NEUTRINO MASSES, MIXING, MOMENTS, AND MATTER*

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ABSTRACT

The present status of neutrino masses, mixing, and electromagnetic moments is surveyed. Potential enhancements of neutrino oscillations, decay, and spin-flavor precession due to their interactions with matter are described.

1. NEUTRINO MASSES

In the standard $SU(2)_L \times U(1)$ model, the photon and all three species of neutrinos have zero mass. For the photon, masslessness is a natural consequence of exact electromagnetic gauge invariance; its validity being well verified experimentally by the present bound¹ $m_\gamma < 3 \times 10^{-27}$ eV. However, the masslessness of neutrinos is not on such firm theoretical or experimental footing. Theoretically, $m_\nu = 0$ because only the left-handed component ν_L of each neutrino species is employed (the right-handed component ν_R is assumed not to exist) and lepton number conservation is required. Relaxing either of these constraints can lead to $m_\nu \neq 0$. Indeed, the present experimental bounds^{2,3,4}

$$m_{\nu_e} < 18 \text{ eV}, \quad (1a)$$

$$m_{\nu_\mu} < 0.25 \text{ MeV}, \quad (1b)$$

$$m_{\nu_\tau} < 35 \text{ MeV}, \quad (1c)$$

leave considerable room for speculation that neutrinos actually do possess mass. I might add that the experimental bounds in (1) have improved by factors of $2 \sim 5$ since I last reviewed⁵ this subject in 1981. Progress is slow, but we can expect further improvement by factors of ~ 3 during the next few years. A worthy goal would be a further factor of 5 to 10 reduction.

In the standard model charged leptons and quarks acquire masses by coupling their left- and right-handed components through a Higgs scalar isodoublet which obtains a vacuum expectation value. (Mass terms change chirality, so one must couple left-to-right in order to generate mass.) The fermion masses generated by the Higgs mechanism are totally arbitrary, their values are chosen to agree with experiment. If right-handed components exist, then neutrinos can also be given arbitrary masses by the standard Higgs mechanism. The right-handed components will be sterile under $SU(2)_L \times U(1)$ gauge interactions; so,

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they do not upset phenomenology. Of course, in such a scenario one must address the question: Why are neutrino masses so much smaller than the masses of other fermions in their generation?

Even without introducing right-handed neutrinos, the neutrino can be given a Majorana mass by coupling ν_L to its (right-handed) charge conjugation ν_L^c (which I subsequently call $\bar{\nu}$, the antineutrino). The mass term would appear as

$$\frac{1}{2} m_\nu (\bar{\nu}_L^c \nu_L + h.c.). \quad (2)$$

(It requires anticommuting fields.) A Majorana mass is not possible for other fermions because it would violate electric charge conservation. In the case of the neutrino, lepton number is violated $\Delta L = 2$ by Eq. (2).

The simplest way of introducing a Majorana mass is to couple the neutrino-charged lepton $SU(2)_L$ doublet to a Higgs triplet.⁶ The implications of such a scenario are the occurrence of neutrinoless double beta decay (so far not observed) and the existence of additional physical scalars associated with the Higgs triplet. In particular, a light scalar and pseudoscalar (Majoron) can exist in such a scenario. This mechanism can accommodate a small neutrino mass, but also fails to naturally explain small masses.

In my opinion, the most compelling rationale for small neutrino masses is found by combining the two mass generating mechanisms mentioned above in the see-saw⁷ mechanism. The basic idea is motivated by grand unified theories (GUTS) which often contain very heavy right-handed Majorana neutrinos, ν_R . Since ν_R is a singlet under $SU(2)_L \times U(1)$, it is expected that large Majorana masses M_R will arise for such fields. On the other hand, the ordinary Higgs mechanism should lead to a Dirac mass, m_f , which couples ν_R to ν_L . Of course, m_f is expected to be a typical fermion mass scale, i.e., of order m_e , m_μ or m_d for the first generation. Therefore, one expects a mass matrix of the form

$$(\bar{\nu}_L^c \quad \bar{\nu}_R) \begin{pmatrix} 0 & m_f \\ m_f & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad (3)$$

