603 TEND=16
   KK = I
   IF (KK .GT. TEND) GO TO 1
   CALL PARAMS(KK) ? IPAR, 1, IEST, IBEG, 80, IRET)
   IBEG = IRET
   IRET + 1) GO TO 5
   READ ANOTHER CARD
   IBEG = 1
   WRITE(10, 101) IEST
   IF (EOF(6), IEST, 1) GO TO 983
   GO TO 4
C ADJUST LOOP TERMS IF LESS THAN 8 RADIUS SPECIFICATIONS
   M = PARAMS(8)
   N = M + 1
   TEND = 8 + M
   GO TO 3

READ TIME AND FLOW DATA
IF <FILE> OPTION, READ FROM FILE; ELSE READ FROM <INPUT>

804 NN = XPAR(2, IEST, 1, 80, IRET)
   IF (NN .LE. 0 OR .GE. 100) NN = 0
   CALL FDNRD(3, IEST, 1, 80, NAME, LEN, IRET)

DATA LOCATED ON A SEPARATE FILE.
GET FORMAT FROM THE <DATA> CARD.
CLOSE UNIT 7, ASSIGN FILE TO UNIT 7 AND READ DATA.

   CALL FDNRD(4, IEST, 1, 80, FORM, LEN, IRET)
   LENF = (LEN - 11) / 10 + 1
   IF (LENF .GE. 0) GO TO 996
   NSKIP = XPAR(5, IEST, 1, 80, IRET)
   ISIG = 1
   CALL FCLOSE (7)
   CALL UGET(7, NAME)
   IF (NSKIP .LE. 0) GO TO 16
   DO 15 I = 1, NSKIP
READ(7,101) DUMMY
15 CONTINUE
16 CONTINUE

C
READ (7,IFROM) (NTIME(I), I=1,N)
 IF (EOF(7),EQ.1) GO TO 992
 GO TO 1
C
DATA IS LOCATED ON THE INPUT UNIT.
READ IN FREE FORMAT, ALL DATA FOR TIME, FOLLOWED BY ALL DATA
FOR FLOW.
C
10 READ (15) (NTIME(I), I=1,N), (Q(I), I=1,N)
 GO TO 1
C
CHECK FOR END OF CARD.

C
<GREEN> CARD
885 GREEN = 1
 GO TO 1
C
<LINES> CARD
886 LINES = XPAR(2,ITEST,1,60,IRET)
 IF (LINES.EQ.0) LINES=81
 GO TO 1
C
= <MULT> AND <MULTEND> CARD
887 IF (IFLAGC(7,IMULTEND,1,ITEST,1),EQ.0) IMULT=2
 GO TO 1
C
= <CONVERT> CARD
888 CALL FMOMRD(2,ITEST,1,20,JCONV,LEN,SEG)
 IF (LEN.EQ.1) GO TO 980
 CALL FMC1C(JCONV,1,1,IFIND,NAMEF,IRET)
 IF (IFIND.GT.1) GO TO 998
 FACTOR = CONV(JNAMEF)
 IF (NAMEF.GT.1) GO TO 37
 FACTOR = XPAR(3,ITEST,1,60,IRET)
 IF (IFACT.LE.0) GO TO 998
 37 DO 36 I=1,N
 36 Q(I,I) = Q(I,I) * FACTOR
 GO TO 1
C
= <FIELD> CARD
889 IPZERO = XPAR(2,ITEST,1,60,IRET)
 IF (IPZERO.LE.0) GO TO 980
 IF (IPZERO.LE.0) GO TO 980
 GO TO 1
C
MANDATORY <END> CARD. START ANALYSIS.
892 IF (IFACTIV.EQ.0) GO TO 997
 IF (IFACTIV.EQ.0) GO TO 1
C
CALCULATE SQUARE OF RADIUS
 IF (NN.EQ.0) GO TO 390
 DO 22 IRAD=1,M
 22 ASQIRAD = X(IRAD) * X(IRAD)
C
CALCULATE SINKING VELOCITY, M.
 IF (MU.EQ.0 .OR. PHI.EQ.0) GO TO 994
 M = (K * GRAVITY * EQ.0) / (MU * PHI)
C
CALCULATE GREENS FUNCTION FOR VALUES OF REAL TIME T AND ALL RADII.
 IF (RHO.EQ.0) GO TO 999
 T = 1.0 / (TMQ* + PHI * RHO)
C
LOOP OVER ALL TIME PERIODS
DO 39 I=1,N
TPRIME = T + .5
TAU = TPRIME / QTEE
G2 = M * TAU + D
G2SQ = G2 * G2
G3 = I + G2
GO TO 36
G3SQ = G3 * G3

