165
4/14
13
PPPL-2111

UC20-G

PPPL-2111
DR#0257-7

NOTICE

PORTIONS OF THIS REPORT ARE ILLEGIBLE. It
has been reproduced from the best available
copy to permit the broadest possible avail-
ability.

THREE CAVEATS FOR LINEAR STABILITY THEORY:
RAYLEIGH-BÉNARD CONVECTION

By

H.S. Greenside

JUNE 1984

MASTER

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America.

Available from:

National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy \$ * ; Microfiche \$3.50

<u>*PAGES</u>	<u>NTIS</u> <u>Selling Price</u>
1-25	\$5.00
26-50	\$6.50
51-75	\$8.00
76-100	\$9.50
101-125	\$11.00
126-150	\$12.50
151-175	\$14.00
176-200	\$15.50
201-225	\$17.00
226-250	\$18.50
251-275	\$20.00
276-300	\$21.50
301-325	\$23.00
326-350	\$24.50
351-375	\$26.00
376-400	\$27.50
401-425	\$29.00
426-450	\$30.50
451-475	\$32.00
476-500	\$33.50
500-525	\$35.00
526-550	\$36.50
551-575	\$38.00
576-600	\$39.50

For documents over 600
pages, add \$1.50 for each
additional 25 page increment.

THREE CAVEATS FOR LINEAR STABILITY THEORY:

RAYLEIGH-BÉNARD CONVECTION

H.S. Greenside

Plasma Physics Laboratory, Princeton University

P.O. Box 451, Princeton, NJ 08544

Recent theories and experiments challenge the applicability of linear stability theory near the onset of buoyancy-driven (Rayleigh-Bénard) convection. This stability theory, based on small perturbations of infinite parallel rolls, is found to miss several important features of the convective flow. The reason is that the lateral boundaries have a profound influence on the possible wave numbers and flow patterns even for the largest cells studied. Also, the nonlinear growth of incoherent unstable modes distorts the rolls, leading to a spatially disordered and sometimes temporally nonperiodic flow. Finally, the relation of the skewed varicose instability to the onset of turbulence (nonperiodic time dependence) is examined. Linear stability theory may not suffice to predict the onset of time dependence in large cells close to threshold.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

1. Introduction

Instability is an essential feature of driven dissipative systems as the driving is increased. Examples of increasing complexity are a buckling beam, a convecting fluid, and a fusion plasma, where the driving is an increasing force, a temperature gradient, and an ohmic current, respectively. When the driving and dissipation are no longer in balance, small fluctuations can grow. What these fluctuations grow into raises several fundamental issues that lie at the forefront of current efforts to understand nonlinear nonequilibrium systems.

These issues are somewhat different depending on whether fluctuations lead to a stationary state or to a time-dependent but statistically steady state. When an unstable system evolves into a stationary state, one would like to know on what time and length scales this evolution occurs. Often the physical system is large in some sense and one would expect a significant degeneracy of possible stationary states. Calculations and experiments show that this intuition is incorrect. Boundary conditions and nonlinear terms sometimes allow only a small number of states to be attained, so that a linear analysis can be misleading.

Similarly, when fluctuations grow and a system becomes time dependent, one would like to understand the relation between the spatial structure and the temporal evolution. That there is a relation is evident since there are often long-wavelength instabilities that are sensitive to the cell geometry and to the spatial patterns. These instabilities may not be intrinsically time dependent; the imaginary part of the fastest growing eigenvalue could be zero. A temporally nonperiodic evolution could then arise only from the interaction of incoherent spatial modes. This interaction is most interesting when the growing fluctuations no longer have small amplitude so that a

nonlinear analysis is needed. A linear stability theory may then be unsuccessful in predicting the onset of time dependence. It is not yet known how to generalize the linear theory to handle this difficult nonlinear regime.

In the following, we will discuss these issues of pattern formation and of the onset of turbulence in the context of convecting fluids [1,2]. This choice has little to do with the practical and widespread applications of thermal convection to engineering, meteorology, astrophysics, and geology. Instead, Rayleigh-Bénard convection comes closest to satisfying the physicist's ideal of a clean experimental system in which a careful comparison can be made between theory and experiment.

This system is ideal in several ways. First, the boundaries of the fluid cell are static and homogeneous and are easily controlled. Only the dynamics of the fluid can introduce a time dependence. Second, the complexity of the flow increases slowly with the driving. Subsequent instabilities are easily isolated and studied. Third, the spatial modes are readily and quantitatively observable, e.g., by laser Doppler velocimetry [3]. Finally, the primary instability, in which the motionless conducting fluid breaks up spontaneously into convecting rolls, is a continuous transition that occurs with a distinguished length scale (roughly the depth of the fluid). There is then a small parameter, the closeness to onset, in which the fluid equations can be expanded. Further, only a few length scales enter even for large driving. Significant theoretical progress can therefore be made.

This experimental and theoretical simplicity can be contrasted with current attempts to understand the stability and confinement of fusion plasmas [4]. While similar questions of pattern formation and the onset of turbulence arise, comparison between theory and experiment is difficult. The boundary conditions are hard to control and sometimes vary with time, the spatial modes

of the plasma are not easily determined, and the complexity of sixteen interacting fields often disguises the underlying physical mechanisms.