Diagonalization leads to a heavy mass $O(M_R)$ Majorana state which is primarily ν_R and a light Majorana state with mass m_f^2/M_R which is the ordinary neutrino. Since $M_R \gg m_f$, the induced mass is naturally very small. In such a scenario, one expects the three neutrinos to have a hierarchical mass relationship

$$m_3^2 \gg m_2^2 \gg m_1^2, \quad (4)$$

since

$$m_1^2 : m_2^2 : m_3^2 :: m_e^4 : m_\mu^4 : m_t^4, \quad (5)$$

(or $m_e^4 : m_\mu^4 : m_t^4$) in many scenarios. If that is the case, then one might (approximately) anticipate

$$m_1^2 : m_2^2 : m_3^2 :: 1 : 10^{10} : 10^{16}. \quad (6)$$

That hierarchy could have important consequences for neutrino oscillations as we shall see. Of course, at issue is what value of M_R sets the neutrino mass scale? Is it the GUT unification scale $\sim 10^{15} \sim 10^{18}$ GeV or something smaller? If the GUT scale enters, then neutrino masses are likely to be very small and extremely hard to determine.

2. MIXING AND OSCILLATIONS

If neutrinos have mass, then one expects them to mix. In analogy with quark mixing, the weak interaction states ν_e , ν_μ and ν_τ will be related to the mass eigenstates ν_1 , ν_2 and ν_3 by a unitary matrix. It is convenient to parameterize the mixing by⁸

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} C_1 C_3 & S_1 C_3 & S_3 e^{-i\delta} \\ -S_1 C_2 - C_1 S_2 S_3 e^{i\delta} & C_1 C_2 - S_1 S_2 S_3 e^{i\delta} & S_2 C_3 \\ S_1 S_2 - C_1 C_2 S_3 e^{i\delta} & -C_1 S_2 - S_1 C_2 S_3 e^{i\delta} & C_2 C_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$C_i \equiv \cos \theta_i, \quad S_i \equiv \sin \theta_i, \quad i = 1, 2, 3. \quad (7)$$

Although it is probably best to keep an open mind about neutrino mixing, if the quark analogy holds, one expects small θ_i .

Neutrino masses and mixing can lead to many interesting phenomena in laboratory experiments, astrophysics, and cosmology. For example, neutrino decay (e.g., $\nu_\mu \rightarrow \nu_e + \gamma$) may occur or neutrinos may have small electromagnetic moments. A particularly popular possibility is neutrino oscillation from one specie to another. I will briefly survey that broad subject.

If neutrinos have masses and mix, then neutrino oscillations will occur. A neutrino ν_ℓ produced at time 0 may at a later time be observed as a $\nu_{\ell'}$. To illustrate the basic features, first consider the simplified case of only two species ν_ℓ and $\nu_{\ell'}$ such that

$$\begin{pmatrix} \nu_\ell \\ \nu_{\ell'} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad (8)$$

with ν_1 and ν_2 mass eigenstates with masses m_1 and m_2 respectively. If at time $t = 0$, a $|\nu_\ell\rangle = |\nu_1\rangle \cos \theta + |\nu_2\rangle \sin \theta$ is produced with momentum $\vec{p}$, then at a latter time t , it will evolve to

$$i \frac{d}{dt} |\nu_i\rangle = E_i |\nu_i\rangle, \quad i = 1, 2$$

$$|\nu(t)\rangle = e^{-iE_1 t} |\nu_1\rangle \cos \theta + e^{-iE_2 t} |\nu_2\rangle \sin \theta \quad (9)$$

The probability of detecting a ν_ℓ at distance $R \simeq t$ (for $\beta \simeq 1$) is therefore given by

$$P(R)_{\nu_\ell \rightarrow \nu_\ell} = |\langle \nu_\ell | \nu(t) \rangle|^2 = 1 - \sin^2 2\theta \sin^2 \left[\frac{\Delta m_{21}^2 R}{4 |\vec{p}|} \right] \quad (10a)$$

while the probability of observing a ν'_ℓ is

$$P(R)_{\nu_\ell \rightarrow \nu'_\ell} = |\langle \nu'_\ell | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left[\frac{\Delta m_{21}^2 R}{4 |\vec{p}|} \right] \quad (10b)$$

where $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$. (I have used $E_i \simeq |\vec{p}| + \frac{1}{2} m_i^2 / |\vec{p}|$.) It oscillates between ν_ℓ and ν'_ℓ with a characteristic oscillation length L_{21}

$$L_{21} = \frac{4\pi |\vec{p}|}{\Delta m_{21}^2} \quad (11)$$

So, low energy neutrinos are more likely to oscillate. If one believes that Δm_{ij}^2 are all small as in the see-saw mechanism, then one must indeed employ neutrinos with an extremely large R/E_ν to have any hope of observing oscillations. At present, laboratory reactor and accelerator experiments looking for $\nu_e \leftrightarrow \nu_\mu$ oscillations have ruled out⁹ $\Delta m_{21}^2 \gtrsim 10^{-2} \text{ eV}^2$ for $\sin^2 2\theta \gtrsim 0.1$ and $\Delta m_{21}^2 \gtrsim 1 \text{ eV}^2$ for $\sin^2 2\theta \gtrsim 0.01$. Bounds on $\nu_\mu \rightarrow \nu_\tau$ are not as good (see fig. 1). To explore much smaller Δm_{21}^2 requires atmospheric or astrophysical neutrino experiments in which R can be very large.