C

INTERNAL LOOP OVER RADIUS

C

GEFUN() IS A STATEMENT FUNCTION

C

FUNCTION TAU(G,B,A290) = (G/A290)**(1.5)**

300 DO 38 IRADE = 1,M
GRN = GFUNC(G2,RSQ(IRAD),G2SQ)
IF(H(N,0,0),GRN = GRN + GFUNC(G3,RSQ(IRAD),G3SQ)
GREENS(IRAD,I) = G1 = GRN
CONTINUE

C

CONTINUE

C

PRINT INITIAL TIME AND HEADER

390 CALL PEJECT(0)

C

NEED = 27
NAMES = DASH(2), DASH(1), DASH
WRITE(6,122) DAT, TIME

102 FORMAT

C

* A CONDITION FOR A FLUID PRESSURE WITH LINEARIZED FREE SURFACE*

* IF I.FORM(I),I=1,LENF)

104 FORMAT

C

READ THE TIME AND FLOW DATA READ FROM FILE I.F, I.P

105 FORMAT

C

READ THE TIME AND FLOW DATA READ FROM INPUT STREAM

NAMES = (NAME(1,NAMEF),I=1,2)

150 FORMAT

C

FLOW DATA WERE CONVERTED FROM TYPE #3, 1, 2, 10, 1, 10, 1, 2, 10, 1, 10

IF(PZERO, I, 1, 1) WRITE(6,131) PZERO

131 FORMAT

C

* A CONDITION FOR A FLUID PRESSURE WITH LINEARIZED FREE SURFACE*

* IF I.FORM(I),I=1,LENF)

WRITE(6,133) N, DEE, DPHI, KRO, KML, M, IX, RII, I=1, M

103 FORMAT

C

READ THE TIME AND FLOW DATA READ FROM FILE I.F, I.P

106 FORMAT

C

READ THE TIME AND FLOW DATA READ FROM INPUT STREAM

WRITE(6,133) N, DEE, DPHI, KRO, KML, M, IX, RII, I=1, M

110 FORMAT

C

WRITE GREENS FUNCTION RESULTS

IF(IGREEN, I, 1, 0) GO TO 41

WRITE(6,108) NEED = NEED + N + 7
CALL PEJECT(NEED), LINES, 6
WRITE(6,112) DASH
WRITE(6,108) TPRIME, GHEAD
WRITE(6,113) (IRAD, IPAD = 1, M)
WRITE(6,112) DASH

116 FORMAT

C

DO 42 I=1,N
TPRIME = T + .5
WRITE(6,115) NTIME(I), TTIME(I), (GREENS(IRAD(I),I),IRAD=1,N)
115 FORMAT(10,F13.1,8(P1E13.3))
42 CONTINUE
WRITE(6,116) OASM
WRITE(6,122) DAT,TIM

WRITE(6,116)
NEED = NEED + M + 11
CALL PEJECT(NEED,LINES,6)
WRITE(6,119) OASM
WRITE(6,106) FLO,FPHEAD
106 FORMAT(17,F13.6,T19,A10,T19,A10)
WRITE(6,117) OASM
WRITE(6,118) (R(IRAD),IRAD=1,N)
WRITE(6,119) (I,J) (KG/SEC)*T35,T50,(METERS OF HEAD)
118 FORMAT(17,F5.1,T19,A10)
119 FORMAT(17,F5.1,A1)