An experimental cell for studying Rayleigh-Bénard convection can be characterized by three dimensionless parameters. The Rayleigh number, R , is a combination of physical constants which compares the strength of the driving (buoyancy forces) to the strengths of the two dissipative mechanisms, viscosity and thermal conduction. When the driving is large compared to the dissipation (i.e., there is a large enough vertical temperature gradient), the fluid becomes unstable to infinitesimal perturbations and breaks up into convection rolls. The critical Rayleigh number for this instability is denoted by R_c . It depends on the wave number of the fluctuations, being a minimum for a critical wave number, K_c (roughly the reciprocal of the depth of the fluid).

Subsequent instabilities of the rolls, at higher values of R , depend on the second dimensionless parameter, the Prandtl number σ . This is the ratio of the viscosity to the thermal conductivity. For large (or infinite) σ , the fluid velocity is not a dynamic variable. The fluid motion is then determined by the nonlinear diffusion of heat, a scalar quantity. For small σ , the fluid motion is dominated by advection of momentum, a vector quantity. For intermediate σ , which is the case for many gases, the advection of heat and momentum both play a role in the fluid motion and the dynamics of the fluid is rich. In particular, for intermediate σ , rolls can become unstable to a skewed varicose instability which has been strongly implicated with the onset of turbulence and with the increase of roll wavelength with increasing R [5].

The final dimensionless parameter is the aspect ratio, Γ . It is the ratio of a typical lateral (horizontal) dimension to the depth of the fluid. Γ measures the influence of lateral boundaries on the convecting flow. For

small Γ , the walls are close compared to the depth of the fluid; only short wavelength modes can be excited. In small aspect ratio experiments, discrete symmetry breaking transitions are seen as the Rayleigh number is increased [6]. Stationary states occur over a large range in R ; the spatial modes often have the symmetry of the cell geometry [7]. Time dependence is usually first observed in the form of periodic or quasiperiodic flow, while nonperiodic behavior occurs only for values of R large compared to R_c . The behavior for these small Γ boxes seems well described by models with few degrees of freedom, e.g., by strange attractors. It is not yet known, however, how to predict the sequence of bifurcations and the onset of time dependence either in the simple models or in experiments, except for the few universal scenarios so far discovered [8].

Large aspect ratio experiments are less understood and are more intriguing. Discrete transitions are rare as R is increased. The stationary states are disordered, consisting of defects and curved rolls; the flow often has no particular symmetry [5]. The onset of time dependence is sometimes nonperiodic and can occur quite close to onset [9, 10]. We note that although the physics is unlikely to be understood by dynamical models with few variables, the number of independent modes participating in the nonperiodic flow may be estimated in the near future if recent algorithms can successfully be applied [11-13].

The basic assumption in the linear stability theory of convecting rolls is that the theory is relevant for large aspect ratio cells, with the calculations being more accurate as the cell size increases. The work of Busse and collaborators [1] has indeed been useful in many experiments for interpreting the possible secondary instabilities and their dependence on R , σ , and K (the roll wave number). However, as experiments and theory become

more sophisticated and thorough, more and more physics is being discovered that seems to lie outside the predictions of the usual stability theory. It is not yet clear how relevant or "universal" these inconsistencies are for re-researchers studying other nonequilibrium systems. For example, some of the physics of convecting fluids is peculiar to the boundary condition that the rolls are normal to the sidewalls. This paper is therefore best summarized as three caveats about linear stability theory:

- (1) Linear stability theory does not predict correctly the possible stable wave numbers in one-dimensional convection cells.
- (2) Linear stability theory does not predict correctly the resolution of spatial degeneracy in two-dimensional cells, in which the rolls can bend through large angles of order one.
- (3) Linear stability theory does not predict correctly the onset of time dependence in large cells.

These three statements might seem too strong; in some cases of Rayleigh-Bénard flow, linear stability theory is qualitatively correct. We shall see in the following sections, though, that for each caveat, linear theory does miss the essential physics.

In Sec. 2, we discuss the first of these caveats and explain how the boundary conditions can have a profound influence on wave number selection, no matter how large the aspect ratio. In Sec. 3, we discuss numerical simulations that illustrate the second caveat. Experiments and these calculations suggest that the motion of defects (irregularities in the rolls) and not the growth of unstable modes resolve the spatial degeneracy of the fluid. In Sec. 4, we discuss experiments and numerical simulations of the onset of turbulence in large cells. Finally, in Sec. 5, we summarize the previous sections and discuss what future experiments and calculations are needed to further our understanding.

2. One-Dimensional Geometries

The simplest cases to examine in which boundary conditions and nonlinearities affect the predictions of linear stability theory are one-dimensional geometries. Here the wave number but not the orientation of the rolls can change. There are two geometries that are readily studied both experimentally and theoretically. The first is a long narrow rectangular cell, in which the rolls evolve to be parallel to the shorter side. The second is axisymmetric convection in a large cylindrical cell, in which the rolls can be made to form concentric circles. The question is then to determine what wave numbers are possible in the final stationary state, and how these wave numbers depend on R , σ , and the initial conditions (if there is hysteresis).

