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MHD STABLE REGIME OF THE TOKAMAK

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Abstract

A broad family of tokamak current profiles is found to be stable against ideal and resistive MHD kink modes for $1 \leq q(0)$, with $q(a)$ as low as 2. For $0.5 \leq q(0) < 1$ and $q(a) > 1$, current profiles can be found that are unstable only to the $m = 1, n = 1$ mode. A specific "optimal" tokamak profile can be selected from the range of stable solutions, by imposing a common upper limit on dj/dr -- corresponding in ohmic equilibrium to a limitation of dT_e/dr by anomalous transport.

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I. INTRODUCTION

A certain family resemblance of experimentally observed tokamak electron temperature profiles $T_e(r)$ has been pointed out [1,2] as providing an important clue to the physical understanding of tokamak confinement. Even in the course of considerable variations in the tokamak auxiliary-heating power deposition profile, the gross energy confinement time τ_E is found to remain roughly constant. For given central and limiter q -values, specific $T_e(r)$ -profiles tend to maintain themselves. In order to understand this "profile consistency" phenomenon and the problem of global energy confinement, Furth [3] and Ohya et al. [4] independently suggested that tokamak profile consistency may be related to the requirement that $j(r)$ -profiles must be stable to both ideal and resistive kink instabilities. This issue has also been addressed in Ref. 5. In order to arrive at a unique $j(r)$ -profile, there must be some auxiliary constraint -- for example, a profile-smoothness requirement relating to anomalous heat conductivity.

In resistive equilibrium, the shape of $T_e(r)/T_e(0)$ follows from $j(r)$ by Ohm's law and corresponds roughly to $j(r)^{2/3}$, neglecting neoclassical effects. The magnitude of $T_e(0)$ is determined by the heat flow equation; the experimental evidence indicates that dT_e/dr in the plasma edge region, outside $q(r) = 2$, provides the critical calibration factor [3,4].

A survey of the total range of MHD-stable $j(r)$ -profiles for specified values of $q(0)$ and $q(a)$ is useful in several respects: First, it provides guidance in regard to MHD-stable current profiles for microscopic theories of anomalous transport, such as those pursued by Perkins [6], Tang [7], and others, which rely on ad hoc models of $j(r)$. Second, for given values of $q(0)$ and $q(a)$, we can calculate the range of permissible internal inductances λ_i and volume-averaged electron temperatures $\langle T_e(r) \rangle / T_e(0)$ for comparison with

experiments. Third, the advent of effective diagnostics for direct measurement of experimental $j(r)$ -profiles will make possible a detailed comparison with the predictions of resistive kink mode theory. Finally, we hope to deduce a physical model for tokamak profile consistency that can account for the observed energy confinement -- possibly including the L-mode/H-mode phenomenon.

The present study has determined the total range of MHD-stable $j(r)$ -profiles, based on the marginal stability of an infinitesimally perturbed pressureless equilibrium [8] in a straight cylinder, using the model described in Section II. The range of stable $j(r)$ -profiles for $q(0) \geq 1$ is described in Section III; a possible method for the selection and amplitude calibration of specific $T_e(r)$ -profiles is presented in Section IV. The range of $j(r)$ -profiles stable against modes other than $m/n = 1$ is investigated in Section V for the $q(0) < 1$ regime, with a view to assessing the potential benefits of $m/n = 1$ feedback stabilization.

II. MODEL

For a perturbed pressureless equilibrium in a straight cylinder, the marginal stability equation [8] is given by

$$\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \Psi - \frac{m^2}{r^2} \Psi = \left[\frac{m}{q(r) B_\theta(r)} \frac{dj}{dr} \right] \frac{[m/q(r) - n]}{[m/q(r) - n]^2 + m^2 \Delta_{mn}^2} \Psi, \quad (1)$$

where $\Psi = r b_r$ and b_r is the radial component of the perturbed magnetic field. In Eq. (1), we have resolved the singularity in a crude way by replacing $(m/q - n)^{-1}$ with $(m/q - n)/[(m/q - n)^2 + m^2 \Delta_{mn}^2]$, where Δ_{mn} represents the finite island width associated with the (m,n) Fourier mode. For vanishing Δ_{mn} , Eq. (1) is the equation for the marginally stable MHD mode outside the

singular resistive layer; it provides Δ' for the assessment of kink-mode stability. For an infinitesimal Δ_{mn} , the plasma is linearly stable to MHD perturbation if and only if Ψ is everywhere positive definite [9]. For a finite value of Δ_{mn} , if the solution b_r of Eq. (1) is such that b_r goes to zero at infinity (or at a conducting wall), then the plasma will form a magnetic island of finite width, approximately Δ_{mn} , on a resistive time scale. Such equilibria are unstable to the (m,n) resistive kink mode.

