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Row Charge Cratering Calculations

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by

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ABSTRACT

Two-dimensional calculations have been done to determine the feasibility of constructing deep canals with nuclear explosives subject to the limitation in the proposed PNE Treaty. The conditions under which a series of explosives set in a row can be approximated by a cylindrical line source have been determined.

Using this result, the possibility of lifting 250 m of overburden with 150-kt charges spaced at 50-m intervals has been investigated. This study shows that for a variety of equations of state for the geological medium, there appears little possibility that such an excavation can be accomplished.

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I. INTRODUCTION

This report discusses a computer study of the impact of the limitations of the proposed PNE treaty on the feasibility of using nuclear explosives spaced in a row to excavate large canals such as the Pechora-Kama project in Russia or the Qattara sea-level canal in Egypt. For part of their length, such canals would have a depth of approximately 250 m. Proposed PNE treaty obligations would limit individual nuclear charges to not more than 150-kt yield and the spacing between charges to not less than 10 m multiplied by the cube root of the yield in kt. The point of this investigation is to examine the feasibility of constructing such a canal with these limits on the nuclear row charges.

Craters produced by rows of charges detonated simultaneously are generally deeper and wider than craters produced by single charges of the same explosive yield. This effect is called enhancement. Toman¹ and Redpath² have obtained semi-empirical relations between enhancement and

charge spacing and depth of burst for the case where the spacing is not greatly different from the depth of burst or from the crater radius produced by a single charge at optimum depth. Enhancement tends to increase as spacing decreases. These enhancement expressions have been compared by Branagan,³ who shows that they agree within the range of spacing indicated above, but predict quite different results where extrapolated into the region of close spacing and large enhancement.

For the problem at hand, it was assumed that charges of maximum permissible yield, 150 kt, with minimum permissible spacing for this yield, 53 m, would be used. The depth of burst would be ~ 300 m for a canal ~ 250 m deep if the shot points are not to be exposed. Thus, in this case the spacing between charges is a much smaller fraction of the depth of burst than it was in the cases studied by Toman or Redpath.

In the limiting case of zero spacing between charges of zero yield but with a fixed ratio of yield to spacing, the row charge becomes a line charge. In

the case of discrete charges of yield W , spacing S , and depth of burst D , if D is much greater than S the shock resulting from the explosion can be expected to approach that due to a line charge of yield W/S kilotons per meter well before reaching the ground surface everywhere except near the individual charges. The shock, as well as the resulting crater, then depends on only two parameters, W/S and D , for a given geological medium, throughout most of the region influenced by the explosion.

This tendency of a row charge shock to approach a line charge shock was used as the basis of an approximation to enable the row charge cratering process to be studied with two-dimensional computer codes. The investigation was done in two phases. In the first phase, which is discussed in Sec. II below, the problem of an infinitely long row charge with infinite depth of burst was run and compared with the problem of an infinite line charge with the same yield per unit length as the row charge. These problems were run until the row charge shock closely approximated the line charge shock throughout most of the volume influenced by the explosion. It was found that the line charge problem becomes an approximation to the row charge problem when the shock radius is greater than 2 to 6 times the row charge spacing. The greater the shock radius, the better the approximation.

It will be noted that when the spacing is close enough for a row charge to be well approximated by a line charge, the enhancement expressions given by Toman¹ and Redpath² are no longer valid. In this limit, the same row crater is produced by charges of yield W and spacing S as, for example, by charges of yield $1/2 \cdot W$ and spacing $1/2 \cdot S$ if the depth of burial is unchanged. This is not in agreement with the expressions of Toman and Redpath.

The second phase of this investigation, which is discussed in Sec. III, is an application of these ideas to the computation of the early stages of row crater formation in geological materials represented by a variety of different equations of state in order to estimate the effectiveness of nuclear explosives, as restricted by treaty limitations, in canal construction.