An alternative way of posing this question -- one that is closer to the theme of this workshop -- is to study the sensitivity of the final stationary state to fluctuations in the initial conditions. We assume that the initial conditions are somehow restricted to evolve only into one-dimensional rolls. For example, an experiment could start with parallel or concentric rolls of varying wavelengths. The subsequent evolution for fixed R would then consist of a gradual adjustment of the local wave number while preserving the roll shape.

We first discuss wave number selection near onset in a long rectangular cell. If the shorter side is not too much longer than a roll diameter, it is found in numerical simulations [14] and is predicted for experiments that the cross-roll, zigzag, and skewed varicose instabilities are unable to grow. These are the only instabilities that can destroy the one-dimensional array of rolls by introducing transverse structure [1]. For a long narrow cell, one would then expect the local wave numbers away from the lateral walls to be

restricted by the one-dimensional Eckhaus instability. The stability diagram (see Fig. 1) predicts a finite band of stable wave numbers for a given value of r , the reduced Rayleigh number, $R/R_c - 1$. The width of the Eckhaus stable band increases as $r^{1/2}$ near onset.

At first thought, the lateral boundaries should only have a qualitative effect on the linear stability theory. The idea is that only the rolls nearest the walls feel the influence of the boundary conditions, which require the fluid velocity to vanish. Thus the amplitude of the convection is diminished near the walls but only on a length scale comparable to a roll diameter. If there are N rolls in the system, the linear stability theory should be wrong for wave numbers that differ only by order $1/N$ from the edge of the stability band. In particular, the theory should be asymptotically correct as the aspect ratio and the number of rolls become arbitrarily large.

A recent calculation by Cross et al. shows that this simple picture of the influence of the boundaries is seriously wrong [15]. Although the amplitude of convection indeed changes only near the walls, the phase of the rolls can wind or unwind by large amounts when the amplitude of convection decreases anywhere in the cell to zero. This leads to the creation or elimination of rolls near the sidewalls and a concomitant change in the local wavenumber. Thus no matter how large the cell is, phase winding near the walls will strongly influence the possible stationary wave numbers in the center of the cell. The band of stable wave numbers for infinite aspect ratio is not continuously attained from finite cells.

The quantitative calculation, based on higher order corrections to the amplitude equation first studied by Segal [16], and Newell and Whitehead [17], shows that the lateral boundaries severely restrict the band of stationary solutions near onset. In fact, the band grows linearly with r . This is shown

in Fig. 1 for a simple model of the onset of convection, the Swift-Hohenberg model [18]. For the full Boussinesq equations, the stable band still grows linearly with r , but is asymmetric. The exact shape depends on the thermal conductivity of the side walls and on the Prandtl number, σ [15].

The crucial property that allows phase winding and the collapsing of the stability band is that the amplitude of the periodicity (e.g., the envelope of the maximum velocity of the locally sinusoidal rolls) goes to zero somewhere in the cell. If the amplitude is not zero at the walls (this can be achieved by creating a small horizontal thermal gradient with a hot wire glued along the perimeter), the phase does not wind and the Eckhaus stability band should be recovered. In Couette flow, where a one-dimensional periodic flow also occurs, the amplitude of the Taylor vortices does not go to zero at the top and bottom of the cylinder. And indeed, wave numbers consistent with the predictions of infinite aspect ratio linear stability theory are found [19].

Cross et al.'s remarkable result has not yet been tested in Rayleigh-Bénard experiments, although it has been confirmed numerically for the Swift-Hohenberg model. For mathematical simplicity, these authors assumed stress-free horizontal boundary conditions, which are difficult (but possible) to realize experimentally [20]. The small band of stable wave numbers and its linear increase in size with r should remain qualitatively correct for the more common rigid boundary conditions. It remains to be demonstrated by experiment that there is a range in R near onset in which these predictions can be observed and that the two-dimensionality of the cell's corners does not affect these conclusions.

An important but subtle point of this analysis is that a finite band of stationary wave numbers remains even after the effects of the lateral walls are considered. This raises the question of whether a unique roll wave number

is ever selected, so that the band collapses to zero width. The answer is yes. However, extra experimental or mathematical constraints must be imposed to achieve this goal. These recent results are sufficiently interesting that we briefly review them here.

The first way to force a unique wave number is to eliminate the lateral boundaries in a devious way. Kramer et al. [21] showed that this could be done by introducing a slow spatial gradient in the bifurcation parameter (here R) in such a way that a subcritical flow ($R < R_c$) is gently connected to a supercritical flow ($R > R_c$). These authors then showed that a unique wave number is selected away from the boundaries in the middle of the supercritical region. While this has not been verified for convection, Ahlers et al. have shown that excellent agreement is found in Couette flow [19]. The small deviations from theory arising from the finite magnitude of the parameter gradient can be explained by adding a forcing term to the equations [22]. Kramer et al.'s clever scheme should be useful to test the influence of the boundary conditions for many other nonequilibrium systems.

The other case where a unique wave number can be forced is in axisymmetric convection. Here the wave number of the rolls is predicted to approach a unique value away from the center of curvature [23, 24]. A recent elegant experiment by Pocheau and Croquette [25] has tested these predictions. They first found that a constraint of a hot wire glued along the perimeter was needed to prevent the circular rolls from breaking up into a less symmetric pattern. They then discovered the surprising result that even with this constraint, the circular rolls are unstable to a symmetry-breaking instability that displaces the center of the rolls a tiny distance away from the geometric center of the cell; a small but finite band of wave numbers is found. Presumably still another constraint, such as a small disk

at the center of the cell, could guarantee axisymmetry and restore the uniqueness of the wave number. Further experiments would be of interest.

