High-Spin Gamma-Ray Spectroscopy

R.M. Diamond

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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R. M. Diamond

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, U.S.A.

ABSTRACT

Nuclei can carry angular momentum by single-particle alignments and by collective motion, as has been well illustrated in discrete-line spectroscopy. From continuum γ-ray studies in still higher spin regions, it appears that these modes both continue. In favorable cases in rare-earth nuclei, particle alignments from the valence shell separate from proton alignments from the next higher shell. A new generation of Compton-suppressed Ge detector arrays will greatly enhance high-spin studies, both continuum and discrete-line.

In recent years the possibility to study nuclei at very high angular momenta has opened up a new dimension along which nuclear properties may vary. Large rotation-induced centrifugal and Coriolis forces are brought into play, and large changes in nuclear structure, pairing, deformation, and shell effects may occur. Nuclei can carry angular momentum mainly in two ways: by collective motion of the whole system, and by single-particle alignment. For quadrupole-deformed nuclei, collective rotation is the lowest-lying mode of excitation, and an example is the ground-band level scheme for $^{238}\text{U}$ shown on the left in fig. 1. This follows surprisingly well the expressions for the energy levels and transitions of a rigid rotor,

$$E = \frac{\hbar^2}{2J} I(I+1)$$  \hspace{1cm} (1.a)

$$E_\gamma = \frac{\hbar^2}{2J} (4I - 2)$$  \hspace{1cm} (1.b)

where $J$ is the moment of inertia. Probably even more significant are the very large (100 - 200 s.p.u.) electric quadrupole transition...
Fig. 1. Level schemes for $^{238}$U and $^{212}$Rn, illustrating collective rotation and aligned-particle motion, respectively (ref. 1,2).

Fig. 2. Ratio of experimental to rigid-rotor matrix elements for the ground band of $^{248}$Cm (ref. 3).
matrix elements shown by the ground-band transitions of a well deformed nucleus, fig. 2, and the fact that they yield an approximately constant quadrupole moment. For nuclei near closed shells, however, deformation is small and excitation of shell-model states occurs; in such cases angular momentum is carried by the alignment of the spin of individual particles along the rotation axis. An example, $^{212}$Rn, is given on the right in fig. 1, and a variety of transition multipolarities are involved with no simple regularity to their energies and transition probabilities.\textsuperscript{2)}

Many, if not most, nuclei, however, take up angular momentum in both modes, and it is the interplay between collective and single-particle motions that leads to the great variety of behavior possible for nuclei at high spin. Consider, for example, the moment of inertia as defined by eq. (1b). If the nucleus were rigid, $\mathcal{J}$ would be a constant. But a nucleus is not rigid; the nucleons move throughout the nuclear volume and values of $\mathcal{J}$ for a deformed nucleus at low spin are smaller than rigid-body values by factors of 2-3. This reduction is due to correlations in the nucleonic motion, particularly to the pairing interactions. With an increase in angular momentum the pairing correlations are reduced by the increasing Coriolis interaction, and so the moment of inertia increases with increasing spin or rotational frequency, $\hbar \omega \sim E_r/2$. This is illustrated by the start of the curve in the upper part of fig. 3. But since the Coriolis force is also proportional to the $j$ of the particle, it acts most strongly on the high-$j$ nucleons in a shell. Thus, on such a plot, the gradually increasing slope is interrupted occasionally by a sharp increase where the nucleus finds it energetically more favorable to align a pair of high-$j$ particles rather completely\textsuperscript{5,6)} while essentially keeping the pairing correlations among the lower-$j$ nucleons. This process corresponds to a bandcrossing between the first band and an excited band with two of the particles aligned and a larger effective moment of inertia. Above the crossing the more highly aligned band becomes the yrast one, and the gain in aligned spin allows the nucleus as a whole to
Fig. 3. Plots of (top) twice the moment of inertia, (middle) the total spin, (bottom) the aligned spin vs. the rotational frequency $\hbar \omega = E_Y/2$, for the yrast sequence in $^{158}\text{Er}$ (ref. 4).

slow down, to decrease its collective rotation. In many rare-earth nuclei it is the alignment of a pair of $i_{13/2}$ neutrons that causes the first sudden jump in the value of $\mathcal{Q}$ at $\hbar \omega \sim 0.25$ MeV, as shown by $^{158}\text{Er}$ in fig. 3.

