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AUTHOR(S): Leon Heller, T-5

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Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Computation of the return current in encephalography: the auto solid angle

Leon Heller

Theoretical Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

In the integral equations that arise in potential theory it is necessary to evaluate the solid angle subtended at a point r on the surface by each element of area of the surface. If the surface is approximated by a set of triangles then those triangles that contain the point r as a vertex require special attention, and this is referred to as the "auto solid angle" problem. We give an exact formula for evaluating the auto solid angle, and a prescription for apportioning it amongst the vertices of these triangles.

INTRODUCTION

The geometric problem that we consider arises in the numerical solution of some integral equations in potential theory. One example comes from encephalography, where the *total* electric current J is composed of two distinct pieces,

$$J = J^P - \sigma \nabla V . \quad (1)$$

J^P is the *primary* current that flows within neurons and $-\sigma \nabla V$ is the *return* current that flows in the extracellular medium, with σ being the electric conductivity and V the electric potential. The "forward" problem consists of assuming a functional form for $J^P(r)$ and $\sigma(r)$ and solving the continuity equation

$$\nabla \cdot (J^P - \sigma \nabla V) = 0 , \quad (2)$$

for $V(r)$. If σ is a different constant in different regions, e.g., brain, skull, and scalp, then the 3-dimensional partial differential equation (2) can be transformed to a 2-dimensional integral equation, where the integration runs over all the surfaces, S_n , on which σ is discontinuous.

Denoting by $\sigma'(\sigma'')$ the value of σ just inside (outside) one of these surfaces, and with r being a point on that surface, this equation can be written¹

$$V(r) = V_0(r) + \frac{1}{2\pi(\sigma' + \sigma'')} \sum_n (\sigma'_n - \sigma''_n) \int_{S_n} d\Omega'(r, r') V(r') . \quad (3)$$

In Eq. (3) $d\Omega'(r, r')$ is given by

$$d\Omega'(r, r') = n(r') \cdot \frac{r' - r}{|r' - r|^3} dS', \quad (4)$$

and represents the element of solid angle subtended at r by an element of surface area dS' at the point r' . $n(r')$ is a unit vector normal (outward) to the surface. The expression for the inhomogeneous term $V_0(r)$ will not be needed in this paper; it is a known function once $JP(r)$ is specified.

An important property of solid angle concerns its integral over a single closed surface,

$$\int d\Omega'(r, r') = \begin{bmatrix} 0 \\ 2\pi \\ 4\pi \end{bmatrix}, \text{ for } r \begin{bmatrix} \text{outside} \\ \text{on} \\ \text{inside} \end{bmatrix} \text{ the surface.} \quad (5)$$

All three possibilities are present in Eq. (3); but for the geometric problem being considered in this paper only the middle case, with r on the surface in question, is of interest.

In any numerical solution of Eq. (3) the integral

$$\int d\Omega'(r, r') V(r') \quad (6)$$

gets converted to a discrete sum

$$\sum_j \Omega_{ij} V_j , \quad (7)$$

and it is important to preserve the normalization from Eq. (5),

$$\sum_j \Omega_{ij} = 2\pi . \quad (8)$$

One illustration of this need can be seen from the homogeneous version of Eq. (3), which has a solution $V(r)=\text{constant}$.² Failure to satisfy Eq. (8) in a discrete approximation to Eq. (3) would result in the loss of this solution.

II. CLOSE AND DISTANT REGIONS

Extra care must be taken³ when the point r' in Eq. (4) gets close to the point r . This can be seen from the example of a sphere, as illustrated in figure 1. Straightforward evaluation of the element of solid angle from Eq.(4) gives

$$d\Omega'(r,r') = \frac{1}{4R^2 \sin \frac{\psi}{2}} dS' , \quad (9)$$

which shows that the amount of solid angle subtended at r *per unit surface area* (at r') becomes arbitrarily large as r' approaches r ! R is the radius of the sphere, ψ is the polar angle of r' , and ϕ is its azimuthal angle. Even if dS' is expressed in terms of increments in ψ and ϕ ,

$$dS' = R^2 \sin \psi d\psi d\phi , \quad (10)$$

and this expression is put into Eq. (9),

$$d\Omega'(r,r') = \frac{1}{2} \cos \frac{\psi}{2} d\psi d\phi , \quad (11)$$

it is still true that the greatest amount of solid angle comes from r' near r ($\psi=0$).