3. Pattern Formation In Two-Dimensional Cells

The difficulties of using linear stability theory to understand pattern formation can be illustrated by the simple two-dimensional Swift-Hohenberg model. This equation shows most of the qualitative features observed in large σ convection experiments near onset [14] without the complexity of three-dimensional flows, several interacting fields, advection, or temporally nonperiodic behavior. Further, this equation is variationally derived from a Lyapunov functional so that much of the physics can be understood by the minimization of a functional integral [24].

The stability diagram for this equation is given in Fig. 1. The band of stable wave numbers is limited on one side by the Eckhaus and on the other side by the zigzag instability. For a given K inside the stability band, no instability appears as r is increased to larger values. No qualitative difference is predicted for small r ($\ll 1$) and larger r (≈ 1) evolutions, except that a larger band of wave numbers should enter the final pattern, allowing more highly curved rolls. The time for the evolution to become stationary should be of order a horizontal thermal diffusion time, since this is the time for disturbances at one end of the cell to diffuse to the other end. This diffusive time should scale with the square of the aspect ratio.

Numerical simulations show that the pattern formation is far more complicated than that suggested by Fig. 1. For large square cells, starting from random initial conditions of all length scales, a complicated transient is found (see Figs. 2 and 3). After a few vertical diffusion times, only wavelengths near the critical value remain, with the wave numbers roughly

filling the stability band. (The distribution of wave numbers at any time can be calculated by a spatial Fourier analysis as shown by Gollub et al. [27].) Subsequent evolution is dominated by the motion, interaction, and annihilation of defects, and cannot be interpreted in terms of the growth and saturation of various linear instabilities.

Near onset, the transient leads to the elimination of most defects and eventually to a symmetric state with curved rolls focusing on diagonally opposite corners (Fig. 2). Repetitions of this numerical experiment with different random initial conditions leads to the same final state; there seems to be only one stationary pattern near onset accessible from random initial conditions. This result sharply contrasts with the large spatial degeneracy suggested by the eigenfunctions of the linear equation at onset [28]. The time scale to become stationary is of order 50 horizontal diffusion times, far longer than a diffusive time scale. The later part of this long evolution is governed by slow adjustments near the wall of grain boundaries (a one-dimensional rather than a point defect). The presence of concentric rolls focused on corners and the slow adjustment of grain boundaries towards the end of the evolution are features seen in many simulations and experiments near onset [29].

For larger values of r (in Fig. 3, $r = 0.9$), the pattern formation is qualitatively different. The same initial conditions lead to a much shorter-lived transient (less than a horizontal diffusion time). The final state has no particular symmetry and many defects are present, with rolls curving sharply over large angles. (As in Fig. 2, the rolls are still normal to the boundaries.) The spatial degeneracy is now much larger; repetitions with different starting conditions lead to different final states. The evolution is thus more rigid in that the final state is not very different from the

early pattern after a few vertical diffusion times; the motion of the defects seems highly constrained. A Fourier spatial analysis shows that only half the band of stable wave numbers is filled in the stationary state, so that some kind of wave number selection is occurring [14]. The details of this mechanism are not yet understood and require further study.

An important insight into the difference between these small and large r evolutions can be obtained by using periodic boundary conditions to eliminate the rigid lateral walls altogether. (Although this cannot be achieved experimentally, a two-dimensional generalization of Kramer et al.'s soft-boundary conditions [21] might be a good approximation.) A numerical run near onset with the same geometry and with the same initial conditions as Figs. 2 and 3 but with periodic boundary conditions (not shown) leads to a short-lived transient (less than a horizontal diffusion time) in which all defects are eliminated and straight or wavy parallel rolls survive [14]. Thus the defects are forced by the lateral walls [26]. Also, the time scale to reach a stationary state is not due to the interaction of defects alone but is specifically due to the interaction of defects with the lateral walls. Simulations for $r=0.9$ with periodic boundary conditions give results similar to Fig. 3. This shows that the rigid evolution is due to the immobility of the defects and is not due to an r -dependent pinning by the lateral wall. Thus the lateral wall is crucial in determining the time scale of evolution near onset and in forcing the presence of defects in the final stationary state.

The above simulations and laboratory experiments emphasize the importance of defect motion. One might inquire whether an understanding of the dynamics of isolated defects or of simple configurations of interacting defects would help to understand the pattern formation. Another deeper side to this inquiry

is to learn how to derive laws of motion for defects and how these laws select wave vectors when no free energy or Lyapunov functional is known. For equilibrium systems, the motion and selection always minimizes the free energy. Close enough to onset, there is a free-energy-like functional which is again minimized by the dynamics [26, 30]. For larger r , however, the possibility of nonperiodic behavior demonstrates that no such functional exists. Thus a careful quantitative study of defects may give insight about how to generalize dynamics to non-potential systems.