Fig. 1: Bounds on neutrino oscillations.⁹ These constraints were summarized by F. Vannucci in 1987. The only update that I have included is deletion of an oscillation signal at the Bugey reactor. Recent bounds from BNL and LAMPF on $\nu_\mu \leftrightarrow \nu_e$ oscillations are similar to the BEBC PS results.

In the real world, there are (at least) three neutrinos, and their mixing can be quite complicated. A full description of neutrino oscillations in the 3 generation scenario will depend on 6 parameters, θ_1 , θ_2 , θ_3 , δ , Δm_{31}^2 and Δm_{21}^2 (see (7)). However, things are considerably simplified in the hierarchy case $m_3^2 \gg m_2^2, m_1^2$. Then oscillations effectively become a 2x2 problem with "effective" mixing parameters. The relevant Δm^2 will depend on R/E_ν . To illustrate how decoupling occurs, consider the case $R/E_\nu \sim 0$ ($1/\Delta m_{31}^2$) $\ll 1/\Delta m_{21}^2$ which may be most relevant for terrestrial experiments. In that case, the oscillation probabilities are simply governed by⁸

$$P(R)_{\nu_\mu \leftrightarrow \nu_e} \simeq \sin^2 \theta_2 \sin^2 2\theta_3 \sin^2 \left[\frac{\Delta m_{31}^2 R}{4E_\nu} \right] \quad (12a)$$

$$P(R)_{\nu_\mu \leftrightarrow \nu_\tau} \simeq \sin^2 2\theta_2 \cos^4 \theta_3 \sin^2 \left[\frac{\Delta m_{31}^2 R}{4E_\nu} \right] \quad (12b)$$

$$P(R)_{\nu_e \leftrightarrow \nu_e} \simeq 1 - \sin^2 2\theta_3 \sin^2 \left[\frac{\Delta m_{31}^2 R}{4E_\nu} \right] \quad (12c)$$

The oscillation length L_{21} is too long to effect the evolution. For small mixing angles, one finds

$$P(R)_{\nu_\mu \leftrightarrow \nu_\tau} \simeq P(R)_{\nu_\mu \leftrightarrow \nu_e} / \sin^2 \theta_3 \quad (13)$$

This suggests that accelerators which produce ν_μ beams may be much better suited to search for $\nu_\mu \rightarrow \nu_\tau$ oscillations rather than $\nu_\mu \rightarrow \nu_e$. So far, most experiments have concentrated on $\nu_\mu \rightarrow \nu_e$.

For the other extreme $R/E_\nu \sim 0$ ($1/\Delta m_{21}^2$) $\gg 1/\Delta m_{31}^2$, (very long distances or small energies) oscillations are governed by Δm_{21}^2 in an effective 2 neutrino system. So, for example one finds

$$P(R)_{\nu_e \leftrightarrow \nu_e} = \sin^4 \theta_3 + \cos^4 \theta_3 \left(1 - \sin^2 2\theta_1 \sin^2 \frac{\Delta m_{21}^2 R}{4E_\nu} \right) \quad (14)$$

It is interesting to compare this formula with the observed flux of solar neutrinos where a long standing problem exists. One expects neutrinos produced in the solar core to give rise to $\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$ at the rate of¹⁰

$$7.9 \pm 2.5 \text{ SNU} \quad (1 \text{ SNU} = 10^{-36} \text{ captures/atom-s}) \quad (15)$$

However, Davis and collaborators have observed for the last 17 years an average flux of¹¹

$$2.2 \pm 0.3 \text{ SNU} \quad (16)$$

The discrepancy constitutes the solar neutrino problem. What happened to the missing flux? A simple solution is that it was never present, i.e., the theoretical prediction in (15) is wrong. More exotic is the possibility that the ν_e flux was depleted by oscillations. That could occur if Δm_{21}^2 is fine-tuned to the earth-sun

distance. More naturally for $\Delta m_{21}^2 \gg R/E_\nu$, one finds from (15) a survival probability average

$$P(R)_{\nu_e \rightarrow \nu_e} = \sin^4 \theta_3 + \cos^4 \theta_3 (\cos^4 \theta_1 + \sin^4 \theta_1) \quad (17)$$

which can be quite small for particular mixing angles. The smallest value of (17) is $1/3$ for $\sin^2 \theta_1 = 1/2$, $\sin^2 \theta_3 = 1/3$. Such a depletion would solve the solar neutrino problem; but it still appears somewhat contrived and requires relatively large mixing which runs counter to our experience with quark mixing and theoretical prejudices.

