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**PROCEEDINGS
OF THE
1977 NATIONAL SYNCHROTRON LIGHT SOURCE
SUMMER THEORY WORKSHOP**

MASTER

**HELD AT BROOKHAVEN NATIONAL LABORATORY
JUNE 20-24, 1977**

INFORMAL REPORT

**BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.**

UNDER CONTRACT NO. EY-76-C-02-0016 WITH THE

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JUNE 20-24, 1977**

ORGANIZED BY: R. Chasman

CHAIRMAN: J.M. Paterson

EDITED BY: S. Krinsky

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INFORMAL REPORT

**BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973**

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PREFACE

The summer study, which resulted in the papers which follow, was organized by Rena Chasman with active help from Ken Green.

Rena and Ken were the team who produced the design for Brookhaven's National Synchrotron Light Source with all of its innovative features. At the time of the summer study we knew that Rena was not well and that Ken was not himself; but we did not imagine that within three months both would be gone.

At Rena's invitation our summer study included a group of electron storage ring experts whose recommendations about our design we felt would be definitive. We were pleased, but not surprised, when the group placed a stamp of approval on our design. In the course of so doing, many wise expressions of opinion emerged. For their help we are indeed grateful. We hope to see our collaborators often during construction of the machine and we shall continually welcome their advice.

John P. Blewett

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Introduction to the Lattice*

R. Chasman and S. Krinsky

Brookhaven National Laboratory, Upton, New York 11973

Existing electron storage rings have been designed for use in colliding beam experiments. The lattices of the NSLS storage rings differ from conventional design, because they are being specifically developed to serve as sources of X-rays and ultraviolet radiation. For experiments requiring high spatial resolution, there is no substitute for high source brightness (photons/sec-Å-mm²-steradian), and while it is not necessary for some types of experiments, it is never a disadvantage. Great emphasis has been placed on maximizing the optical source brightness and the present design will yield a brightness greater by an order of magnitude than that of existing synchrotrons or storage rings. A second important feature of the lattice is that matched insertions have been provided for the inclusion of high field wiggler magnets. The dispersion function X_p is very closely equal to zero in the insertions, so the high magnetic fields of the wigglers have only a small effect on the damping properties of the ring. This implies a large independence of the ring as a whole from the operation of the wigglers.

Let us briefly consider what is involved in obtaining a high source brightness. Synchrotron radiation is emitted about the direction of motion of the emitting electron with an angular spread,

* Work supported by the Department of Energy under Contract No. EY-76-C-02-0016.

$$\sigma_r \sim m c^2 / E, \quad (1)$$

where E is the electron energy, m is its mass, and c is the velocity of light. The transverse density of the electron beam is accurately described by a Gaussian distribution. The spatial widths of the electron distribution are denoted σ_x and σ_y , and the angular widths $\sigma_{x'}$ and $\sigma_{y'}$ (here, x refers to the radial coordinate and y to the vertical). The widths vary with the azimuthal coordinate s around the ring. The angular spread of the radiation beam is $(\sigma_r^2 + \sigma_{y'}^2)^{1/2}$, and since this should not be much greater than the natural width σ_r , we require

$$\sigma_{y'} \leq \sigma_r. \quad (2)$$

If also $\sigma_{x'} \leq \sigma_r$, and we consider a short source (one whose length l satisfies $l\sigma_r < \sigma_y$), the brightness behaves as

$$\frac{d^4 N}{dx dx' dy dy'} \sim \frac{1}{\sigma_r^2 \sigma_{x'} \sigma_y}, \quad (3)$$

for fixed electron current, energy and observed wavelength. To obtain a high brightness we must have σ_x and σ_y small, and to satisfy $\sigma_{x'} < \sigma_r$ and $\sigma_{y'} < \sigma_r$, the angular widths $\sigma_{x'}$ and $\sigma_{y'}$ must also be small. Since,

$$\sigma_x(s) = \sqrt{E_x \beta_x(s)} \quad \text{and} \quad \sigma_y(s) = \sqrt{E_y \beta_y(s)}, \quad (4)$$

the amplitude functions β_x and β_y must not be large at the source. In order to have the angular widths also small, it is necessary that the lattice have small emittances. This is most easily seen at a waist ($\beta_x' = 0$, $\beta_y' = 0$), where

$$E_x = \sigma_{x'} \sigma_{x'}, \quad \text{and} \quad E_y = \sigma_{y'} \sigma_{y'}. \quad (5)$$

The NSLS storage rings have been designed to minimize the electron emittances and to assure small values of the amplitude functions at the sources.

An overall layout of the facility is shown in Fig. 1. There are two storage rings. The X-ray ring (the larger of the two) has 10.2 KG bending magnets and will hold 0.5 Amp at 2.5 GeV, and 1.0 Amp at 2.0 GeV. Radiation of wavelength $1 - 25 \text{ \AA}$ will be readily available from the bending magnets, and the inclusion of 40 KG superconducting wigglers will provide usable intensity down to about $0.1 \text{ \AA}$. The UV-ring has 12.2 KG bending magnets and will store 1.0 Amp at 700 MeV, providing ultraviolet and soft X-radiation in the wavelength range $25 - 1000 \text{ \AA}$.

Injection into the storage rings will be in two stages. First, electrons will be accelerated to 50 MeV in the Linac Injector which has been obtained from Cornell University and is comprised of the electron gun and the first two accelerating sections of the Cambridge Electron Accelerator Injector. The 50 MeV electrons will enter the Booster synchrotron and be accelerated to 700 MeV. The Booster has the necessary damping characteristics to allow injection into the storage rings with small aperture helical wigglers in the insertions.

The Linac vacuum system will be an all metal, ion pumped system. The chamber materials will be stainless steel and copper and will operate at $10^{-7} - 10^{-8}$ torr. The Booster vacuum system will be made of thin wall stainless steel and will be corrugated for strength. The thin wall (0.010 inches) is to reduce eddy current losses. Sputter-ion pumps will bring the Booster to 10^{-7} torr. The 700 MeV and 2.5 GeV storage rings will be similar in design and construction. They will be all metal, UHV systems, 10^{-10} torr without beam and 10^{-9} torr with beam. The vacuum chamber will be an aluminum extrusion, see Fig. 2.

On one side of the extrusion there is a water channel for cooling the wall exposed to synchrotron radiation, and the other side incorporates a chamber for the distributed ion pump which uses the magnetic field of the main bending magnets. The pump section of the extrusion is machined off when the chamber is used in a closed quadrupole or sextupole, thus presenting a single continuous chamber throughout the magnet sections. In addition to the distributed pumps, "lumped" pumps will be used between magnets as required. In order to reduce gas desorption due to the synchrotron radiation, before assembly all parts will be chemically cleaned followed by an in-situ argon glow discharge cleaning. An attempt will be made to use the distributed ion pump anode also as the anode for the glow discharge process.

The X-ray ring will have two RF cavities, while the UV-ring and the booster synchrotron will each have one cavity. The RF system will operate at a frequency of 53.5 MHz. This frequency has advantages over higher frequencies, in particular, one needs a lower over-voltage, there is less higher-mode excitation, less power loss, and less beam instabilities. The major power need is for the X-ray ring which requires a maximum RF power level of greater than 700 KW. About one-half of this power is taken by the two accelerating cavities and the remainder goes to the beam. The proposed cavity design for the X-ray ring is a modified version of that used for Spear I at SLAC. The "Superfish" program developed by Los Alamos has been used to compute higher-order mode frequencies to see that none correspond to multiples of the rotation frequency of the X-ray ring.

The magnet design for the X-ray ring has been strongly influenced by the fact that radiation beams must be taken out from the storage ring to the experimental area. A sextant of the X-ray ring is shown in Fig. 3, and it is seen that certain radiation beams would be obstructed if the dipoles and

quadrupoles were closed structures. The UL-ring is shown in Fig. 4, and one sees that in this ring the dipoles must be open, but the quadrupoles can be closed. The dipoles in both rings will be conventional C-magnets (Fig. 5) allowing radiation beams to pass through the 5 cm gap. A split quadrupole (Fig. 6) has been designed for the X-ray ring so that where necessary radiation can pass through the 4.2 cm gap between the horizontal coils. A bonus associated with having a split quadrupole is that it may be possible to use the magnetic field in the gap between the coils for distributed pumping. The defocusing sextupole SD (Fig. 7) for the X-ray ring has been designed to have a radial dimension of only 15 cm, allowing a radiation beam to pass by closely adjacent to it. It did not seem feasible to design a split sextupole.

The lattice of the 2.5 GeV X-ray ring consists of six arcs matched to six insertions (four having length 3.75 m and two having length 5.0 m). Strictly speaking the lattice has a two-fold symmetry, but its actual deviation from a six-fold symmetry is very small. A sextant of the ring is shown in Fig. 3, and its amplitude and dispersion functions are plotted in Fig. 8, corresponding to a tune $\nu_x = 10.86$ and $\nu_y = 6.85$. The electron beam is brought to a low β focus at the center of each insertion by quadrupole triplets Q4, Q5, Q6 adjacent to the arcs. Conventional three-pole 40 KG wigglers can be located at the center of the 3.75 m insertions, and helical or flat coherent wigglers can be placed at the center of the 5 m straight sections. Each arc is composed of two achromatic bends separated by a quadrupole triplet Q2, Q1, Q2. The horizontally focusing quadrupoles QF localize the dispersion function about the bends, and X_p reaches a maximum of 0.56 m in QF. The lattice has a rather large negative chromaticity ($\xi_x = -25$, $\xi_y = -24$) which must be compensated by the sextupoles SF and SD.

The UV-ring consists of four achromatic arcs connected by straight sections (Fig. 4). The horizontally focusing quadrupoles Q3 between the two dipoles in each arc make the dispersion function zero outside the arcs, and X_p reaches a maximum of 1.1 m in the quadrupoles Q3. The triplets adjacent to the arcs focus the beam at the center of the straight section, and depending on the settings of the triplets high or low β -values can be obtained in the insertion centers. In conventional wigglers low β values are desirable to achieve high brightness, while high β values are necessary for an optical match of the helical wiggler to the ring, and also for injection to avoid an excessively thin septum in the injection magnet. Sextupoles SF and SD will compensate the chromaticity ($\xi_x = -12$, $\xi_y = -8$). In Fig. 9 we plot the β and X_p function for low β insertions (corresponding to a tune of $\nu_x = 4.38$, $\nu_y = 3.37$), and in Fig. 10 we plot the amplitude function for a case of two high and two low β insertions (corresponding to a tune of $\nu_x = 3.62$, $\nu_y = 2.63$).

Some typical values of the source sizes in the two rings are given in the table below.

X-Ray (2.5 GeV)

	$E_x = 4 \times 10^{-8}$ m-rad		$E_y = 4 \times 10^{-10}$ m-rad
<u>typical arc:</u>	$\beta_x = 5$ m		$\beta_y = 20$ m
	$\sigma_x = 0.45$ mm		$\sigma_y = 0.09$ mm
<u>3.75 m insertion:</u>	$\beta_x^* = 0.9$ m		$\beta_y^* = 0.4$ m
	$\sigma_x = 0.2$ mm		$\sigma_y = 0.01$ mm

UV-Ring (700 MeV)

$$E_x = 10^{-7} \text{ m-rad}$$

$$E_y = 10^{-9} \text{ m-rad}$$

typical arc:

$$\beta_x = 2 \text{ m}$$

$$\beta_y = 10 \text{ m}$$

$$\sigma_x = 0.4 \text{ mm}$$

$$\sigma_y = 0.1 \text{ mm}$$

insertion:

$$\beta_x^* = 0.55 \text{ m}$$

$$\beta_y^* = 0.25 \text{ m}$$

$$\sigma_x = 0.24 \text{ mm}$$

$$\sigma_y = 0.02 \text{ mm}$$

The negative chromaticities of each ring will be compensated by focusing and defocusing sextupoles. To bring the chromaticities to zero, the required sextupole strengths are:

$$LB''/BP = 19.3 \text{ m}^{-2} \quad \text{and} \quad -14.5 \text{ m}^{-2}, \text{ (X-ray)}$$

$$LB''/BP = 21.5 \text{ m}^{-2} \quad \text{and} \quad -23.5 \text{ m}^{-2}, \text{ (UV, low } \beta\text{-tune).}$$

These rather large sextupole strengths cause the betatron oscillations to be significantly nonlinear. We have used the computer program SYNCH to study the nonlinear dependence of ν , β , X_p and E_x on momentum (see Figs. 11, 12). The program PATRICIA was used to track particles including synchrotron oscillations, in order to determine the aperture imposed by the sextupoles. The initial energy spread was taken to be $6\sigma_e$, and the initial conditions were $X = X_A$, $X' = 0$, $y = y_A$, $y' = 0$ at the center of Q1. For the X-ray ring the stability limit corresponding to tracking through 7500 turns is shown in Fig. 13. In particular, for $y_A = 0$, we obtain stability for

$$X_A \leq 25 \sigma_x.$$

Since the entire beam is certainly contained in $10 \sigma_x$, this seems adequate. For the UV-ring we track through 100,000 turns and find stability out to more than 100σ .

The strong quadrupole strengths, and the correspondingly high values of the tunes, make the lattices very sensitive to errors. Random gradient errors in the quadrupoles will give rise to a tune shift

$$\langle \Delta \nu \rangle_{rms} \cong \frac{\sqrt{M} L \langle \beta \rangle_{rms} G \langle \Delta G \rangle_{rms}}{4\pi B\rho G}.$$

For the X-ray ring we can approximate the number of quadrupoles $M = 7 \times 12 = 84$, the length of the quadrupoles $L = 0.3$ m, $\langle \beta \rangle_{rms} = 10$ m, the gradient $G = 200$ kG/m, and $B\rho = 83.5$ kG-m. Then,

$$\langle \Delta \nu \rangle_{rms} \cong 5 \frac{\langle \Delta G \rangle_{rms}}{G},$$

so if $\langle \Delta \nu \rangle_{rms} = 10^{-3}$, then $\langle \Delta G \rangle_{rms}/G = 2 \times 10^{-4}$.

The closed orbit distortion resulting from random displacements $\delta\epsilon$ of the quadrupoles is

$$\langle Z_{co} \rangle_{rms} \cong \frac{\sqrt{M} \langle \beta \rangle_{rms} GL}{2B\rho \tan \pi \nu} \langle \delta\epsilon \rangle_{rms}.$$

Here we are using Z to represent either the radial or vertical coordinate, and the corresponding subscripts are understood on the right hand side. In addition to the quantities discussed above, we have $\tan \pi \nu = 0.47$ for $\nu_x = 10.85$ or $\nu_y = 10.85$. Hence,

$$\langle Z_{co} \rangle_{rms} \cong 70 \langle \delta\epsilon \rangle_{rms}$$

implying a quadrupole displacement of only 10 mil will produce an orbit displacement of 18 mm. We plan to develop a closed orbit correction system capable of yielding peak displacements of

$$X_{co} = 3 \text{ mm} \quad \text{and} \quad Y_{co} = 1 \text{ mm}.$$

Another consequence of random quadrupole displacements is the existence of a vertical dispersion function Y_p , which will give rise to an increase of the beam height according to

$$\Sigma_y = (\sigma_y^2 + Y_p^2 \frac{\sigma_e^2}{E^2})^{\frac{1}{2}}.$$

Since $\sigma_e/E = 10^{-3}$ for the X-ray ring, a vertical dispersion of 0.017 m in the insertion would double the beam height of $\sigma_y = 10^{-5}$ m.

In order to explicitly define the lattice for the reader, we close this introduction by giving the input to a SYNCH run for the X-ray ring, and the results of a SYNCH CYEM operation (which computes the electron integrals) at 2.5 GeV and at 700 MeV.

