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EMPIRICAL ESTIMATES OF THE PROTON-NEUTRON INTERACTION AND DIABOLICAL POINTS

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The use of constant- N contours in the (ω, λ) plane is demonstrated both for establishing diabolical points and for studying regularities and fluctuations in the spectrum of single-particle states. Likewise the average properties and fluctuations of empirical proton-neutron interactions, obtained from double differentials of the total energy with respect to N and Z , are discussed for the ground states of all even-even nuclei with $N > 40$ and as a function of the rotational frequency for the lowest even-spin, positive-parity decay sequence of deformed rare earth nuclei.

1 Introduction

The Hamiltonian for independent particle motion in a rotating nucleus can be expressed as:

$$h' = h_0 - \epsilon \cdot Q - \omega \cdot I_z, \quad (1)$$

where the Lagrangian term, $\omega \cdot I_z$, is included to preserve angular momentum conservation, broken by the addition of the nonspherically-symmetric term $-\epsilon \cdot Q$ (ϵ , the deformation parameter). The Lagrange multiplier, ω , can be interpreted as the angular frequency of rotation, and I_z is the angular momentum projection on the rotational axis. In analogy to such rotations in configurations space, it is possible to restore particle-number conservation broken by the pair field $(-\Delta \cdot (P^\dagger + P))$ by including a Lagrangian term in particle-number space (often called gauge space), $-\lambda \cdot N$, in the independent particle Hamiltonian. Thus the Fermi energy, λ , and particle (proton or neutron) number, N , are gauge space analogs of ω and I_z , respectively^{1,2,3}, and the complete Hamiltonian becomes

$$h' = h_0 - \epsilon \cdot Q - \omega \cdot I_z - \Delta \cdot (P^\dagger + P) - \lambda \cdot N$$

This Hamiltonian becomes even more useful since physical observables can be associated with the Lagrange multipliers, i.e. $\hbar\omega = E_\pi/2$ and $\lambda_n = B_{2N}/2$. Here B_{2N} is the two-neutron binding

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energy, λ_n is the neutron Fermi energy, which we will discuss in the following as an example. Indeed the analogy between configuration and gauge space has been utilized to investigate the distribution of nuclear levels in paired and unpaired nuclei^{4,5,6}. A gap in the single-particle level distribution leads to a discontinuity of λ_n , while a high level density gives a nearly constant λ_n for adjacent isotopes. Pair correlations smear out the level occupations.

This paper introduces an efficient technique for systematically displaying the data for an extended mass region. Contours of constant neutron-numbers are plotted in (ω, λ_n) space for deformed rare earth nuclei and the various features, such as experimental evidence for the existence of diabolical points, are discussed. The difference in λ_n values for neighboring isotones differing by two protons can be interpreted as a proton-neutron interaction. Both a global analysis of this p-n interaction for the ground states of all even-even nuclei of $N > 40$ with measured masses and an analysis as a function of the rotational frequency for the yrast configuration of deformed rare earth nuclei are presented.

2 Constant Neutron Contour Plots and Diabolical Points

Fig. 1 shows contours of constant neutron number in the (ω, λ_n) plane for even- Z deformed rare earth nuclei constructed from experimental data. Empirical values of ω and λ_n are extracted from experimental energies and angular momenta using the canonical relations⁶

$$\omega(Z, N, I, \nu) = \frac{E(Z, N, I+1, \nu) - E(Z, N, I-1, \nu)}{I_x(Z, N, I+1, \nu) - I_x(Z, N, I-1, \nu)} \quad (3)$$

$$\lambda_n(Z, N, \omega, \nu) = (E(Z, N+1, \omega, \nu) - E(Z, N-1, \omega, \nu))/2, \quad (4)$$

where ν is an additional label identifying similar intrinsic configurations in the neighboring isotopes, e.g. the lowest even-spin positive-parity sequence in neighboring even-even isotopes, as discussed here. Contours are shown both for the even-spin, positive-parity yrast sequence in even-even isotopes, labelled $(\pi, \alpha) = (+, 0)$, and the lowest negative-parity decay sequence in the odd-mass isotopes $(-, 1/2)$. Band crossings associated with the excitation of pairs of $i_{13/2}$ quasineutrons should be the same in these decay sequences since in the negative parity band of the odd-mass nuclei the $i_{13/2}$ orbitals have not been blocked. Several observations can be made from fig. 1:

