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L. N. Kmetyk*
J. Weertman**
W. V. Green***
W. F. Sommer

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*Guest Scientist. Division 4444, Sandia National Laboratories, Albuquerque, NM 87185.

**Consultant. Department of Materials Science and Engineering, Northwestern University, Evanston, IL 60201.

***Consultant. Eidg. Institut für Reaktorforschung, 5303 Würenlingen, SWITZERLAND.

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VOID GROWTH AND SWELLING FOR
CYCLIC PULSED RADIATION

by

L. N. Kmetyk, J. Weertman,
W. V. Green, and W. F. Sommer

ABSTRACT

The analysis of Ghoniem and Kulcinski of a single radiation pulse has been extended to include the effects of temperature oscillations and multiple pulses by coupling six simultaneous nonlinear ordinary differential equations for point defect concentrations and sink strengths with a heat equation governing radiation-produced temperature fluctuations. The pulsed irradiation considered is that of the 800 MeV proton beam at the Clinton P. Anderson Meson Physics Facility (LAMPF), and numerical calculations have been done for aluminum and molybdenum. The model material is assumed to have been irradiated to a specific microstructure, and the calculation is begun after the material has returned to thermal equilibrium conditions. Void growth is seen to proceed at a nearly linear rate after initial transients (caused by the assumption of initial thermal equilibrium) in both vacancy concentration and vacancy loop size and number density. The temperature pulses associated with cyclic rather than steady irradiation generally enhance void growth if the ambient temperature is below the material peak swelling temperature, and conversely decreases void growth when the ambient temperature is above peak swelling temperature. The exception seen occurs when the temperature pulse is great enough that thermal emission of vacancies, rather than radiation production, is dominant.

I. INTRODUCTION

The fully dynamic rate theory, developed for steady state conditions by Brailsford and Bullough, was used by Kulcinski and Ghoniem to calculate void growth and swelling rates for a single radiation pulse impinging on a constant ambient temperature material. This theory is extended here to follow the effects of arbitrary numbers of repeated pulses and their associated radiation-produced temperature oscillations on material damage structure. The time dependence of such self-consistent temperature changes must be taken into account because the amount of void growth is a sensitive function of temperature. These temperature oscillations could also produce fluctuating thermoelastic and thermoplastic stresses which in turn would have an effect, and which have been neglected here.

Brailsford and Bullough [1] in 1972 first presented rate theory equations for spatially averaged point defect concentrations in irradiated materials containing voids, dislocation loops, deformation dislocations, and coherent or incoherent precipitates. Two coupled nonlinear algebraic equations, one for interstitial and the other for vacancy concentrations, were solved to yield a time-independent first approximation to the volume swelling rate for various ambient temperatures. After an incubation dose the volume swelling rate was found to have a linear dose dependence.

A rate theory for the growth of interstitial loops and the resulting irradiation creep in stressed irradiated material, containing voids, precipitates and network dislocations in addition to loops, was later presented [2]. The two coupled nonlinear algebraic equations for vacancy and interstitial concentrations were modified to include the effects of a tensile applied stress, which enhanced the vacancy emission rates and increased the nucleation rate for favorably oriented loops. These two equations were then solved to yield creep rates due both to the excess population of favored loops and to their faster growth rates.

The void growth rate theory was extended by Bullough, Eyre and Krishan [3] in 1975 to include the possibility of vacancy loop formation due to irradiation-produced cascade damage. Ordinary differential equations (for void radius, interstitial loop radius, number density of vacancy loops, and fraction of vacancies tied up in vacancy loops) were added to the two algebraic point defect concentration equations, and were solved numerically for stress-free situations (although the equations derived were stress-dependent). The resulting

theoretical swelling rates were used to force-fit the interstitial loop number observed by experiment, as a function of temperature.

The rate theory was extended to fully dynamic situations in 1976 by Ghoniem and Kulcinski [4,5] who replaced the two algebraic point defect concentration equations by analogous ordinary differential equations. The resultant six coupled nonlinear ordinary differential equations were solved numerically for steady irradiation and for a single pulse followed by a long annealing period. Although external uniaxial stress effects were included in the equations no results were given. Later reports [6,7] in 1977 compared calculations of both steady-state irradiation void growth and thermal shrinkage of voids undergoing no irradiation to experimental data. Also investigated were the effect of collision cascade collapse and displacement rate on the growth aspect of voids in metal.

Dienes [8] studied special cases of the fully dynamic rate theory (with constant sink density and no recombination, and with zero sink density and constant recombination rate) which allowed analytic solutions of the two differential equations governing point defect concentrations. The concentration oscillations and their time integrals were compared for equivalent net doses of pulsed and steady irradiation.

In the present paper the six coupled simultaneous nonlinear ordinary differential equations solved by Kulcinski and Ghoniem are combined with a heat equation governing material temperature, whose source term includes both direct (radiation-produced lattice excitation) and indirect (annihilation energy of point defects) heating. This heat equation is shown to decouple from the full set of equations only when indirect heating is negligible. Swelling rates in aluminum and molybdenum are calculated for samples irradiated in the LAMPF proton beam, and the effect of varying pulse structure on swelling rates is investigated. (The LAMPF proton beam provides the only known source of pulsed irradiation where significant displacement damage energy levels can be deposited in metals.)

II. THEORY

We are given a material containing concentrations (expressed in number per lattice site, or atom fraction) C_i and C_v of free interstitials and vacancies, a concentration (expressed in number per unit volume) N_v of voids of radius r_v , and a total dislocation density ρ_d given by

$$\rho_d = \rho_d^0 + \rho_d^{il} + \rho_d^{vl} \quad (1)$$

where ρ_d^0 is the dislocation network line density, and ρ_d^{il} and ρ_d^{vl} the line densities of the irradiation produced interstitial and vacancy dislocation loops, given by

$$\rho_d^{i,vl} = 2\pi r_{i,vl} N_{i,vl} \quad (2)$$

N_{il} and N_{vl} are the number per unit volume of interstitial and vacancy loops of radii r_{il} and r_{vl} .

These dislocation loops can change size in time by absorption of free vacancies and interstitials, and by thermal emission. The changes in the loop radii are given by

$$\frac{dr_{il}}{dt} = \frac{1}{b} \left\{ Z_i D_i C_i - Z_v D_v C_v + Z_v D_v C_v^0 \exp \left(- \frac{[\gamma_{sf} + F_{el}(r_{il})] b^2}{kT} \right) \right\} \quad (3)$$

and

$$\frac{dr_{vl}}{dt} = \frac{1}{b} \left\{ Z_v D_v C_v - Z_i D_i C_i - Z_v D_v C_v^0 \exp \left(+ \frac{[\gamma_{sf} + F_{el}(r_{vl})] b^2}{kT} \right) \right\} \quad (4)$$

where D_i and D_v are the diffusion coefficients respectively for interstitials and vacancies, γ_{sf} is the stacking fault energy of the stacking fault of a dislocation loop (set equal to zero if the loop contains no stacking fault), b is the length of the Burger's vector of a dislocation, Z_v is the dislocation bias factor for vacancies ($Z_v = 1$) and Z_i is the bias factor for interstitials (slightly larger than one), k is Boltzmann's constant and T is the temperature, and C_v^0 is the equilibrium vacancy concentration in an unirradiated, unstressed metal. F_{el} is given by

$$F_{el}(r) = \frac{\mu b^2}{4\pi(1-\nu)(r+b)} \log \left(\frac{r+b}{r} \right) \quad (5)$$

where r is the radius of an interstitial or vacancy loop (r_{il} or r_{vl}), μ is the shear modulus and ν is Poisson's ratio. Equation (5) represents the force on a unit length of dislocation that causes dislocation climb in the direction that

decreases the loop radius r . The origin of this climb force is the self-energy or line tension of the dislocation loop.

In equations (3) and (4) a term analogous to the last term on the right-hand-side of these equations has been dropped, since it contains the thermal equilibrium concentration C_i^0 of interstitials. Because the formation activation energy of interstitials generally is very high for all practical applications, the concentration C_i^0 is essentially identical to zero.

Vacancies produced by irradiation may be absorbed directly by voids, network dislocations or interstitial dislocations. Some of these vacancies, however, may first form vacancy dislocation loops, and as these loops shrink during climb, the vacancies are recreated. If ϵP ($0 < \epsilon < 1$) represents the fraction of irradiation produced vacancies that initially condense into vacancy dislocation loops of radius $r_{vl}(0)$ per unit time, the number n_{vl} of vacancy loops created in a unit volume per unit time is given by

$$n_{vl} = \frac{\epsilon P b^2}{\pi [r_{vl}(0)]^2} \quad (6)$$

since there are $\frac{\pi b r_{vl}^2}{b^3}$ vacancies in a dislocation loop of radius r_{vl} . The

average lifetime of a vacancy loop can be approximated by

$$\tau \approx \frac{r_{vl}(0)}{\left. \frac{dr_{vl}}{dt} \right|_0} \quad (7)$$

where $\left. \frac{dr_{vl}}{dt} \right|_0$ is the value of equation (4) when $r_{vl} = r_{vl}(0)$, the initial radius of a vacancy dislocation loop.

