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Informal Report

**Analysis of a dc Commutator Machine
for Exchange of Energy with a
Superconducting Coil**

University of California

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ANALYSIS OF A dc COMMUTATOR MACHINE FOR EXCHANGE OF ENERGY WITH A SUPERCONDUCTING COIL

by

H. F. Vogel

ABSTRACT

A 500-kW dc commutator machine, C, is analyzed for use in an LC circuit with a ringing period of approximately 4 s with a superconducting coil, L. Electrical measurements and the important design equations and characteristics are listed. Attention is paid to the calculation of the commutating voltage, which is an important design feature because it sets a limit to the current allowable in the LC circuit, amounting to 6 times rated machine current.

The equations for the energy loss components of the generator are given and fitted to experimental coast-down data. With a fitting accuracy of 1.2%, the loss coefficients for the bearing loss, brush loss, windage loss, and tooth eddy current loss in the remanent stator field are thus determined.

1. INTRODUCTION

Current reversal will be required in the ohmic-heating (OH) circuits of future tokamaks, because the toroidal plasma current I_p may be induced by changing the OH current from $-I_p/2n$ to $+I_p/2n$, where n is the effective turns ratio. The energy loss in the OH circuit is a minimum when the circuit is operated so the OH coil current reverses rather than increasing from 0 to I_p/n . If the OH coils are superconducting, then the current change $-I_p/2n$ to $+I_p/2n$ constitutes an optimum operating condition, because the current is thus limited to the coil's design value $|I_p/2|$. There are some questions regarding the permissible rate of flux change $|\dot{B}|$ in a superconducting coil

so the coil will remain in the superconducting state. It is, therefore, imperative in a pulsed coil development program to have a pulsed power facility available for testing the coils. A homopolar machine would be suited ideally for this purpose, because no commutator is needed and the machine can thus be built for large currents at much lower cost than a commutator machine.

At the present time and the near future, homopolar machines have the drawback that their use is infrequent and the cost advantage over dc commutator machines of the traction motor type does in reality not exist. Traction motors are being built, rebuilt, and reused in such large quantities for the railroads that their cost per unit of stored energy is quite cheap, i.e., approximately 0.9¢/J.

We report a pilot experiment here in which a surplus motor-generator set rated at 500 kW dc at 2000 A is used for commuting energy in an oscillatory circuit with a superconducting coil storing 400 kJ, with this report giving the details on the machine.

2. SUMMARY OF MOTOR-GENERATOR DATA, BY INSPECTION AND MEASUREMENT

Manufacturer: Westinghouse

Motor-Generator Assembly per Fig. 1

Total rotating mass 4.2t with centers of gravity as determined by levitation indicated in Fig. 1.

2.1 ac Motor Nameplate Data

60 Hz, 8 poles, 178 A/phase, 2300 V

720-HP output

2.2 dc Generator Data

Compound winding (not connected for the experiments)

Commutating Winding, see Fig. 2

2.2.1 Nameplate Data

250 V; 2000 A; 900 rpm

2.2.2 Linear Dimensions (no. of poles, $2p = 6$)
(no. of armature slots, $N = 108$)

Armature length	$\ell_w = 14\text{-}1/2 \text{ in.} = 36.3 \text{ cm}$
(effective armature length ℓ_{wi})	$= 35.6 \text{ cm}$
Armature diameter	$D = 28 \text{ in.} = 71.2 \text{ cm}$
Commutator length	$14 \text{ in.} = 35.6 \text{ cm}$
Commutator diameter	$18\text{-}1/4 \text{ in.} = 46.4 \text{ cm}$
Pole length	$14 \text{ in.} = 35.6 \text{ cm}$
Pole width, main field poles	$7 \text{ in.} = 17.8 \text{ cm}$
interpoles	$2 \text{ in.} = 5.1 \text{ cm}$
Air gap, main field poles	$13/64 \text{ in.} = 0.52 \text{ cm}$
interpoles	$9/16 \text{ in.} = 1.43 \text{ cm}$
Pole pitch $\tau_p = \pi D/2p =$	37.3 cm
Slot pitch $t = \pi D/N =$	2.07 cm
Brush length on commutator	$b = 1.29 \text{ in.} = 3.3 \text{ cm}$
hence, brush- $\beta = b/t$	$= 1.6$

2.2.3 Winding Data

No. of commutator bars	$k = 108$
No. of armature slots	$N = 108$
No. of passes through the same slot	$u = k/N = 1$
Coil pitch forward (slot pitch units)	$y_1 = 18$
backward (slot pitch units)	$y_2 = 17$
Winding pitch (<u>lap winding</u>)	$y = y_1 - y_2 = 1$
No. of independent armature windings	$m = y = 1^a$
No. of main field poles	$2p = 6$
$N/p = 36 \equiv \text{integer}; \text{ hence, there exist } p \text{ bars with no potential difference on the commutator.}$	
Pitch of equalizing winding	$y_v = k/p = 36$
No. of armature conductors	$z = 2N = 216$
No. of turns per armature coil	$s = z/2k = 1$
No. of turns, commutating winding	$W_w = 2$
No. of turns, compound winding	$W_k = 3$
No. of turns, main field winding	$W_F \approx 242$

^aHence, simplex winding.

