DOE/MC/24207--2995

DE91 002076

NOV 1 1991

Development of Algorithms for Capacitance Imaging Techniques for Fluidized Bed Flow Fields

1990 Annual Report

W.J. Loudin

January 1991

Work Performed Under Contract No.: DE-FC21-87MC24207

**For
U.S. Department of Energy
Office of Fossil Energy
Morgantown Energy Technology Center
Morgantown, West Virginia**

**By
West Virginia University
Department of Mathematics
Morgantown, West Virginia**

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1. EXECUTIVE SUMMARY

The objective of this research is to provide support for the instrumentation concept of a High Resolution Capacitance Imaging System (HRCIS). The work involves the development and evaluation of the mathematical theory and associated models and algorithms which reduce the electronic measurements to valid physical characterizations.

The research and development require the investigation of techniques to solve large systems of equations based on capacitance measurements for various electrode configurations in order to estimate densities of materials in a cross-section of a fluidized bed. Capacitance measurements are made for 400 connections of the 32-electrode system; 400 corresponding electric-field curves are constructed by solving a second order partial differential equation. These curves are used to partition the circular disk into 193 regions called pixels, and the density of material in each pixel is to be estimated.

Two methods of approximating densities have been developed and consideration of a third method has been initiated. One method (Method 1) is based on products of displacement currents for intersecting electric-field curves on a cross section. For each pixel one point of intersection is chosen, and the product of the capacitance measurements is found. Both the product and the square-root-of-product seem to yield good relative distribution of densities.

The second mathematical model (Method 2) for the HRCIS project includes a system of 400 linear equations in 193 variables where the variables represent densities in the 193 pixels. Several computer algorithms have been designed for establishing the coefficient matrix of the system and for solving the system. During this reporting period emphasis was placed on this linear transformation method, particularly on the problem of optimizing the 400 by 193 coefficient matrix. The following procedures have been implemented:

- 1 - the basic system of equations has been converted to a similar system with the roles of coefficients and variables interchanged in order to calculate the entries in the coefficient matrix
- 2 - a computer program has been constructed to reduce the sum of squares error in HRCIS calibration by eliminating, one at a time, the coefficients listed in the electrode-connection charts
- 3 - the computer program which eliminated coefficients was modified to adjust coefficients instead of dropping them; one coefficient at a time is incremented twice and sum of squares errors are calculated for each increment
- 4 - a parabola was fitted to the data (coefficient value vs sum of squares) for the prediction of a new low point; the resulting four points are compared, and the one with the least sum is kept
- 5 - a computer program has been updated from 97-pixel application to 193-pixel application to provide theoretical density distribution for a given location of a rod used for calibration
- 6 - a Gaussian digital filter (standard deviation 0.70) has been applied to the one-rod and two-rod density distributions to smooth that data

The principal objective has been to reduce both the one-rod sum of squares and the two-rod sum of squares. Through repeated applications of the programs the one-rod sum has been reduced from 755,000 to 55,800 and the two-rod sum has been reduced from 743,000 to 72,500.

Density distributions have been represented graphically in two ways: by (x,y) rectangular plots showing variation of density with pixel number and by shading the pixels on a circular disk according to variation in density.

The third model (Method 3) is mathematically similar to the second model in that it consists of linear systems of equations. The principal difference is that the Method 2 systems are solved directly for densities while the Method 3 systems are solved for matrix coefficients; after the coefficients are found the calculation of densities requires only a matrix multiplication. Method 3 yields very good least-squares fit to the four sets of calibration data being used.

2. INTRODUCTION

For the High Resolution Capacitance Imaging project the first mathematical problem was to estimate the components of electric-field vectors where the field results from excitation of electrodes surrounding a fluidized bed. Thirty two electrodes are symmetrically placed at the outside diameter of a cross-section of the cylindrical bed; one set of data is made up of displacement currents measured between 400 pairs of electrodes (Fasching, G.E. and N.S. Smith, 1988). Boundary-value problems including the Laplace partial differential equation have been used in the modeling: solutions yield equipotential curves from which twenty-five basic electric-field curves are constructed. Figure 1 shows 50 of these curves separated into Groups 0, 1, 2, and 3, as indicated in Table 1. Rotation of the basic curves is used to determine the 400 curves corresponding to the designated electrode connections.

The circular cross-section of the bed is partitioned into 193 subregions called pixels or cells. Two methods have been developed for estimating the density of material at each pixel. The simpler method uses products of displacement currents to estimate relative densities over the cross-section. In the second method a linear system of equations is established where the variables represent (or are proportional to) the densities of materials within the pixels. The system consists of 400 linear algebraic equations in 193 variables: each equation corresponds to a field curve.

3. PURPOSE

The basic question in the research is, "How can the density of material at each pixel location along the path of an electric-field curve be calculated?" The answer is being sought by two methods which have been tested on several data sets derived from the capacitance imaging system.

The objective in using the linear system of equations is to find a coefficient matrix which may be applied repeatedly to produce density distribution vectors corresponding to multiple current displacement vectors. The product method is being designed to provide intermediate (though probably less accurate) estimates of pixel densities while the coefficient matrix is being optimized.

4. BACKGROUND

The central problem in constructing the mathematical model is to reproduce within a computer program a chart representing the theoretical distribution of capacitance on a circular cross-section of a fluidized-bed. The chart is used to define 400 equations based on a 32-electrode configuration, where each equation represents a connection between two electrodes. For example, the first column of the chart connects the first and fourth electrodes and yields a linear equation of the form

$$\sum_{j=1}^{193} L_{1j} s_j = d_1.$$

For a given set of coefficients L_{ij} , i from 1 to 400, j from 1 to 193, and a given set of d_i , the 400 equations are determined. The number of unique coefficients L_{ij} has been varied from 47 in initial models to 626 in the current model. Each L_{ij} may be used several times; for example, $L_{26,192} = L_{26,163} = L_{1,165} = L_{1,162}$. There are 25 normalizing constraints which require that the sum of the coefficients appearing in any column of the chart be equal to one; if a coefficient appears more than once, it is counted the number of times it occurs.

In the system $As = d$ of 400 equations, the 193 components of vector s represent the density distribution of material in a cross-section of the bed. The circular disk has been partitioned into 193 subregions or pixels as shown in Figure 2. Density in the innermost circular region is also designated by A, i.e., $s_1=A$. Density in the circular ring or annulus surrounding A is also designated by B, i.e., $s_2=B_1$, $s_3=B_2$, ..., $s_{33}=B_{32}$. The C, D, E, F, and G regions are similarly defined with $s_{130}=F_1$, $s_{131}=F_2$, ..., $s_{161}=F_{32}$, and $s_{162}=G_1$, $s_{163}=G_2$, ..., $s_{193}=G_{32}$.

The 400 components of vector d represent displacement currents which are measured experimentally. In each set of experiments there is a test called a calibration which is designed for determining the coefficients. An object such as an empty plastic bottle or a solid nylon rod is placed upright in a certain position within the cylinder so that a circular cross-section of the cylinder will contain a circular cross-section of the bottle or rod. In the most recent configurations for calibration two solid nylon rods have been used instead of one.

The computer program which sets up the matrix of coefficients L_{ij} is called MAT400; all of the computer programs discussed in this report are written in Fortran for use on the VAX 8650 computer and the FPS array processor. In a typical case with 626 unique coefficients L_{ij} , the number of non-zero entries in matrix A is 19,833.

Generally, systems which have more equations (400) than variables (193) are inconsistent, and that may be true here; if so, solutions can be estimated by using a least-squares procedure to minimize the length of the residual vector $r = d - As$. A more limiting problem occurs when the system does not contain at least 193 independent equations perhaps due to the symmetries in the physical system (HRCIS). A change in the position of the nylon rod during calibration has been used to reduce this dependence.