If all the vacancies tied up in vacancy dislocation loops were released into the lattice, the vacancy concentration would be increased by an amount q_{vl} given by

$$q_{vl} = \pi b r_{vl}^2 N_{vl} \quad (8a)$$

which can be rearranged to give

$$r_{v\ell} = \sqrt{\frac{q_{v\ell}}{\pi b N_{v\ell}}} \quad (8b)$$

The rate of change of the number of vacancy loops per unit volume due to cascade collapse formation and shrinkage is given by

$$\frac{dN_{v\ell}}{dt} = \frac{n_{v\ell}}{b^3} - \frac{N_{v\ell}}{\tau} \quad (9)$$

which when combined with equations (4), (6), (7) and (8), becomes

$$\begin{aligned} \frac{dN_{v\ell}}{dt} = \frac{\epsilon P}{\pi b [r_{v\ell}(0)]^2} - \frac{b}{r_{v\ell}(0)} \left[Z_v D_v C_v - Z_i D_i C_i \right. \\ \left. - Z_v D_v C_v^0 \exp \left(\frac{+\{\gamma_{sf} + F_{el}[r_{v\ell}(0)]\}b^2}{kT} \right) \right] \end{aligned} \quad (10)$$

The time dependence of the fraction of vacancies tied up in vacancy loops, $q_{v\ell}$, is similarly given by

$$\begin{aligned} \frac{dq_{v\ell}}{dt} = \epsilon P - \sqrt{\frac{4\pi q_{v\ell} N_{v\ell}}{b}} \left(Z_v D_v C_v - Z_i D_i C_i \right. \\ \left. - Z_v D_v C_v^0 \exp \left(\frac{+\{\gamma_{sf} + F_{el}(r_{v\ell})\}b^2}{kT} \right) \right) \end{aligned} \quad (11)$$

Free point defects are created at a rate i , annihilated by recombination or absorbed by voids, edge dislocations or dislocation loops. The rate of change of the interstitial concentration C_i is given by

$$\frac{dC_i}{dt} = P - \alpha C_i C_v - C_i D_i \left\{ 4\pi r_v N_v + Z_i \left[\rho_d^0 + 2\pi(r_{i\ell} N_{i\ell} + r_{v\ell} N_{v\ell}) \right] \right\} \quad (12)$$

where the recombination coefficient α is given by

$$\alpha = g \left[v_i \exp\left(-\frac{E_m^i}{kT}\right) + v_v \exp\left(-\frac{E_m^v}{kT}\right) \right]$$

$$= (g/b^2) (D_i + D_v) \approx gD_i/b^2 \quad (13)$$

where E_m^i and E_m^v are activation energies for migration of interstitials and vacancies respectively, v_i and v_v are frequencies associated with their motion, and g is a constant of order of magnitude 1. The term in equation (12) containing the constant α represents the destruction of interstitials by mutual annihilation with individual vacancies, while the terms proportional to the product $C_i D_i$ represent the elimination of interstitials at void sinks, dislocation network sinks and dislocation loop sinks.

The change in the vacancy concentration C_v is similarly given by

$$\frac{dC_v}{dt} = (1-\epsilon)P - \alpha C_i C_v - C_v D_v \left\{ 4\pi r_v N_v + Z_v \left[\rho_d^0 + 2\pi(r_{il} N_{il} + r_{vl} N_{vl}) \right] \right\}$$

$$+ C_v^0 D_v \left\{ 4\pi N_v r_v \exp \left[\left(\frac{2\gamma}{r_v} - p_g \right) \frac{b^3}{kT} \right] + Z_v \left[\rho_d^0 + \right. \right. \quad (14)$$

$$\left. \left. 2\pi \left(r_{il} N_{il} \exp \left\{ \frac{[-\gamma_{sf} + F_{el}(r_{il})] b^2}{kT} \right\} + r_{vl} N_{vl} \exp \left\{ \frac{[\gamma_{sf} + F_{el}(r_{vl})] b^2}{kT} \right\} \right) \right] \right\}$$

where γ is the energy per unit surface of a void and p_g is the pressure of the gas that may be present within a void. The terms in equation (14) that contain the constant α or the product $C_v D_v$ are analogous to similar terms in equation (12), while the terms that contain the product $C_v^0 D_v$ represent the continuous creation of vacancies by thermal emission at the voids, network dislocations and dislocation loops. (Note that any sink for point defects can also act as a source for point defects. Equation (12) should contain terms proportional to the product $C_i^0 D_i$, but because the value of C_i^0 is very small these terms can be dropped.) The gas pressure p_g is given by

$$p_g = \frac{n_g kT}{\frac{4}{3}\pi r_v^3 - b_v n_g} \quad (15)$$

where n_g is the number of gas atoms or molecules within one void and b_v is van der Waal's coefficient.

The rate of change of the void radius r_v is given by

$$\frac{dr_v}{dt} = \frac{1}{r_v} \left\{ D_v C_v - D_i C_i - D_v C_v^0 \exp \left[\left(\frac{2\gamma}{r_v} - p_g \right) \frac{b^3}{kT} \right] \right\} \quad (16)$$

where the first two terms represent absorption of free point defects, and the last thermal emission of vacancies at the void surface.

If the swelling S is defined as the change in volume of material divided by the initial volume of material, then the swelling is given by

$$S = \frac{4}{3} \pi r_v^3 N_v \quad (17)$$

The inclusion of a fluctuating temperature $T(t)$ is quite simple, requiring only the substitution of this time-dependent temperature into equations (3), (4), (10), (11), (13), (14), (15) and (16) where T appears explicitly, as well as into the implicit temperature-dependent diffusion coefficients

$$D_{i,v} = D_{i,v}^0 \exp \left(- \frac{E_m^{i,v}}{kT} \right) \quad (18)$$

and the equilibrium vacancy concentration

$$C_v^0 = \exp \left(- \frac{E_f^v}{kT} \right) \quad (19)$$

The temperature fluctuations can be determined from the particular irradiation loading condition specified. Suppose that a cylindrical target of radius a and height l is bombarded by $I(r,t)$ high energy particles per unit area per unit time, and assume that the particles are energetic enough and l small enough that the irradiation damage is essentially uniform within the target in the direction of the beam. The temperature is found from the two-dimensional time-dependent heat conduction equation

$$\frac{1}{K} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\partial^2 T}{\partial z^2} = \frac{1}{K} Q(r, t) \quad (20)$$

where κ is the thermal diffusivity and K the thermal conductivity ($K = \kappa \rho c$ where ρ is the density of the material and c its specific heat).

The heat $Q(r, t)$ deposited per unit volume per unit time by the flux $I(r, t)$ within the target can be divided into

$$Q(r, t) = Q_d(r, t) + Q_i(r, t) \quad (21)$$

The first part $Q_d(r, t)$ is the direct heat produced immediately within the bombarded region by excitation of the lattice. Thus, for the simplest case, $Q_d(r, t)$ is proportional to $I(r, t)$ and can be expressed as

$$Q_d(r, t) = c_d I(r, t) \quad (22)$$

where c_d is the proportionality constant. The second part $Q_i(r, t)$ of the heat produced within the bombarded region is not realized immediately, but is the heat released when point defects are destroyed, and is given by

$$Q_i(r, t) = c_{di} \left(P - \frac{\partial c_i}{\partial t} \right) + c_{dv} \left[(1-\epsilon)P - \frac{\partial c_v}{\partial t} \right] \quad (23)$$

where c_{di} is the heat released by the destruction of a unit concentration of interstitials and c_{dv} the same term for vacancies. The Frenkel pair production rate, P , is of course a function of $I(r, t)$, and in the simplest case is proportional to I , giving

$$P(r, t) = c_p I(r, t) \quad (24)$$

where c_p is the proportionality constant. Combining equations (12), (14), (21), (22) and (23) gives finally for the heat flow

$$Q(r, t) = c_d I + c_{di} \left\{ \alpha c_i c_v + D_i c_i \left[4\pi v N_v + Z_i \left(\rho_d^0 + 2\pi r_{il} N_{il} + 2\pi r_{vl} N_{vl} \right) \right] \right\}$$

$$\begin{aligned}
& + c_{dv} \left\{ \alpha C_v C_i + D_v C_v \left[4\pi r_v N_v + Z_v \left(\rho_d^0 + 2\pi r_{il} + 2\pi r_{vl} N_{vl} \right) \right] \right. \\
& + C_v^0 D_v \left[4\pi r_v N_v \exp \left(\left\{ \frac{2\gamma}{r_v} - \frac{b^3}{kT} \right\} \right) \right. \\
& + Z_v \left(\rho_d^0 + 2\pi r_{il} N_{il} \exp \left[- \left\{ \gamma_{sf} + F_{el}(r_{il}) \right\} \frac{b^2}{kT} \right] \right. \\
& \left. \left. \left. + 2\pi r_{vl} N_{vl} \exp \left[\left\{ \gamma_{sf} + F_{el}(r_{vl}) \right\} \frac{b^2}{kT} \right] \right] \right] \right] \left. \right\} \quad (25)
\end{aligned}$$

APPLICATION TO LAMPF PULSED BEAM

Consider a specific localized flux $I(r,t)$. The LAMPF 800 MeV pulsed proton beam has an elliptically-shaped target area approximated here by a circular area. Since the flux profile is Gaussian, $I(r,t)$ is given by

$$I(r,t) = I(t) \exp \left(- \frac{r^2}{r_0^2} \right) \quad (26)$$

where r_0 is a constant giving the average size of the beam spot at the LAMPF facility and $I(t)$ is a pulsed time function, shown in figure 1, equal to

$$I(t) = \begin{cases} 0 & \text{for } t < 0 \\ I_0 & \text{for } nt^* \leq t \leq nt^* + \Delta t \\ 0 & \text{for } nt^* + \Delta t \leq t \leq (n+1)t^* \\ & \text{where } m\tau \leq t \leq m\tau + \tau_1 \\ 0 & \text{for } m\tau + \tau_1 \leq t \leq (m+1)\tau \end{cases} \quad (27)$$

where $n, m = 0, 1, 2, \dots$ and I_0 is a constant. If n_p is the total number of 800 MeV protons in a pulse of duration Δt , then I_0 is given by

$$I_0 = \frac{n_p / \Delta t}{\int_0^a 2\pi r \exp \left(- \frac{r^2}{r_0^2} \right) dr} \approx \frac{n_p / \Delta t}{\pi r_0^2} \quad (28)$$

The LAMPF accelerator operates at a frequency (τ^{-1}) of 120 Hz with pulse bundles 0.5 ms long (τ_1), containing 0.25 ns (Δt) pulses occurring once every 5 ns (t^*). Thus the average proton flux (i.e., the macropulses) is on 6 percent of the time. (For comparison, calculations will be made with other percent "on-times" ranging from 0.1 percent to 100 percent. The frequency and total damage per pulse will be held constant; the pulse length and annealing time will be adjusted accordingly.)