CALCULATE TIME SERIES FIELD PRESSURE FOR ALL RADII

LOOP OVER TIME PERIOD

DO 44 J=1,N
DO 46 IRAD=1,M
44 CONTINUE
45 Continue
46 DO 46 IRAD=1,M
46 CONTINUE

PRINT RESULTS FOR THIS TIME PERIOD

WRITE(110,118) CENTER(I), CENTER(J), CENTER(K), CENTER(I)
118 FORMAT(10,F13.3)
C
CALCULATE ACTUAL FIELD PRESSURE BASED ON DRAWDOWNS

IF(PZERO.LE.0.0) GO TO 45
45 DO 46 IRAD=1,M
46 CONTINUE
WRITE(6,122) DAT,TIM

PRINT RESULTS FOR ACTUAL FIELD PRESSURE

IF(PZERO.LE.0.0) GO TO 46
46 WRITE(6,116)
CALL PEJECT(NEED,LINES,6)
WRITE(6,119) OASM
WRITE(6,106) FLO,FPHEAD
106 FORMAT(17,F13.6,T19,A10,T19,A10)
WRITE(6,117) OASM
WRITE(6,118) (R(IRAD),IRAD=1,N)
WRITE(6,119) (I,J) (KG/SEC)*T35,T50,(POUNDS PER SQUARE INCH)
118 FORMAT(17,F5.1,T19,A10)
119 FORMAT(17,F5.1,A1)
DO 46 I=1,M
WRITE(6,107) NTIME(I),Q(I),Q(J),Q(K),Q(I)
46 CONTINUE
WRITE(6,112) OASM
WRITE(*,122) DAT, TIM

PRINT RESULTS OF CENTRAL POINT ORIENTATION IF IMULT > 1.
IMULT = 1 IMPLIES NO SUMMATION OCCURRING
IMULT = 2 IMPLIES FINISH SUMMATION, PRINT RESULTS AND RESET COUNTERS
IMULT = 3 IMPLIES E-O-F ENCOUNTERED, PRINT RESULTS AND END.

45 IF(IMULT.EQ.0) GO TO 50
50 NMULT = NMULT + 1
30 IF(NMULT.GT.EQ.11) END.
119 FORMAT(1X,6I11) HDR, R11
IF(NMUL.EQ.6.AND.NMULT.GT.1) BAD = 1
IF(IMULT.EQ.1) GO TO 58
51 REWIND 8
WRITE(6,F11.6) NEED, NEED + N + NMULT * 8
CALL PEJEST(NEED, LINES, 6)
IF(IMULT.EQ.1) PRINT =
IF(IMULT.EQ.0) ERROR DATA SETS HAVE DIFFERENT NUMBER OF TIME STEPS.
BAD = 6
N = 48 I = 1 NMULT
READ(6,F120) HBUF
WRITE(6,F121) HBUF
120 CONTINUE
121 FORMAT(1X,15A10)
43 CONTINUE
139 FORMAT(1X,F11.6) CHEAD
WRITE(6,F139) CHEAD
WRITE(6,F141) DASH
WRITE(6,F121) DASH
WRITE(6,F151) CENTER(I,1,I), CENTER(I), I = 1, N
122 FORMAT(1X,2A10) T1(I10,3X,118)
IF(IMULT.EQ.3) GO TO 991
49 CENTER(I, I) = 0.0
IMULT = 0
90 NMULT = 0
REWIND 8
50 GO TO 1
99 CONTINUE
991 IF(IMULT.EQ.0) GO TO 991
IMULT = 3
GO TO 51
991 CPU TIME = CPU + SECOND (CPU)
910 WRITE(6,F109) TIMEL, FLS, %D, P.F.L.S, PROGRAM UTILIZED, F6.3, % SECONDS CPU TIME.
STOP FEND LFS.