The computer program to solve the system of equations is called SLE (for System of Linear Equations). SLE reads matrix A and vector d ; then SLE calls a NAG (Numerical Algorithms Group) Fortran Library routine which carries out the minimization of the length of r . The NAG routine returns vector s and an error measurement given by,

$\sigma = \sqrt{\frac{r^T r}{400 - k}}$ where $r^T r$ is the residual sum of squares and k is the rank of A. (Lawson, C. L. and R. J. Hanson, 1974)

5. METHODOLOGY, INCLUDING QUALITY CONTROL

The High Resolution Capacitance Imaging System (HRCIS) is a 32-electrode system which measures displacement currents between pairs of electrodes at the rate of 25,000 measurements per second. Table 1 gives a partial list of the electrode connections. The HRCIS charts list 400 pairs of electrode connections to establish a system of 400 linear algebraic equations for a particular time frame so there are about 60 frames of data for each second. Data collections are referred to as standardization, calibration, and fluidization. Calibration data is collected when one or more objects of known size and shape are placed in the six-inch diameter fluidization vessel. Currently, two cylindrical nylon rods are used: a one-rod or one-void calibration is done with a three-inch diameter rod centered at pixel 56 (See Figure 1); a two-rod or two-void calibration is done with the three-inch rod placed as before and a one-and-one-half inch diameter rod centered at pixel 102.

Both the product method and the linear transformation method for determining density distributions use a partitioning of a circular cross-section of the fluidization vessel into subregions called pixels. Figure 2 shows the current partitioning of the 6-inch diameter cross-section into 193 pixels. Numbers 1 to 32 around the circumference of the circle indicate the positions of the electrodes. The notation "D8" corresponds to the pixel with radial measurement between 1.5 and 2.0 inches and angular measurements between $\pi/2$ and $9\pi/16$ radians. Pixel D8 is also denoted by pixel number 73.

Application of the product method (Method 1) required:

- (1) the generation of 25 basic electric-field curves used to represent the 400 connections of electrodes
- (2) a search for the points of intersection of the 400 curves (more than 20,000 points were found; Table 2 lists a few of the intersections)
- (3) sorting of points of intersection according to pixel location and choosing those points which would yield the best representation of data
- (4) calculation of the square root of the product of two displacement current measurements.

The square-root-of-product method is used in the real-time display of HRCIS data on a black and white monitor which shows density variations by shading of pixel areas using white for lowest density and black for highest density; the system also uses a color monitor.

The second mathematical model (Method 2) applied to the HRCIS data consists of a system of linear equations to be established and then to be solved. Currently, the model has 400 equations with the 193 pixel densities as variables. Charts which show

the construction of the system have been provided and have been used to set up the equations. The most significant problem associated with this method is that of finding the coefficients which make up the matrix A in the system of equations denoted by $Ax = b$ or $As = d$ where x or s is the column vector of 193 unknown densities, and b or d is the column vector of displacement current measurements. Computer programs have been written to set up the system $Ax = b$ according to the charts and to solve the system of equations.

In an effort to optimize the coefficients which go into matrix A , the system $Ax = b$ has been converted to a similar system labeled by $DL = b$. The size of this system is 425 by 349, that is, it consists of 425 equations in 349 variables. The first 400 equations were the same as in $Ax = b$, with the roles of coefficients and variables interchanged, and the additional 25 equations imposed so-called normalizing constraints on the coefficients: the sum of the coefficients in any column of the charts must be one unit. The matrix D is an array of theoretical density values based on cross-sections of one or two nylon rods being superimposed on the circular cross-section of the fluidization vessel. Vector L represents the 349 variable coefficients. In both systems the vector b contains the 400 displacement current measurements from the HRCIS; in the second system these 400 measurements are supplemented by appending the 25 ones required for normalization. For the $DL = b$ system, vector b is given by calibration data, a set of measurements taken when the two nylon rods are placed in the vessel. $DL = b$ is solved by the same least-squares procedure that is used to solve $Ax = b$.

Method 2 was enhanced through a column by column search of the HRCIS electrode connection charts. The search showed that in a few columns of the charts the number of variable coefficients exceeded the number of constraints on those coefficients; therefore, coefficients which have been judged least significant were deleted. The density maps produced after this change showed definite improvement over maps produced earlier. Therefore, the process of deleting coefficients was continued in an effort to further improve the maps. A computer program which eliminates, one or two at a time, the coefficients listed in the HRCIS electrode-connection charts has been constructed. The objective was to choose from a basic set of 349 coefficients the subset which would produce the best least-squares fit to two sets of calibration data. Initially, 59 coefficients were dropped because their estimated values had been smallest; 39 of those coefficients have been restored due to indications that some of them were more important than their relative values reflected; finally, contributions of the remaining 25 coefficients were reconsidered.

The computer program which eliminated coefficients was modified to adjust coefficients instead of dropping them. One coefficient at a time was incremented six times (3 increases, 3 decreases), the two-rod sum of squares was calculated for each increment, and a parabola was fitted to the data (coefficient value vs sum of squares) for the prediction of a new low point. For this coefficient, the resulting eight points were compared, the one with the least sum was kept, and the program moved to the next coefficient. After several program runs were made, the number of increments was cut from six to two in order to save computer time; the use of the parabola proved to be very effective, and only the starting point and two new points were required for its determination. Program output includes sum of squares for each set of calibration data, the sum of the absolute values of all coefficients, the sum of the coefficients in each column of the charts, the average change in density over 193 pixels, and the ratio of the two-rod sum of squares to the average change in density.

The third HRCIS mathematical model (Method 3) for determining pixel densities is similar to Method 2, but it is a more extensive system. Each of the six rings and the one center pixel are treated independently: for each ring of pixels a system of 129 equations in 128 variable coefficients is established and solved, and for the center pixel a system of 5 equations in 4 variable coefficients is used, for a total of 779

equations in 772 variables. These equations are based on four sets of calibration data although initially only two calibrations were used, and it is expected that the system will be further expanded to include more than four calibrations.

For Method 3 the matrix equation for a ring of pixels is constructed in the form $I_k = d$, where I is a 129 by 128 matrix of calibration measurements, k is a 128 by 1 column of variable coefficients, and d is a 129 by 1 column of theoretical densities based on the calibration configuration. After the coefficients have been calculated using this system, the system is used again for data other than calibration data. For fluidization data, for example, the matrix I will be made up of fluidization measurements, and corresponding densities will be given by the matrix multiplication $d = Ik$.

For all three methods a computer program has been developed to provide an optional angular digital filter to reduce the effect of measurement-systems noise.

A strong effort has been made to control and assure the quality of work on this project. Post-run analyses are compared with real-time displays to provide a check on all three sets of routines. Results of the three methods are compared in order to assess the strengths and weaknesses of each method. To test the computer programs for solving systems of equations, the programs are applied to systems with known solutions, and the results are verified. Weekly meetings are held with METC personnel working on the project to discuss the validity of numerical results.

6. RESULTS AND DISCUSSION, INCLUDING PROJECTED SCHEDULES

During this reporting period, the principal objective in the numerical analysis for the HRCIS project has been to improve the linear transformation method of estimating densities of materials in a fluidized bed. Calibration measurements are frequently taken to provide bases for the calculations. Initially, calibration was done with one upright cylindrical rod placed in the fluidization vessel, and later two rods were used. Currently, both these calibrations are being used to develop the transformation model. Improvement in the model is dependent on the reduction of the sum of squares errors in matching the assumed calibration positions to the calculated density distributions for calibrations. The computer algorithms for Method 2 have been designed as a two-step process to produce a reduction in the total sum of squares (one-rod plus two-rod): the first is to change the calculated densities by incrementing coefficients in the matrix A for the 400 by 193 system $Ax = b$; the second is to alter the assumed calibration positions.

The number of unique coefficients in matrix A has recently been increased from 349 to 626 through adjustments in the HRCIS charts. The original approach was to solve the system $DL = b$ for 349 variable coefficients, and then to increment them one at a time. As the 277 new coefficients were added to the system they were set at zero, and then incremented and tested along with the original ones.

The change in assumed calibration positions is made in a computer program called CALROD which has been updated from a 97-pixel application to 193-pixel application. This program provides theoretical density distribution for a given location of a rod used for calibration.

A Gaussian digital filter (standard deviation 0.70) has been applied to the theoretical one-rod and two-rod density and to the calculated density distributions to smooth the data.