KRS	RUN	X-RAY RING											
FQX	SUB												
R	=	1.0											
BRHO	=	83.5											
BZ	=	10.2165667											
D2	DRF	0.325											
D3	DRF	0.320											
D6	DRF	1.715											
D7	DRF	0.425											
D8	DRF	2.5											
BB	MAG	1.605	0.	BRHO	BZ	\$							
BBH	MAG	0.535	0.	BRHO	BZ	\$							
BS	MAG	0.08	0.	BRHO	40.	\$							
BT	MAG	0.08	0.	BRHO	-40.	\$							
Q1	MAG	0.3125	180.32	BRHO									
Q2	MAG	0.3125	-193.48	BRHO									
Q4	MAG	0.3125	-225.10	BRHO									
Q5	MAG	0.3125	201.29	BRHO									
Q6	MAG	0.3125	-260.33	BRHO									
QF	MAG	0.2	199.62	BRHO									
Q44	MAG	0.3125	-219.29	BRHO									
Q55	MAG	0.3125	187.66	BRHO									
Q66	MAG	0.3125	-216.02	BRHO									
\$SPH	BML	Q1	D2	Q2	D3	BB	D7	SD	D7	BBH	D3	QF	SF
		QF	D3	BBH	D7	SD	D7	BB	D3	Q4	D2	Q5	Q5
		D2	Q6	D6	BT	BS	BS	BT	D6	Q6	D2	Q5	Q5
		D2	Q4	D3	BB	D7	SD	D7	BBH	D3	QF	SF	QF
		D3	BBH	D7	SD	D7	BB	D3	Q2	D2	Q1	Q1	D2
		Q2	D3	BB	D7	SD	D7	BBH	D3	QF	SF	QF	D3
		BBH	D7	SD	D7	BB	D3	Q44	D2	Q55	Q55	D2	Q66
		D8											
CLO	PVEC												
	SIZE	7											
SD	SXTP	0.0	0.0	R									
SF	SXTP	0.0	0.0	R									
FXP	FXPT	2	-2	CLO	\$SPH								
EPS	CYEM	4	FXP	2.5	24.0515	50.	0.8	.1					
	END												
	CALL	1	FQX										
	REPL	2	SD	-14.326									
	REPL	2	SF	19.154									
	CALL	1	FQX										
	FIN												
	STOP												

X-Ray Ring: $E \approx 2.5$ GeV

Average Radius	$R = .240E+02$ Meters
R F Frequency	$F = .500E+08$ HZ
R F Peak Volts	$VO = .800E-03$ GV
Effective Radius	$R = .817E+01$ Meters
Momentum Compaction	$ALPHA = .561E-02$
Damping Partition	$D = -.175E-02$ Radians
Phase Angle	$PHIS = .430E+02$ Degrees
Energy Oscillation Period	$TE = .227E-03$ Seconds
Energy Aperture	$EPSM = .346E-01$ GeV
$(EPSM**2/SIGE**2)/2$	$XJ = .950E+02$
Sigma (DP/P)	$SIGDP = .100E-02$
	$SIGE = .251E-02$
Synchrotron Radiation	$UO = .584E-03$ GeV/Turn
Circulation Period	$TO = .504E-06$ Seconds
Horizontal Damping	$TAUX = .430E-02$ Seconds
Vertical Damping	$TAUY = .431E-02$ Seconds
Energy Damping	$TAUE = .215E-02$ Seconds
Sigmax (0-Coupling)	$SIGX = .901E-03$ Meters
Quantum Lifetime	$TAUQ = .210E+37$ Seconds
Bunch Length	$SIGPHI = .642E-01$ Radians
Bunch Length	$SIGL = .612E-01$ Meters
Momentum $SPPD*SQRT(6)$	$DP/P = .246E-02$
	$SICBG = .492E+01$

Emittances (Meter Radians) $EPX = 0.411E-07$ $EPY = 0.416E-09$

X-Ray Ring: E = 700 MeV

Average Radius	R = .240E+02 Meters
R F Frequency	F = .500E+08 HZ
R F Peak Volts	VO = .400E-03 GV
Effective Radius	R = .817E+01 Meters
Momentum Compaction	ALPHA = .561E-02
Damping Partition	D = -.175E-02 Radians
Phase Angle	PHIS = .894E+02 Degrees
Energy Oscillation Period	TF = .140E-03 Seconds
Energy Aperture	EPSM = .352E-01 GeV
(EPSM**2/SIGE**2)/2	XI = .159E+05
Sigma (DP/P)	SIGDP = .281E-03
	SIGE = .197E-03
Synchrotron Radiation	UO = .359E-05 GeV/Turn
Circulation Period	TO = .504E-06 Seconds
Horizontal Damping	TAUX = .196E+00 Seconds
Vertical Damping	TAUY = .196E+00 Seconds
Energy Damping	TAUE = .983E-01 Seconds
Sigmax (0-Coupling)	SIGX = .252E-03 Meters
Quantum Lifetime	TAUQ = R Seconds
Bunch Length	SIGPHI = .111E-01 Radians
Bunch Length	SIGL = .106E-01 Meters
Momentum SPRD*SQRT(6)	DP/P = .690E-03
	SIGBG = .386E+00
Emittances (Meter Radians)	EPX = 0.322E-08 EPY = 0.326E-10

FIGURE CAPTIONS

- Fig. 1 Overall layout of the NSLS facility.
- Fig. 2 The vacuum chamber cross-section.
- Fig. 3 One sextant of the X-ray ring.
- Fig. 4 The UV-ring.
- Fig. 5 Cross-section of the C-magnet.
- Fig. 6 Cross-section of the split quadrupole.
- Fig. 7 Cross-section of the defocusing sextupole.
- Fig. 8 Amplitude and dispersion functions for the X-ray ring, corresponding to a tune $\nu_x = 10.86$, $\nu_y = 6.85$.
- Fig. 9 Amplitude and dispersion function for the UV-ring, corresponding to a tune $\nu_x = 4.38$, $\nu_y = 3.37$ with all insertions having low β values.
- Fig. 10 Amplitude function for the UV-ring, corresponding to a tune $\nu_x = 3.62$, $\nu_y = 2.63$ with two insertions having low β values and two having high β values.
- Fig. 11 For the X-ray ring: (a) Dependence of tunes and horizontal emittance on momentum, (b) dependence of β_x on momentum, (c) dependence of β_y on momentum, (d) dependence of X_p on momentum, where for $\Delta P \neq 0$, we take $X_p = X_{co}/(\Delta P/P)$.
- Fig. 12 For the UV-ring (for low β insertions): (a) Dependence of tunes and horizontal emittance on momentum, (b) dependence of β_x and β_y on momentum, (c) dependence of X_p on momentum, where for $\Delta P \neq 0$, we take $X_p = X_{co}/(\Delta P/P)$.
- Fig. 13 Stability limit for X-ray ring with chromaticity compensated to zero, determined by tracking through 7500 turns using PATRICIA with synchrotron oscillations, and initial energy spread $6\sigma_e$. The solid line segments join a stable (inside) to an unstable (outside) point. The dashed curve joints stable points.

NATIONAL SYNCHROTRON LIGHT SOURCE
BROOKHAVEN NATIONAL LABORATORY

Figure 1

Plan of national synchrotron light source.

Figure 2. NSLS VACUUM CHAMBER

Figure 3. Sextant of X-Ray Ring.

Figure 4. 700 MeV RING

Figure 5

DIPOLE

Dimensions in Centimeters

Figure 7

SEXTUPOLEDIMENSIONS ARE IN CENTIMETERS.ALL DIMENSIONS ARE TYPICAL.

X-RAY RING: AMPLITUDE AND DISPERSION FUNCTIONS

UV-RING : LOW BETA INSERTIONS

Figure 9

UV-RING: HIGH AND LOW BETA INSERTION

$$v_g = 2.63$$

Figure 10

X-RAY RING: TUNES AND ENTRANCE VS MOMENTUM

Figure 11a

GRAPHIC

10 X 10 IN. MAT. INER

SQUARE

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X-RAY RING : β_x FOR DIFFERENT MOMENTA

Figure 11b

X-RAY RING : B_y FOR DIFFERENT MOMENTA

Figure 11c

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X-RAY RING : DISPERSION FUNCTION FOR DIFFERENT MOMENTA

Figure 11d

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Q1 Q2 BB SD BBH QF SF BBH SD BB Q4 Q5 Q6 3T 3S 3T

UV-RING: TUNES AND EXTINCTION VS MOMENTUM

Figure 12a

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A-29
1) 1-RING: β_x AND β_y FOR DIFFERENT MOMENTS

Figure 12b

UV-RING : DISPERSION FUNCTION FOR DIFFERENT DATA

Figure 12c

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X-Ray Ring: Aperture Due to Sextapoles

Figure 13

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Partial List of Corrective or Diagnostic Equipment

J. M. Paterson

June 21, 1977

<u>Item</u>	<u>Comments</u>
<u>Position Monitors</u>	<p>a) No great technical problem but require at least four per betatron wavelength (i.e., 40 units in big ring) to get sufficiently accurate information for precise orbit control and η_y measurement. With care one can expect a precision of ± 0.1 mm and an absolute accuracy (taking into account the relative alignment problems between chambers and quads) of ± 0.25 mm.</p> <p>b) For measurements of η_x or η_y, a variation of ± 12 KHz in the RF frequency gives a change in momentum of $\pm 5 \times 10^{-3}$ which with the above precision would translate into an η measurement accuracy of ± 2.5 cms. Even at this level of $\frac{\Delta p}{p}$ variation the software that translates the orbit shift into η will need to correct for the variations in lattice parameters vs $\frac{\Delta p}{p}$.</p> <p>c) Position monitor type signals appropriately gated can be used as inputs to the tune measuring and general diagnostic equipment.</p>
<u>Strip Lines</u> 2 sets, Horizontal and vertical	<p>a) No great technical problems with the exception of handling the beam induced power and protecting attached electronic components.</p> <p>b) They can be used for tickling the beam at low power for tune measurement or at high power and broad band as part of a transverse feedback system.</p>
<u>Synchrotron Light Monitors</u>	<p>a) Could use part of the visible light from existing exit ports or could be independent. Recommend two locations - one where $\eta = 0$ and $\beta_y \geq \beta_x$</p>

<u>Item</u>	<u>Comments</u>
<u>Synchrotron Light Monitors</u> (cont'd.)	and one where η is large. The first is optimum for measuring and minimizing betatron coupling and the latter for measurements of $\frac{\Delta p}{p}$ (incoherent and coherent).
	b) System should include profile scanners with digitized information interfaced to the control computer. In these machines where the source brightness is the measure of success, the software could be a great aid in optimizing performance.
<u>Beam Scrapers</u> Horizontal and Vertical	Again two locations for the reasons given above. They are used to probe the 'tails' of the beam beyond 4σ in x, y and $\frac{\Delta p}{p}$ where the distributions may and usually are different from the core of the beam.
<u>RF Quad</u>	Device used to give successive bunches slightly different betatron tunes to combat multibunch transverse instabilities. Usually this would be a resonant quadrupole at the orbital frequency. For example, in the X-ray ring a 2 MHz quad with a gradient length product of 24 Kg at a point where $\beta = 15$ meters and at 2.5 GeV would give a tune split between successive bunches of .001. This is not large and already the magnet is quite strong, e.g., a quad with a 10 cm bore and 30 cm long would have a pole tip equivalent field of 4 Kg. Not easy at 2 MHz. Conclusion! - every attempt should be made to avoid these instabilities by identifying transverse modes in the RF cavities (the most probable source of high Q resonances) and either designing them away from a multiple of the orbital frequency or damping them by coupling the mode into some load.

Clearing Electrodes

Experience with other storage rings indicate that with high currents and short bunch spacing, beam neutralization by ions is probable. This could lead to an increase in vertical emittance due to the nonlinear forces exerted by the trapped ions. This has been observed in ACO, DORIS and ADONE and in fact the Landau damping associated with these nonlinear forces has often led to stability where a single beam of positrons was unstable. In the synchrotron light rings, the increase in emittance is very undesirable. If one can use the cleaning electrodes for a dual purpose, viz. cleaning and/or argon glow discharge cleaning, then they may be worth the possible penalty of an increased vacuum chamber impedance. These electrodes do not have to hold off a very high voltage ≤ 1 KV and therefore they can be designed in such a way that their impedance is only important at high frequencies or very short bunches. They could still couple out an annoying amount of power which has to be absorbed to prevent resonant buildup.

Another observation is that it is often enough to leave one gap in the circumferential filling factor to prevent ion buildup. If this turns out to be true in these rings then running with a gap of a few bunches may be a solution.

In conclusion, I would look very hard at the engineering problem of installing electrodes (especially in regions of magnetic fields which aid in trapping the ions) but I would probably gamble and leave them out initially. Of course it is obvious that the operating vacuum pressure is important in this problem and at sufficiently low pressures the ion trapping problem becomes less likely.

Bunch Lengthening Cavity

There are many reasons for considering the addition of a high harmonic RF cavity, and there are several ways of using such a device. I believe that running such a cavity powered back phased (i.e., the bunch crosses this cavity at -90°) as shown in Fig. 1 gives several interesting benefits. First selecting the fourth harmonic of the main RF gives a good compromise between the length of flat potential well, the peak voltage of the summed waveforms, the cavity size and the availability of cheap RF power. At the fourth harmonic one needs approximately $1/4$ of the total voltage or 200 KV. This is easily achieved with a single cell cavity of simple shape and <10 KW of power. Note when properly phased this cavity delivers no net power to the beam. Probably to minimize risk of transverse deflecting modes of dangerous Q, one would use very low Q cavity and as much power as is economical.

The benefits include: -

- 1) A lengthened bunch by as much as a factor of three which reduces component heating problems. (See separate PEP Tech Memo on this subject and attached example of dependence of power loss on bunch length.)
- 2) A lengthened bunch helps with Touschek lifetime at low energy and note that peak sum RF voltage is also increased, again helping the lifetime.
- 3) Because the peak current in the bunch is lowered, single bunch transverse instabilities are helped.
- 4) The most significant effect is probably the large spread in synchrotron oscillation

Fig ①

$$\frac{V_{\text{TH}}}{V_1} = \frac{V \sin \phi_s}{N}$$

Case for $\phi_s = 90$ ie 700 MeV.

Bunch Lengthening Cavity
(Cont'd.)

oscillation frequency within the bunch. This gives strong Landau damping to both single and multibunch longitudinal instabilities.

(See ISR report CERN-ISR-RF-TH/77-41.)

- 5) Test results from an operating system in SPEAR will be available late this year.
- 6) Although bunch lengthening would probably occur in any case due to beam interaction with its environment, it is accompanied by a corresponding increase in energy spread which can give trouble with stability and with increased vertical emittance from the residual vertical dispersion.

PEP TECHNICAL MEMO

PTM - 99

AUTHOR: T. Martin/P. Wilson

DATE: 4/28/77

REPLACES PTM #

TITLE: PEP RING IMPEDANCE BUDGET

On the reverse side is the best present estimate of the impedance budget for the PEP ring. For a few of the components, good measured values are already available for prototypes. This category includes the gate valves, distributed ion pump units and extrusion dog legs with position monitors. For the majority of the components, however, values for k have been estimated with varying degrees of certainty. These estimates are based on similarities of the component in question to components already measured, including SPEAR components, or in some cases on simple guesswork. The budget will be updated periodically as more components are measured and firm values of k are obtained.

Values of k are given for each component in units of volts per picocoulomb, and are independent of ring size. The impedance Z is obtained from k by multiplying by the passage time between bunches, 2.446 μsec in the case of PEP and 0.781 μsec for SPEAR. The power dissipation is simply $2I^2Z$ for two beams, where I is the circulating current per beam. All values are given for a bunch length of 2.0 cm, although there may in fact be some bunch lengthening at the design current of 55 mA per beam at 15 GeV. The final column gives the power dissipation for each component at the design current. Not all of this power will necessarily be dissipated in the component; most of the power will in fact be reflected or retransmitted following the pulse.

The total impedance of 126 $\text{M}\Omega$ allows a current of 55 mA per beam to be reached with a total klystron power of 6 MW. Note that two-thirds of the impedance (84 $\text{M}\Omega$) is in the rf cells and cannot be reduced. Out of the 42 $\text{M}\Omega$ remaining for other ring components, a contingency of 13 $\text{M}\Omega$ is available in case some of the losses have been underestimated.

April 28, 1977

PEP Ring Impedance Budget
 $\sigma_z = 2.0$ cm

Component	Number	k per unit (V/pC)	Total k (V/pC)	Z per unit (k Ω)	Pwr @ 2x55 mA (W)
<u>Arcs</u>					
Distributed ion pump units (5 m)	192	.01	1.92	25	150
Dog legs (pump port, bellows, PM's)	108	.004	.43	10	60
Transitions - extrusion to 6" round	12	.01	.12	25	150
Valves - 6" gate with transitions	6	.022	.13	50	300
Wiggler vacuum chambers	3	.01	.03	25	150
Standard instrument modules	12	.003	.04	7	50
Non-standard instr. modules	12	.003	.04	7	50
Tune monitor	1	.02	.02	50	300
Side pump ports	96	.0015	.15	4	25
Symmetry section masking	6	.01	.06	25	150
<u>Straight Sections</u>					
Pump ports on 6" round pipe	~192	.0015	.29	4	25
Fast valves	12	.005	.06	10	60
Slow 6" gate valves	28	.002	.06	5	30
6" bellows with HOM shielding	~242	.002	.48	5	30
Q ₁ chambers	12	.015	.18	40	250
Q ₂ chambers	12	.015	.18	40	250
Separation plates	12	.10	1.20	250	1500
Current monitor (ceramic gap)	1	.10	.10	250	1500
Position monitor sets	24	.002	.05	5	30
Scraper	1	.02	.02	50	300
PPS stopper	1	.01	.01	25	150
High-Z masks	12	.02	.24	50	300
Electric quadrupoles	6	.2	1.2	500	3000
Sync. rad. masks at Q ₃	12	.01	.12	25	150
Kickers	6	.10	.60	250	1500
Septums	2	.10	.20	250	1500
Movable masks	12	.04	.48	100	600
<u>Experimental Insertions</u>					
Interaction region chambers	6	.5	3.0	1200	7000
<u>Contingency</u>					
Vacuum chamber components			5.8		
Totals:				Total Z	Total Power
Vacuum chamber excl. rf cavities			17.2	42 M Ω	250 kW
Rf cells (excl. fund. mode)	120	.29	34.3	84	510

Total ring budget

51.5 126 M Ω 760 kW

These numbers are for already

XRAY RING

$N = 24$

$i_B = 41.7 \text{ mA}$

$\sqrt{I_{THEOR}} = 6 \text{ cm}$

UV Ring.

$N = 7$

$i_B = 143 \text{ mA}$

$\sqrt{I_{THEOR}} = 4 \text{ cm}$

Resistive impedance $R = k \times T_o$ (ohms) T_o is orbital period

Power loss = $P(\text{watts}) = N i_B^2 R$

N = number of bunches i_B is average current per bunch

HPB, June 21, 77

Closed Orbit Errors in the 2.5 GeV X-Ray Source

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A careful calculation of the closed orbit errors to be expected gives the result that the peak vertical orbit error is 41 mm with a 95% probability. The 50% probability peak vertical orbit error is 31 mm. Since the vacuum tank has the vertical dimensions of ± 16 mm, the probability is high that the beam will not go around the accelerator, when the accelerator is first started up. It was assumed in the above that the quadrupole alignment is 6 mil (.15 mm) and the dipole rotational error is 5×10^{-4} radians.

The required vertical closed orbit error is about 1 mm. The initial orbit error of 41 mm will have to be reduced by a factor of 40. This large reduction factor will probably require a good dipole correction system with at least 4 dipole correctors per betatron wavelength. In addition, the orbit will probably have to be corrected by moving the quadrupoles and rotating the dipoles. By using both methods of correction, moving the magnets and separately excited dipole corrections, a reduction factor of 40 is theoretically possible.

The calculation of the closed orbit errors was done as follows. The rms displacement of the orbit at any point in the acceleration is given by

$$\Delta Y_{\text{rms}}^2 = \sum_K \Delta Y_K^2,$$

$$\Delta Y_K = \frac{1}{2.8 \sin \pi \nu_y} \sqrt{N_K} \sqrt{\beta \beta_K} \Delta \theta_K,$$

$$\Delta \theta_K = L_K \Delta B_K / B \rho.$$

The subscript K indicates the different magnets. N_K is the number of these magnets around the ring. L_K is the length, β_K is the β -function at that location, $\Delta \theta$ is the field error present, and β is the β -function at the point where the orbit displacement is being computed.