We have developed a semi-automatic $j(r)$ -profile searching process where all modes up to $m,n = 20$ are examined for stability, and where adjustments in $j(r)$ are made to eliminate unstable modes. For given $q(0)$ and $q(a)$, $j(r)$ is being varied numerically subject to the constraints that $j(r) \geq 0$ and $dj/dr \leq 0$. We determine the total range of stable $j(r)$ -profiles as $q(0)$ and $q(a)$ are varied. In our calculations we use $\Delta_{mn} \leq 10^{-4}$ and assume no conducting shell. Convergence of stable $j(r)$ -profiles has been checked by letting $\Delta_{mn} \rightarrow 0$, with correspondingly greater computational efforts.

III. MHD-STABLE $j(r)$ -PROFILES IN $q(0) \geq 1$ REGIME

In terms of a comparison theorem derived by Furth et al. [10], we will first show that there is no stable $j(r)$ -profile of any shape for $(m-1) < nq(0) < nq(a) < m$. The comparison theorem states the following: For two $j(r)$ -profiles having the same shear $(dq/dr)_s$ and the same safety factor q_s at the singular surface r_s of a given (m,n) mode, if the two profiles everywhere satisfy $|m/q_1(r) - n| \leq |m/q_2(r) - n|$, then the profile $j_1(r)$ is more unstable against the given (m,n) mode than $j_2(r)$. It is well known [11] that for a constant $j(r)$ -profile with no conducting wall, the plasma is unstable to kink modes; each m -mode is unstable for $(m-1) < nq(0) < m$, and for each m all integers n are possible. For integral

$nq(0)$ the plasma is marginally unstable. Now consider a monotonically decreasing current profile $j_1(r)$, which has the same $q(0)$ as the uniform profile $j_2(r)$, but has $q(a)$ such that $(m-1) < nq(0) < nq(a) < m$. Since both $j_1(r)$ -profiles have the same (dq/dr) and q at the singular surface r_s of the (m,n) mode, it follows from the comparison theorem that the $j_1(r)$ -profile must be more unstable than the $j_2(r)$ -profile, which can at best be marginally stable. Accordingly, no monotonically decreasing $j(r)$ -profile can be stable for $(m-1) < nq(0) < nq(a) < m$.

For $q(0) = 1$, there can be no stability when $q(a) < 2$. For $q(a) > 2$, we have been able to obtain a family of stable $j(r)$ -profiles. Note that in the previous studies by Glasser *et al.* [12] the lowest $q(a)$ -value found was $q(a) = 2.6$. Figure 1 shows an example of four stable $j(r)$ -profiles with corresponding $q(r)$ -profiles for $q(0) = 1.01$ and $q(a) = 2.1$. These stable $j(r)$ -profiles have the peculiar feature that they do not converge into a unique solution even as $q(a) \rightarrow 2$, but they become so specialized that barely visible deviations, such as those shown in Fig. 2, can induce unstable resistive kink modes, with mode numbers as listed. A detail worth noting in Figs. 1 and 2 is the slope of $j(r)$ in the edge region. To avoid high- m instabilities at singular points located in the vacuum just outside the plasma, the slope of $j(r)$ near the very edge of the plasma must be kept within an appropriate limit [13]. This result may be related to relaxation phenomena where transient reductions in edge density and temperature are observed in sharp-edged H-mode plasmas [14].