A truly ingenious solution to the solar neutrino puzzle has been proposed by Mikheyev and Smirnov.¹² Employing an analysis of matter effects on neutrino oscillations by Wolfenstein,¹³ they showed that for a large range of neutrino masses and mixing parameters, neutrino oscillations between ν_e and ν_μ or ν_τ in the sun's interior could be significantly enhanced. In fact, for some neutrino energies one can get a nearly complete transformation of ν_e into another neutrino flavor. That scenario (referred to as the MSW effect) provides an elegant natural solution to the solar neutrino puzzle and hence has become very popular. It enters into the planning for future solar neutrino experiments as well as their interpretation. I describe in section 4 some of its salient features and potential implications. I might note that for the last year Ray Davis¹¹ has been averaging 5 SNU. One does not know whether that represents a statistical fluctuation or an important hint regarding resolution of the solar neutrino puzzle.

3. ELECTROMAGNETIC MOMENTS

Although neutrinos are electrically neutral, they can have electromagnetic form factors. In the case of 4 component Dirac neutrino¹⁴

$$\begin{aligned} \langle \nu | J_\mu^{em} | \nu \rangle = & \bar{u}_\nu [(F(q^2) + \gamma_5 G(q^2)) \gamma^\alpha (g_{\alpha\mu} - \frac{q_\alpha q_\mu}{q^2}) \\ & + (M(q^2) + \gamma_5 E(q^2)) i \sigma_{\mu\alpha} q^\alpha] u_\nu \end{aligned} \quad (18)$$

The F and G form factors are charge radii while $M(0)$ is the magnetic dipole moment and $E(0)$ is the electric dipole moment of the neutrino. In the case of Majorana neutrinos (which are self-conjugate), $F = M = E = 0$, i.e., only $G(q^2)$ can be non-vanishing.

Both Dirac and Majorana neutrinos can also have transition moments that are off diagonal, i.e., connect mass eigenstates ν_2 and ν_1 . Magnetic and electric transition moments can give rise to neutrino decay or flavor precession as we shall see.

I will say a few words about the neutrino electromagnetic and transition moments. First consider a transition moment $\kappa_{21} e/2m_e$ between ν_2 and ν_1 such that the decay amplitude for $\nu_2 \rightarrow \nu_1 + \gamma$ is given by¹⁵

$$\mathcal{M} = \kappa_{21} \frac{e}{2m_e} \varepsilon^\mu q^\nu \bar{\nu}_1 \sigma_{\mu\nu} (1 - \gamma_5) \nu_2 \quad (19)$$

In that case, the decay rate for $\nu_{2L} \rightarrow \nu_{1R} + \gamma$ is

$$\Gamma(\nu_{2L} \rightarrow \nu_{1R} + \gamma) = \frac{\alpha}{2m_e^2} \left(\frac{m_2^2 - m_1^2}{m_2} \right)^3 \kappa_{21}^2 \quad (20)$$

Of course, for κ_{21} and $m_2 - m_1$, very small, as expected, the predicted decay rate will be tiny. Indeed, it can easily exceed the age of the universe.

If the neutrino is 4 component (Dirac), it can have a (diagonal) magnetic and/or electric dipole moment. Of course, the electric dipole moment would violate T and would most likely be the smaller. In the standard $SU(2)_L \times U(1)$ with massive Dirac neutrinos, one finds that one loop induced magnetic dipole moment $\mu = \kappa e/2m_e$ is^{15,16}

$$|\mu| = \frac{3eG_\mu m_\nu}{8\sqrt{2}\pi^2} \leq 3 \times 10^{-19} (m_\nu/1\text{eV}) e/2m_e \quad (21)$$

which is very small. (Transition moments are generally even smaller since they involve flavor mixing.) One, therefore, expects $\kappa \lesssim 10^{-19}$ for very light neutrinos, unless some new chiral changing physics enters.