The row charges were approximated by line charges ~ 300 m below the surface of the ground. The problems were run to a time well past the initial interaction of the shock with the ground surface but

not to the time of complete crater formation. There was no gravity in these problems, and there was no consideration of limiting angle of repose or any other such effects as would determine the final ejecta distribution and crater shape because only the initial stages of crater formation are treated.

In these calculations, the likelihood of crater formation was judged by the pressure and velocity distribution in the ground between the shot point and the ground surface. The different equations of state used resulted in large variations in these quantities. For most of the material properties studied, the resulting pressure and velocity distributions were such that formation of the desired crater would be improbable or impossible.

II. COMPARISON OF ROW CHARGE AND LINE CHARGE SHOCKS

To compare the row charge and line charge shocks it is not necessary to calculate the interaction of the shock with the ground surface if the depth of burst is great enough. Thus, the charges were assumed to have infinite depth of burst for this phase of the investigation. The row charge was taken to have infinite length and thereby the problem was made periodic in space along the axis of the row; only one spatial period was calculated. The individual charge yield was 150 kt and the spacing was 53 m; thus the problem done was that of a single 150-kt charge midway between reflecting planes 53 m apart.

The calculation was done in cylindrical coordinates with the Z -axis along the axis of the row. The charge was located at $Z = 26.5$ m with reflecting planes at $Z = 0$ and $Z = 53$ m. The initial zone size was 0.25 m within 4.25 m of the device, changing gradually to 0.5 m beyond 7.5 m from the device. At a time when the shock radius was greater than 7.5 m, the problem was rezoned to 0.5-m zones everywhere. As the shock radius increased, it was necessary to repeatedly rezone the problem in order to keep the total number of zones within limits set by computer storage. Zone size can be judged by the number of plotted points shown on the computer plots below.

For the line charge problem, the yield was 150/53 = 2.83 kt/m in a cylinder along the Z axis. This is a

one-dimensional problem; the shock is computed in a slice of material one zone thick which perpendicularly intersects the Z-axis.

Each of these problems is done in wet tuff as represented by the Tillotson equation of state with the addition of "strength," i.e., deviatoric stress, with a Von Mises yield condition. The equations of state used in these calculations are discussed in detail in the Appendix. The computations were done with W. Johnson's two-dimensional Eulerian hydrodynamics code SOIL.⁴

Figures 1-11 show the evolution of the explosion up to the time when the shock has reached a radius ~ 275 m. In Figs. 4-11, the results of the equivalent line charge are also shown for comparison.

Figures 1 and 2 indicate the positions of the direct and reflected shock and the cavity for the row charge problem at times of 15 ms and 25 ms, respectively. There is little increase in cavity size between these times. The plotted points indicate the positions where the pressure as a function of one of the variables r or z has a maximum, i.e., these points show the positions of ridges in the pressure. These ridges are assumed to be manifestations of direct or reflected shocks. The plotted shock positions in Figs. 1 and 2 can be seen to coincide with pressure peaks shown in Figs. 3 and 4, respectively.

From Figs. 1 and 3, it can be seen that at 15 ms, when the shock has reached a radius ~ 1.25 times the charge spacing, the pressure behind the shock varies strongly with Z even though the radius of the shock doesn't vary strongly with Z . In Fig. 1, in addition to the direct and reflected shocks, there are Mach stems near the reflecting planes at $R \sim 64$ m. At this time, the shock structure from the row charge is rather complex outside the cavity and not similar to that from a line charge. Inside the cavity, the pressure is fairly uniform at about 6 kbar.

Figure 2 shows that at 25 ms the Mach stems from the two reflecting boundaries have met each other and have apparently overtaken the direct shock midway between the boundaries. From Figs. 4-7, it can be seen that the distributions of pressure, density, and velocity near the shock front are becoming less Z -dependent at this time. Within this region the distributions of these quantities are becoming

similar to those for a line charge. The row charge shock radius ~ 92 m is nearly independent of Z and close to the line charge shock radius. The specific internal energy shown in Fig. 7 is large only near the original shot points and varies strongly in the region near $R = 0$. The cavity pressure can be seen from Fig. 4 to have fallen to less than half its value at 15 ms even though the cavity has not increased much in size.