1. Below the band crossing the contours are approximately equally-spaced in λ_n . This reflects the existence of sizable pair correlations, which smear out the level occupations, so that, on average, a given change in N corresponds to roughly the same change in λ_n .
2. Above the band crossing, where neutron pair correlations are greatly reduced, the more irregular spacings are indicative of fluctuations in the single-neutron level distribution. For example, the sizable spacing of the contours for $N=96$ and 106 at large $\hbar\omega$ corresponds to gaps in the single-neutron states near these neutron numbers⁵. In the limit of vanishing pair correlations and stable nuclear shapes for isotopes differing by two neutrons such contours are closely related to the spectrum of single neutron states.

Figure 1: Empirical contours of constant N as a function of λ_n and ω for the lowest states with $(\pi, \alpha) = (+, 0)$ and $(-, 1/2)$ (filled and open circles respectively). The first $i_{13/2}$ quasineutron band crossing frequency in the $(+, 0)$ sequence is indicated by the horizontal dashed lines labelled $\hbar\omega_c$.

3. A series of interesting regions of higher densities of single-neutron levels (i.e. "bunchings" of the constant- N contour lines) are superimposed on the regular features described in items (1) and (2). The features, appearing at $\hbar\omega = 0.27$ MeV for $N = 96-97$ and especially for $N = 90-92$, have been associated⁷ with the diabolical points (discussed below) described by Nikam and Ring⁸. Similar features, observed at $\hbar\omega \approx 0.45$ MeV for $N = 90-92$ and $\hbar\omega \approx 0.40$ MeV for $N = 103-105$ may be associated with diabolical points at larger frequency, perhaps connected with a crossing based on the excitation of an additional pair of quasineutrons, e.g. the second pair of $i_{13/2}$ quasineutrons often referred as the CD pair. Such crossings are unknown in these and other nuclei.

A diabolical point is a singularity where different eigenenergy surfaces of the same Hamiltonian are degenerate (i.e. they touch). But of course in general they can not cross in a quantum system⁹. Zero interaction between configurations of rotating superfluid nuclei differing by two quasiparticles formally meets the requirements for such a diabolical point⁸. These points are simply the minimum interaction band crossings (sharp "backbends") discussed several years ago^{10,11}. Recently it has been suggested⁸, and demonstrated⁷, that these singularities lead to closely-spaced, constant- N .

contours in the (ω, λ_n) plane such as those described in the preceding paragraph. The magnitude of the minimum separation of such contours contains information on the minimum interaction between such intrinsic quantum states differing by two quasineutrons, i.e. the adiabaticity of these two quantal states. Unfortunately, the "grid" imposed by the quantization of the nuclear states both in particle number and angular momentum and the uncertainty in λ_n associated with deformation changes between the $\Delta N=2$ neighboring isotones, see equation (4), complicate the extraction of a minimum interaction. For example, the overlapping contours for the $N=90$ and 92 ytterbium isotopes at $\hbar\omega \approx 0.27$ MeV are attributed to such variations in the nuclear shape.

The phase of the wave function, the Berry phase, when a diabolical point is circumscribed, should change sign^{8,12}. It may be possible to observe consequences of this nuclear equivalent of the Aharonov-Bohm effect¹³ in the next generation of Coulomb excitation plus transfer experiments or in Coulomb excitation studies where weak interband transitions between states differing by two quasiparticles are observed above and below the band crossing. The present constant- N contour analysis identifies candidates for such studies of the Berry phase in nuclei.