Experimentally, the $\gamma$-ray transition energies at a bandcrossing may show a decrease, rather than the smooth increase with spin expected from Eq. (1.b), and this phenomenon has been called "backbending." Some nuclei in this region show a second smaller irregularity at $\hbar \omega \sim 0.4$ MeV when a pair of $h_{11/2}$ protons align to make a four-particle band yrast. At spin $38h$, $^{158}\text{Er}$ may be on the
verge of a third such discontinuity, but with the detectors available until now the intensity of the discrete lines fades at this point.

Part of the reason for this behavior is illustrated in fig. 4, a schematic drawing of the decay modes of a compound nucleus $^{160\text{Er}}$ produced in the reaction shown. The nuclei initially formed with $-67$ MeV of excitation energy have a rounded triangular-like spin distribution and so are concentrated at high spins near 50-60 h, although individual nuclei range from 0 to 65 h. At spins above where the fission barrier approximates a neutron binding energy the nucleus fissions, but at lower spins the nucleus predominantly emits nucleons as long as it has sufficient energy to do so. The amount of angular momentum carried off by particle evaporation is relatively small, so when the nucleus has de-excited to within a neutron binding energy of the yrast line, it must get rid of most of its angular momentum and its remaining excitation energy by $\gamma$-ray emission. It can be seen that the highest spins are associated with the fewest neutrons evaporated and the longest $\gamma$-ray cascades. These $\gamma$-ray

![Figure 4](image_url)

**Fig. 4.** Decay modes in excitation energy-spin space for the (typical) reaction indicated. Long arrows indicate neutron emission, short arrows are $\gamma$ rays.
Cascades have two principal types of transitions. The "statistical" transitions carry off energy but little angular momentum and so cool the nucleus towards the yrast line, while the "yrast-like" transitions follow paths roughly parallel to the yrast line. There is an enormous number of pathways until a region near the yrast line is reached, with the result that no single transition has enough intensity to stand out (with present techniques). This is the "continuum" γ-ray spectrum. But, near the yrast line, the population "condenses" into a few pathways whose transitions can be resolved into discrete lines. This typically happens below spin 35-40 h for masses near 160, and in the lower-spin region one can employ all the techniques of conventional γ-ray spectroscopy and develop detailed information on the nature of the transitions and the states involved.

At the higher spins the individual transitions cannot be resolved, and so one must settle for measuring nuclear properties averaged over the many states of the continuum cascades. A typical spectrum shows, above the low-energy discrete transitions, an "yrast bump" of stretched E2 rotational cascades extending up to ~1.5 MeV (in the rare-earth region), and then at still higher energies, an exponential tail of statistical transitions. An example is shown in fig. 5 taken with a 12.7 x 15.2 cm NaI detector. Average transition multipolarities have been determined by angular distributions, and in a few cases average lifetimes have been measured for a region of the continuum. But the most commonly determined property from continuum spectra is a moment of inertia. Since there are several kinds, let us consider them in more detail. The first distinction to make is between kinematic and dynamic values. A moment of inertia may be defined from the first derivative of the energy with respect to spin:

\[ \frac{\mathcal{D}^{(1)}}{N^2} = \mathcal{I} \left( \frac{dE}{dI} \right)^{-1} = \frac{\mathcal{I}}{\hbar \omega} , \]  

where \( \mathcal{D}^{(1)} \) is called the "kinematic" moment of inertia. The second derivative also leads to a definition:
Fig. 5. NaI γ-ray spectrum for the de-excitation of $^{164}\text{Er}^*$, after unfolding and normalizing to the γ-ray multiplicity. The statistical component, taken to have the form $E_\gamma \exp(-E_\gamma/T)$, is shown as a dashed line.

$$\frac{\mathcal{Q}^{(2)}}{\hbar^2} = \left( \frac{d^2E}{dl^2} \right)^{-1} = \frac{dI}{h\omega},$$

where $\mathcal{Q}^{(2)}$ is called the "dynamic" moment of inertia. If there is only the kinetic energy term as given in Eq. (1.a), these two moments of inertia are equal; but, in general, when there are additional $I$-dependent terms in the Hamiltonian they will differ. In the present case, the Coriolis force perturbs the internal nuclear structure, giving rise, in lowest order, to an $(I-j)$ term, so that $\mathcal{Q}^{(1)} \neq \mathcal{Q}^{(2)}$.

