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A 3-D SLUG FLOW HEAT TRANSFER ANALYSIS OF
COUPLED COOLANT CELLS IN FINITE LMFBR BUNDLES

by

Chung-Nin Wong
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Reports Issued Under This Contract

B. Topical Reports (Available from National Technical Information Service, U.S. Department of Commerce, Springfield, VA 22151)

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E. Khan, W. Rohsenow, A. Sonin and N. Todreas, "A Simplified Approach for Predicting Temperature Distribution in Wire Wrapped Assemblies," COO-2245-5TR, MIT, Sept. 1973.

T. Eaton and N. Todreas, "Instrumentation Methods for Inter-channel Coolant Mixing Studies in Wire-Wrap Spaced Nuclear Fuel Assemblies," COO-2245-9TR, MIT, June 1974.

Y.B. Chen, K. Ip, N.E. Todreas, "Velocity Measurements in Edge Subchannels of Wire Wrapped LMFBR Fuel Assemblies," COO-2245-11TR, MIT, September 1974.

E. Khan, N. Todreas, W. Rohsenow, A.A. Sonin, "Analysis of Mixing Data Relevant to Wire-Wrapped Fuel Assembly Thermal-Hydraulic Design," COO-2245-12TR, MIT, September 1974.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," COO-2245-16TR, MIT, March 1975.

E. Khan, W.M. Rohsenow, A. Sonin, N. Todreas, "Input Parameters to the ENERGY Code (To be used with the ENERGY Code Manual) COO-2245-17TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III," COO-2245-18TR, MIT, May 1975

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III Computer Programs," COO-2245-18TR Revision 1, MIT, July 1976.

P. Carajilescov and N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," COO-2245-19TR, MIT.

B. Chen and N. Todreas, "Prediction of Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," COO-2245-20TR, MIT, May 1975.

B. Chen and N. Todreas, "Prediction of Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," COO-2245-20TR Revision 1, MIT, December 1976.

F. Carre and N. Todreas, "Development of Input Data to ENERGY Code for Analysis of Reactor Fuel Bundles," COO-2245-21TR, MIT, May 1975.

Reports Issued Under This Contract

B. Topical Reports, Continued

H. Ninokata and N.E. Todreas, "Turbulent Momentum Exchange Coefficients for Reactor Fuel Bundle Analysis," COO-2245-22TR, MIT, June 1975.

R. Anoba and N. Todreas, "Coolant Mixing in LMFBR Rod Bundles and Outlet Plenum Mixing Transients," COO-2245-24TR, August 1975.

B. Bosy, "Fabrication Details for Wire Wrapped Fuel Assembly Components," COO-2245-27TR, MIT, November 1975.

Ralph G. Bennett and Michael W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," COO-2245-29TR, MIT, June 1976.

N.E. Todreas, "Analysis Methods for LMFBR Wire Wrapped Bundles," COO-2245-32TR, MIT, November 1976.

K.L. Basehore and N.E. Todreas, "Development of Stability Criteria and an Interassembly Conduction Model for the Thermal-Hydraulics Code SUPERENERGY," COO-2245-33TR, MIT December 1976.

Robert Masterson and Neil E. Todreas, "Analysis of the Feasibility of Implementing an Implicit Temporal Differencing Scheme in the SUPERENERGY Code," COO-2245-35TR, MIT, February 1977.

S. Glazer, C. Chiu and N. Todreas, "Collection and Evaluation of Salt Mixing Data with the Real Time Data Acquisition System," COO-2245-36TR, MIT, April 1977.

B. Mikic, E.U. Khan and N.E. Todreas "An Approximate Method for Predicting Temperature Distribution in Wire Wrapped Fuel Assemblies of a LMFBR," COO-2245-37TR, MIT, April 1977.

C. Chiu and N. Todreas, "Development of a Technique for Subchannel Flow Rate Measurements in LMFBR Wire Wrapped Assemblies," COO-2245-39TR, July 1977

C. Chiu and N. Todreas, "WARD Blanket Assembly Pre-Test Predictions by SUPERENERGY," COO-2245-40TR, July 1977

C. Chiu and N. Todreas, "Flow Split for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-41TR, July 1977

C. Chiu and N. Todreas, "Static Pressure and Pressure Drop for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-42TR, July 1977

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B. Topical Reports, Continued

C. Chiu and N. Todreas, "Mixing Experiments for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-43TR, July 1977

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J. Kelly and N. Todreas, "Turbulent Interchange in Triangular Array Bare Rod Bundles," COO-2245-45TR, July 1977

K.L. Basehore and N.E. Todreas, "Assessment of the Need to Incorporate a Variable Swirl Flow Model into the ENERGY Code," COO-2245-46TR, July 1977.

K.L. Basehore and N. Todreas, "Analysis of the Thermal-Hydraulic Behavior in the CRBR Secondary Control Assembly, Including Interassembly Heat Transfer Effects," COO-2245-47TR, July 1977.

J.G. Bartzis and N.E. Todreas, "Hydrodynamic Behavior of a Bare Rod Bundle," COO-2245-48TR, June 1977.

M.R. Fakori-Monazah and N.E. Todreas, "Measurement and Analysis of Flow Wall Shear Stress in an Interior Subchannel of Triangular Array Rods," COO-2245-49TR, August 1977.

A.S. Hanson and N.E. Todreas, "Fluid Mixing Studies in an Hexagonal 61-Pin, Wire-Wrapped Rod Bundle," COO-2245-51TR, August 1977.

S. Glazer, N. Todreas, W. Rohsenow, and A. Sonin, "TRANSENERGY S,M - Computer Codes for Coolant Temperature Prediction in LMFBR Cores During Transient Events," COO-2245-52TR, January 1977.

C. Chiu, W.M. Rohsenow and N.E. Todreas, "Mixing Experiments in an Alternating Wire Wrapped Assembly," COO-2245-54TR, MIT, December 1977.

C. Chiu, W.M. Rohsenow and N. E. Todreas, "Sweeping Flow Mixing Model for Wire Wrapped LMFBR Assemblies," COO-2245-55TR, MIT, December 1977.

C. Chiu, W.M. Rohsenow and N.E. Todreas, "Lumped Subchannel Flow Model for LMFBR Wire Wrapped Assemblies," COO-2245-56TR MIT, December 1977.

Reports Issued under this Contract

C. Papers and Summaries

Yi Bin Chen, Ka-Lam Ip, Neil E. Todreas, "Velocity Measurements in Edge Channels of Wire-Wrapped LMFBR Fuel Assemblies," American Nuclear Society Transactions Vol. 19, 1974, pp. 323-324.

P. Carajilescov, N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," J. of Heat Transfer, Vol. 98, No. 2, May 1976, pp. 262-268 (Included as Appendix to Quarterly Progress Report, COO-2245-15).

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Fuel Rod Assemblies," Nuclear Engineering and Design, 35 (1975) 1-12.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Rod Assemblies Operating in Combined Forced and Free Convection," Nuclear Engineering and Design, 35 (1975) 199-211.

Ralph G. Bennett and Michael W. Golay, "Development of an Optical Method for Measurement of Temperature Fluctuation in Turbulent Flows," American Nuclear Society Transactions, Vol. 22, 1975, p. 581.

B. Chen and N. Todreas, "Prediction of the Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," Nuclear Engineering and Design 35, (1975) 423-440 (Included as Appendix to Quarterly Progress Report, COO-2245-23).

R. Bennett and M.W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," Accepted for the ASME Winter Annual Meeting, Dec., 1976, (Included as Appendix in Quarterly Progress Report, COO-2245-30).

B.B. Mikic, E.U. Khan, N.E. Todreas, "An Approximate Method for Predicting Temperature Distribution in Wire Wrapped Fuel Assemblies of a Liquid Metal Fast Breeder Reactor," Mech. Res. Comm., Vol. 3, 353-360 (1976).

Reports Issued Under this Contract

C. Papers and Summaries (Continued)

L. Wolf, R. Karimi, I.Y. Kim, C.N. Wong, M.K. Yeung "2-D Thermoelastic Analysis of LMFBR Fuel Rod Claddings," Paper C4/d, 4th International Conf. Structural Mechanics in Reactor Technology, San Francisco, August 1977.

M. Yeung, L. Wolf, "Effective Conduction Mixing Lengths for Subchannel Analysis of Finite Hexagonal LMFBR Bundles," ANS Meeting, New York, June 1977.

C. Chiu and N. Todreas, "Flow Split Measurements In An LMFBR Radial Blanket Assembly," ANS Meeting, New York, June 1977.

ABSTRACT

A three-dimensional single region slug flow heat transfer analysis for finite LMFBR rod bundles using a classical analytical solution method has been performed. According to the isolated single cell analysis, the results show that the peripheral clad temperature variation as well as the thermal entrance length are strongly dependent upon the degree of irregularity displayed by various coolant geometries. Since under the present LMFBR conditions, fully-developed temperature fields may hardly be established in such characteristic rod bundle regions, a 3-D heat transfer analysis seems to be mandatory. This implies that the results of fully developed heat transfer analyses are by far too conservative.

By coupling the corner cell with the internal cell, it is found that higher azimuthal temperature variations and higher Nusselt numbers for either cell will be obtained. For the geometrical conditions studied, results of the isolated cell analysis have been compared to those of the coupled cell analysis. Both approaches were compared with the experimental data obtained from Karlsruhe, Germany. The comparison showed fairly good agreement.

A correction factor for the effective conduction mixing length, L_{eff} , as function of the axial coordinate has been derived. By using this empirical equation as input into the code COBRA-IIIC it could be demonstrated that lumped parameter methods can be improved. The local heat transfer in the cells has been studied under different geometrical and various heating conditions. The results illustrate that azimuthal temperature variations will increase as the radial and/or axial power peaking increases. However, because of the strong influence of the thermal entrance effect, under the present LMFBR conditions, it is not possible to draw final conclusions about the impact of geometrical conditions on heat transfer in the cells. For this purpose, a parametric study has to be initiated. Due to the multi-cell methodology developed in this thesis it was possible to analyze a complete internal cluster of coolant cells for the first time without making the commonly employed assumption of an adiabatic boundary condition at symmetry lines. In this way it was demonstrated that under power tilt conditions the center pin experiences rather large azimuthal temperature difference, a fact that has not been discovered yet.

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CHAPTER 1

INTRODUCTION

1.1. Background

Nowadays, nuclear engineers commonly refer to "sub-channel analysis" for thermal-hydraulic heat-transfer analysis of the nuclear reactor core assemblies. The subchannel methods used today lump the geometry under consideration and use characteristic subchannels as the smallest unit. Within each of these units an average value for the temperature and velocity is assigned. Although this lumping procedure has shown its capabilities for certain design purposes, it has been recently acknowledged that it has the two following decisive disadvantages:

1. Detailed calculations [1] have shown that large uncertainties exist with respect to the transport properties for momentum and energy transfer which are used for the calculation of the respective quantity between subchannels. It was found that these properties cannot be described generally by simple correlations but are dependent upon various boundary conditions.
2. The resultant pin temperature distributions are mostly not accurate enough for subsequent thermal stress analyses because:

- (a) The resolution in the azimuthal direction is not high enough;
- (b) Empirical correlations are used to establish the heat transfer coefficient in order to calculate an average clad temperature over an angular section. Mostly, these correlations do not reflect the realistic geometry and the boundary conditions.

It is a well-known fact that thermal stresses due to azimuthal temperature gradients largely affect LMFBR fuel element and bundle integrity. Spacer design must be such that the overall bundle design can cope with the resulting strains and displacements especially in situations where large power skews exist throughout the bundle. This situation arises mainly at the core/blanket interface regions. Moreover, the fuel element cladding must be designed under various neutronic, thermal-hydraulic and material constraints to meet their economical objectives. For instance, it has been estimated that a decrease of 5°C in the hot spot temperature would result in a saving of $\$3 \times 10^6$ for the CRBR [2]. The savings will be even much higher for a commercial LMFBR. The only way to get this type of pay-off is by performing three-dimensional distributed parameter analyses. Although much work has been devoted to overall bundle analyses on the basis of a so-called lumped parameter approach (subchannel codes) only little progress has been made to predict local fluid temperature, clad temperature and fuel temperature fields.

1.2. Previous Results

After about a decade of research in the field of distributed parameter analyses, the following results can be found in the open literature. For two-dimensional slug flow and laminar flow heat transfer, solutions have been obtained for isolated characteristic cells in a hexagonal bundle. For slug flow, multi-region analyses are presented in [3,4,5]. This included the effect of rod displacement, too. Solutions for the laminar flow heat transfer problem in the same geometries are given in [6,7]. All the aforementioned references treated the heat source density in the fuel as being constant over the cross section. By assuming the heat source density being constant, it is acknowledged that under certain circumstances, these isolated cell analyses may give upper bounds for circumferential temperature distributions. However, in case of strong differences in the power densities of neighboring pins, this is not necessarily true. Despite the needs for a multi-cell calculational scheme, nothing is available in the open literature. Only recently, some work on this subject [8] has been initiated.

On the other hand, some efforts have been undertaken in order to gain some insight into the problem of the thermal entrance length in multiply connected regions. The three-dimensional slug flow heat transfer problem in the various individual cells of a hexagonal bundle was solved in [9] for a constant heat flux around the clad outside boundary. However, the analysis takes into account an arbitrarily axially varying

heat flux distribution by using Duhamel's Theorem [10]. The solution technique in [9] relied again on the point-matching procedure but this time it was used in the least-square sense [11] using an ortho-normalization method for solving the over-determined set of equations [12]. Surprisingly, the outcoming results from this analysis showed a high degree of coincidence with the far more sophisticated turbulent analysis [13] in the thermal entrance region.

So far, no analysis has been reported for the combined hydrodynamic and thermal entrance problem with laminar flow in hexagonal bundle geometries. For fully developed turbulent flow situations, the review shows that a lot has been done in the two-dimensional fluid flow and heat transfer field for the internal subchannels of bare pin bundles, for instance [14] among others. However, the heat transfer problems in the characteristic side and corner cells of a bundle remain still unsolved, with one exception [13].

In the meantime, limited progress has been made in studying the thermal entrance length effect in two coupled internal subchannels. The first study [15] considered only molecular conduction effects and resulted in a very simple best-fit correlation for the ratio of the subchannel averaged temperatures, whereas the second study [16] took into account turbulent convection effects. The same was done in [17,18,19] for fluids with Pr-numbers varying over a large range and different pitch-to-diameter ratios. The underlying physical model which was used in the latter studies was more sophisticated.

On the experimental side, there is fairly little evidence too. A fairly broad study was made in [20] for uniformly heated pins, which pointed out a whole variety of unsolved problems in this field. However, in [21], the clad temperature measurements were made for all types of cells in hexagonal bundles for Pe-numbers as low as 20. Recently, an experiment performed in Karlsruhe, Germany [22] gives detailed three-dimensional temperature fields in side and corner fuel pin clads of a sodium cooled, bare 19-pin bundle. The results show that the temperature variations are even higher than measured by the Russian authors. The data obtained in [22] seems to present a reliable base for comparison with any theoretical, three-dimensional model which solves the heat transfer problem in the bundle wall near regions.

1.3. Scope of Work

The principal work of this research will be concentrated on the three-dimensional coupled coolant cell heat transfer analysis for finite LMFBR rod bundles. At the beginning of this work, the results of the isolated single cell analysis will be rederived for completeness. The purpose of this study is to check how the isolated cell results compare to the measurements and also to prepare some valuable information for further comparisons with the coupled cell approach.

In a next step, the results of the coupled cell approach will be thoroughly studied. The most important work involved in this study can be listed as follows:

1. Comparison of theoretical results with experimental data reported in [22];
2. Check whether the isolated single cell analysis is always conservative with respect to temperature variation around the fuel pin as claimed in the literature;
3. Study of the heat transfer in the cells under different geometric conditions and various heating conditions;
4. Deriving the effective conduction mixing parameter between corner and internal cells as function of the axial co-ordinate for different radial pin powers;
5. Determination of the effect of axial heat flux distribution on circumferential temperature distribution;
6. Presentation of a 3-D analysis of a cluster of rods with superimposed radial power tilt.

CHAPTER 2

ISOLATED CELL ANALYSIS

2.1 General Description

Liquid metals, generally have low specific heats, low viscosities and high thermal conductivities which result in low Prandtl numbers. With the Prandtl number far less than unity, the temperature gradient in the liquid metal coolant near the wall is much smaller than the velocity gradient, Reichardt [23] has studied the thickness of the laminar sublayer for liquid metal flowing through a smooth circular tube and expressed the thickness as follows:

$$\delta_e = 5 \frac{v}{v^*}$$

$$\text{where } v^* = 0.150 u^{7/8} \left(\frac{v}{y}\right)^{1/8}$$

and u is the flow velocity

and y is the radius of the circular tube.

For liquid metal, the viscosity, v is small. If the flow rate is high enough, the thickness of the laminar sublayer, δ_e becomes extremely small. Thus, using a Slug Flow Model for the velocity profile in the channel should be adequate as a first approximation for a distributed parameter analysis for LMFBR subchannel analysis.

As it is demonstrated in Fig. [2.1], the core geometry of an LMFBR is quite different from the geometry of an LWR. In the LMFBR, the rod bundles are positioned in a compact

Fig. 2.1 Schematic Diagram for Fuel Assemblies

hexagonal array. Therefore, in the following distributed parameter analysis, each individual cell has to be identified in terms of geometric parameters and its location within the bundle as it is in Fig. [2.1]. Within each of these cells, the assumption is made that the velocities are everywhere the same. This implies that the slug flow model has been introduced in each individual cell.

At the outer boundary of each individual cell, an adiabatic boundary condition will be applied. In this way, all these cells will be isolated from each other. Thus, the problem to be analyzed has been substantially simplified and reduced.

2.2. Formulation of the Model

The three-dimensional isolated cell model described in this chapter has been originally formulated and solved by Meier [9]. The principal work involved in what follows here is to modify the computer code and to analyze the results which are important for the ongoing research.

To begin the analysis, each isolated cell as shown in Fig. [2.1] is considered. The form of the general energy equation for the coolant can be written as:

$$\rho(\underline{r}) C_p(\underline{r}) \frac{D}{Dt} T(\underline{r}, t) = \nabla k(\underline{r}) \cdot \nabla T(\underline{r}, t) + q'''(\underline{r}, t) + T\beta \frac{DP}{Dt} + \mu\phi(\underline{r}, t) \quad (2.1)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \underline{v} \cdot \underline{\nabla}$

$$\begin{aligned}
\text{and } \phi = & 2 \left[\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right] \\
& + \left[\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right)^2 \right] \\
& - \frac{2}{3} (\nabla \cdot \underline{v})^2
\end{aligned}$$

Eq. 2.1 can be interpreted as follows. The rate of increase in the temperature of a fluid element moving with the stream is governed by heat conduction across its boundaries, heat source effects, compressibility effects and viscous effects. Obviously energy terms due to compressibility and viscous effect will be neglected in the calculation. Since there is no heat source in the coolant, $q'''(\underline{r}, t) = 0$. Furthermore, the following assumptions are made in this analysis to simplify the problem, namely:

- (a). steady state: $\frac{\partial}{\partial t} = 0$;
- (b). Homogeneous and isotropic as well as temperature independent coolant properties;
- (c). Frictionless Fluid. This assumption leads to an approximation for a limited Pe-number which implies axial pressure drop and local velocity field have been neglected;
- (d). Axial conduction effects are neglected in the coolant and the fuel pin.

As a result of these simplifications, the energy equation can be written as follows:

$$k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] = \rho C_p V_z \frac{\partial T}{\partial z} \quad (2.2)$$

For computational purpose, a set of new dimensionless variables is used, namely

$$\text{for the temperature: } \theta(\rho, \phi, \xi) = \frac{T(r, \phi, z) - T_{in}}{q'' r_i / k} \quad (2.3)$$

$$\text{for the radial coordinate: } \rho = \frac{r}{\frac{P}{2}} \quad (2.4)$$

$$\text{for the axial coordinate: } \xi = \frac{z D_h}{\left(\frac{P}{2}\right) 2 Pe} \quad (2.5)$$

After the transformation, the new form of the energy equation reads:

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta}{\partial \phi^2} + \left(\frac{D_h}{\frac{P}{2} Pe} \right)^2 \frac{\partial^2 \theta}{\partial \xi^2} = \frac{\partial \theta}{\partial \xi} \quad (2.6)$$

The effect of the axial energy transport by conduction is described by the last term on the left hand side of Eq. (2.6). This term is mainly dependent upon the Pe-number, which occurs as squared in the denominator. Several author [24,25] have studied this effect and concluded that for $Pe > 100$, the axial heat transport by conduction can be safely neglected. Following this argument, the energy equation can be simplified to:

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{1}{\rho^2} \left(\frac{\partial^2 \theta}{\partial \phi^2} \right) = \frac{\partial \theta}{\partial \xi} \quad (2.7)$$

This equation has to be solved for the boundary conditions as given in Figure [2.2]:

- (a). Inlet condition: at $z = 0$, $T(r, \phi, z = 0) = T_0(r, \phi)$;
- (b). Azimuthal uniform heat flux around the outside cladding surface;
- (c). Uniform or arbitrarily varying axial heat flux;
- (d). Adiabatic boundary condition at the outer cell boundaries. Normally, the temperature profile is set equal to a constant at the inlet (i.e. $z = 0$) of the coolant cell. However, in this model, because of the fact that the inlet temperature distribution will be satisfied at discrete points, it is possible to impose any arbitrary temperature distribution at the inlet. A study of a non-uniform inlet temperature distribution will be undertaken and its result will be discussed in section 2.5.1.3.

As one may notice, the assumption of a circumferentially constant heat flux is an approximation which is about valid only for the internal cell. But, in the cases of very asymmetric coolant channels, like corner and side cells, with or without power gradients across the bundle, this assumption may not be appropriate. However, by assuming an azimuthally constant heat flux at the clad surface, the analysis will be

Fig. 2.2a Boundary Conditions to the Flow Channel

Fig. 2.2b Axial Heat Flux Profile

Fig. 2.2c Initial Condition

enormously simplified because one can avoid a multi-region analysis which would simultaneously take into account of the fuel, the clad and the coolant region. A comparison between single region analysis and multi-region analysis under thermally fully-developed conditions by Yeung [8] has shown that the single region analysis with uniform azimuthal heat flux gives always conservative results. This occurs because the single region analysis always overpredicts the heat flux at the outside clad surface in the coolant region with a hotter bulk temperature but under-predicts the heat flux in the coolant region with a cooler bulk temperature. The difference of the heat flux distribution between these two calculations increases as functions of the axial coordinate. However, for any level in the present LMFBR core design which has a core height of about 36 inches, the assumption of a single region analysis seems to be acceptable and justified.

In summary of the discussion above, the assumptions which are made throughout the following analyses can be stated as follows:

- (1). Only the coolant region is considered, i.e., single region analysis;
- (2). Frictionless flow, i.e., shear stress effects, and dissipation effects are neglected, i.e., the axial velocity will be assumed to be constant over the whole coolant cross-section;
- (3). No heat sources in the coolant;

- (4). Steady state problem;
- (5). Incompressible fluid;
- (6). Material properties (i.e., coolant properties) are independent of direction, position and temperature;
- (7). No pressure gradient between any two points;
- (8). Negligible axial heat conduction in the fluid;
- (9). Constant coolant cross section in the axial direction;
- (10). Single-phase fluid only;
- (11). No energy transport by radiation.

The remaining problem can mathematically be expressed as follows:

(I). Energy Equation in the coolant:

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta}{\partial \phi^2} = \frac{\partial \theta}{\partial \xi} \quad (2.7)$$

(II). Initial Condition:

At the inlet ($\xi = 0$ or $z = 0$)

$$\begin{aligned} \theta(\rho, \phi, \xi) &= 0 & \text{for } 0 \leq \phi \leq 2\pi \\ \text{and } \rho_i &\leq \rho \leq \rho_a \end{aligned} \quad (2.8)$$

where outside clad radius, $\rho_i = \frac{1}{P/D}$

and outside cell boundary, $\rho_a = \frac{a_j}{\frac{P}{2} \cos(\phi - \alpha_j)}$
 $j = 1, 2, 3, 4$

(See Fig.[2.3] for detailed description of geometry).

(III). Boundary Condition

(a). At outside cladding, heat flux is azimuthally uniform:

$$q''(r, \phi) \big|_{r=r_i} = q_o'' = -k \nabla T \big|_{r=r_i} \quad (2.9)$$

at outer cell boundary

$$\hat{n} \cdot \nabla_{r, \rho, z} T \big|_{r=r_a} = 0 \quad (2.10)$$

(b). These two boundary conditions can be brought into the following dimensionless terms,

$$\frac{\partial \theta}{\partial \rho} = - \frac{P}{D} \quad \text{for } \rho = \rho_i \quad (2.11)$$

$$\hat{n} \cdot \nabla_{\rho, \phi, \xi} \theta = 0 \quad \text{for } \rho = \rho_a \quad (2.12)$$

both for $0 < \xi < \infty$

and $0 < \phi < 2\pi$.

(IV). Symmetry Condition

$$\begin{aligned} \frac{\partial \theta}{\partial \phi} &= 0 & \text{for } \phi &= \frac{n\pi}{S} & (2.13) \\ & & \text{and } 0 &\leq \xi < \infty \\ & & \text{and } \rho_i &\leq \rho \leq \rho_a \end{aligned}$$

Internal Cell

Edge Cell

Corner Cell

Fig. 2.3 Sketch for Internal, Edge and Corner Cell

where $n = 0, 1, 2, 3, \dots$

The symmetry parameters characterize the following geometries and assume the following values:

- (a). $s = 1$ for corner and side cells as well as asymmetric internal cells.
- (b). $s = 6$ for symmetric internal cell

2.3. Theoretical Analysis

With the governing equation and the boundary conditions of the problem established in the foregoing Section, the ongoing analysis will be concerned with the methodology of solving this problem, leaving the numerical solution to Section 2.4. The basic tool used throughout this analysis is the Principle of Superposition. For this purpose, it should be noticed that the differential equation after the transformation of the temperature, the radial and the axial terms into dimensionless form is still linear. Thus, the superposition principle is applicable such that one set of boundary conditions will be homogenized. The advantage of expressing the differential equation in dimensionless form is that it can be decomposed into two basic solution forms. One solution form will be the asymptotic solution and the other has the property of decreasing its value with increasing axial height. However, the initial condition at the inlet region requires these two solutions cancel each other. For the purpose of future refer-

ence, the asymptotic solution will be named the thermally fully developed temperature denoted by θ_f and the other solution form will be named as the thermal entrance temperature denoted by θ_e . In mathematical form, the total dimensionless temperature can then be expressed as follows:

$$\theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) + \theta_e(\rho, \phi, \xi) \quad (2.14)$$

And the special properties of the individual forms of the temperature field can be stated as:

$$\text{for } \xi < 0 \quad \theta_f(\rho, \phi, \xi) = -\theta_e(\rho, \phi, \xi) \quad (2.15)$$

$$\text{as } \xi \rightarrow \infty \quad \theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) \quad (2.16)$$

$$\text{or } \theta_e(\rho, \phi, \xi) = 0$$

$$\text{for all } \rho \text{ and } \phi \text{ such that } 0 \leq \phi \leq 2\pi \cdot \rho_i \leq \rho \leq \rho_a$$

Then, the differential equation with its boundary conditions can be split into the following two separate parts:

(a). Thermally Fully Developed Region

$$\frac{\partial^2 \theta_f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_f}{\partial \phi^2} = \frac{\partial \theta_f}{\partial \xi} \quad (2.17)$$

$$\frac{\partial \theta_f}{\partial \rho} = -\frac{P}{D} \quad \text{for } \rho = \rho_i \quad (2.18)$$

$$n \cdot \text{grad } \theta_f = 0 \quad \text{for } \rho = \rho_a \quad \text{and } 0 \leq \phi \leq 2\pi \quad (2.19)$$

$$\begin{aligned} \frac{\partial \theta}{\partial \phi} &= 0 \quad \text{for } \phi = \frac{n}{S} \pi \quad (2.20) \\ &\text{for } 0 \leq \xi \leq \infty \\ &\text{and } \rho_i \leq \rho_a \end{aligned}$$

where $n = 0, 1, 2, \dots$

(b). Thermal Entrance Region

$$\frac{\partial^2 \theta_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_e}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_e}{\partial \phi^2} = \frac{\partial \theta_e}{\partial \xi} \quad (2.21)$$

$$\begin{aligned} \frac{\partial \theta_e}{\partial \rho} &= 0 \quad \text{for } \rho = \rho_i \quad 0 \leq \xi \leq \infty \\ \hat{n} \cdot \text{grad } \theta_e &= 0 \quad \text{for } \rho = \rho_a \quad 0 \leq \phi \leq 2\pi \quad (2.23) \end{aligned}$$

$$\begin{aligned} \frac{\partial \theta_e}{\partial \phi} &= 0 \quad \text{for } \phi = \frac{n}{S} \pi \quad \text{where } n = 0, 1, 2, \dots, \\ &\text{for } 0 \leq \xi \leq \infty \\ &\text{and } \rho_i \leq \rho \leq \rho_a \end{aligned}$$

Figs. [2.4a and 2.4b] summarize the corresponding boundary conditions for the thermally fully-developed solution and for the thermal entrance solution, respectively.

Fig. 2.4a Fully Developed Region for Isolated Cell

$$\frac{\partial^2 \theta_f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_f}{\partial \phi^2} = \frac{\partial \theta_f}{\partial \xi}$$

Boundary Condition

$$\frac{\partial \theta_f}{\partial \rho} = -\frac{P}{D} \text{ for } \rho = \rho_i \text{ and } 0 \leq \phi \leq \pi$$

$$\hat{n} \cdot \text{grad } \theta_f = 0 \text{ for } \rho = \rho_a \text{ and } 0 \leq \phi \leq \pi$$

$$\theta_f(\rho, \phi, \xi = 0) = \theta_{f,o}$$

Fig. 2.4b Thermal Entrance Region for Isolated Cell

$$\frac{\partial^2 \theta_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_e}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_e}{\partial \phi^2} = \frac{\partial \theta_e}{\partial \xi}$$

Boundary Condition

$$\frac{\partial \theta_e}{\partial \rho} = 0 \quad \text{for } \rho = \rho_1 \text{ and } 0 \leq \phi \leq \pi$$

$$\hat{n} \cdot \text{grad } \theta_e = 0 \quad \text{for } \rho = \rho_a \text{ and } 0 \leq \phi \leq \pi$$

$$\theta_e(\rho, \phi, \xi = 0) = \theta_{e,o}$$

$$= -\theta_{f,o}$$

Caution should be taken when transforming the dimensionless solution back to the non-dimensionless temperature (i.e., expression in terms of degree C or degree F). Once the complete solution is solved, then this temperature is given by

$$T(r, \phi, z) = T_{in} + \theta(\rho, \phi, \xi) \frac{q_o'' r_1}{k}$$

$$\text{where } \theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) + \theta_e(\rho, \phi, \xi)$$

However, if the transformation is performed only on either fully-developed solution or thermal entrance solution, the sum of these two results may not necessarily give the real temperature as expected. This can be expressed in mathematical terms as follows:

Suppose the transformation operator, L apply on the temperature such that dimensionless temperature is formed.

$$\theta = L(T) \quad \text{and} \quad T = L^{-1}(\theta)$$

$$\text{where } L(T) = \frac{T - T_{in}}{q_o'' \frac{r_1}{k}}$$

$$\text{given } \theta = \theta_f + \theta_e$$

$$\text{If } T_1 \equiv L^{-1}(\theta_f) \quad \text{and} \quad T_2 \equiv L^{-1}(\theta_e)$$

the result well show that:

$$T_1 + T_2 \neq T$$

$$\text{where } T = L^{-1}(\theta_f + \theta_e)$$

because L or L^{-1} is not a linear operator.