3 Empirical Proton-Neutron Interactions

The difference between values of λ_n for isotones differing by two protons can be interpreted¹⁴ as the average p-n interaction between the pair of protons and the pair of neutrons nearest the Fermi level that are coupled to spin zero and positive parity. That is:

$$\delta V_{pn}(Z, N, \omega) = \frac{1}{2} \cdot (\lambda_n(Z+1, N, \omega) - \lambda_n(Z-1, N, \omega)). \quad (5)$$

This expression can be rewritten as

$$\begin{aligned} \delta V_{pn}(Z, N, \omega) = & \frac{1}{4} \cdot ([B_\omega(Z+1, N+1, \omega) - B_\omega(Z+1, N-1, \omega)] \\ & - [B_\omega(Z-1, N+1, \omega) - B_\omega(Z-1, N-1, \omega)]) \end{aligned} \quad (6)$$

where B_ω is the total energy at given ω , defined as

$$B_\omega(Z, N, \omega) = B(Z, N) + E_{ex}(Z, N, \omega) \quad (7)$$

in terms of the binding energy (negative), B , and the excitation energy E_{ex} at frequency ω . In this form, δV_{pn} can be extracted for any set of intrinsic nuclear states (e.g. ground states, specific quasiparticle configurations, intruder states, gamma bandhead states, etc.). Such an average empirical p-n interaction contains all relevant multipoles. Indeed variations in the mean nuclear field associated with the "last" pair of protons and neutrons are included as well as the nucleon-nucleon interactions. For simplicity, consider the formulae for extracting δV_{pn} in the case of ground states¹⁵. From equations (6,7) it is apparent that δV_{pn} is simply a double difference of the binding energies. If the binding energy is partitioned as follows,

$$B(Z, N) = V_{nn}(N) + V_{pp}(Z) + V_{n,mf}(N) + V_{p,mf}(Z) + V_{pn}(Z, N), \quad (8)$$

where the subscript mf means the mean field, the first difference in equation (6) cancels the

Figure 2: Plot of δV_{pn} for the lowest even-spin, positive-parity configurations of even-even deformed rare-earth nuclei as a function of N for various values of Z and $\hbar\omega$. The proton numbers indicated refer to the last proton pair.

proton-proton interaction, the proton contribution to the mean field and isolates the interaction of the last two neutrons with all $Z+1$ protons and with each other and contains their contribution to the mean field. The terms in the second square bracket give the interaction of the last two neutrons with $Z-1$ protons and with each other and their contribution to the mean field. The difference of these two quantities cancels the p-p and n-n interaction as well as all their contributions to the mean field, giving the effective interaction of the last two neutrons with the last two protons. Dividing by four gives the average interaction of each of the last two protons with each of the last two neutrons. This technique is similar to those discussed by Garvey et al.¹⁶, Sakai¹⁷, Arenas Peris and Federman¹⁸, and Janecke and Comay¹⁹.

δV_{pn} values, extracted for the yrast configuration of neighboring even-even isotopes and isotones with $Z=68-78$ and $N=90-108$, are shown in fig. 2 as a function of N for various values of Z and ω . The average value of this interaction, $\langle \delta V_{pn} \rangle \approx -240$ keV, is reasonable — the nuclear potential for a nucleus with $A=170$ is about -50 MeV, or about -290 keV per nucleon. Therefore, the addition of a nucleon should increase the (negative) potential by about 290 keV. This value is also in qualitative agreement with empirical estimates of the p-n interaction for spherical nuclei²⁰.

However, there are significant variations in δV_{pn} as a function of N , Z , and ω . Many of these variations can be explained by relative overlaps of the proton and neutron configurations near the Fermi surface and the strength of the pair correlations. For example, the largest variations in δV_{pn} occur at large values of ω where the neutron pair correlations are greatly reduced⁵. Therefore, this