For $q(a)$ substantially larger than 2, there is an increase in the degree of latitude for variation in $j(r)$ -profiles. Figure 3 shows four stable $j(r)$ -profiles and corresponding $q(r)$ -profiles for $q(0) = 1.01$ and $q(a) = 8$. To quantify how much latitude in $j(r)$ -profile shape becomes available, we have

calculated the internal inductance λ_i and volume-averaged $\langle T_e(r) \rangle / T_e(0)$ by assuming $T_e \propto j^{2/3}$. Profiles with $j(r)$ concentrated near $r = 0$ correspond to higher λ_i and lower $\langle T_e(r) \rangle / T_e(0)$ values. The highest internal inductance is provided by the case where $j(r)$ is constant up to $r/a = [q(0)/q(a)]^{1/2}$, surrounded by a vacuum region that extends out to $r = a$. The corresponding internal inductance is given by $\text{Max}(\lambda_i) = (1/2) [1 + 2 \ln(q(a)/q(0))]$, and the corresponding $\langle T_e(r) \rangle / T_e(0)$ equals $q(0)/q(a)$.

Figure 4 shows the range of permissible $(\lambda_i/2)$ -values as a function of $q(a)$ for $q(0) = 1.01$. Outside the "jigsaw" boundary, no stable $j(r)$ -profile is possible. The lower bound on $(\lambda_i/2)$ is mainly due to ideal external kink modes. The upper bound on $(\lambda_i/2)$ is due to low-order resistive kinks, mainly $m/n = 2/1$ and $3/2$. When a conducting wall is introduced, the lower bound is relaxed, but the upper bound is little affected. To compare with experimentally observed values of λ_i and $q(a)$, we have plotted about one hundred steady-state data points from TFTR operations and find that all fall inside the stable domain. In Fig. 5 we plot the temporal evolution of $(\lambda_i/2)$ and $q(a)$ for three typical nondisrupted TFTR discharges, where the open circle represents the beginning and the solid circle the end of the plasma evolution. The values of $(\lambda_i/2)$ and $q(a)$ remain inside the permissible domain throughout the entire pulse length. For disrupted TFTR discharges, the values of $(\lambda_i/2)$ and $q(a)$ are found to enter the unstable region just before the plasma disrupts, as shown in Fig. 6, for two typical disrupted TFTR shots.

In Fig. 7, the range of permissible values of $\langle T_e(r) \rangle / T_e(0)$ as a function of $1/q(a)$ for $q(0) = 1.01$ is bounded by two roughly straight lines. The lower line corresponds to $j(r)$ -profiles with higher internal inductance. Also shown in Fig. 7 are the data from TFTR experiments [15]; all data are averaged over several sawtooth periods, except the data shown by a vertical line, which is

bounded by two values corresponding to the $T_e(r)$ -profiles before and after the large internal disruption. Most of the data lie within the permissible domain, except one point at $q(a) \approx 2.4$ which may have $q(0)$ well below one. Note that Waltz et al. [16] have provided a mathematical proof that $\langle T_e(r) \rangle / T_e(0)$ is bounded by $[q(0)/q(a)]^{2/3} \geq \langle T_e(r) \rangle / T_e(0) \geq q(0)/q(a)$, as shown by the two broken curves in Fig. 7.

The experimental results clearly indicate that MHD-stable $j(r)$ -profiles tend to maintain themselves in tokamaks. The profile-shaping mechanisms, however, are not entirely clear. Gross deviations from stability are resisted by MHD oscillations and associated disruptions of magnetic surfaces, but even in the absence of overt MHD activity there appear to be profile-stabilizing mechanisms at work. In such cases, the local ∇T_e , and therefore ∇j , in the vicinity of low-order singular surfaces may be reduced by anomalous microscopic thermal conductivity. Particularly if such flattening of $T_e(r)$ and $j(r)$ occurs mainly towards the smaller- r side of a singular surface, it tends to stabilize the locally resonant resistive kink mode. The stability of other kink modes, with neighboring values of m/n , will tend to be impaired, but a stable compromise solution for the overall $j(r)$ -profile is likely to be reached if ℓ_i and $q(a)$ lie well inside the permissible domain. When ℓ_i approaches its upper bound, however, the current channel becomes very narrow, with sharp gradients of j and T_e . In that case, the $j(r)$ -profile needed for Δ' -stability is less likely to be reached by enhanced local thermal transport, so that partial or total MHD disruptions are triggered. In the TFTR experiments [17], Δ' -stability of high- m modes tends to be violated during current ramp-up, and the resultant magnetic reconnections help to modify $j(r)$ and restore $\Delta' < 0$. The current-plateau phase is normally characterized by $\Delta' \leq 0$ and MHD quiescence. In the exceptional cases that end disruptively, Δ'

becomes positive for $m/n = 2/1$ and $3/2$ and corresponding magnetic oscillations are seen.

