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TITLE: LECTURE 4: APPENDIX - ENERGY STORAGE CAPACITORS

AUTHOR(S): W. J. Sarjeant

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SUBMITTED TO: University of New Mexico
Graduate Course EEC5596

entitled

"HIGH VOLTAGE/PULSE POWER TECHNOLOGY"
(course coordinator- W. Sarjeant)

and

Los Alamos Scientific Laboratory
E Division Training Course

entitled

"POWER CONDITIONING TECHNOLOGY"
(course coordinator - W. Sarjeant)

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HIGH VOLTAGE/PULSE POWER TECHNOLOGY

GRADUATE COURSE EECS596

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Los Alamos Scientific Laboratory
ATTN: W.J. Sarjeant (or Chief Librarian)
P.O. Box 1663
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Dear Sir:

As technical information consultants, we recently obtained a report for one of our clients that was written by W. Sarjeant:

LA-UR-79-1044-App., "Lecture 4: Appendix - Energy Storage Capacitors"

Inside of the report, after Page 47, is a list of 13 lectures (see enclosure) on the subject of HIGH VOLTAGE/PULSE POWER TECHNOLOGY.

It seems that the report we have obtained may be part of this entire lecture series (although its title does not match the title for Lecture 4 in the list). Our client is interested in obtaining the entire series.

The following page (also enclosed) states that there are lecture notes available on these 13 topics -- but they are not included in LA-UR-79-1044-App.

We are interested in knowing if all of these lecture notes are available, and the cost. If available and free, would you kindly send one set to me. If available but not free, please first specify the cost and availability. If not available, please let me know.

Thank you for your help!

Cordially,

Merton Allen
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PREFACE

With the recent increase in technological needs and the interest in the power conditioning arena, one of the problems facing workers in the field is the lack of texts or notes describing recent progress, particularly in the area of repetitive power conditioning. For this reason and because of expanding internal requirements, the University of New Mexico (UNM) and the Los Alamos Scientific Laboratory (LASL) have created a set of lecture notes based upon the graduate course taught recently at UNM. The objective of these notes is to create a record of many of the advances in the field since the last text in the field was published just after World War II. In this context, the lectures presented are oriented toward an introduction of the reader to each of the areas described and present sufficient background information to explain many of these advances. They are not intended to serve as design engineering notes, and thus the reader is referred to the references at the end of each lecture for detailed technical information in specific areas.

The preparation of these writings is a result of a considerable teamwork effort on the part of LASL and Sandia staff. In particular, Cathy Correll, in conjunction with Jo Ann Barnes and the rest of her efficient word processing staff, carried the major responsibility for preparation of the lectures while the lecturers did the proofreading and revisions. As course coordinator, it is a pleasure to acknowledge the strong support of Ray Gore, our E-Division Leader, and Shyam Gurbaxani who is the UNM Graduate Center Director, Los Alamos Campus.

W. J. Sarjeant
Los Alamos Scientific Laboratory
Los Alamos, New Mexico
October 3, 1980

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TITLE (In Caps) LECTURE 1: INTRODUCTION TO POWER CONDITIONING

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☒ TALK

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PARTICULARS This is the first chapter of a set of course notes for a seminar on Pulse Power to be given both inside and outside LASL.

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TITLE: LECTURE 1: INTRODUCTION TO POWER CONDITIONING

AUTHOR(S): W(alter) James Sarjeant

SUBMITTED TO: PULSE POWER COURSE

University of California

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Lecture 1

INTRODUCTION TO POWER CONDITIONING

by

W. J. Sarjeant

LECTURE 1

INTRODUCTION TO POWER CONDITIONING

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LECTURE 1

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Lecture 1
An Introduction to Power Conditioning Systems
by
W. J. Sarjeant

1. Scope of the Course

The purpose of the course is to present an overview of the power conditioning aspects of energy transfer systems. The objective, then, is to develop an understanding and appreciation of the physical processes that govern the performance of these systems, including the role of such elements as switches, capacitors, inductors, and resistors. Secondly, to discuss where are the sources of information once a problem area has been identified? A major effort in this course will be to transfer this information in key areas to you as hand-outs. All the information will be current and state-of-the-art, and directed toward these pulse components, with particular emphasis on the area of high-repetition-rate systems, as these are taking on ever increasing significance.