Physically, this implies that the parameter, θ_f or θ_e has no meaning by itself. In order to assign a physical interpretation to these two terms, one has to consider their summation which is the transform of the temperature in dimensionless form.

The calculational scheme is outlined in Fig.[2.5] and the different forms of the solution will be discussed in the upcoming section, (Section 2.3.1. and 2.3.2).

2.3.1. Solution of the Fully-Developed Temperature Profile

2.3.1.1. Form of the Solution

Since the thermally fully-developed temperature profile is the asymptotic temperature profile, it stays the same in the fully-developed region. There it is not a function of the axial coordinate but only a function of radius and angle. However, in order to preserve the law of conservation of energy, the temperature must contain an axial part which is a linearly increasing function with axial position. That is,

$$\text{Since } \dot{m}C_p (\Delta T) = q'' \pi D(\Delta z)$$

Figure . 2.5 Outline of Calculational Scheme

If transformed into dimensionless form

$\Delta\theta$ is function of only variable, $\Delta\xi$

or $\Delta\theta = K \cdot \Delta\xi$

where K is a variation constant with no dependency on ρ and ϕ .

then $\frac{d\theta}{d\xi} = K$.

As a result, the temperature can be written as the sum of two separate functions:

$$\theta_f(\rho, \phi, \xi) = Z_f(\xi) + G_f(\rho, \phi). \quad (2.25)$$

Since the axial part is a linear function of the axial position due to energy conservation,

$$Z_f(\xi) = C_o \xi \quad (2.26)$$

The right hand side of the differential equation (2.17) becomes a constant. The unknown constant C_o can be derived from an energy balance equation. Consider a volume element $A_c dz$, where heat flow is equal that carried away by the fluid. Then, the energy balance equation can be written as:

$$\rho V_{\infty} C_p A_c \frac{dT_f}{dz} dz = q_o'' u_p dz \quad (2.27)$$

where: u_p : pin perimeter
 A_c : coolant cross-sectional area
 C_p : heat capacity

With the dimensionless variables defined by Eqs. [2.3, 2.4, and 2.5] and the relationship given by Eqs. [2.25 and 2.26]. The constant C_o can be found as

$$C_o = 2 \frac{A_p}{A_c} \left(\frac{P}{D}\right)^2 \quad (2.28)$$

where: A_p : pin cross-sectional area.

With a constant at the right hand side, the differential equation, is a Poisson Equation, namely

$$\frac{\partial^2 G_f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial G_f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 G_f}{\partial \phi^2} = C_o \quad (2.29)$$

The particular solution of this Poisson Equation reads as:

$$P_f(\rho) = C_o \frac{\rho^2}{4} \quad (2.30)$$

and for the remaining homogeneous Laplace's Equation, the method of separation of variables can be applied, namely,

$$G_f(\rho, \phi) = R_f(\rho) \Phi_f(\phi). \quad (2.31)$$

Considering the boundary conditions given by Eqs. [2.18, 2.19, 2.20] for the thermally fully-developed solution, a widely known general solution in terms of a series of cosine functions is obtained.

$$R_f(\rho) \phi_f(\phi) = A_0 + B_0 \ln \rho + \sum_{n=1}^{\infty} [A_n \rho^{sn} + B_n \rho^{-sn}] \cos(sn\phi) \quad (2.32)$$

Thus, the total solution for the thermally fully-developed region becomes

$$\begin{aligned} \theta_f(\rho, \phi, \xi) &= Z_f(\xi) + P_f(\rho) + R_f(\rho) \phi_f(\phi) \\ &= C_0 \xi + C_0 \frac{\rho^2}{4} + A_0 + B_0 \ln \rho \\ &\quad + \sum_{n=1}^{\infty} [A_n \rho^{sn} + B_n \rho^{-sn}] [\cos sn\phi] \end{aligned} \quad [2.33]$$

2.3.1.2. Determination of the Constants for the Fully Developed Solution

Using the solution form, Eq. [2.33], and substituting it into the boundary condition (Eq. [2.11]) yields:

$$\left. \frac{\partial \theta_f}{\partial \rho} \right|_{\rho_i} = - \frac{P}{D}$$

$$\text{and} \quad \frac{\partial \theta_f}{\partial \rho} = C_0 \frac{\rho}{2} + \frac{B_0}{\rho} + \frac{1}{\rho} \sum_{n=1}^{\infty} [A_n \rho^{sn} - B_n \rho^{-sn}] [sn] [\cos(sn\phi)]$$

For $\rho = \rho_i$

$$-\frac{P}{D} = c_o \frac{\rho_1}{2} + \frac{B_o}{\rho} + \frac{1}{\rho} \sum_{n=1}^{\infty} [A_n \rho_1^{sn} - B_n \rho_1^{-sn}] [sn] [\cos(sn\phi)] \quad (2.34)$$

By applying the orthogonal property of the harmonic functions $\cos(sm\phi)$, ($m=0,1,2,\dots,N$) in the interval $[0,\pi/s]$ by multiplying Equation [2.34] with $\cos(sn\phi)$ and integrating it over the interval $[0,\pi/s]$, i.e.,

$$\int_0^{\pi/s} \cos(ms\phi) \cos(ns\phi) d\phi = \begin{cases} 0 & \text{for } m \neq n \\ \pi/2s & \text{for } m=n \neq 0 \\ \pi/s & \text{for } m=n=0 \end{cases} \quad (2.35)$$

The result will be :

for $n = 0$, B_o is determined as:

$$B_o = - \left[\frac{c_o}{2} \frac{1}{(\frac{P}{D})^2} + 1 \right] \quad (2.36)$$

and for $n > 0$, a relationship for B_n in terms of A_n is obtained, namely,

$$B_n = A_n \left(\frac{1}{\frac{P}{D}} \right)^{2sn} \quad (2.37)$$

The remaining coefficients A_n can be determined by using the homogeneous boundary condition (Eq. 2.12), that is, by using

Equations [2.33, 2.36, and 2.37], one gets the following equation:

$$\begin{aligned}
 \hat{n}_j \cdot \text{grad } \theta &= \cos \phi_0 \frac{\partial \phi}{\partial \rho} - \frac{1}{\rho} \sin \phi_0 \frac{\partial \theta}{\partial \phi} \\
 &= (\cos \phi_0) \left(C_0 \frac{\rho}{2} + B_0 \frac{1}{\rho} + \sum_{n=1}^{\infty} \frac{A_n}{\rho} [\rho^{\text{sn}} - \rho_i^{2\text{sn}} \rho^{-\text{sn}}] \right. \\
 &\quad \left. \text{sn} \cos \text{sn}\phi \right) + \left(\frac{1}{\rho} \sin \phi_0 \right) \left(\sum_{n=1}^{\infty} A_n [\rho^{\text{sn}} + \rho_i^{2\text{sn}} \rho^{-\text{sn}}] \right. \\
 &\quad \left. (\text{sn})(\sin \text{sn}\phi) \right)
 \end{aligned} \tag{2.38}$$

From Eq. [2.12], the following set of linear equations results:

$$\begin{aligned}
 \sum_{n=1}^{\infty} \text{sn} A_n [\rho_a^{\text{sn}} \cos(\phi_0 - \text{sn}\phi_a) - \rho_i^{2\text{sn}} \rho_a^{-\text{sn}} \cos(\phi_0 + \text{sn}\phi)] \\
 = - \left[C_0 \frac{\rho_a}{2} + B_0 \frac{1}{\rho_a} \right] \rho_a \cos \phi_0.
 \end{aligned} \tag{2.39}$$

Here, the following relationships have been applied:

$$\cos \phi_0 \cdot \cos(\text{sn}\phi) + \sin \phi_0 \sin(\text{sn}\phi) = \cos(\phi_0 - \text{sn}\phi)$$

$$\cos \phi_0 \cdot \cos(\text{sn}\phi) - \sin \phi_0 \sin(\text{sn}\phi) = \cos(\phi_0 + \text{sn}\phi).$$

Several solution techniques are available to solve the above problem, such as the point-matching technique, the boundary least-square method, the integral boundary least square method etc. After the coefficients, A_n 's have been determined by either

one of these methods, the B_n 's can be obtained by the recursion formula, Eq. [2.37].

The only remaining constant yet to be determined is A_0 which fixes the absolute level of θ_f . One has some freedom to fix A_0 . Here, a decision is made to determine the coefficient A_0 such that the cell average temperature distribution equals zero. The average coolant temperature becomes a linearly increasing function with axial position, i.e.

$$\theta_{f, \text{avg}}(\xi) = Z_f(\xi) = C_0 \xi$$

$$\text{then } A_0 = \frac{\int_0^{\pi/s} d\phi \int_{\rho_i}^{\rho_a} \rho d\rho \left[C_0 \frac{\rho^2}{4} + B_0 \ln \rho + \sum_{n=1}^{\infty} [A_n \rho^{sn} + B_n \rho^{-sn}] \cos sn\phi \right]}{\int_0^{\pi/s} d\phi \int_{\rho_i}^{\rho_a} \rho d\rho} \quad (2.40)$$

Two things need to be mentioned here: first, by evaluating the solution, using the computer as a tool, it is impossible to compute an infinite series because of the limited capacity of the computer. There the summation has to be performed over a finite number of harmonic terms (N_k). The number of terms required mainly depends on how large the truncation error will be. The second item that has to be discussed is the numerical technique of integration which is required to calculate A_0 . Here, the integration (Eq. [2.40]) will be performed by analytical means in the radial direction whereas in the azimuthal direction, a numerical scheme is used. In the computer code

the Gaussian Quadrature Method is applied. More details about this method are discussed in Appendix C. With all the coefficients known, the fully-developed temperature distribution is totally determined.

2.3.2. Solution for the Thermal Entrance Temperature Profile

2.3.2.1. Form of the Solution

Due to the fact that the governing differential equation [2.21] and the associated boundary conditions Eqs. [2.22, 2.23 and 2.24] are homogeneous, the method of separation of variables will be applied again. It is supposed the θ_e can be expressed as

$$\theta_e(\rho, \phi, \xi) = R_e(\rho) \phi_e(\phi) Z_e(\xi). \quad (2.41)$$

Substituting this form into Eq. [2.21], one gets

$$\begin{aligned} \frac{1}{Z_e} \frac{\partial^2 Z_e}{\partial \xi^2} &= - \frac{1}{R_e \phi_e} \left[\left(\frac{\partial^2 R_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial R_e}{\partial \rho} \right) \phi_e + \frac{1}{\rho^2} \frac{\partial^2 \phi_e}{\partial \phi^2} R_e \right] \quad (2.42) \\ &= -\beta^2 \end{aligned}$$

Using the boundary condition, Eq. [2.16], which states that the thermal entrance effect is decreasing with axial position for Z_e , one gets:

$$Z_e(\xi) = E e^{-\beta^2 \xi} \quad (2.43)$$

For the radial and azimuthal solution terms, one obtains

$$\frac{\rho^2}{R_e} \left[\frac{\partial^2 R_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial R_e}{\partial \rho} \right] + \rho^2 \beta^2 = - \frac{1}{\Phi_e} \frac{\partial^2 \Phi_e}{\partial \phi^2} = \pm v^2 \quad (2.44)$$

Due to symmetry property, the positive sign of v^2 is chosen.

As a result, a Bessel Differential Equation for the left hand side is formed. The solution of the equation,

$$- \frac{1}{\Phi_e} \frac{\partial^2 \Phi_e}{\partial \phi^2} = + v^2 \quad (2.45)$$

$$\text{is } \Phi_e(\phi) = K \sin v\phi + H \cos v\phi. \quad (2.46)$$

The solution of the remaining part with parameter R_e , i.e.

$$\frac{\partial^2 R_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial R_e}{\partial \rho} + \left(\beta^2 - \frac{v^2}{\rho^2} \right) R_e = 0 \quad (2.47)$$

$$\text{is: } R_e(\rho) = M \cdot J_v(\beta\rho) + L \cdot Y_v(\beta\rho) \quad (2.48)$$

where J_v and Y_v are linearly independent solutions (eigenfunctions) and β_m are the eigenvalues of the Sturm-Liouville problem with $m = 1, 2, 3, \dots$ with the boundary conditions, Eqs. [2.22, 2.23, and 2.24]. One finds, $K = 0$ and $v = sn$ for $n = 0, 1, 2, \dots$

The product of these separate solutions will constitute the thermal entrance temperature profile in the form

$$\theta_e(\rho, \phi, \xi) = [C J_{sn}(\beta\rho) + D Y_{sn}(\beta\rho)] \cos(sn\phi) E^{-\beta^2 \xi} \quad (2.49)$$

with $C = M \cdot H$ and $D = L \cdot H$.

Additionally, it is necessary to introduce the normalization constant E into Eq. [2.49] because the quantities C and D are determined from homogeneous boundary conditions and thus can only be determined up to a constant factor. Putting all individual solutions together, the final form of the solution is:

$$\theta_e(\rho, \phi, \xi) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} [C_{n,m} J_{sn}(\beta_m \rho) + D_{n,m} Y_{sn}(\beta_m \rho)] \cos(sn\phi) E_m e^{-\beta_m^2 \xi} \quad (2.50)$$

2.3.2.2. Determination of the Constants for the Thermal Entrance Solution

As mentioned in the previous section, due to the limited capacity of the computer, one estimates the summation of the double infinite series to a finite number of terms. Then, the solution becomes:

$$\theta_e(\rho, \phi, \xi) = \sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_{sn}(\beta_m \rho) + D_{n,m} Y_{sn}(\beta_m \rho)] [\cos(sn\phi)] [E_m e^{-\beta_m^2 \xi}] \quad (2.51)$$

By splitting the constant, $C_{n,m}$ and $D_{n,m}$ as following:

$$C_{n,m} = C'_{n,m} \cdot F_{n,m} \quad (2.52)$$

$$\text{and } D_{n,m} = D'_{n,m} \cdot F_{n,m} \quad (2.53)$$

a relationship between $C_{n,m}$ and $D_{n,m}$ is obtained by using the

boundary condition, Eq. [2.22] and the orthogonal property of the harmonic function, namely,

$$\frac{C'_{0,m}}{D'_{0,m}} = \frac{\beta_m Y_1(\beta_m \rho_i)}{\beta_m J_1(\beta_m \rho_i)} = \frac{C_{0,m}}{D_{0,m}} \quad \text{for } n = 0 \quad (2.54)$$

$$\begin{aligned} \frac{C'_{n,m}}{D'_{n,m}} &= - \frac{\left[\frac{\beta_m}{\rho_i} Y_{sn}(\beta_m \rho_i) - \beta_m Y_{sn+1}(\beta_m \rho_i) \right]}{\left[\frac{\beta_m}{\rho_i} J_{sn}(\beta_m \rho_i) - \beta_m J_{sn+1}(\beta_m \rho_i) \right]} \\ &= \frac{C_{n,m}}{D_{n,m}} \quad \text{for } 1 \leq n \leq N_n. \end{aligned} \quad (2.55)$$

The reason for establishing Eqs. 2.52 and 2.53 is that it is impossible to determine two types of constants out of one boundary condition. The proportionality factors $F_{n,m}$ have to be determined from a different equation, in this case, the characteristic equation will be used. To obtain the proportionality factors $F_{n,m}$'s and the eigenvalues β_m 's, the boundary condition Eq.[2.23] is used. In order to obtain the numerical solution, Eq. [2.23] must be satisfied at K discrete points ($K \geq N_n$) at the outer coolant cell boundary. Substituting Equation [2.51] into Equation [2.23] by additionally considering Equation [2.38], results in a set of M homogeneous linear equations, namely

$$\begin{aligned}
& \sum_{n=0}^{N_n} \cos \phi_0 \cdot \cos(\sin \phi_k) F_{n,m} \{ C'_{n,m} \left(\frac{\sin}{\rho_k} J_{\sin}(\beta_m \rho_k) \right. \\
& \quad \left. - \beta_m J_{\sin+1}(\beta_m \rho_k) \right) + D'_{n,m} \left(\frac{\sin}{\rho_k} Y_{\sin}(\beta_m \rho_k) - \beta_m Y_{\sin+1}(\beta_m \rho_k) \right) \} \\
& + \sin \phi_0 \cdot \sin(\sin \phi_k) \frac{\sin}{\rho_k} \cdot F_{n,m} \{ C'_{n,m} J_{\sin}(\beta_m \rho_k) + D'_{n,m} \\
& \quad Y_{\sin}(\beta_m \rho_k) \} = 0 \tag{2.56}
\end{aligned}$$

$$\text{for } \rho_k = \frac{a_j}{\left(\frac{P}{2}\right) \cos(\phi_k - \alpha_j)}$$

$$j = 1, 2, 3, 4$$

$$K = 1, 2, 3, \dots, K$$

$$K > N_n$$

$$m = 1, 2, \dots, M.$$

The condition is only satisfied for specific values of β_m . For $m = \text{constant}$, the sums in Equation [2.56] are the product of matrix W multiplied by a vector F_n . Thus in matrix formulation, one has

$$W_{kn} \cdot \vec{F}_n = 0$$

where the element kn in the matrix W is given by

$$\begin{aligned}
(W)_{kn} = & (\cos \phi_0)(\cos n\phi_k) \{ C'_{n,m} \left(\frac{sn}{\rho_k} J_{sn}(\beta_m \rho_k) \right. \\
& - \beta_m J_{sn+1}(\beta_m \rho_k) \} + D'_{n,m} \left(\frac{sn}{\rho_k} Y_{sn}(\beta_m \rho_k) \right. \\
& \left. - \beta_m Y_{sn+1}(\beta_m \rho_k) \} \} + (\sin \phi_0)(\sin n\phi_k) \\
& \left(\frac{sn}{\rho_k} \{ C'_{n,m} J_{sn}(\beta_m \rho_k) + D'_{n,m} Y_{sn}(\beta_m \rho_k) \} \right)
\end{aligned}$$

It is well known that a homogeneous set of equations has a nontrivial solution only in case that its coefficient matrix vanishes. Because it is only possible to determine the determinants from a quadratic matrix, one has in case of $K > N_n$, to multiply the coefficient matrix W from left side by its transpose, $[W]^*$ in order to generate a quadratic, symmetric and positively definite matrix $[W]$ such that

$$[W]^*[W] = [W] \quad (2.58)$$

This matrix multiplication can also be explained by the method of least squares as follows. Equation 2.57 represents M sets of linear equations

$$\sum_{n=0}^N F_{n,m} \cdot W_{n,m}(\rho_k, \phi_k, \beta_m) = 0 \quad (2.59)$$

$$\text{for } k = 1, 2, 3, \dots, K$$

$$m = 1, 2, 3, \dots, M$$

It can be shown that those $F_{n,m}$ and β_m satisfy the boundary

condition

$$\sum_{n=0}^N [F_{n,m} W_{n,m} (\rho_k, \phi_k, \beta_m)^2] = R. \quad (2.60)$$

Differentiating this form with respect to the coefficients $F_{n,m}$ and setting it to zero results in the same matrix formulation.

$$[W]*[W] = [W].$$

The determinant of the matrix W , also known as the Gram-determinant yields a minimum for those β_m which give a minimum of R . By virtue of the matrix operation, the zeros of the determinant have not changed. Thus, the β_m 's can now be determined.

When the eigenvalues, β_m 's are found, it is easy to generate the $F_{n,m}$. Due to the homogeneity of Eq. [2.56], again, the solution is only possible up to a constant factor, E_m , which has to be fixed later on. In a first step, the m solution vectors F_n ($0 \leq n \leq N$) are arbitrarily normalized by its first elements, (that is, $F_{0,m} = 1$ for $1 \leq m \leq M$). In a next step, the constant $C_{n,m}$ and $D_{n,m}$ are calculated by using Eq. [2.52] and Eq. [2.53] since $F_{n,m}$'s are known. The only remaining unknowns E_m is the solution -- Eq. [2.51] -- can be determined by applying the initial condition, Equation 2.15. At any point of the coolant cross-section, the dimensionless temperature at the inlet should equal the initial inlet temperature profile.

Due to the fact that the outer boundary of the coolant

channels is not a member of the orthogonal cylindrical coordinate system, one cannot use the orthogonality of Bessel functions with respect to the weighting function ρ in the interval. Therefore, the method of internal collocation must be used, that is, one has to satisfy the latter at P points in the interior and at the boundary of the coolant channel. This leads to a linear set of equations for the unknowns E_m , namely,

$$\sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_{sn}(\beta_m \rho_k) + D_{n,m} Y_{sn}(\beta_m \rho_k)] \cos(sn\phi_k) E_m$$

$$= C_0 \frac{\rho_k^2}{4} + A_0 + B_0 \ln \rho_k + \sum_{n=1}^{N_k} [A_n \rho_k^{sn} + B_n \rho_k^{-sn}] \cos(sn\phi_k)$$

for $k = 1, 2, 3, \dots, P$

(2.61)

where $P > M$

The solution vector of this set of inhomogeneous linear equations with known right hand side contains the wanted E_m . Again, use is made of the orthonormalization procedure which is coupled as described with the method of least squares. This procedure results in a solution which is fairly independent upon the position of the collocation points chosen. With the E_m 's found, the total solution for the thermal entrance region is known.

2.3.3. Surface Temperature and Nusselt Number

The clad temperature at the surface of the fuel pin is given by:

$$\theta_w(\rho_i, \phi, \xi) = \theta_{f,w}(\rho_i, \phi, \xi) + \theta_{e,w}(\rho_i, \phi, \xi) \quad (2.62)$$

$$\begin{aligned} \text{with } \theta_{f,w}(\rho_i, \phi, \xi) = & C_o \xi + C_o \frac{\rho_i^2}{4} + A_o + B_o \ln \rho_i \\ & + \sum_{n=1}^{N_k} 2 A_n \rho_i^{sn} \cos(sn\phi) \end{aligned} \quad (2.63)$$

$$\begin{aligned} \text{and } \theta_{e,w}(\rho_i, \phi, \xi) = & \sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_{sn}(\beta_m \rho_i) + \\ & D_{n,m} Y_{sn}(\beta_m \rho_i)] \cos(sn\phi) E_m e^{-\beta_m^2 \xi} \end{aligned} \quad (2.64)$$

The average wall temperature follows from integration over the pin perimeter, i.e.

$$\theta_{w,avg}(\xi) = \frac{\int_0^{\pi/s} \theta_w(\phi, \xi) \rho_i d\phi}{\int_0^{\pi/s} \rho_i d\phi} \quad (2.65)$$

Performing this integration for both parts as given by Equation [2.63] and Equation [2.64] yields

$$\theta_{f,w,avg}(\xi) = C_o \xi + C_o \frac{\rho_i^2}{4} + A_o + B_o \ln \rho_i \quad (2.66)$$

$$\begin{aligned} \text{and } \theta_{e,w,avg}(\xi) = & \sum_{m=1}^M [C_{o,m} J_o(\beta_m \rho_i) + D_{o,m} Y_o(\beta_m \rho_i)] E_m e^{-\beta_m^2 \xi} \end{aligned} \quad (2.67)$$

The average coolant temperature is already known from Equation

[2.26].

$$\theta_{f, \text{avg}}(\xi) = \theta_{\text{avg}}(\xi) = C_o \xi \quad (2.68)$$

and $\theta_{e, \text{avg}}(\xi) = 0$

With the definition of the Nusselt number,

$$Nu = \frac{hD_h}{k} = \frac{D_h}{k} \frac{q''}{T_w - T_{\text{avg}}} \quad (2.69)$$

the local Nusselt number $Nu(\phi, \xi)$ follows with Equation [2.26].

$$Nu(\phi, \xi) = \frac{D_h/r_i}{\theta_w(\phi, \xi) - \theta_{\text{avg}}(\xi)} \quad (2.70)$$

and the azimuthally averaged Nusselt Number, $Nu(\xi)$, follows as

$$Nu(\xi) = \frac{D_h/r_i}{\bar{\theta}_w(\xi) - \theta_{\text{avg}}(\xi)} \quad (2.71)$$

Thus, one obtains for the asymptotic Nu-number, Nu_f

$$Nu_f = \frac{D_h/r_i}{C_o \frac{\rho_i^2}{4} + A_o + B_o \ln \rho_i} \quad (2.72)$$

One considers a coolant flow as being thermally fully developed when the condition $\frac{Nu_f}{Nu(\xi)} = 0.99$ is satisfied. The associated channel length is called thermal entrance length, ξ_{th} . Physically, this does not necessarily imply that the temperature field has been thermally developed up to 99% of its thermally fully-

developed value because the Nusselt number, $Nu(\xi)$ has been azimuthally averaged and is not the local Nusselt number of the fuel pin in the cell.

2.3.4. Axially Variable Heat Flux Density

With an axially non-uniform heat flux distribution, the heat flux becomes a function of the axial coordinate. Fortunately, the solution obtained for the uniform axial heat flux distribution can be used and applied Duhamel's principle [10] to calculate the solution for an axially variable heat flux problem. It must be pointed out here that the axial heat flux distribution has to be a piecewise continuous function in order to apply Duhamel's Principle. This requirement is due to the integration involved. The detailed derivation of Duhamel's principle is presented in Appendix B. The final solution for the non-uniform axial heat flux distribution can be expressed as follows:

$$\theta(\rho, \phi, \xi) = \theta_k(\rho, \phi, \xi)q''(0) + \int_0^{\xi} \theta_k(\rho, \phi, \xi - \chi) \frac{dq''(\chi)}{d\chi} d\chi \quad (2.73)$$

where $\theta_k(\rho, \phi, \xi)$ is the temperature which results for axially $q'' = \text{constant}$

$q''(0)$ is heat flux density at $\xi = 0$

2.4. Computer Code Development

The computer code SCIO has been modified, updated, and renamed MIT3D. The solution procedure of the code mainly consists of three

parts, SCIO I, II and III as before: The first one is to handle the calculation of the fully-developed temperature solution ; the second part solves the eigenvalue problem whereas the third one is to obtain the thermal entrance solution and combines it with the fully-developed solution to form the complete set. Furthermore, the last part provides a solution method for arbitrarily varying axial heat flux problems provided that the distribution is a well-behaved function.

Throughout the problem several adjustments have been made in order to upgrade the efficiency of the code. One of the problems denoted earlier was the efficiency of the searching for the eigenvalues with given accuracy. Upon changing the SCIO II into double precision and using a three-point iteration scheme for the eigenvalue search, the accuracy of the calculated eigenvalues could be improved from 10^{-4} to 10^{-10} at least. Moreover, a new set of Bessel function subroutines were built into the code and resulted in a reduction of the computation time by a factor of 10 if compared to the original version of the code. In addition, three-nested DO-loops were abolished in SCIO III. In order to be able to handle local perturbations in the inlet temperature distribution the respective parts of the code were streamlined.

2.5. Results of the Isolated Cell Analysis

Principally, the code can handle a 3-D temperature field calculation for corner, edge and internal cells of bare hexagonal LMFBR bundles.

In order to confirm the validity of the results as obtained by the code, a comparison was made with recent measurements of 3-D clad temperature profiles in a sodium-cooled 19-pin bundle done by Möller et. al [22]. For this purpose, it was decided to pick the following conditions for the calculation:

$$P/D = 1.3; \quad 2W/D = 1.38; \quad D = 9 \text{ mm}; \quad Pe = 370$$

and

$$T_{in} = 413^{\circ}\text{C} \quad \bar{q}'' = 104 \text{ W/cm}^2 \text{ (flat axial power distribution)}$$

2.5.1. Edge Cell

2.5.1.1. Uniform Axial Heat Flux Distribution

For the edge cell with uniform axial power distribution, Fig. [2.6] summarizes the results of the calculation and includes the results of the experiment taken at different axial levels in the bundle. In view of this comparison, two observations are worth mentioning. First of all, it obviously seems that the calculated results miss the average temperature level at the specific axial heights although this difference steadily diminishes with increasing height. The explanation for this discrepancy is indeed very simple. The experimental results do not really represent outside clad temperatures but profiles as measured by the embedded thermocouples inside the clad. As a result, experimental data must be corrected not only for radial but also for azimuthal heat conduction effects in the

Fig. 2.6 Axial Development of the Clad Temperature Distribution
 of the Edge Cell for Uniform Axial Heat Flux as Predicted by
 Isolated Cell Analysis

(Condition: $P/D=1.30$, $2W/D=1.38$, Dia.=0.9 cm, $Pe=372$, $q''_0=10^4$ W/cm)

clad. Naturally, these corrections vary for both effects at different axial levels and must be accurately found by a fully coupled 3-D temperature analysis of fuel, clad and coolant. Certainly, a 1-D analysis which only accounts for radial heat conduction does not reflect reality although it constitutes a first approximation. Thus, a temperature correction of $16^{\circ}\text{C} \pm 4^{\circ}\text{C}$ as quoted by Möller [22] does not hold equally well for all axial levels. In order to fully show the difficulties which are involved with the reduction of experimental data, it was not tried yet to either modify the original data or the predicted results. Obviously, the radial temperature gradient is largest at the inlet and smooths out at higher levels. On the other hand, the azimuthal temperature gradient increases with increasing axial direction from the channel inlet. Having the aforementioned explanation in mind, these results compare quite favorably with the experiment.

The other point to be mentioned concentrates on the azimuthal temperature variations. Table 2.1 shows that the analytical method always results in conservative values for the azimuthal temperature variations at the outlet region of $z = 65.5$ cm. The conservatism increases with increasing axial distance from the inlet as it is clearly demonstrated in Fig. [2.6].

As can be seen from Table [2.1] as well as Fig. [2.7] Fig. [2.8] and Fig. [2.9], the experimental results indicate slightly larger azimuthal temperature variations in the inlet region for low and medium Pe than predicted. This effect

TABLE 2.1

COMPARISON BETWEEN MEASURED AND CALCULATED
AZIMUTHAL TEMPERATURE DIFFERENCES FOR AN EDGE CELL

Pe = 372.

Axial Height (cm.)	$(T_{\max} - T_{\min})_{\text{meas.}}^{\circ\text{C}}$	$(T_{\max} - T_{\min})$	$(T_{\max} - T_{\min})_{\text{calc}}^{\circ\text{C}}$ <hr/> $(T_{\max} - T_{\min})_{\text{mea}}$
11.5	9.7	3.87	0.3988
29.5	10.3	13.69	1.329
47.5	18.3	22.77	1.244
65.5	20.63	32.10	1.566
Thermally fully developed	_____	121.45	_____

Fig. 2.7 Maximum Clad Temperature Difference of the Isolated Edge Cell vs Axial Coordinate for $Pe=372$ and Uniform q''

Fig. 2.8 Maximum Clad Temperature Difference of the Isolated Edge Cell vs Axial Coordinate for $Pe=450$ and Uniform q''

Fig. 2.9 Maximum Clad Temperature Difference of the Isolated Edge Cell for Various Pe Number and Uniform q''

diminishes with increasing Pe-number. Furthermore, as shown in Fig. [2.10], the difference between measured and calculated results at $z = 65.5$ cm. decreases with increasing Pe. Thus, the calculation agrees quite well with the measurement for $Pe > 800$. The anomalies in the experimental results in the low Pe-range can be possibly attributed to already existing natural convection effects.