Our understanding of defects has recently made significant progress. Simulations and experiments show that the flow is dominated by the motion of two classes of defects: dislocations, a point defect in which a roll ends abruptly; and grain boundaries, a one-dimensional defect in which a patch of rolls with one orientation joins onto a patch with a different orientation. Examples are given in Figs. 2 and 3. Dislocations can move either by climbing parallel to the local roll axis or by gliding perpendicular to this axis. Simulations show that gliding is much more important and that there is an r -dependent barrier that prevents gliding at higher values of r [14]. This qualitatively explains the difference in the final states of Figs. 2 and 3, and of states with periodic boundary conditions.

An analytical theory relating the gliding velocity and barrier height to the local roll curvature (and perhaps to details of the flow far from the core of the dislocation) has not yet been derived. The much simpler climbing motion is better understood, at least near onset. Siggia and Zippelius [31] have shown that compression of the rolls drives climbing at a constant velocity, with a simple relation between the velocity and the compression. The dislocation climbs in such a way so as to select a wave number that is marginally stable to zigzags. (There is also experimental evidence [32] that

grain boundaries act to select wave numbers marginally stable to zigzags.) Further from onset, there is still a preferred wave number but corrections from the generation of vertical vorticity must be taken into account [24, 33]; marginal stability is no longer a selection principle [34]. Further experiments and simulations would be helpful in unraveling the role of defects in wave number selection and pattern formation in the parameter space of Rayleigh number, Prandtl number, and aspect ratio.

4. The Onset of Turbulence

The final caveat we discuss -- the onset of turbulence in large aspect ratio cells with moderate Prandtl number fluids -- is also the one for which there is currently the most serious discrepancy with linear stability theory. For fluids with $\sigma \sim 1$, Busse and Clever [35] have calculated the linear stability of infinite parallel rolls (Fig. 4). This calculation shows that there is a finite band of stable wave numbers near onset and that this stable region vanishes above a certain (σ dependent) value of R (about $3R_c$ for $\sigma \approx 5$).

Figure 4 suggests that a stationary state is possible (although not necessary) in a large cell close to onset since there is a band of stable wave numbers. That a stationary state is also necessary is suggested by a calculation of Schultze, Lortz, and Busse [36]. These authors showed that superpositions of rolls in a Boussinesq fluid are unstable near onset so that no spatial multiperiodicity is possible, only locally singly periodic rolls. Thus fluctuations in the form of rolls lead to unstable superpositions which must decay to periodic rolls whose wave number eventually lies inside the stability band and is stationary. Also, as pointed out previously, Busse [30] and Cross [26] have shown that near onset, to lowest order in an expansion in

$r=R/R_c - 1$, the dynamics are governed by a Lyapunov functional which forces all initial conditions to eventually become time independent.

For R greater than $3R_c$, the vanishing of the stability band (Fig. 4) predicts that rolls become unstable, either to an oscillatory or skewed varicose instability. (The latter actually has greater significance since wave numbers close to K_c become unstable to the skewed varicose as R is increased.) As discussed in the introduction, the onset of instability does not necessarily imply the onset of a statistically steady time dependence since the skewed varicose eigenfunction is not intrinsically time dependent. A nonlinear analysis is required to understand the resulting flow. It is also important to recognize that the absence of a stable band of wave numbers is not equivalent to the onset of turbulence since a stationary state of defects and curved rolls is possible.

Experiments show that the onset of time dependence is a complicated function of σ , Γ , and geometry. For simplicity, we first discuss rectangular and then cylindrical geometries. For moderate Γ rectangular cells, no time dependence is seen even after the skewed varicose instability appears [6]. An initial state of parallel rolls undergoes a complicated transient which leads to parallel rolls of a smaller wave number. This wave number as a function of R seems to trace the skewed varicose instability boundary as R is increased. For large Γ , there is again no time dependence near onset [5]. Instead, there is a complicated long-lived transient leading to a defected stationary state, as described in the previous section. A clear time dependence occurs only for values of r of order 5, which corresponds rather accurately with the onset of the skewed varicose instability [35]. The time dependence manifests itself spatially as a local pinching off and rejoining of rolls, a short wavelength phenomenon that is not related to the linear eigenfunctions of the skewed

varicose instability. Thus this instability leads to stationary or time dependent states depending on the aspect ratio and the initial conditions.

In medium aspect ratio cylindrical cells, there is always a stationary state close enough to onset [9, 37, 38]. This state becomes unstable at much smaller values of R than those predicted by Fig. 4. The instability leads to an oscillatory time dependence [38] but with a period of order a horizontal diffusion time and is thus unlikely to be related to the oscillatory instability of Busse [1], which has a period of order the much shorter vertical diffusion time. The spatial structure of this oscillatory state is presently unknown (as is the case with all the cryogenic experiments using liquid He) because of the difficulties of seeding the fluid with light-scattering impurities. As R is increased, the following sequence is often found: a stationary state, an oscillatory state, another stationary state, then a different oscillatory state, and eventually turbulence. The value of R for which turbulence is first observed decreases with increasing aspect ratio, suggesting that larger cells are more susceptible to nonperiodic flows [38]. The observed sequence of transitions does not seem to follow Fig. 4 even qualitatively.