Through repeated applications of the programs during this reporting period the sums of squares have been reduced by the following amounts:

One-rod calibration from 755,00 to 55,800;

Two-rod calibration from 743,000 to 72,500.

Table 3 shows the current values of coefficients in the system of equations and compares them with the original values. The coefficients are separated into sets labeled M, P, and R, according to the variations in the HRCIS measurement order. Table 4 lists some of the calibration rod positions which have been considered during this period along with the corresponding sum of squares error.

Initial development of Method 3 has produced very good results in terms of sum-of-squares errors in matching calibration data. The method was first set up for the same two calibrations as used in Method 2, but was recently expanded to include two additional calibrations. One advantage of Method 3 over Method 2 is the easy inclusion of additional calibrations; the optimum number of calibrations has not been determined. Another principal question about the method concerns the choices of HRCIS measurements to use for a given ring of pixels; currently, 13 of the 25 columns of the HRCIS charts are being used, and 8 of the 13 columns are chosen for each application of the method.

Figures 3 and 4 show comparisons of the theoretical density distributions with results from each of the three mathematical methods of estimating densities.

Figures 5, 6, and 7 show the application of Methods 1 and 2 to fluidization data where there is no theoretical distribution available; the graphs indicate maximum activity near electrode number 11. Method 3 has not been sufficiently developed for application to fluidization, and so it has not been included in Figure 5. Figures 6 and 7 exhibit the same data as Figure 5.

It is expected that work on both linear transformation methods will continue with the goals of

- (1) improving the resolution in Method 2 (numerically, this means reducing the sum of squares errors to below 50,000 for each of the one-rod and two-rod calibrations), and
- (2) developing Method 3 to optimize the choices of HRCIS columns and calibrations.

It is also expected that the number of pixels will be increased from 193 to 545. Both the square-root-of-product method and the linear transformation methods will be expanded for use with the 545-pixel cross-section.

The HRCIS data used in this project was taken from a fluid bed system with a six inch circular cross-section. METC now has a twelve inch model. Initial data collection has been performed to check operation of the excitation and sense switch circuits in the system. Some of this data has been normalized, plotted, and compared with corresponding data from the six inch system. Testing of calibration and fluidization data will be done.

7. CONCLUSION

As the METC HRCIS has been expanded from a 16-electrode system to a 32-electrode system, progress has been made in providing a mathematical model to complement the System, with revision and enlargement of some of the previously used computer software while new programs were developed as they were needed.

Based on a list of electrode-to-electrode measurements, corresponding electric-field curves have been constructed, and their points of intersection have been located. This process led to implementation of the square-root-of-product method for the HRCIS real-time display monitor and the linear transformation method for post-run analyses. Refinement of both methods has been achieved through least-squares processes applied to (1) choice of points of intersection for the product method, and (2) optimization of the coefficient matrix to set up the system of linear equations in the form $Ax = b$. Improvements in the displays of density distributions for both methods have been made through the application of digital filtering. It is expected that the linear transformation method provides a more accurate model for the physical system than the product method yields.

During this reporting period emphasis has been placed on improvement of the linear transformation method. The two-step process of (1) changing entries in the coefficient matrix, and (2) varying assumed positions of the rods used in calibration has proved effective in reducing the least-squares error in obtaining matches to calibration data. A variation of the linear transformation method which allows for simultaneous use of two, or four, or more than four sets of calibration data is being developed.

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TABLE 1. PARTIAL LIST OF CONNECTIONS OF ELECTRODES

High Resolution Capacitance Imaging System

List of connections between electrodes determining

(a) 400 electric-field curves

(b) 400 linear equations

Electrodes may be numbered in two different ways, from 1A to 16B or from 1 to 32, with the following correspondences

1A or 32	1B or 1	2A or 2	2B or 3	3A or 4	3B or 5	4A or 6
4B or 7	5A or 8	5B or 9	6A or 10	6B or 11	7A or 12	7B or 13
8A or 14	8B or 15	9A or 16	9B or 17	10A or 18	10B or 19	11A or 20
11B or 21	12A or 22	12B or 23	13A or 24	13B or 25	14A or 26	14B or 27
15A or 28	15B or 29	16A or 30	16B or 31			

Measurement number	Connection	Group	Curve number	Number of rotations	(each rotation is $\pi/16$ radians, counter-clockwise)
1	2A to 16B	1	1	16	
2	3A to 15B	1	3	16	
3	4A to 14B	1	5	16	
4	5A to 13B	1	7	16	
5	6A to 12B	1	6	0	
6	7A to 11B	1	4	0	
7	8A to 10B	1	2	0	
8	16A to 2B	1	2	16	
9	15A to 3B	1	4	16	
10	14A to 4B	1	6	16	
11	13A to 5B	1	7	0	
12	12A to 6B	1	5	0	
13	11A to 7B	1	3	0	
14	10A to 8B	1	1	0	
.					
.					
.					
106	7A to 9B	1	2	2	
107	6A to 10B	1	4	2	
108	5A to 11B	1	6	2	
109	4A to 12B	1	7	18	
110	3A to 13B	1	5	18	
111	2A to 14B	1	3	18	
112	1A to 15B	1	1	18	

TABLE 1. Continued

113	2A to 15B	2	1	17
114	3A to 14B	2	3	17
115	4A to 13B	2	5	17
116	5A to 12B	2	6	1
117	6A to 11B	2	4	1
118	7A to 10B	2	2	1
.				
.				
203	6A to 9B	2	2	3
204	5A to 10B	2	4	3
205	4A to 11B	2	6	3
206	3A to 12B	2	5	19
207	2A to 13B	2	3	19
208	1A to 14B	2	1	19
209	3A to 16B	0	2	15
210	4A to 15B	0	4	15
211	5A to 14B	0	6	15
212	6A to 13B	0	7	31
213	7A to 12B	0	5	31
214	8A to 11B	0	3	31
215	9A to 10B	0	1	31
.				
.				
314	8A to 9B	0	1	1
315	7A to 10B	0	3	1
316	6A to 11B	0	5	1
317	5A to 12B	0	7	1
318	4A to 13B	0	6	17
319	3A to 14B	0	4	17
320	2A to 15B	0	2	17
321	2A to 14B	3	1	18
322	3A to 13B	3	3	18
323	4A to 12B	3	5	18
324	5A to 11B	3	4	2
325	6A to 10B	3	2	2
.				
.				
396	5A to 9B	3	2	4
397	4A to 10B	3	4	4
398	3A to 11B	3	5	20
399	2A to 12B	3	3	20
400	1A to 13B	3	1	20

TABLE 2. SHORT LIST OF INTERSECTIONS OF ELECTRIC-FIELD CURVES

Pixel Number	Curve #1	Curve #2	Sum of Squares	Angle of Intersection	Distance from Centroid
1	109	200	363.	11.40	0.06415
1	200	323	735.	32.90	0.05742
1	11	102	1892.	22.60	0.05142
1	95	109	2042.	22.60	0.05142
1	11	116	2572.	11.40	0.06415
1	102	205	2728.	11.40	0.06415
1	102	338	2768.	22.00	0.05139
1	95	323	2771.	22.00	0.05139
1	205	328	3210.	10.30	0.06549
1	328	338	3254.	21.30	0.05138
16	302	316	257810.	30.90	0.05548
16	12	101	294103.	25.40	0.05735
16	122	302	310454.	31.00	0.04252
16	294	308	312303.	39.10	0.04513
16	87	117	317756.	37.90	0.08756
16	12	87	320921.	51.10	0.07513
16	101	288	325593.	47.20	0.05701
16	302	324	327264.	22.70	0.04256
16	288	308	337918.	62.00	0.03274
16	302	334	341442.	39.30	0.04250
17	12	101	257810.	25.40	0.05735
17	5	316	268145.	13.30	0.05855
17	213	302	275823.	62.00	0.04243
17	12	221	293100.	21.30	0.04774
17	101	221	293287.	48.50	0.05721
17	108	213	294696.	40.10	0.05030
17	117	294	301830.	39.30	0.08729
17	122	213	311559.	31.00	0.04253
17	134	302	329806.	55.60	0.02977
48	301	315	138852.	49.00	0.05669
48	93	123	160832.	42.70	0.03576
48	280	315	169210.	76.90	0.05305
48	6	93	170935.	57.90	0.04100
48	6	107	171600.	28.90	0.04586
48	315	392	180966.	30.30	0.05275
48	123	280	187725.	70.20	0.03923
48	6	280	196089.	84.10	0.01120
48	107	280	205015.	54.70	0.03619
49	107	214	138852.	57.60	0.01722
49	123	301	149434.	48.20	0.04133
49	87	315	157021.	33.00	0.05299