The details of the calculation are given in Table I for the dipoles in the ring and the quadrupoles. It is found that the quadrupole misalignments cause an rms error of $\Delta Y_{\text{rms}} = 12.6$ mm as the Q_2 quadrupole where β_y has its maximum value. The dipole misalignments give $\Delta Y_{\text{rms}} = 5.1$ mm. The total rms orbit error is $\Delta Y_{\text{rms}} = 13.6$ mm at the Q_2 quadrupole.

Computing the peak orbit error to be expected with a certain probability from ΔY_{rms} is not easy to do without a computer simulation study. A reasonable guess, based on experience with ISABELLE, shows the ΔY_{rms} should be multiplied by 3 for the expected peak error with 95% probability and by 2.25 for the peak error with 50% probability. This gives 41 mm for the expected peak error with 95% probability and 31 mm for 50% probability.

TABLE I

Quadrupoles

Type	N_K	L_K (m)	Gradient (KG/m)	$\beta_{Y,K}$ (m)	$\Delta\theta_K/10^{-4}$	ΔY_K (mm)
Q ₁	6	.625	180	11.8	2.0	4.8
Q ₂	12	.3125	-193	22.1	1.1	5.1
Q _F	12	.4	200	.971	1.4	1.4
Q ₄	12	.3125	-225	21.7	1.26	5.8
Q ₅	12	.625	201	9.1	2.2	6.1
Q ₆	12	.3125	-260	19.2	1.4	6.0

$$\Delta Y_{rms} = 12.6 \text{ mm}$$

Dipoles

Type	N_K	L_K (m)	B_K (kG)	$\beta_{y,K}$ (m)	$\Delta\theta_K/10^{-4}$	θY_K (mm)
BB	24	1.605	10.2	13.7	.96	5.0
BBH	24	.535	10.2	3.51	.32	.7
BT	12	.08	-40	.31	.20	.5
BS	6	.16	40	.25	.39	.14

$$\Delta Y_{rms} = 5.1 \text{ mm}$$

$$\Delta Y_{rms, total} = 13.6 \text{ mm}$$

Linear Horizontal-Vertical Coupling,
Vertical Dispersion, and Control of the
Vertical Closed Orbit at the Wiggler

S. Ohnuma

Fermilab

I. Linear Horizontal-Vertical Coupling

The coupling parameter κ for resonances of the type $\nu_x \pm \nu_y = N$ is¹

$$\kappa = \frac{1}{4\pi(B\rho)} \oint ds \sqrt{\beta_x \beta_y} B' e^{iF}$$

where B' is the skew quadrupole field gradient and

$$F = \int_0^s ds [(1/\beta_x - \nu_x/R) \pm (1/\beta_y - \nu_y/R) + N/R].$$

The amount of the coupling in amplitude is

$$\kappa / \sqrt{(\Delta/2)^2 + \kappa^2}, \quad \Delta \equiv (\nu_x \pm \nu_y) - N.$$

Note that, for $\nu_x + \nu_y = N$, the motion is unstable if $|\kappa| \geq |\Delta/2|$. If one disregards the construction errors of magnets, major contributions to the skew quadrupole field come from the rotating of quadrupoles and from the vertical closed orbit displacement at sextupoles. In the following, rms are indicated by $\langle \rangle$.

rotation of quadrupoles $\langle \theta \rangle$

$$\langle \theta \rangle = \frac{\langle \theta \rangle}{2\pi} \sqrt{\sum \beta_x \beta_y (B' l / B\rho)_{\text{quad}}^2}.$$

$$\text{X-ray ring}^2: \quad \langle \kappa \rangle = 13.6 \langle \theta \rangle$$

$$\text{UV ring}^2: \quad \langle \kappa \rangle = 7.6 \langle \theta \rangle$$

Vertical orbit errors at sextupoles $\langle \Delta y \rangle$

$$\langle \kappa \rangle = \frac{\langle \Delta y \rangle}{4\pi} \sqrt{\sum_x \beta_x \beta_y (B''L/B\rho)^2_{\text{sext.}}}$$

For the X-ray ring, $(B''L/B\rho) = 21 \text{ m}^{-2}$ for SF and -16 m^{-2} for SD
give $\Delta v/(\Delta p/p) = +2$ for both directions. With these sextupole
strengths,

$$\langle \kappa \rangle = 17.5 \langle \Delta y \rangle; \quad (\Delta y) \text{ in meters}$$

For the UV ring, $(B''L/B\rho) = \pm 20 \text{ m}^{-2}$ to get the chromaticity
 $\Delta v/(\Delta p/p) = +2$ and

$$\langle \kappa \rangle = 6.4 \langle \Delta y \rangle.$$

Correction skew quadrupole system should be able to provide

$$|\kappa| = 0.05 \text{ for the X-ray ring and } 0.025 \text{ for the UV ring.}$$

1. X-ray ring $(v_x + v_y = 18 - 0.29, v_x - v_y = 4 + 0.01)$

Twelve skew quadrupoles with the same value of $|B'L|$ are considered.

Each resonance is controlled by two "knobs", one for the real part and
the other for the imaginary part of κ .

#1 ~ 2, # 5 ~ 8, #11 ~ 12: placed at D6 close to quadrupole Q6

#3 ~ 4, #9 ~ 10 : placed at D8 close to quadrupole Q66

$$a) v_x - v_y = 4, \text{ real part of } \kappa$$

$$(3 + 4 + 9 + 10) - (1 + 6 + 7 + 12)$$

$$b) \nu_x - \nu_y = 4, \text{ imag. part of } \kappa$$

$$(5 + 6 + 11 + 12) - (1 + 2 + 7 + 8)$$

$$c) \nu_x + \nu_y = 18, \text{ real part of } \kappa$$

$$(1 + 2 + \dots + 12)$$

$$d) \nu_x + \nu_y = 18, \text{ imag. part of } \kappa$$

$$(1 - 2 + 3 - 4 + \dots - 12)$$

For $|\kappa| \leq 0.05$, $|B'l/B\rho| \leq 0.011 \text{ m}^{-1}$. At 2.5 GeV ($B\rho = 83.4 \text{ kG-m}$),

$|B'l| \leq 0.92 \text{ kG}$. This is not a very difficult requirement. For example, correction skew quadrupoles at Fermilab are $l = 6''$ and

$$|B'l| = 0.064 \text{ kG/amp.}$$

$$2. \text{ UV Ring } (\nu_x + \nu_y = 6 + 0.25, \nu_x - \nu_y = 1 - 0.01)$$

Eight skew quadrupoles are placed at D3,

#1, 4, 5, 8 near Q4

#2, 3, 6, 7 near Q44.

$$a) \nu_x - \nu_y = 1, \text{ real part of } \kappa$$

$$(1 + 3 + 6 + 8) - (2 + 4 + 5 + 7)$$

$$b) \nu_x - \nu_y = 1, \text{ imag. part of } \kappa$$

$$(2 + 3 + 5 + 8) - (1 + 4 + 6 + 7)$$

$$c) \nu_x + \nu_y = 6, \text{ real part of } \kappa$$

$$(1 + 2 + \dots + 8)$$

$$d) \nu_x + \nu_y = 6, \text{ imag. part of } \kappa$$

$$(1 + 2 + 5 + 6) - (3 + 4 + 7 + 8).$$

For $|\kappa| \leq 0.025$, $|B'l/B\rho| \leq 0.014 \text{ m}^{-1}$. At 700 MeV ($B\rho = 23.3 \text{ kG-m}$),

$$|B'l| \leq 0.33 \text{ kG}.$$

II. Vertical Dispersion

Two types of vertical dispersion exist, one coming from the coupling with the horizontal dispersion η_x and the other independent of η_x .

A. Proportional to η_x

A.1. Quadrupole roll $\langle \theta \rangle$

This is not important since $\eta_x = 0$ at all quadrupoles except at QF in the X-ray ring and at Q3 in the UV ring.

$$\langle \eta_y \rangle = \frac{\langle \theta \rangle \sqrt{\beta_y}}{\sin(\pi \nu_y)} \sqrt{\frac{1}{2} \Sigma \beta_y \eta_x^2 (B' \ell / B \rho)^2}$$

$$\langle \eta'_y \rangle = \frac{\langle \theta \rangle}{\sqrt{\beta_y} \sin(\pi \nu_y)} \sqrt{\frac{1}{2} \Sigma \beta_y (1 + \alpha_y^2) \eta_x^2 (B' \ell / B \rho)^2}$$

$$\text{X-ray ring: } \langle \eta_y \rangle = 2.8 \sqrt{\beta_y} \langle \theta \rangle$$

$$\langle \eta'_y \rangle = 3.1 \langle \theta \rangle / \sqrt{\beta_y}$$

A.2. Vertical orbit errors at sextupoles $\langle \Delta y \rangle$

$$\langle \eta_y \rangle = \frac{\sqrt{\beta_y} \langle \Delta y \rangle}{2 \sin(\pi \nu_y)} \sqrt{\frac{1}{2} \Sigma \beta_y \eta_x^2 (B'' \ell / B \rho)^2}$$

$$\langle \eta'_y \rangle = \frac{\langle \Delta y \rangle}{2 \sqrt{\beta_y} \sin(\pi \nu_y)} \sqrt{\frac{1}{2} \Sigma \beta_y (1 + \alpha_y^2) \eta_x^2 (B'' \ell / B \rho)^2}$$

$$\text{X-ray ring: } \langle \eta_y \rangle = 45.0 \sqrt{\beta_y} \langle \Delta y \rangle$$

$$\langle \eta'_y \rangle = 91.0 \langle \Delta y \rangle / \sqrt{\beta_y}$$

$$\text{UV ring: } \langle \eta_y \rangle = 24.2 \sqrt{\beta_y} \langle \Delta y \rangle$$

$$\langle \eta_y' \rangle = 47.2 \langle \Delta y \rangle / \sqrt{\beta_y}$$

These could be important if $\langle \Delta y \rangle$ is larger than ~ 1 mm.

It is important to minimize Δy where $\eta_x \neq 0$.

B. Independent of η_x

The vertical equation of motion is

$$d^2 y / ds^2 = B_x / B\rho$$

where

$$B_x = B_o^\theta$$

dipole roll θ

$$+ B_o'(y-\delta)$$

quads displaced by δ

$$+ b_x$$

steering dipoles

$$(B\rho)^{-1} \simeq (B\rho)_o^{-1} (1 - \delta_p), \quad \delta_p \equiv \Delta p/p$$

The vertical closed orbit is made of two parts,

$$y \equiv y_o + \eta_y \delta_p$$

and

$$y_o'' + K_y \cdot y_o = \frac{1}{(B\rho)_o} (B_o^\theta - B_o' \delta + b_x),$$

$$\eta_y'' + K_y \cdot \eta_y = - \frac{1}{(B\rho)_o} (B_o^\theta - B_o' \delta + b_x + B_o' y_o)$$

The closed orbit distortion y_o , which is independent of the momentum, is due to $(B_o^\theta - B_o' \delta + b_x)$ and the vertical dispersion η_y arises from $(B_o^\theta - B_o' \delta + b_x + B_o' y_o)$. If the closed orbit is zero at quadrupoles, minimizing y_o also minimizes η_y . Otherwise, it is not enough to adjust b_x to minimize y_o alone.

For X-ray ring, without correction ($b_x = 0$)

$$\langle y_o \rangle = \sqrt{\beta_y} \sqrt{7.82 \langle \theta \rangle^2 + 529. \langle \delta \rangle^2}$$

For $\langle \eta_y \rangle$, one can probably assume that $\theta_o' y_o$ is dominant after an overall correction of y_o . Then

$$\langle \eta_o \rangle = 23 \beta_y \langle y_o \rangle_{\text{quad}}$$

III. Control of the Vertical Closed Orbit at the Wiggler

The control of the vertical closed orbit is not straightforward (three- or four-magnet bumps) because of the unusual lattice structure. Most likely one must depend on computer programs similar to one used at SLAC or Fermilab in order to find the optimum setting of steering elements. The following is a simple scheme of controlling the vertical orbit (y and y') at the wiggler (BS) position in the X-ray ring.

There are two sextupoles, SD_1 and SD_2 , upstream of BS and two more, SD_3 and SD_4 , downstream of BS. It is assumed here that vertical steering elements are placed near SD_1 , SD_2 and at two quadrupoles (Q4) between them.

<u>location</u>	<u>vertical phase</u>	<u>β_y</u>	<u>α_y</u>	<u>kick</u>
SD_1	0°	5.13 m	2.42	θ_1
Q4	100.1°	23.16	4.74	θ_2
BS	190.9°	.251	0	
Q4	281.7°	23.16	-4.74	θ_3
SD_4	381.8°	5.13	-2.42	θ_4

a) Δy only at BS

$$\theta_1 = -.0302 \theta_2, \theta_3 = \theta_2, \theta_4 = \theta_1$$

$$(\Delta y) = 2.42 \theta_2$$

b) $\Delta y'$ only at BS

$$\theta_2 = .0890 \theta_1, \theta_3 = -\theta_2, \theta_4 = -\theta_1$$

$$(\Delta y') = -4.45 \theta_1$$

For $(\Delta y) = \pm 5$ mm, $\theta_2 = \pm 2.1$ mr which corresponds to $(B^L) = \pm 0.17$ kG-m at 2.5 GeV.

The angular change $(\Delta y')$ is unfortunately limited by the resulting orbit displacements at SD_2 and SD_3 . If this is limited to ± 1 mm, max. $|\theta_1| = 0.20$ mr and max. $|\Delta y'| = 0.87$ mr. For larger values of $|\Delta y'|$, one must suppress the distortions at SD_2 and SD_3 by another set of steering elements, for example at Q1, $SD_{2,3}$ and Q6. However, the phase advance between $SD_{2,3}$ and Q6 is not too large and one can possibly extend the range of achievable $|\Delta y'|$ value by at most a factor of ~ 2 .

REFERENCES

1. G. Guignard, CERN Report ISR-MA/75-23, June 1975.
2. The lattice of the X-ray ring is the one provided by R. Chasman and S. Krinsky during the workshop. Tunes are $\nu_x = 10.861$ and $\nu_y = 6.850$. The UV ring used here was kindly provided to me by S. Krinsky. There are two low- β insertions ($\beta_x^* = 0.55$ m, $\beta_y^* = 0.22$ m) and two high- β insertions ($\beta_x^* = 14.15$ m, $\beta_y^* = 6.59$ m). Tunes are $\nu_x = 3.620$ and $\nu_y = 2.630$.

Bunch Lengthening and Longitudinal Stability

Frank J. Sachserer

CERN

SUMMARY

An inductive-wall type impedance of $Z/n = 1$ ohm for the X-ray ring and $1/4$ ohm for the UV ring is assumed. This causes substantial bunch lengthening and widening for the X-ray ring at 700 MeV, but none at 2.5 GeV, and also none for the UV ring at 700 MeV. The same impedance is sufficient to move the coherent dipole frequency outside the band of incoherent synchrotron frequencies, so that Landau damping is lost. The thresholds for the resulting coupled-bunch instabilities are of the order of a few milliamperes or less for both rings due to the small bunch lengths. An additional RF cavity at the 4th harmonic of the RF (Landau cavity) prevents these coupled-bunch instabilities by lengthening the bunches and increasing the frequency spread within the bunch.

1. Bunch length for low intensities.

The Hamiltonian for the equations of motion

$$\frac{dE}{dt} = \frac{\omega_0}{2\pi} V(\varphi) \quad (1)$$

$$\frac{d\varphi}{dt} = h\eta\omega_0 \frac{\Delta p}{p} = \frac{h\eta\omega_0}{\beta^2\gamma E_0} E \quad (2)$$

is

$$H = \frac{h\eta\omega_0}{2\beta^2\gamma E_0} E^2 + \frac{\omega_0}{2\pi} U(\varphi) \quad (3)$$

where
$$V(\varphi) = - \frac{dU}{d\varphi} .$$

With a fourth harmonic Landau cavity,¹⁾

$$\begin{aligned} V(\varphi) &= V_0 \left[\sin \varphi - \frac{1}{4} \sin 4\varphi \right] \\ U(\varphi) &\approx V_0 \left[\frac{5}{8} \varphi^4 - \frac{1}{3} \varphi^6 + \dots \right] \end{aligned} \quad (4)$$

so the φ^6 term can be neglected for bunch lengths less than one radian.*) With the Landau cavity turned off

$$U(\varphi) \approx \frac{1}{2} V_0 \varphi^2$$

and the bunch length is given by

$$\frac{1}{2} V_0 \varphi_0^2 = \frac{\pi h \eta}{\beta^2 \gamma E_0} E_m^2$$

*For simplicity, the synchronous phase φ_s is assumed to be zero. For finite φ_s replace V_0 by $V_0 \cos \varphi_s$ in all expressions.

or
$$\varphi_0 = \left[\frac{2\pi h \eta}{V_0 \beta^2 \gamma E_c} E_m^2 \right]^{1/2}. \quad (5)$$

With the Landau cavity on,

$$\frac{5}{8} V_0 \varphi_m^4 = \frac{\pi h \eta}{\beta^2 \gamma E_0} E_m^2.$$

or
$$\varphi_m = \left(\frac{4}{5} \right)^{1/4} \sqrt{\varphi_0} = 0.946 \sqrt{\varphi_0}. \quad (6)$$

The bunch boundary is assumed to be the 2σ value.