Magnetic, electric, and transition moments can be bounded by laboratory experiments and/or astrophysics experiments. In the case of $\nu - e$ scattering, the existence of any such moment would increase the cross-section by (neglecting neutrino mass)¹⁷

$$\Delta \frac{d\sigma(\nu e)}{dy} = \frac{\pi \alpha^2 \kappa^2}{m_e^2} \left(\frac{1}{y} - 1 \right) \quad (22)$$

where $y = (E'_e - m_e)/E_\nu$. Note that it is rather insensitive to the initial ν energy. Therefore, low energy experiments where the usual cross-section $\sim E_\nu$ are most sensitive to κ . From existing experimental data, one finds^{18,19}

$$|\kappa_{\nu\mu}| < 10^{-9} \quad (23)$$

$$|\kappa_{\nu e}| < 4 \times 10^{-10} \quad (24)$$

(Those bounds also apply to electric and transition moments.) It will be difficult to push those direct experimental bounds much farther, since the effect in (22) goes like κ^2 . Better bounds,²⁰

$$|\kappa_\nu| < 8.5 \times 10^{-11} \quad (25)$$

are obtainable from stellar evolution arguments. Recently, Fukugita and Yazaki²¹ have argued that such bounds can be extended to $\kappa \lesssim 10^{-11}$ using observed data on Red Giants. A value of $\kappa \gtrsim 10^{-11}$ would have important implications for solar neutrinos, as we shall see in the next section. Therefore, laboratory experiments should also aim to achieve that level of sensitivity. I might note that rather conservative analyses²² of supernova dynamics combined with the observation of expected neutrino flux from 1987a give the bound $\kappa < 10^{-12}$. That bound does not apply to Majorana transition moments.

4. MATTER EFFECTS

To understand the effect of matter on neutrino propagation, consider the amplitude for low energy neutrino, ν_ℓ , scattering off a fermion f .

$$\mathcal{M}(\nu_\ell f \rightarrow \nu_\ell f) = -i \frac{G_\mu}{\sqrt{2}} \bar{\nu}_\ell \gamma^\alpha (1 - \gamma_5) \nu_\ell \bar{f} \gamma_\alpha \left(C_{\nu_\ell f}^V + C_{\nu_\ell f}^A \gamma_5 \right) f \quad (26)$$

where C^V and C^A are constants and $G_\mu = 1.16636 \times 10^{-5} \text{ GeV}^{-2}$. In the case of an unpolarized medium, the coherent forward scattering of neutrinos with momentum p_ν can be described by an index of refraction n_{ν_ℓ} given by²³

$$p_\nu (n_{\nu_\ell} - 1) = -\sqrt{2} G_\mu \sum_{f=e,u,d} C_{\nu_\ell f}^V N_f \quad (27)$$

where the N_f is the fermion number density and

$$C_{\nu_\ell f}^V = T_{3f} - 2Q_f \sin^2 \theta_W \quad f \neq \ell \quad (28a)$$

$$C_{\nu_\ell \ell}^V = 1 + T_{3\ell} - 2Q_\ell \sin^2 \theta_W \quad (28b)$$

with $T_{3f} = \pm \frac{1}{2}$ the weak isospin. (Quantum loop corrections to (28) are given in ref. 24). Note that $\nu_e e$ scattering differs from $\nu_\mu e$ and $\nu_\tau e$ scattering because of an additional charged current component in the amplitude. That effect leads to an important difference in indices of refraction

$$-p_\nu (n_{\nu_e} - n_{\nu_\mu}) = -p_\nu (n_{\nu_e} - n_{\nu_\tau}) = \sqrt{2} G_\mu N_e \quad (29)$$

which can significantly modify oscillations. An alternative description of the difference in (29) can be given by writing the amplitude in (26) as an effective Lagrangian

$$\mathcal{L} = -\sqrt{2} G_\mu \bar{\nu}_\ell \gamma^\alpha \frac{1 - \gamma_5}{2} \nu_\ell \bar{f} \gamma_\alpha \left(C_{\nu_\ell f}^V + C_{\nu_\ell f}^A \gamma_5 \right) f \quad (30)$$

We can average the Lagrangian over the background matter medium using

$$\langle C_{\nu_\ell f}^V \bar{f} \gamma_0 f \rangle = C_{\nu_\ell f}^V N_f \quad (31)$$

whereas all other terms average to zero. One can, therefore, interpret the medium as providing an external potential

$$V = \sqrt{2} G_\mu \sum_f C_{\nu_\ell f}^V N_f \quad (32)$$

felt by neutrinos. With such a potential, neutrino propagation is described by replacing

$$i \frac{d}{dt} \rightarrow i \frac{d}{dt} - V \quad (33)$$

in the equation of motion. (For antineutrinos $V_{\bar{\nu}} = -V_{\nu}$.)