Walt Willis, Bob Butcher, and Bill Nunnally will cover the areas within this course on "Power Conditioning Systems" (PCS) with which they have had considerable experience. Jim Sarjeant will handle the course integration and the discussions of the component technology-base (for example, the present status of power conditioning systems and thyatron switching in general). As we progress, it will be arranged for several experienced individuals to come in and discuss special topics. Some such suggested topics are grounding and shielding, safety, accelerators, and E beams. Ken Prestwich will discuss E-beam accelerators, and Gerry Rohwein will present detailed information on pulse transformers and high-voltage insulation. Tom Boyd and Don Swenson will discuss accelerator systems with us, noting in particular current PCS needs arising from present accelerator research.

The text on which the course is based is Pulse Generators by Glasoe and Lebacqz, the World War II radar modulator text, primarily concerned with charging capacitors or pulse-forming networks (PFN), and then discharging the energy stored therein at high repetition rates, either directly or through a transformer into a load. The secondary load was generally a magnetron. This text will serve as a reference base and then technology developments over the subsequent 30 years will be molded around it. Extensive use will be made of handout information to bring the overall technology base up to date.

The field of "Power Conditioning" is fraught with differing definitions as to what it means, as well as defining the branching points within each area. For the purposes of this course, we will define the concept of "Power Conditioning" as "the shaping of electrical power from the conventional 60- to 400-Hz mains, into temporally well-defined pulses of electrical energy having reproducible voltage and current time histories." (See Figure 1.) In this context there is no specific mention of the repetition rate at which these pulses of energy are deposited into whatever the load may be. Normally, such systems are divided into single-shot (several times per hour or day) and repetitively pulsed (1 Hz upwards), primarily depending upon load requirements. A "Power Conditioning System" (PCS) can then be defined as "an energy transfer system that stores energy, either mechanically or electrically, and then discharges a determined fraction of this as electrical energy into the load." There are two main types of energy storage techniques:

1. Mechanical: The mechanical energy, W_r , is stored in rotary motion of machinery, such as a flywheel dc source device or a pulsed compensated alternator:

$$W_r = \frac{1}{2} I_0 \omega^2$$

INTRODUCTION TO POWER CONDITIONING SYSTEMS

DEFINITION:

AN ENERGY TRANSFER SYSTEM THAT STORES ENERGY, EITHER MECHANICALLY OR ELECTRICALLY, AND THEN DISCHARGES A DETERMINED FRACTION OF THIS AS ELECTRICAL ENERGY INTO THE LOAD.

MECHANICAL

ENERGY, W_R , IS STORED IN ROTARY MOTION OF MACHINERY.

$$W_R = \frac{1}{2} I_0 \omega^2$$

ELECTRICAL

ENERGY IS STORED IN ELECTROSTATIC OR MAGNETIC FIELDS.

ELECTROSTATIC:

$$W_E = \frac{1}{2} \int_{\text{volume}} \bar{D} \cdot \bar{E} \, DV$$

E.G. FOR CAPACITOR
OF CAPACITANCE C:

$$W_E = \frac{1}{2} CV^2 \quad \frac{1}{T_0} + V$$

MAGNETIC:

$$W_M = \frac{1}{2} \int_{\text{volume}} \bar{B} \cdot \bar{H} \, DV$$

E.G. FOR INDUCTOR
OF INDUCTANCE L:

$$W_M = \frac{1}{2} LI^2$$

FIG. 1

2. Electrical: The energy is stored in electrostatic or magnetic fields:

Electrostatic: $W_e = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} dv$ For example, for a capacitor,
 $W_e = \frac{1}{2} CV^2$

Magnetic: $W_m = \frac{1}{2} \int_V \vec{B} \cdot \vec{H} dv$ For an inductor, as an example,
 $W_m = \frac{1}{2} LI^2$

These two classes of PCS, mechanical and electrical, are schematically illustrated in the block diagram in Fig. 2. Mechanical energy PCS of both types A and B are being actively pursued at the Naval Research Laboratory (NRL) and the University of Texas at Austin, the latter under Lawrence-Livermore Laboratory (LLL) support. It has been projected that these devices will be capable of transferring energies of up to 100 megajoules per unit, in a time span of several hundred microseconds for flashlamp pumping applications. It may be possible to extend the concept of a compensated pulsed alternator (type B in Fig. 2) to pulse-charging applications, up to 10-kHz repetition rates, for charging times around 10 microseconds. If this is possible, these devices, following the normal time scale of high technology development, may well make excellent second-generation (1985-1990) charging units for fusion reactor drivers -- charging high-energy-density transmission lines, which could then be discharged either in microseconds for laser drivers of the CO_2 type or in tens of nanoseconds for Electron Beam/Light Ion driven (EB/LI) or excimer (e.g., KrF) laser-driven fusion systems. It is far from clear that the leakage flux of these rather low voltage alternators can be reduced to the level that will allow efficient operation in the required ~ 10 microsecond time scale. This point is currently under study at the University of Texas.