In Fig. [2.8], the calculated results are compared against the experimental results obtained by Möller [22] and Subbotin, et al. [21]. In the first place, it is surprising to see that the difference between the two measurements is of the same order as the difference between the calculation and the Karlsruhe measurement. A possible explanation for this discrepancy was given by Möller [22] who indicated that the Russian experiment was performed for q'' being only $\bar{q}'' = 2-6 \text{ W/cm}^2$. Thus, a large uncertainty in the maximum clad temperature difference has to be assigned to the results for these experiments. Furthermore, the thickness of the bundle wall was appreciably larger than normally used in reactors which leads to large azimuthal conduction effects and thus decreasing coolant temperature asymmetries. From all of the aforementioned figures, it becomes quite clear that the thermally fully-developed region is by far not reached, especially for higher Pe-numbers. This can also be seen from Fig. [2.11] where the Nu-number averaged over the cross sectional area is shown as function of the axial coordinate. Nu still decreases and does not reach its asymptotic value, Nu_{∞} , throughout the whole core. As a results, 2-D thermally fully-developed analysis do result in too conserva-

Fig. 2.10 Maximum Clad Temperature Difference of the Isolated Edge Cell as Function of Pe - number for $z=65.5$ cm and Uniform q''

Fig. 2.11 Axial Variation of the Nusselt Number of the Isolated Edge Cell for Uniform Heat Flux

Fig. 2.12 Nusselt Number of the Isolated Edge Cell as Function of Pe - Number for $z = 65.5$ cm and Uniform q''

tive values for azimuthal temperature differences as depicted in the last entry of the Table 2.1. For the special case under consideration, the real temperature variation is only $\frac{1}{6}$ that of the result as calculated by a two-dimensional analysis. Thus, the 3-D analysis as used in this study is indeed a big step forward into the right direction. Fig. [2.13] clearly shows how good the agreement between experiment and calculation for $Pe = 622$ is. This agreement is indeed quite surprising when one realizes what effects really affect the measurement and what effects have been neglected in the analysis. Table [2.2] gives an indication about these discrepancies.

2.5.1.2. Non-Uniform Axial Heat Flux Distribution

This section summarizes the finding of an investigation about the effects of an axially variable heat flux. At the same time the difference to the results of the uniform axial heat flux profile are reported. The sinusoidal axial heat flux function used here has the following form:

$$q''(z) = q''_{\max} \cos \frac{\pi z}{H_0} \quad (2.74)$$

where $q''_{\max}$ = maximum heat flux

$$= (f_A)(q''_0)$$

and f_A = axial peaking factor

and q''_0 = average heat flux

and H_0 = extrapolated distance

In order to find the extrapolated distance when the axial peaking

$$\frac{T - T_{\min}}{T_{\max} - T_{\min}}$$

Fig. 2.13 Dimensionless Temperature Variation of the Isolated Edge Cell for $Pe=622$ and Uniform q''

TABLE 2.2

LISTING OF EFFECTS WHICH ARE CONSIDERED IN THE
MEASUREMENT AND THE ANALYSIS

<u>Effect</u>	Measurement	Slug-Flow Analysis	Nijssing, Eifler [13]
conduction	yes	yes	yes
axial conduction	yes	no	no
conduction in walls	yes	no	no
turbulent convection	yes	no	yes
mixed convection at low Pe	yes	no	no
grid effects	yes	no	no
temperature dependent properties	yes	no	no
coupling to neighboring cells	yes	no	no
perturbation due to TCs	yes	no	no

factor is known, one has to perform the following calculation:

$$\begin{aligned}
 q''_0 &= \frac{\int_{-z_0/2}^{z_0/2} q''(z) dz}{z_0} \\
 &= \frac{(f_A)(q''_0) \int_{-z_0/2}^{z_0/2} \cos \frac{\pi z}{H_0} dz}{z_0} \\
 &= (f_A)(q''_0) \left(\frac{2H_0}{\pi z_0} \right) \left(\sin \frac{\pi z_0}{2H_0} \right) \quad (2.75)
 \end{aligned}$$

$$\text{therefore} \quad (f_A) \left(\frac{2H_0}{\pi z_0} \right) \left(\sin \frac{\pi z_0}{2H_0} \right) = 1 \quad (2.76)$$

where z_0 is the total core-height

The axial peaking factor that will be used here is 1.24. This value of 1.24 is chosen because it closely reflects the Clinch River Fast Breeder Reactor Design. All other parameters will be kept the same as used for the flat heat flux distribution.

Figures [2.14] and [2.15] show a comparison of the azimuthal temperatures due to the flat and the sinusoidal heat flux profiles, whereas Fig. [2.16] depicts the comparison of the maximum temperature variations for both profiles as functions of the axial coordinate for $Pe = 372$. From the set of curves in Figs. [2.14] and [2.15], one can conclude that the average temperature level for axially varying heat flux is lower than that of the

Fig. 2.14 Axial Development of Clad Temperature Distribution of the Isolated Edge Cell for Non-Uniform q''

(Condition: $P/D=1.3$, $2W/D=1.38$, Dia.=0.9 cm, $Pe=372$, $Ht=100$ cm, Peaking Factor=1.24, $q''_0=104$ W/cm²)

Fig. 2.15 Axial Development of Clad Temperature Distribution of the Isolated Edge Cell for Non-Uniform q'' (continued)

Fig. 2.16 Maximum Clad Temperature Difference of the Isolated Edge Cell vs Axial Coordinate for Non-Uniform q''

flat profile in the inlet region. Further downstream the average temperature level for the sinusoidal profile slowly overtakes that of the flat profile but finally in the outlet region the average temperature level for the flat profile is almost the same as the sinusoidal profile. This occurs because the total energy supply is the same in both cases. So far, no experimental data exist in the open literature to compare these findings. However, it is believed that the results for the sinusoidal profile are very sensitive to the following parameters: axial peaking factor, Pe-number and heated length of the core. Further sensitivity studies of these parameters will provide more conclusive results. Fig. [2.16] summarizes the results with respect to the azimuthal temperature differences for both profiles and the experimental results for a flat heat flux profile. As can be seen from this figure, the difference between the two profiles is larger at the inlet and the outlet regions.

Compared to the uniform axial heat flux profile, the maximum circumferential clad temperature difference is smaller at the lower levels of the core for the sinusoidal heat flux profile because of the comparatively smaller heat flux in this region. But at the outlet, this difference is higher than for the uniform axial profile, even though the average heat flux is the same in both cases. As a results, one can deduce that for any cosine axial heat flux distribution, as long as the average heat flux remains the same, the higher the axial power peaking is, the larger the maximum clad temperature

difference will be at the outlet. Fig. [2.17] shows how the Nusselt number varies as function of the axial distance for the two different heat flux distribution. Under the condition of cosine axial profile, there is a substantial decrease in the Nusselt number near the outlet region. This may create some heat transfer problem for the clad outside near the outlet region.

2.5.1.3. Non-uniform Inlet Temperature and Axially Uniform Heat Flux Profile

An interesting case for demonstrating the flexibility of the computer code is by introducing a perturbation in the dimensionless inlet temperature over a certain section of the cross section as shown in the insert of Fig. [2.18]. This can be thought of as initiating a hot lump of fluid into the edge cell. Fig. [2.18] shows how this disturbance spreads around the circumference of the pin as a function of the axial distance. For convenience the solution for a uniform inlet temperature distribution is given again. The comparison shows that the disturbance creates a rather severe clad temperature peak at the inlet. The peak is then reduced by the azimuthal conduction effect in the coolant. At the same time the temperature field of the coolant in the unperturbed section of the cell starts to develop. Thus, there are two competing effects. Following the axial development of the temperature fields through the different axial levels, one realizes that the perturbed temperature profile approaches the unperturbed temperature pro-

Fig. 2.17 Axial Variation of the Nusselt Number of the Isolated Edge Cell for Non-Uniform q''

Fig. 2.18 Axial Development of Clad Temperature Distribution of the Isolated Edge Cell with Perturbed Inlet Temperature and Uniform q''

($P/D=1.3$, $2W/D=1.38$, Dia.=0.9 cm, $Pe=372$, $q''_0=104 \text{ W/cm}^2$)

file for large axial distance. Naturally, the higher temperature asymmetry introduced by the perturbation throughout the cell reflects itself in a further decreases of the average Nusselt number as shown in Fig. [2.19].

2.5.2. Results for the Corner Cell

2.5.2.1. Uniform Axial Heat Flux Distribution

The geometric parameters and the thermal input data for the following study are the same as those already used for the edge cell study. An additional feature of this study as compared to what has been done elsewhere thus far is that a rounded corner of the duct wall has been introduced in order to check its positive effect on the local temperature distribution. A radius of $r_0 = 0.55$ cm. has been chosen which coincides with the experimental set-up used in Ref. [22].

Figures [2.20] and [2.21] show the development of the azimuthal temperature distributions around the pin for the sharp-edged and the rounded corner cases. As can be seen from these distributions, the rounded corner lifts the temperature in the vicinity of the corner but what is more important, it reduces the maximum temperature at 180 degrees. Thus it has indeed an overall beneficial effect upon reducing the overall temperature variation which can be also seen from Table [2.3].

By comparing the results in Table [2.3] with those of the Table [2.1] presenting the results for the edge cells, one notes that the latter show larger temperature variations.

Figures [2.22] and [2.23] compare the calculational results

Fig. 2.19 Axial Variation of the Nusselt Number of the Isolated Edge Cell with Perturbed Inlet Temperature and Uniform q''

Fig. 2.20 Fully-Developed Clad Temperature Distribution of the Isolated Corner Cell for Uniform q''

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $q''=104$ W/cm²)

Fig. 2.21 Axial Development of Clad Temperature Distribution of the Isolated Corner Cell for Uniform q''

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $q''=104$ W/cm²)

TABLE 2.3

COMPARISON OF MAXIMUM AZIMUTHAL TEMPERATURE DIFFERENCES OF
SHARP-EDGED AND ROUNDED CORNER CELLS

Axial Height	$(T_{\max} - T_{\min})_{\text{sharp}}$	$(T_{\max} - T_{\min})_{\text{round}}$	$(T_{\max} - T_{\min})_{\text{exp}}$
11.5 cm.	3.02°C	2.76°C	6.5°C
29.5 cm.	11.19°C	10.93°C	13.2°C
47.5 cm.	19.26°C	18.17°C	18.1°C
65.5 cm.	27.49°C	24.54°C	21.7°C
Thermally fully developed	81.75°C	68.05°C

Fig. 2.22 Maximum Clad Temperature Difference of the Isolated Corner Cell vs Axial Coordinate for Uniform q'' and $Pe=372$

Fig. 2.23 Maximum Clad Temperature Difference of the Isolated Corner Cell vs Axial Coordinate for Uniform q'' and $Pe=450$

with the experimental evidence as reported by Möller, et al. [22] and Subbotin [21]. Again, the agreement is quite satisfactory, even though some differences among these 3 curves still exist due to the reasons explained in Section 2.5.1.1. Figures [2.24] and [2.25] show that the deviation between the calculated results and the experimental results is diminishing for higher Pe-numbers.

Finally, Fig. [2.26] and Fig. [2.27] show a comparison of the Nusselt numbers as function of the axial position on Pe-number for both types of corner cells. From both graphs it becomes apparent that the rounded corner cell always show overall a better heat transfer behavior than the sharp-edged corner cell. This occurs due to the fact that the sharp-edged corner creates a larger coolant temperature asymmetry, so that the resultant Nusselt number is slightly less than that in case of the rounded corner.

2.5.2.2. Non-Uniform Axial Heat Flux Distribution

The same sinusoidal function as applied in the study of the edge cell is used here to represent an axially variable heat flux. The function is:

$$q''(z) = q''_{\max} \cos \frac{\pi z}{H_0}$$

Again, the axial peaking factor used is 1.24. Fig. [2.28] and Fig. [2.29] show the development of the temperature profile around the clad for the non-uniform axial heat flux distribution.

Fig. 2.24 Maximum Clad Temperature Difference of the Isolated Corner Cell vs Axial Coordinate for Different Pe-Number

Fig. 2.25 Maximum Clad Temperature Difference of the Isolated Corner Cell as Function of Pe - Number for $z = 65.5$ cm

Fig. 2.26 Axial Variation of the Nusselt Number of the Isolated Corner Cell for Uniform q'' and $Pe = 372$

Fig. 2.27 Nusselt Number of the Isolated Corner Cell as Function of Pe - Number at $z = 65.5$ cm

Fig. 2.28 Axial Development of Clad Temperature Distribution of the Isolated Corner Cell for Non-Uniform q''

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $q''_0=104$ W/cm²,
 $Ht=100$ cm, Peaking Factor=1.24)

Fig. 2.29 Axial Development of Clad Temperature Distribution of the Isolated Corner Cell for Non-Uniform q'' (continued)

Similarly to the finding in case of the uniform axial heat flux, the graphs show that the corner cell with the round-edged duct wall always depicts a smaller azimuthal temperature difference and a lower maximum clad temperature at all levels. Fig. [2.30] shows the axial variation of the maximum clad temperature difference. Again, the graph proves that the round-edged corner cell does give improved overall heat transfer capability which results in a smaller maximum clad temperature difference. Moreover, when a comparison between the Nusselt number of these two cases is made, the results of which are drawn, in Fig. [2.31], it can be shown that the effect of a round-edged corner of the duct wall instead of a sharp-edged corner is beneficial in the sense that a higher average Nu-number results for the isolated corner cell.

2.5.3. Results for the Internal Cell

2.5.3.1. Uniform Axial Heat Flux Distribution

The internal cell, compared to the edge and the corner cell shows a higher degree in geometric symmetry but has a smaller coolant cross-sectional area. Because of this larger fuel-to-coolant ratio, the average temperature at the clad surface and the coolant temperature are always higher than in the other cells. Due to the high degree in geometrical symmetry, the internal cell has a smaller circumferential temperature variation. In general, the maximum circumferential clad temperature variation is about $2-4^{\circ}\text{C}$. All these results are clearly shown in Figs. [2.32] and [2.33]. Fig. [2.34] depicts the Nusselt number as function of the axial coordinate. As

Fig. 2.30 Maximum Clad Temperature Difference of the Isolated Corner Cell vs Axial Coordinate for Non-Uniform q''

Fig. 2.31 Axial Variation of the Nusselt Number of the Isolated Corner Cell for Non-Uniform q''

Fig. 2.32 Axial Development of Clad Temperature Distribution of the Isolated Internal Cell for Uniform q''

($P/D=1.3$, Dia.=0.9 cm, $Pe=372$, $q''=104 \text{ W.cm}^2$)

Fig. 2.33 Maximum Clad Temperature Difference of the Isolated Internal Cell vs Axial Coordinate with Different Pe - Number

Fig. 2.34 Axial Variation of the Nusselt Number of the Isolated Internal Cell for Uniform q''

can be seen from this figure, the thermal entrance length has been achieved at a distance of about only 25 cm. from the inlet for $Pe = 372$. This results implies that at any level above 25 cm., asymptotic thermally fully-developed temperature profiles exist. In general, thermal entrance effects are less significant for the isolated internal cell. This is in contrast to the finding in Sections [2.5.1] and [2.5.2], where it had been clearly shown that for the edge and the corner cells, the thermal entrance effect plays the key role in the distributed parameter analysis. Having a higher bulk temperature and a smaller coolant cross-sectional area, the heat transfer capability of the internal cell is the best among all three types of cells. The average Nusselt number over the core of axial height = 100 cm. in the internal cell is 13.8 while for the edge and corner cells, the values are only 8.5 and 7.5 respectively for the geometry considered in this study.

2.5.3.2 Non-uniform Axial Heat Flux Distribution

The same sinusoidal function for the axial heat flux distribution is used as already employed in the study of the edge and corner cell. The result in Fig. [2.35] shows the azimuthal clad temperature profile at different axial levels. With the application of a non-uniform axial power distribution, the maximum temperature variation is still insignificant -- less than 3°C . The sinusoidal profile causes the curve to shift and deform into a distorted sinusoidal function which is shown in Fig. [2.36]. In Fig. [2.37], the Nusselt number of

Fig. 2.35 Axial Development of Clad Temperature Distribution of the Isolated Internal Cell for Non-Uniform q''

($P/D=1.3$, Dia.=0.9 cm, Ht=100 cm, $Pe=372$, $q''_0=104 \text{ W/cm}^2$,
Peaking Factor=1.24)

Fig. 2.36 Maximum Clad Temperature Difference of the Isolated Internal Cell vs Axial Axial Coordinate for Non-Uniform q''

Fig. 2.37 Axial Variation of the Nusselt Number of the Isolated Internal Cell for Non-Uniform q''

the internal cell is plotted against the axial coordinate. The curve no longer approaches an asymptotic value at the higher levels of the core as in the condition of uniform axial profile. Instead, similar to what has been observed for the corner and the edge cells, the Nusselt number decreases towards the core outlet.

2.5.3.3. A Comparison of the Predictions with Results from the Open Literature

It will be of general interest to check the predictions presented before with those available from the open literature. So far, the only existing information on the same subject which has been discussed throughout Chapter 2 has been provided by Nijssing [13] and Slagter and Roodherger [26]. Table 2.6 summarizes the methodologies which have been used in each analysis. The results of their predictions are summarized in the series of figures [2.38 through 2.41]. Figs. [2.38] and Fig. [2.39] show the results for the uniform axial heat flux distribution. [2.39] compares the result of the analytical method presented here with Nijssing's results and show that they closely resemble the results of Nijssing's calculation for the maximum clad temperature difference. Slagter's results seem to be slightly off the other two. The Nusselt numbers generated by the three different methods are almost the same. Figs. [2.40] and [2.41] summarize the results for the case of a sinusoidal axial heat flux distribution. Again, the prediction of the maximum clad temperature difference, by the analytical method resembles

TABLE 2.6

LISTING OF THE METHODOLOGY USED IN EACH ANALYSIS

	<u>Methodology</u>
Present Analysis	Classical Analytical method.
Nijsing and Eifler [13]	Finite Difference approach
Slagter and Roodherger [26]	Finite Element method

Fig. 2.38 Maximum Clad Temperature Difference of the Internal Cell vs Axial Coordinate for Uniform q'' as Predicted by Different Analyses

Fig. 2.39 Axial Variation of the Nusselt Number of the Internal Cell for Uniform q'' as Predicted by Different Analyses

Fig. 2.40 Maximum Clad Temperature Difference of the Internal Cell vs Axial Coordinate for Non-Uniform q'' as Predicted by Different Analysis

Fig. 2.41 Axial Variation of the Nusselt Number of the Internal Cell for Non-Uniform q'' as Predicted by Different Analyses

closely the results obtained by Nijssing. However, Slagter's results are still overconservative. The results for the Nusselt number for any of these three methods are almost the same.

2.6. Discussion

Overall, it can be claimed that the results of the calculations presented before are quite encouraging. So far, the agreement between calculated values and experimental data is reasonably good, even though some adjustment is needed for the reasons listed in Section 2.5.1. The great achievement of the slug-flow model proves that the distributed parameter method is a useful tool for detailed heat transfer analysis within the subchannel. However, several remarks about the method seem to be necessary and are listed below.

First, unlike all other subchannel codes such as COBRA-IIIC, no heat transfer correlation is used to calculate the wall temperature of the cladding. As it has been shown throughout this chapter, the Nusselt number is a function of various variables such as pitch-to-diameter ratio, wall-distance-to-radius ratio, Pe-number, axial heat flux distribution and also the detailed geometry of the cell. Most of the heat transfer correlations rely upon results which are usually obtained by averaging the experimental measurements in an infinite array of fuel pins. They do not account for local effects. In this way, the value for Nusselt number is equivalent to the asymptotic value of the isolated internal cell in our analysis. Obviously, applying a single Nusselt number for local design purposes

will probably be misleading, especially for the cell with highly irregular geometry. Table [2.4] shows these differences for the resultant Nusselt number for different geometries under different conditions. Furthermore, from the same table, it can be observed that using the results for fully-developed heat transfer obtained for the conditions of both slug and laminar flow may be far too conservative. Thus, the 3-D distributed parameter analysis really provides an improved knowledge about the variation in the heat transfer characteristics. Moreover, from the results of these calculations, one can evaluate the heat transfer coefficient as a function of Pe-number at any particular level. An example of this calculation is shown in Figs. [2.12 and 2.27]. In this way, a reasonably accurate clad temperature prediction is assured which is especially important in low Pe-number situations like the loss of flow.

Detailed knowledge of the circumferential clad temperature variation is important for the thermo-mechanical design. Severe temperature variation will possibly induce rod bowing. However, the results obtained so far show that this problem will arise predominantly in the edge or the corner cell. If the corner of the duct wall is rounded rather than sharp-edged, the temperature variation becomes smaller and the maximum clad temperature is lowered. The axial heat flux profile has only a small effect on the temperature variation of the cladding especially for the internal cell. In the case of the internal cell, the differences caused by the axially variable heat flux are fairly small. These results have been illustrated clearly

TABLE 2.4

COMPARISON OF NUSSELT NUMBERS FOR ROD BUNDLES WITH $P/D=1.3$,
 $2W/D = 1.38$, $Pe = 372$ HEATING LENGTH = 100 cm.

	Nu (Edge rod)	Nu (Sharp Corner)	Nu (Round Corner)	Nu (Internal Rod)
Condition 1	8.126	7.141	7.2102	13.63
Condition 2	8.496	7.430	7.531	13.79
Condition 3	4.6924	5.1083	5.3664	13.647
Condition 4	3.01	5.05	---	9.06

Condition 1: Slug Flow, Sinusoidal power, Average over height

Condition 2: Slug Flow, Flat power, average over height

Condition 3: Slug Flow, Fully Developed

Condition 4: Laminar Flow; Fully Developed [6]

in Section 2.5.3 and are additionally supported by the findings of Nijssing and Eiffer. On the other hand, for the corner or the edge cell, one should not neglect the effect of the axial heat flux distribution because increasing the peaking factor will result in an increase of the azimuthal clad temperature variation.

Naturally, the flow rate has also a substantial effect on the heat transfer capability of the cell. The axial coordinate denoted as ξ as introduced in Section 2.2 can be expressed as follows:

$$\xi = \frac{z D_h}{\left(\frac{P}{2}\right)^2 Pe}$$

where $Pe = Re \cdot Pr$.

As the flow rate decreases, the Reynolds Number, Re , will also decrease and as a result, the axial coordinate, ξ increases. Therefore, at any particular level of the core, when the flow rate decreases, it will initiate a decrease in the heat transfer coefficient but an increase in the clad temperature variation.

2.7. Additional Remarks about the Isolated Cell Analysis

For an LMFBR fuel subassembly analysis, the assumption of adiabatic boundary conditions along each cell boundary seems to be a reasonable approximation in the first place. However, by checking the results obtained from the internal, edge and corner cell analysis, the average coolant temperature is much

higher in the internal cell than in the others. Thus from an individual cell analysis, a temperature discontinuity is generated at the common boundary shared by two different cell types. This problem will get worse for non-uniform radial power distribution. Table [2.5] shows that the temperature difference at the common boundary shared by two adjacent cells is possibly as high as 30°C at $z = 65.5$ cm. Therefore, in order to satisfy the continuity of temperature and heat flux at the common boundary, heat transport must take place across the common boundary into the adjacent cells with lower enthalpy. This conclusion automatically leads forward to another topic which is the coupling of two adjacent cells and which will be addressed in the next chapter.

TABLE 2.5

COMPARISON OF DIMENSIONLESS TEMPERATURE* AT CORNER BOUNDARY OBTAINED
FROM DIFFERENT GEOMETRIES

$z = 65.5 \text{ cm.}$

Angle	Internal	Sharp Corner	Rounded Corner
180°	1.5609	1.1931	1.1554
172°	1.5475	1.1799	1.1420
164°	1.5143	1.1445	1.1059
157°	1.4788	1.1028	1.0631
150°	1.4621	1.0833	1.0426

Max. Temp. Diff.:

$(\Delta\theta)_{\max}$	(Sharp Corner)	0.3788
	(Round Corner)	0.4195
$(\Delta T)_{\max}$	(Sharp Corner)	26.896°C
	(Round Corner)	29.786°C

$$* \quad \theta(\rho, \phi, \xi) = \frac{T(r, \phi, z) - T_{in}}{q''r_i/k}$$

CHAPTER 3

3-D COUPLED CELL ANALYSIS

3.1. General Description

As mentioned at the end of the last chapter, using an isolated cell approach to calculate the local temperature field of two adjacent cells with different geometry will result in a discontinuity of the temperature distribution and the heat flux at the common boundary. Furthermore, one can imagine that this problem will become worse in cases where a radial power distribution exists across the bundle. This basically implies that imposing an adiabatic boundary condition on the cell boundary may be unrealistic in some particular cases as for instance power-tilting, or near-the-wall region. In order to satisfy the continuity of temperatures and heat fluxes at the common boundary, one must relieve the adiabatic constraint by allowing heat transport across the common boundary. The amount of heat flowing past the outer boundary of a particular cell is principally determined by the thermal condition of its neighboring cell. Obviously, this implies that the heat flow across the common boundary will depend on the thermal and hydraulic conditions such as heat flux, bulk temperature, flow rate, cell geometry, etc. of the other cells. In addition this heat flux is also governed by geometric parameters such as P/D , $2W/D$, radius of the pin and especially the axial coordinate. With the consideration of the thermohydraulic impact

from the neighboring cell, this automatically extends the single-cell method used in the last chapter to 3-D multi-cell calculations.

The necessity of a multi-cell analysis can also be shown by the following argument. It is a well-known fact that not only the coolant region irregularities give rise to strong azimuthal clad temperature variations. But internal fuel pin distortions such as an eccentric pellet position and non-uniform radial power distribution will also contribute to these variations. For the first time, this study will try to incorporate the majority of these contributors in order to check what effect might contribute most and under what circumstances. Thus far, the internal disturbances have only dealt with by thermally decoupling the coolant region and using the concept of heat transfer coefficient. Only one exception has been reported in the literature [13] where the combined effects of power tilt and coolant geometry were studied, with respect to their influence on thermoelastic stress and displacement fields.

For the multi-cell analysis, a new parameter, namely the ratio of the heat fluxes of neighboring rod rings, must be added to the already existent parameters as pitch-to-diameter ratio and wall distance-to-diameter ratio. As a result, mixing length and conduction form factors will be established and presented in terms of these parameters in what follows.

3.2. Formulation of the Model

To attack the problem of coupling two cells in three

dimensions, two solution schemes seem to be feasible and worth consideration. The first one is to apply the iteration method developed by M.K. Yeung for the two-dimensional analysis [8]. His solution procedure obtains a local temperature field from an isolated cell analysis first. Then the average value of the temperature distributions at the common boundary obtained from the result of the isolated cell calculations is generalized. With this temperature distribution as new boundary condition at the common boundary, the temperature of one particular cell is recalculated. Then, a heat flux distribution at the common boundary from the new result is determined by taking the temperature gradient of that particular cell. Using this new heat flux at the common boundary as a new condition of the outer boundary of the other cell, the calculation is performed again. Finally the result of the temperature distributions at the common boundary of these two cells are compared and the continuity of the temperature checked. Then, the whole procedure will be repeated again until the continuity for the temperature field and the heat flux distribution is met by a predetermined fraction. Yeung [8] showed that the solution procedure converges quickly and needs only few iterations. However, by applying this methodology in a 3-D distributed parameter coupled-cell calculation, some problems may occur. Since the heat flux across the common boundary varies with the axial coordinate. This will naturally affect the thermal entrance solution which in turn will not allow the analytical scheme to be used.

As was already discussed in the last chapter, using the

Principle of Superposition, the 3-D solution can be decomposed into two parts: a thermally fully developed solution and a thermal entrance solution. Applying an iteration scheme on the fully developed solution basically works the same way as what has been done by Yeung [8]. However, problems arise when applying the iteration on the thermal entrance solution. The trade-off is the temperature continuity at the common boundary will introduce non-zero heat flux at that outer boundary. This non-zero heat flux condition will make the original Sturm-Liouville eigenvalue problem not applicable and thus results in a severe problem, how to determine the unknown coefficients in the thermal entrance solution. The major concern is to decide what exact value of the eigenvalues, β 's to use. As what has been indicated in Chapter 2, the eigenvalues, β 's determine the importance of the thermal entrance effect in the overall calculation. When coupling two different types of cells together, the degree of irregularity obviously will change. In order to determine the right value of β to use, the geometry of each cell is needed to be studied carefully. So far, there is not a well-established way to find a good set of β 's which can satisfy the given condition. An example for the coupled internal and corner cells will show the details of this problem. In the isolated cell situation, each cell has its own set of eigenvalues, β 's. They are quite different due to the differences in geometry. The internal cell with its highest degree of symmetry, reaches the asymptotic solution of temperature much faster than the corner cell. However, since the internal

cell has a higher average bulk temperature, the allowance of heat flux across the common boundary into the corner will delay the thermal development of the internal cell. Therefore the set of the eigenvalues β 's will change in the case of coupling two cells together. Thus far, no method is known which could handle this eigenvalue problem effectively.

The other problem which needs considerations is that in order to generate a reliable solution, iterations have to be performed at every axial level. This may result in a time-consuming process and is certainly costly.

Since the aforementioned arguments seem to indicate that this iteration scheme for 3-D calculation may not be practical or as highly efficient as in the 2-D calculation, the already available analytical tool is being re-considered. To extend this analytical tool for a 3-D coupled cell analysis, the change which has to be made involves the coolant geometry representation in the code. Instead of studying the cells individually, the geometry as shown in Figure [3.1] is considered. That is, the main idea is to treat the two cells as a whole new unit. Some other typical forms of coolant geometries which can be handled in this way are described in Figure [3.1], too. This concept works well as long as the coolant channel geometry can be easily described by a rod-centered polar-coordinate system located in one of the rods. Although this limits the application of the method to smaller regions, it is felt that the most important bundle-wall-near regions together with the next row of cells can be effectively analyzed. In fact, there are strong

Table 3.1 Comparison of Eigenvalues Obtained from Isolated
Cell Analysis and Coupled Cell Analysis

	<u>Isolated Internal Cell</u>	<u>Isolated Corner Cell</u>	<u>Coupling Cell</u>
β_1	6.271	1.110	0.973
β_2	10.179	2.020	1.978
β_3	12.817	3.337	6.429
β_4	14.301	4.353	9.169
β_5	---	4.915	11.301
β_6	---	6.107	12.839
β_7	---	7.606	13.588
β_8	---	10.230	14.434

Fig. 3.1 Geometrics of the Coupled Cells

indications in the literature [8] that the wall effect may not propagate substantially beyond the next row of internal cells. Therefore, among all those forms of coolant geometries illustrated in Fig. [3.1], the simplest geometry which couples the internal and the corner cell as shown in Fig. [3.2] is selected for the detailed study in this chapter. After the thorough study of the combination of the internal and the corner cell, the geometry as shown in Fig. [5.1] will be analyzed next. This latter geometry couples 7 internal cells together and the results will be discussed in Chapter 5.

For the purpose of making the analysis in its initial stage as simple as possible, the case of coupling internal and corner cells together will be studied first and the input geometrical parameters for both cells are assumed to be the same, such as equal diameter of fuel pins, no rod displacement, etc. Furthermore, the heat fluxes are assumed to be axially and azimuthally uniform for both pins while the radial power tilting is allowed in this analytical study. Other assumptions are listed as follows:

1. Only the coolant region is considered (single region problem);
2. Frictionless Fluid, that is, no dissipation, no shear stress, constant axial velocity at all levels;
3. No heat source in the coolant;
4. Steady State Problem;
5. Incompressible fluid;
6. Material properties (i.e. coolant properties) are

Isolated Cell Analysis

Coupled Cell Analysis

Fig. 3.2 Geometry of the Coupling Between the Corner Cell and the Internal

independent of direction, position and temperature;

7. No pressure gradient between any two points;
8. Negligible axial heat conduction in the fluid;
9. Constant coolant cross section in the axial direction;
10. Single-phase fluid only;
11. No energy transport by radiation;
12. Adiabatic condition at all cell boundaries.