Fig. 1. A portion of a sphere of radius R with r at the pole and r' at polar angle ψ and azimuthal angle ϕ . $n(r')$ is the outward unit normal to the surface at r' .

This point becomes even more important when a smooth surface is approximated by a collection of planar elements, such as triangles. Two different choices are made in the literature for the discrete variables V_i . In one approach the potential is evaluated at an *interior* point of the triangle, e.g., at its center of mass. (The consequences are similar if the *average* value of V over a single triangle is taken as the discrete variable.) For this choice the one triangle that contains the point in question, r_0 , subtends *zero* solid angle at that point, since the normal to such a triangle, $n(r')$, is perpendicular to every vector $r'-r_0$ that lies in the triangle. Since the discussion above showed that the portion of the surface nearest r_0 actually subtends the most solid angle, this may produce a significant error. It is true, however, that all the other triangles on that same surface subtend a total solid angle of 2π at r_0 , thereby satisfying Eq. (8). We shall not be concerned with this approach any further in this paper.

The situation is quite different in the approach that chooses the V_i to be the values of the potential at the vertices of the triangles. All the triangles that share a particular vertex, r_0 , subtend zero solid angle at that point, and the remaining triangles subtend a total solid angle at r_0 that is *less* than 2π . (If the surface were concave at r_0 rather than convex, the total would be *more* than 2π .) In the derivation of Eq. (5) it is assumed that the surface is *smooth*

at the point $r (=r_0)$, but choosing it to be one of the vertices violates that assumption. With this approach, therefore, there is no choice but to treat the triangles that contain the point r_0 separately from all the others; the former have been referred to as the "close" region and the latter as the "distant" region.⁴ Figure 2 shows the close region.

It is necessary, therefore, to treat the close region more carefully; this has been referred to as the "auto solid angle" problem.³ There are two aspects to this problem, which we will consider in order. (i) How much solid angle is *missing*, Ω_m , as a result of triangulation, and (ii) how should Ω_m be apportioned amongst the vertices of the triangles in the close region? Both of these questions were considered in reference 3 but we shall generalize and slightly modify the results obtained there.

III. THE MISSING SOLID ANGLE

To compute Ω_m first find the total solid angle subtended at r_0 by all the triangles of the *distant* region, Ω_d ; the missing solid angle, Ω_m , is just the difference between Ω_d and 2π ,

$$\Omega_m = 2\pi - \Omega_d. \quad (12)$$

If the solid angle subtended at r_0 by every single triangle of the distant region has already been computed, then the sum of all those values gives Ω_d . There is a direct way to calculate Ω_d , however, just from a knowledge of the boundary that separates the close region from the distant region. Figure 2 shows this boundary as the (generally) nonplanar polygon with vertices $r_1, r_2, \dots, r_N$, which are the other vertices of the triangles that contain r_0 . It is helpful to think of the pyramidal surface that has r_0 as its apex and which passes through the perimeter of this polygon; this surface is shown shaded in Fig. 2. It is seen that Ω_d is just the solid angle subtended at r_0 by the portion of space that lies *below* this pyramid.

(a)

(b)

Fig. 2. All the triangles that contain a given vertex, r_0 , constitute the "close" region to that point. (a) A view of the close region from the front, with the triangles that contain r_0 shown shaded. The bold lines that connect the other vertices of the close region form a nonplanar polygon. (b) The polygon seen from r_0 . r^* is any convenient point below r_0 , e.g., the center of mass of the vertices of the polygon.

Now pick any convenient point, r^* , e.g., the center of mass of the vertices of the polygon, and draw straight lines from it to each of the vertices, thereby subdividing the polygon into triangles. See Fig. 2(b). Then use a formula for the solid angle subtended by each of these triangles at r_0 , and sum the result over all the triangles in the polygon to obtain Ω_d .