The pressure near the reflecting planes at small R has fallen to zero. This is due to the low specific internal energy and low density in this region. The material has rebounded from the reflecting plane here. Other equations of state give low but finite pressure in this region. The pressure at the shock front is ~ 25 to 30 kbar. Secondary shocks behind the shock front now have somewhat lower pressures.

Figures 8-11 show the row charge and line charge shocks at 95 ms at which time it can be seen that there is very little difference except very near the reflecting planes at small R . The noise that occurs near 120 m is probably due to the transition in the Tillotson EOS between the solid and vapor phases. The pressure in the cavity is less than 0.1 kbar for these results obtained with the Tillotson EOS. Other equations of state gave pressures both in the cavity and in the solid material outside the cavity that varied considerably from those shown in Fig. 8. This is discussed further below. The small step in pressure just ahead of the shock is an elastic precursor. It is not known why the elastic precursor is more pronounced in the line charge problem than in the row charge problem. Except for the elastic precursor, the row charge and line charge shocks have reached the same radius at 95 ms. At this time, the largest difference between these shocks is in the specific internal energy, which still varies strongly with Z for small R in the row charge case. However, throughout most of the volume influenced by the explosion, it can be seen that at this time there is little difference between the row charge problem and the line charge problem. From this, it was concluded that in the case of close charge spacing, the row charge cratering problem could be well approximated by a line charge problem.

Fig. 1.

One spatial period of the row charge problem plotted in cylindrical coordinates. The Z axis is along the axis of the row with a shot point at $Z = 26.5$ m. The reflecting boundaries are at $Z = 0$ and $Z = 53$ m. The plotted points show the positions of the direct and reflected shocks. The cross-hatched area is the cavity.

Fig. 2.

The same as Fig. 1 but for a time of 25 ms.

Fig. 3.
Plots of pressure and radial velocity for the row charge problem at 15 ms.

Fig. 4.

Pressure plots for three positions along the row charge axis compared with a pressure plot for the line charge at 25 ms.

Fig. 5.
Density plots for three positions along the row charge axis compared with a density plot for the line charge at 25 ms.

Fig. 6.
Radial velocity plots for three positions along the row charge axis compared with a radial velocity plot for the line charge at 25 ms.

Fig. 7.

Specific internal energy plots for three positions along the row charge axis compared with a specific internal energy plot for the line charge at 25 ms.

Fig. 8.
Pressure plots for three positions along the row charge axis compared with a pressure plot for the line charge at 95 ms.

Fig. 9.

Density plots for three positions along the row charge axis compared with a density plot for the line charge at 95 ms.

Fig. 10.
Radial velocity plots for three positions along the row charge axis compared with a radial velocity plot for the line charge at 95 ms.

Fig. 11.
Specific internal energy plots for three positions along the row charge axis compared with a specific internal energy plot for the line charge at 95 ms.

III. ESTIMATES OF CRATERING POSSIBILITIES

The next phase of this investigation was a computer study of the explosion of a line charge at a uniform depth underground where the resulting shock was allowed to penetrate the surface of the ground. This was done in a two-dimensional Cartesian coordinate system with the Y-axis vertical and both the X- and Y-coordinate axes perpendicular to the axis of the line charge. The line charge intersected the vertical coordinate axis. Several problems of this type were run to study the effects of variations in material properties and to determine if different computer codes gave similar results. These problems were not run to the point where crater formation was complete, but only far enough to determine whether the velocity distribution in the material directly above the line charge was such that the material could be thrown high enough for crater formation to be possible.