The energy equation for the coolant becomes:

$$k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] = \rho C_p V_z \frac{\partial T}{\partial z} \quad (3.1)$$

with the following boundary conditions

- a. Inlet region: at $z = 0$

$$T(r, \phi, z = 0) = 0 \quad (3.2)$$

- b. At the clad-surface of the corner pin

$$\text{at } r = r_o, \quad -k \frac{\partial T}{\partial r} \Big|_{r=r_o} = q_c'' \quad (3.3)$$

where q_c'' is the heat flux of corner pin

- c. At the clad-surface of the internal pin

$$\text{at } r = r_a, \text{ for } \phi_o < \phi < \pi \quad (3.4)$$

$$-k \hat{n} \cdot \nabla T \Big|_{r=r_a} = q_i''$$

where q_i'' is the heat flux at the outside clad sur-

face internal pin. At the outer boundary of the coupled cell as shown in Fig. [3.1]

$$\text{at } r = r_a, \quad 0 < \phi < \phi$$

$$-k \hat{n} \cdot \nabla T \Big|_{r=r_a} = 0 \quad (3.5)$$

d. Symmetric property

$$\frac{\partial T}{\partial \phi} \Big|_{\phi = n\pi} = 0 \quad \text{where } n = 0, 1, 2, \dots \quad (3.6)$$

Again, for computational purpose, the same transformation as used in Section 2.3 is applied:

$$\text{for the temperature: } \theta(\rho, \phi, \xi) = \frac{T(r, \phi, z) - T_{in}}{q_o'' r_i / k} \quad (3.7)$$

$$\text{for the radial coordinate: } \rho = \frac{r}{\frac{P}{2}} \quad (3.8)$$

$$\text{for the axial coordinate: } \xi = \frac{z D_h}{(\frac{P}{2})^2 Pe} \quad (3.9)$$

As a result, the dimensionless form of the energy equation reads:

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta}{\partial \phi^2} + \left(\frac{D_h}{\frac{P}{2} Pe} \right)^2 \frac{\partial^2 \theta}{\partial \xi^2} = \frac{\partial \theta}{\partial \xi} \quad (3.10)$$

By neglecting the axial heat conduction term, Eq. [3.10] simplifies to:

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta}{\partial \phi^2} = \frac{\partial \theta}{\partial \xi} \quad (3.11)$$

with the following boundary conditions

a. Initial condition:

$$\text{at } \xi = 0, \quad \theta(\rho, \phi, \xi = 0) = 0 \quad (3.12)$$

b. At the clad-surface of the corner pin

$$\begin{aligned} &\text{at } \rho = \rho_1, \quad 0 \leq \phi \leq \pi \\ &\left. \frac{\partial \theta}{\partial \rho} \right|_{\rho = \rho_1} = - \frac{P}{D} \frac{q_c''}{q_o''} \end{aligned} \quad (3.13)$$

where q_o'' is a reference heat flux

c. At the clad-surface of the internal pin

$$\begin{aligned} &\text{at } \rho = \rho_a, \quad \phi_o \leq \phi \leq \pi \\ &\hat{n} \cdot \text{grad } \theta \Big|_{\rho = \rho_a} = \frac{P}{D} \frac{q_1''}{q_o''} \end{aligned} \quad (3.14)$$

At the outer boundary of the coupled cell as shown in Fig.[3.1]

$$\text{at } \rho = \rho_a, \quad 0 < \phi < \phi_0 \quad (3.15)$$

$$\hat{n} \cdot \text{grad } \theta \Big|_{\rho = \rho_a} = 0$$

d. Symmetric property

$$\frac{\partial \theta}{\partial \phi} \Big|_{\phi = n\pi} = 0 \quad (3.16)$$

$$n = 0, 1, 2, 3, \dots$$

3.3. Theoretical Analysis

In order to solve the governing energy equation under the given boundary conditions, the same analytical approach used in Section 2.3 is applied here again. By using the Principle of Superposition, the total solution composes of thermally fully developed solution and thermally entrance solution

$$\theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) + \theta_e(\rho, \phi, \xi) \quad (3.17)$$

These particular solution-forms have the following properties:

$$\text{for } \xi \leq 0 \quad \theta_f(\rho, \phi, \xi) = -\theta_e(\rho, \phi, \xi) \quad (3.18)$$

$$\text{for } \xi \rightarrow \infty \quad \theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) \quad (3.19)$$

$$\text{or } \theta_e(\rho, \phi, \xi) = 0$$

for all ρ and ϕ such that

$$0 \leq \phi \leq 2\pi, \quad \rho_i \leq \rho \leq \rho_a$$

That is, at the inlet, the thermal entrance solution will off-

set the fully developed solution. At a distance far away from the inlet, the fully-developed solution is the asymptotic solution while the thermal entrance effect diminishes. Then the differential equation and its boundary conditions will split into the two following parts:

a. Thermally Fully-Developed Region.

$$\frac{\partial^2 \theta_f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_f}{\partial \phi^2} = \frac{\partial \theta_f}{\partial \xi} \quad (3.20)$$

$$\frac{\partial \theta_f}{\partial \rho} = - \frac{P}{D} \frac{q_c''}{q_o''} \quad \text{for } \rho = \rho_i \text{ and } 0 \leq \phi \leq 2\pi \quad (3.21)$$

$$\hat{n} \cdot \text{grad } \theta_f = \frac{P}{D} \frac{q_i''}{q_o''} \quad \text{for } \rho = \rho_a \text{ and } \phi_o \leq \phi \leq (2\pi - \phi_o) \quad (3.22)$$

$$\hat{n} \cdot \text{grad } \theta_f = 0 \quad \text{for } \rho = \rho_a \text{ and } 0 \leq \phi \leq \phi_o \quad (3.23)$$

$$\text{and } (2\pi - \phi_o) \leq \phi \leq 2\pi$$

$$\frac{\partial \theta_f}{\partial \phi} = 0 \quad \text{for } \phi = n\pi \quad \text{and } \rho_i < \rho < \rho_a$$

$$n = 0, 1, 2, 3 \dots$$

$$\text{for all } \xi$$

b. Thermal Entrance Region

$$\frac{\partial^2 \theta_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_e}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_e}{\partial \phi^2} = \frac{\partial \theta_e}{\partial \xi} \quad (3.25)$$

$$\frac{\partial^2 \theta_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_e}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_e}{\partial \phi^2} = \frac{\partial \theta_e}{\partial \xi} \quad (3.25)$$

$$\frac{\partial \theta_e}{\partial \rho} = 0 \quad \text{for } \rho = \rho_i \text{ and } 0 < \phi < 2\pi \quad (3.26)$$

$$\hat{n} \cdot \text{grad } \theta_e = 0 \quad \text{for } \rho = \rho_a \text{ and } 0 \leq \phi \leq 2\pi \quad (3.27)$$

$$\frac{\partial \theta_e}{\partial \phi} = 0 \quad \text{for } \phi = n\pi \quad n = 0, 1, 2, 3, \dots \quad (3.28)$$

and $\rho_i < \rho < \rho_a$

for all ξ

Figure [3.3] gives a pictorial description of these two regions, the governing differential equations and their boundary conditions.

Since the solutions for the coupled cell calculation is nearly the same as that for the isolated cell calculation, the discussion of solving the equation and determining the unknown coefficients will be very brief in what follows. For more details about the theoretical analysis, the reader should refer to Section 2.3.

3.3.1. Solution of the Fully-Developed Temperature Profile

3.3.1.1. Form of the Solution

Because the fully developed temperature profile is an

$$\frac{\partial^2 \theta_f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_f}{\partial \phi^2} = \frac{\partial \theta_f}{\partial \xi}$$

Boundary Condition

$$\frac{\partial \theta_f}{\partial \rho} = -\frac{P}{D} \frac{q_c''}{q_o''} \quad \text{for } \rho = \rho_1 \quad 0 \leq \phi \leq 2\pi$$

$$\hat{n} \cdot \text{grad } \theta_f = \frac{P}{D} \frac{q_i''}{q_o''} \quad \text{for } \rho = \rho_a \quad \phi_o \leq \phi \leq \pi$$

$$\hat{n} \cdot \text{grad } \theta_f = 0 \quad \text{for } \rho = \rho_a \quad 0 \leq \phi \leq \phi_o$$

$$\theta_f(\rho, \phi, \xi=0) = \theta_{f,o}$$

Fig. 3.3a Fully Developed Region for Coupled Cell

$$\frac{\partial^2 \theta_e}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_e}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \theta_e}{\partial \phi^2} = \frac{\partial \theta_e}{\partial \xi}$$

Boundary Condition

$$\frac{\partial \theta_e}{\partial \rho} = 0 \quad \text{for } \rho = \rho_1 \quad \text{and } 0 \leq \phi \leq \pi$$

$$\hat{n} \cdot \text{grad } \theta_e = 0 \quad \text{for } \rho = \rho_a \quad \text{and } 0 \leq \phi \leq \pi$$

$$\theta_e(\rho, \phi, \xi=0) = \theta_{e,o}$$

$$= -\theta_{f,o}$$

Fig. 3.3b Thermal Entrance Region for Coupled Cell

asymptotic profile, it does not vary with the axial coordinate but will simply be a function of radius and azimuthal angle. However, in order to satisfy the conservation of energy, the average coolant temperature will be linearly increasing with axial height while the profile remains constant throughout all levels. That is,

$$\dot{m} C_p \Delta T = Q \Delta z \quad (3.29)$$

$$\text{or } \Delta T = \frac{Q}{\dot{m} C_p} \Delta z \quad (3.29)$$

where Q is the total energy supplied in the combined channel

$$Q = q_c'' \pi D + q_1'' \frac{\pi D}{6}$$

In dimensionless form,

$$\Delta \theta = C_o \Delta \xi \quad (3.30)$$

$$\text{with } C_o = 2 \frac{A_p}{A_c} \left(\frac{P}{D} \right)^2 \left(\frac{q_c''}{q_o''} + \frac{q_1''}{q_o''} \right) \quad (3.31)$$

where A_p is total area of corner fuel pin
and A_c is total coolant cross-sectional area.

As a result of this, the fully-developed solution can be written as:

$$\theta_f(\rho, \phi, \xi) = Z_o(\xi) + G_f(\rho, \phi) \quad (3.32)$$

$$\text{and } Z_o(\xi) = C_o \xi \quad (3.33)$$

by substituting this solution into the differential equation (eq. 3.20), a Poisson Equation, results, namely

$$\frac{\partial^2 G_f}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial G_f}{\partial \rho} + \frac{1}{\rho} \frac{\partial^2 G_f}{\partial \phi^2} = C_o. \quad (3.34)$$

The particular solution of the Poisson Equation is

$$(G_f)_p = C_o \frac{\rho^2}{4} \quad (3.35)$$

and the homogeneous solution will be

$$\begin{aligned} (G_f)_H &= R_f(\rho) \Phi_f(\phi) \\ &= A_o + B_o \ln \rho + \sum_{n=1}^{\infty} [A_n \rho^n + B_n \rho^{-n}] [\cos n\phi]. \end{aligned}$$

Then, the total solution for the fully developed region is:

$$\begin{aligned} \theta_f(\rho, \phi, \xi) &= Z_f(\xi) + [G_f(\rho, \phi)]_p + [G_f(\rho, \phi)]_H \\ &= C_o \xi + C_o \frac{\rho^2}{4} + A_o + B_o \ln \rho \\ &\quad + \sum_{n=1}^{\infty} [A_n \rho^n + B_n \rho^{-n}] [\cos n\phi] \end{aligned} \quad (3.37)$$

3.3.1.2. Determination of the Constants for the Thermally-Fully Developed Solution

Substituting the solution form [Eq. 3.37] into boundary

condition [eq. 3.21] will yield the following equation. The derivative in the radial direction of the temperature at the clad surface of the central rod becomes:

$$\begin{aligned} \left. \frac{\partial \theta_f}{\partial \rho} \right|_{\rho=\rho_i} &= C_o \frac{\rho_i}{2} + B_o \frac{1}{\rho_i} + \frac{1}{\rho_i} \sum_{n=1}^{\infty} [A_n \rho_i^n - B_n \rho_i^{-n}] [\cos n\phi] \\ &= -\frac{P}{D} \end{aligned} \quad (3.38)$$

Then, both sides are multiplied $(\cos m\phi)$ and integrated over the interval from 0 to π . Due to the orthogonal property of the harmonic function, i.e., $\cos n\phi$, the results will be:

$$\text{for } n = 0, \quad B_o = - \left[\frac{C_o}{2} \frac{1}{\left(\frac{P}{D}\right)^2} + \frac{q_c''}{q_o} \right] \quad (3.39)$$

$$\text{for } n > 0, \quad B_n = A_n \left[-\frac{1}{\frac{P}{D}} \right]^{2n} \quad (3.40)$$

The remaining coefficients, A_n 's can be found by applying the other boundary conditions [Eq. 3.27 and 3.28], that is,

$$\begin{aligned} \hat{n} \cdot \text{grad } \theta_f \Big|_{\rho=\rho_a} &= \left[\cos \beta' \frac{\partial \theta_f}{\partial \rho} - \sin \beta' \frac{1}{\rho} \frac{\partial \theta_f}{\partial \phi} \right]_{\rho=\rho_a} \\ &= \sum_{n=1}^{\infty} n A_n [\rho_a^n \cos (\beta' - n\phi_a) - \rho_i^{2n} \rho_a^{-n} \\ &\quad (\cos (\beta' + n\phi_a))] \end{aligned}$$

$$= \begin{cases} -[C_o \frac{\rho_a}{2} + B_o \frac{1}{\rho_a}] [\rho_a \cos \beta'] & \text{for } 0 \leq \phi \leq \phi_o \\ -[C_o \frac{\rho_a}{2} + B_o \frac{1}{\rho_a}] [\rho_a \cos \beta'] + (\frac{q_1''}{q_1''}) (\frac{P}{D}) (\rho_a) & \text{for } \phi_o \leq \phi \leq \pi \end{cases}$$

(3.41)

$$\text{where } \hat{n} \cdot \hat{e}_r = \cos \beta$$

Once again, the boundary least-square method is used to determine the set of coefficients, A_n 's. With the A_n 's known, the B_n 's are obtained by the recursion formula Eq.[3.40].

The last coefficient to be determined is A_o which fixes the level of θ_f . A_o is evaluated such that the cell average temperature distribution is zero, that is, the integral of the second term in equation 3.32 over the total coolant area is zero. It follows then that the average fully developed temperature is a linear function with axial height.

$$\text{i.e. } \theta^f(\rho, \phi, \xi) = Z_f(\xi) = C_o \xi$$

$$\text{then } A_o = \frac{- \int_0^\pi d\phi \int_{\rho_i}^{\rho_a} d\rho [C_o \frac{\rho^2}{4} + B_o \ln \rho + \sum_{n=1}^{\infty} A_n (\rho^n + \rho_i^{2\rho-n}) (\cos n\phi)]}{\int_0^\pi d\phi \int_{\rho_i}^{\rho_a} \rho d\rho}$$

(3.42)

Coupling two cells together as a single unit does change substantially the coefficients of the fully developed solution because the geometry is now quite different. One of the charges is that a new set of coefficients, A_n 's, must be developed. The value of the A_n 's are primarily governed by the condition at the outer boundary of the internal cell which is no longer adiabatic in the coupled cell situation.

3.3.2. Solution for the Thermal Entrance Temperature Profile

3.3.2.1. Form of the Solution

For the thermal entrance temperature profile, the governing differential equation has a homogeneous condition. Applying the method of Separation of Variables, the solution form will be:

$$\theta_e(\rho, \phi, \xi) = R_e(\rho) \Phi_e(\phi) Z_e(\xi) \quad (3.43)$$

By substituting this form, Eq. [3.43] into the governing Eq. [3.25] and by solving for the final solution with the given boundary condition Eqs. [3.26, 3.27 and 3.28], one obtains the following functional form:

$$\theta_e(\rho, \phi, \xi) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [C_{n,m} J_n(B_m \rho) + D_{n,m} Y_n(B_m \rho)] [\cos n\phi] [E_m e^{-\beta_m^2 \xi}]. \quad (3.44)$$

3.3.2.2. Determination of the Constant for the Thermal Entrance Solution

As could be seen in Section 3.3.2.3., the thermal entrance solution contains a double summation. The first are sums over the number of harmonics kept in the solution and the second one sums over the number of eigenvalues. With the limitation of computer capacity, the harmonic series is truncated at a value, N_N and the number of eigenvalue is terminated at a value, M . Then the approximation for the solution becomes:

$$\theta_e(\rho, \phi, \xi) \approx \sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_n(\beta_m \rho) + D_{n,m} Y_n(\beta_m \rho)] [\cos n\phi] [E_m e^{-\beta_m^2 \xi}]. \quad (3.45)$$

Because of the homogeneity of the boundary condition in this solution, the coefficient constants $C_{n,m}$ and $D_{n,m}$ will be split into two factors, namely:

$$C_{n,m} = C'_{n,m} \cdot F_{n,m} \quad (3.46)$$

and $D_{n,m} = D'_{n,m} \cdot F_{n,m}$

With the boundary condition Eq. [3.26], a relationship between the constants $C'_{n,m}$ and the constants $D'_{n,m}$ can be found.

$$\text{for } n = 0, \quad \frac{C'_{0,m}}{D'_{0,m}} = - \frac{\beta_m Y_1(\beta_m \rho_i)}{\beta_m J_1(\beta_m \rho_i)} \quad (3.47)$$

$$\text{for } n > 0, \quad \frac{C'_{n,m}}{D'_{n,m}} = - \frac{\left[\frac{n}{\rho_i} Y_n(\beta_m \rho_i) - \beta_m Y_{n+1}(\beta_m \rho_i) \right]}{\left[\frac{n}{\rho_i} J_n(\beta_m \rho_i) - \beta_m J_{n+1}(\beta_m \rho_i) \right]} \quad (3.48)$$

The other factors $F_{n,m}$'s will be treated as the eigenvectors corresponding to eigenvalues β_m 's. In order to find the eigenvalues β_m 's, consider the adiabatic conditions Eq. [3.27] at the outer boundary. With K discrete points defined at the outer boundary, a set of K homogeneous linear equations for a single eigenvalue β_m is formed. They are:

$$\begin{aligned} & \sum_{n=0}^{N_n} [(\cos \phi_0)(\cos n\phi_K)(F_{n,m}) C'_{n,m} \left(\frac{n}{\rho_K} J_n(\beta_m \rho_K) - \right. \\ & \left. \beta_m J_{n+1}(\beta_m \rho_K) \right) + D'_{n,m} \left(\frac{n}{\rho_K} Y_n(\beta_m \rho_K) - \beta_m Y_{n+1}(\beta_m \rho_K) \right) \\ & + (\sin \phi_0)(\cos n\phi_K)(F_{n,m}) \left(\frac{n}{\rho_K} \right) (C_{n,m} J_n(\beta_m \rho_K) + D_{n,m} Y_n(\beta_m \rho_K)) \\ & = 0 \end{aligned} \quad (3.49)$$

for $K = 1, 2, 3, \dots, K$

$m = 1, 2, 3, \dots, M$

$K \geq N_n$

In Eq. [3.49], $\hat{n} \cdot \hat{e}_r = \cos \phi_0 = \cos \beta'$

These equations can be written in a simple matrix form as

$$[W_{kn}][F_n] = 0 \quad \text{for any particular } \beta_m \quad (3.50)$$

where the element k_n of matrix $[W]$ is given by

$$\begin{aligned} W_{kn} = & (\cos \phi_0)(\cos n\phi_K) \{C_{n,m}(\frac{n}{\rho_K} J_n(\beta_m \rho_K) \\ & - \beta_m J_{n+1}(\beta_m \rho_K)) + D'_{n,m}(\frac{n}{\rho_K} Y_n(\beta_m \rho_K) \\ & - \beta_m Y_{n+1}(\beta_m \rho_K))\} + (\sin \phi_0)(\sin n\phi_K) \\ & (\frac{n}{\rho_K}) \{C'_{n,m} J_n(\beta_m \rho_K) + D'_{n,m} Y_n(\beta_m \rho_K)\} . \end{aligned} \quad (3.51)$$

For the purpose of generating a quadratic, symmetric and positively defined matrix W such that its determinant vanishes at the correct eigenvalues, β , a matrix operation is performed, namely,

$$[W]^t[W] = [W] \quad (3.52)$$

As a result, the matrix equation becomes

$$[W W_{nn}][F_n] = 0 \quad (3.53)$$

The solution vector, $\vec{F}_n$ exists if and only if the $\det [W W_{nn}]$

vanishes.

$$\det (W_{nn}) \Big| = 0 \quad (3.54)$$

Then a set of eigenvalues β_m 's and the corresponding eigenvectors F_n 's can be easily found.

With the problem of eigenvalues and eigenvectors being solved, the only unknowns left in the solution are the E_m 's. The unknown coefficients, E_m 's are determined by satisfying the initial condition, Eq. [3.18], such that the thermal entrance solution will cancel the fully developed solution at the inlet level. In mathematical terms this can be expressed as:

$$\begin{aligned} & - \sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_n(\beta_m \rho_k) + D_{n,m} Y_n(\beta_m \rho_k)] [\cos n\phi_k] [E_m] \\ & = C_0 \frac{\rho_k^2}{4} + A_0 + B_0 \ln \rho_k + \sum_{k=1}^{N_k} [A_n (\rho_k^n + (-\frac{1}{P})^{\frac{2n}{D}} \rho_k^{-n})] [\cos n\phi_k] \end{aligned}$$

for $k = 1, 2, 3, \dots, P$

where $P \geq M$

and P is the total number of allocated points at the inlet.

Again, the boundary least-square method is used to calculate a solution to an over-determined system of m linear equations in n unknowns, that is, given equations

$$\sum_{j=1}^n A_{ij} X_j = b_j \quad (3.56)$$

where $i = 1, 2, 3, \dots, m$

and $m \geq n$,

the solution vector $\underline{X} = X_j$ is calculated such that the sum of squares of the residuals

$$F(X) = \sum_{i=1}^m \left\{ \sum_{j=1}^n A_{ij} X_j - b_i \right\}^2 \text{ is minimized.} \quad (3.57)$$

Applying this methodology to Eqs [3.49], and [3.55] the unknown coefficients $F_{n,m}$'s and E_m 's are found.

3.3.3. Surface Temperature and Nusselt Number

In this particular geometry, with the center of the corner pin as the origin of the polar-coordinate system, the clad temperature at the surface of the corner fuel pin is given by:

$$\theta_w^c(\rho_1, \phi, \xi) = \theta_{f,w}^c(\rho_1, \phi, \xi) + \theta_{e,w}^c(\rho_1, \phi, \xi) \quad (3.58)$$

$$\begin{aligned} \text{with } \theta_{f,w}^c(\rho_1, \phi, \xi) = & C_0 \xi + C_0 \frac{\rho_1^2}{4} + A_0 + B_0 \ln \rho_1 \\ & + \sum_{n=1}^{N_k} (2A_n \rho_1^n \cos n\phi) \end{aligned} \quad (3.59)$$

$$\text{and } \theta_{e,w}^c(\rho_1, \phi, \xi) = \sum_{m=1}^M \sum_{n=0}^{N_n} [C_{n,m} J_n(\beta_m \rho_1) + D_{n,m} Y_n(\beta_m \rho_1)] [\cos n\phi] [E_m e^{-\beta_m^2 \xi}]. \quad (3.60)$$

The average clad temperature is obtained by integrating over the pin circumference, that is,

$$\theta_{w,avg}^c(\xi) = \frac{\int_0^\pi \theta_{w,avg}^c(\rho_1, \phi, \xi) \rho_1 d\phi}{\int_0^\pi \rho_1 d\phi} \quad (3.61)$$

After the integration has been completed, Eqs. [3.59] and [3.60] become

$$\theta_{f,w}^c(\xi) = C_0 \xi + C_0 \frac{\rho_1^2}{4} + A_0 + B_0 \ln \rho_1$$

$$\text{and } \bar{\theta}_{e,w}^c(\xi) = \sum_{m=1}^M [C_{0,m} J_0(\beta_m \rho_1) + D_{0,m} Y_0(\beta_m \rho_1)] [E_m e^{-\beta_m^2 \xi}]$$

For the small section of the internal cell, the average wall temperature can be expressed as follows:

$$\theta_{w,avg}^i(\xi) = \frac{\int_{5\pi/6}^\pi \theta_w^i(\rho_a(\phi), \phi, \xi) \rho_a(\phi) d\phi}{\int_{5\pi/6}^\pi \rho_a(\phi) d\phi} \quad (3.62)$$

$$\text{with } \theta_w^i(\rho_a, \phi, \xi) = \theta_{f,w}^i(\rho_a, \phi, \xi) + \theta_{e,w}^i(\rho_a, \phi, \xi) \quad (3.63)$$

$$\text{and } \theta_{f,w}^i(\rho_a, \phi, \xi) = C_0 \xi + C_0 \frac{\rho_a^2}{4} + A_0 + B_0 \ln \rho_a$$

$$+ \sum_{n=1}^{N_k} \left[A_n \left(\rho_a^n + \left(\frac{1}{\frac{P}{D}} \right)^{2n} \rho_a^{-n} \right) \right] [\cos n\phi] \quad (3.64)$$

$$\text{and } \theta_{e,w}^i(\rho_a, \phi, \xi) = \sum_{m=1}^M \sum_{n=1}^{N_n} [C_{n,m} J_n(\beta_m \rho_a) + D_{n,m} Y_n(\beta_m \rho_a)]$$

$$[\cos n\phi] [E_m e^{-\beta_m^2 \xi}] \quad (3.65)$$

Unlike Eq. [3.61], Eq. [3.62] has an extra factor which is function of the azimuthal angle. This factor is ρ_a . In order to solve the integrals in Eq. [3.62], a numerical method for integration called Gaussian Quadrature is used. From Eq. [3.33] it is known that the average bulk temperature of both cells together is $C_0 \xi$, that is,

$$\theta_{\text{avg}}^{\text{tot}}(\xi) = \theta_{f,\text{avg}}(\xi) = C_0 \xi \quad (3.64)$$

For the evaluation of the bulk temperature in either the interval or the corner region, double integration is needed. This is achieved as follows:

for the corner cell,

$$\bar{\theta}_b^c(\xi) = \frac{\int_0^\pi d\phi \int_{\rho_i}^{\rho_x} \rho d\rho \theta(\rho, \phi, \xi)}{\int_0^\pi d\phi \int_{\rho_i}^{\rho_x} \rho d\rho}$$

for the internal cell

$$\theta_b^i(\xi) = \frac{\int_{5\pi/6}^{\pi} d\phi \int_{\rho_x}^{\rho_a} \rho d\rho \theta(\rho, \phi, \xi)}{\int_{5\pi/6}^{\pi} d\phi \int_{\rho_x}^{\rho_a} \rho d\rho} \quad (3.66)$$

where ρ_x is the radial distance from the center of the origin to the common boundary. With the definition of the Nusselt number

$$Nu = \frac{hD_h}{k} = \frac{D_h}{k} \frac{q''}{T_w - T_{avg}} \quad (3.67)$$

where D_h is the equivalent diameter of the cell, the azimuthally averaged Nusselt number, $Nu(\xi)$, follows as

$$Nu(\xi) = \frac{D_h/r_i}{\theta_w(\xi) - \theta_{avg}(\xi)} \frac{q''}{q_o''} \quad (3.68)$$

where q_o'' is the assigned reference-heat flux and q'' is the average heat flux for each individual cell.

In order to compare the results of the Nusselt number for each individual cell in the case of the coupled cell analysis with the case of the isolated cell analysis, the average wall temperature, the average bulk temperature and the average heat flux of each individual cell will be determined additionally. Thus, the Nusselt number for each cell will be calculated. For instance, the Nusselt number of the corner

cell can be expressed as follows:

$$Nu^c(\xi) = \frac{D_h/r_i}{\bar{\theta}_w^c(\xi) - \bar{\theta}_b^c(\xi)} \frac{q_c''}{q_o''}$$

where $\bar{\theta}_w^c$, $\bar{\theta}_b^c$, q_o'' are the average clad temperature, the average bulk temperature and the heat flux of the corner cell, respectively.

For the internal cell,

$$Nu^i(\xi) = \frac{D_h/r_i}{\bar{\theta}_w^i(\xi) - \bar{\theta}_b^i(\xi)} \frac{q_i''}{q_o''}$$

where $\bar{\theta}_w^i$, $\bar{\theta}_b^i$, q_i'' are the average clad temperature, the average bulk temperature and the heat flux of the internal cell respectively.

However, for the determination of the Nusselt number for the combination of these two cells, one has to find the average clad temperature, the average bulk temperature and the average heat flux at the clad surface of the total cell first. The definitions of these various parameters are expressed in mathematical terms as follows:

For the average clad temperature,

$$\bar{\theta}_w(\xi) = \frac{\int_0^\pi \theta_w^c(\rho_i, \phi, \xi) \rho_i d\phi + \int_{5\pi/6}^\pi \theta_w^i(\rho_a, \phi, \xi) \rho_a d\phi}{\int_0^\pi \rho_i d\phi + \int_{5\pi/6}^\pi \rho_a d\phi} \quad (3.70)$$

where ρ_i is the distance from the center of the fuel pin to the clad surface of the corner pin.

and ρ_a is the radial distance from the center of the corner pin to the outer clad surface of the internal cell.

For the average bulk temperature of the combined cell,

$$\bar{\theta}_b(\xi) = C_o \xi \quad (3.71)$$

where C_o is the coefficient that has been evaluated in section 3.3.1.

The average heat flux at the outer clad surface of the combined cell is:

$$\bar{q}'' = \frac{\int_0^\pi q_c'' \rho_i d\phi + \int_{5\pi/6}^\pi q_i'' \rho_a d\phi}{\int_0^\pi \rho_i d\phi + \int_{5\pi/6}^\pi \rho_a d\phi} \quad (3.72)$$

Then, the Nusselt number of the combined cell will be as follows:

$$Nu^{tot}(\xi) = \frac{D_h/r_i}{\bar{\theta}_w(\xi) - \bar{\theta}_b(\xi)} \frac{q''}{q_o''} \quad (3.73)$$

with the quantities as given above.

3.4. Results of the Coupled Analysis for Internal and Corner Cells

3.4.1. Study of a Sample Problem

3.4.1.1. Uniform Axial Heat Flux Distribution

The same geometric data which have been already used in Chapter 2 are chosen again for the test case. These geometric parameters are:

$$\frac{P}{D} = 1.3 \quad 2W/D = 1.38 \quad D = 9\text{mm}$$

$$Pe = 372 \quad T_{in} = 415^{\circ}\text{C}$$

$$q'' = 104\text{W/cm}^2$$

and it should be recalled that they reflect one of the experiments conducted at Karlsruhe. It will be interesting to see how the results from the coupled cell calculation compare to the isolated cell results as well as the experimental results. In this chapter, the analysis assumes uniform heat flux in the axial and azimuthal directions.

Fig. [3.4] shows the comparison between the results of the single and coupled cell calculations for the clad temperature distribution. Surprisingly, the coupled cell calculation gives an even larger temperature variation around the clad as predicted by the single cell calculation. This interesting result complies perfectly with what has been already discovered in the 2-D coupled cell analysis by M.K. Yeung [8]. The implication of this result is that under certain geometric condition the clad temperature variations of the corner pin are amplified rather than smoothed out as one would intuitively expect in the first place.