TABLE 2. Continued

Pixel Number	Curve #1	Curve #2	Sum of Squares	Angle of Intersection	Distance from Centroid
49	135	198	159843.	48.90	0.08486
49	135	301	161223.	74.20	0.05774
49	198	307	162001.	22.20	0.08477
49	214	301	163926.	88.30	0.04277
49	288	315	172954.	53.90	0.05228
49	315	349	175312.	30.30	0.05274
80	13	86	109400.	69.50	0.05790
80	293	307	110177.	65.70	0.08470
80	118	293	110540.	61.20	0.07328
80	214	293	122888.	78.10	0.06086
80	13	100	124920.	33.80	0.08430
80	274	307	128812.	74.30	0.08307
80	86	118	133698.	51.00	0.05404
80	13	186	137316.	74.10	0.04665
80	86	214	138520.	76.60	0.05432
81	13	307	109400.	29.00	0.08344
81	13	301	119315.	66.20	0.08002
81	13	198	133169.	39.80	0.07573
81	222	301	134898.	78.10	0.06066
81	13	93	136183.	61.60	0.07621
81	222	293	136790.	54.10	0.08674
81	100	222	140359.	71.30	0.04047
81	27	301	144459.	75.70	0.05432
81	13	192	147261.	66.40	0.07632
81	47	307	150432.	74.50	0.08329
112	287	314	94722.	65.30	0.04314
112	86	106	103822.	37.90	0.06913
112	7	86	105810.	78.40	0.06833
112	274	314	106883.	63.70	0.04412
112	7	287	107330.	64.50	0.02335
112	106	274	113278.	76.00	0.06497
112	106	261	113549.	79.70	0.06114
112	106	248	118270.	82.50	0.05971
112	53	106	119149.	82.10	0.05983
112	7	106	120049.	40.60	0.08095
113	7	293	94722.	84.20	0.07289
113	27	293	109363.	58.00	0.06834
113	27	106	110109.	78.40	0.06832
113	130	293	118584.	86.10	0.07802
113	7	314	119832.	32.70	0.06002
113	280	314	120084.	61.50	0.06419

TABLE 2. Continued

Pixel Number	Curve #1	Curve #2	Sum of Squares	Angle of Intersection	Distance from Centroid
113	27	314	120683.	65.20	0.07313
113	106	147	123101.	80.90	0.07560
113	147	293	123980.	56.00	0.07642
113	267	314	128746.	62.50	0.05418
144	99	287	199101.	85.20	0.04572
144	79	99	206583.	87.30	0.04637
144	99	274	208961.	83.00	0.04526
144	99	222	212672.	75.90	0.04453
144	66	99	213113.	83.30	0.04545
144	99	261	214757.	82.00	0.04503
144	99	248	215384.	80.70	0.04486
144	99	300	217389.	88.00	0.04783
144	53	99	217921.	80.90	0.04496
145	14	86	199101.	73.10	0.04466
145	14	186	209994.	74.10	0.04461
145	14	73	213754.	77.80	0.04463
145	14	382	213922.	77.80	0.04463
145	14	175	219482.	79.00	0.04468
145	14	60	220922.	80.90	0.04496
145	14	373	223461.	81.00	0.04499
145	14	164	225903.	82.50	0.04512
145	14	47	227920.	83.20	0.04544
176	124	197	28770.	59.10	0.03978
176	197	350	28907.	39.10	0.05145
176	124	350	31023.	76.10	0.00644
176	124	136	31703.	62.60	0.01146
176	14	124	31865.	63.90	0.01069
176	124	191	37970.	85.90	0.00617
176	124	300	39202.	88.50	0.00580
176	92	124	43661.	85.90	0.00641
176	197	300	47960.	59.30	0.03069
177	99	124	28770.	58.10	0.01068
177	136	391	29302.	39.20	0.05145
177	124	391	30302.	70.20	0.00644
177	124	136	30373.	53.40	0.03978
177	124	197	30582.	57.00	0.01146
177	124	355	39560.	84.70	0.00582
177	124	142	43383.	88.30	0.00616
177	124	215	46655.	85.80	0.00580
177	136	382	46821.	53.20	0.03523
177	136	186	47802.	51.20	0.03778

TABLE 3. CALCULATED COEFFICIENTS, INCLUDING CHANGES

GROUPS 1 AND 2 (or M) COEFFICIENTS

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
1	-0.1353	0.0525	-0.1878	46	-0.1170	0.0000	-0.1170
2	0.7531	0.3059	0.4471	47	0.0028	0.0000	0.0028
3	3.1081	0.2591	2.8489	48 *	-0.0157	0.0000	-0.0157
4	-0.2560	-0.0936	-0.1624	49	0.0331	0.0804	-0.0473
5	-0.1186	-0.0295	-0.0891	50	-0.9827	-1.1300	0.1473
6	-2.5111	0.0000	-2.5111	51	0.4301	0.6812	-0.2511
7	-1.6283	0.0000	-1.6283	52	0.5388	0.2843	0.2545
8	0.8539	0.0000	0.8539	53	-0.4549	-0.4947	0.0398
9 *	0.4342	0.0000	0.4342	54	1.8926	1.7811	0.1116
10	-0.8883	0.4326	-1.3208	55	-1.6451	-1.1235	-0.5215
11	-1.7423	-0.7287	-1.0136	56	1.2106	1.1231	0.0875
12	1.3405	0.5731	0.7674	57	-0.2652	-0.4970	0.2319
13	1.7025	0.4098	1.2927	58	-1.9700	-1.7769	-0.1931
14	-0.9416	-0.3266	-0.6150	59	2.0651	1.9852	0.0799
15	0.7422	0.3163	0.4259	60	-1.6681	-1.8896	0.2215
16	-0.8760	-0.2069	-0.6691	61	1.5249	1.3848	0.1401
17	0.4485	0.0282	0.4204	62	-0.1164	0.0000	-0.1164
18	-0.2461	0.0000	-0.2461	63	0.0513	0.0000	0.0513
19	0.7088	0.0000	0.7088	64	0.0531	0.0000	0.0531
20	-0.2046	0.0000	-0.2046	65	-0.0058	0.0000	-0.0058
21	0.1111	0.0000	0.1111	66	-0.1238	0.0000	-0.1238
22	0.1416	0.0000	0.1416	67	0.0705	0.0000	0.0705
23	0.1470	0.0000	0.1470	68	-0.0741	0.0000	-0.0741
24 *	0.0566	0.0000	0.0566	69	-0.1134	0.0000	-0.1134
25	-0.2967	-0.2667	-0.0301	70	0.0538	0.0000	0.0538
26	-0.9389	-0.5013	-0.4376	71	0.0533	0.0000	0.0533
27	-0.3123	-0.1738	-0.1386	72	-0.0009	0.0000	-0.0009
28	2.4195	1.7188	0.7007	73 *	-0.0569	0.0000	-0.0569
29	-0.2150	0.3781	-0.5931	74	-0.0383	0.0172	-0.0555
30	-1.2243	-1.5019	0.2776	75	-0.4404	-0.5892	0.1489
31	1.5430	1.1150	0.4280	76	0.3551	0.2368	0.1184
32	-2.2505	-1.7657	-0.4848	77	0.3367	0.4945	-0.1578
33	2.1423	1.2106	0.9317	78	0.0117	0.0000	0.0117
34	1.6197	1.6636	-0.0439	79	-0.3866	0.2847	-0.6713
35	-1.3427	-1.3826	0.0400	80	0.1534	-0.0422	0.1956
36	-0.3007	0.0000	-0.3007	81	1.2555	0.9188	0.3367
37	0.2227	0.0000	0.2227	82	-1.0966	-1.0894	-0.0072
38	-0.6303	0.0000	-0.6303	83	-0.0036	-0.3438	0.3402
39	-0.0007	0.0000	-0.0007	84	0.2110	0.4741	-0.2631
40	0.2041	0.0000	0.2041	85	0.0056	0.4426	-0.4370
41	-0.1373	0.0000	-0.1373	86	-0.3038	-1.0134	0.7096
42	0.0008	0.0000	0.0008	87	-1.3661	-0.7499	-0.6163
43	0.0947	0.0000	0.0947	88	1.4280	1.4585	-0.0305
44	-0.0244	0.0000	-0.0244	89	0.0538	0.0000	0.0538
45	0.0568	0.0000	0.0568	90	0.1790	0.0000	0.1790