The electrical PCS fall into two categories, differing primarily in how the energy storage capacitors and/or transmission lines are charged.

POWER CONDITIONING SYSTEMS

I. MECHANICAL

II. ELECTRICAL

For purposes of illustration we shall discuss them with particular reference to some of the present repetitive PCS work underway at both LASL and Sandia. In the type II-A of Fig. 1, a conventional dc power supply feeds energy into the energy-storage unit through the charging system. For single-shot systems, the latter may only be a string of charging resistors (giving a 50% energy-charging efficiency), while for repetitively pulsed applications, the unit is usually an inductive type of resonant charge network to ensure near-unity energy transfer efficiency. These inductive charging systems, with several high-regulation variations, are currently under development at LASL for laser isotope separation (LIS) applications, pulsed medical accelerators, the free-electron laser, and also are under study for use in the very large PCS required for high-reliability fusion laser systems. In addition to the above, pulse-charging transformers form a third charging technique used primarily where high-voltage gains and efficiencies are required. The development of large-scale air-core pulse-charging transformer techniques was pioneered at Sandia for high-voltage gain, fast charging (0.1 to 1 microsecond) and triggering applications. They have developed an extensive expertise in these air-cored transformers, which are complementary to the iron-cored units under research and development at LASL. These compact iron-cored transformers, designed with very high coupling coefficients and transfer efficiencies, have the potential of being vital components in future high-efficiency, cost-effective multikilohertz lasers as well as in numerous related applications, such as research accelerators.

The energy storage device required for high-reliability, long-life systems will likely remain capacitors for some considerable period of time until other technologies become sufficiently mature to displace them in a cost-effective way. At LASL, research and development is underway on capacitor structures directed towards satisfying the very long lifetime requirements in LIS, laser fusion, accelerators, and nuclear particle diagnostic systems. These efforts have clearly identified the need for a considerable technology base development and effective

laboratory-industry technology transfer in this area, which may soon take on considerable strategic significance. It has been recognized, through a number of LASL programs, that the future systems evolving from the research described above will demand component performance levels vastly beyond those obtainable from our current technology base. For this reason, the research emphasis is being placed upon the two weakest areas in PCS, namely, capacitors and switches for lifetimes in the region of 10^{10} to 10^{12} shots, as needed in the mature versions of all the above systems. In this field of high-repetition rate capacitor development there has been exceedingly sparse data available in the area of long lifetime systems except for small radar modulator mica capacitors developed for the F-111 ($> 10^{11}$ shot life) and polysulfone/polypropylene-silicone oil capacitors ($> 10^{10}$ shot life) specifically created for driving industrial CO_2 TEA lasers. The results of the work to date here have shown that the present research and development approach to developing very long lifetime capacitor systems can be applied to PCS requirements for full-scale fusion laser driver applications.

In the area of switching, several laboratories are working on complementary parallel efforts. At this time it appears that the first feasibility demonstration fusion reactor systems, operating in the 1- to 10-Hz region can be cost effectively switched utilizing spark gaps under development at Sandia and at LASL. At Sandia, components are currently being tested in the 30-kW range and will soon be into the > 100 -kW test-stand configurations. These are all Marx Bank devices and the Sandia facility will yield valuable scaling and lifetime data. Studies at LASL have shown that it is highly probable that alternative PCS configurations can be developed, utilizing extensions of adiabatic Blumlein techniques to continuous operation in the 10- to 40-Hz region as noted above, but yielding an MTBF of 3 to 5 years in contrast to months to a year ($\sim 10^7$ shots) for spark gap systems. For second generation fusion drivers of the future, further development of switch and capacitor technology from the base currently being established through present programs is necessary to achieve industrial reliability levels for these systems.

The repetitive switching techniques undergoing research and development at LASL primarily utilize thyatron technology, because programmatic needs demand long lifetime operation in all of the high-repetition-rate systems, (although some systems can function with spark gap switches to meet near-term objectives). Through LASL contract support, the demonstration of current pulse switching rates comparable to many single-channel spark gaps has been accomplished this year at a level in excess of 10^{12} A/s. Scaling these new thyatron structures to exceedingly large power levels has been carefully studied by both EG&G and LASL staff. It has been concluded that sufficiently large devices could be developed to allow a full-scale CO_2 fusion laser driver or EB/LI pulse-charging unit to be switched with only a few thyatrons (two to six depending upon voltage needs), each passing multimewatts of average power. It is to be noted that these thyatrons represent a direct scaling from megawatt devices produced through the ERADCOM program and the ultrafast tubes being developed for the LASL LIS program and show the significant offshoot benefits of both technology-base development programs.