The preceding discussion does not refer to stabilization by an external conducting wall. Present-day tokamak experiments are rarely equipped with sufficiently close-fitting shells to have much impact on δ' -stability, but the potential of the wall-stabilization technique deserves to be kept in mind in the future. Its effectiveness will depend on the degree to which plasma rotation or active feedback control can compensate for the finite skin-relaxation time of a resistive conducting shell.

IV. SELECTION AND CALIBRATION OF $T_e(r)$ -PROFILES

In resistive equilibrium, the normalized electron temperature $T_e(r)/T_e(0)$ is roughly proportional to $[j(r)/j(0)]^{2/3}$. A way to determine the magnitude of $T_e(r)$, independently suggested by Furth [3] and Ohya et al. [4], calls for the magnitude of $T_e(r)$ to be calibrated by plasma transport at some critical radius r_c , where the thermal conductivity $\chi_e(r)$ is mainly due to local microturbulence.

The steady-state heat flow equation is given by

$$-n(r) \chi_e \frac{dT_e}{dr} = S(r) = \frac{1}{r} \int_0^r dr' r' [P_H(r') - P_R(r')] \quad , \quad (2)$$

where $S(r)$ is the power flux, $P_H(r)$ is the heating power, $P_R(r)$ the radiation cooling power, and $n(r)$ the plasma density. Experimental evidence [15,18] shows that χ_e must be a function of $S(r)$, and that the global confinement time τ_E is insensitive to the precise radius of auxiliary power deposition out to some critical r_c that appears to be located outside the $q = 2$ surface. At the point r_c , we can write

$$-n(r_c) \chi_m(r_c) \left. \frac{dT_e}{dr} \right|_{r_c} = S(r_c) \quad , \quad (3)$$

where $\chi_e = \chi_m$ is the local transport due to microturbulence. The gross energy confinement time τ_E is then determined by [19]

$$\tau_E \propto \frac{r_a^2 \langle n \rangle}{\chi_m(r_c) n(r_c)} \quad , \quad (4)$$

where $\langle n \rangle$ is a temperature-weighted volume-average of n . The confinement model given by Eq. (4) implies that there are two multiplicative τ_E -enhancement factors: (1) increase of the MHD-stable energy content in the plasma core by increasing $n(0)$; (2) improvement of thermal insulation in the outer shell by reducing $n(r_c)$ and $\chi_m(r_c)$. Unlike the theories of anomalous transport by Perkins [6], Tang [7], and others, where gross confinement is determined by an average of χ_e between the $q = 1$ and $q = 2$ surfaces, Eq. (4) implies that gross confinement depends on the region $q > 2$.

Equation (3) offers a prescription for calibrating the amplitude of the $T_e(r)$ -profile, as well as a means for selecting an "optimal" $j(r)$ -profile from the family of stable cases. We consider a model where dT_e/dr -- and therefore dJ/dr -- is constrained by Eq. (3), and the optimal profile is defined as that which permits the highest plasma energy content for given $q(0)$ and $q(a)$. Figure 8 illustrates how this model can be used to select a particular $j(r)$ -profile from a broad family of stable profiles (curves H, K, L) for $q(0) = 1.01$ and $q(a) = 4$. $T_e(r)$ is assumed to be proportional to $[j(r)]^{2/3}$ and the magnitude of the $T_e(r)$ -profile is calibrated by a common limit on dT_e/dr in an outer region (marked by arrows in the figure) between the $q = 2$ surface and the limiter. The amplitude $T_e(0)$ is found to be highest for the

L curve. If $T_e(r)$ in the $q < 2$ region is assumed to be determined by $j(r)$ -profile-shaping requirements, rather than by χ_m , one might expect that, for sufficiently strong central heating, $T_e(r)$ will rise to some marginally stable solution of type L, independent of the detailed form of the power-deposition profile.