Fig. 3.4 Thermally Fully-Developed Clad Temperature Distribution of the Corner Pin as Predicted by Coupled Cell Analysis

($P/D=1.3$, $2W/D=1.38$, Dia.=0.9 cm, $Pe=372$, $q''=104 \text{ W/cm}^2$)

Figs. [3.5, 3.6, 3.7, and 3.8] show the axial development of the peripheral clad temperature distributions for corner and internal cells. Whereas some remarkable changes in the profile can be noticed for the corner cell, the distribution in the internal cell does not change that much. Since both channel geometries are quite similar in the regime around $\phi = 180^\circ$; the clad temperature distribution of the corner and the internal pins around this particular regime are about the same. For $\phi < 180^\circ$ the specific geometry of the coolant channel and the thermal entrance effect dominate the development of the temperature profile in the corner cell. Thus, by checking the peripheral temperature distribution, a significant increase of the clad temperature at $\phi = 180^\circ$ can be easily observed while the rest of the temperature profile of the corner pin is overall resembling that of the single cell analysis. Normally, the temperature at $\phi = 180^\circ$ is the hot spot of the corner pin. An increase of temperature in this hot spot will surely result in a higher maximum temperature variation. Meanwhile, for the internal cell, the allowance of heat flow across the common boundary of these two cells will practically drop the average temperature level of the internal cell when compared to the results of a single cell calculation. On the other hand, allowing heat-flowing from the internal cell also increases its temperature variation. However, this value is still not large enough to compare with that of the corner or edge rods. (At $z=100$ cm, $\Delta\theta$ is only 0.145 or ΔT is about 7°C). With these results on hand, it can be stated that the temperature distri-

Fig. 3.5 Axial Development of the Clad Temperature Distribution
 of the Corner Pin for Uniform q'' as Predicted by Coupled Cell
 Analysis

Fig. 3.6 Axial Development of the Clad Temperature Distribution of the Corner Pin for Uniform q'' as Predicted by Coupled Cell Analysis (continued)

Fig. 3.7 Axial Development of the Clad Temperature Distribution of the Internal Pin for Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.8 Axial Development of the Clad Temperature Distribution of the Internal Pin for Uniform q'' as Predicted by Coupled

Cell Analysis (continued)

bution in the neighborhood of $\phi = 180^\circ$ is quite different from what has been predicted by the isolated cell analysis. In the coupled two cells analysis, the behavior of the curve in this region is pretty much governed by the internal cell even at small z . This is a remarkable difference between the results obtained from the simplistic adiabatic boundary condition as compared to the non-adiabatic one. By removing the adiabatic condition at the common boundary, the thermal entrance length of the corner cell increases from 543 cm. to 796.6 cm. This implies that the thermal entrance effect becomes more important in the case of coupling the two cells together.

A good idea about what happens along the imaginary boundary between the two cells can be obtained by calculating the heat flux along this line as function of the axial distance. This is explicitly shown in Fig [3.9]. These results indicate that in the inlet regime of the coolant, heat is transferred from the corner cell to the internal cell. However, about mid-way through the core this behavior reverses and heat flows from the internal cell towards the corner cell. This trend increases with increasing axial position in the channel. Naturally, the largest amount of energy is transferred across a line segment of this imaginary boundary which is characterized by the interval $150^\circ \leq \phi \leq 170^\circ$, whereas for $\phi > 170^\circ$ the transfer rate drops due to the similarity and symmetry of the coolant channel cross-section in this regime. Fig. [3.9] shows that the assumption of zero heat flux along the interface between the two cells as applied in the adiabatic single-cell

Fig. 3.9 Heat Flux Across the Common Boundary for Different Axial Levels (Provided $q''(z) = \text{Uniform}$)

Table 3.2 Comparison of Dimensionless Temperature* Distribution at
Common Boundary Obtained from Isolated Cell
Analysis and Coupled Cell Analysis at $z = 65.5$ cm

<u>Angle</u>	<u>Isolated Internal</u>	<u>Isolated Corner</u>	<u>Coupling</u>
180°	1.5609	1.1554	1.2661
172°	1.5475	1.1420	1.2737
164°	1.5143	1.1059	1.2608
157°	1.4788	1.0631	1.2433
150°	1.4621	1.0426	1.2528

*Dimensionless Temperature $\theta(\rho, \phi, \xi) = \frac{T(r, , z) - T_{in}}{q'' r_i / k}$

calculation is not valid in a strict sense at all. On the other hand, one can observe in Fig. [3.9] that up to an axial level of $z = 47.5$ xm., the adiabatic analysis seems to be a reasonable first approximation. The change of heat flux distribution as function of the axial coordinate can be explained in terms of the vastly different thermal entrance behaviors of the two cells.

It was stated at the beginning of this chapter that the thermal condition of the internal cell will highly affect the azimuthal temperature variation of the corner pin. This is shown explicitly in Figs. [3.11, 3.12, and 3.13]. In Fig. [3.11] the temperature variation is plotted versus the axial coordinate for different Pe-numbers, it illustrates that at the inlet, there is a sharp increase in the temperature variation whereas at higher elevations this seems to approach an asymptotic value. This explains why in the inlet region the effect of the coupling of the two cells contributes most to the temperature variation. At higher levels of the core, the irregularity in the geometry then becomes the main contributor to the temperature variation in the corner pin.

Surprisingly, by comparing the results of the coupled cell analysis, with the experimental data with respect to the maximum clad temperature difference, the former always gives larger values. This can be clearly seen from Figs. [3.10, and 3.11]. The cause for these differences can be well explained by the following arguments. First, the turbulent mixing has been totally neglected. Second, the experimental result is

Fig. 3.10 Maximum Clad Temperature Difference of the Corner Pin for $Pe=372$ and Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.11 Maximum Clad Temperature Difference of the Corner Pin for Various Pe-Number and Uniform q'' as Predicted by Coupled Cell Analysis

not exactly equivalent to the temperature distribution at the outside surface of the cladding.

As has been already explained in Section 2.5, the experimental data are the result of the clad temperature measurement inside the clad because the thermocouple is embedded inside the clad. However, the coupled cell analysis, gives the temperature difference directly at the clad outside surface. Therefore, azimuthal heat conduction in the clad will lower the difference in measurement. Moreover, the assumption of a uniform azimuthal heat flux at the outside surface of the clad (single-regime analysis) instead of at the inside surface (two-region analysis) results in an overprediction of the temperature distribution at the clad outside surface because conduction effects in the clad are not taken into account. This will contribute to the difference in maximum clad temperature between measurement and prediction.

It has been mentioned earlier in Section 2.5, that convection and conduction are the two major heat transport mechanisms which are being considered in the analysis. Turbulent mixing within the cell has been totally neglected. As shown in Fig. [3.11], at low Pe-number such as Pe-170, the predictions of the coupled cell analysis are in remarkable agreement with the experimental results especially in the inlet region. It should be noticed that for the single cell calculation, the predictions were off by a substantial margin. This proves that for low Pe-number flows the contribution of turbulent mixing is indeed small. Heat flow from the internal cell across the common

boundary is the main contributor for this large temperature variation. While at higher Pe-number, turbulent mixing becomes more important, the coupled cell analysis gets worse when compared to the experimental results.

In Fig. [3.12], the maximum clad temperature difference at the axial level of $z = 65.5$ cm. is plotted versus the Pe-number for the coupled cell analysis and the isolated single cell analysis respectively. The graph shows that there is a substantial increase in the temperature difference for the former for all Pe-numbers.

The axial variation of the average Nusselt number for either cell is shown in Fig. [3.13 and 3.14] respectively. By permitting heat transfer across the common boundary, the film temperature difference of the corner cell in the region characterized by $120^\circ < \phi < 180^\circ$ has decreased substantially. This implies that there will be a big gain in the local value of the heat transfer coefficient within this particular region. As a result of this, the azimuthally averaged Nusselt number of the corner cell will also be increased that has been clearly shown in Fig. [3.13]. For the internal cell, a drop in the film temperature difference also has been observed. Thus a substantial gain in heat transfer coefficient shows up in Fig. [3.14]. Therefore, the result of removing the adiabatic condition at the common boundary seems to positively affect the heat transfer coefficients in both cells.

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Fig. 3.12 Maximum Clad Temperature Difference of the Corner Pin as Function of Pe-Number at $z=65.5$ cm as Predicted by Coupled Cell Analysis (Uniform q'')

Fig. 3.13 Axial Variation of the Nusselt Number of the Corner Cell for Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.14 Axial Variation of the Nusselt Number of the Internal Cell for Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.15 Nusselt Number as Function of Pe-Number at $z=65.5$ cm as Predicted by Coupled Cell Analysis

Fig. 3.15a Comparison of Predicted and Experimental Results for $P/D=1.3$

3.4.1.2. Non-Uniform Axial Heat Flux Distribution

To simplify the problem, only the case in which both the corner and the internal pins have exactly the same axial heat flux profiles is considered. This assumption seems reasonable because it is hard to imagine any substantial change in the axial heat flux profile for two neighboring cells. The only possible change is the change in the power level due to radial power tilting. In this case, only the magnitude changes but not the shape. Therefore, the same cosine function as used in the isolated cell calculation is applied here again. This function can be written in the following form;

$$q''(z) = q_1'' \cos\left(\frac{\pi z}{H_0}\right) \quad (3.74)$$

for the corner pin, $q_1'' = (q_c'')_{\max}$

$$\frac{(q_c'')_{\max}}{\bar{q}_c''} = \text{axial peaking factor} = 1.24$$

and $\frac{\bar{q}_c''}{\bar{q}_0''} = \text{radial power peaking ratio.}$

For the internal pin, $q_1'' = (q_i'')_{\max}$

$$\frac{(q_i'')_{\max}}{\bar{q}_i''} = \text{axial peaking factor} = 1.24$$

and $\frac{\bar{q}_i''}{\bar{q}_0''} = \text{radial power peaking ratio.}$

Then Duhamel's Principle is applied again to obtain the solution for the axially variable heat flux problem.

The results for the non-uniform axial heat flux calculations are quite similar to those which have been found before for the uniform axial heat flux condition. Fig. [3.16 and 3.17] show the axial development of the azimuthal temperature distribution for both the corner and the internal cells. Again, the temperature distributions around $\phi = 180^\circ$ for both cells are about similar. An increase in the clad temperature of the corner pin at $\phi = 180^\circ$ is clearly shown as the result of the addition of energy from the internal cell. On the other hand, premitting heat flowing across the common boundary will substantially drop the temperature level in the internal cell.

It is interesting to notice that the temperature variation of the corner pin which is shown in Fig. [3.18] appears to be slightly flattened at the outlet region which happens to be the place where the axial sinusoidal heat flux reaches a low value. This effect has not been observed in the isolated cell calculation. Obviously, it is caused by the coupling effect. This trend can be explained as follows: when the outlet region is approached, the axial sinusoidal heat flux at the clad surface reaches a low value. As a result of this decreasing heat flux, less heat will be transported across the common boundary from the internal cell to the corner cell. Thus, it lowers the increase of the clad temperature variation in the corner cell. Although the axial variation of the heat flux across the common boundary has not been plotted to prove this statement, there is some other evidence which can support this statement which is illustrated in Fig. [3.21]. This figure shows that the average

Fig. 3.16 Axial Development of Clad Temperature Distribution of the Corner Pin with Non-Uniform q'' as Predicted by Coupled Cell Analysis

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $Ht=100$ cm, peaking factor=1.24, $q''_0=104$ W/cm²)

Fig. 3.17 Axial Development of Clad Temperature Distribution of the Internal Pin with Non-Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.18 Maximum Clad Temperature Difference of the Corner Pin for Non-Uniform q'' as Predicted by Coupled Cell Analysis

heat flux across the gap between two adjacent subchannels does decrease near the outlet region.

In Figs. [3.19 and 3.20], the axial variation of the Nusselt number for the corner cell and the internal cell have been plotted respectively. These two graphs clearly indicate that among all four cases listed in the figures, the case of the coupled cell analysis with a sinusoidal axial heat flux distribution will give the best heat transfer coefficient for both the corner cell and the internal cell. However, for either cell, the curve of the coupled cell analysis with the sinusoidal axial heat flux distribution drops substantially near the outlet region. This decrease in the Nusselt number is affected by the increase of the film temperature difference as a result of the decreasing heat flux at the clad surface near the outlet region.

3.4.1.3. Calculation of the Effective Conduction Mixing Length

Subchannel codes usually "lump" the assembly into a finite grid of subchannels each of which is characterized by a bulk temperature, flow rate and pressure. Energy transport between adjacent channels is governed by the mechanism of diversion cross-flow, turbulence mixing and conduction mixing. For LMFBR design calculation, the energy transport by conduction constitutes a major contribution to the overall energy transport. This effect is even more pronounced for flows which are characterized by low Reynold numbers. These flow situations

Fig. 3.19 Axial Variation of the Nusselt Number of the Corner Pin for Non-Uniform q'' as Predicted by Coupled Cell Analysis

Fig. 3.20 Axial Variation of the Nusselt Number of the Internal Pin for Non-Uniform q'' as Predicted by Coupled Cell Analysis

especially arise in the case of a loss-of-flow accident. Despite the importance of this effect, only few information is available for the internal cells but nothing is known for the bundle-wall-near regions of finite bundles.

The heat transfer rate due to conduction between adjacent subchannel i and j as calculated in subchannel codes is given by:

$$Q_{ij} = k \frac{S_{ij}(\bar{T}_i - \bar{T}_j)}{l_{ij}}$$

$$= k \frac{S_{ij}}{l_{ij}^*} (\bar{T}_i - \bar{T}_j) \frac{1}{L_{ij}} \quad (3.75)$$

where S_{ij} is the length of the common boundary, l_{ij} is the effective conduction mixing length, l_{ij}^* is the centroid-to-centroid distance and L_{ij} is a dimensionless correction factor which represents the ratio of the effective conduction mixing length to the centroid-to-centroid distance ($L_{ij} = l_{ij}/l_{ij}^*$). In subchannel calculations, L_{ij} is usually set to be unity due to the lack of knowledge of the local temperature field. It is obvious that the lumped parameter subchannel codes will give reliable results only if reasonably correct effective mixing lengths are known. The calculation of the local temperature field in 3-D thus provides a tool for the determination of axially variable correction factor for effective mixing length, L_{ij} . After some rearrangements, Eq. [3.75] can be written as:

$$L_{ij} = k \frac{S_{ij}}{l_{ij}^*} \frac{(\bar{T}_i - \bar{T}_j)}{Q_{ij}}$$

$$\text{with } Q_{ij} = \int_{S_{ij}} q_s'' dS$$

$$L_{ij} = \frac{S_{ij}}{l_{ij}^*} \frac{(\bar{T}_i - \bar{T}_j)}{\int_{S_{ij}} \frac{q_s'' dS}{S_{ij}}} \quad (3.75a)$$

where the channel averaged temperature and the heat transfer at the channel boundary can be calculated explicitly from the local temperature field of the coolant, and the ratio S_{ij}/l_{ij}^* for hexagonal bundles which appears in Eq. [3.75] and Eq. [3.76] can be simply expressed by geometric parameter as:[8]

$$\frac{S_{ij}}{l_{ij}^*} = \frac{\frac{1}{2} \left\{ \frac{P}{D} - 1 \right\}}{\left\{ \frac{\frac{1}{16} \left(\frac{2W}{D} \right)^2 \left(\frac{P}{D} \right) + \frac{1}{24} \left(\frac{2W}{D} \right)^3 - \frac{1}{16}}{\frac{1}{4} \left(\frac{P}{D} \right) \left(\frac{2W}{D} \right) + \frac{1}{8\sqrt{3}} \left(\frac{2W}{D} \right)^2 - \frac{\pi}{12}} + \frac{\frac{1}{2} \left(\frac{P}{D} \right)^3 - \frac{\sqrt{3}}{12} \pi \left(\frac{P}{D} \right)}{\sqrt{3} \left(\frac{P}{D} \right)^2 - \frac{\pi}{2}} \right\}}$$

Furthermore, Eq. [3.76] can be simplified as:

$$\begin{aligned} L_{ij} &= k \frac{S_{ij}}{l_{ij}^*} \frac{\frac{\bar{T}_i - \bar{T}_j}{q_o'' r_i/k}}{\int_{S_{ij}} \frac{q_s''}{q_o'' r_i/k} dS} \\ &= \frac{S_{ij}}{l_{ij}^*} \frac{\frac{\bar{T}_i - \bar{T}_j}{q_o'' r_i/k}}{\int_{S_{ij}} \frac{q_s''}{q_o''} \left(\frac{P}{D} \right) d \left(\frac{S}{P} \right)} \end{aligned}$$

where q''_0 is the reference heat flux

r_i and D are the radius and the diameter of the fuel pin respectively

and P is the pitch.

$$q''_s = -k \hat{n} \cdot \Delta T \Big|_{\text{gap}}$$

$$= -\frac{1}{\rho} \frac{1}{\left(\frac{P}{D}\right)} \frac{\partial}{\partial \phi} \left(\frac{T}{r_i/k} \right) \Big|_{\text{gap}}$$

thus $\frac{q''_s}{q''_0} = -\frac{1}{\rho} \frac{1}{\left(\frac{P}{D}\right)} \frac{\partial}{\partial \phi} \left(\frac{T}{q''_0 r_i/k} \right) \Big|_{\text{gap}}$

$$= -\frac{1}{\rho} \frac{1}{\left(\frac{P}{D}\right)} \frac{\partial \theta}{\partial \phi} \Big|_{\text{gap}}$$

or $-\frac{1}{\rho} \frac{\partial \theta}{\partial \phi} \Big|_{\text{gap}} = \frac{q''_s}{q''_0} \left(\frac{P}{D}\right)$

therefore $L_{ij} = \frac{S_{ij}}{l_{ij}^*} \frac{\theta_i - \theta_j}{\int_{s_{ij}} \left[-\frac{1}{\rho} \frac{\partial \theta}{\partial \phi} \right] ds}$ (3.76c)

where $s = \frac{S}{\frac{P}{2}}$

For the case of uniform axial heat flux, Figs. [3.21, 3.22 and 3.23] show the results of the effective mixing length calculations. Fig. [3.21] gives the correction factor L_{ij} as function of the axial coordinate. It indicates that L_{ij}

Fig. 3.21 Correction Factor for Effective Mixing Length vs Axial Coordinate for Uniform q''

(Condition: $P/D=1.30$, $2W/D=1.38$, Dia.=0.9 cm, $Pe = 372$, $q''_0=10^4$ W/cm)

Fig. 3.22 Average Heat Flux Across the Gap vs Axial Coordinate for Uniform q''

Fig. 3.23 Bulk Temperature Difference Between Two Adjacent Subchannels vs Axial Coordinate for Uniform q''

initially starts at zero, increases and approaches an asymptotic value. The behavior of this curve may be thought of as being the image of the heat transfer coefficient in the thermal entrance region. Although the numerical value is naturally completely different, there is some similarity between both parameters. The heat transfer coefficient represents the ratio of the amount of energy deposited into the coolant to the film temperature difference. At the inlet level, there is no film temperature difference, but energy distribution has already started. Therefore, the heat transfer coefficient becomes singular in this region and decreases asymptotically towards a limiting value. On the other hand, the correction factor includes a factor which is the ratio of the bulk temperature difference between two subchannels and the heat flux across the boundary separating these two subchannels. It is difficult to predict how this ratio behaves at the inlet because no temperature difference and heat flux have been developed yet. But at a small distance away from the inlet, as shown in Fig. [3.22] and Fig. [3.23], the heat flux has outgrown the temperature differences. Therefore, L_{ij} approaches zero at the entrance. Physically, this can be explained as follows: once the temperature difference exists, it initiated an instantaneous heat transport progressing through the gap in order to decrease the difference. However, since the gap spacing is comparatively small, an increase in the heat flux is necessary. As a result, by moving upstream towards the entrance, the heat flux is decreasing at a lower rate than the bulk tempera-

ture difference. Thus, L_{ij} becomes zero at the inlet and increases towards its asymptotic value.

Fig. [3.23] illustrates the importance of using the accurate correction factor for effective mixing length in the calculation. The solid line represents the calculation of the bulk temperature difference between two adjacent channels as obtained from the three-dimensional temperature calculation, whereas, all broken lines represent cases of using different correction factors for effective mixing lengths in COBRA-IIIC [38]. For all these COBRA-IIIC calculations, an attempt has been made to suppress the diversion cross-flow by adjusting the wetted parameter of the corner subchannel in such a way that both subchannels will have an equal hydraulic diameter. In addition to this, the cross-flow resistance factor K_{ij} has been set to 1000 and the turbulent mixing factor, β , has also been set to zero. The result seems quite successful. But a very small amount of diversion cross-flow still exists because it seems to be impossible to suppress totally the pressure difference between these two adjacent subchannels. From Fig. [3.23], one notices that the isolated channel approach yields the highest temperature difference at the core exit (i.e. $z = 150$ cm.), indicating that it is over-conservative because it neglects the conduction transport at all. It is obvious that conductive mixing is especially high in this region. On the other hand, setting L_{ij} equal to unity which is mostly done in subchannel calculations, results in lowering the temperature difference by 9°C to 46.3°C at $z = 36$ in.,

since conductive mixing is allowed between the two channels. Furthermore, using a fully-developed value of L_{ij} which has been checked with the 2-D analysis by Yeung [8], who derived independently a value of L_{ij} as 0.70102, one discovers that this gives a further drop in the bulk temperature difference to 43.1°C at $z = 36$ in. But still, these results are quite conservative compared to the result of the 3-D analysis which gives 33.9°C at $z = 36$ in. Finally, for more convenient use, the correction factor for effective mixing length is formulated as an empirical function of the axial coordinate and is applied to COBRA-IIIC. The expression of the empirical equation which has been obtained by curve fitting technique is as follows:

$$\begin{aligned}
 L_{ij} = & 0.1849523 + 0.2136824 \log_{10} (z_1) \\
 & - (0.0063282)(\log_{10} z_1)^2 - (0.00574214)(\log_{10} z_1)^3 \\
 & + (5.1819 \times 10^{-4})(\log_{10} z_1)^4
 \end{aligned} \tag{3.78}$$

where z_1 is expressed in terms of centimeters.

By inputting Eq. [3.78] into COBRA-IIIC calculation, it gives an excellent result. As it can be seen in Fig. [3.23], there is a very good agreement between the result obtained by COBRA-IIIC with the accurate correction factor for effective mixing length and the results of the analytical 3-D calculation. Only near the outlet region, the curve of the COBRA-IIIC calculation with accurate L_{ij} appears to underpredict the bulk temp-

erature difference between two adjacent subchannels. A careful check on the overall analysis shows that a small diversion cross-flow still exists and it seems to be the cause of this under-prediction.

To summarize the results of the COBRA-IIIC calculations, it can be concluded that applying an accurate correction factor for effective mixing length in subchannel analysis is very important. By using an inaccurate L_{ij} , will give wrong though conservative results in subchannel heat transfer analysis.

Next, the case of non-uniform heat flux is being considered. The assumption of a cosine shape for the axial heat flux is applied again. The cosine function used here is the same as given by Eq. [3.74].

Figs. [3.24 and 3.25] present the results for the effective mixing length and heat flux across the gap both of which are plotted as function of the axial coordinates respectively. When the axial heat flux profile varies, both curves do not increase as sharp in the inlet region as in the case of uniform axial heat flux. Furthermore, the curves start to diverge at an axial level close to mid-core height where the heat flux is maximum. Due to the decrease of the cosine function near the outlet region, the heat flux at the gap drops slightly at the end. But the curve of the effective mixing length which is inversely proportional to the heat flux at the gap, linearly increases with axial position due to the balancing effect of a similar decrease in both bulk temperature difference and heat flux across the gap. It is shown in Fig. [3.26] that the bulk

Fig. 3.24 Correction Factor for Effective Mixing Length vs Axial Coordinate for Non-Uniform q''

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $z_0=100$ cm, $q''_0=104$ W/cm², Peaking Factor=1.24)

$$\bar{q}_{ij} = \frac{q''_{ij}}{q''_0} \left(\frac{P}{D} \right)$$

Fig. 3.25 Average Heat Flux Across the Gap vs Axial Coordinate for Non-Uniform q''

Fig. 3.26 Bulk Temperature Difference Between Two Adjacent Subchannels vs Axial Coordinate for Non-Uniform q''

temperature difference levels off slightly as the heat flux decreases towards the core outlet.

Once again, different correction factors for the effective mixing length, L_{ij} , are fed into COBRA-IIIC for comparison of the bulk temperature difference between two adjacent subchannels in the case of sinusoidal axial heat flux profile for both pins. Similar results are observed in Fig. [3.26] as in the case of uniform axial heat flux. Setting $L_{ij} = \infty$ in COBRA-IIIC will result in an overprediction of the temperature difference because of the assumption that no heat is transferred between these two subchannels is over-conservative. By setting $L_{ij} = 1$ which is commonly used by the COBRA-IIIC users still results in conservative values. Both curves show that they are still deviate from the solid line which results from the 3-D calculation. To reduce this margin, the method of polynomial regression is used to fit an equation for the correction factor, L_{ij} , for the case of non-uniform axial heat flux. The empirical equation which has been found for the conditions under consideration reads:

$$L_{ij} = 0.307931 + (0.034405)(\ln z_1) + (0.0075934)(\ln z_1)^2 + (6.733481 \times 10^{-4})(\ln z_1)^3 \quad (3.79)$$

where z_1 is expressed in terms of centimeters.

This equation is fed into COBRA-IIIC and the comparison shows excellent agreement with the result of the 3-D calculation. Again, similar to the case of uniform axial heat flux, the effect of the diversion cross-flow is suspected to cause

the slight difference between these two results near the outlet region.

So far, the correction factor for the effective mixing length that has been studied in detail is only concerned with the specific geometry under consideration. It is reasonable to believe that the present results may change when the geometry of the rod bundles is changed. However, no matter what geometry is considered, the correction factor for the effective mixing length has been shown to be an important parameter governing the heat transfer along the channels. Generating enough knowledge about this correction factor for the effective mixing length L_{ij} is necessary for input into codes such as COBRA-IIIC in order to achieve more reliable solutions. With the assistance of the analytical method presented here L_{ij} can be determined effectively for any geometry and power distribution.

3.4.2. Sample Problem with The Geometry Typical of the Clinch River Fast Breeder Reactor Design

In section 3.4.1., the only geometry which has been studied thoroughly resembles the geometry of the experiment performed in Karlsruhe, Germany. As it has been shown and discussed in Section 3.4.1., the results are favorably good. However, by comparing with the present designs of existing or proposed fast reactor fuel pin bundles, this geometry seems to have a higher pitch-to-diameter ratio and larger pin radius. In order to check the validity of the code, predictions for various designs, i.e., a lower pitch-to-diameter ratio will

by studied in what follows. In addition, it will be very interesting to see how a decrease in the pitch-to-diameter ratio affects the azimuthal temperature distribution, the thermal entrance length, the Nusselt number, the correction factor for the effective mixing length and other parameters of interest.

The geometry which has been selected for a detailed analysis is the present design of the Clinch River Fast Breeder Reactor Demonstration Plant. The reason to choose the CRBR design lies in the fact of its popularity and high interest in the industry. The following study will cover both the fuel and the blanket elements. The geometrical parameters of the fuel element of the CRBR are listed as follows:

$$P/D = 1.24; \quad 2W/D = 1.42; \quad \text{Diameter} = 0.23 \text{ in.};$$

$$\text{Height} = 36 \text{ in.}; \quad T_{\text{inlet}} = 730^{\circ}\text{F}; \quad Pe = 357$$

$$\text{Average linear power} = 6.6 \text{ KW/ft.}$$

$$\text{Axial power peaking factor} = 1.24.$$

For the blanket element of the CRBR design, the values of the parameters are quite different. They are:

$$P/D = 1.077; \quad 2W/D = 1.096; \quad \text{Diameter} = 0.52 \text{ in.}$$

$$\text{Height} = 36 \text{ in.}; \quad T_{\text{inlet}} = 730^{\circ}\text{F}; \quad Pe = 250$$

$$\text{Average linear power} = 2.02 \text{ KW/ft}$$

$$\text{Axial power peaking factor} = 1.24$$

Before any further discussion of the results, it is necessary to make an important comment about the setup of the

sample problem with Clinch River Fast Breeder Reactor geometry. It is generally known that both the fuel element and the blanket element of the CRBR are wire-wrapped rod bundles. However, the study presented here will totally neglect the effect of the wire at all. Thus, the bundles are considered as bare rod bundles. One possibility to account for the effect of the wire-wrap is to determine the effective conduction mixing length by the analytical method as shown in the foregoing section and to implement this into COBRA-IIIC code which then is run with the diversion cross-flow option. In this way, it is assured that at least the conduction effects are accurately accounted for.

The results of the study of the bare rod bundle in CRBR geometry are summarized and plotted in the figures [3.27 to 3.33]. First, in Figures [3.27 and 3.28], the axial development of the peripheral clad temperature distributions of the corner pin for both the fuel element and the blanket element are shown respectively. Obviously, these results are quite different from the results with the wide pin spacing studied in Section 3.4.1. In the case of smaller pitch-to-diameter ratio such as in the case of the CRBR fuel design, the corner pin has a much larger average clad temperature at $\phi > 120^\circ$ than at $\phi < 120^\circ$. This region is the segment of the corner pin which is in closest distance to the neighboring cell. Therefore, this part of the corner pin is very sensitive to the thermal condition of its neighboring cell. As it has been explained in the previous section, by permitting heat transfer across the imaginary common boundary, the thermal condition of the internal cell greatly affects the

Fig. 3.27 Clad Temperature Distribution of Corner Pin
in CRBR Fuel Assembly ($P/D=1.24$, $2W/D=1.42$, $D=0.23$ in, $Pe=357$, $\bar{q}'=6.6$ KW/ft)

Fig. 3.28 Clad Temperature Distribution of the Corner Pin in CRBR Blanket Assembly ($P/D=1.077$, $2W/D=1.096$, $\text{Dia.}=0.52$ in, $Pe=357$, $\bar{q}'=6.6$ KW/ft)

clad temperature of the corner pin in the region of $\phi=180^\circ$. This largely explains the large increase in temperature within this segment of the corner pin. Another reason for this increase is naturally the smaller pitch-to-diameter ratio. As the value of the pitch-to-diameter ratio becomes smaller, the coolant area of the two cells becomes smaller too. Less heat will be carried away by convection in this case provided that the flow rate remains constant. As an overall result, the clad temperature will increase in the region of $\phi>120^\circ$.

Fig. [3.28] shows the clad temperature distribution of the corner pin of the blanket element. The maximum clad temperature appears to be at the angle of 135° instead of being at $\phi=180^\circ$ as observed for sample problems with larger P/D. This result seems to be surprising at first. However, in Section 4.2, Fig. [4.24] clearly depicts that when thermally fully-developed condition prevails, the maximum clad temperature shifts back to the expected position of $\phi=180^\circ$. As a result, the occurrence of the hot spot temperature at $\phi=120^\circ$ is a direct consequence of the thermal entrance effects.

The temperature fluctuations which are observed in Fig. [3.28] in the region characterized by $20^\circ \leq \phi \leq 90^\circ$ may be the result of an insufficient number of eigenvalues taken. Furthermore, the number of harmonic functions may also be insufficient. For this calculation, the number of eigenvalues is 7 and the number of harmonics used is 15. This may be not sufficient to give a perfect solution for such a low value of P/D and 2W/D. But the results are considered to be acceptable because the

maximum magnitude of fluctuation is only 4°F . However, one can infer from these results that as the pitch-to-diameter ratio becomes smaller, a higher number of eigenvalues and harmonic function is seemingly required for accurate solutions.

In figure 3.29, the maximum clad temperature variation is plotted versus the axial coordinate for both the fuel element and the blanket element. Although these two curves are quite distinctive, such as the maximum temperature difference which for the fuel element is 158°F while for the blanket element, it is only 41.5°F . However, their profiles are quite similar to each other. Both of them grow larger until they peak at about three-quarter of the core-height. Towards the end of the channel they drop off slightly. For the bulk coolant temperature difference between the two subchannels, the fuel element rises up to the value of 113.3°F at the outlet while the blanket element reaches a bulk temperature difference of 26.5°F at the outlet. The difference between both elements occur mainly because their respective linear heat generation rates differ substantially. Similar differences can be observed for the average heat flux across the gap for the two elements as shown in Fig. [3.31]. The heat flux across the gap of the fuel element increases, peaks at mid-core and drops off drastically towards the channel outlet. The correction factor for mixing length calculation, L_{ij} as shown in Fig. [3.32] behaves as follows: it starts initially as linearly increasing function of the axial coordinate then curves upward sharply as the heat flux decreases drastically. But for the blanket element,

Fig. 3.29 Maximum Clad Temperature Difference
of the Corner Pin for CRBR Design

Fig. 3.30 Bulk Temperature Difference Between Two Adjacent Subchannels for CRBR Design

Fig. 3.31 Average Heat Flux Across
the Gap for CRBR Design

Fig. 3.32 Correction Factor for Effective Mixing Length for CRBR Design

L_{ij} is substantially smaller than one and can be approximated by a linear function of axial coordinate throughout the core. The small value of L_{ij} indicates that the effective mixing length is much smaller than the centroid-to-centroid distance. Thus, the heat transfer across the gap is enormously high in the case of low pitch-to-diameter ratios. More details about the effective mixing length for different geometrical parameters will be given in the next chapter.