These two moments of inertia can be defined in principle for any sequence of states, but certain ones occur rather naturally. If the particle configuration is frozen, so there is no perturbation (alignment, shape change, etc.) of the internal structure, e.g. a true collective rotational band, the appropriate moments of inertia are $\mathcal{Q}^{(1)}_{\text{band}}$ and $\mathcal{Q}^{(2)}_{\text{band}}$. In general, however, a single pathway involves
a sequence of bands having different alignments or shapes, and we define \( Q_1(t) \) and \( Q_2(2) \) to include both the collective contribution and contributions caused by changes in particle alignment and shape. For the unresolved spectra from the highest spin states, the population is spread over many bands in many decay sequences. Nevertheless, the average values of \( Q_1(t) \) for the states populated can be determined, and we shall call them "band" and "effective" values.

Since \( Q_1(1) \) at a frequency \( \omega \) is an average of \( Q_2(2) \) over the frequency range 0 to \( \omega \), it is much less sensitive to the local changes in the internal nuclear structure than \( Q_2(2) \). Thus the latter is a better indicator of change. Values of \( Q_2(2) \) can be determined from the width of the valley in a two-dimensional \( E_1(1) - E_2(2) \) array for a rotational nucleus, and such measurements are of great importance in indicating the collectivity remaining in the nucleus at that frequency. But such measurements suffer from either the poor resolution of NaI detectors, which smears out the valley ridges, or the poor response function of Ge detectors, which gives mostly Compton-scattered transitions rather than full-energy peaks. The new generation of Compton-suppressed Ge detectors will surely lead to big advances in these measurements.

Values of \( Q_2(2) \) are relatively easier to determine for a good rotor whose spectrum in the yrast-bump region is predominantly stretched E2. For then the number of transitions \( dN \) in the \( \gamma \)-ray spectrum (normalized to the \( \gamma \)-ray multiplicity) in a given transition energy interval \( dE_\gamma \) is just half the spin \( dI \) removed in that interval. The spectrum height \( dN/dE_\gamma \) thus equals \( dI/4d\omega = Q_2(2)/4 \) if the cascades are already fully fed. The interest is in determining \( Q_2(2) \) at high frequencies, but that is just where feeding does occur and so must be corrected for. A method \(^9\) has been developed to make this correction (subject to certain constraints) by using the difference between two spectra from similar but slightly shifted spin distributions (selected in coincidence with neighboring total \( \gamma \)-energy slices) as the feeding curve in that frequency range. For
Fig. 6. Plots of $2g^{(2)}_{\text{eff}}/\hbar^2$ vs. $\hbar\omega$ obtained from spectra like the one shown in fig. 5 after subtracting the statistical component and correcting for partial feeding. The curves are for compound nuclei made by 185 MeV $^{40}$Ar on: $^{124}$Sn (thick solid line), $^{126}$Te (dotted line), $^{130}$Te (thick dashed line). Also shown are the values of $g^{(2)}$ for $^{124}$Sn (thin solid lines) and $^{130}$Te (thin dashed lines) targets (ref. 8).

example, after subtracting the statistical $\gamma$-ray spectrum shown dashed in fig. 5 (leaving essentially pure stretched E2 transitions) and then correcting the result for feeding, gives the solid curve in fig. 6. Two other cases, $^{162}$Yb and $^{166}$Yb, are also shown. The irregularities below $\hbar\omega \approx 0.3$ MeV result from partially resolved individual $\gamma$-ray transitions and the known first backbonds. The band moments of inertia from $\gamma$-$\gamma$ correlation data are also plotted as lighter lines in the regions where they have been determined. The most impressive feature is the large rise of $g^{(2)}_{\text{eff}}$ for the Yb nuclei starting around $\hbar\omega \approx 0.5$ MeV. Such an increase is most likely due to either a shape change or a particle alignment, or both. Since the
\( J_{\text{band}}^{(2)} \). Values in this region seem to be decreasing, new alignments are suggested. And since the two Yb nuclei behave alike while the Er isotone of one of them has only a small delay rise, it would appear that high-\( j \) proton orbitals from the next shell are involved; the Fermi level for protons is higher in Yb (\( Z = 70 \)) than in Er (\( Z = 68 \)). This agrees with calculations\(^{10,11} \) that predict \( \pi_{9/2} \) and \( \pi_{13/2} \) crossings and hence alignments occur in this frequency region.