ERROR EXITS
992 PRENT = *** PREMATURE END-OF-FILE ENCOUNTERED ON DATA FILE.
GO TO 1000
993 PRENT = *** #END# CARD NOT FOUND, RUN ABORTED.
994 PRENT = *** VALUE FOR RHO, MU, OR PHI IS EQUAL TO ZERO.
995 PRENT = *** PREMATURE END-OF-FILE ENCOUNTERED ON INPUT UNIT.
GO TO 999
996 PRENT = *** ERROR IN #DATA# CARD.
GO TO 1000
997 PRENT = *** ERROR. #BEGIN# AND #END# CARDS OUT OF ORDER.
GO TO 999
998 PRENT = *** ERROR IN #CONVERT# CARD.
GO TO 1000
999 PRENT = *** INITIAL PRESSURE SPECIFICATION IS ABSENT, ZERO OR #.
PRENT = *** AN INTEGER. FIELD PRESSURES CANNOT BE CALCULATED.
PZERO = 0.0
GO TO 1000
981 PRENT = *** ERROR IN #CONVERT# CARD. NO DECIMAL POINT IN #.
PRENT = *** FACTOR SPECIFICATION.
GO TO 1000
982 PRINT **** NO #BEGIN# CARD, OR OUT OF PLACE#
   GO TO 999
983 PRINT **** ERROR IN PARAMETER LIST#
   GO TO 995
984 PRINT **** ERROR IN CONTROL CARDS#
   GO TO 1000
985 PRINT **** NUMBER OF DATA POINTS EXCEEDS MAXIMUM OF 100#
   GO TO 1000
999 PRINT **** RUN ABORTED#
   GO TO 991
1000 PRINT **** ANALYSIS ON THIS DATA SET ABORTED#
   IACTIVE=-1
   GO TO 1
END
--- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ---

**L.F.S. FOR WAIKAKE GROUP 1, 1953 TO 1976**

REFERENCE DATE AND TIME: 05/05/71. 17:35.36.

TIME AND FLOW DATA READ FROM FILE #MLFSG1 # WITH FORMAT (T10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 3 (1000 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43379926E-02

### INPUT PARAMETERS:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>NUMBER OF DATA POINTS (N)</td>
<td>24</td>
</tr>
<tr>
<td>DELTA-TAU</td>
<td>3.156E+07 SECONDS</td>
</tr>
<tr>
<td>FLUID LEVEL (SG)</td>
<td>1.610E+02 METERS</td>
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<tr>
<td>DENSITY (GMD)</td>
<td>6.438E+01 KG/CM3</td>
</tr>
<tr>
<td>PERMEABILITY (KI)</td>
<td>4.000E-14 SG-METERS</td>
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<tr>
<td>DYNAMIC VISCOSITY (MU)</td>
<td>1.699E-04 KG/CM3-SEC</td>
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<tr>
<td>SINKING VELOCITY (U11)</td>
<td>5.445E-06 METER/SEC</td>
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<td>RADIUS SPECIFICATIONS</td>
<td>990.00</td>
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### F.L.S. FOR WAIKAKE GROUP 1, 1953 TO 1976

<table>
<thead>
<tr>
<th>TIME (KG/SEC)</th>
<th>FLOW FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>METERS 900.00 0.00 941.00 1692.00 1540.00 1586.00 1512.00</td>
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<tr>
<td>1953</td>
<td>19.779 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1954</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1955</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1956</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1957</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1958</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1959</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1960</td>
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<td>1962</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1963</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<tr>
<td>1964</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1966</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1967</td>
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<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1972</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1973</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
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<td>1976</td>
<td>19.344 0.253 9.448 0.277 0.297 0.115 0.103 0.093</td>
</tr>
</tbody>
</table>

REFERENCE DATE AND TIME: 05/05/71. 17:35.36.
### Field Point Pressures with Linearized Free Surface Condition

**L.F.S. for Wairakei Group 2, 1953 to 1970**

**Input Parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Data Points (N)</td>
<td>24</td>
</tr>
<tr>
<td>Delta-Tau</td>
<td>3.15E-07 Seconds</td>
</tr>
<tr>
<td>Fluid Level (ft)</td>
<td>2.07E-01 Meters</td>
</tr>
<tr>
<td>Density (g/cc)</td>
<td>9.94E-03 Kg/Cu. Meters</td>
</tr>
<tr>
<td>Permeability (Mu)</td>
<td>1.08E-03 Kg/Cu.-Second</td>
</tr>
<tr>
<td>F.P. Thickness (ft)</td>
<td>6.20E-02 Meters</td>
</tr>
<tr>
<td>Starting Fluid Velocity (ft)</td>
<td>2.34E-08 Meters/Second</td>
</tr>
</tbody>
</table>