The resultant Nusselt number of the combined cell which consists of the internal and the corner cell is plotted versus the axial coordinate for both the fuel element and the corner element in Fig. [3.33], respectively. The diagram shows that the fuel element has a higher average heat transfer coefficient than the blanket element because it has a larger pitch-to-diameter ratio. More detailed discussion about the effect of varying geometric parameters on the heat transfer capability of the cells such as the Nusselt number will be presented later on in Section 4.2.

3.5. Conclusion

It has been found in this foregoing chapter that by coupling the corner cell together with the internal cell, the clad temperature variation of the corner pin will increase substantially. Surprisingly, this result does not agree with the experimental result at large Pe-numbers. A full explanation about this discrepancy has been given in Section 3.4. For the internal cell, a small increase in clad temperature variation has been observed

Fig. 3.33 Nussett Number of the
Combined Cell for CRBR Design

too. But its value is still very small and can be neglected for the specific geometrical condition under consideration.

However, for the Nusselt number calculation, the coupling effect results in better results for both cells. Similar effects of the coupling have been observed for the clad temperature variation and the Nusselt number by switching from the uniform axial heat flux profile to a sinusoidal one.

The correction factor for the effective mixing length, L_{ij} as function of the axial coordinate has been derived. By feeding this empirical equation into the COBRA-IIIC calculation, led to promising results. As has been demonstrated in Section [3.4.3.] the application of the correct L_{ij} leads to an accurate solution for the bulk temperature difference between two adjacent subchannels. Therefore, it must be concluded that it is very important to obtain an accurately determined L_{ij} in order to get more accurate solutions from subchannel codes.

CHAPTER 4

A PRELIMINARY PARAMETRIC STUDY FOR THE COUPLED CELL ANALYSIS

4.1 Radial Power Tilting

For various reasons, it is interesting to know how the radial power tilt affects the 3-D coolant temperature distribution. A term, called radial heat flux ratio will be defined next as the ratio of the heat flux of the internal cell to the heat flux of the corner cell and will be used throughout this section. The following study will mainly consist of several cases with various heat flux ratios between the internal pin and the corner pin. This ratio is defined as

$$\text{Radial Heat Flux Ratio} = \frac{q_i'' / q_o''}{q_c'' / q_o''} = \frac{q_i''}{q_c''}$$

where q_o'' is a reference heat flux. It should be pointed out that the only change involved here is the magnitude of the heat flux for either fuel pin. The assumption is made that no parameter will be changed other than the radial power tilt. The geometry which will be investigated is the same as that of the experiment performed in Karlsruhe [22]. Its parameters and their values are listed again for convenience:

$$P/D = 1.3, \quad 2W/D = 1.38; \quad \text{Diameter} = 0.9 \text{ cm.}$$

$$\text{Heating length} = 100 \text{ cm.} \quad Pe = 372$$

Uniform axial heat flux distribution or non-uniform axial

heat flux with cosine function and peaking factor = 1.24

Reference heat flux, $q_0'' = 104 \text{ W/cm}^2$

The values for the radial heat flux ratio that have been studied are summarized in Table [4.1].

The strategy of this analysis will emphasize radial heat flux ratios larger than 1 because since the corner pin is positioned next to the duct wall, it is reasonable to expect that the radial power is tilted this way. Besides, it has been observed that power tilt does occur over the core regions due to various reasons. The results of these studies are summarized and graphed in Fig. [4.1] to [4.22]

4.1.1 Uniform Axial Heat Flux

4.1.1.a Clad Temperature Distribution

Figure [4.1] illustrates the clad temperature distribution of the corner pin for thermally fully-developed conditions for various radial heat flux ratios. As it is shown in this figure, the larger the heat flux ratio is, the higher the peripheral temperature variation will be. It is interesting to follow through the results of distributed parameter analysis. Beginning with an isolated cell approach it was shown in Section 2.4.2. that the corner cell has the largest peripheral clad temperature variation due to the high irregularity of its geometry. Then, with the coupling of the internal cell, the variation increases. This occurs as has been already explained in Chapter 3 due to the fact that the enthalpy of the internal cell is higher than the corner cell, the removal of the adiabatic condition at

TABLE 4.1

THE CASES OF STUDY AND THEIR CORRESPONDING
RADIAL HEAT FLUX RATIO

	Internal Cell $\frac{q_i''}{q_o''}$	Corner Cell $\frac{q_c''}{q_o''}$	Radial Heat Flux Ratio $\frac{q_i''}{q_c''}$
Case 1	1.2	0.8	1.500
Case 2	1.1	0.9	1.2222
Case 3	1.0	1.0	1.00
Case 4	0.9	1.1	0.8185

Fig. 4.1 Fully-Developed Clad Temperature Distribution for Corner Pin under Condition of Different Radial Heat Flux Distribution

($P/D=1.3$, $2W/D=1.38$, $\text{Dia.}=0.9$ cm, $Pe=372$, $\bar{q}_0''=104$ W/cm²)

the common boundary will permit heat transport between the two cells and induce an increase in the maximum clad temperature of the corner pin in the region of $\phi = 180^\circ$. In addition, the power tilt is in the direction such that the internal cell has a higher heat flux than the corner cell, it gives an even higher peripheral temperature variation because more heat is transported across this common boundary as shown in Fig. [4.7]. This implies that the degree of the clad temperature variation of the corner pin is directly proportional to the radial heat flux ratio. Therefore, from a design point of view, it seems to be desirable to keep the radial heat flux ratio less than one to avoid any further increase in the clad temperature variation.

Figs. [4.2, 4.3, and 4.4] show sequentially the axial development of the peripheral clad temperature distribution of the corner pin for various heat flux ratios. Surprisingly, even though the heat flux ratio varies between 0.8182 and 1.50, the clad temperature level in the region around $\phi = 180^\circ$ does not change that much as expected. The explanation for this is due to the designated condition of the heat fluxes for both cells in these three cases of study. As long as the heat flux of the corner pin is lowered by a factor of 10% of the reference heat flux, the heat flux of the internal pin will go up by a factor of 10%, too. The result of this does not alter the average clad temperature much because an increase of the radial heat flux ratio will generate an increase of the heat flux across the common boundary and this additional heat will contribute to the increase of the average clad temperature in the region

Fig. 4.2 Clad Temperature Distribution
 of Corner Pin for $q''_i/q''_c = 1.2/0.8$

Fig. 4.3 Clad Temperature Distribution
 of Corner Pin for $q_i''/q_c'' = 1.1/0.9$

Fig. 4.4 Clad Temperature Distribution
 of Corner Pin for $q_1''/q_c'' = 0.9/1.1$

around $\phi = 180^\circ$.

Obviously, for the low-angled region which is far away from the internal cell, the clad temperature level will drop as the heat flux of the corner pin decreases. A similar result is observed when the radial heat flux ratio is smaller than one. The average clad temperature in the region around $\phi = 180^\circ$ is more or less the same as Case 1 and 2. The additional energy is transported across the common boundary in the opposite direction as in Case 1 and 2 as shown in Fig. [4.5, 4.6 and 4.7] and increases the temperature of the internal pin. The other segment of the cladding has an increase in the temperature level due to the increase of heat flux.

It is interesting to observe in Figs. [4.5, 4.6 and 4.7] that the increments of the heat flux across the common boundary are almost the same if the heat flux ratio changes by a factor of 12%. This implies that the heat flux across the common boundary is linearly dependent on the heat flux ratio between the two cells. However, in the case of the heat flux ratio being less than one, these graphs show that at the lower axial level, the transport of heat is in the reverse direction, that is from the corner cell into the internal cell. At the higher elevations such as $z = 100$ cm., a portion of the heat flux at the common boundary goes the other way which is from the internal cell to the corner cell. This phenomenon extends throughout the rest of the core up to the thermally fully-developed level. At this elevation, the reverse heat flow direction ranges from $\phi = 150^\circ$ to $\phi = 160^\circ$. This result illustrates that at the upper

Fig. 4.5 Heat Flux Across the Common Boundary at $z = 29.5$ cm under Different Radial Heat Flux Distribution (Positive Heat Flux Implies Heat-Flow from Internal Cell to Corner Cell)

Fig. 4.6 Heat Flux Across the Common Boundary
 at $z = 100$ cm under Different Radial Heat
 Flux Distribution

Fig. 4.7 Heat Flux Across the Common Boundary at Fully-Developed Region under Different Radial Heat Flux Distribution

level, the geometry of the cell will overcome other factors and has a big influence on the shaping of the temperature profile. The explanation of this phenomenon is explained by Fig. [4.7a]. T_1 and T_2 are supposed to be the bulk temperatures at position 1 and 2, respectively. Even if the corner cell has a higher heat flux, at the upper level when the geometry becomes important, the bulk temperature difference, that is $(T_1 - T_2)$ may become negative and a negative heat flux across the common boundary may result. This occurs as the rate of bulk temperature increase of T_1 slows down because of the azimuthal heat conduction through the gap.

The clad temperature of the internal cell is shown in Fig. [4.8]. At the lower level of $z = 11.5$ cm., there is no appreciable change in the temperature as the heat flux ratio varies. However, at the higher axial position when $z > 45.5$ cm. a distinction among all the clad temperature distribution varies according to the different heat fluxes of the internal cell can be seen easily.

4.1.1.b. Maximum Clad Temperature Difference

As it has been mentioned earlier, the irregularity in the geometry of the cell contributes the most to the maximum azimuthal clad temperature difference. As for the corner cell, the relative thermal condition of the adjacent internal cell plays a major role, too. The effect of various heat flux ratios on the maximum azimuthal clad temperature difference will be

FIGURE 4.7a

Fig. 4.8 Clad Temperature Distribution of the Internal Pin under Different Radial Heat Distribution

analyzed according to the results shown in Fig. [4.7].

This figure shows that for larger radial heat flux ratio, the temperature variation will increase. Surprisingly, it is found that as the heat flux ratio changes by about 20%, there is a similar increase in the temperature variations among all these four cases.

For $z = 100$ cm., the increment is about

$$\Delta\theta = 0.035 \quad \text{or} \quad T = 3.4^{\circ}\text{C}$$

For $z = 150$ cm., it will be:

$$\Delta\theta = 0.05 \quad \text{or} \quad T = 4.8^{\circ}\text{C}$$

Another finding is that all the curves approach a single one at the lower axial elevations. This implies that no matter how the adjustment of the radial heat flux ratio will be, it seems to be impossible to obtain a zero temperature variation. Thus, it can be concluded that a positive, non-zero minimum limit in the maximum clad temperature difference can be obtained by lowering the radial heat flux ratio. This again shows the overall strong influence of the irregularity in the geometry upon the temperature variation.

4.1.1.c Effective Mixing Length Calculation

The effect of various radial heat flux ratios on the correction factor for the effective mixing length calculation is shown in Fig. [4.10]. Judging from these results, it seems that the effect is small and can be neglected in this particu-

Fig. 4.9 Maximum Clad Temperature Difference for Corner Pin under Different Radial Heat Flux Distribution

Fig. 4.10 Axial Variation of the Correction Factor for Effective Mixing Length under Different Radial Heat Flux Distribution

lar geometry as long as the core height is less than 100 cm. Throughout the core up to 150 cm., all four curves in Fig. [4.10] are closely together. Even at the level of 100 cm., the difference of L_{ij} is only 0.01 between the heat flux ratio of 0.8182 and 1.5. Therefore, it can be safely concluded that L_{ij} is mainly a function of the geometry and does not depend on the thermal conditions of both pins.

To more easily comprehend why the heat flux ratio has less effect on the correction factor for the mixing length calculation L_{ij} , it is necessary to look into the behavior of the two factors which govern L_{ij} under the condition of different heat flux ratios. These two factors are the bulk temperature difference between two adjacent channels and the average heat flux across the gap. Both these two factors are plotted separately for various heat flux ratios in Figs. [4.11] and [4.12]. By carefully checking these two families of curve, it is noticed that their responses to the change of radial heat flux ratio are quite similar. A change in the heat flux ratio of about 20%, results in the following increments:

$$\text{for } z = 100 \text{ cm.} \quad (\Delta\theta)_{\text{bulk}} = 0.035 \quad \text{or} \quad (\Delta T)_{\text{bulk}} = 3.4^{\circ}\text{C}$$

$$\text{and} \quad \frac{(q'')_{\text{gap}}}{(q'')_0} = 0.3846$$

$$\text{for } z = 150 \text{ cm.,} \quad (\Delta\theta)_{\text{bulk}} = 0.05 \quad \text{or} \quad (\Delta T)_{\text{bulk}} = 4.8^{\circ}\text{C}$$

$$\text{and} \quad \frac{q''_{\text{gap}}}{q''_0} = 0.4615$$

Fig. 4.11 Bulk Temperature Difference between Two Adjacent Subchannels under Different Radial Heat Flux Distribution

Fig. 4.12 Average Heat Flux Across the Gap under
 Different Radial Heat Flux Distribution

and these increments are almost the same if the heat flux ratios are the same. With this similar response, the effect of changing the heat flux ratio on the ratio of these two factors will cancel out. Thus, L_{ij} will not vary at all.

It was mentioned earlier in Section 4.1.1.b that in this particular geometry, the value of the maximum clad temperature difference of the corner pin is always positive and non-zero. But for the bulk temperature difference between two adjacent subchannels, it is possible to adjust the radial heat flux ratio such that the result will give a zero value. By a crude extrapolation, the ratio is believed to be about 0.5/1.5. However, under this condition of $q_1''/a_c'' = 0.5/1.5$, the heat flux across the gap is found to be approximately equal to zero too. Again, linear extrapolation is used for calculating this value.

4.1.1.d Nusselt Number Calculation

Fig. [4.13] shows that as the heat flux ratio increases, the Nusselt number of the internal cell increases throughout the core of 150 cm. However, the fully developed Nusselt number indicates that there is a decrease in its value while the heat flux ratio increases. In order to explain this phenomenon, it is necessary to refer back to the figure showing the axial development of the clad temperature distribution of the internal cell for various heat flux ratios, Fig. [4.8]. At the lower axial level, the average wall temperature of the internal cell has not been affected as much as the upper axial level of the core as the heat flux ratio changes. On the other

Fig. 4.13 Axial Variation of the Nusselt Number of the Internal Cell under Different Radial Heat Flux Distribution

it is required that the overall bulk temperature in the internal cell changes in the same direction as the power of the internal cell changes, due to the conservation of energy. Therefore, the result will cause the film temperature difference getting smaller at the lower level and bigger at the extremely high elevations. This implies larger Nusselt numbers throughout the core and smaller values at the fully-developed stage under the condition of larger radial heat flux ratio.

In contrast to what has been observed in the internal cell, the Nusselt number of the corner cell behaves as expected when changing the heat flux ratio. In Fig. [4.14], it is shown that as the heat flux ratio increases, the heat flux level of the corner pin relatively decreases and so does the Nusselt number. Throughout the whole core up to the fully-developed stage, the behavior of the curves for different heat flux ratios remains the same.

It will be interesting to see how the resultant Nusselt number of the combined cell will behave. The results is plotted in Fig. [4.15]. It shows that the overall Nusselt number will increase as the heat flux ratio decreases. The average value of the differences is small, only about 0.3 for a change of heat flux ratio by 20% of its initial value.

The effect of various heat flux ratios on the thermal entrance length is listed in Table 4.2. As the heat flux ratio becomes larger, the thermal entrance length becomes smaller. However, for the heat flux ratio being larger than one, the difference is very small. For the ratio being smaller than

Fig. 4.14 Axial Variation of the Nusselt Number of the Corner Cell under Different Radial Hat Flux Distribution

Fig. 4.15 Axial Variation of the Nusselt Number of the Combined Cell
(Internal & Corner Cell) under
Different Radial Heat Flux Distribution

TABLE 4.2

THERMAL ENTRANCE LENGTH FOR VARIOUS HEAT FLUX RATIO

Heat Flux Ratio	$\frac{1.2}{0.8}$	$\frac{1.1}{0.9}$	$\frac{1.0}{1.0}$	$\frac{0.9}{1.1}$
Thermal Entrance Length ξ_{th}	3.8982	3.9631	4.0183	4.6788
Z/D_h	1143.09	1162.10	1178.29	1371.97

one, the difference becomes larger. No matter how this thermal entrance length varies as a result of the different heat flux ratios, they are still far larger than the expected core-height in the present LMFBR designs.

4.1.2. Non-uniform Axial Heat Flux Distribution

In this section, the effect of radial power peaking on the heat transfer analysis in the case of non-uniform axial heat transfer will be studied. Again, the same cosine function which has been used in the last two chapters is applied in the following analyses. The axial heat flux function is

$$q''(z) = q''_{\max} \cos \frac{\pi z}{H_0}$$

Obviously, an important assumption has already been made such that both the corner pin and the internal pin will have the same axial profile and the same axial power peaking factor even though the average magnitude of the heat flux may be different. As in the case of power tilting, a difference in the average value of the heat flux does exist. However, for solving the problem of the non-uniform axial heat flux, this will cause no problem. As long as the axial functions are the same and are perfectly piecewise continuous throughout the range, Duhamel's Principle can still be applied.

The effect of the power tilting on the coupled cell analysis under the condition of non-uniform axial heat flux distribution has been summarized and graphed in the sequence of

figures [4.16 to 4.23].

4.1.2a Clad Temperature Distribution of the Corner Pin

Fig. [4.16] shows the clad temperature distribution of the corner pin for various radial heat flux ratios at several axial levels. As may be seen in the graph, it seems that there is no spectacular temperature difference created by varying the radial heat flux ratio at any of these levels.

Particularly, at the large angle where $\phi \approx 180^\circ$, all curves are so closely together that it is hard to distinguish among them. This result occurs because a higher radial heat flux ratio will induce an increase of heat transfer across the common boundary and this will overcome the reduction of energy due to the decrease of the heat flux density of the corner pin. However, for the region of $\phi < 120^\circ$, the influence from the internal cell is small. Thus, when the radial heat flux ratio increases, a decrease of heat flux density will result and the temperature level around that region will drop. But this difference is comparatively smaller than what has been observed in the case of uniform axial heat flux.

4.1.2.b Maximum Clad Temperature Difference

The effect of varying the radial heat flux ratio on the maximum clad temperature difference can be seen from Fig. [4.17]. Fig. [4.17] shows that the differences among the curves are very small at the lower axial elevations. The curves

Fig. 4.16 Clad Temperature Distribution of the Corner Cell for $q'' = f(z)$ under Different Radial Heat Flux Distribution

Fig. 4.17. Maximum Clad Temperature Difference for Corner Pin
for $q'' = f(z)$ under Different Radial Heat Flux Distribution

behave in this way throughout most of the core region. Even at the outlet at $z = 100$ cm., the differences among the curves are not that significant as one would probably expect. The largest difference among these curves is about 3.7°C on the average with an increase in the heat flux ratio by 20%. This implies that the effect of varying the heat flux ratio on the temperature variation is smaller in the case of sinusoidal axial heat flux profile than in the case of uniform axial heat flux profile.

4.1.2.c. Effective Mixing Length Calculation

Similar to what has been seen in the case of uniform axial heat flux distribution, the correction factor for the effective mixing length L_{ij} again does not seem to respond much to the radial power tilting as shown in Fig. [4.18]. These four curves which correspond to the four different situations are all closely together throughout three-quarters of the core. Then at the outlet, $z = 100$ cm., small differences among the curves can be observed. The curve of the largest heat flux ratio stays on the top of the others while the one corresponding to the lower heat flux ratio lies underneath the other curves. The difference is only 0.06 between these two. Therefore, it seems feasible to neglect the effect of varying heat flux ratio on L_{ij} under these circumstances.

Again, as before, if tracing back into the behavior of the two corresponding factors which are the bulk temperature difference between two channels and the average heat flux across

Fig. 4.18 Correction Factor for Effective Mixing Length
Calculation for $q'' = f(z)$ under Different Radial
Heat Flux Distribution

the gap upon changing heat flux ratio, it is easy to comprehend this behavior. Figs. [4.19 and 4.20] clearly show the response of these two factors under the condition of different radial heat flux ratio. If compared, Figs. [4.19] and [4.20] together, they show that these two factors behave quite similarly to each other. The correction factor for effective mixing length, L_{1j} , which is the ratio of the two remains unchanged for changes in the heat flux ratio because both effects will offset each other.

4.1.2.d. Nusselt Number Calculation

For the Nusselt number calculation of each individual cell in the case of non-uniform axial heat flux, the effect of changing the radial heat flux ratio gives no unexpected answers. As shown in Figures [4.21 and 4.22] the Nusselt number will increase as the heat flux density of that particular cell increases. Similarly, the value will drop when the heat flux density decreases. For the combined cell, the average heat transfer coefficient will go up if the radial heat flux ratio decreases. This results is clearly depicted in Fig. [4.23].

4.2. Study of Changes Due to Different Geometrical Cell Parameters

4.2.1. Introduction

Throughout the discussion of the coupled cell analysis in Chapter 3, three different geometries have been analyzed in considerable detail. The first one which has been thoroughly studies resembles the geometry of the experiment performed in Karlsruhe. The other two geometries chosen are the fuel assembly

Fig. 4.19 Bulk Temperature Difference between Two Adjacent Subchannels for $q'' = f(z)$ under Different Radial Heat Flux Distribution

Fig. 4.20 Average Heat Flux Across the Gap for $q'' = f(z)$
under Different Radial Heat Flux Distribution

Fig. 4.21 Axial Variation of the Nusselt Number of the Internal Cell for $q'' = f(z)$ under Different Radial Heat Flux Distribution

Fig. 4.22 Axial Variation of the Nusselt Number of the Corner Cell for $q'' = f(z)$ under Different Radial Heat Flux Distribution

Fig. 4.23 Axial Variation of the Nusselt Number of the Combined Cell for $q'' = f(z)$ under Different Radial Heat Flux Distribution

and the blanket assembly of the Clinch River Fast Breeder Reactor Design, respectively. With the results of these three calculations on hand, the following section will be devoted to the study of the effect of geometry changes upon the heat transfer.

The on-going discussion will be divided into two parts. The first one is related to the condition of uniform axial heat flux distribution and the second part is related to the condition of non-uniform axial heat flux distribution for which a cosine function is used. For the purpose of comparison among these different geometries, all the graphs in the uniform axial heat flux condition have been plotted against the dimensionless parameter, Z/D_h . However, for the condition of the cosine axial heat flux the dimensional variable of z is used instead of z/D_h . The reason for this is because for all cases of study, the core-heights have been designated to be equal to 100 cm. Since among these cases, each has its own distinct value of the equivalent diameter of the cell, the axial parameter z_o/D_h , where z_o is the total core-height, is different under different geometrical conditions. So it is more feasible to use the variable, z . The conditions for all cases considered for comparison are listed in Table [4.3].

4.2.2. Study of the Problem with Uniform Axial Heat Flux Distribution

4.2.2.a Clad Temperature Distribution of the Corner Cell

In this analysis, only the comparison of the thermally fully-developed clad temperature distribution of the corner

TABLE 4.3

DESCRIPTION OF THE GEOMETRICAL CONDITIONS OF DIFFERENT
CASES

	<u>P/D</u>	<u>2W/D</u>	<u>2W/P</u>	<u>Dia.</u>	<u>Pe</u>	<u>Ht.</u>	<u>Axial Peaking Factor</u>
Case 1	1.30	1.38	1.062	0.9 cm.	372	100cm.	1.24
Case 2	1.24	1.42	1.145	0.5842 cm.	357	100cm.	1.24
Case 3	1.077	1.096	1.018	1.3208 cm.	250	100cm.	1.24

pin will be shown. As it has been illustrated in Fig. [4.24], the clad temperature variation is the largest for case 3 and is the smallest for case 1. Basically, as what has been shown in Table [4.3], the ratio of $2W/P$ of these three cases are approximately the same. Then, it can be concluded that as the pitch-to-diameter ratio decreases, the clad temperature variation will increase if $2W/P$ stays constant. However, since there is not a large change of the wall-distance-to-pitch-diameter ratio, $2W/P$, the maximum clad temperature stays at $\phi = \pi$ while the minimum clad temperature remains in the region characterized by low angles. For case 1, with the corner of the duct wall being designed to be rounded, the minimum clad temperature appears at $\phi \approx 60^\circ$. For case 3, it shifts to $\phi \approx 30^\circ$. The reason why there is a temperature rise for case 3 at small azimuthal angle is that the distance between the duct wall and the center of the pin is too small. The geometrical factor of the cell prevents any temperature drop at the small angle.

4.2.2.b Maximum Circumferential Clad Temperature Difference

In Fig. [4.25], the maximum clad temperature difference of the corner pin has been plotted versus the dimensionless parameter Z/D_h . Since case 1 has a wider spacing among the rods if compared to the other two cases, its curve shows that it has the smallest temperature difference throughout the core. But for case 2 and 3, a surprising result can be observed.

Fig. 4.24 Fully-Developed Clad Temperature of the Corner Pin under Various Geometrical Conditions

Fig. 4.25 Maximum Clad Temperature Difference of the Corner Cell
for q'' = Uniform under Various Geometrical Conditions

Near the inlet, i.e., $Z/D_h < 50$, curve 3 gives a larger temperature difference. However, above this level, i.e., $Z/D_h > 50$, curve 2 has a larger temperature difference between these two. The explanation of this result can be stated as follows: near the inlet, when coupling the internal cell with the corner cell, the maximum clad temperature will increase and the degree of change seems to be inversely proportional to the pitch-to-diameter ratio. But, as will be shown in Table [4.4] and be discussed in Section 4.2.2.d, the thermal entrance length of case 3 is much longer than case 2. Even though case 3 has a larger asymptotic value of the maximum clad temperature difference than case 2, it does take a longer distance to achieve this value. As a results, at the level far below the asymptotic value, it is possible for case 2 to have a larger maximum clad temperature difference than that for case 3. This is what has been observed in Fig. [4.25]. Therefore, by judging only from the asymptotic solution without considering the thermal entrance effect, there exists the danger of reaching a wrong conclusion.

4.2.2.c Correction Factor for Effective Mixing Length Calculation

Fig. [4.26] clearly shows that the curves for the correction factor, L_{ij} are widely spread apart for different geometrical conditions. Curve 1 shows a sudden increase of the value, L_{ij} near the inlet and almost linearly increases for the rest of the core. Curve 2 steadily increases throughout the core with a smaller value. But curve 3, shows a very small

TABLE 4.4

THERMAL ENTRANCE LENGTH UNDER DIFFERENT GEOMETRICAL CONDITIONS

	<u>P/D</u>	<u>2W/D</u>	<u>Dia.</u>	<u>Pe</u>	<u>Z/D_h</u>
Case 1	1.30	1.38	0.9 cm.	372	1178.12
Case 2	1.24	1.42	0.5842 cm.	357	2029.90
Case 3	1.077	1.096	1.3208 cm.	250	4811.25

Fig. 4.26 Correction Factor for Effective Mixing Length Calculation for $q''(z) = \text{Uniform}$ under Various Geometrical Conditions

value and always stays below 0.04 for $Z/D_h < 350$. In order to explain these results, it is advisable to carefully check the results of Figs. [4.27] and [4.28] respectively. These are the figures of the bulk temperature difference between two adjacent subchannels as function of the axial coordinate and the average heat flux across the gap as function of the axial coordinate, respectively.

As one may observe in Fig. [4.27], the behavior of the curves for the bulk temperature difference surprisingly is very similar to the curves of the maximum clad temperature difference. Only the intersection point between curve 2 and curve 3 has shifted up to $Z/D_h = 100$. But for the average heat flux across the gap, the figure shows that this value is highly dependent on the geometrical conditions. For case 3 with the smallest P/D value, the heat flux has increased enormously when compared to a larger P/D ratio such as used in case 2. This large value of the heat flux causes the correction factor, L_{ij} to be close to zero in case 3.

4.2.2.d. Nusselt Number

The Nusselt number of the combined cell has been plotted in Fig. [4.29]. Basically, the graph shows that at the elevations characterized by $Z/D_h \geq 200$, the Nusselt number decreases if the pitch-to-diameter ratio decreases. But for $Z/D_h < 200$, the thermal entrance effect becomes important. As one may observe in the graph, even in case 2 which has a smaller P/D ratio than in case 1, the Nusselt number is the

Fig. 4.27 Bulk Temperature Difference between Two Adjacent Subchannels
for $q''(z) = \text{Uniform}$ under Various Geometrical Consideration

Fig. 4.28 Average Heat Flux Across the Gap for $q''(z) = \text{Uniform}$ Under Various Geometrical Conditions

Fig. 4.29 Nusselt Number of the Combined Cell for $q''(z) = \text{Uniform}$
 Under Various Geometrical Conditions

largest. Naturally, these results can only be discovered by a 3-D coupled cell analysis.

The results of the calculation of the thermal entrance length, are listed in Table [4.4]. The results are obvious and show that case 3 has the longest and case 1 has the shortest thermal entrance length.

4.2.3. Study of the Problem with Non-uniform Axial Heat Flux Distribution

4.2.3.a. Maximum Clad Temperature Distribution

What has been already observed for the case of uniform axial heat flux can be observed again in Fig. [4.30]. Curve 1 which has the largest P/D ratio increases steadily with the axial coordinate. At the outlet of $z = 100$ cm., it reaches its maximum value of $\Delta\theta = 0.70$. But for curves 2 and 3, although curve 2 has a smaller value at the inlet, it overtakes curve 3 at $z = 37$ cm., and peaks at $z = 80$ cm., with the value of $\Delta\theta = 1.74$. And for curve 3, its maximum value is only 1.54 located at the axial elevation of $z = 70$ cm. The effect of the cosine axial profile which results in an increase of the film temperature difference near the outlet region can easily be seen because curve 2 and curve 3 decrease substantially towards the outlet.

4.2.3.b Correction Factor for Effective Mixing Length Calculation

The correction factors, L_{1j} for different geometries under the condition of cosine axial heat flux profile are shown

Fig. 4.30 Maximum Clad Temperature Difference of the Corner Cell for $q'' = f(z)$ Under Various Geometrical Conditions

in Fig. [4.31]. With this addition of cosine axial profile, there is a reduction of the heat flux across the subchannel gap which can be seen in Fig. [4.33]. As has been explained earlier in Section 4.2.2.c, the average heat flux across the gap increases as the pitch-to-diameter decreases. The drop of this heat flux across the gap will result in an increase of correction factor, L_{ij} . For curve 1, the decrease of heat flux is small at the outlet. Therefore, the correction factor, L_{ij} does not change much. But for cases 2 and 3, both heat fluxes decrease substantially, a sharp increase of L_{ij} can be found in curve 2 but for curve 3, the value of L_{ij} is too small to be recognized. Fig. [4.32] shows that the behavior of the bulk temperature difference between the two adjacent channels. The results show case 1 has the smallest temperature difference. But for the largest bulk temperature difference, it depends upon what axial level one refers to. Near the inlet region, case 3 has the largest bulk temperature difference but at the higher elevation, case 2 does have the largest difference. The explanation of this occurrence has been discussed in detail in Sections 4.2.2.b. and 4.2.2.c. Finally, it can be seen from Fig. [4.32] that all these three curves are peaked at the outlet level.