2.2.4 Inductance Calculations

Carter factor for the air gaps $k_c = t/(t-\gamma\delta)$

Main field: $s/\delta = 0.635/0.52 = 1.22$; $\gamma = 0.24$; $k_c = 1.064$

Interpole field: $0.635/1.43 = 0.036$; $\gamma = 0.0367$; $k_c = 1.026$

Effective air gap, main field $\delta_F^I = (k_c \delta)_F = 0.55 \text{ cm}$

interpole field $\delta_W^I = (k_c \delta)_W = 1.47 \text{ cm}$

Main pole excitation $\Theta_F = B_{\text{air}}/\mu_o \times \delta^I = 2.9 \text{ kA}$

I_F measured for $B_{\text{air}} = 0.75 \text{ T}$ under

the main poles:

$$I_F \approx 12 \text{ A}$$

Hence, approx. no. of turns in field winding

$$W_F \approx 242$$

Slot leakage inductance¹

Refer to Fig. 3, where:

$$\text{Inductance number } \lambda = \frac{L}{\mu_o s^2 \ell}$$

s = no. of turns

ℓ = length

Slot inductance number $\lambda_N = \lambda_{su} + \lambda_{so}$

$$\text{Lower level inductance number } \lambda_{su} = \frac{4}{3} \frac{h_1'}{a} + \frac{h_1' + h_2}{a} + \frac{h_4}{a}$$

$$\text{Upper level inductance number } \lambda_{so} = \frac{h_1'}{3a} + \frac{h_2}{a} + \frac{h_4}{a}$$

$$\text{Mutual inductance number } \lambda_g = \lambda_{gu} = \lambda_{go} = \frac{h_1'}{2a} + \frac{h_2}{a} + \frac{h_4}{a}$$

Approximate dimensions: $a_4 = \frac{1}{4} \text{ in.} = 0.635 \text{ cm}$; $a = \frac{3}{8} \text{ in.} = 0.95 \text{ cm}$

$$h_2 = 1/8 \text{ in.} = 0.32 \text{ cm}$$

$$h_3 = 1/16 \text{ in.} = 0.16 \text{ cm}$$

$$h_4 = 1/16 \text{ in.} = 0.16 \text{ cm}$$

$$h_1' = 5/8 \text{ in.} = 1.59 \text{ cm}$$

$$h' = 1/8 \text{ in.} = 0.32 \text{ cm}$$

¹R. Richter, Elektrische Maschinen, Vol. I, Birkhäuser, p. 269.

$$\lambda_{su} = 2.23 + 0.67 + 0.17 = 3.07$$

$$\lambda_{so} = 0.558 + 0.337 + 0.17 = 1.063$$

$$\lambda_N = 4.133$$

$$\lambda_g = 0.837 + 0.337 + 0.252 = 1.426$$

$$\text{Total slot inductance number } \lambda'_N = \frac{1}{4} (\lambda_N + 2\lambda_g) = 1.746$$

$$\ell = \ell_i = 0.356 \text{ m}$$

$$\text{Total slot inductance } L_N = 3.13 \text{ } \mu\text{H}$$

End Turn Inductance

Refer to Fig. 4, where:

$$1/2 \cdot \ell_s = 33 \text{ cm}$$

$$\ell_s = 66 \text{ cm}$$

$$\tau = 37.3 \text{ cm}$$

$$\beta = b/t_K = 2.44$$

$$b + \delta = 1.6 \text{ cm}$$

$$L_s = 2.55 \beta s^2 \ell_s \left(\frac{\ell_s + \tau}{h + \beta(b + \delta) + \delta} \right)$$

$$= 1.92 \text{ } \mu\text{H}$$

$$\text{Total leakage inductance } L_\sigma = L_N + L_s = 5 \text{ } \mu\text{H}$$

$$\text{Armature inductance } L = \frac{L_\sigma}{2} \frac{z}{(2a)^2} = 15 \text{ } \mu\text{H}$$