Divertor experiments [14,20] clearly show that improved global energy confinement (H-mode operation) can be achieved by architectural improvements in the outer plasma shell. For a fixed total plasma current, divertor plasmas appear to have two basically different MHD-stable $j(r)$ -profile options: (1) the usual L-mode profile, where dj/dr is forced to become small toward the plasma edge, since dT_e/dr is depressed by the normal pattern of radially increasing $\chi_m(r)$ and "atomic" cooling effects; and (2) the H-mode "pedestal" profile, where dT_e/dr is large at the plasma edge -- perhaps by virtue of the proximity of the separatrix, which may be helpful in limiting χ_m and/or reducing atomic effects. Figure 9 shows what happens to the $T_e(r)$ -profile competition of Fig. 8 if the region of common dT_e/dr -limitation (marked by the arrows in the figure) is shifted away from the plasma edge. Now the H-type profile allows the highest core temperature and can therefore provide the best gross energy confinement.

Note that the above discussion suffers from the inconsistency that our stability analysis is one-dimensional, while the presence of a separatrix implies at least a two-dimensional geometry. According to Ref. 21, the impact of the separatrix on the one-dimensional kink-stability results may not be very great; in any case, a mild generalization of the stability-comparison theorem of Ref. 10 suggests that the separatrix effect could only be further stabilizing. A true two-dimensional resistive-kink-mode analysis would be desirable to reach quantitative conclusions.

V. NEARLY MHD-STABLE $j(r)$ -PROFILES IN THE $q(0) < 1$ REGIME

For $q(0) < 1$, conventional tokamak plasmas are characterized by the sawtooth mechanism [22], where the central plasma temperature, current density, and rotational transform q^{-1} exhibit a roughly linear rise on a slow resistive time scale, interrupted periodically by a fast dynamic process of magnetic reconnection that is associated with the $m = 1, n = 1$ resistive kink mode [23,24]. Obtaining stable plasma operation at the lowest possible value of $q(0)$, by suppressing the $m = 1, n = 1$ mode would have the obvious advantage of allowing higher plasma current, thus providing improved confinement, a higher beta limit, and stronger ohmic heating. Assuming that the $m = n$ modes may be controllable by special techniques [25] such as rf-feedback control or lower hybrid current drive, we look for $j(r)$ -profiles that are stable against all other m,n modes. (No conducting shell is assumed.)

For $0.5 \leq q(0) < q(a) < 1$, there is an unstable $m = 1, n = 1$ external kink mode, which would not lend itself to stabilization by rf techniques. From the comparison theorem [10], it can also be shown that, due to the unstable $m = 1, n = 2$ mode, there can be no stability for $q(0) < 0.5$. As $q(a)$ is lowered down to 1 for $0.5 < q(0) < 1$, higher-order resistive modes with $m/n = 3/5, 2/3, 3/4, 4/5$, etc., tend to be unstable. Our $j(r)$ -profile optimization code has been able to find stable $j(r)$ -profiles with $q(0)$ down to 0.5 and $q(a)$ down to 1 -- a significant improvement over previous work [26], where a lower limit on $q(0)$ of about 0.67 was imposed, so as to exclude the $2/3$ mode, and a lower limit on $q(a)$ of about 1.5 was needed to stabilize the $3/2$ mode.

Figure 10 shows an illustrative $j(r)$ -profile that is unstable only to the $1/1$ mode, and the associated $q(r)$ -profile with $q(0) = 0.5$ and $q(a) = 2$. The

broken curve represents a neighboring profile with additional unstable modes ($m/n = 2/3, 3/5$). Note that the more stable $j(r)$ -profile is quite peaked near $r = 0$ and flattened near the $q = 2/3$ and $3/5$ surfaces. Figure 11 shows that, as $q(a)$ is reduced to 1.1, the $j(r)$ -profile (solid curve) remains peaked near $r = 0$ and becomes broader between the $q = 0.6$ and $q = 0.75$ surfaces, so as to stabilize the $m/n = 3/5, 2/3$, and $3/4$ modes.

VI. CONCLUSIONS

In this paper we have determined the total range of MHD stable $j(r)$ -profiles as $q(0)$ and $q(a)$ are varied in the region of maximum experimental interest. For $q(0) \geq 1$, stable $j(r)$ -profiles can be obtained only when $q(a) > 2$. For $0.5 < q(0) < 1$ and $q(a) > 1$, $j(r)$ -profiles have been found that are unstable only to the $m = 1, n = 1$ mode. To indicate the degree of latitude in regard to stable $j(r)$ -profile shapes, we have plotted boundaries for the permissible β_1 and $\langle T_e(r) \rangle / T_e(0)$, as functions of $q(a)$ for given $q(0)$. Experimental TFTR results indicate that nondisrupted tokamak discharges are normally confined within these boundaries.