The first problem used the SOIL code and the same Tillotson equation of state used in the investigation of the equivalence of the row charge and line charge shocks, with the exception that strength was turned off so the problem was one of pure hydrodynamics. This had the effect of reducing the shock speed as well as changing the velocity and pressure distributions from those obtained above. However, these changes were quantitative rather than qualitative. For this problem, the shock propagated a distance of 214 m in 100 ms as compared to 275 m in 95 ms for the case with strength. This represents $\sim 35\%$ difference in shock speed averaged over about the first 100 ms. For these problems the shock speed is not greatly different from the sound speed after ~ 10 ms. Figure 12 shows the vertical component of material velocity and pressure along the vertical axis at a time of ~ 180 ms. In this problem the bomb position was at 400 m and the ground surface at 712 m. The material near 600 m does not have a sufficient vertical velocity component to reach the height of the ground surface without additional acceleration if the sum of its kinetic plus gravitational energy is conserved. From the pressure plot, it appears unlikely that this material will receive much additional acceleration and it was therefore concluded that the desired canal probably could not be produced by this means in the material represented by this equation of state.

Figure 13 shows the vertical velocity component and the pressure along the vertical axis at ~ 180 ms for a problem the same as above, except for the use of an equation of state obtained from LASL Group J-9.⁶ The J-9 EOS in its form with strength but no void crushing represents water-saturated tuff. Again, the strength was turned off. In this problem, the bomb is at 2000 m and the ground surface at 2312 m. In this case, all the material above ~ 2070 m has sufficient velocity to be ejected from the crater. The velocity distribution of the material outside the cavity is distinctly different from that obtained in the previous problem. This is due to the difference in the equations of state used in these two problems in the region of greater than normal density. These equations of state also differ in that the J-9 equation gives a finite pressure for low values of density and specific internal energy where the Tillotson EOS indicates the material has condensed and the pressure has fallen to zero. With the J-9 EOS, expanding cavity gas can contribute more to the acceleration of the material. To investigate this effect, this problem was run to ~ 553 ms.

Some results are shown in Figs. 14-16 that are plots in which the density of dots is proportional to the square root of the pressure for times of 180 ms, 260 ms, and 550 ms. These show the shock reaching the surface of the ground and a rarefaction wave propagating back down through the bomb position. Behind the rarefaction the pressure in the solid material falls to zero. In the cavity the pressure remains finite because here the material is a gas. The cavity pressure then causes a pressure wave to propagate again outward into the surrounding solid material. This is indicated by the gray central area of Fig. 16.

Figures 17 and 18 show the velocity and pressure at 260 ms and 553 ms. These indicate that after the rarefaction from the surface passes downward through the material, the material receives an upward acceleration. In Fig. 18, it can be seen that even the cavity gas near 2000 m has received an upward acceleration. The secondary pressure wave due to cavity gas pressure can be seen in Figs. 17 and 18. In Fig. 18, this pressure wave is indicated by the velocity spike near 2120 m. The pressure in this wave at ~ 550 ms is ~ 0.1 kbar, very small compared to the pressure in the initial shock, and at this time it has affected only a small fraction of the material through which the initial shock has passed.

Fig. 12.

Vertical velocity and pressure at 180 ms obtained with the Tillotson wet-tuff equation of state without strength.

Fig. 13.

Vertical velocity and pressure at 180 ms obtained with the J-9 water-saturated-tuff equation of state without strength.

Fig. 14.

The shock at 180 ms. The density of dots is proportional to the square root of the pressure. The dashed line indicates the original position of the ground surface.

Fig. 16.

The shock at 553 ms. The density of dots is proportional to the square root of the pressure. The dashed line indicates the original position of the ground surface.

Fig. 15.

The shock at 260 ms. The density of dots is proportional to the square root of the pressure. The dashed line indicates the original position of the ground surface.