2.2.5 Electric and Magnetic Data

$$\text{Electric loading}^* \text{ of the armature } A = \frac{\theta_{\text{slot}}}{t_{\text{slot}}} = \frac{zJ}{\pi D \cdot 2a} = 322 \text{ A/cm}$$

$$\text{Armature resistance measured between machine terminals } R_a = 1.9 \text{ m}\Omega$$

$$\text{Mean value of the induced EMF } E = U + JR_a = 253.8 \text{ V}$$

(i.e., generator duty; in motor mode U is different, i.e.,

$$253.8 + 3.8 = 257.6 \text{ V})$$

Mean value of the EMF induced in the coil by the stator field

$$e_W = \frac{2p}{k} E = 14.1 \text{ V}$$

Mean flux density linked with a coil terminated under brush centers

$$B_m = \frac{e_W}{2s\ell_{wi} v} = 0.59 \text{ T } (\ell_{wi} \approx 0.356 \text{ m; } s = 1 \text{ turn/coil})$$

* I.e., conductor current per centimeter of armature periphery.

MMF of armature field

$$\Theta_A = \frac{1}{2} A \tau_p = 6000 \text{ A}$$

MMF of interpole field

$$\Theta_F = J W_w = 4000 \text{ A}$$

Flux density of interpole gap field $B_F = \mu_o \Theta_F / \delta_F = 0.34 \text{ T}$

$$(\delta_F = 0.0147 \text{ m})$$

Flux density of quadrature gap field $B_Q = \mu_o \frac{-\Theta_F + \Theta_A}{\delta_F} = -0.17 \text{ T}$

EMF of motion of a coil shorted by brushes of length b_j in the quadrature

$$\text{field } e_F = 2 B_F v \ell_i s = 4.1 \text{ V}$$

$-L \frac{di}{dt}$ due to commutation by brushes of length b_j

$$e_r = - \frac{J}{a} \frac{v}{b_j} L = -5.1 \text{ V}$$

EMF of commutation $e_F + e_r = -1.0 \text{ V}$

Short-circuit resistance of coil $R_s = 2 R_a \frac{(2a)^2}{z} = 0.63 \text{ m}\Omega$

The emf of commutation manifests itself as voltage between adjacent commutator bars under the bushes. It may give rise to sparking and arcing if it exceeds more than a few volts. For this reason, the emf of commutation sets a limit to the machine's maximum operating current. On the present machine, operating in a ringing circuit with 4.4-s period, we observed spark-free operation to 8 kA (corresponding to $e_F + e_r \approx 4 \text{ V}$) and sparks leaving no visible traces on the commutator to the highest current tested, i.e. 12.2 kA (corresponding to $e_F + e_r \approx 6 \text{ V}$).

3. EQUATIONS OF MOTION AND CIRCUIT

The EMF $e = \frac{\omega \Psi}{2\pi}$

$$\Psi = 16.9 \text{ [Wb]}$$

$$E_o = 253.8 \text{ [V]}$$

Power balance $e i = \Theta \omega \dot{\omega}$

$$\omega_o = 94.2 \text{ [rad/s]}$$

$$\Theta = 466 \text{ [Js}^2\text{]} \text{ (See Sec. 3.3)}$$

Circuit Equation $L \ddot{i} + R \dot{i} + \dot{e} = 0$

Hence, $L \ddot{i} + R \dot{i} + \frac{\Psi^2}{(2\pi)^2 \Theta} i = 0$

Undamped oscillating frequency $\omega_o = (LC)^{-1/2}$

Damping factor

$$\delta = R/2L$$

Equivalent Capacitance

$$C = \Theta \left(\frac{2\pi}{\Psi} \right)^2 = 64.41 [F]$$

3.1 Coil Charged Initially, Machine at Rest

$$i = I_o \frac{\omega_o}{\omega} e^{-\delta t} \sin(\omega t + \alpha)$$

$$\alpha = \pi - \text{Arc tan } \frac{\omega}{\delta}$$

3.2 Coil and Machine Charged Initially

$$i(t) = [I_o \cos \omega t - (I_o \frac{\delta}{\omega} + \frac{V_o}{\omega L}) \sin \omega t] e^{-\delta t}$$

I_o = Initial current in coil

V_o = Initial voltage on capacitor (i.e., machine)

The maximum current occurs for

$$\omega t = \text{arc tan } \frac{V_o C + I_o R C}{V_o \delta C - I_o}$$

For the 300-kJ coil, $L = 6 \text{ mH}$; hence, $\omega_o = 1.61 \text{ rad/s}$

$$R = 2 \text{ m}\Omega; \delta = 0.17 [\text{sec}^{-1}]$$

$$\omega = 1.60 \text{ rad/s}$$

3.3 Determination of the Moment of Inertia

$$\text{Equation of motion: } \Theta \omega \dot{\omega} = ui - i^2 R_a - P_m$$

Where P_m is the sum of all mechanical losses

In the stationary state where $\dot{\omega} = 0$, we get for $\omega = \omega_{\text{ref}}$

$$(P_m)_{\text{ref}} = (ui)_{\text{ref}} - (i^2 R_a)_{\text{ref}}$$

During coast-down ($ui = 0$) we get

$$\Theta \omega_c \dot{\omega}_c = (I^2 R_a)_{\text{ref}} - (P_m)_{\text{ref}}$$

from which Θ may be determined.