Fig. 7. Plots of \( 2\epsilon_{\text{band}}^{(2)}/\hbar^2 \) vs. \( \hbar \omega \) for the systems \( {}^{40}\text{Ar} \) plus: \( {}^{124}\text{Sn} \) (solid line), \( {}^{122}\text{Sn} \) (long-dashed line), \( {}^{120}\text{Sn} \) (short-dashed line). In each curve, the last point corresponds to 25% feeding (correction by a factor 4). The thin solid lines are a plot of \( \epsilon_{\text{band}}^{(2)} \) or the \( {}^{124}\text{Sn} \) target.
Figure 7 shows the results for $J^2$ obtained for several Er isotopes. The solid line is again $^{160}$Er, with the first $\nu_{13/2}$ backbend at $h\omega \sim 0.27$ MeV, the blocked backbend in $^{159}$Er as a small peak at $\sim 0.35$ MeV, and then probably the second backbend in $^{160}$Er, of $h\omega_{11/2}$ protons, at $\sim 0.45$ MeV. The spectrum shows a minimum around 0.5 MeV and then rises. The long- and short-dashed curves are principally $^{158}$Er and $^{156}$Er, respectively, at this relatively high sum-energy cut (25-27.5 MeV), and illustrate the effect of changes in the neutron Fermi level and in deformation. These nuclei become less deformed with loss of neutrons, and this and their increased softness to changes in the shape parameter $\gamma$ make valence-shell alignments easier. The valence backbends are compressed in frequency which makes the lower bump in these spectra taller and narrower than for the heavier Er, and so creates a lower valley or minimum before the rise at higher frequency due to alignment of protons from the next shell. Thus in the lighter Er isotopes the alignments of the valence shell separate from those of the next higher shell, and this is even visible in the raw spectra.

We have not, however, ruled out the possibility that an increased deformation also contributes to the high-energy bumps observed in Er and Yb nuclei. This is particularly true for the lighter isotopes of Er, as they are softer towards deformation at high spin and are closer to the region of superdeformation (prolate shape with a 2:1 axis ratio) that calculations predict around $N \sim 84-86$ and $Z \sim 64-66$. Very recently, the first experimental evidence for such superdeformation in a nucleus in this region has been presented. A high-resolution $E_\gamma(1) - E_\gamma(2)$ coincidence array measured at high spin in $^{152}$Dy with Compton-suppressed Ge detectors showed a valley with ridges characteristic of rotational behavior for $\gamma$-ray transitions from 0.8 - 1.35 MeV. The valley width gives a value of $\sqrt{\langle 2 \rangle}_{\text{band}} = (85 \pm 2)h^2$ MeV$^{-1}$; this corresponds to a quadrupole deformation of $\epsilon = 0.5$ for a rigid symmetric rotor (the spin range covered is 34 - 58 h).
Thus, although continuum studies are beginning to uncover valuable and exciting information about the structure of high-spin states and indicate that particle alignments and collective rotational motion both continue to play roles in carrying angular momentum, much more detail and a better understanding of the nuclear structure would be obtained if the use of discrete $\gamma$-ray techniques were possible. It does seem likely that by one or both of two methods the limits of discrete-line spectroscopy will in the future be pushed up some (perhaps ten) units of $\hbar$. One technique is to cut down the number of $\gamma$ cascades under observation. Referring back to fig. 4 it can be seen that if a cut is made on the excitation energy by requiring a coincidence with a particular interval in the output of a total-energy spectrometer, and at the same time a cut is made on the spin axis by requiring a small range of $\gamma$-ray multiplicities, the initial population of excited states is strongly restricted. This is the idea behind using the NaI crystal balls for this purpose, and although this will surely help, it does not appear to be the complete answer. Very high resolution for distinguishing the lines is also necessary, and for that NaI is inadequate. Germanium detectors are the best high-resolution $\gamma$-detectors available today, but they suffer from a very poor response function or peak/total ratio, namely 15-20\% for $\sim 1.2$ MeV $\gamma$ rays for a moderately large detector. In a $\gamma$-$\gamma$ coincidence experiment then only 2-4\% of the events recorded are useful peak-peak coincidences. The answer to this problem is to use Compton-suppression, and the six NaI-suppressed Ge detectors of TESSA2\textsuperscript{16} at Daresbury have provided, during the past year, probably the best $\gamma$-$\gamma$ coincidence spectra ever recorded in-beam, e.g., the $^{152}$Dy study mentioned in the previous paragraph. But present-day technology allows even more. The effective resolution of a system depends upon the number of resolvable points in the spectrum. If there are $N$ resolvable points in an energy interval of a singles spectrum, there will be $N^2$ such points in a $\gamma$-$\gamma$ array covering the same energy interval for each detector, and $N^3$ points in a triple-coincidence spectrum. That is, setting two gates to
bring back a particular triple-coincidence projection gives effectively a much higher resolution than a simple singles spectrum. Certainly it can be expected that the use of high-order coincidences, triples and even quadruples, should permit pushing discrete-line studies to higher spins. To achieve this goal requires a large number of detectors placed close to the target, but not so close as to cause appreciable Doppler broadening or summing in an individual detector.