Suppose each 800 MeV proton loses an amount of energy E^* per unit distance traversed in the target, so that if one 800 MeV proton hits a unit area of the target it gives up the same energy E^* per unit volume of material. Of this lost energy a fraction α^* goes directly into heat through lattice excitation and the other fraction $(1-\alpha^*)$ goes into point defect production. Thus c_d is given by

$$c_d = \alpha^* E^* \quad (29)$$

The proportionality constant c_p that appears in the Frenkel pair production rate given by equation (24) is found by noting that the energy lost by an 800 MeV proton which does not immediately appear as heat is stored in the point defect formation energy, so that

$$c_p = \frac{(1-\alpha^*) E^* b^3}{E_f^i + E_f^v} \quad (30)$$

where E_f^i and E_f^v are again the formation energies respectively of interstitials and vacancies. (The term b^3 insures that the generation rate $P(r,t)$ is expressed in units of atom fraction per second.)

The fraction α^* of energy appearing directly as heat is found by equating the above expression with the standard point defect production rate [9]

$$P(r,t) = \frac{.8\sigma_D}{2E_D} I(r,t) = c_p I(r,t) \quad (31)$$

in which σ_D is the damage energy cross section in b-keV and E_D the threshold energy in eV. Once α^* is found, the proportionality constant c_d is also

determined, and the total heat generation per unit time per unit volume in each individual pulse is given by

$$Q(r,t) = c_d I(r,t) = \alpha^* E^* I_0 \exp\left(-\frac{r^2}{r_0^2}\right) \quad (32)$$

The heat released by the destruction of a unit concentration of interstitials (c_{di}) or vacancies (c_{dv}) used in equation (23) is given by

$$c_{di,v} = E_f^{i,v}/b^3 \quad (33)$$

III. APPROXIMATIONS

Several special cases of the fully dynamic rate theory can be studied analytically for temperature fluctuations so small they can be neglected. One such case is that of constant recombination rate and zero sink density. The other, studied here, is that of zero recombination and constant total sink density.

The point defect concentration equations (12) and (14) become, if α is set equal to zero,

$$\frac{dC_i}{dt} = P - D_i C_i \rho_{\text{sink}} \quad (34)$$

$$\frac{dC_v}{dt} = (1-\epsilon)P - D_v C_v \rho_{\text{sink}} + P_{\text{th}} \quad (35)$$

where ρ_{sink} is the total sink density given by

$$\rho_{\text{sink}} = 4\pi r_v N_v + \rho_d^0 + 2\pi(r_{il} N_{il} + r_{vl} N_{vl}) \quad (36)$$

and P_{th} is the rate of thermal emission of vacancies from the various sinks, given by

$$\begin{aligned}
P_{th} = C_v^0 D_v \left\{ 4\pi N_v r_v \exp \left[\left(\frac{2\gamma}{r_v} - p_g \right) \frac{b^3}{kT} \right] + Z_v \left[\rho_d^0 \right. \right. \\
+ 2\pi \left(N_{i\ell} r_{i\ell} \exp \left[\frac{-[\gamma_{sf} + F_{el}(r_{i\ell})] b^2}{kT} \right] \right. \\
\left. \left. + N_{v\ell} r_{v\ell} \exp \left[\frac{+[\gamma_{sf} + F_{el}(r_{v\ell})] b^2}{kT} \right] \right) \right] \right\} \quad (37)
\end{aligned}$$

With all the sink properties held constant the solution of these approximate point defect concentration equations for the time during which the material is being irradiated is

$$\begin{aligned}
C_i(k\tau \leq t \leq k\tau + \tau_1) = \frac{P}{D_i \rho_{sink}} \left\{ 1 - \exp \left[-D_i \rho_{sink} (t - k\tau) \right] \right\} \\
+ C_i(k\tau) \exp \left[-D_i \rho_{sink} (t - k\tau) \right] \quad (38)
\end{aligned}$$

$$\begin{aligned}
C_v(k\tau \leq t \leq k\tau + \tau_1) = \frac{(1-\epsilon)P + P_{th}}{D_v \rho_{sink}} \left\{ 1 - \exp \left[-D_v \rho_{sink} (t - k\tau) \right] \right\} \\
+ C_v(k\tau) \exp \left[-D_v \rho_{sink} (t - k\tau) \right] \quad (39)
\end{aligned}$$

The initial conditions for $k = 0$ are taken to be $C_i(t = 0) \equiv 0$ (since C_i^0 is very small) and $C_v(t=0) = C_v^0$, the thermal equilibrium concentration of vacancies. The solution for the annealing period is similarly

$$C_i[k\tau + \tau_1 \leq t \leq (k+1)\tau] = C_i(k\tau + \tau_1) \exp \left[-D_i \rho_{sink} (t - k\tau - \tau_1) \right] \quad (40)$$

$$\begin{aligned}
C_v[k\tau + \tau_1 \leq t \leq (k+1)\tau] = \frac{P_{th}}{D_v \rho_{sink}} \left\{ 1 - \exp \left[-D_v \rho_{sink} (t - k\tau - \tau_1) \right] \right\} \\
+ C_v(k\tau + \tau_1) \exp \left[-D_v \rho_{sink} (t - k\tau - \tau_1) \right] \quad (41)
\end{aligned}$$

For many situations the recombination rate is negligible compared to the sink absorption rate, and the change in sink densities and sizes (and thus total strength) is small for any given pulse. The approximate expressions derived above are then valid, and can be used to study the small changes in various sink properties during and between single pulses, although repeated applications to multiple pulses lead to large cumulative errors. The void radius equation (16) can be written as

$$\frac{dr_v}{dt} = \frac{1}{r_v} (D_v C_v - D_i C_i - P_{th}^v) \quad (42)$$

where P_{th}^v represents the thermal emission of vacancies at the void surface given by

$$P_{th}^v = D_v C_v^0 \exp \left[\left(\frac{2\gamma}{r_v} - p_g \right) \frac{b^3}{kT} \right] \quad (43)$$

If the thermal emission term is assumed constant (neglecting the effect of changing r_v and indirectly p_g , which is given in equation (15) as a function of r_v), the solution of equation (42) for the irradiation periods, found by substituting equations (38) and (39) for C_i and C_v and integrating, is

$$\begin{aligned} r_v^2(k\tau \leq t \leq k\tau + \tau_1) &= \frac{2}{v}(k\tau) + 2 \left(\frac{P + P_{th}}{\rho_{sink}} - P_{th}^v \right) (t - k\tau) \\ &+ \frac{2}{\rho_{sink}} \left[C_i(k\tau \leq \tau \leq k\tau + \tau_1) - C_i(k\tau) - C_v(k\tau \leq t \leq k\tau + \tau_1) + C_v(k\tau) \right] \end{aligned} \quad (44)$$

The change in void radius during the annealing period is likewise found by substituting equations (40) and (41) for C_i and C_v into equation (42) and integrating, and is given by

$$\begin{aligned}
r_v^2 \left[k\tau + \tau_1 \leq t \leq (k+1)\tau \right] &= r_v^2(k\tau + \tau_1) + 2 \left(\frac{p_{th}}{\rho_{sink}} - p_{th}^v \right) (t - k\tau - \tau_1) \\
&+ \frac{2}{\rho_{sink}} \left\{ C_i \left[k\tau + \tau_1 \leq \tau \leq (k+1)\tau \right] - C_i(k\tau + \tau_1) \right\} \\
&- \frac{2}{\rho_{sink}} \left\{ C_v \left[k\tau + \tau_1 \leq t \leq (k+1)\tau \right] + C_v(k\tau + \tau_1) \right\}
\end{aligned} \tag{45}$$

The interstitial loop radius, described by equation (3), is similarly approximated during the irradiation and annealing periods by the equations

$$\begin{aligned}
r_{il}(k\tau \leq t \leq k\tau + \tau_1) &= r_{il}(k\tau) - \frac{1}{b} \left(\frac{P + P_{th}}{\rho_{sink}} - p_{th}^{il} \right) (t - k\tau) \\
&- \frac{1}{b\rho_{sink}} \left[Z_i C_i(k\tau \leq t \leq k\tau + \tau_1) - Z_i C_i(k\tau) \right] \\
&- \frac{1}{b\rho_{sink}} \left[-Z_v C_v(k\tau < t < k\tau + \tau_1) + Z_v C_v(k\tau) \right]
\end{aligned} \tag{46}$$

and

$$\begin{aligned}
r_{il} \left[k\tau + \tau_1 \leq t \leq (k+1)\tau \right] &= r_{il}(k\tau + \tau_1) - \frac{1}{b} \left(\frac{p_{th}}{\rho_{sink}} - p_{th}^{il} \right) (t - k\tau - \tau_1) \\
&- \frac{1}{b\rho_{sink}} \left\{ Z_i C_i \left[k\tau + \tau_1 \leq t \leq (k+1)\tau \right] - Z_i C_i(k\tau + \tau_1) \right\} \\
&- \frac{1}{b\rho_{sink}} \left\{ -Z_v C_v \left[k\tau + \tau_1 \leq t \leq (k+1)\tau \right] - Z_v C_v(k\tau + \tau_1) \right\}
\end{aligned} \tag{47}$$

where p_{th}^{il} represents the emission of vacancies at the surface of the interstitial loops, given by

$$p_{th}^{il} = D_v C_v^0 \exp \left\{ \frac{-[\gamma_{sf} + F_{el}(r_{il})]b^2}{kT} \right\} \tag{48}$$