We get $\Theta = 466 \text{ [Js}^2\text{]}$ for the stationary values measured at $\omega = 92.7 \text{ [rad/s]}$ (885 rpm): $V = 251 \text{ [V]}$, $I = 132 \text{ [A]}$, hence, $(P_m)_{\text{ref}} = 33.0 \text{ [kW]}$ and for a coast-down value $\dot{\omega}_c = -0.773 \text{ [rad/s}^2\text{]}$ measured at $\omega_c = 92.7 \text{ [rad/s]}$.

4. DETERMINATION OF THE BEARING LOSS, WINDAGE LOSS, AND BRUSH FRICTION LOSS FROM A COAST-DOWN TEST

4.1 Derivation of the Physical Model

The following loss mechanisms should be considered in a physical model

a) Bearing friction loss

$$P_B = 2.3 \ell \sqrt{\eta p d (1 + 4 \frac{d}{\ell})} v^{3/2},$$

Where $\eta = 0.0017 \text{ [kps/m}^2\text{]}$ (viscosity)

$\ell = 6.1 \text{ [cm]}$ = bearing length (end bearings total) see Fig. 4
 4.3 [cm] bearing length (center bearing).

$p = 33.9 \text{ [bar]}$ for the end bearings pressure

38.3 [bar] for the center bearing pressure

$d = 10.2 \text{ [cm]}$ for the end bearing diameter

12.7 [cm] for the center bearing diameter

This yields approximately 360 W for each of the two sets of bearings. Note that the equation for P_B is normalized and MKS units may be used with the same coefficient, 2.3, if the viscosity is chosen as $0.017 \text{ [Ns/m}^2\text{]}$.

b) Brush friction loss

$$P_{BR} = \mu p A N v,$$

where μ = brush friction coefficient = 0.1 ... 0.5

p = brush load

A = brush area

N = no. of brushes = 42

v = peripheral velocity $\approx 22 \text{ m/s}$ (at full speed)

we measured $pAN = 964 \pm 163 \text{ [N]}$

This yields for (for $\mu = 0.1$) 2.1 kW at full speed.

c) Windage loss

$$P_W = \lambda_R \frac{\rho D^5}{8\pi^3} \omega^3,$$

Where $\lambda_R \approx 4.6$

$$\rho = 1.3 \text{ [kg/m}^3\text{]}$$

D = equivalent aerodynamic diameter ≈ 0.9 [m]

$\omega = 30 \pi$ [rad/s] at full speed.

This yields approximately 17 kW at full speed, where $\lambda_R = 6.6$ because we have two machines on the same shaft.

d) Teeth - eddy loss

$$P_T = 0.13 G \left(\frac{N_T}{100} \right)^2 \cdot \left(\frac{6}{2\pi} \right)^2 B^2 \omega^2,$$

where

G = weight of the teeth eddy current region

N_T = no. of teeth

B = induction in the air gap-use remanence if the machine is not excited ≈ 0.9 [T]

ω = angular velocity, 30π (at full speed)

The teeth eddy-loss amounts to approximately $P_T = 17.8$ kW.

The three loss equations scale therefore with the angular velocity as follows:

$$P_B \propto \omega^{3/2}; P_{BR} \propto \omega; P_W \propto \omega^3; P_T \propto \omega^2.$$

The energy flow equation can thus be written

$$M\dot{\omega} = 0\omega\dot{\omega} = \alpha\omega + \beta\omega^{3/2} + \gamma\omega^2 + \delta\omega^3$$

from which a differential equation for the angular velocity follows, i.e.