**Radius Specifications:**

- 211.00 Ft
- 9.00 Kg
- 94.10 Meters
- 1017.00 Kg
- 734.00 Kg
- 709.00 Kg
- 306.00 Kg

**L.F.S. for Wairakei Group 2, 1953 to 1970**

**Time**

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>Flow (Kg/Sec)</th>
<th>Field Drawdown at Specified Distance from Borehole (Meters of Head)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.708</td>
<td>211.00</td>
</tr>
<tr>
<td>110</td>
<td>1.730</td>
<td>9.00</td>
</tr>
<tr>
<td>120</td>
<td>1.750</td>
<td>94.10</td>
</tr>
<tr>
<td>130</td>
<td>1.770</td>
<td>1017.00</td>
</tr>
<tr>
<td>140</td>
<td>1.790</td>
<td>734.00</td>
</tr>
<tr>
<td>150</td>
<td>1.810</td>
<td>709.00</td>
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<tr>
<td>160</td>
<td>1.830</td>
<td>306.00</td>
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**Reference Date and Time:** 06/05/51. 17.16.38.
### FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION

#### L.F.S. FOR NAIKKEI GROUP 3, 1953 TO 1976

**REFERENCE DATE AND TIME: 09/05/11. 17.10.61.**

**TIME AND FLOW DATA READ FROM FILE #MLFSG3 # WITH FORMAT: (11S,15X,F15.2)**

**FLOW DATA WERE CONVERTED FROM TYPE 6 (1065 POUNDS/YEAP) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.437792E-32**

#### INPUT PARAMETERS:

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<thead>
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<th>Parameter</th>
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<tbody>
<tr>
<td>NUMBER OF DATA POINTS (N)</td>
<td>24</td>
</tr>
<tr>
<td>DELTA-FLUID (D1)</td>
<td>1.1564E+17 SECONDS</td>
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<tr>
<td>POROSITY (PH1)</td>
<td>5.791E-12 METERS</td>
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<tr>
<td>DENSITY (KG/M3)</td>
<td>6.174E+02 KG/M3</td>
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<tr>
<td>DYNAMIC VISCOSITY (MU)</td>
<td>1.092E+14 KG/M3-SECOND</td>
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<tr>
<td>SINKING VELOCITY (M1)</td>
<td>6.492E-03 METER/SECOND</td>
</tr>
<tr>
<td>RADIUS SPECIFICATIONS</td>
<td>427.00</td>
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</tbody>
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#### L.F.S. FOR NAIKKEI GROUP 3, 1953 TO 1976

**REFERENCE DATE AND TIME: 09/05/11. 17.10.61.**

<table>
<thead>
<tr>
<th>TIME (METERS)</th>
<th>FLOW (KG/SEC)</th>
<th>FIELD DROPOUT AT SPECIFIED DISTANCE FROM BOREHOLE (METERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1955</td>
<td>0.000</td>
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</tr>
<tr>
<td>1975</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**REFERENCE DATE AND TIME: 09/05/11. 17.10.61.**
## FIELD POINT Pressures With Linearized Free Surface Condition

**L.F.S. FOR HAIRAKEI GROUP 4, 1953 TO 1976**

**REFERENCE DATE AND TIME: 80/05/11, 17.36.47.**

**TIME AND FLOW DATA READ FROM FILE JULPS14 # WITH FORMAT (110,19X,F9.2)**

**FLOW DATA WERE CONVERTED FROM TYPE 8 (110.65 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.4379296E-02**

### INPUT PARAMETERS:

<table>
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<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Number of Data Points (N)</td>
<td>24</td>
</tr>
<tr>
<td>Delta-Z (ft)</td>
<td>3.186E+02 SECONDS</td>
</tr>
<tr>
<td>Fluid Level (D1)</td>
<td>2.186E+02 SECONDS</td>
</tr>
<tr>
<td>Density (rho)</td>
<td>1.08E+03 KG/CM CUBED</td>
</tr>
<tr>
<td>Radius (R)</td>
<td>6.18E+02 KG/CM CUBED</td>
</tr>
<tr>
<td>Dynamic Viscosity (mu)</td>
<td>1.999E-05 KG/CM-SECOND</td>
</tr>
<tr>
<td>Core Thickness (h)</td>
<td>2.30E-02 METERS</td>
</tr>
<tr>
<td>Drilling Velocity (Vz)</td>
<td>1.999E-02 SECONDS</td>
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<tr>
<td>RADIUS SPECIFICATIONS</td>
<td>576.00 0.00 194.00 734.00 368.00 194.00 368.00</td>
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</table>