One convenient version of this formula appears in van Oosterom and Strackee⁵ as their Eq. (8). With R_1 , R_2 , and R_3 , the vectors from the point r_0 to the three vertices of any triangle, their formula for the solid angle Ω subtended by that triangle at r_0 is

$$\tan\left(\frac{1}{2}\Omega\right) = - \frac{R_1 \cdot R_2 \times R_3}{R_1 R_2 R_3 + (R_1 \cdot R_2) R_3 + (R_2 \cdot R_3) R_1 + (R_3 \cdot R_1) R_2} . \quad (13)$$

To eliminate a twofold ambiguity in Ω would require another condition, but we shall only be interested in small triangles for which $-\pi < \Omega < \pi$. We have chosen the opposite sign convention with Ω positive if R_1 , R_2 , and R_3 occur in counterclockwise order as seen from r_0 .

To illustrate these ideas, consider the special case in which the close region is bounded by N points $r_1, r_2, \dots, r_N$, with the latter forming a regular N -sided plane polygon and r_0 symmetrically located directly above the center of mass r^* . Due to the symmetry it is sufficient to compute the solid angle Ω subtended at r_0 by any one triangle; multiplying the result by N gives Ω_d . Figure 3 shows the geometry for the triangle with vertices r_1, r_2 , and r^* ; the central angle is $\beta = 2\pi/N$. We now proceed to evaluate Eq. (13) in terms of β and θ .

Fig. 3. Geometry for the symmetric case of a regular N -sided plane polygon with central angle $\beta=2\pi/N$. The points are labelled as in Fig. 2(b).

From Fig. 3 it is seen that under a rotation about the direction $\mathbf{u}=-\mathbf{R}_3/R_3$ by an angle β , $\mathbf{R}_1$ becomes $\mathbf{R}_2$. The algebraic expression for this rotation is

$$\mathbf{R}_2 = \cos\beta \mathbf{R}_1 + (1-\cos\beta)(\mathbf{R}_1 \cdot \mathbf{u})\mathbf{u} + \sin\beta \mathbf{u} \times \mathbf{R}_1. \quad (14)$$

With the help of Eq. (14) and Fig. 3, all the products in Eq. (13) can be evaluated. For example, when working out the triple product in the numerator, only the term in $\mathbf{R}_2$ involving $\sin\beta$ survives, and yields

$$-\mathbf{R}_1 \cdot \mathbf{R}_2 \times \mathbf{R}_3 = \sin\beta \frac{(\mathbf{R}_3 \times \mathbf{R}_1)^2}{R_3} = \sin\beta \sin^2\beta R_3 R_1^2 = \sin\beta \sin^2\theta \cos\theta R_1^3. \quad (15)$$

Evaluating the other products in Eq. (13) in similar fashion, it becomes

$$\tan\left(\frac{1}{2}\Omega\right) = \frac{\sin\beta \sin^2\theta}{(1+\cos\theta)^2 + \cos\beta \sin^2\theta}. \quad (16)$$

One check on Eq. (16) is obtained in the limit that the number of sides of the polygon, N , becomes large so that it approaches a circle. In that limit $\beta=2\pi/N$ becomes small and it follows directly from Eq. (16) that

$$\Omega_d = N\Omega \cong 2\pi(1-\cos\theta) \quad (\text{large } N) \quad (17)$$

which is the correct expression for the solid angle subtended at r_0 by a cone of vertical angle 2θ . Except for an opposite sign convention, Eq. (17) is the same as Eq. (A1) in reference 3.

In practice it is common for each point on the triangulated surface to be surrounded by 6 triangles, so we now want to see how different Ω_d is for $N=6$ as compared with the large N result given in Eq. (17). Putting $\beta=\pi/3$ into Eq. (16) and considering only small triangles, i.e., values of θ that are near $\pi/2$, leads to

$$\Omega_d = 6\Omega \cong 2\pi \left(1 - \frac{4\sqrt{3}}{2\pi} \cos\theta \right) = 2\pi (1 - 1.10 \cos\theta) . \quad (18)$$

Comparing Eq. (18) for $N=6$ (a hexagon) with Eq. (17) for large N (a circle), it is seen that the respective missing solid angles, $\Omega_m=2\pi-\Omega_d$, differ by 10% if θ is close to $\pi/2$.

