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Velocity-Space Particle Loss in
Field-Reversed Configurations

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The particle confinement criteria for axisymmetric field-reversed configurations (FRCs) are obtained from the two constants of motion, H and P_θ (1-3). Here, H and P_θ are the total energy and canonical angular momentum of a particle. It is found that only part of (H, P_θ) space is available for particle confinement. As with the loss in velocity space for a simple mirror, a loss region in (H, P_θ) space, or equivalently in velocity space, have a significant influence on plasma behavior in FRCs. In present work we study the effect of this loss region on particle loss.

For simplicity, approximate particle confinement criteria⁽¹⁾ are used. Furthermore, the lower bound on P_θ/q , which represents a constraint for particle confinement and reduces the confinement region in (H, P_θ) space⁽¹⁾, consequently in (r, y) space, is neglected. The particle confinement criteria then become:

$$\left. \begin{aligned} \frac{(P_\theta - \frac{q\psi}{c})^2}{2mr^2} \leq H < -\frac{qB_0}{mc} P_\theta, \\ P_\theta/q < 0. \end{aligned} \right\} \quad (1)$$

With a fast loss of unconfined particles the constraints on the particle distribution function, f , are $f = 0$ on the loss boundary and f finite for all y . In present case, after shifting the origin of (v_r, v_θ, v_z) space from $(0, 0, 0)$ to $(0, -qB_0r/mc, 0)$, this condition becomes $f(r, y, t) = 0$ for $|y| = v = v_c(r, z)$, where $v_c(r, z) = |q|B_0r(1-2\psi/B_0r^2)^{1/2}/mc$, and all other notations follow convention. This represents a spatially dependent

but isotropic boundary condition in velocity space for f .

Let us consider the particle loss due to like-particle collisions. Making use of the assumption that the spatial gradient is relatively small and the fact that $\underline{y} \times \underline{B} \cdot \nabla_{\underline{y}} f$ vanishes for an isotropic f , the Fokker-Planck kinetic equation in spherical velocity coordinates, (v, θ_v, ψ_v) , reduces to $\partial f(r, z, v, t) / \partial t = (\partial f / \partial t)_c$. By using a model distribution function, f_0 , for f in evaluating the Rosenbluth potentials, g and h , where

$$f_0(r, \underline{y}) = n(r, z) C_1(r, z) \left(1 - \frac{v^2}{v_c^2}\right) \exp\left(-\frac{v^2}{v_c^2}\right) U\left(1 - \frac{v}{v_c}\right), \quad (2)$$

$$C_1(r, z) = \left\{ \int_0^{v_c(r, z)} 4\pi v^2 \left(1 - \frac{v^2}{v_c^2}\right) \exp\left(-\frac{v^2}{v_c^2}\right) dv \right\}^{-1},$$

$U(x)$ and n being the heavy-side unit step function and particle number density, respectively, and changing variable $\underline{y} = \underline{y}' v_c$, the kinetic equation becomes

$$\frac{\partial F(r, z, v', t)}{\partial t} = \frac{\Gamma n}{v_c^3} D\{F\}, \quad 0 \leq v' \leq 1, \quad (3)$$

with

$$\left. \begin{aligned} F(r, z, v', t) &= 0 \text{ for } v' = 1, \\ F(r, z, v', t) &\text{ finite for all } v'. \end{aligned} \right\} \quad (4)$$

Here D is a second order differential operator depending on v' only, $F(r, z, v', t) \equiv f(r, z, v \equiv v' v_c, t)$ and $\Gamma = 4 q^2 \ln \Lambda / m^2$. Equation (3) is linear in F and separable in $\underline{r}$ and v' .

In solving the boundary-value problem, defined by Eqs. (3-4), it is useful to solve the following eigenvalue problem first:

$$D\{V(v')\} = \alpha V, \quad 0 \leq v' \leq 1, \quad (5)$$

with $V(1) = 0$ and finiteness of V . This is done numerically. As evident from Eq. (3), the i -th mode decays exponentially with time constant $v_c^3 / |\alpha_i| \Gamma n$; the larger $|\alpha_i|$, the faster decay. Therefore, near collisional equilibrium the only dominant mode is the fundamental mode. Figure 1 compares the

fundamental mode, $V_1(v')$, with the model distribution function, $V_0(v')$.

The comparison is favorable and demonstrates that: (1) the linearization of the Fokker-Planck collisional term by using the model distribution function, Eq. (2), is valid, (2) the model distribution function is a good analytic representation of the actual distribution near collisional equilibrium, i.e. between V_0 and V_1 .

Figure 1. Comparison between $V_0(v')$ and $V_1(v')$.

Based on Eq. (3), the local particle confinement time, τ_p , defined by $-n/(dn/dt)$, is given by

$$\tau_p(r, z) = v_c^3 / C_3 \Gamma n, \quad (6)$$

where $C_3 = 0.51$ for $V(v') = V_1(v')$, 0.97 for $V(v') = V_0(v')$. Since the equilibrium distribution lies between V_0 and V_1 , for the FRX-A experiment⁽⁴⁾ data the τ_p at field null is between $350-700 \mu s$. Assuming rigid-rotor equilibrium density profile and magnetic field, i.e., $n(r) = n_m \text{sech}(K(r^2/R^2-1))$, $B(r) = B_0 \tanh(K(r^2/R^2-1))$, the $\langle \tau_p \rangle$ (averaged over $0 \leq r \leq R_s = \sqrt{2} R_0$) for the FRX-A data is between $3.6-7.2 \mu s$. This is shorter than the observed

value, $58 \mu\text{s}$ measured at $10 \mu\text{s}$ after implosion, probably due to the assumption of a fast loss. Still, this shows that the velocity-space particle loss, more precisely, the particle loss due to the existence of the loss region in velocity space, is very important and can be large enough to account for the observed short particle confinement time.

In conclusion, near collisional equilibrium, the particle distribution subjected to a loss boundary in velocity space is obtained. A simple analytic expression is found to be a good approximation for the distribution function. The velocity-space particle loss rate and the corresponding particle confinement time due to like-particle collisions are calculated. The importance of this mechanism is demonstrated by the relatively short $\langle\tau_p\rangle$ prediction.

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