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
91	0.1347	0.0000	0.1347	138	0.3086	0.5404	-0.2318
92	-0.0709	0.0000	-0.0709	139	-0.3746	-0.3747	0.0001
93	0.0528	0.0000	0.0528	140	0.4119	0.6609	-0.2490
94	0.0343	0.0000	0.0343	141	-0.0711	-0.1865	0.1154
95	-0.2366	0.0000	-0.2366	142	-0.1515	-0.0942	-0.0573
96	-0.0420	0.0000	-0.0420	143	-0.1617	-0.0717	-0.0901
97	0.0295	0.0000	0.0295	144	0.3508	0.0000	0.3508
98	-0.0549	0.0000	-0.0549	145	-0.4989	-0.6493	0.1504
99	0.0628	0.0000	0.0628	146	1.3142	1.4651	-0.1509
100	0.0709	0.0000	0.0709	147	-0.9148	-1.2568	0.3421
101	0.0362	0.0000	0.0362	148	0.2716	0.4863	-0.2147
102	-0.0267	0.0000	-0.0267	149	-0.0411	0.0000	-0.0411
103	0.0924	0.0000	0.0924	150	0.1530	0.3654	-0.2124
104	-0.0842	0.0000	-0.0842	151	0.2390	0.0000	0.2390
105 *	0.2941	0.0000	0.2941	152	0.0242	0.0000	0.0242
106	0.6067	0.6327	-0.0260	153	-0.0507	0.0000	-0.0507
107	-0.1148	-0.0856	-0.0292	154	0.0109	0.0000	0.0109
108	-0.2054	-0.1430	-0.0624	155	0.0119	0.0000	0.0119
109	-1.3584	-1.3749	0.0165	156	0.0293	0.0000	0.0293
110	0.1851	0.1448	0.0404	157	-0.0472	0.0000	-0.0472
111	2.2454	2.0928	0.1525	158	-0.0016	0.0000	-0.0016
112	-1.1918	-1.0498	-0.1420	159	-0.0619	0.0000	-0.0619
113	1.9934	1.6015	0.3919	160	-0.0155	0.0000	-0.0155
114	-2.7566	-2.5738	-0.1828	161	-0.1354	0.0000	-0.1354
115	0.4054	0.0000	0.4054	162	0.0255	0.0000	0.0255
116	1.6027	1.8770	-0.2743	163	0.0768	0.0000	0.0768
117	-1.6859	-1.6630	-0.0229	164	0.0166	0.0000	0.0166
118	0.1415	0.0285	0.1130	165	-0.0269	0.0000	-0.0269
119	0.5104	0.5168	-0.0064	166	0.0269	0.0000	0.0269
120	-2.0741	-2.0541	-0.0200	167 *	0.0190	0.0000	0.0190
121	2.5186	2.5506	-0.0319	168	-2.3783	-1.9387	-0.4396
122	-0.2026	0.0000	-0.2026	169	1.5396	1.0268	0.5128
123	-0.1227	0.0000	-0.1227	170	-0.8444	-1.4068	0.5624
124	0.0546	0.0000	0.0546	171	0.1295	-0.2311	0.3607
125	-0.0297	0.0000	-0.0297	172	1.4582	1.7775	-0.3194
126	0.0274	0.0000	0.0274	173	0.6671	0.5404	0.1266
127	-0.0567	0.0000	-0.0567	174	-0.4271	-0.0004	-0.4267
128	0.0730	0.0000	0.0730	175	-0.1338	-0.2394	0.1057
129	-0.0081	0.0000	-0.0081	176	4.9504	5.2690	-0.3186
130	-0.0110	0.0000	-0.0110	177	-3.9211	-4.5718	0.6507
131	-0.0690	0.0000	-0.0690	178	6.9227	7.4893	-0.5666
132	-0.0114	0.0000	-0.0114	179	-7.1047	-8.4636	1.3590
133	-0.0261	0.0000	-0.0261	180	3.2930	3.6082	-0.3151
134	-0.0076	0.0000	-0.0076	181	-0.0470	0.0000	-0.0470
135	0.0028	0.0000	0.0028	182	-1.6447	-1.4282	-0.2165
136 *	0.0035	0.0000	0.0035	183	0.8926	1.0304	-0.1379
137	-0.1607	-0.2043	0.0436	184	-4.0965	-3.9220	-0.1744

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
185	-0.4916	0.0000	-0.4916	232	-0.7945	-0.4299	-0.3647
186	-0.6305	0.0000	-0.6305	233	1.0949	0.3592	0.7357
187	0.1909	0.0000	0.1909	234	-0.4064	0.1798	-0.5862
188	-0.1106	0.0000	-0.1106	235	-0.2973	-0.2398	-0.0576
189	-0.0228	0.0000	-0.0228	236	0.3472	-0.1508	0.4981
190	-0.0720	0.0000	-0.0720	237	0.3257	0.7399	-0.4142
191	0.1144	0.0000	0.1144	238	-0.0661	-0.0787	0.0126
192	0.0385	0.0000	0.0385	239	0.0400	0.3996	-0.3595
193	0.0553	0.0000	0.0553	240	-1.2037	-0.7949	-0.4088
194	-0.0324	0.0000	-0.0324	241	2.1300	1.1012	1.0289
195	0.0384	0.0000	0.0384	242	-1.6163	-0.9777	-0.6387
196	0.1134	0.0000	0.1134	243	0.6548	0.6852	-0.0304
197	-0.0162	0.0000	-0.0162	244	-0.1166	-0.0422	-0.0744
198 *	0.0213	0.0000	0.0213	245	0.9766	0.0000	0.9766
199	0.4541	0.1235	0.3306	246	-0.1651	-0.7176	0.5525
200	-0.4559	-0.3918	-0.0640	247	-0.7816	0.4669	-1.2484
201	0.3709	-0.0702	0.4411	248	0.5385	0.0000	0.5385
202	0.1822	1.3415	-1.1592	249	-0.1020	0.0000	-0.1020
203	-0.9637	-0.3567	-0.6070	250	0.1156	0.0000	0.1156
204	-0.2900	-0.7131	0.4231	251	0.2006	0.0000	0.2006
205	0.9749	0.2893	0.6855	252	0.0151	0.0000	0.0151
206	0.8018	0.1749	0.6269	253	-0.0213	0.0000	-0.0213
207	-0.3839	-0.2149	-0.1690	254	-0.2302	0.0000	-0.2302
208	-0.0541	-0.4818	0.4277	255	-0.0822	0.0000	-0.0822
209	-0.8198	-0.0319	-0.7880	256	-0.0304	0.0000	-0.0304
210	2.3362	2.8118	-0.4756	257	-0.1278	0.0000	-0.1278
211	-2.9901	-3.5669	0.5768	258	-0.0232	0.0000	-0.0232
212	1.3863	1.1459	0.2404	259	0.0660	0.0000	0.0660
213	-0.2068	0.1035	-0.3103	260	0.3409	0.0000	0.3409
214	0.3903	0.3369	0.0534	261	-0.2579	0.0000	-0.2579
215	-0.4264	0.0000	-0.4264	262	-0.1278	0.0000	-0.1278
216	0.3004	0.0000	0.3004	263 *	0.2087	0.0000	0.2087
217	-0.5592	0.0000	-0.5592	264	-0.0903	-0.3125	0.2222
218	0.0970	0.0000	0.0970	265	-0.0223	0.0000	-0.0223
219	0.0244	0.0000	0.0244	266	0.3270	0.2723	0.0547
220	0.2321	0.0000	0.2321	267	-0.0062	0.1923	-0.1985
221	0.2433	0.0000	0.2433	268	-0.0598	-0.1694	0.1096
222	-0.0191	0.0000	-0.0191	269	0.1948	0.1164	0.0783
223	-0.0394	0.0000	-0.0394	270	0.1165	0.1974	-0.0809
224	0.2142	0.0000	0.2142	271	-0.0741	0.1570	-0.2311
225	-0.2627	0.0000	-0.2627	272	-0.2316	-0.1104	-0.1212
226	0.0203	0.0000	0.0203	273	0.0139	0.1257	-0.1118
227	0.2126	0.0000	0.2126	274	0.0615	0.0000	0.0615
228	0.1188	0.0000	0.1188	275	0.0422	0.0286	0.0136
229	0.0532	0.0000	0.0532	276	-0.2022	0.0000	-0.2022
230	-0.2262	0.0000	-0.2262	277	0.1547	0.0000	0.1547
231 *	-0.0658	0.0000	-0.0658	278	0.2868	0.0000	0.2868