interaction should be characteristic of an overlap of the wave function of a specific pair of neutrons (instead of a pair of quasineutrons based on an ensemble of neutron pairs near the Fermi level) with the quasiproton pair nearest the proton Fermi level. Much of the N and Z dependence of δV_{pn} seems to be associated with the overlaps of the proton and neutron configurations nearest the respective Fermi levels. For instance, the 71st and 72nd protons seem to interact more strongly (larger $|\delta V_{pn}|$ values) with the 91st and 92nd, the 97th and 98th, and the 99th and 100th neutrons than with the 93rd and 94th and the 95th and 96th neutrons. This can be qualitatively explained as follows: The occupation probability of the $\Omega=1/2 \pi(h_{9/2})$ prolate orbital (downsloping on the Nilsson plot) is large for the 71st and 72nd protons. This proton orbital has a large overlap with the 91st and 92nd neutrons where the $\Omega=3/2$ component of the prolate $\nu(i_{13/2})$ orbital is near the neutron Fermi level. When the neutron number increases, the less downsloping neutron orbitals, the $\Omega=3/2$ and $5/2$ components of the $\nu(h_{9/2})$ and $\nu(f_{7/2})$ configurations are near the Fermi level; therefore, $|\delta V_{pn}|$ decreases for $N=93,94$ and $95,96$. When the $\Omega=5/2$ components of the $\nu(i_{13/2})$ configuration are near λ_n for $N=97,98$ and $99,100$ δV_{pn} again becomes more negative.

Figure 3: Ground state δV_{pn} values from experimental (not extrapolated) binding energies for all even-even nuclei with $N > 40$, for which the uncertainty of δV_{pn} is 50 keV or less. The inset gives an expanded view for nuclei with $Z=35-49$ and $N=51-63$.

A systematic analysis of the global variation of the ground state p-n interaction was made recently²¹ for all even-even nuclei with measured masses²². Fig. 3 shows the δV_{pn} values of the ground states of all nuclei with $N > 40$. We see that δV_{pn} varies smoothly from about -200 keV

in the actinides to about -600 keV near $N=40$. This general trend has long been known²³, and can be qualitatively understood by the increase of the radii of the proton and neutron orbits with increasing mass. Since the range of the p-n interaction is roughly constant, its strength decreases as the average distance between the last proton and neutron increases. For heavy nuclei, this effect of lower overlap is exacerbated by the increase of $(N-Z)$ so that the neutron radii increase faster than the proton radii. The figure gives another example of the sensitivity of the p-n interaction to the configuration. The inset shows δV_{pn} for nuclei with $Z=35-49$, and $N=51-63$. The staggering of $|\delta V_{pn}|$ for the $Z=41$ chain, decreasing from $N=51$ to 55, increasing for $N=57$, then decreasing again for $N=59$ and 61, reflects the overlap of proton and neutron orbitals in corresponding nuclei. From the Nilsson diagram at the small deformations appropriate to these nuclei, one can see that the 41st proton is generally in a strongly downsloping low Ω $\pi(g_{9/2})$ orbit, while the 51st to 55th neutrons fill subsequently $\nu(d_{5/2})$ orbitals with $\Omega=1/2$ to $5/2$ (decreasing the down slope) and then the $\nu(g_{7/2})$ orbit starting from the 57th neutron. For $Z=39$ the δV_{pn} values vary in the opposite manner because the last proton mainly occupies the $\pi(p_{1/2})$ upsloping orbital.

4 Summary

The use of constant- N contours as a function of the empirical Lagrangian multipliers, ω and λ , (single differentials of the total energy with respect to I and N or Z) is demonstrated as a technique both for locating diabolical points and for studying regularities and fluctuations in the spectrum of single-particle states. Some general comments are made on the physical basis of diabolical points and the possibility of observing the Berry phase associated with these quantum mechanical singularities in Coulomb excitation and transfer experiments.

These empirical analyses are extended to double differentials of the total energy with respect to N and Z , which is interpreted as an effective proton-neutron interaction, δV_{pn} . Such empirical interactions are presented for the ground states of all even-even nuclei with $N>40$ and as a function of the rotational frequency ω for the lowest even-spin, positive-parity decay sequence of deformed rare earth nuclei. The general decrease of the δV_{pn} values of the ground states with increasing mass is attributed to the decreased overlap of valence proton and neutron orbitals due to the occupancy of shells with different average radii. Many of the fluctuations associated with δV_{pn} as a function of N , Z , and ω can be correlated with expected variations in the overlap of the valence proton and neutron configurations. However, care must be exercised in the interpretation of such empirical interactions since these quantities also include modifications of the mean nuclear field associated with the valence nucleons^{24,25}.