Since the nearest vertices to an arbitrary point r_0 generally do not form a regular polygon, it is necessary to follow the more general procedure described below Eq. (12) in order to compute Ω_m . Once this has been done the next question is how to apportion Ω_m amongst the vertices $r_0, r_1, \dots, r_N$ of the close region.

IV. APPORTIONMENT OF Ω_m

There are two steps to be considered in deciding how to divide up Ω_m . The first one is what fraction of Ω_m should be assigned to each triplet of vertices, $(r_0, r_1, r_2), (r_0, r_2, r_3), \dots, (r_0, r_N, r_1)$; and then, how should each fraction be further divided amongst the members of that triplet. To answer these questions we shall approximate the surface near any one triplet by a portion of a sphere. Once this has been done an approximate value for the solid angle subtended by that triplet at r_0 can be calculated. Repeating this process for the other triplets and taking ratios determines the proper fraction for each triplet. Assuming that $V(r)$ varies linearly with distance on the surface of each spherical portion then determines how to further subdivide the solid angle amongst the three

vertices of each triplet. Even though approximations are made in these calculations, since they only involve ratios the total solid angle subtended by the close region at r_0 will sum precisely to Ω_m , and the total solid angle subtended by the entire surface at r_0 will be 2π .

Since three points do not determine a unique sphere, we use the following procedure. To find a suitable sphere that passes through the triplet of vertices (r_0, r_1, r_2) , for example, require that it also pass through the next vertex of the close region, r_3 . See Fig. 2. The location of the center of this sphere, r_c , can be found by solving the three simultaneous linear equations

$$\left(r_c - \frac{r_0 + r_i}{2} \right) \cdot (r_0 - r_i) = 0, \quad i=1,2,3 \quad (19)$$

for the three components of r_c . The radius of this sphere has the value

$$R = |r_0 - r_c| = |r_1 - r_c|. \quad (20)$$

Fig. 4. A portion of a sphere that passes through r_0 (the pole), r_1 , and r_2 . The curve connecting r_0 and r_1 has fixed longitude, ϕ_1 , and the one connecting r_0 and r_2 has longitude ϕ_2 . The curve connecting r_1 and r_2 is given by Eq. (24). The point $\bar{r}$ is on the same longitude as r_1 , and on the same latitude as r_2 . ϕ_{12} is the difference of the longitudes.

As shown on Fig. 4, with the pole of this sphere located at r_0 the polar angles of the points r_1 and r_2 can be gotten from

$$\sin \frac{\psi_i}{2} = \frac{|r_i - r_0|}{2R}, \quad i=1,2. \quad (21)$$

In order to find ϕ_{12} , the difference in the azimuthal angles of r_1 and r_2 , it is convenient to introduce the point $\bar{r}$ that is on the same longitude as r_1 but has the same latitude as r_2 [see Fig. 4]. After some algebra it can be shown that

$$\bar{r} = r_C + \frac{1}{\sin \psi_1} [\sin (\psi_1 - \psi_2) (r_0 - r_C) + \sin \psi_2 (r_1 - r_C)], \quad (22)$$

and again referring to Fig. 4,

$$\sin \frac{\phi_{12}}{2} = \frac{|\bar{r} - r_2|}{2R \sin \psi_2}. \quad (23)$$

The spherical region containing the points r_0 , r_1 , and r_2 is bounded on one side by the line of longitude (ϕ_1) that connects r_0 and r_1 , and on the second side by the line of longitude (ϕ_2) that connects r_0 and r_2 . To complete the specification of this region it is necessary to determine the nature of the boundary that connects r_1 and r_2 . From the definitions of Ω_d and Ω_m the proper way to do this is to project from r_0 through the straight line that connects r_1 and r_2 , and see where that projection intersects the spherical surface. For small values of ψ_1 and ψ_2 , the resulting curve $\psi(\phi)$ contains a term linear in $\sin \phi$ and a term linear in $\cos \phi$. For simplicity we shall also assume that ϕ_{12} is small and approximate this by

$$\psi(\phi) = \psi_1 + \frac{\phi - \phi_1}{\phi_{12}} (\psi_2 - \psi_1). \quad (24)$$

The curve on Fig. 4 connecting r_1 and r_2 represents Eq. (24).