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
279	0.1333	0.0000	0.1333	308	0.1655	0.0000	0.1655
280	-0.1502	0.0000	-0.1502	309	-0.3426	0.0000	-0.3426
281	0.1253	0.0000	0.1253	310	-0.0202	0.0000	-0.0202
282	-0.1425	0.0000	-0.1425	311	0.3856	0.0000	0.3856
283	0.1292	0.0000	0.1292	312	-0.7913	0.0000	-0.7913
284	0.0145	0.0000	0.0145	313	0.2534	0.0000	0.2534
285	0.0688	0.0000	0.0688	314	0.3911	0.0000	0.3911
286	0.0012	0.0000	0.0012	315	-0.3167	0.0000	-0.3167
287	0.0237	0.0000	0.0237	316 *	-0.1992	0.0000	-0.1992
288	-0.1229	0.0000	-0.1229	317	0.0137	-0.1238	0.1375
289	-0.0143	0.0000	-0.0143	318	0.4966	0.2792	0.2174
290	-0.0648	0.0000	-0.0648	319	0.1249	0.0066	0.1184
291	-0.0340	0.0000	-0.0340	320	-0.7844	-0.2084	-0.5760
292 *	0.0217	0.0000	0.0217	321	0.0139	0.2595	-0.2456
293	-2.1113	-1.6729	-0.4384	322	0.0682	0.0000	0.0682
294	1.4979	1.5643	-0.0664	323	0.0756	0.2280	-0.1524
295	0.2301	0.0000	0.2301	324	-0.0551	0.0734	-0.1285
296	1.3277	-0.0029	1.3306	325	0.0183	-0.0169	0.0352
297	-0.7344	0.1836	-0.9180	326	0.2363	0.0000	0.2363
298	0.4835	0.4505	0.0329	327	0.2815	0.0000	0.2815
299	-0.4528	0.5930	-1.0457	328	-0.0068	0.0000	-0.0068
300	1.1885	-0.1864	1.3749	329	-0.0428	0.0000	-0.0428
301	-0.6648	-0.6447	-0.0201	330	0.0475	0.0000	0.0475
302	0.2373	-0.1589	0.3962	331	0.0416	0.0000	0.0416
303	1.4683	0.5996	0.8688	332	0.0415	0.0000	0.0415
304	-2.1176	-1.3639	-0.7537	333	-0.0092	0.0000	-0.0092
305	0.8769	1.0989	-0.2220	334	-0.0373	0.0000	-0.0373
306	-0.3088	0.0000	-0.3088	335 *	-0.0239	0.0000	-0.0239
307	0.0538	0.0000	0.0538				

GROUP 0 (or P) COEFFICIENTS

1	-0.6282	-0.8201	0.1919	15 *	-0.0092	0.0000	-0.0092
2	-0.3547	0.0000	-0.3547	16	-0.1604	0.0000	-0.1604
3	1.5999	1.1462	0.4537	17	-0.4235	-0.7987	0.3752
4	-0.7352	-0.2003	-0.5349	18	0.5243	0.7335	-0.2091
5	0.9951	0.3520	0.6431	19	0.1059	0.0000	0.1059
6	-0.5362	0.0000	-0.5362	20	-0.6598	-0.7018	0.0420
7	0.1533	0.0201	0.1333	21	-0.2267	-0.2666	0.0399
8	-0.0163	0.0000	-0.0163	22	1.7086	1.7523	-0.0438
9	-0.0221	0.0000	-0.0221	23	-0.7989	-0.8333	0.0344
10	-0.0034	0.0000	-0.0034	24	1.0444	1.1666	-0.1222
11	0.0292	0.0000	0.0292	25	-0.2350	-0.1882	-0.0469
12	-0.0343	0.0000	-0.0343	26	-1.6488	-1.5475	-0.1012
13	0.0599	0.0000	0.0599	27	1.4135	1.4807	-0.0672
14	0.0021	0.0000	0.0021	28	-1.1915	-1.3900	0.1985

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
29	1.1860	1.0877	0.0984	76	1.2084	1.1051	0.1033
30	0.0159	0.0000	0.0159	77	4.0054	4.0321	-0.0267
31	-0.0212	0.0000	-0.0212	78	2.0548	1.9998	0.0550
32	-0.0017	0.0000	-0.0017	79	-0.7382	-0.9015	0.1633
33	0.0369	0.0000	0.0369	80	-6.9396	-6.5458	-0.3938
34	-0.2538	0.0000	-0.2538	81	0.5877	0.8995	-0.3118
35	0.0300	0.0000	0.0300	82	8.9870	8.9940	-0.0070
36	0.0423	0.0000	0.0423	83	-6.8321	-6.9720	0.1399
37	0.0022	0.0000	0.0022	84	11.4923	11.2959	0.1963
38	0.0270	0.0000	0.0270	85	0.0291	0.0000	0.0291
39	-0.0242	0.0000	-0.0242	86	-21.5849	-21.6149	0.0300
40	-0.0090	0.0000	-0.0090	87	33.3806	33.3394	0.0412
41	0.0150	0.0000	0.0150	88	-29.1867	-29.2400	0.0533
42 *	0.0024	0.0000	0.0024	89	12.2223	12.1902	0.0321
43	0.9841	1.6479	-0.6638	90	-7.6349	-8.1093	0.4744
44	2.8022	2.1039	0.6983	91	-0.2445	0.0000	-0.2445
45	-1.7062	-1.0709	-0.6353	92	0.0287	0.0000	0.0287
46	-6.2067	-5.9017	-0.3049	93	0.0210	0.0000	0.0210
47	-2.8112	-2.6058	-0.2054	94	-0.0235	0.0000	-0.0235
48	11.3185	8.9910	2.3275	95	0.0489	0.0000	0.0489
49	-2.0251	-1.1307	-0.8943	96	-0.0560	0.0000	-0.0560
50	4.8628	3.7324	1.1304	97	-0.5941	0.0000	-0.5941
51	-10.1317	-9.0501	-1.0816	98	0.0750	0.0000	0.0750
52	-0.2854	-0.0864	-0.1990	99	0.0326	0.0000	0.0326
53	-1.4401	0.4522	-1.8923	100	0.0321	0.0000	0.0321
54	1.1893	-1.2050	2.3943	101	0.0220	0.0000	0.0220
55	6.9013	6.4156	0.4857	102	0.0141	0.0000	0.0141
56	-4.5440	-3.7523	-0.7917	103 *	0.0288	0.0000	0.0288
57	-3.1483	-2.0217	-1.1267	104	-4.4265	-21.3662	16.9397
58	4.2716	3.9818	0.2898	105	4.8545	23.5245	-18.6701
59	-0.0641	0.0000	-0.0641	106	0.0656	-0.3491	0.4147
60	0.0175	0.0000	0.0175	107	-3.0354	-7.5299	4.4945
61	0.2238	0.0000	0.2238	108	2.3459	12.8406	-10.4946
62	-0.1971	0.0000	-0.1971	109	0.8922	-9.7648	10.6571
63	0.1472	0.0000	0.1472	110	-1.0005	-1.0737	0.0733
64	0.0295	0.0000	0.0295	111	-1.5751	-7.1847	5.6096
65	-0.0012	0.0000	-0.0012	112	5.6482	15.6728	-10.0245
66	0.1396	0.0000	0.1396	113	-5.8733	-15.9619	10.0886
67	0.0828	0.0000	0.0828	114	8.4896	33.5885	-25.0989
68	-0.1157	0.0000	-0.1157	115	-1.6800	-2.6091	0.9291
69	0.0763	0.0000	0.0763	116	-0.4885	0.3288	-0.8173
70	0.4465	0.0000	0.4465	117	1.0515	0.9910	0.0604
71	-0.3732	0.0000	-0.3732	118	-3.9258	-19.2882	15.3624
72 *	0.1140	0.0000	0.1140	119	3.9809	34.6920	-30.7111
73	-3.3099	-3.3828	0.0728	120	-4.4831	-36.0104	31.5273
74	2.2227	2.0981	0.1246	121	0.0921	0.0000	0.0921
75	-2.6664	-2.7425	0.0761	122	-0.1530	0.0000	-0.1530