Ring			$\eta (= \alpha)$	$\Delta E/E$	φ_0 (rad)	φ_m (rad)
UV	700	MeV	$.272 \times 10^{-1}$	$.868 \times 10^{-3}$	.0816	.270
X-ray	700	MeV	$.56 \times 10^{-2}$	$.564 \times 10^{-3}$	.0314	.166
X-ray	2.5	GeV	$.56 \times 10^{-2}$	2×10^{-3}	.128	.338

2. Potential-well distortion for inductive-wall impedance.

The additional potential is

$$U(\varphi) = h\beta c e \frac{Z}{n} \lambda(\varphi) \quad (7)$$

where $\lambda(\varphi)$ is the line density (particles/meter). For the elliptic distribution

$$\psi(\varphi, E) = \sqrt{H_0 - H} = \sqrt{E_B^2(\varphi) - E^2}, \quad (8)$$

the line density

$$\lambda(\varphi) = \int \psi dE = \frac{\pi}{2} E_B^2(\varphi) \propto \frac{H}{F_0} - U(\varphi) \quad (9)$$

has the same form as the external potential.²⁾ Therefore

$$\lambda(\varphi) = \frac{5}{8} \frac{hN}{R\varphi_m} \left[1 - \left(\frac{\varphi}{\varphi_m} \right)^4 \right] \quad (\text{Landau cavity on}) \quad (10)$$

$$\lambda(\varphi) = \frac{3}{4} \frac{hN}{R\varphi_m} \left[1 - \left(\frac{\varphi}{\varphi_m} \right)^2 \right] \quad (\text{Landau cavity off})$$

where N is the number of particles in a bunch and R is the machine radius. The bunch length with the potential (7) included is given by

$$\frac{5}{8} V_0 \varphi_m^4 = \frac{\pi h \eta}{\beta^2 \gamma E_0} E_m^2 + \frac{5}{8} \frac{2\pi h}{\varphi_m} I \frac{Z}{n}$$

or

$$\varphi_m^4 = \frac{4}{5} \varphi_0^2 + \frac{2\pi h}{V_0 \varphi_m} I \frac{Z}{n} \quad (\text{Landau cavity on}) \quad (11)$$

$$\varphi_m^2 = \varphi_0^2 + \frac{3\pi h}{V_0 \varphi_m} I \frac{Z}{n} \quad (\text{Landau cavity off})$$

where $I = ehNf_0$ is the total current in h bunches.

3. Bunch lengthening and widening due to microwave instability.

For high frequency instabilities with many wavelengths within one bunch length and with growth times shorter than a synchrotron period, the threshold is given by the usual coasting-beam criterion.^{3,4,5)}

$$\hat{I} \frac{Z}{n} = \frac{0.7\pi}{2} \gamma E_o |\eta| \left(\frac{\Delta E}{E}\right)^2 \quad (12)$$

but using the peak bunch current $\hat{I}$,

$$\hat{I} = 1.5 \frac{I}{B} = \frac{1.5\pi}{\phi_m} I .$$

The current threshold is therefore given by

$$I = \frac{\gamma E_o |\eta| \phi_m}{4.29 \frac{Z}{n}} \left(\frac{\Delta E}{E}\right)^2 \quad (13)$$

where $\Delta E/E$ is the 2 σ value. The threshold currents for $Z/n = 1$ ohm are:

Ring			Landau cavity off	Landau cavity on
UV	700	MeV	272 mA	900 mA
X-ray	700	MeV	9.1 mA	48.7 mA
X-ray	2.5	GeV	1.67 A	4.42 A

Above threshold, the energy spread increases until (13) is satisfied, and the bunch length formula (11) becomes

$$\phi_m^4 = \underbrace{4.29 \frac{4}{5} \frac{2\pi h}{V_o \phi_m} I \frac{Z}{n}}_{\text{microwave}} + \underbrace{\frac{2\pi h}{V_o \phi_m} I \frac{Z}{n}}_{\text{potential-well}} . \quad (14)$$

Since the potential-well term is considerably smaller than the microwave term, there will be very little bunch lengthening below the microwave threshold, and the low intensity expressions (5) and (6) can be used. For higher intensities

$$\begin{aligned}\varphi_m &= 1.95 \left(\frac{hI}{V_o} \frac{Z}{n} \right)^{1/5} && \text{(Landau cavity on)} \\ \varphi_m &= 3.31 \left(\frac{hI}{V_o} \frac{Z}{n} \right)^{1/3} && \text{(Landau cavity off).}\end{aligned}\tag{15}$$

The bunch half-lengths and energy spread are plotted in Figures 1 and 2 for the X-ray ring at 700 MeV assuming $Z/n = 1$ ohm.

Above the microwave threshold, the effective voltage including potential-well depression is given by

$$\begin{aligned}V &= V_o - \frac{2\pi h}{5} I \frac{Z}{n} = 0.774 V_o && \text{(Landau cavity on)} \\ V &= V_o - \frac{3\pi h}{3} I \frac{Z}{n} = 0.740 V_o && \text{(Landau cavity off),}\end{aligned}\tag{16}$$

while below the threshold, the potential-well depression can be neglected.

4. Coupled-bunch instability for dipole modes.

With the Landau cavity on, the threshold for coupled-bunch instability is⁶⁾

$$hI \frac{Z}{n} \frac{1}{V\varphi_m} \leq 0.151\tag{17}$$

so the bunch length should be greater than

$$\begin{aligned}\varphi_m &\geq 1.46 \left(\frac{hI}{V_o} \frac{Z}{n} \right)^{1/5}, \quad I < \text{microwave threshold} \\ &\geq 1.54 \left(\frac{hI}{V_o} \frac{Z}{n} \right)^{1/5}, \quad I > \text{microwave threshold}.\end{aligned}\tag{18}$$

However, due to the microwave instability

$$\varphi_m \geq 1.95 \left(\frac{hI}{V_0} \frac{Z}{n} \right)^{1/5} \quad (19)$$

so coupled-bunch modes are always stable with the Landau cavity on. The coherent dipole frequency given by⁶⁾

$$\begin{aligned} y &= \frac{\omega}{1.34 \omega_{so} \varphi_m} = 1 + 2.9 hI \frac{Z}{n} \frac{1}{V \varphi_m^5} \\ &= 1 + 0.103 V_0/V \end{aligned} \quad (20)$$

is always well within the range $\frac{1}{2} < y < 1\frac{1}{2}$ required for Landau damping. Here ω_{so} is the synchrotron frequency without Landau cavity, $1.34 \omega_{so} \varphi_m$ is the synchrotron frequency for the particles with amplitude φ_m , and $V \approx V_0$ or $0.740 V_0$ for intensities below or above the microwave threshold.

With the Landau cavity off, the threshold for coupled-bunch instability is⁶⁾

$$hI \frac{Z}{n} \frac{1}{V \varphi_m^3} \leq 0.106 \frac{S}{\omega_{so}} \quad (21)$$

where the spread in synchrotron frequencies is given by

$$\frac{S}{\omega_{so}} \approx \frac{2}{3} \frac{1 - \Gamma^2}{1 - \Gamma^2} \left(\frac{\varphi_m}{\pi} \right)^2 \quad (22)$$

with $\Gamma = \sin \varphi_s$. For non-zero φ_s , the V in (21) should be replaced by $V \cos \varphi_s$. The bunch length required for coupled-bunch stability given by (21) is plotted in Fig. 3 for $\varphi_s = 0$ and 45° . Also plotted is the bunch length (15) that results from the microwave instability, which is too small to prevent coupled-bunch instability. The thresholds determined from (21) with $\frac{Z}{n} = 1$ ohm are:

Ring			V_0 (kV)	φ_0 (rad)	φ_s	I
UV	700	MeV	100	.0816	6.4°	3.6 mA
X-ray	700	MeV	200	.0314	0	1.8×10^{-6} A
X-ray	2.5	GeV	800	.128	47°	18.3 mA

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FIG. 1. X-ray ring at 700 MeV for $\frac{Z}{R} = 1$ ohm

Longitudinal Stability with a Landau Cavity

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SUMMARY

An additional RF cavity operating at a higher harmonic of the RF can substantially increase the bunch length and spread in synchrotron frequencies within a bunch.¹⁾ The effect is most pronounced when the additional cavity (Landau cavity) is phased opposite to the main RF to cancel the usual φ^2 parabolic potential well, leaving a φ^4 potential. A stability criterion for coherent dipole oscillations is derived for this case.

1. Particle orbits in phase space.

For the stationary non-oscillating bunch, the single-particle orbits are determined by the Hamiltonians

$$\begin{aligned} H_0 &= \frac{1}{2} (ap^2 + bq^2) & (\text{Landau cavity off}) \\ H_0 &= \frac{1}{2} (ap^2 + bq^4) & (\text{Landau cavity on}) \end{aligned} \quad (1)$$

where, for example, $b = \frac{1}{2}V$ or $\frac{5}{8}V$ for the Landau cavity off or on.²⁾ It is convenient to use the action-angle variables J, w instead of p, q where

$$\begin{aligned} J &= \oint p dq \\ \dot{w} &= \frac{\partial H_0}{\partial J} = \frac{\omega_s}{2\pi} \end{aligned} \quad (2)$$

and ω_s is the synchrotron frequency. The results are shown in Table I. With the Landau cavity on, the synchrotron frequency of a particle is proportional to its amplitude,

$$\omega_s = 1.34 \omega_{s0} q_m \quad (3)$$

where ω_{s0} is the synchrotron frequency without Landau cavity. The coordinate q is given by a Jacobi elliptic function,³⁾

$$q = q_m \operatorname{cn}(7.4w) \quad (4)$$

which can be expanded as

$$q = q_m [0.956 \cos 2\pi w + 0.042 \cos 6\pi w + \dots]. \quad (5)$$

TABLE I

	Landau cavity off	Landau cavity on
angle variable	$J = \pi \sqrt{\frac{b}{a}} q_m^2$	$J = 3.49 \sqrt{\frac{b}{a}} q_m^3$
particle amplitude	$q_m = \left(\frac{2H}{b}\right)^{\frac{1}{2}}$	$q_m = \left(\frac{2H}{b}\right)^{\frac{1}{3}}$
Hamiltonian	$H_o = \frac{\sqrt{ab}}{2\pi} J$	$H_o = .0945 \left(\frac{a}{b}\right)^{2/3} b J^{4/3}$
angle variable	$w = \frac{\sqrt{ab}}{2\pi} t$	$w = 0.191 \sqrt{ab} q_m t$
synchrotron freq.	$\omega_{so} = \sqrt{ab}$	$\omega_s = 1.2 \sqrt{ab} q_m$
coordinate	$q = q_m \cos 2\pi w$	$q = q_m \operatorname{cn}(7.4w)$

The motion is almost sinusoidal, with 4% third harmonic and less than 1% higher harmony content.

2. Dispersion relations.

For an oscillating bunch,

$$\psi = \underbrace{\psi_o}_{\text{stationary}} + \underbrace{\psi_i e^{j\omega t}}_{\text{perturbation}} \quad (6)$$

and

$$H = H_o + H_i e^{j\omega t} \quad (7)$$

where H_o includes the fields due to the stationary distribution ψ_o , while H_i includes the fields due to the perturbation ψ_i . The distribution ψ is determined by the Vlasov equation

$$\frac{\partial \psi}{\partial t} + \dot{w} \frac{\partial \psi}{\partial w} + j \frac{\partial \psi}{\partial J} = 0 \quad (8)$$

$$\text{or} \quad \frac{\partial \psi}{\partial t} + \frac{\partial H}{\partial J} \frac{\partial \psi}{\partial w} - \frac{\partial H}{\partial w} \frac{\partial \psi}{\partial J} = 0 \quad (9)$$

where action-angle variables J, w for the stationary Hamiltonian H_0 have been used. For these variables,

$$\frac{\partial H_0}{\partial J} = \nu(J), \quad \frac{\partial H_0}{\partial w} = 0. \quad (10)$$

Substitute (6) and (7) into (9) to find the linearized Vlasov equation for ψ_i ,

$$j\omega \psi_i + \nu(J) \frac{\partial \psi_i}{\partial w} - \frac{d\psi_0}{dJ} \frac{\partial H_i}{\partial w} = 0 \quad (11)$$

where nonlinear terms in ψ_i have been dropped.

The perturbation ψ_i can be expanded in multipoles,

$$\psi_i = \sum_m R_m(J) e^{2\pi j m w}$$

where the last term in (11) couples the different multipoles together. As long as the frequencies of two modes are not shifted too close together, this coupling can be neglected, and

$$j(\omega - m\nu_s) R_m(J) - \frac{d\psi_0}{dJ} \int_0^1 e^{2\pi j m w} \frac{\partial H_i}{\partial w} dw = 0. \quad (12)$$

Integrate this by parts to find

$$[\omega - m\nu_s(J)] R_m(J) + 2\pi m \frac{d\psi_0}{dJ} \int_0^1 e^{2\pi j m w} H_i(J, w) dw = 0. \quad (13)$$

In the following, we specialize to $m = 1$ rigid-bunch oscillation and drop the subscript on $R(J)$. For constant Z/n such an inductive-wall impedance, the perturbed Hamiltonian is

$$H_i = h\omega_0 I \frac{Z}{n} \lambda_i(q) \quad (14)$$

where
$$\lambda_i(q) = \int \psi(p, q) dp \quad (15)$$

and the normalization

$$1 = \int \psi_0(p, q) dp dq = \int \psi_0(J) dJ \quad (16)$$

is assumed. Here I is the total current in h bunches, and the relation $dpdq = dJdw$ for canonical variables has been used in (16).

For rigid-bunch oscillation,

$$\lambda_i(q) = \frac{d\lambda_0}{dq} \bar{q} = \int R(J) e^{2\pi j w} dp \quad (17)$$

and $\bar{q}$ is found as follows:

$$\begin{aligned} \int q \lambda_i dq &= \bar{q} \int q \frac{d\lambda_0}{dq} dq = -\bar{q} \\ &= \int q R(J) e^{2\pi j w} dJ dw, \end{aligned}$$

so
$$\bar{q} = -\frac{1}{2} a_1 \int q_m(J) R(J) dJ \quad (18)$$

where the expansion (5) has been used with $a_1 = 0.956$ for the φ^4 potential and $a_1 = 1$ for the φ^2 potential. For the φ^2 potential,

$$\lambda_0 = \frac{3}{4\varphi_m} \left[1 - \left(\frac{q}{\varphi_m} \right)^2 \right]$$

and
$$H_i = \frac{3}{4} h w_0 I \frac{Z}{n} \frac{q}{\varphi_m} \int q_m(J) R(J) dJ \quad (19)$$

while for the φ^4 potential

$$\lambda_0 = \frac{5}{8\varphi_m} \left[1 - \left(\frac{q}{\varphi_m} \right)^4 \right]$$

$$\text{with } y = \frac{\omega - \omega_{so}}{\omega_{so}}$$

$$\text{and } 1 = 38.7 \, hI \, \frac{Z}{n} \frac{1}{V \varphi_m} \int \frac{x^7 e^{-1.83x^4}}{y-x} dx \quad (\varphi^4 \text{ potential}) \quad (25)$$

$$\text{with } y = \frac{\omega}{1.2 \sqrt{ab} \varphi_m}.$$

Some nonlinearity has been added to the φ^2 potential to give a spread in synchrotron frequencies,

$$\omega = \omega_{so} - S(q_m/\varphi_m)^2$$

where S is the spread for two standard deviations.

For given beam parameters and coupling impedance, relations (24) and (25) determine the coherent dipole frequency ω or equivalently y , which can be complex. This frequency has a strong damping component (Landau damping) as long as the pole from the denominator lies in the range where the numerator is large, or in other words, as long as the coherent dipole frequency lies within the band of incoherent synchrotron frequencies. Inductance shifts the coherent frequency up relative to the incoherent single-particle frequencies, so the stability limit for the φ^2 potential occurs when ω approaches ω_{so} , or y approaches zero, and (24) reduces to

$$hI \frac{Z}{n} \frac{1}{V \varphi_m} \leq 0.106 \frac{S}{\omega_{so}} \quad (\varphi^2 \text{ potential}) \quad (26)$$

For the φ^4 potential, the numerator in the dispersion relation is non-zero only in the range $\frac{1}{2} < x < 1\frac{1}{2}$ (see Figure 1), so the threshold occurs at $y = \frac{1}{2}$ or $1\frac{1}{2}$, and (25) reduces to

$$hI \frac{Z}{n} \frac{1}{V\varphi_m^5} \leq 0.151 \quad (\varphi^4 \text{ potential}) \quad (27)$$

An approximate expression for the coherent frequency ω for the φ^4 potential is

$$y = \frac{\omega}{1.2 \sqrt{ab} \varphi_m} = 1 + 2.9 hI \frac{Z}{n} \frac{1}{V\varphi_m^5}, \quad (28)$$

plus a damping term when y lies within the range $\frac{1}{2} < y < 1\frac{1}{2}$.

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Fig. 1. Numerator of the dispersion relation (25)

Transverse Coupled-bunch Instabilities

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SUMMARY

A Q-spread from octupoles of 10^{-3} over 10σ should stabilize the beam provided the inductive-wall impedance Z/n is less than 1 ohm in both rings. In the absence of a Q-spread, a few ohms of shunt resistance is sufficient to drive coupled-bunch instabilities, provided the resonator bandwidth is less than the RF frequency. Wall resistance results in e-folding times of the order of 40 msec, or considerably less if resistive coatings are used.

The general formula for the frequency shift caused by an arbitrary transverse coupling impedance $Z_{\perp}$ is¹⁾

$$\Delta\omega_m = \frac{j}{1+m} \frac{e\beta I}{2Q\omega_0 \gamma m_0} \frac{1}{2\pi R} \sum_n Z_{\perp}(\omega_n) F'_m(\omega_n) \quad (1)$$

for head-tail mode m . For M bunches, there are M coupled-bunch modes of oscillation, with phase shift $2\pi|n|/M$ between adjacent bunches. The sum in (1) is over the frequencies

$$\omega_n = (n+Q)\omega_0 \pm \text{multiples of the RF} \quad (2)$$

that are associated with mode n .

The transverse impedance $Z_{\perp}$ (ohms/meter) is often related to the longitudinal impedance Z by

$$Z_{\perp} = \frac{2c}{b^2} \frac{Z}{\omega} \quad (3)$$

where b is an effective vacuum chamber radius and $c = 3 \times 10^8$ m/s. Equation (3) is exact for inductive-wall or resistive-wall impedances, but not for a cavity with different transverse and longitudinal resonant frequencies, although the peak values are related approximately by (3). Reactive impedance such as wall inductance causes a real frequency shift $\Delta\omega$ that may be sufficient to shift the coherent betatron frequency outside the band of incoherent betatron frequencies so that Landau damping is lost. Then a relatively small resistive component due to a resonant element in the vacuum chamber, for example, can cause instability provided its bandwidth is less than the RF frequency so that the wake field couples adjacent bunches, and provided the resulting growth rate is larger than the radiation damping rate.