We consider a model for selection of specific $j(r)$ -profiles from the family of stable solutions, in terms of maximization of the plasma energy content. If dT_e/dr is limited by transport at the plasma edge, this approach selects profiles that resemble the usual I-mode pattern. If dT_e/dr is limited further inside the plasma, profiles of the H-mode type are preferred.

Finally, we note that our $j(r)$ -profile-optimization study is based on a one-dimensional analysis of resistive and ideal MHD stability that assumes a pressureless cold plasma in a straight cylinder with no conducting shell. More realistic calculations should involve finite- β plasmas in a two-dimensional toroidal equilibrium with appropriate boundary conditions. A

number of studies have indicated that toroidicity and finite- β [27], as well as the presence of a separatrix [21], or a conducting shell [10] can have stabilizing effects on the resistive kink modes, though finite β can also be destabilizing to ideal kink modes [11] and can induce ballooning modes [28]. On the whole, the present profile constraints for resistive kink stability may turn out to be somewhat conservative.

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FIGURE CAPTIONS

FIG. 1 An example of four MHD-stable $j(r)$ -profiles with corresponding $q(r)$ -profiles.

FIG. 2 Unstable $j(r)$ -profiles and corresponding $q(r)$ -profiles with barely visible deviations from the profiles of Fig. 1. The unstable mode numbers are also shown.

FIG. 3 Four illustrative MHD-stable $j(r)$ and $q(r)$ profiles for $q(0) = 1.01$ and $q(a) = 8$.

FIG. 4 The range of permissible internal inductance $\ell_i/2$ as a function of $q(a)$ for $q(0) = 1.01$ is contained by a jigsaw boundary. The maximum $\ell_i/2$, which corresponds to a uniform current profile up to $r/a = [q(0)/q(a)]^{1/2}$, is also shown. Steady-state data from TFTR operations are found to fall inside the permissible domain.

FIG. 5 Temporal evolution of $\ell_i/2$ and $q(a)$ for three typical nondisrupted TFTR discharges, where the open circles represent the beginning and the solid circles the end.

FIG. 6 Temporal evolution of $\ell_i/2$ and $q(a)$ for two typical disrupted TFTR discharges, where the open circles represent the beginning of evolutions that end up in the unstable region as the solid circles, just before the plasma disrupts.

FIG. 7 The range of permissible $\langle T_e(r) \rangle / T_e(0)$ as a function of $1/q(a)$ for $q(0) = 1.01$ is bounded by two roughly straight lines. Also shown are TFTR data averaged over several sawtooth periods. The two broken curves represent the boundaries given by Waltz et al. in Ref. 15.

FIG. 8 Given a family of MHD-stable $j(r)$ -profiles (curves H,K,L) with the same $q(0)$ and $q(a)$, the L-type profile has the highest temperature $T_e(0) \propto j(0)^{2/3}$ on the basis of a common limit of dT_e/dr in the outer region marked by the arrows in (a).

FIG. 9 When the profile-comparison of Fig. 9 is repeated with the region of dT_e/dr -calibration narrowed as indicated in (a), the highest $T_e(0) \propto j(0)^{2/3}$ is reached by the H-profile.

FIG. 10 A $j(r)$ -profile that is unstable only to the $m = 1, n = 1$ kink mode, and its corresponding $q(r)$ -profile for $q(0) = 0.5$ and $q(a) = 2.0$. An unstable $j(r)$ -profile is shown by the broken curve.

FIG. 11 $j(r)$ and $q(r)$ -profiles as in Fig. 11 for $q(0) = 0.5$ and $q(a) = 1.1$.

STABLE PROFILES

#86X0635

Fig. 1

UNSTABLE PROFILES

86X0636

Fig. 2

Fig. 3

86X0630

Fig. 4

86X0631

Fig. 5

#86X0632

Fig. 6

Fig. 7

Fig. 8

#86X0625

Fig. 9

Fig. 10

#86X0629

Fig. 11

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