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References

- [1] R.A. Broglia, O. Hansen and C. Riedel, *Adv. Nucl. Phys.* 6 (1973) 287.
- [2] A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975).
- [3] R. Bengtsson, J.-Y. Zhang and S. Åberg, *Phys. Lett.* 105B (1981) 5.
- [4] J.-Y. Zhang, *Nucl. Phys.* A421 (1984) 353c.
- [5] J.C. Bacelar, M. Diebel, C. Ellegaard, J.D. Garrett, G.B. Hagemann, B. Herskind, A. Holm, C.X. Yang, J.-Y. Zhang, P. O. Tjøm, and J.C. Lisle, *Nucl. Phys.* A442 (1985) 509.
- [6] J.-Y. Zhang, J.D. Garrett, J.C. Bacelar, and S. Frauendorf, *Nucl. Phys.* A453 (1986) 104.
- [7] J.-Y. Zhang and J.D. Garrett, *Phys. Rev. Lett.* 62 (1989) 693.
- [8] R.S. Nikam and P. Ring, *Phys. Rev. Lett.* 58 (1987) 980; P. Ring, in: *Contemporary Topics in Nuclear Structure Physics*, Cocoyoc, Mexico, 1988, eds. R.F. Casten et al. (World Scientific, Singapore, 1988) p. 677.
- [9] J. von Neumann and E.P. Wigner, *Z. Phys.* 30 (1929) 467.
- [10] R. Bengtsson, I. Hamamoto, and B. Mottelson, *Phys. Lett.* 73B (1978) 259.
- [11] F. Grümmer, K.W. Schmid, and A. Faessler, *Nucl. Phys.* A236 (1979) 1.
- [12] M.V. Berry, *Proc. Roy. Soc. (London)*, Ser. A 392 (1984) 45.
- [13] Y. Aharonov and D. Bohm, *Phys. Rev.* 115 (1959) 485.
- [14] J.-Y. Zhang, C.-S. Wu, C.-H. Yu and J.D. Garrett, in: *Contemporary Topics in Nuclear Structure Physics*, Cocoyoc, Mexico, 1988, Abstract Volume, p.109.
- [15] J.-Y. Zhang, R.F. Casten and D.S. Brenner, *Phys. Lett.* 227B (1989) 1.
- [16] G.T. Garvey and I. Kelson, *Phys. Rev. Lett.* 16 (1966) 197.
- [17] M. Sakai, *Nucl. Phys.* A345 (1980) 232, and references therein.
- [18] G.E. Arenas Peris and P. Federman, *Phys. Lett.* 173B (1986) 359; *Phys. Rev. C* 38 (1988) 493.
- [19] J. Janecke and E. Comay, *Nucl. Phys.* A436 (1985) 108.
- [20] J.P. Schiffer and W.W. True, *Rev. Mod. Phys.* 48 (1976) 191.
- [21] D.S. Brenner, C. Wesselborg, R.F. Casten, D.D. Warner and J.-Y. Zhang, *Phys. Lett. B*, in print.
- [22] A.H. Wapstra, G. Audi and R. Hoekstra, *At. Data and Nucl. Data Tables* 39 (1988) 281.
- [23] see e.g. N. Zeldes, M. Gronau and A. Lev, *Nucl. Phys.* 63 (1965) 1.
- [24] J.D. Garrett, G.B. Hagemann and B. Herskind, *Nucl. Phys.* A400 (1983) 113c.
- [25] S. Frauendorf, L.L. Riedinger, J.D. Garrett, J.J. Gaardhøje, G.B. Hagemann and B. Herskind, *Nucl. Phys.* A431 (1984) 511.

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