Although the magnitude of the potential is small

$$\sim 4 \times 10^{-14} \text{ eV } (N_f/6 \times 10^{23} \text{ cm}^{-3}),$$

it can have truly remarkable consequences when it interferes with an equally small vacuum energy difference $\Delta m^2/2E_{\nu}$. Several examples will be illustrated below.

4.1 MSW Oscillations

Only the difference of potentials affects neutrino oscillations in matter. In ordinary matter that difference is due to an additional repulsive interaction for the ν_e with electrons in the medium coming from charged current interactions. In that way, the coupled equations of motion in matter become (neglecting quantum corrections)²⁴ up to common diagonal terms

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2/2p_{\nu} & 0 \\ 0 & 0 & \Delta m_{31}^2/2p_{\nu} \end{pmatrix} U^{-1} + \begin{pmatrix} \sqrt{2}G_{\mu}N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} \quad (34)$$

where U is the unitary matrix in (7).

To illustrate the MSW effect, consider the two generation limit, i.e., $\nu_e - \nu_{\mu}$ oscillations with $S_2 = S_3 = 0$ in (7). For that case (34) becomes

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} = \begin{pmatrix} \sqrt{2} G_{\mu} N_e & \frac{-\Delta m_{21}^2}{4p_{\nu}} \sin 2\theta \\ -\frac{\Delta m_{21}^2}{4p_{\nu}} \sin 2\theta & \frac{\Delta m_{21}^2}{2p_{\nu}} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} \quad (35)$$

As we saw before, in a vacuum with $N_e = 0$, $\nu_e \rightarrow \nu_{\mu}$ oscillations are governed by (see (10))

$$P(R)_{\nu_e \rightarrow \nu_{\mu}} = \sin^2 2\theta \sin^2 [\pi R/L_{12}] \quad (36a)$$

with

$$L_{21} = 4\pi p_{\nu}/\Delta m_{21}^2 \quad (36b)$$

the vacuum oscillation length. Solving (35) for constant N_e , one finds the modified matter oscillation result

$$P(R)_{\nu_e \rightarrow \nu_{\mu}} = \sin^2 2\theta_m \sin^2 \left(\frac{\pi R}{L_m} \right) \quad (37a)$$

where

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{1 - 2 \frac{L_{21}}{L_0} \cos 2\theta + \left(\frac{L_{21}}{L_0} \right)^2} \quad (37b)$$

$$L_m = \frac{L_{21}}{\left[1 - 2 \frac{L_{21}}{L_o} \cos 2\theta + \left(\frac{L_{21}}{L_o}\right)^2\right]^{1/2}} \quad (37c)$$

$$L_o = \frac{2\pi}{\sqrt{2} G_\mu N_e} \quad (37d)$$

so, we see that the matter mixing angle θ_m can be large even if the vacuum angle θ is small. In fact, if the condition

$$L_{21}/L_o = \cos 2\theta \quad (\text{Resonance Condition}) \quad (38)$$

which corresponds to an electron density

$$N_e = \frac{\Delta m_{21}^2 \cos 2\theta}{2\sqrt{2} G_\mu p_\nu} \quad (39)$$

is satisfied, one finds $\theta_m = 45^\circ$, maximal mixing. Of course, with the full 3 generation mixing matrix, the situation is more complicated but the physics is basically the same. The potential in matter can cause energy level crossings and thereby enhance oscillations. (It can also, of course, quench oscillation if the splitting is increased by the matter potential.)

The MSW effect actually comes about from a changing electron density profile in the sun. Neutrinos produced in the solar core start out at a high electron density, but experience a decreasing density as they propagate outward (i.e., the potential changes). If the resonance condition in (39) is satisfied during transit, ν_e can resonate completely into ν_μ (or ν_τ). Assuming that the vacuum angle θ is small, they will not convert back to ν_e as they propagate to the earth.

The MSW solution to the solar neutrino problem is very elegant and naturally attractive. It can resolve the longstanding low neutrino flux observations for Δm^2 in the range $10^{-7} \sim 10^{-4} \text{ eV}^2$ and for a large range of mixing angles $0.001 < \sin^2 \theta \leq 0.4$ (see fig. 2). It has also inspired many other applications of the effect of matter on neutrinos. Two applications, neutrino decay and spin-flavor precession will be briefly surveyed.