**L.F.S. FOR HAIRAKEI GROUP 4, 1953 TO 1976**

### TIME

<table>
<thead>
<tr>
<th>Flow</th>
<th>Field Drawdown at Specified Distance from Borehole</th>
</tr>
</thead>
<tbody>
<tr>
<td>(KG/SEC)</td>
<td>METERS OF RADIUS</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
</tr>
<tr>
<td>1956</td>
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<tr>
<td>1957</td>
<td>8.000</td>
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**REFERENCE DATE AND TIME: 80/05/11, 17.36.47.**
### Field Point Pressures with Linearized Free Surface Condition

**L.F.S. for Wairakei Group 5, 1953 to 1976**

**Reference Date and Time:** 08/05/11, 17:16:49

**Time and Flow Data Read from File #MLFS05 # with Format (U10.15, X, F15.2)**

Flow data were converted from type 6 (10866 pounds/year) to kg/sec by the multiplicative factor 1.4377929E-02

**Input Parameters:**

- Number of Data Points (N): 24
- Delta-Tau (sec): 3.56E+07 seconds
- Fluid Level (L): 3.36E-01 meters
- Porosity (Phi): 4.00E-01
- Density (ρ₀): 2.14E+02 kg/cu. m
- Permeability (K): 4.06E-14 m²
- Dynamic Viscosity (μ): 1.09E-06 kg/meter-second
- P.I. Thickness (H): 2.28E-06 meters
- Sinking Velocity (W): 9.99E-06 meter/second
- Radius Specifications: 682.00 4.90 1588.00 788.00 533.00 194.00 311.00

**L.F.S. for Wairakei Group 5, 1953 to 1976**

<table>
<thead>
<tr>
<th>Time (KD/sec)</th>
<th>Field Drawdown at Specified Distance from Borehole (Meters of Head)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>0.000</td>
</tr>
<tr>
<td>1962</td>
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<tr>
<td>1966</td>
<td>37.77</td>
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<td>1969</td>
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**Reference Date and Time:** 08/05/11, 17:16:49
--- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ---

REFERENCE: FOR HAIRAKEI GROUP 6, 1953 TO 1976

REFERENCE DATE AND TIME: 26/05/11. 17:16.51.


FLOW DATA WERE CONVERTED FROM TYPE B (1086 POUNDS/Year) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779250 E-02.

INPUT PARAMETERS:

<table>
<thead>
<tr>
<th>NUMBER OF DATA POINTS (N)</th>
<th>24</th>
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</thead>
<tbody>
<tr>
<td>DELTA-TAU (sec)</td>
<td>3.156E-07</td>
</tr>
<tr>
<td>FLUID LEVEL (m)</td>
<td>3.306E+02</td>
</tr>
<tr>
<td>DENSITY (kg/m^3)</td>
<td>0.080E+03</td>
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<td>K (m/s)</td>
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<td>PERMEABILITY (m^2)</td>
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<td>DYNAMIC VISCOSITY (kg/m/s)</td>
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<td>SINKING VELOCITY (m/s)</td>
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LFS FOR HAIRAKEI GROUP 6, 1953 TO 1976

<table>
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<tr>
<th>TIME (sec)</th>
<th>FLOW (kg/sec)</th>
<th>FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE</th>
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REFERENCE DATE AND TIME: 26/05/11. 17:16.51.
<table>
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<tr>
<th>TIME</th>
<th>FIELD DECLINATION AT CENTRAL POINT IN METERS OF HEAD</th>
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REFERENCE DATE AND TIME: 30/05/11. 17.16.51.

L.F.S. PROGRAM UTILIZED 3.415 SECONDS CPU TIME