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
123	0.1895	0.0000	0.1895	144	0.2878	0.4017	-0.1139
124	0.1493	0.0000	0.1493	145	0.1916	-0.0023	0.1939
125	-0.3688	0.0000	-0.3688	146	-0.0620	0.0000	-0.0620
126	-0.2151	0.0000	-0.2151	147	-0.0006	0.0000	-0.0006
127	0.1648	0.0000	0.1648	148	0.0588	0.0000	0.0588
128	-0.2928	0.0000	-0.2928	149	0.0121	0.0000	0.0121
129	-0.5998	0.0000	-0.5998	150	0.0526	0.0000	0.0526
130	0.6816	0.0000	0.6816	151	0.0615	0.0000	0.0615
131	0.4461	0.0000	0.4461	152	0.0503	0.0000	0.0503
132	0.0339	0.0000	0.0339	153	0.0134	0.0000	0.0134
133	-0.1807	0.0000	-0.1807	154 *	-0.0474	0.0000	-0.0474
134 *	-0.5745	0.0000	-0.5745	155	0.7976	0.3622	0.4354
135	0.0346	-0.6426	0.6772	156	-0.6513	-0.3845	-0.2668
136	-0.1738	0.3349	-0.5087	157	-0.1672	0.1897	-0.3569
137	-0.9566	-0.0515	-0.9051	158	0.2923	0.3341	-0.0418
138	1.1083	0.3448	0.7635	159	-0.1423	0.0000	-0.1423
139	0.4928	0.4084	0.0844	160	0.1385	0.0000	0.1385
140	0.0387	-0.0783	0.1170	161	0.2789	0.0000	0.2789
141	-0.2007	0.0668	-0.2675	162	-0.0080	0.0000	-0.0080
142	-0.1416	-0.1028	-0.0388	163 *	-0.0386	0.0000	-0.0386
143	-0.3198	-0.1786	-0.1412				

GROUP 3 (or R) COEFFICIENTS

1	0.1711	0.5061	-0.3349	23	-0.7357	-0.5335	-0.2022
2	0.0131	-0.1283	0.1415	24	-1.1532	-1.2482	0.0951
3	-0.0168	-0.0015	-0.0153	25	0.8214	0.8637	-0.0423
4	0.3196	0.1095	0.2101	26	-0.2208	-0.2491	0.0282
5	-0.1290	-0.0259	-0.1031	27	-0.5718	-0.3937	-0.1781
6	0.0510	-0.1595	0.2105	28	2.1123	2.0967	0.0156
7	0.0628	0.1096	-0.0468	29	0.3733	0.1643	0.2089
8	-0.2428	0.0000	-0.2428	30	0.8489	0.8081	0.0408
9	0.2153	0.0414	0.1739	31	0.5417	0.8956	-0.3540
10	0.1552	0.1459	0.0093	32	-1.6408	-1.7082	0.0674
11	-0.1277	-0.1016	-0.0262	33	-0.0484	-0.0330	-0.0154
12	0.0334	0.0000	0.0334	34	1.0020	0.8925	0.1095
13	0.0414	0.0000	0.0414	35	0.7324	0.6622	0.0702
14	-0.0935	0.0000	-0.0935	36	0.0152	0.0000	0.0152
15	-0.0173	0.0000	-0.0173	37	-2.8313	-2.7961	-0.0352
16	0.0962	0.0000	0.0962	38	-1.4833	-1.5335	0.0501
17	-0.1219	0.0000	-0.1219	39	2.6347	2.6116	0.0232
18	0.0112	0.0000	0.0112	40	0.0113	0.0000	0.0113
19	0.0049	0.0000	0.0049	41	0.0325	0.0000	0.0325
20	0.0358	0.0000	0.0358	42	-0.0382	0.0000	-0.0382
21	0.0169	0.0000	0.0169	43	0.0040	0.0000	0.0040
22 *	0.0212	0.0000	0.0212	44	-0.0399	0.0000	-0.0399

TABLE 3. Continued

NO.	FINAL	START	CHANGE	NO.	FINAL	START	CHANGE
45	0.0419	0.0000	0.0419	87	1.2156	1.3994	-0.1838
46	-0.0189	0.0000	-0.0189	88	-2.7330	-2.8327	0.0997
47	0.0766	0.0000	0.0766	89	4.9260	4.9169	0.0091
48	0.0029	0.0000	0.0029	90	-7.4253	-7.6623	0.2370
49	0.0287	0.0000	0.0287	91	6.7646	7.0493	-0.2847
50 *	0.0050	0.0000	0.0050	92	-4.8464	-5.0776	0.2312
51	0.8813	0.8875	-0.0061	93	2.3607	2.4788	-0.1181
52	0.6082	0.3272	0.2810	94	0.0495	0.0000	0.0495
53	-1.8958	-1.5093	-0.3865	95	-0.1594	0.0000	-0.1594
54	1.4431	1.3730	0.0702	96	-0.0350	0.0000	-0.0350
55	-0.9912	-1.4278	0.4366	97	0.0159	0.0000	0.0159
56	0.5982	1.3581	-0.7600	98	0.0300	0.0000	0.0300
57	-1.9726	-1.6262	-0.3464	99	0.0207	0.0000	0.0207
58	-1.0547	-0.7763	-0.2784	100	-0.0022	0.0000	-0.0022
59	3.3884	3.5609	-0.1725	101	-0.0006	0.0000	-0.0006
60	-3.8320	-4.0807	0.2488	102	-0.0751	0.0000	-0.0751
61	0.1540	0.0000	0.1540	103 *	0.0268	0.0000	0.0268
62	1.2002	1.2317	-0.0315	104	-0.2799	0.0112	-0.2911
63	2.9654	2.3622	0.6031	105	0.1941	0.0000	0.1941
64	0.0044	0.0000	0.0044	106	0.2238	0.1142	0.1096
65	0.1459	0.0000	0.1459	107	-0.2011	0.2134	-0.4145
66	0.1898	0.0000	0.1898	108	-0.0029	0.0728	-0.0757
67	-0.2337	0.0000	-0.2337	109	0.1122	-0.1886	0.3008
68	-0.0293	0.0000	-0.0293	110	0.3819	0.3906	-0.0087
69	0.1734	0.0000	0.1734	111	-0.3396	-0.2371	-0.1025
70	0.0201	0.0000	0.0201	112	0.5157	0.1173	0.3984
71	-0.0396	0.0000	-0.0396	113	0.1076	0.1177	-0.0101
72	0.0487	0.0000	0.0487	114	-0.2737	-0.4420	0.1683
73	0.1126	0.0000	0.1126	115	0.1134	0.7088	-0.5954
74	0.0912	0.0000	0.0912	116	-0.1367	-0.3772	0.2405
75	0.0189	0.0000	0.0189	117	-0.1381	0.0000	-0.1381
76 *	-0.0122	0.0000	-0.0122	118	0.1369	0.0000	0.1369
77	0.3380	0.2101	0.1279	119	0.0226	0.0000	0.0226
78	-0.5856	-0.5057	-0.0799	120	-0.0566	0.0000	-0.0566
79	-0.0399	-0.0385	-0.0014	121	0.0476	0.0000	0.0476
80	0.5824	0.5440	0.0384	122	0.0228	0.0000	0.0228
81	-0.4816	-0.4776	-0.0040	123	0.0224	0.0000	0.0224
82	-0.2316	-0.5998	0.3682	124	0.0336	0.0000	0.0336
83	0.1430	0.3800	-0.2369	125	-0.0050	0.0000	-0.0050
84	1.3323	1.8142	-0.4819	126	-0.0290	0.0000	-0.0290
85	-0.5892	-0.6751	0.0859	127	0.0238	0.0000	0.0238
86	-0.0875	-0.4235	0.3360	128 *	0.0044	0.0000	0.0044