For a very large aspect ratio cylindrical cell ($\Gamma \sim 60$), the fluid becomes turbulent for r as small as 0.1 and no value of r is found for which the flow becomes stationary [9]. Thus the onset of convection and the onset of nonperiodic motion are the same. This result seems completely at odds with the linear stability analysis and with the lowest order calculations of Busse [30] and Cross [26]. It also seems incompatible with the trend established in large rectangular cells. The spatial nature of this turbulence is not yet known for reasons mentioned previously. Recent optical experiments with water at the same Prandtl number (≈ 3) in large cylindrical cells of half the

diameter ($\Gamma \approx 30$) showed no turbulence near onset [39], in accord with Gollub et al.'s work on rectangular cells [5]. The spatial flow was, in fact, in remarkable accord with the simulations of Greenside and Coughran [14] and Manneville [40], and with the analytical work of Cross [26], strongly supporting the lowest order perturbation calculations: all defects eventually anneal out leading to a single domain of parallel rolls, with a few cross rolls near the lateral walls. The discrepancy between this optical experiment and Ref. 9 is disturbing. Further optical experiments on still larger cells will be necessary to understand why convecting fluids can become time dependent so close to onset.

5. Conclusions

In the previous sections, we have discussed three clear cases in which analytical, numerical, and experimental studies of Rayleigh-Bénard convection near onset seem to contradict the physics suggested by a linear stability analysis of infinite parallel rolls. In one-dimensional geometries (long narrow rectangular cells), the lateral walls strongly restrict the band of possible wave numbers no matter how large the aspect ratio. In two-dimensional cells, the lateral walls require rolls to be perpendicular to the boundaries and force the presence of defects, which dominate the evolution. The time and length scales suggested by a linear stability analysis (Fig. 1) are relevant only for the earliest part of the evolution (much less than a horizontal diffusion time). Finally, in the fourth section, we saw how the fluid can become turbulent as soon as it starts to convect in a large enough cylindrical cell. Details of the flow are lacking because of the difficulties of observing liquid He at low temperatures.

These three cases should be useful as warnings to investigators who use linear stability theory to understand the physics of driven dissipative systems as a function of the driving. Boundary conditions can have a profound influence on the time and length scales of the evolution, on the possible spatial degeneracy, and on the onset of temporally nonperiodic behavior. The lateral walls and the nonlinear growth of fluctuations disrupt the local periodicity of the cellular flow, introducing defects. The fundamental element of linear stability theory, the periodicity of infinite parallel rolls, is therefore lost and the linear analysis breaks down. There is no comprehensive understanding of the consequences of this disruption of periodicity. For this reason, further experiments and theories on different physical systems are needed.

Only the third case raises a fundamental issue which is presently unresolved: how are we to understand the existence of turbulence close to onset in large cells? Experiments and simulations suggest that the answer involves defects and geometry in a complicated way. Turbulence near onset is seen only in the largest cylindrical cells [9], while an unambiguous relaxational evolution towards a time-independent state is seen in all other cases. Optical observations of the skewed varicose instability [5] show that the nonlinear growth leads to a defected pattern, with dislocations constantly appearing and disappearing as rolls pinch off and rejoin. A similar behavior is suspected for Ahlers and Behringer's experiment [9]. The significance of a cylindrical as opposed to a rectangular geometry, the difference of a factor of 2 in aspect ratio for the presence of turbulence [39], and the peculiar role of the skewed varicose instability are not yet understood.

Theory has not yet been successful in going beyond the linear stability theory but has given some insight. Several global details of the evolution

near onset, such as the presence of focal singularities (concentric curved rolls with a corner of the cell as a focus) and the long nondiffusive time scale to become stationary, can be derived using an elegant rotationally invariant generalization of multiscale perturbation theory [29]. Simulations have shown [14, 40] that the absence of a Lyapunov functional is not a sufficient criterion for the onset of nonperiodic behavior, even for large cells with defects. Other simulations of the Boussinesq equations [33] or of simpler model equations which include the generation of vertical vorticity [41,42] can reproduce the qualitative features of the time-dependent experiments [5], but have not explained why some initial conditions lead to stationary states while others give nonperiodic behavior. A first modest step will be a systematic study with controlled initial conditions and controlled perturbations of model equations that are far simpler than the Boussinesq equations but have some of the same ingredients, such as a skewed varicose instability [43]. Such studies are currently under way.

ACKNOWLEDGMENTS

I would like to thank W. M. Coughran Jr., M. Cross, J. Gollub, and J. Tesauro for stimulating discussions related to this paper.

This work was supported by the U. S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