The cavity expansion nearly stops at a time that is small compared to the time for the shock to reach the surface of the ground; therefore the cavity volume is very small compared to the volume through which the shock has propagated. Because of the low cavity pressure and relatively small volume, it appears that acceleration by cavity gas can be of very little significance. The average velocity of the material above the bomb has not changed by more than a few tens of meters per second in the time interval from 180 ms to 550 ms.

In order to be certain that these results were not influenced by some peculiarity of the SOIL code, this problem was also run to a time ~ 180 ms, using the Lagrangian code TOODY of J-9. The results were the same as those obtained above with the SOIL code. In addition, TOODY was used to investigate the effects of strength and void crushing. Figures 19-21 show velocity distributions in the material above the bomb at 180 ms for three different cases. For these problems, the ground surface was 310 m above the bomb. Comparison of these figures with each other and with Fig. 13, for which the scalar pressure part of the EOS was the same, shows that the effects of strength and void crushing can be large.

Fig. 17.
Vertical velocity and pressure at 260 ms obtained using the J-9 EOS without strength.

Fig. 18.
The same as Fig. 17 but for a time of 553 ms.

Fig. 19

Velocity distribution at 180 ms for the case with strength but no void crushing TOODY computation

Fig. 21

Velocity distribution at 180 ms for the case with strength and 25 per cent void crushing TOODY computation

Fig. 20

Velocity distribution at 180 ms for the case with strength and 5 per cent void crushing TOODY computation

The velocity in the region of the minimum 100 to 200 meters above the shot point is 40 to 45 m/s in Figs. 19 and 20, compared to 30 m/s in Fig. 13. In Fig. 21 the velocity is not only lower than that in Fig. 13 but its distribution is qualitatively different.

These results indicate that it is impossible to compute the velocity distribution between the shot points and the surface of the ground without knowledge of the deviatoric components of the stress tensor and their yield condition as well as knowledge of the percentage of void in the material. The yielding of the material and the crushing of voids are each mechanisms for dissipation of mechanical energy, i.e., they both lead to nonisentropic motion. Thus, it is to be expected that yielding and void crushing will result in lower values of material velocity.

IV. CONCLUSION AND DISCUSSION

If one argues that in order to dig a canal of a certain depth, the material above the charge must acquire an upward velocity at least sufficient to cause

it to rise against the force of gravity a distance equal to twice the canal depth above its initial position, a 250 m deep canal would require an initial upward velocity ≥ 100 m/s. The velocity distribution in Fig. 17 is the only one found in this investigation that satisfies this condition. This velocity distribution was based on a pure hydrodynamics (i.e., no void crushing, no strength) calculation. Such a model is unrealistic for a problem of this sort. This is clear from the fact that the average shock velocity over the first 100 ms is 36% different for cases with and without strength. The shear modulus of the material and the yield condition associated with it can be expected to play an important role in the behavior of the shock for almost the entire time of the cratering process. The problems run with void crushing indicate that energy dissipative effects of this sort can also be important. Thus, of the models investigated here, none of the ones that could be realistic enough to be meaningful would permit cratering according to the above criterion.

At this point in this study, no effort has been made to determine the accuracy with which the various equations of state used describe the relevant properties of the geological material of interest here. Such an investigation is, of course, necessary to determine a definitive answer to the cratering question.

The excavation discussed here is of much greater depth than any nuclear excavation that has been done in the past by the U.S. It appears very unlikely that any nuclear excavation program of this magnitude will be undertaken by the U.S. in the foreseeable future. For this reason, existing experimental data can be used to predict results of this excavation only with a very large and very unreliable extrapolation.

Hydrodynamic computer codes have been used extensively to calculate the effects of underground nuclear explosions under conditions where the pressure is so high that geological materials behave in a way that is indeed well approximated by hydrodynamics. However, for almost the entire time required for a crater to be formed by an underground nuclear explosion, the pressure is so low that the difference between the material behavior and hydrodynamics is definitely not a small perturbation. If the pressure is less than a few tens of kilobars, the equation of state of a geological

material must determine not only the scalar pressure but also the deviatoric part of the stress tensor which gives the shear stresses in the material. It must also specify a yield condition for the deviatoric stresses and provide for other dissipative processes such as crushing of voids in porous material. The parameters that describe these effects vary widely in different geological materials and are generally poorly known.