4.2.3.c. Nusselt Number

Fig. [4.34] shows that curve 3 which has the smallest P/D ratio has in turn the lowest value of the Nusselt number throughout the core. But curve 2 which has a smaller P/D ratio than curve 1, depicts a better heat transfer at the

Fig. 4.31 Correction Factor for Effective Mixing Length for $q'' = f(z)$ Under Various Geometrical Conditions

Fig. 4.32 Bulk Temperature Difference Between Two Adjacent Subchannels for $q'' = f(z)$ Under Various Geometrical Conditions

$$= \frac{Q_{ij}}{q_o} \left(\frac{P}{D} \right)$$

Fig. 4.33 Average Heat Flux Across the Gap for $q'' = f(z)$
Under Various Geometrical Conditions

Fig. 4.34 Nusselt Number of the Combined Cell for $q'' = f(z)$
Under Various Geometrical Conditions

lower half of the core but shows a worse heat transfer at the upper half of the core. This unexpected results can be only displayed by a 3-D coupled cell analysis.

4.3. Conclusions

The conclusions can be split into two different parts.

The first part emphasizes the effect of the radial power tilting on the heat transfer capability of each cells. The latter part contains the discussion about the changes of the heat transfer capability of the cells under different geometrical conditions such as P/D , $2W/D$, Pe , etc.

When a radial heat flux distribution is imposed on the corner and the internal cell, it has been found that under the conditions that have been studied, the clad temperature variation will increase and the Nusselt number of the combined cell will decrease when the radial heat flux ratio becomes larger. The correction factor L_{ij} does not seem to be affected much by changing the radial heat flux ratio. According to this result, one may expect that the correction factor L_{ij} is only a function of the geometrical parameters. All the aforementioned findings are valid for the condition of a uniform axial heat flux distribution. However, by imposing a sinusoidal function instead of a constant function for the axial heat flux distribution, under the condition of heating length = 100 cm. and axial peaking factor = 1.24, the radial power tilting effect on these various parameters discussed earlier such as clad temperature variation, etc. is not that distinguishable. These results have

been discussed at some length in Section 4.1.2.

In the study of the heat transfer capability of each cell under different geometrical conditions, some of the results obtained were predictable whereas others are quite complicated. As for the correction factor, L_{ij} , the value becomes smaller as expected under the conditions of smaller P/D and smaller $2W/D$ because the value of the average heat flux gets larger as the condition changes accordingly. However, parameters such as the maximum clad temperature difference and the Nusselt number, behave quite differently. It has been shown in Table [4.4] that under the condition of smaller P/D and smaller $2W/D$, the thermal entrance length gets longer. As a results of this, the thermal entrance effect does alter the behavior of these parameters. This has been discussed in detail in Section 4.2.

These unexpected results can only be observed from a thorough 3-D coupled cell analysis.

CHAPTER 5

A STUDY OF COUPLED INTERNAL CELLS WITH RADIAL POWER TILTING

5.1. Introduction

In both Chapter 3 and Chapter 4, the only geometrical shape that has been analyzed so far is the coupling of the corner cell and the internal cell together. The concern about the corner cell is understandable because as it has been shown in Chapter 2, from the isolated cell approach, that the corner cell has the most severe problem with the peripheral temperature variation. Then, the effects of coupling the corner cell with an internal cell and the additional impact of the radial power tilting have been studied thoroughly in Chapter 3 and Chapter 4 respectively. The results show that these two effects are of major importance for detailed heat transfer analysis of the corner cell. In the following analysis, a new geometry will be studied in order to show the capability of the analytical method developed in Chapter 3. The following study is basically concerned about the impact of radial power tilting on the internal cells. Obviously, under the condition of flat radial power distribution, it seems justified to treat each individual internal cells as isolated cells if they are reasonably far away from the bundle duct wall. The reason why the isolated cell approach is sufficient is because the symmetrical property of the internal cells will result in an adiabatic condition at the common boundary. However, by imposing a non-uniform radial power distribution, the adiabatic condition will no longer exist

at these common boundaries. Thus, the coupled internal cell analysis is the only way to solve this problem.

5.2. Formulation of the Sample Problem

The formulation of the sample problem which will be discussed in Section 5.3. starts with the lay-out of the geometry of the coupled internal cells as shown in Fig. [5.1]. It has a total of 7 internal cells coupled together with the imposed symmetric condition of $\phi=\pi$. That is, by labelling the cells accordingly to what is shown in Fig. [5.1], the thermal condition of pin 6 is exactly equivalent to that of pin 4. Similarly, pin 7 and pin 3 are identical to each other. Another assumption is that the heat fluxes of each individual pin are azimuthally uniform even though there is a radial power distribution among these pins.

The solution scheme of the coupled internal cells analysis is exactly the same as has been used for the case of coupling internal and corner cells in Chapter 3. Again, the solution can be decomposed into two parts: a thermally fully-developed solution and a thermal entrance solution. Their solution forms and the corresponding boundary conditions are as follows:

$$\theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi) + \theta_e(\rho, \phi, \xi) \quad (5.1)$$

$$\begin{aligned} \text{where } \theta_f(\rho, \phi, \xi) = & C_0 \xi + C_0 \frac{\rho^2}{4} + A_0 + B_0 \ln \rho \\ & + \sum_{n=1}^{\infty} [A_n(\rho^n + \rho_1^{-2n} \rho^{-n})][\cos n\phi] \end{aligned} \quad (5.2)$$

Fig. 5.1 Geometry of the Coupled
Internal Cells

Fig. 5.2 Radial Heat Flux Distribution
of the Internal Cells

and the boundary condition for $\theta_f(\rho, \phi, \xi)$

a. at $\rho = \rho_i$

$$\frac{\partial \theta_f}{\partial \rho} = -\frac{P}{D} \frac{q_1''}{q_0''} \quad \text{for } 0 \leq \phi \leq 2\pi \quad (5.3)$$

b. at $\rho = \rho_a$

$$\hat{n} \cdot \text{grad } \theta_f = \frac{P}{D} \frac{q_n''}{q_0''} \quad \text{for } \frac{(n-2)\pi}{3} - \phi_0 \leq \phi \leq \frac{(n-2)\pi}{3} + \phi_0 \quad (5.4)$$

and $n = 2, 3, 4, 5, 6, 7$

and the angle, ϕ_0 , is shown in Figure [5.1].

for other angle, ϕ

$$\hat{n} \cdot \text{grad } \theta_f = 0 \quad (5.5)$$

$$c. \quad \frac{\partial \theta_f}{\partial \phi} = 0 \quad \text{for } \phi = n\pi \text{ and } \rho_i \leq \rho \leq \rho_a$$

$$n = 0, 1, 2, 3, \quad (5.6)$$

for all ξ .

For the thermal entrance solution $\theta_e(\rho, \phi, \xi)$

$$\theta_e(\rho, \phi, \xi) = \sum_{m=1}^{N_{\text{eig}}} \sum_{n=0}^{N_n} [C_{n,m} J_n(\beta_m \rho) + D_{n,m} Y_n(\beta_m \rho)] [\cos n\phi] [E_m e^{-\beta_m^2 \xi}] \quad (5.7)$$

and the boundary condition for $\theta_e(\rho, \phi, \xi)$

a. at $\rho = \rho_1$

$$\frac{\partial \theta_e}{\partial \rho} = 0 \quad \text{for all } \phi \quad (5.8)$$

b. at $\rho = \rho_a$

$$\hat{n} \cdot \text{grad } \theta_e = 0 \quad \text{for all } \phi \quad (5.9)$$

c. $\frac{\partial \theta_e}{\partial \phi} = 0$ for $\phi = n\pi$ and $\rho_1 \leq \rho \leq \rho_a$

$$n = 0, 1, 2, 3, \dots$$

$$\text{for all } \xi \quad (5.10)$$

For the initial condition

at $\xi = 0$

$$\theta_f(\rho, \phi, \xi) = -\theta_e(\rho, \phi, \xi) \quad (5.11)$$

at $\xi \rightarrow \infty$

$$\theta(\rho, \phi, \xi) = \theta_f(\rho, \phi, \xi)$$

$$\text{or } \theta_e(\rho, \phi, \xi) = 0 \quad (5.12)$$

The designated radial heat flux distribution of the sample problems are described as follows:

$$\frac{q_1''}{q_0''} = 1.00 ; \quad \frac{q_2''}{q_0''} = 1.10 ; \quad \frac{q_3''}{q_0''} = 1.05,$$

$$\frac{q_4''}{q_0''} = 0.95; \quad \frac{q_5''}{q_0''} = 0.90$$

Again, this distribution is clearly illustrated in Fig. 5.2.

Technically, there is no particular reason to pick this specific radial heat flux distribution. Overall, the heat flux is designed to tilt in the direction of pin 2 which can be considered in the same direction as towards the middle of the core. Comparing the sample problem to the present CRBR design, the degree of radial power tilting may be too conservative for either the fuel assembly or the blanket assembly. However, under certain circumstances a more severe power tilting may result. Therefore, this assumption made for the sample problem seems to be adequate for the purpose of this study.

Throughout the analysis in Section 5.2, only the case of the uniform axial heat flux will be considered. No further analysis corresponding to the non-uniform axial heat flux condition will be discussed here because as it has been shown earlier in section 4.1.2., the effect of power tilting in the case of non-uniform axial heat flux seems to be minor when compared to the case of uniform axial heat flux.

5.3. Results of the Coupled Internal Cells Analysis With Radial Power Tilting

The geometry of this sample problem is the same as that

of the experiment performed at Karlsruhe [22]. The effect of the power tilting is summarized in the diagrams, [Figs. 5.3 to 5.14]. These results are compared to those which resulted for a flat radial power distribution.

5.3.2 Clad Temperature Distribution

A detailed study of the clad temperature distribution of each individual pin has been made and the results are plotted in Figs. [5.3, 5.4, 5.5, 5.6 and 5.7]. From these figures, it can be realized that pin 1 has the most severe peripheral temperature variation as a result of the power tilt. This occurs because pin 1 is the central pin in this geometry and has four neighboring pins surrounding it with various heat flux distributions, varying from $\frac{q_1''}{q_0''} = 1.10$ to $\frac{q_1''}{q_0''} = 0.90$. Since pin 2 has a heat flux density which is larger by 10%, the cladding of pin 1 around $\phi = 0$ becomes excessively heated by the heat transported across their common boundary from pin 2. Meanwhile, the average temperature of pin 1 around $\phi = 180^\circ$ will be lowered substantially because some of the heat is being extracted away towards pin 5 which has the lowest heat flux density.

The pin with the next larger temperature variation will be either pin 3 or pin 4. For both pin 3 and pin 4, each has only 3 neighboring pins with different heat flux distributions. Their variations are not as large as for pin 1. Therefore, they do not pose as big a problem as pin 1. Since pin 3 has an above-average heat flux density, when compared to the case of flat radial power distribution, there occurs an increase

Fig. 5.3 Clad Temperature Distribution of Pin 1 as
as Predicted by the Coupled Internal Cell Analysis

Fig. 5.4 Clad Temperature Distribution of Pin 2
as Predicted by the Coupled Internal Cell
Analysis

Fig. 5.5 Clad Temperature Distribution of Pin 3 as predicted by the Coupled Internal Cell Analysis

Fig. 5.6 Clad Temperature Distribution of Pin 4 as predicted by the Coupled Internal Cell Analysis

Fig. 5.7 Clad Temperature Distribution of Pin 5 as predicted by the Coupled Internal Cell Analysis

in the average clad temperature for pin 3. However, for pin 4, there is a drop in the average clad temperature because of the reduced heat flux. Finally, for pin 2 and pin 5, these have only two neighboring pins with different heat fluxes and their variations are quite small. As a result of this, the clad temperature variations can be neglected.

Throughout Fig. [5.3 to 5.7], only two axial levels are shown. This limitation was done because at the axial level above 45 cm., the temperature field of the internal cells have already reached the fully-developed stage. Therefore, two levels are sufficient to illustrate the outcome of the radial power tilting effect.

5.3.b. Maximum Temperature Difference

Since pin 1 has a higher temperature variation as a result of the variation in the heat flux distributions of its neighboring pins, it has the largest maximum clad temperature difference among the pins. At the axial level of $z = 20$ cm., the temperature difference of pin 1 has reached a value as high as 48°C . At that level pin 3 and pin 4, have a maximum clad temperature of about 25°C . But pins 2 and 5, show a very small value for this parameter.

This discovery has proven that in the case of radial power tilting, a large maximum clad temperature difference exists around the center rod. This finding is only possible by a coupled cell analysis.

Fig. 5.8 Maximum Clad Temperature Difference for Various Internal Pins under Radial Power Tilting

5.3.c. Correction Factor for Effective Mixing Length Calculation

In Fig. [5.8], two correction factors for the effective mixing lengths are shown. Each corresponds to a different subchannel gap between two adjacent channels. Curve 1 stands for the gap between subchannel I and J and curve 2 stands for the gap between subchannel J and K. As it was shown in Fig. [5.8], these two curves diverge in an opposite direction as they approach the inlet. Curve 2 which decreases to zero at the inlet resembles the earlier result of the internal and corner cell coupling. But surprisingly, curve 1 goes up to infinity when it approaches the inlet. This result looks quite distinct from the early result presented in Section 3.4.1.3. The explanation for this finding is that due to the allowance of heat transfer across the common boundary of subchannel J and subchannel K. In this calculation, subchannel J is connected to both subchannel I and subchannel K and heat is permitted to be transported across these common boundaries. As the original problem stated, subchannel I has the highest enthalpy and subchannel K has the lowest, heat will flow through the common boundary from subchannel I to subchannel J and also from subchannel J and subchannel . Without letting a portion of heat flow away from subchannel J into subchannel K, the bulk temperature of subchannel J will be higher and the bulk temperature difference between the two subchannels will drop substantially. Therefore, the resultant correction factor L_{ij} will increase from zero to an asymptotic limit. However, by permitting a portion of the heat extracted away from subchannel J, it will lower the bulk temperature in subchannel J. Thus, an

Fig. 5.9 Correction Factor for Effective Mixing Length
for Internal Coupled Cells

increase in the bulk temperature difference between subchannel I and J will be observed. As a result of this, the axial gradient of the bulk temperature difference in this case will become larger than in the case of having an adiabatic condition at the common boundary between subchannel J and K. But for the average heat flux across the gap between subchannel I and J, the thermal condition at the common boundary between subchannel J and K does not influence its value much because this heat flux is a totally localized value. So when the axial gradient of the bulk temperature difference is larger than the axial gradient of the average heat flux at the inlet such as in this case of study, the correction factor for effective mixing length is possible to go up to infinity at $z = 0$ cm. These results have been shown in Fig [5.10 and 5.11].

5.2.d. Nusselt Number

The effect of radial power tilting on the Nusselt number will be analyzed in this section. Before any results will be discussed, the definition of Nusselt number needs to be clarified adequately because it is not clear that under the condition of radial power tilting, which heat flux should be used for the Nusselt number calculation. The answer simply depends upon what Nusselt number should be calculated. The local Nusselt number for each pin is defined as:

$$Nu(r, \phi, z) = \frac{h}{k} = \frac{k}{D_h} \frac{q''(r, \phi, z)}{T_w(r, \phi, z) - T_b} \quad (5.1)$$

Fig. 5.10 Bulk Temperature Difference between Two Adjacent Subchannels
 for Internal Coupled Cell

Fig. 5.11 Average Heat Flux across the Gap for the
 Internal Coupled Cells

where T_w is the wall temperature of the pin, T_b is the bulk temperature of the corresponding subchannel and q'' is the heat flux of the pin. Then, for the average Nusselt number of the subchannel, one can define this parameter as follows:

$$\overline{Nu}(z) = \frac{k}{D_h} \frac{\overline{q''}}{\overline{T_w} - T_b} \quad (5.2)$$

where $\overline{q''}$ is the overall average heat flux of the pins within the subchannel. It can be calculated as follows:

$$\overline{q''} = \frac{\sum_{m=1}^M q''_m A_m}{\sum_m A_m} \quad (5.3)$$

where A_m and q''_m , respectively, are the clad surface area and the heat flux of any pin within the subchannel. The indice, M represents the total number of pins in the subchannel.

But for the average wall temperature, $\overline{T_w}$, it is calculated in a different way as:

$$\overline{T_w} = \frac{\sum_{m=1}^M (T_w)_m l_m}{\sum_m l_m} \quad (5.4)$$

where $(T_w)_m$ and l_m are the wall temperature and the circumferential length of the pin within the subchannel, respectively. The difference between Eq. 5.3 and Eq. 5.4 is that the surface area of the pins is used as the weighting function to find the average heat flux of the pins within the subchannel. But for the average wall temperature of the pin within the subchannel, cir-

cumferential length of the pins is used as the weighting function.

Moreover, under various circumstances, it is also interesting to know the average Nusselt number of each individual pin within the subchannel. It can be defined as:

$$[Nu(z)]_m = \frac{k}{D_h} \frac{q_m''}{(T_w)_m - T_b}$$

where q_m'' and $(T_w)_m$ are the heat flux and the wall temperature of this particular pin, respectively.

In what follows, both the resultant Nusselt number of each individual subchannel and the Nusselt number of each pin in the corresponding subchannel have been obtained to illustrate their differences.

Figs. [5.12, 5.13 and 5.14] show the results of these Nusselt number calculations. Among all subchannels, subchannel I has the highest heat transfer coefficient and subchannel K has the lowest one. This implies that the average Nusselt number of the subchannel is the largest if the overall average heat flux at the pin surface within the subchannel is the largest.

Within each subchannel, the following results can be observed. That is, excluding the central pin, the pin which has the smallest heat flux density within the subchannel will have the highest heat transfer coefficient. On the other hand, the one with the largest heat flux will have the smallest heat transfer coefficient. Therefore, in subchannel I, pin 3

Fig. 5.12 Nusselt Number of Subchannel I for the
Coupled Internal Cell Analysis

Fig. 5.13 Nusselt Number of Subchannel J for the Coupled Internal Cell Analysis

Fig. 5.14 Nusselt Number of Subchannel K for the Coupled Internal Cell Analysis

has the smallest value. In subchannel K, pin 5 has the highest value of the Nusselt number and pin 4 has the smallest value. For the central pin 1, the Nusselt number always seems to be close to the resultant Nusselt number of the subchannel.

5.4. Conclusion

Throughout this chapter, it has been shown that although the internal cell is comparatively symmetric in shape, severe clad temperature variation may occur if a particular radial heat flux distribution is imposed. In the sample problem, the power is tilted in one direction such that the maximum-to-average heat flux ratio is 1.10 while the minimum-to-average heat flux ratio is 0.90. This results in a maximum clad temperature difference of about 45°C for the central pin at the core outlet as compared to about 4°C for uniform radial heating. This severe temperature variation will possibly induce rod bowing which may endanger the reactor core. This discovery can only be found by 3-D coupled internal cell analysis.

Under the radial power tilting effect, the behavior of the correction factor for effective mixing length of the gap between subchannel I and J is quite distinct from the one between subchannel I and J. For the gap between subchannel I and J, the correction factor goes to infinite when approaching the inlet level where $z = 0$ cm. But for the gap between subchannel J and K, the correction factor becomes zero at the inlet level. The explanation of these results have been fully discussed in Section 5.2.c.

Next, the Nusselt numbers of the subchannels, the Nusselt numbers of the fuel pins in their corresponding subchannels have been calculated. Their values are quite different from each other. So, in order to choose the right Nusselt number for the correct answer, it really depends upon what specific problem one wants to tackle. Applying a wrong Nusselt number, obviously, will result in a wrong solution.

CHAPTER 6

CONCLUSION AND RECOMMENDATION

6.1. Conclusion

With the results of the available calculations, the following conclusions can be reached.

(a). Clad Temperature Variation

From the results of the isolated single cell analysis, it has been found that both the corner and the edge cell have severe problems with respect to the clad temperature variations mainly as a result of the high degree of irregularity in the coolant geometries. Furthermore, because of this high degree of irregularity in the coolant geometry, both the corner cell and the the edge cell have a much longer thermal entrance length than the internal cell. This implies that under the present LMFBF conditions, thermally fully-developed temperature fields may hardly be established in these regions, which are the corner cell and the edge cell. As a result, the three dimensional heat transfer analysis demonstrates that the fully-developed heat transfer analysis gives a far too conservative result such as over-predicting the clad temperature variation throughout the reactor-core.

By coupling the corner cell with the internal cell and/or by introducing a radial power tilting such that the radial heat flux ratio becomes larger than 1, results in a further increase of the clad temperature variation of the corner pin. If a cosine axial heat flux distribution is used instead of an

uniform axial heat flux distribution, this will only increase the clad temperature variation of the corner pin near the outlet region. Moreover, one can also deduce that by increasing the axial peaking factor will also result in an increase in the clad temperature variation at the outlet region.

The internal cell which has a higher degree of symmetry with respect to its coolant geometry does not have any problem of severe clad temperature variation under the condition of a flat radial power distribution. However, with a slight radial power tilting such that the maximum-to-minimum heat flux ratio of its neighboring pins goes up from 1.00 to 1.222, severe clad temperature variation must be expected especially for the central pin.

Finally, by decreasing P/D and $2W/D$, one normally will expect that the clad temperature variation of the corner pin will increase throughout all the axial levels of the core. However, this is not necessarily the case under all possible circumstances. The cases that have been studied in Section 4.2 show that the fully-developed value of the clad temperature variation becomes larger when the geometrical parameters, P/D and $2W/D$, become smaller. But within certain portions of the core-height, it has been found that the clad temperature variation decreases as the parameters, P/D and $2W/D$ gets smaller. This surprising result can only be obtained by the 3-D heat transfer analysis because this model thoroughly considers the thermal entrance effect for all the distributed parameters calculation. As what has been discussed in Section 4.2, the

thermal entrance length gets longer as the parameters, P/D and $2W/D$ become smaller. This implies that the thermal entrance effect at any particular axial level, comparatively becomes more important in the case of smaller P/D and $2W/D$. As a result, it is possible to obtain an unexpected result such that the clad temperature variation within certain portions of the core height will be less as P/D and $2W/D$ gets smaller. So the 3-D heat transfer analysis is the only reliable tool for analyzing the clad temperature variation.

(b) Nusselt Number

By analyzing the axial variation of the Nusselt number for both the corner cell and the edge cell, again, one discovers that the result of the fully-developed heat transfer analysis is too conservative under the condition of the present LMFBR design.

The results of the isolated single cell analysis show that among all cells, the corner cell has the smallest Nusselt number. By coupling the corner cell together with the integral cell, this will result in an improved Nusselt number for both cells. However, the effect of the radial power tilting will reduce the heat transfer capability of the cells in a way that the Nusselt number of the combined cell will decrease as the radial heat flux ratio gets larger than 1.

When a cosine function for the non-uniform axial heat flux distribution is imposed, the result shows a substantial decrease of the Nusselt number near the outlet region. This behavior of the Nusselt number at the outlet region has never been incorporated into any of the heat transfer correlations

used in subchannel analyses.

By changing the geometric parameters, the 3-D analysis is the only way to predict the change of the Nusselt number because the thermal entrance effect has a strong influence on the heat transfer capability of the cells throughout the core.

For the case of coupling internal cells under the condition of radial power tilting, the Nusselt number changes. Even in the same subchannel, each pin does have a different Nusselt number. Applying an incorrect Nusselt number will give an incorrect prediction of the film temperature difference of the fuel pin.

(c) Correction Factor for Effective Mixing Length

The correction factor for the effective mixing length L_{ij} is a very important parameter which has to be supplied to subchannel codes such as COBRA-IIIC in order to obtain more accurate solutions for the bulk temperature differences between two adjacent subchannels. It has been found that the radial heat flux ratio, has not much effect on this correction factor L_{ij} . But by changing the axial heat flux distribution from a constant profile to a sinusoidal profile, the correction factor L_{ij} changes its shape. From the results of the geometrical parametric study, it can be concluded that L_{ij} is very sensitive to the geometrical parameters P/D and $2W/D$.

In conclusion, the 3-D coupled cell analysis is a valuable tool to calculate a variety of quantities which can be inputted

into subchannel codes such as COBRA-IIIC, for instance the axial variation of the Nusselt number and also the axial variation of the correction factor for the effective mixing length.

6.2. Recommendation

Throughout these analyses, the effect of the rod displacement has not been discussed. With some small modification, this 3-D coupled cell analysis is capable to handle the problem of a displaced rod. Unlike all other papers in the open literature on the same topic, the result of this 3-D coupled cell analysis will give not only the heat transfer coefficient of the cell with a displaced pin but also will include the effect on its neighboring cells too. This analysis is worth looking into in the future as a possible continuation of this type of work.

It is not a simple matter to reach a solid conclusion at the end of these analyses because there are too many variables involved and not enough number of calculations that have been performed. In order to obtain more valuable data for precise conclusion many more calculations are required.

APPENDIX A

Least Square Method

Basically, the problem is to determine a solution vector of an over-determined linear set of equations by the least square method.

A boundary condition or an initial condition is usually formed as:

$$\sum_{i=1}^N (W_0)_i X_i = A_0 \quad (A.1)$$

Where $(W_0)_i$'s are the coefficients which result from the boundary condition;

and A_0 is the given function at the boundary.

Assume $\vec{X}$ is the vector which contains all unknown X_i .

If the boundary condition, Eq. [A.1] is discretized at K number of points such that $K > N$, the result can be written as

$$\sum_{i=1}^N W_{ji} X_i = A_j \quad (A2.a)$$

for $j = 1, 2, 3, \dots, K$.

In matrix form, this becomes

$$\begin{aligned} [W] \{X\} &= \{A\} \\ \text{or } [W] \vec{X} &= \vec{A} \end{aligned} \quad (A.2b)$$

$[W]$ is a $K \times N$ matrix where $K > N$. $\vec{X}$ and $\vec{A}$ are vectors of dimensions N and K , respectively. Because $K > N$, the set of linear equations is overdetermined and there must exist N linearly independent solutions. From different sets of N independent sets of these K equations, one would get somewhat different solution vectors, $\vec{X}$. In order to make these equations consistent, one uses a factor V_k such that it has the following property:

$$R = \sum_K V_k^2 = \vec{V}' \cdot \vec{V} \Rightarrow \text{minimum value} \quad (\text{A.3})$$

Where R can be considered as the residual. As a result, instead of dealing with the inconsistent set of Eq. [A.2.b], one can use the following equation.

$$W \vec{X} - \vec{A} = \vec{V} \quad (\text{A.4})$$

The condition, Eq. [A.3] leads to the well-known fact that in order to minimize the residual, the following must hold

$$\frac{\partial R}{\partial \vec{X}} = 2(W'W \vec{X} - W'A) = 0$$

$$\text{or } W'W \vec{X} = W'A \quad (\text{A.5})$$

$W'W$ is called Gaussian Transformation.

$$\text{Let } W = W' \cdot W$$

from Eq. [A.5] one gets

$$\vec{X} = W^{-1} \cdot W'A \quad (\text{A.6})$$

In order to obtain this solution form, one has to find the inverse of the big matrix W and this involves complicated calculations which are time-consuming. Instead of using this direct approach, another solution technique has been developed [27,28].

By using the second solution technique, one makes use of the fact that the column vectors can be orthonormalized by applying Schmidt's orthonormalization Procedure. That is

$$U_i = \sum_{j=1}^i p_{i,j} \vec{W}_j \quad (A.7)$$

for $i = 1, 2, 3, \dots, N$

The vectors $\vec{U}_i$ of the matrix $[U]$ form a normalized orthogonal system

$$[U]'[U] = I \quad (A.8)$$

where I is an identity matrix

The coefficients $p_{i,j}$ are the coefficients of the upper triangular matrix $[P]$. Thus, from Eq. [A.7], one gets

$$[U] = [W][P] \quad (A.9)$$

Re-writing Eq. [A.7] into the following form

$$\vec{W}_i = \sum_{j=1}^N q_{i,j} \vec{U}_j = [U] \vec{q}_i \quad (A.10)$$

for $i = 1, 2, \dots, N$

results in

$$[W] = [U][Q] \quad (A.11)$$

By starting with Eq. [A.9], one gets

$$\begin{aligned} [U] &= [W][P] \\ [U][P]^{-1} &= [W][P][P]^{-1} = [W] \end{aligned}$$

$$\text{therefore } [Q] = [P]^{-1} \quad (\text{A.12})$$

Then, the vector $\vec{A}$ in Eq. [A.2] can be represented as a linear combination of the $\vec{U}_i$'s.

$$\vec{A} = \sum_{j=1}^N b_j \vec{U}_j = [U] \vec{b} \quad (\text{A.13})$$

Substituting Eq.s [A.11] and [A.13] into Eq. [A.5], one gets

$$\begin{aligned} [W]'[W]\vec{X} - [W]'\vec{A} &= [Q]'[U]'[U][Q]\vec{X} - [Q]'[U]'\vec{A} \\ &= [Q]'([Q]\vec{X} - [U]'\vec{A}) \\ &= 0 \end{aligned}$$

Thus an equivalent condition as given by Eq. [A.6] has been obtained, namely,

$$[Q]\vec{X} - [U]'\vec{A} = 0 \quad (\text{A.14})$$

From Eq. [A.13] it follows

$$[U]'\vec{A} - [U]'[U]\vec{b} = \vec{b}$$

As a result, Eq. [A.14] becomes

$$\begin{aligned} [Q]\vec{X} &= \vec{b} \\ \text{or } \vec{X} &= [Q]^{-1}\vec{b} \\ &= [P]\vec{b} \end{aligned} \quad (\text{A.15})$$

This solution vector satisfies the Gaussian condition. Solving the equation in the aforementioned way makes it unnecessary to explicitly determine the matrice. Instead, it is sufficient to know $[U]$ and $[P]$ in order to determine $\vec{X}$.

The whole procedure has been implemented into subroutine LLSQ.

APPENDIX B

DUHAMEL'S PRINCIPLE OF SUPERPOSITION [10,29]

A linear partial differential equation as Eqs. [2.7] and [3.11], has the following form:

$$\frac{\partial \theta}{\partial \xi} = \nabla^2 \theta \quad (\text{B.1})$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2}$$

with the non-homogeneous boundary condition

$$\nabla \theta(\rho_0, \phi_0, \xi) = q(\xi) \quad (\text{B.2})$$

$$\text{where } \nabla = \frac{\partial}{\partial \rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi}$$

and homogeneous initial condition

$$\theta(\rho, \phi, 0) = 0 \quad (\text{B.3})$$

where (ρ_0, ϕ_0) are order pairs of numbers describing boundary point locations and (ρ, ϕ) are ordered pairs of numbers describing interior points of the region. The solution of this partial differential equation, by Duhamel's Theorem, can be obtained as :

$$\theta(\rho, \phi, \xi) = \int_0^\xi q(\chi) \frac{\partial \theta_k(\rho, \phi, \xi - \chi)}{\partial \xi} d\chi \quad (\text{B.4})$$

with χ being a dummy variable in axial coordinate and $\theta_k(\rho, \phi, \xi)$ is the solution to the auxiliary problem

$$\frac{\partial \theta_k}{\partial \xi} = \nabla^2 \theta_k \quad (\text{B.5})$$

$$\text{where} \quad \nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2}$$

with the homogeneous conditions

$$\nabla \theta_k(\rho_0, \phi_0, \xi) = 1 \quad (\text{B.6})$$

and

$$\theta_k(\rho, \phi, 0) = 0 \quad (\text{B.7})$$

The proof of this Duhamel's Theorem will be discussed in the latter part of this Appendix B. Now, with the solution of the partial equation is given as Eq. [B.4], the following derivation is to show how this solution (Eq. B.4) transforms into a different form which has the same form as in Eqs. [2.73].