We now want to calculate the solid angle $\Omega(r_0; r_1, r_2)$ subtended at r_0 by the spherical region bounded by r_0 , r_1 , and r_2 , which is shown in bold outline on Fig. 4. The element of solid angle is given in Eq. (10), and since we are only

interested in small values of ψ the linear variation of ψ with ϕ in Eq. (24) leads to

$$\Omega(r_0; r_1, r_2) = \frac{1}{2} \int_0^{\phi_{12}} c \phi \int_0^{\psi(\phi)} d\psi = \frac{\psi_1 + \psi_2}{4} \phi_{12}, \quad (25)$$

which says that $\Omega(r_0; r_1, r_2)$ equals 1/2 the average polar angle multiplied by the difference in the azimuthal angles. After repeating this calculation for all the other triplets of the close region and summing,

$$\Omega(r_0) \equiv \Omega(r_0; r_1, r_2) + \Omega(r_0; r_2, r_3) + \dots + \Omega(r_0; r_N, r_1), \quad (26)$$

the fraction of the missing solid angle Ω_m to be assigned to each triplet, e.g., (r_0, r_1, r_2) , is

$$f(r_0; r_1, r_2) = \frac{\Omega(r_0; r_1, r_2)}{\Omega(r_0)}. \quad (27)$$

The final task is to apportion each individual solid angle amongst its three vertices; for example, what fractions of $f(r_0; r_1, r_2)\Omega_m$ should be assigned to each of the points r_0 , r_1 , and r_2 . The goal here, as always, is to choose the respective solid angles Ω_0 , Ω_1 , and Ω_2 so as to make the discrete sum a good approximation to the integral that appears in Eq. (3),

$$\int d\Omega'(r_0, r') V(r') \equiv \Omega_0(r_0; r_1, r_2)V_0 + \Omega_1(r_0; r_1, r_2)V_1 + \Omega_2(r_0; r_1, r_2)V_2, \quad (28)$$

where the integration is over the spherical region bounded by r_0 , r_1 , and r_2 , shown in Fig. 4. In Eq. (28) $V_0 = V(r_0)$, etc., and the three Ω 's sum to the total,

$$\Omega_0 + \Omega_1 + \Omega_2 = f(r_0; r_1, r_2) \Omega_m. \quad (29)$$

To decide this matter requires an assumption about how $V(r)$ varies with ψ and ϕ . One would expect terms involving $\psi \cos \phi$ and $\psi \sin \phi$ near the pole of a sphere, but, as before, we shall approximate this by

$$V(r) = a + b\psi + c\psi\phi. \quad (30)$$

The second term in Eq. (30) represents a linear variation with *distance on the sphere* along a given longitude, and the last term represents the same thing along a given longitude or latitude. (Recall that we are only interested in small values of ψ .) After expressing the coefficients a , b , and c in Eq. (30) in terms of V_0 , V_1 , and V_2 , inserting $V(r)$ into Eq. (28), and using the expression for the element of solid angle given in Eq. (25), the following formulas are obtained for Ω_0 , Ω_1 , and Ω_2 :

$$\Omega_0(r_0; r_1, r_2) = \frac{\phi_{12}}{48} \left(7\psi_1 + 7\psi_2 - \frac{\psi_1^2}{\psi_2} - \frac{\psi_2^2}{\psi_1} \right) \frac{\Omega_m}{\Omega(r_0)},$$

$$\Omega_1(r_0; r_1, r_2) = \frac{\phi_{12}}{48} \left(3\psi_1 + 2\psi_2 + \frac{\psi_2^2}{\psi_1} \right) \frac{\Omega_m}{\Omega(r_0)},$$

and

$$\Omega_2(r_0; r_1, r_2) = \frac{\phi_{12}}{48} \left(3\psi_2 + 2\psi_1 + \frac{\psi_1^2}{\psi_2} \right) \frac{\Omega_m}{\Omega(r_0)}. \quad (31)$$