The vacancy loop properties, described by equations (10) and (11), are slightly more complicated. The vacancy loop number density equation is solved as an example. During the irradiation period the vacancy loop number density is given by

$$\begin{aligned}
& N_{v\ell}(k\tau \leq t \leq k\tau + \tau_1) \\
& = N_{v\ell}(k\tau) - \frac{b}{r_{v\ell}(0)} \left(\frac{p + p_{th}}{\rho_{sink}} - \left\{ p_{th}^{v\ell} [r_{v\ell}(0)] + \frac{\epsilon P}{\pi b^2 r_{v\ell}(0)} \right\} \right) (t - k\tau) \\
& \quad - \frac{b}{r_{v\ell}(0)} \left[Z_i C_i(k\tau \leq t \leq k\tau + \tau_1) - Z_i C_i(k\tau) \right] \\
& \quad - \frac{b}{r_{v\ell}(0)} \left[-Z_v C_v(k\tau \leq t \leq k\tau + \tau_1) + Z_v C_v(k\tau) \right] \quad (49)
\end{aligned}$$

where the thermal emission of vacancies at the surface of the vacancy loop is given by

$$p_{th}^{v\ell} = D_v C_v^0 \exp \left\{ \frac{+[\gamma_{sf} + F_{el}(r_{v\ell})] b^2}{kT} \right\} \quad (50)$$

The vacancy loop number density during the annealing period is given by

$$\begin{aligned}
& N_{v\ell}[k\tau + \tau_1 \leq t \leq (k+1)\tau] \\
& = N_{v\ell}(k\tau + \tau_1) - \frac{b}{r_{v\ell}(0)} \left(\frac{p_{th}}{\rho_{sink}} - \left\{ p_{th}^{v\ell} [r_{v\ell}(0)] + \frac{\epsilon P}{\pi b^2 r_{v\ell}(0)} \right\} \right) (t - k\tau - \tau_1) \\
& \quad - \frac{b}{r_{v\ell}(0)} \left\{ Z_i C_i[k\tau + \tau_1 \leq t \leq (k+1)\tau] - Z_i C_i(k\tau + \tau_1) \right\} \\
& \quad - \frac{b}{r_{v\ell}(0)} \left\{ -Z_v C_v[k\tau + \tau_1 \leq t \leq (k+1)\tau] + Z_v C_v(k\tau + \tau_1) \right\} \quad (51)
\end{aligned}$$

The expressions giving $q_{v\ell}$ are much more elaborate and are not given here.

IV. NUMERICAL SOLUTION

Equations (3), (10), (11), (12), (14), (16) and (20) constitute a system of seven non-linear interdependent first order differential equations. The indirect heating term in the temperature equation, which would normally couple the six ordinary differential equations for $r_{i\ell}$, $N_{v\ell}$, $q_{v\ell}$, C_i , C_v , and r_v to a partial differential equation for T , is shown in Appendix I to be negligible in the situations considered in this paper. The temperature equation thus completely decouples from the other six equations, and can be solved analytically, as discussed in detail in Appendix II. Since this analytic solution provides the temperature at any spatial point as a function of time, the problem reduces back to a system of ordinary rather than partial differential equations. One of the difficulties of solving time-dependent materials problems such as this is the "stiffness" of the resulting system of equations, requiring special numerical techniques.

A system of ODE's is generally regarded as being stiff if it involves both very rapidly changing terms and very slowly changing terms, all of a decaying nature, so that it models a physical system with widely differing time constants. More precisely, consider the eigenvalues λ_i of the $N \times N$ Jacobean matrix

$$J = \left. \frac{\partial f_i}{\partial y_j} \right|_{i, j = 1}^N$$

and suppose that all the λ_i have negative real parts. If the time constants

$$1/|\text{Re}(\lambda_1)|, 1/|\text{Re}(\lambda_2)|, \dots, 1/|\text{Re}(\lambda_N)|$$

are widely varying, the system is called stiff. Since J varies with time the eigenvalues do, too, and consequently stiffness is a local property. A system may be stiff in some regions of t and not in others. Stiffness is also a relative property, measured by the ratio of the maximum to minimum time constants, $(\max \tau_i / \min \tau_i)$. The difficulty in solving stiff problems is that, while most conventional ODE solvers require time increments commensurate with $\min \tau_i$, the

size $|t-t_0|$ of the problem range is commensurate with $\max \tau_i$. Thus, the problem cannot run to completion in a reasonable number of steps.

The numerical method used is based on ideas of C. W. Gear [10] and A. Hindmarsh [11] for the solution of the initial-value problem for systems of ODE's. The package of subroutines SDRIVE [12] includes both the implicit Adams method and the linear multistep methods, or backwards differentiation formulas, of Gear as options. With Gear's methods, the increment h is restricted to small values (by the requirements of accuracy), only where the solution is relatively active. Since by definition the problem is not stiff in such regions, accuracy is achieved at minimum cost by allowing both h and the order of the method to vary. In regions of stiffness where the solution is inactive, Gear's methods have the property of "stiff stability", which assures that h is no longer restricted by the small time constants unless or until the corresponding rapidly decaying terms again become active. Among other things, this property requires that the method be implicit and a system of generally non-linear algebraic equations be solved at each step. Stiffness, moreover, means that a fairly powerful iteration method (variants of Newton's methods called chord methods) must be used to solve this system.

The main program, HANS, first sets all the required material constants and the parameter options for the equation solver SDRIVE (e.g., number of equations, method of solution, error control, etc.), as well as the initial conditions given. Then, for each pulse and each annealing period, the subroutine package SDRIVE is called. When return occurs, either the solution and the numerical statistics are printed out and the next integration is begun, or (if an error occurs during the attempted integration) an error message is printed and the program stops.

Subroutine DIFFUN, required by the SDRIVE package, calculates $\dot{y} = f(y, t)$ for the irradiation period. An analogous subroutine, DIFF, calculates $\dot{y} = f(y, t)$ for the annealing periods.

The irradiation load itself is calculated in function BEAM. If the irradiation is constant or slowly varying in time, the integration routine's usual mode of solution, using the largest possible (and hence most efficient) choice of time-step to overshoot and then interpolating back to desired output times, can be used with no difficulty. However, if a singularity exists beyond the desired solution time (as would occur, for example, when the irradiation load consists of a step function as described earlier), the solution must be

obtained by integrating exactly to the desired time point, limiting the variable time-step to the order of magnitude of the characteristic pulse time and thus increasing the computer time required for solution.

V. RESULTS

The fully dynamic rate theory with self-consistent temperature effects described above has been used to analyze the behavior of both aluminum and molybdenum during cyclic pulsed radiation and to compare the results with those for equivalent dose steady state irradiation. The base case pulsed irradiation is taken to be that of the LAMPF 800 MeV proton beam.

The beam at LAMPF is assumed to have a Gaussian radial flux profile given by equation (26). Its stepwise time function, described in equation (27), consists of 0.25 ns micropulses occurring every 5 ns in macropulse bundles 0.5 ms wide at a frequency of 120 s^{-1} , as shown in Figure 1. To see whether this structure must be taken into account, the various pulse times must be compared to the characteristic time constants for the differential equations. (If the pulsing is much more rapid than the variables' reaction times, the system will not see any new effects.) The vacancy and interstitial characteristic reaction times (the shortest in the system) can be approximated by

$$\tau_{i,v} = D_{i,v} / \rho_{\text{sink}} \quad (52)$$

from equations (34) and (35). For the material data given in Tables 1 and 2 this leads to the ordering

$$\begin{aligned} \Delta t, t^* &\ll \tau_i, \tau_v \\ \tau_i &\ll \tau_l, \tau_{\text{TT}} \ll \tau_v \end{aligned} \quad (53)$$

The sample cannot react to the micropulses, but is affected by the macropulses. To test whether the beam microstructure can in fact be neglected, two calculations were done, whose results are shown in Fig. 2. In the first calculation, the first 10^{-4} s of a macropulse was run with its 2×10^4 micropulses modelled accurately, while in the second an average macropulse was used. The material was taken to be aluminum at an ambient temperature of 400 K. The point defect

concentrations are almost identical, and confirm that the LAMPF beam microstructure is indeed negligible. Therefore, all following calculations have only average macropulses modelled.

The concentrations shown in Fig. 2 are for the center point of a thin cylindrical target, as described in Appendix III. The radiation damage due to high energy particles such as 800 MeV protons is uniform in depth for thin samples. The effect of the Gaussian radial flux profile is shown in Fig. 3. The vacancy concentration is simply proportional to the current at any given location, since there has not been time for any diffusion to occur. The interstitials, being much more mobile, as seen in equation (53), have reached an equilibrium concentration early in the pulse ($\sim 10^{-6}$ sec). The enhanced diffusion due to higher radiation produced temperatures at the sample center causes the equilibrium concentration there to decrease, leading to interstitial concentration profiles peaked at some intermediate radius. The vacancy concentration is peaked at the sample center, and both the vacancy and interstitial concentrations are within a few percent of the values predicted by equations (38) - (41). The interstitial concentration very rapidly decays back to thermal equilibrium at the end of the pulse, while the vacancy concentration remains constant during the annealing period, also as predicted by the approximate equations.