Suppose the solution form of the linear partial differential equation (Eq. B.1], by Duhamel's Theorem, is expressed as:

$$\theta(\rho, \phi, \xi) = \int_0^\xi q(\chi) \frac{\partial \theta_k(\rho, \phi, \xi - \chi)}{\partial \xi} d\chi \quad (\text{B.4})$$

Integrating this solution by parts

$$\int v du = uv - \int u dv$$

$$\text{If } v = q(\chi) \quad (\text{B.8})$$

$$\text{and } du = \frac{\partial \theta_k(\rho, \phi, \xi - \chi)}{\partial \xi} d\chi \quad (\text{B.9})$$

$$\text{then } dv = \frac{dq(\chi)}{d\chi} d\chi \quad (\text{B.10})$$

$$\begin{aligned} \text{and } u &= \int_0^\xi \frac{\partial \theta_k(\rho, \phi, \xi - \chi)}{\partial \xi} d\chi \\ &= \frac{\partial}{\partial \xi} \left[\int_0^\xi \theta_k(\rho, \phi, \xi - \chi) d\chi \right] - \theta_k(\rho, \phi, \xi - \chi) \end{aligned} \quad (\text{B.11})$$

$$\text{Let } F(\xi) = \frac{\partial}{\partial \xi} \left[\int_0^\xi \theta_k(\rho, \phi, \xi - \chi) d\chi \right] - \theta_k(\rho, \phi, \xi - \chi) \quad (\text{B.12})$$

$$\text{thus } u = F(\xi) - \theta_k(\rho, \phi, \xi - \chi) \quad (\text{B.13})$$

$$\begin{aligned} \text{therefore } \theta(\rho, \phi, \xi) &= [F(\xi) q(\chi) - \theta_k(\rho, \phi, \xi - \chi) q(\chi)]_{\chi=0}^\xi \\ &\quad - \int_0^\xi F(\xi) \frac{dq(\chi)}{d\chi} d\chi + \int_0^\xi \theta_k(\rho, \phi, \xi - \chi) \frac{dq(\chi)}{d\chi} d\chi \\ &= F(\xi) q(\xi) - \theta_k(\rho, \phi, 0) q(\xi) - F(\xi) q(0) \\ &\quad + \theta_k(\rho, \phi, \xi) q(0) - F(\xi) q(\xi) \\ &\quad + F(\xi) q(0) + \int_0^\xi \theta_k(\rho, \phi, \xi - \chi) \frac{dq(\chi)}{d\chi} d\chi \end{aligned} \quad (\text{B.14})$$

But the initial condition for the auxiliary problem states

$$\theta_k(\rho, \phi, 0) = 0 \quad (\text{B.7})$$

Therefore

$$\begin{aligned} \theta(\rho, \phi, \xi) = & \theta_k(\rho, \phi, \xi) q(0) \\ & + \int_0^\xi \theta_k(\rho, \phi, \xi - \chi) \frac{dq(\chi)}{d\chi} d\chi \end{aligned} \quad (\text{B.15})$$

This form of solution is known as Rouse's form of Duhamel's Theorem.

Next, the proof of the Duhamel's Theorem will be discussed. This proof shows how the solution for Eq. [B.1] with the conditions (Eqs. B.2 and B.3) can be obtained as the form of Eq. [B.4].

Proof of the Duhamel's Theorem

[10, 29, 30, 31]

Consider a boundary value problem of the form

$$\frac{\partial \theta_1}{\partial \xi} = \nabla^2 \theta_1 \quad (\text{B.16})$$

with the boundary condition

$$\nabla \theta_1(\rho_0, \phi_0, \chi) = \begin{cases} 0, & 0 \leq \chi \leq \chi_{i-1} \\ 1, & \chi_{i-1} < \chi \leq \xi \end{cases} \quad (\text{B.17})$$

(See Fig. B.1)

and initial condition

$$\theta_1(\rho, \phi, 0) = 0 \quad (\text{B.18})$$

where χ is a variable in ξ such that $0 \leq \chi \leq \xi$

FIG. B.1

From the auxiliary problem the solution is

$$\theta_1(\rho, \phi, \xi) = \begin{cases} 0 & 0 < x \leq x_{i-1} \\ Q(\rho, \phi, \xi - x_{i-1}) & x_{i-1} < x \leq \xi \end{cases} \quad (\text{B.19})$$

Consider the linear differential equation

$$\frac{\partial^2 \theta}{\partial \xi^2} = \nabla^2 \theta_2 \quad (\text{B.20})$$

with the conditions

$$\nabla \theta_2(\rho_o, \phi_o, x) = \begin{cases} 0 & 0 < x \leq x_1 \\ 1 & x_1 < x \leq \xi \end{cases} \quad (\text{B.21})$$

(See Fig. B.2)

and

$$\theta_2(\rho, \phi, 0) = 0 \quad (\text{B.22})$$

for which the solution is

$$\theta_2(\rho, \phi, \xi) = \begin{cases} 0 & 0 < x \leq x_1 \\ Q(\rho, \phi, \xi - x_1) & x_1 < x \leq \xi \end{cases} \quad (\text{B.23})$$

FIG. B.2

Now consider a boundary value problem of the form

$$\frac{\partial \theta_3}{\partial \xi} = \nabla^2 \theta_3 \quad (\text{B.14})$$

with the conditions

$$\nabla \theta_3(\rho_0, \phi_0, \xi) = \begin{cases} 0, & 0 < x \leq x_{i-1} \\ 1, & x_{i-1} < x \leq x_i \\ 0, & x_i < x \leq \xi \end{cases} \quad (\text{B.25})$$

(See Fig. A.3)

$$\text{and } \theta_3(\rho, \phi, 0) = 0 \quad (\text{B.26})$$

FIG. B.3

Since the differential equation for the $\theta_3(\rho, \phi, \xi)$ problem is linear the solution is obtained by the superposition of the $\theta_1(\rho, \phi, \xi)$ and $\theta_2(\rho, \phi, \xi)$ problem solutions. The solution of the $\theta_3(\rho, \phi, \xi)$ problem then is

$$\theta_3(\rho, \phi, \xi) = \begin{cases} 0 & 0 < \chi \leq \chi_{i-1} \\ \theta_1(\rho, \phi, \xi - \chi_{i-1}) & \chi_{i-1} < \chi \leq \chi_i \\ \theta_1(\rho, \phi, \xi - \chi_{i-1}) - \theta_2(\rho, \phi, \xi - \chi_i) & \chi_i < \chi \leq \xi \end{cases}$$

Therefore after the "unit pulse" at $\chi_{i-1} \leq \chi \leq \chi_i$ the solution for $\xi > \chi_i$ is

$$\theta_3(\rho, \phi, \xi) = Q(\rho, \phi, \xi - \chi_{i-1}) - Q(\rho, \phi, \xi - \chi_i) \quad (\text{B.28})$$

The more general boundary value problem

$$\frac{\partial \theta_{pi}}{\partial \xi} = \nabla^2 \theta_{pi} \quad (\text{B.29})$$

with the conditions

$$\nabla \theta_{pi}(\rho_o, \phi_o, \xi) = \begin{cases} 0 & 0 < \chi \leq \chi_{i-1} \\ q_i(\chi_i) & \chi_{i-1} < \chi \leq \chi_i \\ 0 & \chi_i < \chi \leq \xi \end{cases} \quad (\text{B.30})$$

(See Fig. B.4)

FIG. B.4

The solution is obtained by multiplying the $\theta_3(\rho_0, \phi_0, \xi)$ solution by $q_i(x_i)$. Thus the solution to the $\theta_{pi}(\rho, \phi, \xi)$ is found to be

$$\theta_{pi}(\rho, \phi, \xi) = \begin{cases} 0, & 0 < x \leq x_{i-1} \\ q_i(x_i)[Q(\rho, \phi, \xi - x_{i-1}) - Q(\rho, \phi, \xi - x_i)] & x_{i-1} < x < \xi \end{cases} \quad (B.31)$$

The $\theta_p(\rho, \phi, \xi)$ solution is for a constant pulse of magnitude $q_i(x_i)$ lasting from x_{i-1} to x_i .

Now consider a boundary value problem which has a boundary condition consisting of a series of such pulses. The interval $0 \leq x \leq \xi$ is divided into number of points $x_0, x_1, x_2, \dots, x_n$ where $x_0 = 0$ and $x_n = \xi$. Let x_i denote the interval $x_{i-1} - x_i$ and the magnitude of the pulse in the interval $x_{i-1} < x \leq x_i$ is given by $q_i(x_i)$.

Define the function $q_i(x)$ as

$$q_i(x) = \begin{cases} q_i(x_i) & x_{i-1} < x \leq x_i \\ 0 & \text{elsewhere} \end{cases} \quad (B.32)$$

and form the same function as

$$q_s(x) = \sum_{i=1}^n q_i(x) \quad (B.33)$$

(See Fig. B.5)

FIGURE B.5

Now consider the problem

$$\frac{\partial \theta_s}{\partial \xi} = \nabla^2 \theta_s \quad (B.34)$$

$$\nabla \theta_s(\rho_0, \phi_0, x) = q_s(x) \quad 0 < x \leq \xi \quad (B.35)$$

and

$$\theta_s(\rho, \phi, 0) = 0 \quad (B.36)$$

The solution for the $\theta_s(\rho, \phi, \xi)$ problem is obtained by superposing the solutions for each of the pulse-intervals $x_{i-1} < x < x_i$

The solution for $\theta_s(\rho, \phi, \xi)$ then is

$$\theta_s(\rho, \phi, \xi) = \sum_{i=1}^n \theta_{pi}(\rho, \phi, \xi)$$

or

$$\theta_S(\rho, \phi, \xi) = \sum_{i=1}^n q_i(\chi_i) [Q(\rho, \phi, \xi - \chi_{i-1}) - Q(\rho, \phi, \xi - \chi_i)] \quad (\text{B.37})$$

The $\theta_S(\rho, \phi, \xi)$ solution is the solution for a "staircase" boundary condition $q_S(\chi)$. The solution for a problem involving general axial varying boundary conditions is obtained by partitioning the interval $0 < \chi < \xi$ such that $n \rightarrow \infty$, while $\Delta\chi_i \rightarrow 0$ for all $i=1, 2, 3, \dots, n$, and the values of $q_i(\chi_i)$ are chosen such that $q_i(\chi_i) = q(\chi_i)$ where $q(\chi)$ is a continuous function of χ . $q(\chi)$ does not have to be a continuous function but may have only a finite number of ordinary discontinuities.

It follows that

$$\theta(\rho, \phi, \xi) = \lim_{\substack{n \rightarrow \infty \\ \Delta\chi_i \rightarrow 0}} \theta_S(\rho, \phi, \xi) \quad (\text{B.38})$$

is the solution to the problem when $q(\chi)$, $0 \leq \chi \leq \xi$, replaces $q_S(\chi)$ as the axial-varying boundary condition. To evaluate $\theta(\rho, \phi, \xi)$, consider the form of the limit

$$\theta(\rho, \phi, \xi) = \lim_{\substack{n \rightarrow \infty \\ \Delta\chi_i \rightarrow 0}} \theta_S(\rho, \phi, \xi)$$

or

$$\theta(\rho, \phi, \xi) = \lim_{\substack{n \rightarrow \infty \\ \Delta x_i \rightarrow 0}} \sum_{i=1}^n q(x_i) \frac{Q(\rho, \phi, \xi - x_{i-1}) - Q(\rho, \phi, \xi - x_i)}{(\xi - x_{i-1}) - (\xi - x_i)} \Delta x_i$$

(B.39)

By the definition of the partial derivative

$$\lim_{\Delta x_i \rightarrow 0} \frac{Q(\rho, \phi, \xi - x_{i-1}) - Q(\rho, \phi, \xi - x_i)}{(\xi - x_{i-1}) - (\xi - x_i)} = \left. \frac{\partial Q(\rho, \phi, \xi - x)}{\partial (\xi - x)} \right|_{x=x_i}$$

(B.40)

for any particular value of x , the derivative is evaluated as

$$\frac{\partial Q(\rho, \phi, \xi - x)}{\partial (\xi - x)} = \frac{\partial Q(\rho, \phi, \xi - x)}{\partial \xi} \quad (\text{B.41})$$

Then the solution for $\theta(\rho, \phi, \xi)$ is

$$\theta(\rho, \phi, \xi) = \lim_{n \rightarrow \infty} \sum_{i=1}^n q(x_i) \frac{\partial Q(\rho, \phi, \xi - x)}{\partial \xi} \Delta x_i \quad (\text{B.42})$$

If the product $Q(x_i) \cdot \frac{\partial Q(\rho, \phi, \xi - x)}{\partial \xi}$ is piecewise continuous, the limit converges to

$$\theta(\rho, \phi, \xi) = \int_0^\xi q(x) \frac{\partial Q(\rho, \phi, \xi - x)}{\partial \xi} dx \quad (\text{B.43})$$

which proves Duhamel's theorem.

APPENDIX C
GAUSSIAN QUADRATURE

(GAUSS-LEGENDRE QUADRATURE)

In order to estimate the value of the integral $\int_a^b f(x)dx$, one can approximate the function $f(x)$ with an n th-degree interpolating polynomial $p_n(x)$ and integrate as follows:

$$\int_a^b f(x)dx = \int_a^b p_n(x)dx + \int_a^b R_n(x)dx \quad (C.1)$$

Here, $R_n(x)$ is the error term for the n th-degree interpolating polynomial. If the Lagrangian form of the interpolating polynomial is used, it becomes:

$$\begin{aligned} f(x) &= p_n(x) + R_n(x) \\ &= \sum_{i=0}^n L_i(x)f(x_i) + \prod_{i=0}^n (x-x_i) \frac{f^{(n+1)}(\xi)}{(n+1)!} \end{aligned} \quad (C.2)$$

$a < \xi < b$

where

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \left(\frac{x-x_j}{x_i-x_j} \right)$$

To simplify the problem, the interval of integration will be changed from $[a,b]$ to $[-1,1]$ by a suitable transformation of variables. Then the new variable z will be defined by

$$z = \frac{2x - (a+b)}{b-a}$$

Similarly, the new function will be defined as

$$F(z) = f(x) = f\left(\frac{(b-a)z + (a+b)}{2}\right) \quad (C.4)$$

As a result, Eq. [C.2] becomes

$$F(z) = \sum_{i=0}^n L_i(z) F(z_i) + \left[\sum_{i=0}^n (z-z_i) \right] \frac{F^{(n+1)}(\bar{\xi})}{(n+1)!} \quad (C.5)$$

where

$$L_i(z) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{(z-z_j)}{(z_i-z_j)} \quad \text{and} \quad -1 < \bar{\xi} < 1.$$

Here, z_i is simply the base-point value x_i transformed by Eq. [C.3]. Now, if $f(x)$ is assumed to be a polynomial of degree $2n+1$, then the term $F^{(n+1)}(\bar{\xi})/(n+1)!$ must be a polynomial of degree n , since $\sum_{i=0}^n L_i(z) F(z_i)$ is a polynomial of degree n at most, and $\prod_{i=1}^n (z-z_i)$ is a polynomial of degree $n+1$. Let

$$\frac{F^{(n+1)}(\bar{\xi})}{(n+1)!} = q_n(z)$$

where $q_n(z)$ is a polynomial of degree n . Then

$$F(z) = \sum_{i=0}^n L_i(z) F(z_i) + \left[\sum_{i=0}^n (z-z_i) \right] q_n(z) \quad (C.6)$$

By integrating both sides of Eq. [C.6] between the integration limits -1 and 1 , it follows that

$$\int_{-1}^1 F(z) dz = \int_{-1}^1 \sum_{i=0}^n L_i(z) F(z_i) dz + \int_{-1}^1 \left(\prod_{i=0}^n (z - z_i) \right) q_n(z) dz \quad (C.7)$$

Since the $F(z_i)$ are fixed values, the integral can be simplified as

$$\begin{aligned} \int_{-1}^1 F(z) dz &= \sum_{i=0}^n F(z_i) \int_{-1}^1 L_i(z) dz \\ &= \sum_{i=0}^n W_i F(z_i) \end{aligned} \quad (C.8)$$

where

$$W_i = \int_{-1}^1 L_i(z) dz = \int_{-1}^1 \prod_{\substack{j=0 \\ j \neq i}}^n \left[\frac{z - z_j}{z_i - z_j} \right] dz \quad (C.9)$$

One has to select z_i in such a way that the second integral on the right hand side of Eq. [C.7], i.e.,

$$\int_{-1}^1 \sum_{i=0}^n \left(\prod_{i=0}^n (z - z_i) \right) q_n(z) dz \quad (C.10)$$

vanishes in order to have a perfectly working procedure, i.e.,

Eq. [C.8]. The orthogonality property of the Legendre polynomials will be used to establish such values as z_i . By expanding the two polynomials $q_n(z)$ and $\prod_{i=0}^n (z-z_i)$ in terms of the Legendre polynomials, one obtains:

$$\begin{aligned} \prod_{i=0}^n (z-z_i) &= b_0 P_0(z) + b_1 P_1(z) + \dots + b_n P_n(z) + b_{n+1} P_{n+1}(z) \\ &= \sum_{i=0}^{n+1} b_i P_i(z) \end{aligned} \quad (C.11)$$

$$\text{and } q_n(z) = c_0 P_0(z) + c_1 P_1(z) + \dots + c_n P_n(z)$$

$$= \sum_{i=0}^n c_i P_i(z) \quad (C.12)$$

The product $q_n(z) \cdot \prod_{i=0}^n (z-z_i)$ becomes

$$\sum_{i=0}^n \sum_{j=0}^n b_i c_j P_i(z) P_j(z) + b_{n+1} \sum_{i=0}^n c_i P_i(z) P_{n+1}(z). \quad (C.13)$$

The integral of Eq. [C.10] will assume the form

$$\int_{-1}^1 \left[\sum_{i=0}^n \sum_{j=0}^n b_i c_j P_i(z) P_j(z) + b_{n+1} \sum_{i=0}^n c_i P_i(z) P_{n+1}(z) \right] dz. \quad (C.14)$$

Because of the orthogonality properties of the Legendre polynom-

ials, all terms of this integral that are of the form

$$b_i c_j \int_{-1}^1 P_i(z) P_j(z) dz \quad i \neq j \quad (C.15)$$

will vanish. Thus the error term (Eq. C.10) for the quadrature formula of Eq. [C.8] may be written:

$$\begin{aligned} \int_{-1}^1 \left[\prod_{i=0}^n (z - z_i) \right] q_n(z) dz &= \int_{-1}^1 \sum_{i=0}^n b_i c_i [P_i(z)]^2 dz \\ &= \sum_{i=0}^n b_i c_i \int_{-1}^1 [P_i(z)]^2 dz. \end{aligned} \quad (C.16)$$

One way to make this expression vanish is to specify that the first $n+1$ of the b_i , $i = 0, 1, \dots, n$ are zero. The coefficient b_{n+1} of $P_{n+1}(z)$ is still unspecified, but from Eq. [C.12] it must be given by:

$$\prod_{i=0}^n (z - z_i) = b_{n+1} P_{n+1}(z) \quad (C.17)$$

The important feature of this equation is that the polynomial

$\prod_{i=0}^n (z - z_i)$ is already in factored form, that is, it has the $n+1$ roots z_i , $i = 0, 1, 2, \dots, n$. Since $b_{n+1} P_{n+1}(z)$ is the same polynomial, the z_i must be the roots of $b_{n+1} P_{n+1}(z)$ as well, or equivalently, of $P_{n+1}(z)$. Thus the $n+1$ base points to be used

in the integration formula of Eq. [C.8] are the $n+1$ roots of the appropriate $[(n+1)\text{th-degree}]$ Legendre polynomial. The relative weight assigned to each functional value $F(z_i)$ is given by Eq. [C.9]. Values of the appropriate base points (roots) and weight factors are shown in Table C.1.

Table C.1 Roots of the Legendre Polynomials $P_{n+1}(z)$ and the Weight Factors for the Gauss-Legendre Quadrature

$\int_{-1}^1 F(z) dz \doteq \sum_{i=0}^n w_i F(z_i)$			
Roots (z_i)	Weight Factors (w_i)		
<i>Two-Point Formula</i> $n = 1$			
$\pm 0.57735\ 02691\ 89626$	1.00000 00000 00000		
<i>Three-Point Formula</i> $n = 2$			
0.00000 00000 00000	0.88888 88888 88889		
$\pm 0.77459\ 66692\ 41483$	0.55555 55555 55556		
<i>Four-Point Formula</i> $n = 3$			
$\pm 0.33998\ 10435\ 84856$	0.65214 51548 62546		
$\pm 0.86113\ 63115\ 94053$	0.34785 48451 37454		
<i>Five-point Formula</i> $n = 4$			
0.00000 00000 00000	0.56888 88888 88889		
$\pm 0.53846\ 93101\ 05683$	0.47862 86704 99366		
$\pm 0.90617\ 98459\ 38664$	0.23692 68850 56189		
<i>Six-Point Formula</i> $n = 5$			
$\pm 0.23861\ 91860\ 83197$	0.46791 39345 72691		
$\pm 0.66120\ 93864\ 66265$	0.36076 15730 48139		
$\pm 0.93246\ 95142\ 03152$	0.17132 44923 79170		
<i>Ten-Point Formula</i> $n = 9$			
$\pm 0.14887\ 43389\ 81631$	0.29552 42247 14753		
$\pm 0.43339\ 53941\ 29247$	0.26926 67193 09996		
$\pm 0.67940\ 95682\ 99024$	0.21908 63625 15982		
$\pm 0.86506\ 33666\ 88985$	0.14945 13491 50581		
$\pm 0.97390\ 65285\ 17172$	0.06667 13443 08688		
<i>Fifteen-Point Formula</i> $n = 14$			
0.00000 00000 00000	0.20257 82419 25561		
$\pm 0.20119\ 40939\ 97435$	0.19843 14853 27111		
$\pm 0.39415\ 13470\ 77563$	0.18616 10001 15562		
$\pm 0.57097\ 21726\ 08539$	0.16626 92058 16994		
$\pm 0.72441\ 77313\ 60170$	0.13957 06779 26154		
$\pm 0.84820\ 65834\ 10427$	0.10715 92204 67172		
$\pm 0.93727\ 33924\ 00706$	0.07036 60474 88108		
$\pm 0.98799\ 25180\ 20485$	0.03075 32419 96117		

APPENDIX D

EIGENVALUE SEARCHING PROCEDURE

It was mentioned in Section 2.3.2.2 that when the thermal entrance solution (Eq. 2.50) is substituted into the boundary condition (Eq. 2.23), a set of M homogeneous linear equations is formed. In matrix form, this can be written as:

$$[W_{kn}] \{F_n\} = 0 \quad (D.1)$$

By multiplying both sides by the transpose of $[W_{kn}]$, this becomes:

$$[W_{nn}] \{F_n\} = 0 \quad (D.2)$$

where $[W_{nn}] = [W_{kn}]^t [W_{kn}]$.

for the vector solution $\{F_n\}$ to exist, the following condition must be satisfied:

$$\det |W_{nn}| = 0 \quad (D.3)$$

Because W_{nn} is function of variable, β , the determinant of the coefficient matrix W will vanish at some particular values of β 's which are known as eigenvalues. A graph showing the value of the determinant of the coefficient matrix W as function of β is given in Fig. [D.1] for the case of a coupled cell analysis with $P/D = 1.3$.

FIGURE D.1. Value of the Determinant of the Coefficient Matrix W as Function of β , for Coupled Cell Analysis with $P/D = 1.3$, $2W/D = 1.38$

To determine the eigenvalue β 's, the subroutine SHERIN has been developed. The method which has been applied is by allocating three points in the domain of β and searching for the minimum value of the determinants. The flow chart for the eigenvalue searching scheme is given in Fig. [D.2].

In order to obtain a reliable solution for this 3-D heat transfer analysis, one needs to get these eigenvalues β 's with an accuracy of 10^{-5} . Therefore, as shown in Fig. [D.2], the conditional block that determines which determinant (that is d_1 or d_2 or d_3 or d_4 or d_5) has the minimum value becomes very important for the searching of an accurate eigenvalue. If the required accuracy is small as 10^{-5} , it is impossible to obtain an accurate eigenvalue by running the program in IBM machine with single precision. The problem arises because the determinant calculated is incorrect due to the floating-error in the computer machine. As a result, the determinants are inaccurately compared, wrong prediction of eigenvalue will be obtained. By changing the computer program into double precision, this problem has been solved.

FIGURE D.2 OUTLINE OF EIGENVALUE SEARCHING

REFERENCES

1. France, D.M.; Ginsberg, T.: "Evaluation of Lumped Parameter Heat Transfer Techniques for Nuclear Reactor Applications" Nucl. Sci. Eng. 51. (1973), 41-51.
2. Carelli, M.D.: "FATHOM-360," Presentation at LMFBR Rod Bundle T & H Working Group Meeting, Pittsburg, Dec. 1975.
3. Wolf, L; Johannsen, K.: "Two-Dimensional Multiregion Analysis of Temperature Fields in Finite Rod Bundles Cooled by Liquid Metals." In: Proc. First Int. Conf. on Structural Mechanics in Reactor Technology (SMIRT), Berlin, Germany, 1971, Vol. 6, 57-64, also TUBIK-23.
4. Wolf, L; Johannsen, K.: "Theoretical Prediction of the Effect of Rod Displacement on Heat Transfer in Unbaffled LMFBR Fuel Assemblies". Atomwirtschaft 1971, 193-194.
5. Hsu, C.J.: "Effect of Rod Displacement on Heat Transfer in Slug Flow Unbaffled Rod Bundles." Liquid Metal Heat Transfer and Fluid Dynamics, ASME, N.Y., 1970, 58-68.
6. Hsu, C.J.: "Laminar-and Slug-Flow Heat Transfer Characteristic of Fuel Rods Adjacent to Fuel Subassembly Walls". Nuc. Sci. Engng. 49 (1972), 398-404.
7. Hsu, C.J.: "The Effects of Lateral Rod Displacement on Laminar - Flow Heat Transfer", J. Heat Transfer 94 (1972), 169-173.
8. Yeung, M.K.: "Two-Dimensional Multi-cells Heat Transfer Analysis in Rod Bundles", Ph.D. Thesis, Dept. Nucl. Engng. MIT, to be published.
9. Meier, J.: "An Analytical Solution for the Temperature Fields in Liquid-Metal Cooled Fuel Pin Bundles with Arbitrary Heat Flux Profile," (in German) Ph.D. Thesis, Institute fur Kerntechnik, Berlin, Germany, TUBIK-37, 1974.
10. Hepworth, H.K.: "Duhamel's Theorem for Two-dimensional Unsteady Laminar Incompressible Fluid Flow Problems with Regular and Non-regular Boundaries." Ph.D. Thesis, Arizona State University, Tempe, Arizona, 1970.
11. Ojalvo, I.V.; Linzer, F.D.: "Improved Point-matching Techniques". Quart. J. of Mech. and App. Math. 18 (1965), 41-56.
12. Davis, P.J.; Rabinowitz, P.: "Advances in Orthonormalizing Computation". In: Advances in Computers (Alt, F.L., ed.), Vol. 2, Academic Press, N.Y., 1961, 56.

13. Nijsing, R.; Eifler, W.: "Axial Development of Temperature Fields in Hexagonal Fast Reactor Fuel Rod Assemblies with Liquid-Metal Cooling." In: Proc. Int. Meeting on Reactor Heat Transfer, Karlsruhe, Germany, 1973, 252-280.
14. Nijsing, R.; Eifler, W.: "Analysis of Liquid Metal Heat Transfer in Assemblies of Closely Spaced Fuel Rods." Nucl. Eng. Des. 10 (1969), 21.
15. Malang, et al: "Procedure for the calculation of the Thermal Entrance Length in Bare Fuel Pin Bundles" (In German). Atomwirtschaft 18 (1973) 247.
16. Vonka, V.; Boonstra, B.H.: "Calculated Heat Transfer Development in Bundles." Nucl. Engng. Des. 31(1974), 337-345.
17. Ramm, H.; Johannsen, K.: "Prediction of Local and Integral Turbulent Transport Properties from Liquid-Metal Heat Transfer in Equilateral Triangular Rod Arrays." J. Heat Transfer, August 1975.
18. Ramm, H.; Johannsen, K.: "A Phenomenological Turbulence Model and its application to Heat Transfer in Infinite Rod Arrays with Axial Turbulent Flow." J. Heat Transfer, August, 1975.
19. Ramm, H.; Johannsen, K.; and Todreas, N.E.; "Single Phase Transport Within Bare Rod Arrays at Laminar, Transition and Turbulent Flow Conditions." Nucl. Engng. and Des. Vol. 30, 1974. P. 186-204.
20. Dutton, J.C.; Welty, J.R.: "An Experimental Study of Low Prandtl Number Natural Convection in an Array of Uniformly Heat Vertical Cylinders."
21. Subbotin, V.I. et al.; "Experimental Research on Model Temperature Conditions of Operations of Fuel Elements of BOR-60 Reactor." AEC-tr-7221, 1971.
22. Möller, Tschöke, H., "Experimental Determination of Temperature Fields in Sodium-Cooled Pin Bundles," (In German), Nucl. Conf., Dusseldorf, Germany, 1976.
23. Reichardt., Schlichting, "Turbulent Flow Through Pipes--Boundary Layer Theory" McGraw Hill, p. 568.
24. Hennecke, K.: "Heat Transfer by Hager-Poiseuille Flow in the Thermal Development Region with Axial Conduction." Wärme - u. Stoffübertragung 1, 177-184(1968).
25. Schneider, P.J.: "Effect of Axial Fluid Conduction on Heat Transfer in the Entrance Region of Parallel Plates and Tubes." Trans. ASME, Ser. C, 79, 765-723, (1957).

26. Slagter, W., Roodbergen, H.A.,: "Numerical Calculation of Temperature Fields in Rod Assemblies with Liquid Metal Cooling." RCN (Reactor Centrum Nederland) -227, Netherland, July, 1975.
27. Lawson C., Hanson, R., "Solving Least Squares Problems" Prentice-Hall, Inc., N.J., 1974.
28. IBM System/360 Scientific Subroutine Package, Version III, Programmer's Manual (H20-0205).
29. Von Karman, T., Biot, M.A.: "Mathematical Methods in Engineering." McGraw-Hill, New York, 1940.
30. Kays, W.M., "Convection Heat and Mass Transfer," McGraw-Hill, New York, 1966.
31. Rouse, Hunter, "Advanced Mechanics of Fluids," Wiley, 1959.
32. Carnahan, Luther, Wilkes, "Applied Numerical Method, " John Wiley & Son, Inc., 1969.
33. Dwyer, O.Z., Berry H., Hlavac P., "Heat Transfer to Liquid Metals Flowing Turbulently Through Closely Spaced Rod Bundles, Part II", Nucl. Eng. Design 23, 295 (1972).
34. Borishanskii, V.M., Grotovsky, M.A. and Firsova, E.V., "Heat Transfer for Liquid Metal Flowing Longitudinally in Wetted Bundles of Rods," Sov. At. Energy 27, 549 (1969).
35. Graber H. and Rieger M., "Experimental Study of Heat Transfer to Liquid Metals Flowing In-Line Through Tube Bundles", Progress in Heat and Mass Transfer, 7, 151, (1973).
36. West M., "Heat Transfer Correlation Recommendation for FFTF", WARD-FRT-60, July, 1969.
37. Schad, H.O., "Heat Transfer Correlation for LMFBR Design," ARD-TH-426, Nov. 1969.
38. Rowe, D.S., "COBRA-III-C - A Digital Computer Program for Steady State and transient Thermal-Hydraulic Analysis of Rod Bundle Nuclear Fuel Elements.", BNWL-1695, Mar. 1973.
39. Yeung, M., Wolf, L., "Effective Conduction Mixing Lengths for Subchannel Analysis of Finite Hexagonal LMFBR Bundles," ANS Meeting, New York, June, 1977, P-463-464.
40. Wolf, L., Wong, C., Yeung, M.K., Kazimi, R, Kim, J.T., "2-D Thermoelastic Analysis of LMFBR Fuel Rod Claddings," Paper C4/d, 4th International Cong. Structural Mechanics in Reactor Technology, San Francisco, August, 1977.