REFERENCES

1. F. H. Busse: Rep. Prog. Phys. 41, 1929 (1978).
2. J. P. Gollub: Nonlinear Dynamics and Turbulence, ed. by D. Joseph and G. Iooss (Pittman, New York, 1982).
3. J. P. Gollub and S. V. Benson: J. Fluid Mech. 100, 449 (1980).
4. E. Teller: Fusion, Vols. I and II (Academic Press, New York, 1981).
5. J. P. Gollub, A. R. McCarriar, and J. F. Steinman: J. Fluid Mech. 125, 259 (1982).
6. C. M. Surko, P. Kolodner, R. W. Walden, A. Passner, and G. M. Dimino: Bull. Am. Phys. Soc. 28, 1360 (1983).
7. K. Stork and U. Muller: J. Fluid Mech. 54, 599 (1972).
8. J.-P. Eckmann: Rev. Mod. Phys. 53, 643 (1981).
9. G. Ahlers and R. P. Behringer: Prog. Theor. Phys. Suppl. 64, 186 (1979).
10. H. S. Greenside, G. Ahlers, P. C. Hohenberg, and R. W. Walden: Physica 5D, 322 (1982).
11. A. Brandstater, J. Swift, H. L. Swinney, A. Wolf, J. D. Farmer, E. Jen, J. P. Crutchfield: Phys. Rev. Lett. 51, 1442 (1983).
12. B. Malraison, P. Atten, P. Berge, and M. Dubois: Phys. Lett. 44, 897 (1983).
13. J. Guckenheimer and G. Buzyna: Phys. Rev. Lett. 51, 1438 (1983).
14. H. S. Greenside and W. M. Coughran, Jr.: Phys. Rev. A (in press); H. S. Greenside, W. M. Coughran, Jr., and N. L. Schryer: Phys. Rev. Lett. 49, 726 (1982).
15. M. C. Cross, P. G. Daniels, P. C. Hohenberg, and E. D. Siggia: Phys. Rev. Lett. 45, 898 (1980); J. Fluid Mech. 127, 155 (1983).
16. L. Segal: J. Fluid Mech. 38, 203 (1969).

17. A. C. Newell and J. A. Whitehead: J. Fluid Mech. 38, 279 (1969).
18. J. Swift and P. C. Hohenberg: Phys. Rev. A15, 319 (1977).
19. G. Ahlers, D. S. Cannell, and M. A. Dominguez-Lerma: Phys. Rev. A27, 1225 (1983).
20. R. J. Goldstein and D. J. Graham: Phys. Fluids 12, 1133 (1969).
21. L. Kramer, E. Ben-Jacob, H. Brand, and M. C. Cross: Phys. Rev. Lett. 49, 1891 (1982).
22. M. C. Cross: Phys. Rev. A29, 391 (1984).
23. Y. Pomeau and P. Manneville: J. Phys. (Paris) 42, 1067 (1981).
24. M. C. Cross: Phys. Rev. A27, 490 (1983).
25. V. Croquette and A. Pocheau: "Wavenumber Selection in Rayleigh-Bénard Convective Structure," preprint (1984).
26. M. C. Cross: Phys. Rev. A25, 1065 (1982).
27. J. P. Gollub and A. R. McCarriar: Phys. Rev. A26, 3470 (1982).
28. C. Normand: J. Appl. Math. Phys. 32, 81 (1981).
29. M. C. Cross and A. C. Newell: Physica D (1984) (in press).
30. F. H. Busse: J. Fluid Mech. 30, 625 (1967).
31. E. D. Siggia and A. Zippelius: Phys. Rev. A24, 1036 (1981).
32. A. Pocheau and V. Croquette: J. Phys. (Paris) 45, 35 (1984).
33. A. Zippelius and E. D. Siggia: Phys. Fluids 26, 2905 (1983).
34. J. S. Langer: Rev. Mod. Physics 52, 1 (1980).
35. F. H. Busse and R. M. Clever: J. Fluid Mech. 91, 319 (1979).
36. A. Schluter, D. Lortz, and F. Busse: J. Fluid Mech. 23, 129 (1965).
37. R. W. Walden: Phys. Rev. A27, 1255 (1983).
38. R. P. Behringer, H. Gao, and J. N. Shaumeyer: Phys. Rev. Lett. 50, 1199 (1983).

39. J. Gollub, article presented in the Workshop on Fluctuations and Sensitivity in Nonequilibrium Systems, Austin, TX, March 1984.
40. P. Manneville: J. Phys. (Paris) 44, 563 (1983).
41. P. Bjorstad, W. M. Coughran, Jr., H. S. Greenside, D. J. Rose, and N. L. Schryer: Elliptic Problem Solvers II, ed. by G. Birkoff and A. Schoenstadt (in press) p. 514 (1984).
42. P. Manneville: "Towards An Understanding of Weak Turbulence Close To The Convection Threshold In Large Aspect Ratio Systems," preprint (1983)
43. H. S. Greenside and M. C. Cross: Phys. Rev. A (to be published, 1984).

FIGURES

Fig. 1. Linear stability diagram of infinite parallel rolls of wave number K as a function of the reduced Rayleigh number, $r=R/R_c - 1$, for the Swift-Hohenberg model [18]. (This equation accurately models convection near onset for infinite Prandtl number fluids [14].) The labels refer to the neutral (N), cross-roll (CR), Eckhaus (E), and zigzag (ZZ) instabilities, respectively [1]. The shaded area is the region of stable wave numbers. The large bar at $r = 1.0$ is the band of stable wave numbers for one-dimensional cells without lateral boundaries. The smaller bar is the band of stable wave numbers in the presence of lateral boundaries [15].

Fig. 2. Evolution of the Swift-Hohenberg model near onset ($r = 0.1$) in a large square cell of aspect ratio 16 at three consecutive times (in units of a vertical diffusion time). The evolution is dominated by the motion, interaction, and annihilation of defects, regions where the local periodicity of the rolls is undefined. A one-dimensional defect, a grain boundary, is easily seen in the lower left and upper right corners of (b). The time scale to attain the stationary state in (c) is far longer than the diffusive time scale suggested by linear stability theory. The final state (c) is independent of the random initial conditions.