The vacancy concentration will not increase linearly with time (as in Fig. 3) indefinitely, but will also come to an equilibrium concentration after many pulses, as shown in Fig. 4. The vacancy concentration oscillates between limits, whose separation depends on the relative duration of the irradiation and annealing periods. For the LAMPF beam macrostructure the fractional "on-time" or $\frac{\tau_1}{\tau}$ ($= f$) is 0.06. (The pulse structure in this study is varied by keeping τ fixed to the LAMPF value of 1/120 seconds, and changing τ_1 . While the vacancy concentration is approaching its equilibrium value, the voids are primarily absorbing interstitials and hence shrinking, as shown in Fig. 4. As the vacancy concentration stabilizes, the voids begin to grow at a nearly uniform rate. This initial void shrinkage is also well predicted by the approximate sequence of pulses. Molybdenum requires many more pulses than aluminum before the vacancy concentration limits are reached, because of its lower vacancy diffusivity. The length of this initial void shrinkage is also a sensitive function of the ambient temperature, which strongly influences the diffusion rate - the higher the temperature, the fewer pulses required for the vacancy concentration to saturate.

A second and much slower initial "transient" is shown in Figure 5. The vacancy loops take approximately ten times as many pulses to reach an eventual oscillating steady state for both size and number density as does the vacancy concentration. (The oscillations are too small to be seen on the scale of Figure 5.) The final value of the vacancy loop average radius ($\sim 3A_0$) represents a small cluster of vacancies rather than a true vacancy dislocation loop. This illustrates a defect in the theory, that no term is present to correctly account for the disappearance of vacancy loops as they shrink down to individual vacancies.

When the vacancy loop size and number density reach equilibrium values, void growth rate becomes constant as shown in Figure 6. The void growth rate will change only gradually under further irradiation, as shown in Figure 7, as the total sink strength slowly increases. The approximate equations gave excellent agreement for short sequences of pulses when initial conditions are taken from the numerical calculations, but completely fail to duplicate the results in Figures 6 and 7, due to cumulative error, if they are iterated for many pulses.

The above calculations were repeated with the temperature pulsing suppressed. Since Dienes' work [8] concluded that it was the nonlinear recombination term that would be responsible for the difference between steady state and cyclic pulsed irradiation behavior, the calculations were also rerun with no recombination ($\alpha = 0$), with the results shown in Figure 8. Nearly identical void growth rates are obtained in the presence or absence of recombination, and a similar lack of sensitivity on very small temperature excursions is seen. The calculated results only show a significantly lower void growth rate when the "on time" is very short, the temperature excursion large and the displacement damage rate very large for a short period of time. (The maximum temperature rise for the various pulse structures considered is given in table 3).

The ambient temperature used in the above calculation ($T_0 = 400$ K) is actually above the peak swelling temperature for the "simulated Al", as is shown in Fig. 9. We expect that temperature pulsing would produce enhanced void growth if the ambient temperature were below the peak swelling temperature. A series of calculations for aluminum at an ambient temperature of 352 K did indeed show this effect, as seen in in Fig. 10. The early void shrinkage is not shown, but both the vacancy concentration and the vacancy loop size and number

density transients were present, and lasted much longer than their analogs in 400 K aluminum, due to the decrease in vacancy diffusivity at lower temperatures.

A parallel set of calculations was done for a molybdenum target. Ambient temperatures of 1200 K and 1600 K were chosen as bracketing the peak swelling temperature, as shown in Fig. 11. The lower temperature calculations ($T_0 = 1200$ K) gave the expected result of increasing temperature pulsing increasing void growth, as shown in Fig. 12, but the higher temperature calculations ($T_0 = 1600$ K) gave contradictory results, with the voids growing faster than more slowly for $f = 0.1\%$ than for $f = 1\%$, as seen in Fig. 13. Upon investigation, the temperature excursion for the short pulse length was found to be so large that, when added to the high ambient temperature, the thermal emission of vacancies dominated the radiation production during the latter part of each pulse. Thermal emission also remained important during the early part of each annealing period. The result was, as seen in Fig. 14, to decrease the total sink density as a function of time rather than to increase it, and hence to increase the void growth rate.

VI. CONCLUSIONS

The fully dynamic rate theory developed by Ghoniem and Kulcinski has been used to study the behavior of both aluminum and molybdenum under cyclic pulsed irradiation. Several important conclusions can be drawn:

- (1) The pulsed nature of irradiation is important only when the characteristic pulse times are greater than or comparable to the vacancy and interstitial characteristic reaction times (the shortest in the system). If the pulsing is much more rapid than the variables' reaction time, the system will not see any new effects.
- (2) The assumption that the material initially contains a given microstructure and is at thermal equilibrium results in transients in both vacancy concentration and vacancy loop size and number density. Since void growth does not proceed at a fairly linear rate until these transients have ended, single or few pulse behavior cannot be extrapolated to estimate long-term behavior, as was done by Ghoniem and Kulcinski.
- (3) Pulsing the irradiation has only an indirect effect on material swelling.
 - (a) When the cyclic radiation is assumed to produce self-consistent

temperature pulses, the material "sees" enhanced ambient temperatures. This temperature enhancement increases void growth rate if the ambient temperature is below the material peak swelling temperature, and conversely decreases void growth rate when the ambient temperature is above peak swelling temperature.

- b) When radiation-produced temperature pulses are suppressed so that the material remains at a constant ambient temperature, the void growth rate does not change as the radiation is fragmented into various pulse structures. Although Dienes' analytical work concluded that the nonlinear recombination term would be affected by the pulsed nature of the irradiation, our calculations did not confirm this. In the absence of recombination, the void growth rate was greater since vacancies formerly destroyed by recombination were now available for absorption by voids, but no effect was seen as the pulse structure was varied.
- (4) For high ambient temperatures and short intense radiation pulses, which cause large temperature excursions, thermal emission of vacancies greatly overshadows radiation production during the latter part of each pulse, and the general effect of pulse structure and ambient temperature on void growth summarized above is no longer applicable.

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APPENDIX I.

INDIRECT HEATING IN ALUMINUM AND MOLYBDENUM

The heat Q deposited per unit time per unit volume in the target by a flux I consists of the direct heat produced immediately within the bombarded region by excitation of the lattice, and the heat released when point defects are destroyed by recombination or absorption at sinks. The second term is in most applications negligible.

The direct heat is given by

$$Q_d = E^* I \quad (I-1)$$

where E^* is the amount of energy lost by an 800 MeV proton per unit distance traversed in the target. (We have assumed $\alpha^* \approx 1$.) The indirect heating term is

$$Q_i = \frac{E_f^i}{b^3} \left(P - \frac{\partial C_i}{\partial t} \right) + \frac{E_f^v}{b^3} \left[(1-\epsilon)P - \frac{\partial C_v}{\partial t} \right] \quad (I-2)$$

where $\frac{E_f^{i,v}}{b^3}$ is the heat released by the destruction of a unit concentration of interstitials or vacancies with formation energies $E_f^{i,v}$. Using equations (12) and (13), this expression can be rewritten as

$$Q_i = \frac{E_f^i}{b^3} \left(\alpha C_i C_v + D_i C_i \rho_{\text{sink}} \right) + \frac{E_f^v}{b^3} \left(\alpha C_i C_v + D_v C_v \rho_{\text{sink}} - P_{\text{th}} \right) \quad (I-3)$$

where ρ_{sink} is the total sink density defined in equation (36).

The maximum possible values that C_i and C_v can have are, from equations (38) - (41),

$$\begin{aligned} C_i &= \frac{P}{D_i \rho_{\text{sink}}} \\ C_v &= \frac{(1-\epsilon)P + P_{\text{th}}}{D_v \rho_{\text{sink}}} \end{aligned} \quad (I-4)$$

If the total sink density is assumed constant and if the thermal emission of vacancies is neglected (a good assumption for the conditions we are interested in), the direct and indirect heating for the aluminum and molybdenum samples described in Tables 1 and 2 are

		Al			Mo
		325 K	400 K	1200 K	1600 K
Q_d	$\sim 3 \cdot 10^{11}$	$\sim 3 \cdot 10^{11}$	$\sim 3 \cdot 10^{12}$	$\sim 3 \cdot 10^{12}$	
Q_i	$\sim 1 \cdot 10^8$	$\sim 3 \cdot 10^7$	$\sim 1 \cdot 10^9$	$\sim 1 \cdot 10^9$	

The indirect heating is always at least three orders of magnitude lower than the direct heating, and obviously negligible while the direct heating is present.

In between pulses the maximum possible temperature rise due to indirect heating can be estimated from equation (30) by assuming the absence of conduction, so that

$$\Delta T_{\max} = \frac{\kappa}{K} Q \tau \quad (\text{I-5})$$

giving a worst possible temperature rise of .3 and 3 degrees Kelvin for aluminum and molybdenum, respectively. These are negligible since the samples can easily conduct away such small amounts of heat.