In spite of these complexities, the above mentioned computer codes have also been used with appropriate equations of state to calculate the late time effects of underground nuclear explosions where the departure from pure hydrodynamic behavior is large. Successful predictions of cratering and containment effects at NTS have been made under conditions that were not too far removed from past experience. However, it would be very unwise to assume that comparable success can be achieved in predicting cratering effects in an area of the world and in a yield range where there is no experimental data on the effects of underground nuclear explosions.

For the calculations described here, it seemed most useful to investigate a variety of different equations of state in order to determine the effects of variations in some of the poorly known parameters. By this means, it is possible to estimate the extent to which the uncertainties in material properties lead to uncertainties in the results of cratering calculations. Unfortunately, these uncertainties are so large that only rather loose bounds on the cratering effectiveness of the nuclear charges in the atom considered can be given.

The investigation of the line charge as an approximation to a row charge has yielded encouraging results that should be useful in future problems of this sort. For example, the results of this work may be related to the empirical cratering algorithms mentioned in Sec. 1. Figures 4-7 indicate that a row charge may be at least roughly approximated by a line charge if the depth of burst is $\geq$ twice the spacing. If the depth of burst is 300 m, the effect of a row of 150 kt charges spaced 53 m should not differ significantly from a row of 300 kt charges spaced at 100 m since both W/D and D are unchanged and D is still greater than $2 \cdot H$. Then

$$W/D = 100/300 = 0.33$$

This is in the range where the enhancement expressions of Toman¹ and Redpath² may be valid. With this value of S/D and $W = 300$ kt, Toman's expression

$$S/D = 1.2 (W/W_{ap})^{0.83}$$

gives

$$W_{ap} \cong 3000 \text{ kt},$$

while Redpath's expression as rewritten by Branagan³

$$S/D = 1.2 (W/W_{ap})^{0.86},$$

gives

$$W_{ap} \cong 1200 \text{ kt}.$$

If the depth d of a crater is given in meters⁴ by

$$d = 27.4 W_{ap}^{1/3.4},$$

where W_{ap} is in kt, then a 250-m-deep crater requires

$$W_{ap} = 1800 \text{ kt}.$$

Since this row charge would probably not produce a 250-m-deep crater for most of the models investigated, the W_{ap} for the row charge must be somewhat less than this. The results obtained here thus seem more consistent with the theory of Redpath than with that of Toman.

APPENDIX

COMPARISON OF THE EQUATIONS OF STATE

In these calculations, geological material properties were specified in terms of an elastic-plastic model similar to that described by Wilkins.⁷ In this model, the symmetric stress tensor is written in the form

$$\sigma_{ik} = S_{ik} - \delta_{ik} P, \quad (A-1)$$

where the subscripts i, k refer to coordinate axes in a Cartesian coordinate system. The components S_{ik} form the deviatoric part of the stress tensor and P represents the hydrostatic pressure; P is the negative mean of the three diagonal components of σ_{ik} .

$$P = -\frac{1}{3} \sum_i \sigma_{ii}. \quad (A-2)$$

Consequently, the trace of the stress deviator is zero,

$$\sum_i S_{ii} = 0. \quad (A-3)$$

The pressure P is regarded as a function of density and specific internal energy. (In this Appendix, only models with no void crushing will be discussed.)