Fig. 3. Evolution of the Swift-Hohenberg model away from onset ($r = 0.9$) for the same cell and initial conditions as Fig. 2. The final stationary state, (b), is now reached on a much shorter time scale. The final state has highly curved rolls and many defects compared to Fig. 2c. A point defect, a dislocation, is evident on the lower right side of (b). The final state is similar to the flow at early times, (a), suggesting that the evolution is constrained. Simulations show that this constraint arises from r -dependent barriers to dislocation gliding [14].

Fig. 4. Linear stability diagram of infinite parallel rolls of wave number K as a function of the Rayleigh number, R , for the Boussinesq equations. The calculation was done for a Prandtl number of 7 [1]. The labels are the same as for Fig. 1 except for the oscillatory (OS), the skewed varicose (SV), and the knot (K) instabilities [1]. The area of stable wave numbers is shaded. There is a stable band of wave numbers near onset that vanishes for $R \geq 3R_c$. The skewed varicose instability seems to play an essential role in wave number selection [6] and the onset of turbulence [5].

Fig. 1

#84T0080

(a) $\tau = 4/9$

(b) $\tau = 5000$

(c) $\tau = 15000$

Fig. 2

#84T0081

(b) $\tau = 6430$

(a) $\tau = 11$

Fig. 3

#84T0082

Fig. 4

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20.

Plasma Res Lab, Austre Nat'l Univ, AUSTRALIA
 Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Prof. F. Cap, Inst Theo Phys, AUSTRIA
 Prof. Frank Verheest, Inst theoretische, BELGIUM
 Dr. D. Palumbo, Dg XII Fuslon Prog, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Dr. P.M. Sakanaka, Univ Estadual, BRAZIL
 Dr. C.R. James, Univ of Alberta, CANADA
 Prof. J. Teichmann, Univ of Montreal, CANADA
 Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Hannes Bernard, Univ British Columbia, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Zhengwu Li, SW Inst Physics, CHINA
 Library, Tsing Hua University, CHINA
 Librarian, Institute of Physics, CHINA
 Inst Plasma Phys, Academia Sinica, CHINA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Prof. Schatzman, Observatoire de Nice, FRANCE
 J. Radet, CEN-BP6, FRANCE
 AM Dupas Library, AM Dupas Library, FRANCE
 Dr. Tom Muai, Academy Bibliographic, HONG KONG
 Preprint Library, Cent Res Inst Phys, HUNGARY
 Dr. S.K. Trehan, Panjab University, INDIA
 Dr. Indra, Mohan Lal Das, Baneres Hindu Univ, INDIA
 Dr. L.K. Chavda, South Gujarat Univ, INDIA
 Dr. R.K. Chhajlani, Var Ruchi Marg, INDIA
 P. Kaw, Physical Research Lab, INDIA
 Dr. Phillip Roseau, Israel Inst Tech, ISRAEL
 Prof. S. Cuperman, Tel Aviv University, ISRAEL
 Prof. G. Rustegni, Univ Di Padova, ITALY
 Librarian, Int'l Ctr Theo Phys, ITALY
 Miss Ciella De Palo, Assoc EURATOM-CNEN, ITALY
 Biblioteca, del CNR EURATOM, ITALY
 Dr. H. Yameto, Toshiba Res & Dev, JAPAN
 Prof. M. Nishikawa, JAERI, Tokai Res Est, JAPAN
 Prof. T. Chida, University of Tokyo, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Prof. Sigeru Mori, JAERI, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Ichiro Kawakami, Nihon Univ, JAPAN
 Prof. Setoshi Itoh, Kyushu University, JAPAN
 Tech Info Division, Korea Atomic Energy, KOREA
 Dr. R. England, Ciudad Universitaria, MEXICO
 Bibliothek, Fon-inst Voor Plasma, NETHERLANDS
 Prof. B.S. Lilley, University of Waikato, NEW ZEALAND
 Dr. Suresh C. Sharma, Univ of Calabar, NIGERIA
 Prof. J.A.C. Cabral, Inst Superior Tech, PORTUGAL
 Dr. Octavian Petrus, ALI GUZA University, ROMANIA
 Prof. M.A. Hellberg, University of Natal, SO AFRICA
 Dr. Johan de Villiers, Atomic Energy Bd, SO AFRICA
 Fusion Div. Library, JEN, SPAIN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Dr. Erik T. Karlson, Uppsala Universitet, SWEDEN
 Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
 Dr. W.L. Weise, Nat'l Bur Stand, USA
 Dr. W.M. Stacey, Georg Inst Tech, USA
 Dr. S.T. Wu, Univ Alabama, USA
 Prof. Norman L. Oleson, Univ S Florida, USA
 Dr. Benjamin Ma, Iowa State Univ, USA
 Prof. Magne Kristiansen, Texas Tech Univ, USA
 Dr. Raymond Askew, Auburn Univ, USA
 Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electrophysical, USSR
 Institute Gen. Physics, USSR
 Prof. T.J. Boyd, Univ College N Wales, WALES
 Dr. K. Schindler, Ruhr Universitat, W. GERMANY
 Nuclear Res Estab, Jülich Ltd, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Dr. H.J. Kaepler, University Stuttgart, W. GERMANY
 Bibliothek, Inst Plasmforschung, W. GERMANY