APPENDIX II
GREEN'S FUNCTION METHOD FOR CALCULATING TEMPERATURE

Equations

The two-dimensional time-dependent heat conduction equation in cylindrical coordinates is

$$\frac{1}{\kappa} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa} Q \quad (\text{II-1})$$

The region of interest is a finite cylinder of height ℓ and radius a as shown in Fig. A-1, whose surface is held at a fixed temperature $T_0 = T(t=0)$. Treating the right-hand side of equation (II-1) as a source term allows us to write the formal solution in terms of the Green's function for the homogeneous equation [Carslaw and Jaeger, p. 380] as

$$T(r, z, t) = \frac{\kappa}{K} \int_0^a \int_0^\ell \int_0^t G(r, r'; z, z'; t, t') Q(r', z', t') 2\pi r' dr' dz' dt' \quad (\text{II-2})$$

To find the necessary Green's function we use the method of separation of variables on the modified heat conduction equation

$$\frac{1}{\kappa} \frac{\partial G}{\partial t'} - \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial G}{\partial r'} \right) - \frac{\partial^2 G}{\partial z'^2} = 0 \quad (\text{II-3})$$

Assuming $G(r', z', t') = R(r')Z(z')\Theta(t')$ gives

$$0 = \frac{1}{\kappa \Theta(t')} \frac{\partial \Theta(t')}{\partial t'} - \frac{1}{r' R(r')} \frac{\partial}{\partial r'} \left(r' \frac{dR(r')}{dr'} \right) - \frac{1}{Z(z')} \frac{d^2 Z(z')}{dz'^2} \quad (\text{II-4})$$

which breaks up into the three ordinary differential equations

$$\frac{d\Theta(t')}{dt'} = - (c_r^2 + c_z^2) \kappa \Theta(t') \quad (\text{II-5})$$

$$\frac{d^2 Z(z')}{dz'^2} = -c_z^2 Z(z') \quad (\text{II-6})$$

$$\frac{1}{r'} \frac{d}{dr'} \left(r' \frac{dR(r')}{dr'} \right) = -c_r^2 R(r') \quad (\text{II-7})$$

Equation (II-7) is the standard equation obeyed by Bessel functions of order zero

$$r'^2 \frac{d^2 R}{dr'^2} + r' \frac{dR}{dr'} + (r'^2 - (n=0)^2) R = 0 \quad (\text{II-8})$$

where $r^* = c_r r'$. The boundary condition on this radial equation is

$$T(a, z, t) = 0 \text{ (or a constant)} \quad (\text{II-9})$$

giving immediately $r' = a \leftrightarrow r^* = \alpha_n$ where α_n is the nth zero of $J_0(r^*)$, so that

$$c_r = \frac{\alpha_n}{a} \quad (\text{II-10})$$

(The zeroeth order Bessel function of the second kind is eliminated since it cannot satisfy boundedness at $r = 0$.) The solution to the radial equation is then

$$R(r) = \sum_n A_n J_0 \left(\alpha_n \frac{r'}{a} \right) \quad (\text{II-11})$$

where the coefficient A_n is determined from the conditions

$$R(r' = r) = \delta(r' - r) = f(r') \quad (\text{II-12})$$

to be

$$f(r') = \sum_{\alpha_n} A_n J_0 \left(\alpha_n \frac{r'}{a} \right)$$

$$\int_0^a 2\pi r' dr' J_0 \left(\alpha_m \frac{r'}{a} \right) \delta(r'-r) = \int_0^a 2\pi r' dr' \sum_{\alpha_n} A_n J_0 \left(\alpha_n \frac{r'}{a} \right) J_0 \left(\alpha_m \frac{r'}{a} \right)$$

$$J_0 \left(\alpha_m \frac{r}{a} \right) = \pi a^2 A_m \left[J_1 \left(\alpha_m \right) \right]^2$$

$$A_m = \frac{1}{\pi a^2} \frac{J_0 \left(\alpha_m \frac{r}{a} \right)}{\left[J_0'(\alpha_m) \right]^2} \quad (\text{II-13})$$

giving finally

$$R(r, r') = \frac{1}{\pi a^2} \sum_{\alpha_n} \frac{J_0 \left(\alpha_n \frac{r'}{a} \right) J_0 \left(\alpha_n \frac{r}{a} \right)}{\left[J_0'(\alpha_n) \right]^2} \quad (\text{II-14})$$

Equation (II-6) is even simpler in that the solution are trigonometric rather than Bessel functions. Using the boundary condition

$$Z(z' = 0) = Z(z' = l) = 0 \quad (\text{II-15})$$

gives the solution

$$Z(z') = \sum_m B_m \sin \left(\frac{m\pi z'}{l} \right) \quad (\text{II-16})$$

where the cosines disappear because of the $z = 0$ condition, and the $z = l$ condition gives

$$c_z l = m\pi \text{ or } c_z = \frac{m\pi}{l} \quad (\text{II-17})$$

The coefficients B_m are evaluated using

$$Z(z'=z) = \delta(z-z') = f(z') \quad (\text{II-18})$$

to give

$$\int_0^{\ell} \sin\left(\frac{n\pi z}{\ell}\right) \delta(z-z') dz' = \int_0^{\ell} \sum_m A_n \sin\left(\frac{m\pi z'}{\ell}\right) \sin\left(\frac{n\pi z'}{\ell}\right) dz'$$

$$\sin\left(\frac{n\pi z}{\ell}\right) = \frac{\ell}{2} A_n$$

$$A_n = \frac{2}{\ell} \sin \frac{n\pi z}{\ell} \quad (\text{II-19})$$

giving finally

$$Z(z, z') = \frac{2}{\ell} \sum_m \sin\left(\frac{m\pi z}{\ell}\right) \sin\left(\frac{m\pi z'}{\ell}\right) \quad (\text{II-20})$$

Utilizing the results of equations (II-10) and (II-17) allows equation (II-5) to be written as

$$\frac{d\theta(t')}{dt'} = -\kappa \left[\frac{\alpha_n^2}{a^2} + \frac{(m\pi)^2}{\ell^2} \right] \theta(t') \quad (\text{II-21})$$

whose solution is

$$\theta(t, t') = \sum_{\alpha_n} \sum_m \exp \left\{ -\kappa \left[\frac{\alpha_n^2}{a^2} + \frac{(m\pi)^2}{\ell^2} \right] (t-t') \right\} \quad (\text{II-22})$$

Thus the Green's function for this problem is

$$G(r, r'; z, z'; t, t') = \frac{2}{\pi a^2 \ell} \sum_{m=1}^{\infty} \exp \left[-\kappa \frac{m^2 \pi^2}{\ell^2} (t-t') \right] \sin\left(\frac{m\pi z}{\ell}\right) \sin\left(\frac{m\pi z'}{\ell}\right)$$

$$\times \sum_{\alpha_n} \exp \left[-\kappa \frac{\alpha_n^2}{a^2} (t-t') \right] \frac{J_0(\alpha_n \frac{r'}{a}) J_0(\alpha_n \frac{r}{a})}{[J_0'(\alpha_n)]^2} \quad (\text{II-23})$$

Application

The LAMPF 800 MeV pulsed proton beam has an elliptically-shaped target area approximated here by a circular area of radius a . Irradiation and heating of a thin rectangular foil whose edges and surface are held constant at the initial temperature can be approximated by considering heat conduction and generation in a finite cylinder large enough that the radial edges see little irradiation. The proton flux for the LAMPF beam is Gaussian in profile and hence can be described as

$$I(r,z,t) = I(t) \exp \left(-\frac{r^2}{r_0^2} \right) \quad (\text{II-24})$$

where r_0 is a constant ($= \sqrt{2}\sigma$ where σ is the standard deviation of the Gaussian profile) giving the average size of the beam spot and $I(t)$ is a pulsed time function equal to

$$I(t) = \begin{cases} 0 & \text{for } t < 0 \\ I_0 & \text{for } m\tau \leq t \leq m\tau + \tau_1 \\ 0 & \text{for } m\tau + \tau_1 \leq t \leq (m+1)\tau \end{cases} \quad (\text{II-25})$$

where $m = 0, 1, 2, \dots$, $\tau_1 = 0.5$ ms and $\tau = \frac{1}{120 \text{ Hz}}$. For a net current of 1 mA, I_0 is given by

$$I_0 = \frac{n_p / \tau_1}{\int_0^a 2\pi r \exp \left(-\frac{r^2}{r_0^2} \right) dr} \approx \frac{n_p / \tau_1}{\pi r_0^2} \quad (\text{II-26})$$

where n_p is the total number of protons in a pulse occurring every τ seconds, given by

$$n_p = 1 \text{ mA} \times \frac{10^{-3} \text{ C}}{\text{sec-mA}} \times 6.28 \times 10^{18} \frac{\text{P}}{\text{C}} \times \frac{1}{120 \text{ pulse/sec}} = 5.233 \times 10^{13} \frac{\text{P}}{\text{pulse}} \quad (\text{II-27})$$

for a net current of 1 mA with 120 pulses per second. If each 800 MeV proton loses an amount of energy E^* per unit distance traversed in the target, and if all this energy is assumed to produce heat by direct excitation of the lattice,

the heat generation rate per unit volume is

$$Q = E^* \frac{n_p / \tau_1}{\pi r_0^2} \exp \left(-\frac{r^2}{r_0^2} \right) = Q_0 \exp \left(-\frac{r^2}{r_0^2} \right) = c_Q I_0 \exp \left(-\frac{r^2}{r_0^2} \right) \quad (\text{II-28})$$

whenever the beam is on, and zero otherwise. To insure that most of the beam is accounted for we set $a = 3\sigma = 2.12132r_0$ (by which time we are using 99% of the current).