In the following, we specialize to zero machine chromaticity, and to the rigid-bunch mode $m = 0$. This mode is driven by resonators with frequencies less than the reciprocal bunch length, which is typically a few GHz. The higher modes are driven by higher frequency resonators. Equation (1) becomes

$$\Delta\omega = - \frac{I}{2\pi Q \gamma E_0} \frac{cR}{b^2} \left[\frac{1}{B} \left(\frac{Z}{n} \right)_\ell - j \left(\frac{Z}{n} \right)_r F'_0 \right] \quad (4)$$

where $(Z/n)_\ell$ is a broadband inductive impedance, $(Z/n)_r$ is a resonator shunt impedance with bandwidth less than the RF frequency, $F'_0 \approx \frac{1}{2}$ and falls to zero for frequencies higher than the reciprocal bunch length, B is the bunching factor, R is the machine radius, I is the total current, and $E_0 = 0.511 \times 10^6$ volts.

As a rule-of-thumb, the required transverse Q-spread over 2σ of the beam for Landau damping is about $2|\Delta\omega|/\omega_c$. The Q-spread for 10σ is shown in Table I for an inductive-wall impedance of $Z/n = 1$ ohm in both rings, $b \approx 5$ cm, $I = 1$ A,

Table I. The 10σ Q-spread required for Landau damping.

					Landau cavity off		Landau cavity on	
Ring			V_0 (kV)	Q	σ (cm)	Q-spread (10^{-3})	σ (cm)	Q-spread (10^{-3})
UV	700	MeV	100	2.8	3.9	3.6	12.9	1.1
X-ray	700	MeV	200	6.8	1.5 (8.1)	8 (1.5)	7.8 (15.6)	1.5 (.7)
X-ray	2.5	GeV	800	6.8	6.1	.5	16.2	.2

and for the natural bunch length σ with and without the Landau cavity.²⁾ Considerable bunch lengthening is expected for the X-ray ring at 700 MeV, and the numbers in parenthesis refer to this case.

The shunt impedance required to overcome the radiation damping is shown in Table II, where τ is the damping time.

Table II. Limiting resonator shunt impedance.

Ring			V_0 (kV)	Q	τ (msec)	Z/n (ohms)
UV	700	MeV	100	2.8	20	1.47
X-ray	700	MeV	200	6.8	200	.104
X-ray	2.5	GeV	800	6.8	4	18.5

The resistance in the vacuum chamber wall can also cause instability. The impedances at the revolution frequency for an Aluminum vacuum chamber of $b = 5$ cm radius and resistivity $\rho = 3 \times 10^{-8}$ ohm-m are shown in Table III.

Table III. Resistive-wall impedance.

	UV 700 MeV	X-ray 700 MeV
skin depth (mm)	.033	.059
surface resistance (ohm/square)	$.92 \times 10^{-3}$	$.51 \times 10^{-3}$
Re Z (ohm)	.12	.24
Re $Z_{\perp}$ (ohm/m)	$.67 \times 10^3$	$.41 \times 10^4$

Because of the skin depth variation with frequency, Z/n varies as $1/\sqrt{n}$ and becomes large at low frequencies. The growth rate from (2) and (3) is

$$\begin{aligned}
 \text{Im } \Delta\omega &= \frac{70}{Q \sqrt{|n+Q|}} && \text{X-ray ring} \\
 &= \frac{11.4}{Q \sqrt{|n+Q|}} && \text{UV ring}
 \end{aligned} \tag{5}$$

for $I = 1$ A at 700 MeV. Instability requires negative resistive impedance $Z_{\perp}$, and therefore negative ω (from 3) or negative $n+Q$. The growth rates for the fastest growing modes at 700 MeV are:

	UV	X-ray
Q	2.8	6.8
n	-3	-7
$\text{Im } \Delta\omega$	9.1 rad/s	23 rad/s
e-folding time	110 msec	43 msec

Considerably faster growth rates occur if high resistivity coatings are used. If a 1 meter ceramic tube with radius 5 cm and a resistive coating of 0.1 ohm/square is added, the additional coupling impedance is $Z = 0.32$ ohms, and

$$\begin{aligned}
 \text{Im } \Delta\omega &= \frac{94}{Q|n+Q|} && \text{X-ray ring} \\
 & && (6) \\
 &= \frac{30}{Q|n+Q|} && \text{UV ring}
 \end{aligned}$$

at 700 MeV. The e-folding times become 19 msec for the UV ring and 14 msec for the X-ray ring.

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Coupling Impedance and Power Dissipation
for a Step Change in the Vacuum Chamber

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In reference 1, the inductive and resistive components of the longitudinal coupling impedance Z due to a step change in the vacuum chamber are computed for a rectangular geometry with a vertical step. The general

trends are sketched in Figure 1. Here S is the relative

Fig. 1.

change in vacuum chamber dimensions,

$$S = \frac{h_2 - h_1}{h_2} \quad (1)$$

and f_c is the lowest TM cutoff frequency

$$f_c = \frac{c}{4h_1}, \quad (2)$$

which is typically a few GHz. For frequencies below f_c/S , both inductive and resistive components are approximately

$$\frac{Z}{n} = 100 \frac{S^2}{n_c^2} \quad (3)$$

with

$$n_c = \frac{\pi R}{2 h_1}$$

where R is the machine radius. For a 20% step in the X-ray ring with $h_1 = 17.5$ mm,

$$\frac{Z}{n} = 1.86 \text{ m}\Omega \quad (4)$$

up to frequencies around 20 GHz. In addition, there are peaks in the impedance curves three to five times the values shown in Fig. 1, which occur near the lower cutoff frequencies.

As pointed out in reference 1, the impedances can also be roughly estimated from the excess electric or magnetic field in the perturbed region, using the relations

$$\begin{aligned} \frac{1}{2} CV^2 &= \frac{1}{2} \epsilon E^2 \\ \frac{1}{2} LI^2 &= \frac{1}{2} B^2/\mu. \end{aligned} \quad (5)$$

For a 45° electrostatically shielded corner region, the added inductance due to the excess magnetic field is

$$\Delta L = \frac{1}{I^2} \int \frac{B^2}{\mu_0} dv \quad (6)$$

where the integral is over the volume of the shielded region. For a circular geometry,

$$B \approx \frac{\mu_0 I}{2\pi r_2} \quad (7)$$

in the shielded region of volume $\pi r_2 (r_2 - r_1)^2$, so the added inductance is

$$\Delta L = \frac{\mu_0}{4\pi} r_2 S^2 \quad (8)$$

where S is again the relative step height. In terms of the lowest TM cutoff $f_c = n_c f_0$,

$$\frac{Z}{n} = 72 \frac{S^2}{n_c} \quad (9)$$

with
$$n_c = 2.405 \frac{R}{r_1},$$

which is somewhat smaller than the computer result (3).

The resistive impedance in the high frequency region $f > f_c/S$ where there are several wavelengths in one step height is also easy to estimate. The power flow associated with the beam fields is found by integrating the Poynting vector $S = E \times H$ over the vacuum chamber aperture. For a circular geometry, one finds

$$\int S \, dA = \frac{Z_0 I^2}{2\pi} \ln \frac{b}{a} \quad (\text{watts}) \quad (10)$$

where $Z_0 = 120\pi$ ohms, a is the beam radius and b is the vacuum chamber radius.

The power reflected from a step at high frequencies is given by the ratio of the step area to vacuum chamber aperture,

$$\frac{r_2^2 - r_1^2}{r_2^2} \frac{Z_0 I^2}{2\pi} \ln \frac{b}{a}, \quad (11)$$

and since only half the steps reflect in this high frequency region, the equivalent resistive impedance is

$$Z = 60 S \ln \frac{b}{a}, \quad (12)$$

which is in reasonable agreement with Figure 1.

The power dissipated by a step is

$$P = \frac{1}{2} I^2 \sum_n \operatorname{Re} Z(n) e^{-n^2 \sigma^2 \omega_0^2} \quad (13)$$

where I is the total current, σ is the one-standard-deviation half-bunch-length in seconds, and ω_0 is the revolution frequency c/R . For a step with impedance (3),

$$P = 50 \frac{S^2}{n_c} \frac{1}{(\sigma \omega_0)^2}. \quad (14)$$

The power dissipated by a 20% step in a vacuum chamber with $h_1 = 17.5$ mm, $I = 1$ A and the natural bunch length is:

			Landau cavity off		Landau cavity on	
Ring		V_0 (kV)	σ (cm)	P(watts)	σ (cm)	P(watts)
UV	700 MeV	100	3.9	211	12.9	19.3
X-ray	700 MeV	200	1.5	2,380	7.8	85.2
X-ray	2.5 GeV	800	6.1	144	16.2	20.4

If the bunch lengthening expected for the X-ray ring at 700 MeV is taken into account,²⁾ the power dissipated in a single 20% step is about 100 watts with the Landau cavity off and about 20 watts with the cavity on. A total resistive component of $Z/n = 1$ ohm corresponds to about 600 such steps and leads to 60 kW dissipation with the Landau cavity off and 12 kW with the cavity on. A resistive component one quarter as large in the UV ring leads to 28 kW dissipation with the Landau cavity off and 2.6 kW with it on.

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Bunch Lengthening and Widening

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Bunch lengthening and widening has been observed in electron storage rings for large bunch currents. Since these effects would increase the synchrotron radiation source size and require more accurate field compensation, they have to be understood and cured for both electron rings, the ultraviolet source (UV-ring) and the X-ray source (XR-ring).

Recently¹ two empirical formulae were derived based on computer simulation of the motion of a bunch of electrons under the effect of an external RF system, the synchrotron radiation (damping and quantum fluctuations) and current induced effects coupled to the existence of a resistive broad-band impedance. The formulae are

$$S = 0.7 \times 10^3 \left(\frac{Q v_s}{ch} \right)^{\frac{1}{2}} \frac{\rho^{1/2}}{E} \quad (1)$$

$$R = 1.9 \times 10^3 \left(\frac{Q\rho}{ch} \right)^{\frac{1}{2}} \frac{v_s}{E} \quad (2)$$

where

$$S = \frac{\text{final bunch length}}{\text{equilibrium bunch length}}$$

$$R = \frac{\text{final bunch width}}{\text{equilibrium bunch width}}$$

and

$$Q = 10^{-3} \frac{IZ/\sigma}{V_c/2\pi} \quad (3)$$

I is the average beam current, Z the impedance, V_c the RF peak voltage and σ the equilibrium rms bunch length. The coefficients at the r.h. side of (1) and (2) are derived from the fitting with the computer results. The other parameters are: ν_s the number of phase oscillations per turn, α the momentum compaction factor, h the RF harmonic number, ρ the bending radius in meter, and E the total energy of a particle in GeV.

At this point we warn the reader that equations (1) and (2), which correspond to equations (3a and b) of ref. [1], are correct,^(*) whereas eqs. (4a and b) of the same reference are wrong because they are derived with the wrong assumption on σ ; in fact σ in eq. (3) is the equilibrium bunch length (rms) in the limit of zero current and henceforth a number which depends only on the machine parameters. As a consequence the lengthening and widening factors, that can be calculated directly from (1) and (2) by means also of (3), have a dependence on the current like $I^{\frac{1}{2}}$ and not $I^{1/3}$ as erroneously stated in the discussion of ref. [1].

The fitting of SPEAR II data² with (1) and (2) would give for the impedance Z a value around 10 K Ω . If one assumes that the impedance is linearly proportional to the size of the ring then we take about the same value for the XR-ring and about one-third of this for the UV-ring as shown in Table I. In the same table we show their S and R as calculated by means of (1) and (2).

It seems that both machines would suffer large lengthening and practically no widening. Actually the fact that $R < 1$ for the XR-ring seems to indicate that this ring is below the threshold of instability at 2.5 GeV,

(*) Observe the change of power dependence on the bending radius ρ from $1/3$ to $1/2$, which seems to be more correct.

whereas the UV-ring is just about at the threshold. Then we remind that both equations (1) and (2) have been derived for the case where one is well above the threshold where also $R \sim S$. Thus, because of this result, we should not apply eq. (2) for the UV-ring and the XR-ring. To see the implication of all this we did a few computer runs where we simulated the motion of an electron bunch in the X-ring at 2.5 GeV. We plotted the final bunch growth (S and R) versus $Q^{\frac{1}{2}}$ in Fig. 1. Each run lasted about three times the energy damping time. One can observe that there is indeed a threshold on the bunch widening around $Q = 0.6 \times 10^{-3}$. The bunch lengthening does not seem to possess any threshold and its dependence on the current is indeed $I^{\frac{1}{2}}$. In the last two rows of Table I we show the growth factors for the parameters listed on the same table, as extrapolated from the computer results. The lengthening is equal to the one calculated by means of eq. (1), but the widening is now substantially higher.

In order to take into account the threshold one should modify the scaling equations (1) and (2) by adding a constant so to have now

$$S = K_1 \left(\frac{Q \nu_s}{\alpha h} \right)^{\frac{1}{2}} \frac{\rho^{1/2}}{E} + K_2 \quad (4)$$

$$R = h_1 \left(\frac{Q \rho}{\alpha h} \right)^{\frac{1}{2}} \frac{\nu_s}{E} + h_2 \quad (5)$$

where the four constants K_1 , K_2 , h_1 and h_2 should be calculated by fitting the computer results discussed in ref. [1]. For the present we simply calculate them by fitting the data shown in Fig. 1. One has

$$K_1 = 0.8 \times 10^3, \quad K_2 = 0$$

and

$$h_1 = 1.5 \times 10^4, \quad h_2 = -1$$

The growth factors calculated by means of (4) and (5) for the UV-ring are shown in round brackets in Table I.

A 700 MeV beam is directly transferred to the UV-ring so there is no need of acceleration in this ring. The same beam is transferred also to the XR-ring where this can be accelerated to any energy up to 2.5 GeV. The worst case is then expected to be in the XR-ring at the injection energy. In fact both growth factors S and R scale with the energy like E^{-2} and they are an order of magnitude larger than at full energy.

In conclusion quite a large bunch dilution is expected for both machines. One obvious cure is to reduce the environmental impedance Z, though this is rather difficult because of the geometry of the vacuum chamber with so many ports and flanges for the radiation extraction. Also a reduction of the impedance by an order of magnitude would only reduce the bunch growth in either dimension by a factor 3.

A more effective cure is the installation of a higher harmonic cavity which operates at an integer times the fundamental RF frequency. This type of cavity has already been proposed for SPEAR and an experiment is going to be performed this fall.³

Using the same program we discussed above we made a computer simulation for the XR-ring at 2.5 GeV with the addition of a cavity working at the harmonic number 4×24 . The voltage and the phase of the cavity were adjusted so that the potential well remained flat across the bunch length. The main effect is that, also in the limit of zero current, the bunch distribution changes from a Gaussian to a rectangular one, with a longer length but the same width. This was

predictable. Not much extra growth was found on our computer simulation when the bunch current and the impedance were taken into account.

In conclusion, our investigation led to quite an alarming bunch growth for both rings, especially at low energies. We recommend the use of a higher harmonic cavity to reduce this growth and careful design of the vacuum chamber to reduce the surrounding impedance. This sort of investigation as outlined here should, of course, continue.

References

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3. J. M. Paterson, private communication, June 1977.

TABLE I.

List of Parameters for

	<u>UV-ring</u>	and	<u>XR-ring</u>
I	1 A		1 A
V_c	0.1 MV		0.8 MV
σ	0.0346 rad		0.054 rad
ρ	1.9 m		5.9 m
α	0.02		0.0056
E	0.7 GeV		2.5 GeV
v_s	0.0018		0.0022
Z	3 K α		10 K α
h	7		24
Q (eq. (3))	5.4×10^{-3}		1.4×10^{-3}
S (eq. (1))	11		3.3
R (eq. (2))	1.3		0.4
S (computer)	(12)		2.9
R (computer)	(9)		2.1

XR-Ring (2.5 GeV)

FIG. 1

Aperture Limitations by Sextupoles

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I. Introduction

The unusual lattice structure of the X-ray ring and the UV ring makes it necessary to use sextupoles of a high field gradient for controlling chromaticities. In the X-ray ring, chromaticities are related to sextupole strengths by (lowest-order terms only)

$$\Delta v_x / (\Delta p/p) = 1.668 \left(\frac{B'' l}{B\rho} \right)_{SF} + 0.471 \left(\frac{B'' l}{B\rho} \right)_{SD} - 25.38,$$

$$\Delta v_y / (\Delta p/p) = -0.517 \left(\frac{B'' l}{B\rho} \right)_{SF} - 2.360 \left(\frac{B'' l}{B\rho} \right)_{SD} - 24.35$$

where $(B'' l/B\rho)$ is in m^{-2} . For the UV ring,¹

$$\Delta v_x / (\Delta p/p) = 0.887 \left(\frac{B'' l}{B\rho} \right)_{SF} + 0.339 \left(\frac{B'' l}{B\rho} \right)_{SD} - 8.79,$$

$$\Delta v_y / (\Delta p/p) = -0.444 \left(\frac{B'' l}{B\rho} \right)_{SF} - 0.802 \left(\frac{B'' l}{B\rho} \right)_{SD} - 5.74$$

In order to make both chromaticities positive, the sextupole strength $|B'' l/B\rho|$ must be $\sim 20 m^{-2}$. Tunes are of course chosen such that structural (inherent) resonances of the third order

$$3v_x = N, \quad v_x \pm 2v_y = N \quad (N = \text{multiples of ring periodicity})$$

are far from the expected operating point. Nevertheless, computer runs made by S. Krinsky clearly show a limited stable area in the transverse phase space. This is the case even when the effects of the phase oscillation and of the horizontal-vertical coupling are ignored.