An artist's sketch of such a system under construction at Berkeley is shown in fig. 8. An inner castle of ~44 bismuth germanate (BGO) sectors provides a small (20 cm diameter) 4π ball as a sum spectrometer with almost as good energy efficiency and resolution as the larger NaI balls, but with poorer multiplicity resolution due to the small number of detectors. Outside this ball are 21 Ge detectors arranged in three circles; seven in a horizontal plane, seven pointing slightly downward at the target from a circle above the horizontal plane, and seven pointing up from a circle below. The Ge detectors are ~15 cm from the target, and each one is Compton-suppressed by a BGO shield. This compact arrangement is possible because of the high density of BGO (7.13 g/cm³) compared to NaI. When the full array is completed, it should be possible to obtain ~10⁶ quadruple coincidences a day in an in-beam study such as the one that produced the spectrum in fig. 5. We have performed two experiments, one with four and one with seven of the BGO-suppressed Ge detectors (but with no central ball) at ~13 cm from the target. A portion of the spectrum of 156Er taken in coincidence with the gate shown in the insert on the figure is given in fig. 9 to illustrate the cleanliness of the resulting spectrum; without the Compton suppression the background would be 4-5 times higher in the gate spectrum (making it most improbable to be able to set this particular gate) as well as in the gated spectrum displayed. With our shield design, the peak/total ratio for a 1 MeV γ ray counted above 300 KeV is ~50% compared to ~20% unshielded; this rises to ~55% with a NaI cover on the front of the BGO shield, and it is clear that for higher
Fig. 8. Sketch of the Berkeley array. It consists of an inner $4\pi$ ball of 44 sectors of bismuth germanate, surrounded by 21 Compton-suppressed Ge detectors with BGO shields (ref. 17).

Fig. 9. Portion of the spectrum of $^{156}$Er in coincidence with the gate shown in the insert after subtraction of the background gate also indicated. These spectra were taken with BGO-suppressed Ge detectors. The starred transitions are new.
order coincidences this represents an enormous gain in useful events. Thus this system will contribute both to resolving the lower end of the continuum region and to more detailed studies of the discrete-line spectra.

The previous speaker has told you about some of the latest developments in discrete-line studies and I have described some recent continuum experiments. There has been almost an explosion in the amount and type of data becoming available. These are giving a much more detailed picture of the changes that occur in nuclear properties with increase in spin. But many questions remain to be answered. Where and how are the pairing interactions for both neutrons and protons finally quenched by rotation? How and why does the nuclear shape vary with spin, with neutron and proton number, and with temperature? Does superdeformation exist in the predicted regions of the Periodic table and at the predicted spins? How large are shell effects at high spin, and to how high an excitation energy do they remain? Do collective bands occur up to the highest spins, and if so, what is their nature? How frequent and important are high-j particle alignments? We have much to learn, and the new Compton-suppressed arrays will open up still more exciting vistas; in the future, still more powerful tools will surely be developed to help us find the answers.

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