The temperature is given by equations (2) and (23) as

$$T(r, z, t) - T_0 = \frac{\kappa}{K} Q_0 \frac{2}{\pi a^2 \ell} \sum_{m=1}^{\infty} \sum_n \int_0^t \exp \left\{ -\kappa \left[\frac{m^2 \pi^2}{\ell^2} + \frac{\alpha_n^2}{a^2} \right] (t - t') \right\} f(t') dt' \\ \times \int_0^a 2\pi r' \exp \left(-\frac{r'^2}{r_0^2} \right) \frac{J_0(\alpha_n \frac{r'}{a}) J_0(\alpha_n \frac{r}{a})}{[J_0'(\alpha_n)]^2} dr' \int_0^\ell \sin \left(\frac{m\pi z}{\ell} \right) \sin \left(\frac{m\pi z'}{\ell} \right) dz' \quad (\text{II-29})$$

where $f(t')$ is a pulsed time function such as given by equation (25) but normalized to unity.

The integration in z' is readily carried out to give

$$\int_0^\ell \sin \left(\frac{m\pi z}{\ell} \right) \sin \left(\frac{m\pi z'}{\ell} \right) dz' = \frac{2\ell}{m\pi} \sin \left(\frac{m\pi z}{\ell} \right) \quad m = 1, 3, 5, \dots \quad (\text{II-30})$$

and the summation over odd m can be rewritten as $m \rightarrow 2m+1$. The integration in r' can only be approximated, using the definite integral

$$\int_0^\infty r'^{\nu+1} e^{-a^2 r'^2} J_\nu(br) dr' = \frac{b^\nu}{(2a)^{\nu+1}} \exp \left(-\frac{b^2}{4a^2} \right) \quad (\text{II-31})$$

to give

$$\int_0^a 2\pi r' \frac{J_0(\alpha_n \frac{r'}{a}) J_0(\alpha_n \frac{r}{a})}{[J_0'(\alpha_n)]^2} \exp \left[-\frac{r'^2}{r_0^2} \right] dr' \approx 2\pi \frac{J_0(\alpha_n \frac{r}{a})}{J_1^2(\alpha_n)} \sigma^2 \exp \left[-\frac{\alpha_n^2}{18} \right]$$

$$\approx \pi r_0^2 \frac{J_0(\alpha_n \frac{r}{n})}{J_1^2(\alpha_n)} \exp\left(-\frac{\alpha_n^2}{18}\right) \quad (\text{II-32})$$

where we have used the relations $J_0'(\alpha_n) = -J_1(\alpha_n)$ and $2\sigma^2 = r_0^2 = \frac{a^2}{4.5}$.

Due to the rapidly decaying nature of the Gaussian the approximation $a \approx \infty$ in the integration limit is quite reasonable.

The integration in t' is done by defining the integral

$$I_{\alpha m} = \beta_{\alpha m} \int_0^t \exp(\beta_{\alpha m} t') f(t') dt' \quad (\text{II-33})$$

where

$$\beta_{\alpha m} = \left(\frac{\alpha_n^2}{a^2} + \frac{(2m+1)^2 \pi^2}{l^2} \right) \kappa \quad (\text{II-34})$$

Since $f(t')$ only exists for $\kappa\tau \leq t \leq \kappa\tau + \tau_1$ the integral $I_{\alpha m}$ reduces to

$$\begin{aligned} I_{\alpha m} = & \beta_{\alpha m} \int_0^{\tau_1} \exp(\beta_{\alpha m} t') dt' + \int_{\tau}^{\tau+\tau_1} \beta_{\alpha m} \exp(\beta_{\alpha m} t') dt + \dots + \\ & + \beta_{\alpha m} \int_{(k-1)\tau}^{(k-1)\tau+\tau_1} \exp(\beta_{\alpha m} t') dt' \\ & + \int_{\kappa\tau}^{t \text{ or } \kappa\tau+\tau_1} \beta_{\alpha m} \exp(\beta_{\alpha m} t') dt' \end{aligned} \quad (\text{II-35})$$

The lower limits give the sum

$$-1 - \exp(\beta_{\alpha m} \tau) - \exp(2\beta_{\alpha m} \tau) - \dots - \exp(k\beta_{\alpha m} \tau) = \frac{\exp[(k+1)\beta_{\alpha m} \tau] - 1}{1 - \exp(\beta_{\alpha m} \tau)} \quad (\text{II-36})$$

while the upper limits give

$$\begin{aligned}
& \exp (\beta_{\alpha m} \tau_1) \left\{ 1 + \exp (\beta_{\alpha m} \tau) + \exp (\beta_{\alpha m}^2 \tau) + \dots + \exp [(k-1)\beta_{\alpha m} \tau] \right\} \\
& + \left\{ \frac{\exp (\beta_{\alpha m} \tau)}{\exp [\beta_{\alpha m} (k\tau + \tau_1)]} \right\} \quad (II-37) \\
& = \exp (\beta_{\alpha m} \tau_1) \left[\frac{\exp (k\beta_{\alpha m} \tau) - 1}{\exp (\beta_{\alpha m} \tau) - 1} \right] + \left\{ \frac{\exp (\beta_{\alpha m} \tau)}{\exp [\beta_{\alpha m} (k\tau + \tau_1)]} \right\}
\end{aligned}$$

Combining these results yields

$$\begin{aligned}
I_{\alpha m} &= \frac{1 - \exp [(k+1)\beta_{\alpha m} \tau] + \exp [\beta_{\alpha m} (k\tau + \tau_1)] - \exp (\beta_{\alpha m} \tau_1)}{\exp (\beta_{\alpha m} \tau) - 1} \\
& + \left\{ \frac{\exp (\beta_{\alpha m} \tau)}{\exp [\beta_{\alpha m} (k\tau + \tau_1)]} \right\} \quad (II-38)
\end{aligned}$$

where the two final terms refer respectively to times during and between pulses.

Putting together equations (II-29), (II-30), (II-32) and (II-38) gives as the entire solution

$$\begin{aligned}
T(r, z, t) - T_0 &= \frac{\kappa}{K} Q_0 \frac{2}{\pi a^2 \ell} \sum_m \left[\frac{2\ell}{(2m+1)\pi} \sin \left(\frac{(2m+1)\pi z}{\ell} \right) \right] \\
& \times \sum_{\alpha_n} \pi r_0^2 \exp \left(-\frac{\alpha_n^2}{18} \right) \frac{J_0(\alpha_n \frac{r}{a})}{J_1^2(\alpha_n)} \times \frac{1}{\kappa} \left(\frac{a^2 \ell^2}{\alpha_n^2 k^2 + (2m+1)^2 \pi^2 a^2} \right) \\
& \times \exp (-\beta_{\alpha m} t) \left\{ \frac{1 - \exp [(k+1)\beta_{\alpha m} \tau] + \exp [\beta_{\alpha m} (k\tau + \tau_1)] - \exp (\beta_{\alpha m} \tau_1)}{\exp (\beta_{\alpha m} \tau) - 1} \right. \\
& \quad \left. + \exp (\beta_{\alpha m} t) \right\} \quad (II-39a) \\
& \text{for } k\tau \leq t \leq k\tau + \tau_1
\end{aligned}$$

$$\begin{aligned}
T(r, z, t) - T_0 &= \frac{\kappa}{K} Q_0 \frac{2}{\pi a^2 \ell} \sum_m \frac{2\ell}{(2m+1)\pi} \sin \frac{(2m+1)\pi z}{\ell} \\
&\times \sum_{\alpha_n} \pi r_n^2 \exp \left(-\frac{\alpha_n^2}{18} \right) \frac{J_0(\alpha_n \frac{r}{a})}{J_1^2(\alpha_n)} \times \frac{1}{\kappa} \left(\frac{a^2 \ell^2}{\alpha_n^2 \ell^2 + (2m+1)^2 \pi^2 a^2} \right) \\
&\times \exp(-\beta_{\alpha m} t) \left\{ \frac{1 - \exp[(k+1)\beta_{\alpha m} \tau] + \exp[\beta_{\alpha m}(k\tau + \tau_1)] - \exp(\beta_{\alpha m} \tau_1)}{\exp(\beta_{\alpha m} \tau) - 1} \right. \\
&\quad \left. + \exp[\beta_{\alpha m}(k\tau + \tau_1)] \right\} \\
&\text{for } k\tau + \tau_1 \leq t \leq (k+1)\tau
\end{aligned} \tag{II-39b}$$

Rearranging the constants and returning to the $I_{\alpha m}$ notation gives

$$\begin{aligned}
T(r, z, t) &= T_0 + \\
&\frac{Q}{K} \frac{8}{9\pi} \sum_m \frac{\sin \left(\frac{(2m+1)\pi z}{\ell} \right)}{(2m+1)} \sum_{\alpha_n} \exp \left(-\frac{\alpha_n^2}{18} \right) \frac{J_0(\alpha_n \frac{r}{a})}{J_1^2(\alpha_n)} \frac{\kappa}{\beta_{\alpha m}} \exp(-\beta_{\alpha m} t) I_{\alpha m}
\end{aligned} \tag{II-39c}$$

Results

Two limiting cases of equation (II-39) are of particular interest. The sample may be constrained to return to its initial temperature T_0 throughout before the arrival of the next heating pulse, resulting in perfectly periodic and identical temperature oscillations (studied by setting $k = 0$). If all the heat deposited in the sample cannot be conducted away before the arrival of the next pulse on the other hand, the temperature will rise in a stepwise fashion until at some possibly much higher average temperature the conduction losses balance the irradiation heating, and the temperature oscillations are again periodic and identical (studied by setting $k = \infty$). This case is of interest since the final temperatures may be above the sample melting temperature. Only the first is considered here.