The stress deviators were calculated at each time step with the equation

$$S_{ik}(t + \delta t) = S_{ik}(t) + 2\mu \int_t^{t+\delta t} \dot{\epsilon}_{ik} dt. \quad (A-4)$$

Here μ is the shear modulus of the material and the $\dot{\epsilon}_{ik}$ are strain rate deviators.

$$\dot{\epsilon}_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) - \frac{1}{3} \delta_{ik} \nabla \cdot \vec{u},$$

where $\vec{u}$ is the material velocity. The stress deviators were required to satisfy a von Mises yield condition. In the SOIL code, the yield condition was written

$$\sum_{ik} S_{ik}^2 \leq 2 y^2 .$$

If the S_{ik} as calculated at some time step with Eq. (A-4) did not satisfy this condition, each S_{ik} was multiplied by

$$\sqrt{2} y \left(\sum_{ik} S_{ik}^2 \right)^{-1/2}$$

so that the resulting new S_{ik} would satisfy Eq. (A-5).

The quantity y was assumed to be a function of the specific internal energy of the material.

$$y = y_0 \left(1 - \frac{E}{E_m} \right) . \quad (A-6)$$

Thus, in SOIL the stress deviators were determined by the three constants

$$\mu = 1.25 \times 10^{11} \text{ dyn/cm}^2$$

$$y_0 = 0.5 \text{ kbar} = 5.0 \times 10^8 \text{ dyn/cm}^2$$

$$E_m = 3.2 \times 10^{10} \text{ erg/g} .$$

In TOODY, the yield condition was written such that the S_{ik} must satisfy equations equivalent to Eqs. (A-4)-(A-6) but with

$$\mu = 2.73 \times 10^{10} \text{ dyn/cm}^2 ,$$

$$y_0 = \frac{1 \text{ kbar}}{\sqrt{3}} = 5.77 \times 10^8 \text{ dyn/cm}^2 ,$$

and

$$E_m = 2.511 \times 10^{10} \text{ erg/g} .$$

Thus, the main difference between the stress deviators calculated by the two codes is due to the much larger shear modulus used in the SOIL calculation.

The scalar pressure was calculated with either the Tillotson⁸ equation of state or the J-9 equation of state obtained from Fred App.⁵

For $E < E_s$ or $\rho > \rho_0$, the Tillotson equation of state is

$$(A-5) \quad P_s = \left[a + \frac{b}{\frac{E}{E_0 \xi^2} + 1} \right] E \rho + A(\xi - 1) + B(\xi - 1)^2 ,$$

where

$$\xi = \rho/\rho_0 .$$

For states with $\xi < 1$ and $E > E_s'$, the pressure is

$$P_g = a E \rho + \left[\frac{b E \rho}{\frac{E}{E_0 \xi^2} + 1} + A(\xi - 1) e^{-\beta \left(\frac{\rho_0}{\rho} - 1 \right)} \right] \times e^{-\alpha \left(\frac{\rho_0}{\rho} - 1 \right)^2} .$$

When $\rho < \rho_0$, a smooth transition between these forms is achieved with the formula

$$P_T = \frac{(E - E_s) P_g + (E_s' - E) P_s}{E_s' - E_s} ,$$

where

$$E_s < E < E_s' .$$

The constants used, representing wet tuff, were

$$\rho_0 = 1.97 \text{ g/cm}^3 \quad E_s = 3.2 \times 10^{10} \text{ erg/g}$$

$$a = 0.5 \quad E_s' = 1.6 \times 10^{11} \text{ erg/g}$$

$$E_0 = 1.1 \times 10^{11} \text{ erg/g} \quad \alpha = 5.0$$

$$b = 1.3 \quad \beta = 5.0$$

$$A = 1.0 \times 10^{11} \text{ dyn/cm}^2 \quad B = 6.0 \times 10^{10} \text{ dyn/cm}^2 .$$

Here A represents the bulk modulus. The bulk sound speed $c = \sqrt{A/\rho_0}$ is $2.25 \times 10^5 \text{ cm/s}$. The specific internal energy required for vaporization is

represented by E_0 , and E_s represents the specific internal energy of the unvaporized material at a temperature equal to the vaporization temperature. If $E < E_s$ and $\rho < \rho_0$ then with the above constants $E_p < A$, and as the density or specific internal energy decreases further the dominant term in the pressure will become $A(\xi - 1)$, which is negative for these conditions. This represents condensation. Negative pressures computed in the program are set equal to zero.