4.2 Neutrino Decay

If neutrinos have mass and mix, then the heavier neutrino species should decay into lighter ones. Some vacuum decays such as $\nu_{2L} \rightarrow \nu_{1R} + \gamma$ (see (20)) will, however, be significantly altered in dense matter because ν_{2L} and ν_{1R} experience different potentials. Even more important, in a hot dense plasma, photons acquire an effective mass, the plasma frequency. That mass will inhibit and can quite easily eliminate the possibility of radiative neutrino decay in matter.

A more interesting scenario considered by Berezhiani and Vsotsky²⁶ involves the decays $\nu_e \rightarrow \bar{\nu}_e + M$ (or $\bar{\nu}_e \rightarrow \nu_e + M$ in neutron-rich matter)

Fig. 2: 30% survival contour for solar ν_e neutrinos undergoing MSW oscillations. For mixing and mass differences inside the contour more than 70% of the ν_e flux is lost.

and $\bar{\nu}_\mu \rightarrow \nu_\mu + M$ in matter with M the hypothetical spin-0 Majoron previously mentioned. Such decays are forbidden in the vacuum because CPT requires ν and $\bar{\nu}$ to have the same mass. However, ordinary neutral matter with $N_e > N_n$ provides a repulsive potential for ν_e and $\bar{\nu}_\mu$ and attractive potential for $\bar{\nu}_e$ and ν_μ (see (28)). Those potentials give rise to effective mass differences that govern the decay rate. The rates for such decays in the earth, sun and supernova were estimated by Berezhiani and Vysotsky using experimental bounds on the Majoron couplings to neutrinos. The most interesting possibility seems to be in a supernova. There the neutron-rich core region can lead to $\bar{\nu}_e \rightarrow \nu_e + M$, $\bar{\nu}_\mu \rightarrow \nu_\mu + M$, $\bar{\nu}_\tau \rightarrow \nu_\tau + M$ and thus skew the flux toward a higher neutrino and depleted antineutrino flux rate. Such an effect would be energy dependent and could be affected by flavor mixing.

4.3 Neutrino Spin-Flavor Precession

One speculative solution to the solar neutrino flux problem was suggested long ago (1971) by Cisneros.²⁷ He argued that if neutrinos had a magnetic moment $\kappa \sim 10^{-14}$ and large transverse magnetic fields $\sim 10^6 G$ existed in the sun, then neutrinos could undergo spin precession $\nu_{eL} \rightarrow \nu_{eR}$ as they traversed the solar radius. Since ν_{eR} is sterile, one would observe an effective depletion of neutrino flux by $\sim 1/2$ in accord (approximately) with the Davis experiment. That scenario has been revived and improved by Okun, Voloshin and Vysotsky (OVV).²⁸ Their motivation came from the observation that there appears to be an anticorrelation between neutrino flux and variations in sunspot activity, i.e., large magnetic field disturbances.²⁸ Indeed, there is mounting evidence that solar neutrino flux may follow the 11-year solar cycles.

The precession scenario for solar neutrinos has been extensively studied by OVV. They noted that magnetic or electric dipole moments could give $\nu_{eL} \rightarrow \nu_{eR}$ precession, while flavor transition moments could result in the combined spin-flavor precession $\nu_{eL} \rightarrow \nu_{\mu R}$ or $\nu_{\tau R}$ (for Dirac neutrinos) or $\nu_e \rightarrow \bar{\nu}_\mu$ or $\bar{\nu}_\tau$ (for Majorana neutrinos). In all cases, however, they concluded that a relatively large moment $\kappa \sim 10^{-10} \sim 10^{-11}$ was necessary for realistic solar magnetic fields $10^3 \sim 10^4 G$. Such a scenario renders $\kappa \lesssim 10^{-11}$ an interesting domain for direct experimental searches.

I would like to conclude by describing work done in collaboration with C-S. Lim²⁹ on the effect of matter on neutrino spin-flavor precession (similar observations have been made by E. Akhmedov).³⁰ We noted that in a vacuum, neutrino mass differences can quench spin flavor precession. The different potentials felt by distinct flavor neutrinos in matter is also capable of quenching precession. However, when both a mass difference and potential difference exist, they can cancel one another. In that case, a resonance situation can arise and $\nu_L \rightarrow \nu'_R$ can proceed unimpeded (provided magnetic fields are present). For a varying density, one can even have an MSW effect in which essentially complete conversion can occur. Let me illustrate how that could come about for Majorana neutrinos (the more realistic case) by considering $\nu_e \rightarrow \bar{\nu}_\mu$ spin-flavor precession. In that case, (ignoring other species and components), the precession evolution in a strong magnetic field B is governed by²⁹

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} V_{\nu_e} & \mu^* B \\ \mu B & \frac{\Delta m_{21}^2}{2E_\nu} \cos 2\theta - V_{\nu_\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix} \quad (40)$$

where θ is the mixing angle, and V_{ν_e} , V_{ν_μ} are matter potentials and μ is a transition moment.