The average changes in the absolute values of the coefficients were

M 0.1730 P 1.3425 R 0.0990

* Indicates the end of a column of coefficients in the HRCIS charts

**TABLE 4. SHORT LIST OF POSITIONS OF CALIBRATION RODS
INCLUDING CORRESPONDING SUM OF SQUARES ERROR**

OPTIMIZE POSITION OF SINGLE ROD

FILTER CONSTANT = 0.65

Note. There are three position variables: (1) Number of the electrode in line with the center of the vessel's cross section and the center of the rod's cross section, (2) Assumed radius of the rod (in inches), and (3) Distance of the rod from the wall of the vessel (in inches). A set of random numbers is used in incrementing the variables.

Sum of Squares Error	Electrode No.	Radius of Rod	Distance from Wall
80419.	22.0500	1.4650	0.2400
80525.	21.9875	1.4650	0.2380
80588.	21.9875	1.4650	0.2375
80590.	22.0500	1.4648	0.2390
80672.	21.9875	1.4638	0.2400
80678.	21.9250	1.4640	0.2395
80704.	22.0500	1.4646	0.2385
80727.	21.8625	1.4642	0.2400
80739.	21.9875	1.4636	0.2400
80785.	22.0500	1.4640	0.2395
80806.	22.0500	1.4636	0.2405
80816.	21.8000	1.4650	0.2400
80874.	21.9875	1.4646	0.2365
80942.	21.9875	1.4630	0.2405
80954.	21.8625	1.4634	0.2405
81007.	21.9875	1.4632	0.2395
81018.	21.9250	1.4646	0.2360
81046.	21.9250	1.4630	0.2400
81107.	21.9250	1.4632	0.2390
81154.	21.9875	1.4634	0.2380
81158.	21.8000	1.4644	0.2385
81177.	21.8625	1.4632	0.2395
81180.	21.9250	1.4640	0.2365
81256.	21.8000	1.4648	0.2370
81442.	21.7375	1.4646	0.2395
81455.	21.8625	1.4634	0.2375
81490.	21.9250	1.4630	0.2375
81497.	21.7375	1.4648	0.2385
81498.	21.9250	1.4638	0.2355
81556.	22.1125	1.4634	0.2380
81559.	21.8625	1.4638	0.2360
81563.	21.8000	1.4640	0.2370
81576.	22.0500	1.4632	0.2370
81582.	22.0500	1.4630	0.2375
81608.	22.1750	1.4642	0.2380
81706.	22.0500	1.4630	0.2370
81841.	21.9250	1.4632	0.2355

TABLE 4. Continued

OPTIMIZE POSITIONS OF TWO RODS

FILTER CONSTANT = 0.65

SMALL ROD				LARGE ROD		
Error	Elect. No.	Radius	Wall	Elect. No.	Radius	Wall
95587.	4.7755	0.8011	0.0385	22.3000	1.4801	0.3110
98182.	4.7755	0.8005	0.0385	22.2999	1.4795	0.3110
98187.	4.7755	0.8011	0.0410	22.3000	1.4801	0.3135
99009.	4.8380	0.7997	0.0365	22.2999	1.4787	0.3090
99905.	4.5880	0.7995	0.0385	22.2999	1.4785	0.3110
100346.	4.6505	0.7999	0.0390	22.2999	1.4789	0.3115
101303.	4.8380	0.7997	0.0390	22.2999	1.4787	0.3115
105533.	4.7755	0.8015	0.0365	22.3000	1.4805	0.3090
105861.	4.7130	0.8001	0.0390	22.2999	1.4791	0.3115
106733.	4.7130	0.8005	0.0375	22.2999	1.4795	0.3100
107660.	4.8380	0.8005	0.0380	22.2999	1.4795	0.3105
108328.	4.9005	0.8015	0.0380	22.3000	1.4805	0.3105
108499.	4.5880	0.8007	0.0375	22.2999	1.4797	0.3100
109097.	4.5255	0.8013	0.0370	22.3000	1.4803	0.3095
109280.	4.7755	0.8011	0.0380	22.3000	1.4801	0.3105
109537.	4.7755	0.8011	0.0375	22.3000	1.4801	0.3100
110096.	4.5880	0.8001	0.0375	22.2999	1.4791	0.3100
110505.	4.7130	0.8015	0.0405	22.3000	1.4805	0.3130
112220.	4.6505	0.7999	0.0365	22.2999	1.4789	0.3090
112476.	4.5255	0.8005	0.0405	22.2999	1.4795	0.3130
112495.	4.9630	0.8003	0.0365	22.2999	1.4793	0.3090
114062.	4.9630	0.8015	0.0385	22.3000	1.4805	0.3110
115630.	4.5255	0.8013	0.0360	22.3000	1.4803	0.3085
115930.	4.9005	0.8009	0.0390	22.3000	1.4799	0.3115
116697.	4.9630	0.8001	0.0400	22.2999	1.4791	0.3125
116736.	4.8380	0.7999	0.0390	22.2999	1.4789	0.3115
117353.	4.4630	0.7995	0.0380	22.2999	1.4785	0.3105
117861.	4.5255	0.8011	0.0405	22.3000	1.4801	0.3130
118896.	4.4630	0.8003	0.0405	22.2999	1.4793	0.3130
119739.	4.5255	0.7995	0.0370	22.2999	1.4785	0.3095
121083.	4.8380	0.7999	0.0380	22.2999	1.4789	0.3105
121394.	4.5880	0.8005	0.0375	22.2999	1.4795	0.3100
122077.	4.6505	0.8007	0.0400	22.2999	1.4797	0.3125
122780.	4.5880	0.8011	0.0370	22.3000	1.4801	0.3095
124239.	4.9005	0.7999	0.0370	22.2999	1.4789	0.3095
125184.	4.7130	0.7997	0.0390	22.2999	1.4787	0.3115
126536.	4.9005	0.8005	0.0365	22.2999	1.4795	0.3090
127964.	4.4630	0.8009	0.0405	22.3000	1.4799	0.3130
129001.	4.5255	0.8011	0.0385	22.3000	1.4801	0.3110
129415.	4.4630	0.8015	0.0385	22.3000	1.4805	0.3110
129604.	5.0880	0.8001	0.0395	22.2999	1.4791	0.3120
129846.	5.0880	0.8005	0.0365	22.2999	1.4795	0.3090

FIGURE 1. ELECTRIC FIELD CURVES

Theoretical Density Distribution

Method 1 Density Distribution

Method 2 Density Distribution

Method 3 Density Distribution

FIGURE 3. ONE-ROD CALIBRATION

FIGURE 4. TWO-ROD CALIBRATION

FIGURE 5. FLUIDIZATION

FIGURE 6. FLUIDIZATION METHOD 1

FIGURE 7. FLUIDIZATION METHOD 2

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