The purpose of this note is to see if one can understand the sextupole effects from the standard analytical investigation. Since the operating point is far from the resonances, it may be meaningless to retain the so-called resonance-driving terms alone in the Hamiltonian. Although these terms are nominally the weakest in the dependence on phase, they are not even approximately phase-independent. One would have to go to the next order of approximation in which oscillating terms in the lowest order are combined to produce phase-independent terms as well as driving terms of different resonances. With regular (non-skew) sextupoles alone, resonances in the lowest order are

$$3\nu_x = N, \quad \nu_x = N \text{ and } \nu_x \pm 2\nu_y = N$$

whereas, in the next order, one finds

$$6\nu_x = N, \quad 4\nu_x = N, \quad 2\nu_x = N,$$

$$4\nu_y = N, \quad 2\nu_y = N,$$

$$4\nu_x \pm 2\nu_y = N, \quad 2\nu_x \pm 2\nu_y = N \text{ and } 2\nu_x \pm 4\nu_y = N.$$

These are similar to but not identical to resonances produced by regular 12-pole field in the lowest order,

$$6\nu_x = N, \quad 4\nu_x \pm 2\nu_y = N, \quad 2\nu_x \pm 4\nu_y = N, \text{ and } 6\nu_y = N$$

The amplitude dependence of the driving terms and the phase independent terms is identical to the one with the lowest-order octupole field, i.e., (amplitude)².

II. Lowest-Order Sextupole Resonances

It is easy to see that the effect of sextupoles cannot be understood in the lowest-order approximation. Coupling resonances of the type

$v_x \pm 2v_y = N$ are ignored in the following demonstration for the sake of simplicity and clarity. They will be included in Section III where the second-order approximation is considered.

The equation of motion in the horizontal direction is²

$$d^2\xi/d\varphi^2 + v^2\xi = v^2\beta^{3/2}F$$

where

$$\xi = x/\sqrt{\beta},$$

$$\varphi = \int ds/(v\beta),$$

and
$$F = -\frac{1}{2}(B''/B\beta)x^2 = -\frac{1}{2}(B''/B\beta)\beta\xi^2.$$

Introducing η which is the canonical conjugate of ξ ,

$$\eta = \frac{1}{v}(d\xi/d\varphi),$$

one can describe the motion with the Hamiltonian

$$H = v(\xi^2 + \eta^2)/2 + v\beta^{5/2}(B''/B\beta)\xi^3/6.$$

The canonical transformation from (ξ, η) to (ψ, I) can be generated by

$$S(\xi, \psi) = \frac{1}{2}\xi^2 \cot \psi$$

and

$$\xi = \sqrt{2I} \sin \psi,$$

$$\eta = \sqrt{2I} \cos \psi,$$

$$H_1 = vI + \frac{1}{24} v\beta^{5/2}(B''/B\beta)(2I)^{3/2}[3 \sin \psi - \sin (3\psi)].$$

Note that

$$x = \sqrt{\beta} \xi = \sqrt{2I\beta} \sin \psi,$$

$$dx/ds = (\eta - \alpha\xi)/\sqrt{\beta} = \sqrt{2I} (\cos \psi - \alpha \sin \psi)/\sqrt{\beta}.$$

Since $\rho^{5/2}(B''/B\rho)$ is a periodic function of ϕ with the period 2π , one can express the Hamiltonian H_1 in the form

$$H_1(\psi, I; \phi) = \nu I + (2I)^{3/2} \sum_n \sum_m A_{nm} \cos(n\psi - m\phi + a_{nm})$$

where $n = 1$ and 3 , $m = -\infty \sim \infty$ and

$$A_{3m} \exp(ia_{3m}) = (i/48\pi B\rho) \sum_k \rho_k^{3/2} (B''/B)_k \exp(im\phi_k)$$

$$A_{1m} = 3A_{3m}, \quad a_{1m} = a_{3m} + \pi.$$

Constant parameters A_{nm} and a_{nm} are fixed by the lattice and by the sextupoles $(B''/B)_k$ at the location ϕ_k . If the periodicity of the lattice is p , m must be an integer multiple of p . With the construction errors, the periodicity is always 1 so that m can take any integer value. In the absence of sextupole field, $A_{nm} = 0$,

$$d\psi/d\phi = \partial H_1 / \partial I = \nu$$

so that the argument of $\cos$ in H_1 is approximately $(n\nu - m)\phi + a_{nm}$. One expects that the effect of $\cos$ terms will not be important over many revolutions unless $|n\nu - m| \ll 1$.

The lowest-order approximation simply considers one $\cos$ term in the Hamiltonian with the smallest value of $|n\nu - m|$, which is called the driving term for the resonance $n\nu - m = 0$.

$$H_1 = \nu I + (2I)^{3/2} A_{nm} \cos(n\psi - m\phi + a_{nm}).$$

This is still a function of the independent variable ϕ . One must transform the variables from (ψ, I) to (θ, J) with

$$S(\psi, J) = (\psi - \frac{m}{n} \phi + \frac{a_{nm}}{n})J.$$

Then (subscripts n, m are dropped from A_{nm} and a_{nm})

$$I = \partial S / \partial \psi = J,$$

$$\Theta = \partial S / \partial J = \psi - (m/n)\phi + a/n.$$

The new Hamiltonian K is independent of ψ ,

$$K = H_1 + \partial S / \partial \phi = (\nu - m/n)J + (2J)^{3/2} A \cos(n\Theta).$$

Fixed points in the phase space (Θ, J) are obtained from the equations of motion ($\epsilon \equiv \nu - m/n$)

$$d\Theta/d\phi = \partial K / \partial J = \epsilon + 3(2J)^{1/2} A \cos(n\Theta) = 0$$

$$dJ/d\phi = -\partial K / \partial \Theta = n(2J)^{3/2} A \sin(n\Theta) = 0$$

The origin $J = 0$ ($x = x' = 0$) is obviously a stable fixed point. Unstable fixed points (Θ_U, J_U) are

$$a) \quad \Theta_U = 0, 2\pi/n, \dots, 2\pi \text{ and } \cos(n\Theta) = 1$$

$$\sqrt{2J_U} = -\epsilon/(3A), \quad \epsilon < 0$$

$$b) \quad \Theta_U = q\pi/n \text{ with } q = \text{odd integers, } \cos(n\Theta) = -1$$

$$\sqrt{2J_U} = \epsilon/(3A), \quad \epsilon > 0.$$

The boundary of the stable area is given by

$$K(\Theta, J) = K(\Theta_U, J_U) = (\epsilon^3/54A^2) \equiv K_U$$

One can always choose the origin of φ such that $\varphi = 0$ at $\varphi = 0$. Boundary values of (x, x') at the $\varphi = 0$ location of the ring are

$$\begin{aligned}x_0 &= \sqrt{\beta} \sqrt{2I_0} \sin \psi_0 \\&= \sqrt{\beta} \sqrt{2I_0} \sin (\Theta_0 - a/n), \\x_0' &= (\sqrt{2J_0}/\sqrt{\beta}) \cos(\Theta_0 - a/n)\end{aligned}$$

with $K(\Theta_0, J_0) = K_U$. The boundary for $x_0 > 0$ and $x_0' = 0$ is given by $\Theta_0 - a/n = \pi/2$.

$$x_0 = \sqrt{\beta} \sqrt{2J_0}$$

with $K(\Theta_0 \equiv \pi/2 + a/n, J_0) = K_U$.

Strictly speaking, the periodicity of the X-ray ring is two and one must consider all even integers for m . On the other hand, the deviation from the sixfold symmetry is so small that A_{nm} with $m = 2, 4, 8, \dots$ are negligible. For example, if one takes $m = 32$ for the resonance $3\nu = 32$ ($\epsilon \equiv \nu_x - 32/3 = 0.195$ with $\nu_x = 10.8614$), the stable boundary at $\varphi = 0$ is ($x_0 = 0.68$ m, $x_0' = 0$) or ($x_0 = -0.34$ m, $x_0' = 0$), clearly a meaningless result. If m is taken to be an integer multiple of six, the closest resonances are $\nu = 12$ and $3\nu = 30$.

$$a) \quad \nu = 12, \epsilon = 10.8614 - 12 = -1.139, n = 1$$

$$Ae^{ia} = i(16.94/\sqrt{m})$$

$$A = 16.94/\sqrt{m}, a = -\pi/a$$

$$\sqrt{2J_U} = .0224 \sqrt{m}, K_U = -0.953 \times 10^{-4} \text{ m}$$

At $\varphi = 0$, the boundary ($x_0 > 0$, $x_0' = 0$) is obtained with

$$\Theta_0 = \pi/2 + a = 0. \text{ One can easily find that}$$

$$\sqrt{2J_0} = \sqrt{2J_u} = 0.0224 \sqrt{m}$$

$$\text{At } \varphi = 0, \beta_x = 19.53m \text{ and } x_0 = \sqrt{\beta} \sqrt{2J_0} = 0.099m.$$

In terms of σ_x ($= \sqrt{\beta_x} \sqrt{0.4117 \times 10^{-7} m} = 0.897 \text{ mm at } 2.5 \text{ GeV}$),
 $x_0 = 110\sigma$. This should be compared with the numerical result
 $x_0 \approx 32\sigma$.

$$b) \quad 3\nu = 30, \epsilon = 10.8614 - 30/3 = .8614, n = 3$$

$$Ae^{ia} = i(10.38/\sqrt{m}), A = 10.38/\sqrt{m}, a = \pi/2$$

$$\sqrt{2J_u} = 0.0277 \sqrt{m}, K_u = 0.110 \times 10^{-3} m.$$

$$\sqrt{2J_0} = 0.0138 \sqrt{m}, x_0 = 0.061 m = 68\sigma.$$

Since two resonances $\nu = 12$ and $3\nu = 30$ seem to be of equal importance (approximately the same boundary x_0), one should really consider two driving terms in the Hamiltonian $H_1(\psi, I; \varphi)$ even in the lowest-order approximation,

$$H_1 = \nu I - (2I)^{3/2} [A_{1,12} \sin(\psi - 12\phi) + A_{3,30} \sin(3\psi - 30\phi)]$$

where $a = \pi/2$ is used in both terms. Unfortunately, it is now impossible to make the transformation such that the new Hamiltonian is independent of φ . The only way to see the effect of two coexisting resonances seems to be a numerical integration of the equations of motion

$$d\psi/d\varphi = \nu - 3(2I)^{1/2} [A_{1,12} \sin(\psi - 12\phi) + A_{3,30} \sin(3\psi - 30\phi)]$$

$$dI/d\varphi = (2I)^{3/2} [A_{1,12} \cos(\psi - 12\phi) + 3A_{3,30} \cos(3\psi - 30\phi)]$$

where $A_{1,12} = 16.94$, $A_{3,30} = 10.38$ and the initial conditions are

$$\psi(\varphi = 0) = \pi/2, I(\varphi = 0) = x_0^2 / (2\beta_x).$$

For a numerical integration, it may be more convenient to use the revolution number $n \equiv \varphi/2\pi$ instead of φ as the independent variable.

III. Higher-Order Resonances²

It is entirely possible that combined resonances $\nu = 12$ and $3\nu = 30$ can explain the result obtained by the numerical orbit tracing. One should then include the coupling term and analyze its effect on the nearest coupling resonance $\nu_x + 2\nu_y = 24 + 0.56$ either analytically³ or numerically. Another important factor that should be taken into account in trying to find the extent of the stable area is the possibility of phase-independent terms in the higher-order approximation. These terms do not drive resonances by themselves but reduce the stable area of the resonance driven by other terms. In the lowest-order approximation, such terms exist with octupole fields. In this section, steps leading to the second-order Hamiltonian are presented and the coupling terms are retained for the sake of completeness.

The immediate consequence of the coupling is that it is no longer possible to use the angle φ as the independent variable. Since φ is related to β by the relation

$$\varphi = \int_0^s ds / (\nu\beta),$$

$\varphi_x(s)$ and $\varphi_y(s)$ are different quantities and there is no way to make the Floquet transformation which transforms both horizontal and vertical oscillations into simple harmonic motions. One is forced to use s or $\theta \equiv s/R$ as the independent variable and the effect of the modulation in β must be included in evaluating A_{nm} .

Equations of motion with the sextupole field $B''(s)$ are

$$d^2x/ds^2 + K_x x = \frac{1}{2} \frac{B''}{B\rho} (x^2 - y^2)$$

$$d^2y/ds^2 + K_y y = \frac{B''}{B\rho} xy$$

These can be derived from the Hamiltonian

$$H(x, p_x, y, p_y; s) \\ = \frac{1}{2}(p_x^2 + p_y^2 + K_x x^2 + K_y y^2) + \frac{1}{6} \frac{B''}{B\rho} (x^3 - 3xy^2)$$

where $p_x = dx/ds$ and $p_y = dy/ds$.

The canonical transformation from (x, p_x, y, p_y) to $(\psi_x, I_x, \psi_y, I_y)$ is generated by

$$S(x, \psi_x, y, \psi_y) = (x^2/2\beta_x)(-\alpha_x + \cot F_x) + (y^2/2\beta_y)(-\alpha_y + \cot F_y)$$

where

$$F_{x,y} = (\psi_{x,y} - v_{x,y} s/R + \int ds/\beta_{x,y}).$$

The original variables are then

$$x = \sqrt{2I_x \beta_x} \sin F_x,$$

$$p_x = \sqrt{2I_x/\beta_x} (\cos F_x - \alpha_x \sin F_x)$$

and similarly for y and p_y . The new Hamiltonian is

$$H_1 = H + \partial S/\partial s \equiv H_{10} + H_{11} ,$$

$$H_{10} = (v_x/R)I_x + (v_y/R)I_y ,$$

$$H_{11} = \frac{1}{6} \frac{B''}{B\rho} (x^3 - 3xy^2) \text{ where } x = x(\psi_x, I_x), y = y(\psi_y, I_y).$$

It is more convenient to use $\theta \equiv s/R$ as the independent variable. This simply means that the new Hamiltonian is R times the old one. The sextupole contribution to the Hamiltonian is

$$H_{11} = \frac{1}{6} \frac{B''}{B\rho} R [(2I_x \beta_x)^{3/2} \sin^3(\psi_x + Q_x) - 3 \sqrt{2I_x \beta_x} (2I_y \beta_y) \times \\ \times \sin(\psi_x + Q_x) \sin^2(\psi_y + Q_y)]$$

where

$$Q_{x,y}(\theta) = \int (R/\beta_{x,y}) d\theta - \nu_{x,y} \theta$$

represent the modulation of β around the ring. At sextupole locations, these angles could be as large as $\sim 40^\circ$ in the X-ray ring so that one should not ignore their effects. On the other hand, they simply produce more terms in the Hamiltonian without adding anything fundamentally different.⁴ In the following, $Q_x = Q_y = 0$ at the sextupole locations is assumed to show the essential feature of the second-order effects.

$$H_{11} = \frac{1}{24} (B''/B\rho) R(\beta_x)^{3/2} (2I_x)^{3/2} [3 \sin \psi_x - \sin(3\psi_x)] \\ - \frac{1}{8} (B''/B\rho) R\sqrt{\beta_x} \beta_y \sqrt{2I_x} (2I_y) (2 \sin \psi_x - \sin \psi_+ - \sin \psi_-)$$

where

$$\psi_{\pm} = \psi_x \pm 2\psi_y.$$

By using the Fourier components of periodic quantities, one finds

$$H_{11} = (2I_x)^{3/2} [\Sigma A_m \cos(3\psi_x - m\theta + a_m) - 3\Sigma A_m \cos(\psi_x - m\theta + a_m)] \\ + \sqrt{2I_x} (2I_y) [2\Sigma B_m \cos(\psi_x - m\theta + b_m) \\ - \Sigma B_m \cos(\psi_+ - m\theta + b_m) - \Sigma B_m \cos(\psi_- - m\theta + b_m)]$$

$$A_m \exp(ia_m) = (i/48\pi) \int d\theta R(B''/B\rho) \beta_x^{3/2} \exp(im\theta).$$

$$B_m \exp(ib_m) = (i/16\pi) \int d\theta R(B''/B\rho) \sqrt{\beta_x} \beta_y \exp(im\theta).$$

It is easy to see in H_{11} which term drives what resonance. For example, the driving term for $\nu_x + 2\nu_y = 24$ is driven by

$$- \sqrt{2I_x} (2I_y) B_{24} \cos(\psi_+ - 24\theta + b_{24}).$$

Note that the resonance $\nu_x = N$ is affected by the vertical motion because of the term

$$2 \sqrt{2I_x} (2I_y) B_N \cos(\psi_x - N\theta + b_N).$$

At this point, there are several ways of studying the sextupole effects, some of which may be too involved to be practical.

1) Take one resonance at a time and find the stable area. This was done in Section II for $\nu_x = 12$ and $3\nu_x = 30$. The resonance $\nu_x + 2\nu_y = 24$ is another one.

2) Take all three resonances simultaneously and solve the Hamiltonian equations of motion for ψ_x , I_x , ψ_y and I_y . Four terms are then kept in H_{11} .

3) Retain the driving term (or terms) for one resonance and transform the remaining terms to see if the second-order effects can modify the result of the lowest-order approximation.