For a neutral medium $N_e = N_p$, one finds (N_n = neutron density)

$$\begin{aligned} V_{\nu_e} &= \frac{G_\mu}{\sqrt{2}} (2N_e - N_n) \\ V_{\nu_\mu} &= \frac{G_\mu}{\sqrt{2}} (-N_n) \end{aligned} \quad (41)$$

Assuming that B and the number densities are static, one finds that the transition probability for spin-flavor precession is given by

$$P(t)_{\nu_e \rightarrow \bar{\nu}_\mu} = \frac{(2\mu B)^2}{\Delta^2 + (2\mu B)^2} \sin^2 \left(\sqrt{\Delta^2 + 4\mu^2 B^2} t / 2 \right) \quad (42)$$

$$\Delta = \sqrt{2} G_\mu (N_e - N_n) - \frac{\Delta m_{21}^2}{2E_\nu} \cos 2\theta \quad (43)$$

For the special condition $\Delta = 0$, spin-flavor precession is unimpeded and (42) becomes

$$P(t)_{\nu_e \rightarrow \bar{\nu}_\mu} = \sin^2 (\mu B t) \quad (44)$$

i.e., maximal spin-flavor precession.

That condition is achieved when the density $N_e - N_n$ just allows a cancellation in (43). Of course, for an adiabatically changing density profile, it is possible to start with $\Delta > 0$ and encounter a resonance region, $\Delta \simeq 0$, where essentially all ν_e of a given energy are converted to $\bar{\nu}_\mu$ and then reach a region where either $\Delta < 0$ or B is no longer large enough to give a significant precession. In that case essentially total conversion of $\nu_e \rightarrow \bar{\nu}_\mu$ can occur.

For the above resonant spin-flavor precession scenario to proceed, one needs non-zero flavor transition moments, large magnetic fields and dense matter. In the sun, where $\langle B \rangle \simeq 10^3 \sim 10^4 G$ is expected, the transition moment κ must be relatively large $\sim 10^{-10} - 10^{-11}$ for large spin-flavor precession of neutrinos with $E_\nu \sim 10$ MeV to occur. Nevertheless, it will be interesting to see whether the hints of a time variation in the neutrino flux correlated with magnetic field fluctuations is confirmed by subsequent experiments. A more likely candidate to look for spin-flavor precession is a supernova. In that case, very large $B \sim 10^{12} \sim 10^{15} G$ can occur. Hence, one is sensitive to $\kappa \sim 10^{-19} \sim 10^{-23}$ a realistic range of transition moments. The signature of such a phenomenon will be an interchange of ν_e and $\bar{\nu}_\mu$ (or $\bar{\nu}_\tau$) supernova spectra. In addition, for $N_n \gtrsim N_e$ one may have $\bar{\nu}_e \rightarrow \nu_\mu$ or ν_τ near the core. Observing such an effect would be extraordinary. Will we be so lucky to have one occur close enough to observe and will our detectors be ready if one occurs? I hope so.

5. CONCLUSION

Neutrino physics research has entered an exciting new era. The potential effects of matter on oscillations, decay and spin-flavor precession have inspired many interesting theoretical conjectures with testable experimental consequences. In most cases high densities over large distances are required for such effects to be manifested. That means that we must use the sun, earth, or some other heavenly body (supernova) as a catalyst.

The coming of age of neutrino astronomy nicely complements speculations on neutrino exotica. Following up the patient pioneering work of Ray Davis with new solar neutrino detectors will tell us more about the internal dynamics of our sun and provide a unique laboratory for probing neutrino properties.

Furthermore, if a supernova in our galaxy should occur, a wealth of data will become available.

At the terrestrial level, new experiments are now starting to be mounted at accelerator facilities. Those experiments will probe the standard model parameters and search for new physics in scattering processes such as $\bar{\nu}_{\mu} + e \rightarrow \bar{\nu}_{\mu} + e$. In addition, they provide an alternative more controlled means of searching for oscillations and electromagnetic moments. In my opinion, the next generation of such experiments should focus on $\nu_{\mu} - \nu_{\tau}$ oscillations. An appearance experiment in which one observed ν_{τ} by τ production would be particularly welcome. Many theoretical and experimental ideas have been put forward; but I would guess that new clever possibilities will be found as experiments provoke creative thinking.

Neutrino physics is very hard, but the discovery potential is high and well worth the effort.

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