The J-9 EOS for material with $\rho > \rho_0$ is

$$P = P_H + \Gamma_0 \rho_0 (E - E_H) ,$$

with

$$\eta = 1 - \rho_0/\rho$$

$$P_H = \kappa_0 \eta (1 + \kappa_1 \eta + \kappa_2 \eta^2 + \kappa_3 \eta^3)$$

$$E_H = P_H \eta / 2\rho_0$$

and for $\rho < \rho_0$

$$P = \rho \left\{ H + (\Gamma_0 - H) \sqrt{\frac{\rho}{\rho_0}} \right\}$$

$$\times \left\{ E - E_s \left[1 - \exp(N\eta\rho_0/\rho) \right] \right\} .$$

The constants used are

$$\rho_0 = 1.88 \text{ g/cm}^3$$

$$\Gamma_0 = 0.5$$

$$\kappa_0 = 7.1318 \times 10^{10} \text{ dyn/cm}^2$$

$$\kappa_1 = -2.1934$$

$$\kappa_2 = 101.46$$

$$\kappa_3 = -111.82$$

$$H = \gamma - 1 = 0.25$$

$$E_s = 1.0 \times 10^6 \text{ erg/g}$$

$$c_0 = \sqrt{\frac{\kappa_0}{\rho_0}} = 1.9477 \times 10^5 \text{ cm/s}$$

$$N = \frac{c_0^2}{\Gamma_0 E_s} = 75.8707 .$$

Here the bulk modulus κ_0 and bulk sound speed c_0 are somewhat smaller than those of the Tillotson equation. The function P_H is a fit to the observed Hugoniot for water-saturated tuff at a certain location in the NTS. The vaporization energy is represented by E_s , which is much smaller than the vaporization energy used with the Tillotson expression. The consequences of this are best seen by comparing the plots of the Tillotson and J-9 equations of state shown in Figs. A-1 and A-2.

These show that for densities between 10^{-2} g/cm³ and normal density, the Tillotson equation predicts condensation, i.e., a rapid decrease in pressure when E drops below the E_s value used in that equation of 3.2×10^{10} erg/g. The J-9 equation shows no indication of condensation in this region. Over most of the region of these plots outside of the condensation region, the J-9 equation gives, for the same pressure and density, a somewhat higher internal energy than the Tillotson equation, usually by less than a factor of 2.

Fig. A-1.
Tillotson equation of state.

Fig. A-2.
J-9 equation of state.

Butkovich⁹ has calculated equations of state for the mixture SiO_2 plus 1 to 20% water. His results indicate that at pressures above ~ 10 kbar and densities less than $\sim 1 \text{ g/cm}^3$, up to 20% water makes no significant difference. At lower pressures and at specific internal energies less than a few times 10^{11} erg/g, water content makes a significant difference. The more water, the lower the specific internal energy is for a given density and pressure. The reason for this is that SiO_2 condenses at these low temperatures so the pressure is due to water vapor.

Outside the rock condensation region, the J-9 equation of state agrees fairly well with Butkovich's results. Within the condensation region, the Butkovich results for any water content from 1% to 20% tend to lie between the Tillotson and J-9 curves. The Tillotson equation seems to ignore the effect of water vapor while the J-9 equation seems to exaggerate its effect. This is largely due to the values of the constants E_0 used in these equations.

The comparison of the Tillotson and J-9 equations of state with the Butkovich results indicates that the equations of state used in this calculation

are comparable to those used elsewhere to represent geological materials. Without appropriate measurements at the sites in question, little more than this can be said about how well these materials are represented.

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