4) Take the standpoint that, in the lowest order, all terms are oscillating so that resonances can be studied only in the second-order approximation. The procedure is the same as 3) but all terms are transformed. Assume that the driving term for $3\nu_x = 30$ is retained and others are considered to be oscillating terms. The transformation from $(\psi_x, I_x, \psi_y, I_y)$ to $(\Theta_x, J_x, \Theta_y, J_y)$ is generated by

$$\begin{aligned} S(\psi_x, \psi_y, J_x, J_y) &= \psi_x J_x + \psi_y J_y + (2J_x)^{3/2} \left(\sum_m \frac{A_m}{m-3\nu_x} \sin q_{3m} - 3 \sum_m \frac{A_m}{m-\nu_x} \sin q_{1m} \right) \\ &+ \sqrt{2J_x} (2J_y) \left(\sum_m \frac{2B_m}{m-\nu_x} \sin q_m - \sum_m \frac{B_m}{m-\nu_+} \sin q_{+m} - \sum_m \frac{B_m}{m-\nu_-} \sin q_{-m} \right) \end{aligned}$$

where

$$v_{\pm} = v_x \pm 2v_y$$

$$q_{km} = k\psi_x - m\theta + a_m \quad (k = 1 \text{ and } 3)$$

$$q_m = \psi_x - m\theta + b_m$$

$$q_{+m} = \psi_x + 2\psi_y - m\theta + b_m$$

$$q_{-m} = \psi_x - 2\psi_y - m\theta + b_m.$$

The symbol Σ' indicates that the sum does not include the resonance term, $m = 30$. If all terms are treated as oscillating, the term with $m = 30$ will also be in the sum.

$$\Theta_x = \frac{\partial S}{\partial J_x} = \psi_x + 3(2J_x)^{\frac{1}{2}} (---) + (2J_y)/\sqrt{2J_x} (---),$$

$$\Theta_y = \psi_y + 2\sqrt{2J_x} (---),$$

$$\begin{aligned} I_x = \frac{\partial S}{\partial \psi_x} = J_x + 3(2J_x)^{3/2} (\Sigma' \frac{A_m}{m-3v_x} \cos q_{3m} - \Sigma \frac{A_m}{m-v_x} \cos q_{1m} \\ + \sqrt{2J_x} (2J_y) (\Sigma \frac{2B_m}{m-v_x} \cos q_m - \Sigma \frac{B_m}{m-v_+} \cos q_{+m} \\ - \Sigma \frac{B_m}{m-v_-} \cos q_{-m})) \end{aligned}$$

$$I_y = \frac{\partial S}{\partial \psi_y} = J_y + 2\sqrt{2J_x} (2J_y) (\Sigma \frac{B_m}{m-v_-} \cos q_{-m} - \Sigma \frac{B_m}{m-v_+} \cos q_{+m}).$$

The new Hamiltonian is

$$K(\Theta_x, J_x, \Theta_y, J_y) = H_{10} + H_{11} + \partial S / \partial \theta$$

$$= v_x J_x + v_y J_y + (2J_x)^{3/2} A_{30} \cos(3\psi_x - 30\theta + a_{30})$$

$$+ [(2I_x)^{3/2} - (2J_x)^{3/2}](\Sigma A_m \cos q_{3m} - 3\Sigma A_m \cos q_{1m})$$

$$+ [\sqrt{2I_x} (2I_y) - \sqrt{2J_x} (2J_y)](2\Sigma B_m \cos q_m - \Sigma B_m \cos q_{+m} - \Sigma B_m \cos q_{-m}).$$

The sum $\Sigma A_m \cos q_{3m}$ now is over all m ($-\infty \sim +\infty$), including $m = 30$ term. The original resonance term is still there but with J_x instead of I_x .

What is done here is to make the oscillating terms depend on higher powers of the amplitudes J_x and J_y . In the lowest-order, one can put

$$\Theta_x = \psi_x, \Theta_y = \psi_y \text{ in all terms except in the resonance term.}$$

$$(2I_x)^{3/2} - (2J_x)^{3/2} = (3\epsilon_1/2)(2J_x)^2 + (3\epsilon_2/2)(2J_x)(2J_y),$$

$$\sqrt{2I_x} (2I_y) - \sqrt{2J_x} (2J_y) = (\epsilon_1/2 + \epsilon_3)(2J_x)(2J_y) + (\epsilon_2/2)(2J_y)^2,$$

$$\epsilon_1 = 6(\Sigma' \frac{A_m}{m-3v_x} \cos q_{3m} - \Sigma \frac{A_m}{m-v_x} \cos q_{1m})$$

$$\epsilon_2 = 2(\Sigma \frac{2B_m}{m-v_x} \cos q_m - \Sigma \frac{B_m}{m-v_+} \cos q_{+m} - \Sigma \frac{B_m}{m-v_-} \cos q_{-m})$$

$$\epsilon_3 = 4(\Sigma \frac{B_m}{m-v_-} \cos q_{-m} - \Sigma \frac{B_m}{m-v_+} \cos q_{+m}).$$

$$K \equiv K_0 + K_1,$$

$$K_0 = v_x J_x + v_y J_y + (2J_x)^{3/2} A_{30} \cos(3\Theta_x - 30\theta + a_{30})$$

The higher-order term K_1 is made of two parts, one coming from the resonance term with ψ_x expressed in terms of Θ_x and the other from products of two cos functions, all of them having $(2J_x)^2$, $(2J_y)^2$ or $(2J_x)(2J_y)$ dependence. Possible phase combinations are

$$(2J_x)^2 \text{ term: } 6\Theta_x, 4\Theta_x, 2\Theta_x, (3\Theta_x - 30\theta) \text{ from the resonance term,}$$

phase-independent terms

$(2J_y)^2$ term: $2\Theta_x, 2\Theta_x \pm 2\Theta_y, 2\Theta_x \pm 4\Theta_y, 2\Theta_y, 4\Theta_y,$
phase-independent terms

$(2J_x)(2J_y)$ term: $4\Theta_x, 2\Theta_x, (3\Theta_x - 3\Theta_y), 4\Theta_y, 2\Theta_y,$
 $2\Theta_x \pm 2\Theta_y, 2\Theta_x \pm 4\Theta_y, 4\Theta_x \pm 2\Theta_y,$
phase-independent terms.

It is probably impractical to consider more than a few terms even if one is willing to solve the equations for $(\Theta_x, J_x, \Theta_y, J_y)$ numerically. The simplest approach would be to keep the resonance term $(3\Theta_x - 3\Theta_y)$ in the lowest order and add the phase independent terms. The Hamiltonian is

$$\begin{aligned}
 K = & \nu_x J_x + \nu_y J_y + (2J_x)^{3/2} A_{30} \cos(3\Theta_x - 3\Theta_y + a_{30}) \\
 & + \frac{9}{2}(2J_x)^2 \left(\Sigma' \frac{A_m^2}{m-3\nu_x} + 3\Sigma \frac{A_m^2}{m-\nu_x} \right) \\
 & + \frac{1}{2}(2J_y)^2 \left(4\Sigma \frac{B_m^2}{m-\nu_x} + \Sigma \frac{B_m^2}{m-\nu_+} + \Sigma \frac{B_m^2}{m-\nu_-} \right) \\
 & + 2(2J_x)(2J_y) \left(\Sigma \frac{B_m^2}{m-\nu_+} - \Sigma \frac{B_m^2}{m-\nu_-} - 6\Sigma \frac{A_m B_m}{m-\nu_x} \cos(\alpha_m - \beta_m) \right)
 \end{aligned}$$

This is still a function of the independent variable θ but the transformation to the θ -independent system is trivial as shown in the preceding section.

IV. Remarks

In view of the complexity of the final Hamiltonian, it is somewhat uncertain if one can see the effect of the second order quantitatively. It is necessary to evaluate all amplitudes A_m and B_m for $m \approx$ integer multiple of six. With a reasonable shape of the amplitude dependence on m , one may be able to replace the sum over m by an integral. For some cases, it may be possible to make use of the relation

$$\Sigma \frac{1}{n \pm a} = \mp (\pi/2) \tan(a\pi/2) \text{ for } n = \text{positive and negative odd integers,}$$

$$= \pm (\pi/2) \cot(a\pi/2) \text{ for } n = 0, \text{ positive and negative even integers.}$$

REFERENCES

- 1) The UV ring used here has two low- β ($\beta_x^* = 0.55$ m, $\beta_y^* = 0.22$ m) and two high- β ($\beta_x^* = 14.15$ m, $\beta_y^* = 6.59$ m) insertions. Tunes are $\nu_x = 3.620$ and $\nu_y = 2.630$.
- 2) K. R. Symon, NAL FN-140, April 29, 1968; F. T. Cole, NAL TM-179, June 13, 1969; S. Ohnuma, Proceedings of the U.S.-Japan Seminar on High Energy Accelerator Science, 5-9 November, 1973, p. 114.
- 3) P. A. Sturrock, Annals of Physics 3 (1958) 113; S. Ohnuma, NAL TM-507, July 15, 1974.
- 4) The first term of H_{11} includes $(\cos Q) \sin \psi$, $(\sin Q) \cos \psi$, $\cos(3Q) \sin(3\psi)$ and $\sin(3Q) \cos(3\psi)$. Functions of Q are combined with the factor

$$\frac{1}{6} \frac{B''}{B\rho} R(\beta_x)^{3/2}$$

to calculate the Fourier amplitudes A_m and angles a_m .

Second-Order Effect of Correction Sextupoles

- Horizontal Aperture in X-Ray Ring -

S. Ohnuma

Fermilab

I.

The case reported here is for the horizontal motion only; coupling effects are not included. Strengths of correction sextupoles are

$$(B''L)/(B\rho) = 19.33 \text{ m}^{-2} \text{ and } -14.55 \text{ m}^{-2}.$$

The origin of the ring, $\varphi = 0$, is at the center of two Q1 quadrupoles. Horizontal orbits have been traced for 8,000 revolutions with the initial conditions $(x_0, x'_0 = 0)$. This corresponds approximately to the horizontal damping time at 2.5 GeV. By varying x_0 and recording the subsequent values of (x, x') at $\varphi = 0$, one can find the boundary of the stable area in (x, x') phase space. This has been checked with a few cases of the initial conditions $(x_0 = 0, x'_0)$. The stable area is shown in Fig. 1 (the smaller of two areas). Because of the symmetry of the lattice, the area with $x' < 0$ is the mirror image of the area with $x' > 0$. On the other hand, there is a strong asymmetry for x . From the shape of this, one suspects the resonance $6\nu_x = 66$ which can be driven by a sextupole field in the second-order approximation.

II.

The Hamiltonian in the second-order approximation with the average term and the driving term for $6\nu_x = 66$ is*

*Subscript x is omitted.

$$K = vJ + (9/2) \left(\sum_m \frac{A_m^2}{m-3v} + 3 \sum_m \frac{A_m^2}{m-v} \right) (2J)^2 \\ + (9/2) \left(\sum_m \frac{A_m A_n}{m-3v} \right) (2J)^2 \cos (6\Theta - 66\varphi + a_m + a_n)$$

where $m + n = 66$ and the summations over m are for $m = 0$ and all positive and negative integers,

$$A_m \exp(ia_m) = (i/48\pi) \sum_k \beta_k^{3/2} (B''_k / B\rho)_k \exp(im\varphi_k).$$

The summation over k here is for all sextupoles. Note that one can use the phase angle

$$\varphi = \int ds / (v\beta)$$

instead of the azimuthal angle θ since coupling effects are ignored. The symmetry of the lattice gives $a_m = \pi/2$ or $-\pi/2$ for all m and the amplitude A_m is large when m is a multiple of six. One can easily evaluate all summations in the Hamiltonian up to a certain maximum value of $|m|$. Beyond $|m| = 500$ or so, there is no essential difference in the final results. The Hamiltonian is (K and J in meters)

$$K = vJ + [21.77 \cos (6\Theta - 66\varphi) + 339.0] (2J)^2.$$

This still contains the independent variable φ . The canonical transformation from (Θ, J) to (Θ', J') is generated by

$$S(\Theta, J') = (\Theta - 11\varphi)J'.$$

One finds

$$J = \partial S / \partial \Theta = J'$$

$$\Theta' = \partial S / \partial J' = \Theta - 11\varphi$$

and the new Hamiltonian is

$$K' = \epsilon J + [21.77 \cos(6\Theta') + 339.0] (2J)^2$$

where $\epsilon \equiv \nu - 11 = -0.13852$. Relations between (x, x') and (Θ, J) are¹

$$x = \sqrt{B} \xi, \quad x' = (\eta - \alpha \xi) / \sqrt{B}, \quad (\alpha = 0 \text{ at } \varphi = 0)$$

$$\begin{aligned} \xi = X + 3 \left[\sum_m \frac{A_m}{m-\nu} \sin(a_m) + \sum_m \frac{A_m}{m-3\nu} \sin(a_m) \right] X^2 \\ + \left[9 \sum_m \frac{A_m}{m-\nu} \sin(a_m) - 3 \sum_m \frac{A_m}{m-3\nu} \sin(a_m) \right] Y^2, \end{aligned}$$

$$\eta = Y - 6 \left[\sum_m \frac{A_m}{m-\nu} \sin(a_m) + \sum_m \frac{A_m}{m-3\nu} \sin(a_m) \right] XY$$

with $\sin(a_m) = 1$ or -1 and $X = \sqrt{2J} \sin(\Theta)$, $Y = \sqrt{2J} \cos(\Theta)$. Summations over m are easy to evaluate and one finds (ξ, η, X and Y in $\sqrt{m}$)

$$\xi = X - 25.92X^2 - 48.32Y^2,$$

$$\eta = Y + 51.84XY$$

Note that $\Theta = \Theta'$ at $\varphi = 0$.

III.

Conditions for the fixed points in the phase space are

$$dJ/d\varphi = -\partial K'/\partial \Theta' = 6 \times 21.77 (2J)^2 \sin(6\Theta') = 0,$$

$$d\Theta'/d\varphi = \partial K'/\partial J = \epsilon + 4[21.77 \cos(6\Theta') + 339.0](2J) = 0.$$

$$1) \sin(6\Theta') = 0, \quad \cos(6\Theta') = 1, \quad \sqrt{2J} = 0.00980 \sqrt{m}$$

$$2) \sin(6\Theta') = 0, \quad \cos(6\Theta') = -1, \quad \sqrt{2J} = 0.0105 \sqrt{m}$$

For the boundary of the stable area, one substitutes $\sqrt{2J} = 0.00980 \sqrt{m}$ in the Hamiltonian K' and solves for (Θ', J) . By transforming (Θ', J) to (x, x') , one can find the stable area in (x, x') space and this is shown in Fig. 1 (the larger of two areas).

In principle, one can investigate the behavior outside the stable area from the second set of fixed points. However, the approximation becomes poor as the value of J increases. As one can see in Fig. 1, the agreement is not perfect but the overall characteristics of the stable area are reproduced by this mechanism. Extending this to the coupled case will be very tedious but not impossible.

Reference

1. S. Ohnuma, Proceedings of the U.S. - Japan Seminar on High Energy Accelerator Science, Tokyo and Tsukuba, 5-9 November 1973, p. 114.

K-5 5 X 5 TO THE CENTIMETER 18 X 24 CM.
NEUFEL & ESSER CO. MAULIN, U.S.A.

46 1610

Introduction of Sinusoidal Wigglers into the X-ray Lattice

H. Winick

SLAC

An important feature of the lattice design for the NSLS storage rings is the provision of matched insertions in which to place high field wiggler magnets. The availability of the synchrotron radiation from such wiggler magnets will open up very exciting research possibilities. The SYNCH runs provided the workshop participants have three-pole hard-edge wiggler magnets in each of the four 3.75 m insertions. Al Garren has recently introduced the capability into the SYNCH program of using magnetic fields depending on the azimuthal coordinate as a sum of a few sinusoidal harmonics. Therefore, it seemed of interest to make a SYNCH run with the hard-edged wigglers replaced by a sinusoidal field distribution. The results of these calculations are shown in the following figure.

Effect of 4 such
wigglers in x-ray ring

$$\Delta U_x = .00036$$

$$\Delta U_y = .00191$$

2.5 GeV

Max. excursion in
Wiggler = 1.2 mm

Max. Angle in
Wiggler = 16.3 mrad

$$W1 = 40 \left[-0.5 \sin \frac{2\pi}{\lambda} z + 0.25 \sin \frac{4\pi}{\lambda} z \right]$$

$$W2 = 40 \left[\sin \frac{2\pi}{\lambda} z \right] \quad W3 = 40 \left[\cos \frac{2\pi}{\lambda} z \right]$$

$$W4 = 40 \left[-0.5 \sin \frac{2\pi}{\lambda} z - 0.25 \sin \frac{4\pi}{\lambda} z \right]$$

Hard Edge
Wiggler

Effect of 4 such
wigglers in x-ray ring
2.5 GeV

$$\Delta U_x = .00002$$

$$\Delta U_y = .00677$$

Max. excursion in
Wiggler = 12 mm

Max. Angle in
Wiggler = 76.9 mrad

PPH-10 2 1/2 x 1 1/2 INCH
WIND LINE PLAT

$B_w (KG)$

$$\lambda = 32 \text{ cm}$$

0.1m 0.2m 0.3m

H. Winick
6/23/77

The Touschek Effect (a la Bruck)

Lloyd Smith

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Elastic scattering of electrons within a bunch can cause transverse betatron momenta to be transformed to longitudinal momenta, as viewed in the rest frame of the bunch. In the laboratory system, this can result in electrons acquiring large momentum deviations with respect to the momentum of the synchronous particle. These deviations may exceed the momentum acceptance of the RF accelerating system, leading to a loss of electrons from the bunch.

We denote the momentum of the synchronous particle by $m_0 c \beta_0 = m_0 c \gamma$ and the maximum momentum deviation accepted by the accelerating RF by $m_0 c \Delta p_{RF}$. For large overvoltages ($e\hat{V}/U_0$), one has (Bruck Eq. 30.2):

$$\left(\frac{\Delta p_{RF}}{\gamma} \right)^2 = \frac{2e\hat{V}}{\pi \alpha k E_0} ,$$

where $\hat{V}$ is the peak RF voltage, U_0 is the energy lost per turn, α is the momentum compaction, k is the harmonic number, and $E_0 = \gamma m_0 c^2$. In the laboratory frame, the loss rate $1/\tau$ is given by Bruck, Eq. 30.17:

$$\frac{1}{\tau} = \frac{\sqrt{\pi} r_0^2 c N C(\epsilon)}{\delta p_x (\Delta p_{RF})^2 (4\pi)^{3/2} \sigma_x \sigma_z \sigma_\ell} ; \quad (1)$$

where

$$\epsilon = \left(\frac{\Delta p_{RF}}{\gamma \delta p_x} \right)^2$$

$$\delta p_x = \frac{1}{m_0 c} \langle p_x \rangle_{rms} = \gamma \sigma_{x'} ,$$

$$r_0 = 2.8 \times 10^{-15} \text{ m}, \quad c = 3 \times 10^8 \text{ m/sec},$$

N = number of electrons per bunch

σ_z = r.m.s. bunch length,

σ_x and σ_z = radial and vertical bunch widths,

$C(\epsilon)$ is defined in Eq. 30.11 of Bruck, and plotted in Fig. 52.