In a controlled-temperature experiment the maximum temperature rise (occurring at the end of each pulse $t = \tau_1$ and at the sample midpoint $r = 0$ and $z = \frac{\ell}{2}$) would be limited to a small fraction of the melting temperature, and the sample would ideally return to its initial state before the arrival of the next heating pulse. The behavior of the temperature during and immediately after the first ($k = 0$) pulse would then describe the behavior for all subsequent times. The temperature rise at the sample center ($r = 0$ and $z = \frac{\ell}{2}$) is given by

$$T(0, \frac{\ell}{2}, t) = T_0 + \frac{Q}{k} \frac{8}{9\pi} \sum_m \frac{(-1)^m}{(2m+1)} \sum_{\alpha_n} \frac{\exp\left(\frac{-\alpha_n^2}{18}\right)}{J_1^2(\alpha_n)} \left(\frac{a^2 \ell^2}{\alpha_n^2 \ell^2 + (2m+1)^2 \pi^2 a^2} \right) \\ \times [1 - \exp(-\beta_{\alpha_m} t)] \\ \text{for } 0 \leq t \leq \tau_1 \quad (\text{II-40a})$$

$$T(0, \frac{\ell}{2}, t) = T_0 + \frac{Q}{k} \frac{8}{9\pi} \sum_m \frac{(-1)^m}{(2m+1)} \sum_{\alpha_n} \frac{\exp\left(\frac{-\alpha_n^2}{18}\right)}{J_1^2(\alpha_n)} \left(\frac{a^2 \ell^2}{\alpha_n^2 \ell^2 + (2m+1)^2 \pi^2 a^2} \right) \\ \times \exp(-\beta_{\alpha_m} t) \left[\exp(\beta_{\alpha_m} \tau_1) - 1 \right] \\ \text{for } \tau_1 \leq t \leq \tau \quad (\text{II-40b})$$

The maximum centerline temperature rise (occurring at $t = \tau_1$) is shown for aluminum in figure A-2 and molybdenum in figure A-3, for various values of beam current, Gaussian width and sample thickness. The centerline temperature at the beginning of the next heating pulse ($t = \tau$) is shown in figures A-4 and A-5, for aluminum and molybdenum respectively.

The maximum temperature rise occurs at the center of the sample ($r = 0$ and $z = \frac{\ell}{2}$) and the temperature then falls off both radially and axially to the fixed surface temperature T_0 . The central temperature is shown in figure A-6 as a function of time for aluminum, using the parameters given in tables 1 and 2.

The radial profiles at $z = \frac{\ell}{2}$ and the axial profiles at $r = 0$ are shown in figures A-7 and A-8 for several different times. Figures A-9, and A-10 and A-11 give the same results for molybdenum, using the parameters given in tables 1 and 2.

The temperature solution developed is applicable to an arbitrary beam time structure (varying τ_1 and τ). The effect of changing τ_1 is shown in figures A-12 and A-13, where the centerline temperature rise in aluminum and molybdenum is shown as a function of time for various values of τ_1 , for the same parameters used above. The maximum temperature rise (given in table 3), occurring at $t = \tau_1$, increases as the heating pulse becomes shorter and more intense (the net amount of heat deposited is held fixed). A limiting value of this maximum temperature rise is reached when essentially none of the heat can be conducted away during the pulse itself, and is given by

$$\lim_{\frac{\tau_1}{\tau} \rightarrow 0} \Delta T_{\max} = \frac{\kappa}{k} c_Q \frac{n_p}{\pi r_0^2} \quad (\text{II-41})$$

The temperature rise at $t = \tau_1 = 10^{-3} \tau$ has indeed reached this limiting value.

TABLE 1
Assumptions and Constants

Dislocation Density	ρ_d^0	10^{11}	m/m^3
Void Number Density	N_v	10^{20}	m^{-3}
Interstitial Loop Number Density	N_{il}	10^{21}	m^{-3}
Initial Vacancy Loop Number Density	$N_{vl}(0)$	0	m^{-3}
Initial Void Radius	$r_v(0)$	10^{-8}	m
Initial Interstitial Loop Radius	$r_{il}(0)$	$2.4 \cdot 10^{-8}$	m
Vacancy Loop Formation Radius	$r_{vl}(0)$	$1.5 \cdot 10^{-9}$	m
Gas Atoms per Void	n_g	10	-
Van der Waal's Coefficient	b_v	$8.3 \cdot 10^{-29}$	m^3
Boltzmann's Constant	b	$1.38 \cdot 10^{-23}$	$J/^{\circ}K$
Vacancy Frequency of Motion	v_v	10^{13}	s^{-1}
Interstitial Frequency of Motion	v_i	10^{13}	s^{-1}
Combinatory Number	g	1	-

TABLE 2
Material Properties

		<u>Aluminum</u>	<u>Molybdenum</u>	<u>Units</u>
Burgers vector	b	$2.32 \cdot 10^{-10}$	$2.73 \cdot 10^{-10}$	m
Vacancy formation energy	E_f^v	$1.06 \cdot 10^{-19}$	$4.8 \cdot 10^{-19}$	J
Interstitial formation energy	E_f^i	$5.19 \cdot 10^{-19}$	$1.44 \cdot 10^{-18}$	J
Vacancy migration energy	E_m^v	$9.12 \cdot 10^{-20}$	$2.45 \cdot 10^{-19}$	J
Interstitial migration energy	E_m^i	$1.6 \cdot 10^{-20}$	$1.36 \cdot 10^{-20}$	J
Vacancy diffusion pre-exponential	D_v^0	$4.0 \cdot 10^{-6}$	$3.0 \cdot 10^{-6}$	m^2/s
Interstitial diffusion pre-exponential	D_i^0	$8.0 \cdot 10^{-6}$	$1.6 \cdot 10^{-5}$	m^2/s
Shear modulus	μ	$2.43 \cdot 10^{10}$	$1.15 \cdot 10^{11}$	Pa
Poisson's ratio	ν	0.347	0.32	-
Stacking fault energy	γ_{sf}	0.28	0.574	J/m^2
Surface energy	γ	1.0	2.05	J/m^2
Cascade efficiency	ϵ	0.001	0.001	-
Vacancy dislocation bias factor	Z_v	1.0	1.0	-
Interstitial dislocation bias factor	Z_i	1.08	1.08	-
Thermal conductivity	K	222.0	142.0	$J/ms^{\circ}K$
Thermal diffusivity	κ	$9.13 \cdot 10^{-5}$	$5.04 \cdot 10^{-5}$	m^2/s
Damage energy cross section	σ_D	63.0	731.0	b-keV
Threshold energy	E_D	17.0	40.0	eV
800 MeV proton energy loss	E^*	$8.0 \cdot 10^{-11}$	$2.47 \cdot 10^{-10}$	J/mp
Current	I	1	0.5	mA
Gaussian size	r_0	$3.1 \cdot 10^{-3}$	$1.18 \cdot 10^{-3}$	m
Sample thickness	ℓ	$2.5 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	m

TABLE 3

Radiation-produced Temperature Rises

Aluminum:	$f(\%)$	$\Delta T_{\max}$ (K)
	100	0.58
	50	1.17
	25	2.33
	10	5.84
	6	9.72
	1	40.28
	0.1	56.94
Molybdenum:	$f(\%)$	$\Delta T_{\max}$ (K)
	100	9.70
	50	19.39
	25	38.78
	10	96.83
	6	158.58
	1	457.12
	0.1	526.71

Figure 1. Time structure of LAMPF 800 MeV proton beam.

Figure 3. Vacancy and interstitial concentrations during first pulse and annealing period. For each set of curves, the upper curve corresponds to the center ($r = 0$ of Fig. A1) of the beam and the lower curve to the edge ($r = 3\sigma$ of Fig. A1) of the beam. The intermediate curves correspond to equispaced positions from center to edge of the beam.

Figure 4. Vacancy concentration initial transient and resulting void shrinkage at early time.

Figure 5. Vacancy loop size and number density transient.

Figure 6. Void growth for various irradiations in aluminum at an ambient temperature of 400 K.

Figure 7. Long-term void growth in aluminum at 400 K.

Figure 8. Void growth rate in 400 K (ambient) aluminum as a function of pulse structure for various assumptions.

Figure 9- Void growth as a function of ambient temperature during steady irradiation of aluminum.

Figure 10. Void growth for various irradiations in aluminum at an ambient temperature of 325 K.

Figure 11. Void growth as a function of ambient temperature during steady irradiation of molybdenum.

Figure 12. Void growth for various irradiations in molybdenum at an ambient temperature of 1200 K.

Figure 13. Void growth for various irradiations in molybdenum at an ambient temperature of 1600 K.

Figure 14. Total sink strength for various irradiations in molybdenum at an ambient temperature of 1600 K.

Figure A-1, Gaussian beam structure and target geometry,

Figure A-2. Temperature rise at the end of a radiation pulse in aluminum.

Figure A-3. Temperature rise at the end of a radiation pulse in molybdenum.

Figure A-4. Temperature rise at the end of an annealing period in aluminum.

Figure A-5. Temperature rise at the end of an annealing period in molybdenum.

Figure A-6. Temperature response of an aluminum target during the first pulse and annealing period.

Figure A-7. Radial temperature profiles in aluminum at various times.

Figure A-8. Axial temperature profiles in aluminum at various times.

Figure A-9. Temperature response of a molybdenum target during the first pulse and annealing period.

Figure A-10. Radial temperature profiles in molybdenum at various times.

Figure A-11. Axial temperature profiles in molybdenum at various times.

Figure A-12. Temperature response of aluminum to various pulse structures.

Figure A-13 . Temperature response of molybdenum to various pulse structures.

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