For the special case that $\psi_1 = \psi_2$, Eq. (31) leads to the result that the proportions of $f(r_0; r_1, r_2)\Omega_m$ assigned to r_0 , r_1 , and r_2 are $\Omega_0: \Omega_1: \Omega_2 = 1/2: 1/4: 1/4$. The fact that the point r_0 itself gets the largest fraction is a reflection of the discussion in Eqs. (9)-(11), where it is shown that the portion of the surface closest to a given point subtends the most solid angle at that point. This is to be contrasted with the treatment of the triangles in the distant region³, where each vertex of a given triangle is assigned 1/3 of the solid angle subtended by that triangle. (We note that the latter apportionment could also be improved to reflect a linear variation of the potential over the triangle.)

The ratios do change, however, if the polar angles of the other two points are unequal. If $\psi_1 = 2\psi_2$, for example, then the optimal proportions are $\Omega_0: \Omega_1: \Omega_2 = 0.46: 0.24: 0.30$. It is seen from Eq. (31) that the fraction assigned to the point with the larger polar angle is practically constant ($\approx 1/4$) for moderate ratios ψ_2/ψ_1 , but that r_0 gets less than 1/2, in general, and the point with the smaller polar angle gets more than 1/4.

The appearance of some negative terms in the expression for Ω_0 in Eq. (31) may seem strange, especially if ψ_1 is much smaller than ψ_2 , (or vice versa). In that event, however, r_1 is very close to r_0 and hence V_1 and V_0 are nearly

equal, so only the sum of Ω_0 and Ω_1 would be relevant. Finally, the apportionment process described in Eqs. (25), (26), and (31) must be repeated for each of the other triplets (r_0, r_2, r_3) through (r_0, r_N, r_1) .

V. SUMMARY

The 2-dimensional integral equations that arise in potential theory, such as Eq. (3) from electro- and magneto-encephalography, involve a kernel that is equal to the solid angle per unit surface area subtended at a point r on the surface. This quantity is largest for the points that are closest to r [Eqs. (9)-(11)], and actually becomes infinite as the element of surface area approaches r .

If the surface is approximated by a set of triangles, and the vertices of those triangles are taken as the discrete set of points at which the potential is evaluated, then particular attention must be paid to the triangles that share the vertex r_0 , at which the solid angle is being evaluated; they are referred to as the "close" region to that point. Since these triangles subtend zero solid angle at r_0 , the total solid angle subtended by that surface is just that due to the "distant" region, Ω_d , and the difference between this and 2π is the missing solid angle Ω_m . We have presented a direct way to calculate Ω_m just from a knowledge of the nearest neighbor vertices to r_0 . We have shown in Eq.(18), for the special case that these nearest neighbor points form a regular hexagon, that Ω_m differs by 10% from the approximation in reference 3, [which is the same as our Eq. (17)].

We have next given a prescription for apportioning Ω_m , first amongst the various triangles of the close region, and then amongst the three vertices of each triangle. With r_0 taken to be the pole of a different sphere for each triangle, that triangle's share of Ω_m is proportional to the average value of the polar angles of its other two vertices multiplied by the difference in their azimuthal angles [Eq. (25)]. The further subdivision of this solid angle amongst the three vertices of the given triangle is expressed in Eq. (31).

If the nearest neighbor points to r_0 are symmetrically located, then $1/2$ of Ω_m is assigned to r_0 and the other half is divided equally amongst the other vertices of the close region; this division is the same as in reference 3. In general, however, there will be deviations from the symmetrical case, and they are prescribed by Eq. (31).

We plan to see how much of an effect each of these modifications of the auto solid angle has on the numerical solution of the integral equation.

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