The distribution function for the radial betatron oscillations is:

$$f(x, x') = \frac{1}{2\pi \epsilon_x} \exp \left\{ \frac{-1}{2\beta_x \epsilon_x} [x^2 + (\beta_x x' + \alpha_x x)^2] \right\} ,$$

where $\langle x^2 \rangle = \sigma_x^2 = \beta_x \epsilon_x ,$

$$\langle x'^2 \rangle = \sigma_{x'}^2 = \epsilon_x (1 + \alpha_x^2) / \beta_x .$$

Eq. (1) can be written

$$\frac{1}{\tau} = \frac{N r_0^2 c}{8\pi \gamma^3 (\Delta p_{RF}/\gamma)^2} \frac{C \left[\frac{(\Delta p_{RF}/\gamma)^2 \beta_x}{\gamma^2 \epsilon_x (1 + \alpha_x^2)} \right]}{\sigma_z \sqrt{\beta_z \epsilon_z} \epsilon_x \sqrt{1 + \alpha_x^2} \sqrt{1 + \frac{\eta^2 \sigma_p^2}{\beta_x \epsilon_x}}} , \quad (2)$$

where η = the dispersion function, and σ_p = r.m.s. $(\Delta p/p)$.

Since α , β , and η are functions of position, Eq. (2) should be averaged around the ring. For an order-of-magnitude estimate, we let $\alpha_x = \eta = 0$, and $\beta = R/v$. We also take $E = 700$ MeV, $I = 1A \rightarrow N = 10^{11}$, $\gamma = 1370$, $\beta_x = 2.4$ m, $\beta_y = 3.5$ m, $\hat{V} = 400$ keV, $(\Delta p_{RF}/\gamma) = 0.05$, $\sigma_p = 2.8 \times 10^{-4}$, $\epsilon_x = 3.2 \times 10^{-9}$ m-rad, $\epsilon_z = 3.2 \times 10^{-11}$ m-rad, and $\sigma_L = 0.01$ m. Then

$$\epsilon = \frac{1}{\gamma^2} \left(\frac{\Delta p_{RF}}{\gamma} \right)^2 \frac{\beta_x}{\epsilon_x} = 1.0$$

and

$$C(1.0) = 0.045 \text{ from Bruck, Fig. 52,}$$

so we obtain

$$\tau = 86 \text{ min, for } \hat{V} = 400 \text{ keV.}$$

For $\hat{V} = 200$ keV, $\epsilon = 0.5$ and $C(0.5) = 0.15$, hence

$$\tau = 13 \text{ min, for } \hat{V} = 200 \text{ keV.}$$

The averaging of Eq. (2) should be done - there are probably factors of two lurking there. Also, the effect of multiple scattering on beam size should be examined.

We close by checking the consistency of Bruck's assumption of negligible p_{11} in the electron frame:

$$p_1/m_0c = \gamma \sqrt{\epsilon_x/\beta_x}$$

$$p_{11} \sim \frac{1}{\gamma} (\Delta p)_L = \left(\frac{\Delta p}{p} \right)_L \frac{p_L}{\gamma} = \sigma_p m_0 c$$

$$\frac{p_{11}}{p_1} = \sqrt{\frac{\beta_x}{\epsilon_x}} \frac{\sigma_p}{\gamma} = \sqrt{\frac{2.4}{3 \times 10^{-19}}} \times \frac{2.8 \times 10^{-4}}{1370} = 5.7 \times 10^{-3}, \quad \text{o.k.}$$

Also:

$$\frac{p_1}{m_0 c} = 1370 \sqrt{\frac{3 \times 10^{-9}}{2.4}} = 5 \times 10^{-2} = |\beta_1|_{\text{c.m.}},$$

so non-relativistic approximation is o.k.

Reference

H. Bruck, Circular Particle Accelerators, published by the Presses Universitaires de France.

Possible Mode of Injection into the X-ray Ring

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Since the NSLS design report is not very explicit on the details of injection into the 2.5 GeV electron ring, it was thought worthwhile to explore a few aspects of this during the workshop session June 20-24, last.

A) Some numbers

Linac: 50 MeV, 20 mA

Booster: Chopped beam from Linac, single bunch filling in booster in order to relax injection/ejection kicker rise time/fall time requirements. n_{bunch} , booster, assuming one turn transverse 100% efficient injection, $n_b = 3.7 \cdot 10^8 e^-$

Xray ring: With a 1 A beam current objective, $n_{\Sigma} = 2.8 \cdot 10^{11} e^-$. Consequently, assuming 100% efficient transfer from the booster, with a cycling time of 0.75 sec $\approx$ 756 booster charges are required, demanding a charging time of 9-10 minutes. Regarding the total charging time; it is essential that the Touschek lifetime at 700 MeV be long enough to avoid significant particle loss. Also, regarding the 0.75 sec period between injection of charge into the Xray ring and considering the detailed mode of injection, the transverse damping time at 700 MeV must not be larger than given by $3\tau_T \leq 0.75 \text{ sec.}$

B) Touschek Lifetime

Since the Touschek lifetime for the Xray ring at 1.5 GeV was calculated to be ~ 1 hour but decreases strongly as a function of energy ($\propto \gamma^5$, see Bruck, Ch. 31) some concern existed with regard to the actual lifetime at 700 MeV. (The Touschek lifetime is defined as $N_b(\tau_{\text{Tou}}) = (\frac{1}{2}) N_b(0)$, i.e. 50% particle emigration from the longitudinal phase stable domain in $t = \tau_{\text{Tou}}$). Direct scaling would suggest $\tau_{\text{Tou}}(700 \text{ MeV}) \approx 2 \text{ min.}$, in which case a charging time of 9 min. would be unacceptable. Consequently, direct calculation at 700 MeV, taking into account bunch lengthening (A. Ruggiero) and assuming fully coupled transverse

motion $\sigma_r \approx \sigma_z$, proved to be necessary. Several workshop participants (including this writer) ventured in this direction, a "final" number was calculated by L. Smith as $\tau_{\text{Tou}} (\frac{V}{V_{\text{rf}}} = 800 \text{ kV}, 700 \text{ MeV, Xray ring}) \approx 40 \text{ min.}$ It is reported, however, that general experience has shown an experimental lifetime shorter by about a factor of 4 compared with the calculated value (V.P. Suller, Daresbury, private communication, reported in Pampus design report).

In the case of excessive particle loss at injection associated with Touschek lifetime, this could be alleviated, other than by further increase of the rf bucket area, by compromising somewhat on the (circumferentially averaged) bunch density (lower tune value). For the present it will be assumed $\tau_{\text{Tou}} = k\tau_{\text{charging}}$ with $k \geq 3$.

C) Radiation damping times at 700 MeV

The damping rates are given by (Sands, SLAC 121, 1970)

$(1/\tau_i) = J_i U f_0 / 2E$ where J_i is the relevant partition damping coefficient, U is the synchrotron radiation loss per turn and f_0 the revolution frequency. Note that

$$U = C_\alpha E^4 / 2\pi \oint (\rho^{-2}) ds \text{ with } C_\alpha = 8.85 \cdot 10^{-5} \text{ m GeV}^{-3}.$$

Further $J_r = 1 - D$, $J_z = 1$, $J_e = 2 + D$, for no coupling, with, for an isomagnetic system $D = \alpha R / \rho$. For the Xray ring $D = 0.165$, consequently $J_r = 0.84$, $J_z = 1$, $J_e = 2.16$. [Using these relationships yield damping times at 2.5 GeV of $\tau_r = 7 \text{ msec}$, $\tau_z = 6 \text{ msec}$, $\tau_e = 2.7 \text{ msec}$ compared with computer printout values from "Synch" of $\tau_r = 4.3 \text{ msec}$, $\tau_z = 4.3 \text{ msec}$, $\tau_e = 2.1 \text{ msec}$].

At 700 MeV, the above yields: $\tau_r \approx \tau_z \approx 200 \text{ msec}$ and $\tau_e \approx 100 \text{ msec}$. Consequently, with a booster cycling time of 0.75 sec., sufficient time allowance exist between Xray ring injection times for damping of the betatron oscillations, essential in the process of charge built up in this ring.

- D) In order to explore details of possible injection trajectories, it is necessary to establish the Xray ring clearance profile. In a first approach, the maximum aperture demand is given by:

$$A_{x,y} = F_{x,y} \hat{\sigma}_{x,y} + C_{x,y}$$

where $F_{x,y}$ are multipliers guided by the requirement for adequate quantum lifetimes and $C_{x,y}$ are allowances for errors. Reasonable values are thought to be $F_{x,y} \approx 10$ and $C_{x,y} \approx 20$ mm.

Alternatively, one might accept the present vacuum chamber aperture and fit in for the presently calculated lattice, the largest possible acceptance. This has been done, using a hypothetical phase space given by:

$$E_H = 7.5 \cdot 10^{-5} \text{ mrad}, \quad E_V = 9.8 \cdot 10^{-6} \text{ mrad}, \quad (\Delta p/p) = 10^{-3}.$$

The resulting "stay clear" requirement is given for the bending arc of the Xray lattice in Fig. 1 and for the straight segment in Fig. 2 in agreement with a chamber aperture of ± 16 mm vertical, ± 40 mm horizontal, but indicating, also, that the good field aperture requirement in the dipoles is only ± 20 mm.

- E) Using the equilibrium emittance in the Xray Ring at 700 MeV: $\hat{\sigma}_x \approx 0.3$ mm, assuming full coupling: $\hat{\sigma}_y \approx \hat{\sigma}_x / \sqrt{3} \approx 0.2$ mm indicating that during injection, a septum location at $r \approx 20$ mm for a $\beta = 16$ m, value is not an unreasonable choice. At higher energy, the $\hat{\sigma}_x$ and $\hat{\sigma}_y$ values are larger and the septum might have to be moved out radially in order to provide adequate quantum lifetime.

- F) Using the AGS program, a number of injection trajectories have been explored. Two of these are illustrated in Fig. 3. Since it is more favorable to locate the septum on the inside of the ring because of synchrotron radiation heating of the septum, case B, as illustrated, is favored. For this case, more details, illustrating also the external trajectory are given in Fig. 4.

- G) A quick exploration of the external beam optics indicated the need for external quadrupoles located close to the insertion straight in order to avoid excessive external β values. Also, in addition to the two septum units, i.e. a pulsed, 20 mrad, septum and a possibly dc, 70 mrad septum, an external bend of approximately 70 mrad is needed to arrive at a dispersion free transport between booster and Xray ring.

With the septum located as shown, ($r = 22$ mm), properly adequate within the clearance aperture to permit injection of the booster beam, the maximum radial clearance aperture during injection is 26 mm. which is sufficient at 700 MeV. As is evident from the illustrated trajectories, only the "good" field domain of the quadrupoles of the quadrupole triplet adjacent to the Xray ring free straight section has been used. An alternative approach of using an injection trajectory through the "slots" of special quads is not favorable because of the need to locate external quads close to the location where the beam exits from the insertion straight.

- H) The essential conclusion of the above quick exploration of injection possibilities is that beam injection in the Xray ring, with a septum located at a near β maximum as indicated, is straight forward, with no need for special insertion optics, specifically for beam injection.

XRAY BENDING ARC

X RAY STRAIGHT

X ray ring injection

Injection trajectory options.

A (outside septum)

fast kick ($\approx 1 \text{ mm/turn}$):

$\Theta_1 \approx 1 \text{ mrad}$; $\Theta_2 \approx 3.3 \text{ mrad}$

(3 σ separation).

Septum A : $\approx 30 \text{ mrad}$.

B (inside septum)

fast kick : $\Theta_1 \approx 1 \text{ mrad}$.

$\Theta_2 \approx 3.3 \text{ mrad}$.

Septum B : $\approx 20 \text{ mrad}$.

X-ray ring injection (external trajectory)

An Inquiry into the Flexibility of the X-ray Lattice

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The high tune values ($\nu_x = 10.86$, $\nu_y = 6.85$) being considered at this workshop are a consequence of the desire to obtain optical sources having central brightness an order of magnitude larger than available in existing storage rings. The quadrupole strengths corresponding to this tune are very high, as are the sextupole strengths necessary to compensate the large negative chromaticity. A result is that the lattice is extremely sensitive to displacement errors in positioning the quadrupoles and sextupoles. A rough estimate is that an rms quadrupole displacement of only 10 mil will lead to an rms closed orbit displacement of about 2 cm. It is planned to develop a closed orbit correction system capable of reducing the closed orbit displacement to peak values of 3 mm (radial) and 1 mm (vertical). However, even if this is done a future detailed study must determine whether the high brightness of the ideal machine will be realized in the actual machine. In particular, it remains to be determined whether the momentum dispersion functions X_p and Y_p will be substantially increased over their ideal values by the closed orbit errors. Such an increase could reduce the actual brightness by increasing the electron emittances and source sizes.

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We feel that it is quite possible that in order to optimize the brightness of the real imperfect machine, one should be able to operate it in configurations with lower tunes having less sensitivity to errors. In this note we present a first look at the flexibility of the lattice and we emphasize that future work on this subject should be pursued.

In order to minimize the dependence of the damping properties of the lattice on the high field wiggler magnets placed in the insertions, it is important that X_p be zero in the insertions. In the original tune this was accomplished by choosing the strength of the focusing quadrupole QF such that X_p was localized about the bending magnets (Fig. 1). However, it is not necessary that X_p be zero in the quadrupoles Q1 and Q2, so we shall consider weakening the strength of QF. We first consider a sixfold symmetric lattice with all insertions having length 3.75 m. We demand $\alpha_x^* = \alpha_y^* = 0$, $\beta_x^* = 0.65$ m, $\beta_y^* = 0.251$ m, and $X_p = X_p' = Y_p = Y_p' = 0$ at the insertion center. We fix the strength of QF, and use the SOLV feature of the SYNCH program to vary the strengths of Q1, Q2, Q4, Q5, Q6 to find a solution to the five constraints:

$$\nu_x = 9.86, \quad \nu_y = 6.85$$

$$\alpha_x = \alpha_y = 0, \quad X_p' = 0 \text{ at center of Q1.}$$

We then do the same for a different strength of QF. In Fig. 2 we plot the radial emittance E_x vs the gradient of QF, and see that a minimum exists for a gradient of 150 KG/m. We now fix the gradient of QF at this value and introduce the two 5 m long insertions. This makes some minor changes, e.g. increasing the radial emittance from 4.0×10^{-8} m-rad to 4.46×10^{-8} m-rad, and changing the tune to $\nu_x = 9.875$, $\nu_y = 6.865$.

The amplitude and dispersion functions are shown in Fig. 3. Both β_x and β_y are quite similar to the β -functions of the original tune. The

dispersion function has changed considerably. Rather than vanishing at 01 it reaches its maximum value there of 0.8 m. The uncorrected chromaticities are $\xi_x = -21.8$ and $\xi_y = -21.3$, and the sextupole strengths necessary to bring the chromaticities to zero are:

$$B''L/BP = 16.1 \text{ m}^{-2} \quad \text{and} \quad -11.0 \text{ m}^{-2},$$

which are somewhat smaller values than for the original tune. The results of a SYNCH CYEM command (evaluation of the electron integrals) is given in Table I. We note that the momentum compaction is a factor of 2 larger than that of the original tune. The momentum dependence of the tunes and the horizontal emittance (w/o coupling) is shown in Fig. 4a, while the dependence on momentum of β_x , β_y , and X_p is shown in Figs. 4b, c, d respectively. The dependence on momentum is in general qualitatively similar to that of the original tune, however, the dependence of ν_x is slightly stronger. Also, we wish to note that this tune lies quite close to the third order coupling resonance

$$2\nu_x + \nu_y = 24,$$

and the effect of this will have to be studied.

In summary we would conclude that this first inquiry into the flexibility of the lattice is encouraging. Relaxing the constraint that X_p be localized about the bending magnets enabled us to decrease ν_x by one unit without adversely affecting the emittances and amplitude functions. By decreasing the tune further, and hence presumably increasing the emittances, we may be able to arrive at a lattice significantly less sensitive to errors. It may turn out that the maximum brightness of the imperfect machine corresponds not to the highest tune, but to such a lower one.

TABLE I: X-Ray Ring, E = 2.5 GeV

Average Radius	R = .240E+02 Meters
R F Frequency	F = .500E+08 HZ
R F Peak Volts	VO = .800E-03 GV
Effective Radius	R = .817E+01 Meters
Momentum Compaction	ALPHA = .114E-01
Damping Partition	D = -.174E-02 Radians
Phase Angle	PHIS = .430E+02 Degrees
Energy Oscillation Period	TE = .160E-03 Seconds
Energy Aperture (EPSM**2/SIGE**2)/2	FPSM = .243E-01 GeV XT = .468E+02
Sigma (DP/P)	SIGDP = .100E-02 SIGE = .251E-02
Synchrotron Radiation	UO = .584E-03 GeV/Turn
Circulation Period	TO = .504E-06 Seconds
Horizontal Damping	TAUX = .430E-02 Seconds
Vertical Damping	TAUY = .431E-02 Seconds
Energy Damping	TAUE = .216E-02 Seconds
Sigmax (O-Coupling)	SIGX = .653E-03 Meters
Quantum Lifetime	TAUQ = .480E+16 Seconds
Bunch Length	SIGPHI = .914E-01 Radians
Bunch Length	SIGL = .872E-01 Meters
Momentum SPRD*SQRT(6)	DP/P = .246E-02 SIGBG = .492E+01

Emittances (Meter Radians) EPX = 0.446E-07 EPY = 0.451E-09

X-RAY RING : AMPLITUDE AND DISPERSION FUNCTIONS

RADIANT EMITTANCE VS GRADIENT OF ∂F

$$V_x = 7.86, V_y = 6.85$$

Fig. 2

 $E_x(10^{-4} \text{ m} \cdot \text{m}^{-2})$

7

6

5

4

3

2

1

140

150

160

170

180

190

200

GRADIENT OF $\partial F (K/m)$

7 1 0-7 AMPLITUDE AND DISPERSION FUNCTIONS

Fig. 3

$S(m)$

30

20

10

0

B_y

B_x

B_y

B_x

B_x

$S(m)$

Q1 Q2 BB SD BBH QF SF BBH SD BB Q4 Q5 Q6 BT BS

TUNES AND EMITTANCE VS MOMENTUM

Fig. 7a

3x FOR DIFFERENT MOMENTA

F₁ = 423x
(x^{1/3})

Q1 Q2 BB SD BBH QF BBH SD BB Q4 Q5 Q6 BT BS BT

FOR DIFFERENT 0-10 MOMENTA

Fig 4c

DISPERSION FUNCTION FOR DIFFERENT MOMENTA

Fig 4d