$$\dot{\omega} = \frac{\alpha}{0} + \frac{\beta}{0}\omega^{1/2} + \frac{\gamma}{0}\omega + \frac{\delta}{0}\omega^2, \quad (1)$$

where

$\beta = \mu p A N r_{BR}$, where r_{BR} is the collector radius

$$\alpha = 0.813 \ell d^2 \sqrt{\eta p (1 + 4 \frac{d}{\ell})}$$

$$\gamma = 0.13 G \left(\frac{N_T}{100} \right)^2 \left(\frac{6}{2\pi} \right)^2 B^2$$

$$\delta = \lambda_R \rho \frac{D^5}{8\pi^3}$$

Equation 1 is solved numerically for $\omega = \omega(t)$ and the integral $n = \int_0^t \omega d\tau$ obtained numerically also and fitted to the experimentally determined set n_i , where the n_i -set is provided by a revolutions counter. The parameters in the fitting problem are ω_0 , α , β , γ , and δ , where $\omega_0 \equiv \omega(t=0)$. Numerical analysis is done by program DIFF, which plots and lists $n(t)$, $\omega(t)$, $\dot{\omega}(t)$, α , β , γ , δ . The plots for the coast-down run are shown in Fig. 5, for which we obtained

$$\alpha/\theta = 0.003 [s^{-2}]$$

$$\beta/\theta = 0.1009 [s^{-3/2}]$$

$$\gamma/\theta = 0.0043 [s^{-1}]$$

$$\delta/\theta = 44 \times 10^{-6}$$

$$\omega_0 = 108.6 [\text{rad/s}]$$

From this set of fitting parameters, we conclude that the coefficient in the relationship for the bearing loss is 30% high (i.e., 3.0 instead of 2.3). For this result, we assumed $\ell = 0.104$ m; $d = 0.117$ m; and $p = 3.6 \times 10^6$ N/m². An indeterminate configuration is used for the shaft support, where three bearings exist, thus rendering the bearing pressure components indeterminate. It is a possibility that this may account for the 30% increase.

We further conclude that the brush friction coefficient is 0.10 for $p_{AN} = 964$ [N]. We obtain $\lambda_R = 6.6$ for the windage coefficient if we set $D = 0.9$ m. The latter result exceeds the $\lambda_R = 4.6$ value suggested in Section 4c, a fact explained by the presence of two rotating machines (i.e., the induction motor and the dc generator, both mounted on the same shaft). The fitting parameter for the tooth eddy loss implies a tooth mass - remanent flux density product $GB^2 = 14.5$ [kgT²], or approximately half of this number for each of the two machines.

4.2 Fitting of the Physical Model

Equation 1 describes the coasting machine set adequately because the experimental data were fitted with a standard deviation $\sigma = 1.2\%$ for all but the initial two points.^b Program DIFF allows thus to compute the coefficients of the bearing loss, brush friction loss, windage loss, and teeth eddy loss with good accuracy from the experimental data of one coast-down test. Being able to fit the coast-down data with this degree of accuracy leads us to the following conclusions:

- a) One can indeed obtain the bearing, brush, windage, and remanent field - induced eddy loss, each from one coast-down run.
- b) The windage loss shows the ω^3 dependency described by Richter and not the ω^2 dependency described by Schuisky.

5. Acknowledgments

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6. Bibliography

All electrical and mechanical relationships used in this report, except for the equation of motion, Eq. 1, are discussed in

Rudolf Richter, Elektrische Maschinen, Vol. 1: Allgemeine Berechnungselemente, die Gleichstrommaschinen, 3rd Ed. (Birkhäuser, Basel, 1967).

The subjects of tooth eddy loss, windage loss, and bearing loss are discussed also in

W. Schuisky, Induktionsmaschinen (Springer-Verlag, 1957).

^bThe first point $n_1 = 0$ is a null and its vicinity therefore not suitable for evaluating the normalized standard deviation.

DIMENSIONS IN INCHES
WEIGHTS IN METRIC †

Fig. 1. 500-kW motor-generator set.

THE COMMUTATING WINDING IS TO PERMIT SPARK FREE OPERATION WITH THE BRUSHES FIXED IN THE GEOMETRICALLY NEUTRAL ZONE. THE COMMUTATING WINDING DOES NOT PREVENT FIELD DISTORTION UNDER THE MAIN FIELD POLES AND THE RESULTING OUTPUT VOLTAGE DROP.

POLARITY AND SENSE OF ROTATION

QUARTER CYCLE	VOLTAGE	CURRENT	MODE	ROTATION	
I	+	+	MOTOR	↶	MAIN FIELD NOT CHANGED
II	+	-	GENERATOR	↷	
III	-	+	MOTOR	↶	
IV	-	-	GENERATOR	↷	

Fig. 2. Polarities and effect of commutating winding.

Fig. 3. Armature slot, definition of linear dimensions.

Fig. 4. Armature winding and turn, definition of $b+\delta$.

Fig. 5. Coast-down test, plot of revolutions count n , velocity ω , angular deceleration $-\dot{\omega}$.