As noted above for thyatrons, military tube developments are underway to meet adiabatic applications from 50 to 120-Hz repetition rates, for 60 to 90 seconds on-time, long off-times, a limited number of total cycles (a few hundred). Thus, all components -- thyatrons, capacitors, charging systems, and power supplies -- need to be lightweight and of limited life. These conditions are diametrically opposed to the long-life needs of all long-life, industrial systems as well as numerous other program needs as described above, and dramatically illustrate the need for a research and technology base development program to create systems meeting these conditions that are so at variance with the military requirements.

In part II-B of Fig 1, the power supply/charging system is shown as one block, connected directly to the mains. This type of configuration was chosen to illustrate charging systems utilizing high-frequency inverters and polyphase cycloinverters, which directly charge the

capacitor banks in times somewhat shorter than the pulse repetition period. In the area of large-scale inverter technology, this is primarily at the research level (Gilmour, University of Buffalo). Cycloinverters are well established in the power industry (AC-DC and vice versa) and they may have application to single-shot systems. In most cases, their cost is so high that it is unclear just where they could be effectively deployed.

A Power Conditioning System (PCS), then, stores electrical or mechanical energy and, on command, deposits a predetermined amount of this energy into a load (either a resistor, an inductor, or a more complex load system). As we saw above, storage can be either electrical or mechanical and either all or a portion of the energy can be discharged into the load. This includes, for example, rotating machinery, pulsed alternators, and the like as storage elements. In the mechanical sense, we're thinking of rotational energy, stored in rotary motion. In the electrical case above, there are two classes: (1) storing the energy as electrostatic energy in the dielectric of the medium and (2) in a magnetic case, where the magnetic energy is stored in the magnetic fields inside on the inductor. This discussion does not take into account time-varying capacitance or time-varying inductance. These relations above (Fig. 1) apply, then, to geometrically invariant systems, not varying in spatial dimensions or position with time. Otherwise, the C or L cannot be moved from under the integral sign. If one is dealing with nonlinear cases or time-varying C and L , it is necessary to return to the complete formulation of Maxwell's equations.

Let us expand somewhat upon several facets of the above pulse power systems. In mechanical systems, mechanical energy is often stored in a rotating machine and then discharged through a switch that couples the storage component to a load. A pulsed mechanical energy discharger, being developed at the University of Texas (Austin) is called a compulsator. It is essentially a magnetic-flux compressor. When the flux is compressed to a maximum amount, a burst of energy is emitted into

the load. The real advantage of this system is the enormous amount of energy that can be stored. At NRL, several megajoules have been routinely stored in a rotary machine. Mechanical stress analysis indicates that storage of up to several hundred megajoules in a superconducting rotary motion machine, or 100 megajoules in a straight mechanical wheel, is possible. The present difficulty with this system is the repetition rate for a fixed energy per pulse: they are extremely slow to recharge, with bursts once every second to once every two minutes.

As discussed in the Introduction above, electrical systems are of two major types: firstly (II-A), the PCS can have an AC or DC power supply (AC is relatively rare), connected to a charging system that charges the energy storage device (capacitor, inductor, or transmission line) until the assigned stored energy is reached. At that time the output switch is turned on and the energy is dumped into the load. (Fig. 2) The second electrical case (II-B) is the high-frequency inverter power supply. It charges the energy system in a few milliseconds, permitting very high rep rates. Some other types of systems in this class are the constant-current DC supplies that use series vacuum tubes, and command charging systems that use a series tube in the output or a thyatron turned full on to initiate the start of the (resonant) charge cycle. The system then charges the energy storage system in a very short period of time, the series tube turns or is turned off, and the output switch then closes, transferring the stored energy into the load.

If the energy storage device is charged rapidly, the probability of switch prefire is reduced, as it is proportional to the time the switch is at full voltage. When it is necessary to stress the switch to the maximum voltage, charging must take place very quickly (to decrease this prefire probability). Corona formation times are on the order of 1-10 microseconds. If the system is charged in times of that magnitude the probability of the switch itself having a significant internal corona

current, which gives you a small electron density and thus a prefire, is quite small. The other advantage of pulse charging is that the dielectric strength of the energy storage medium is increased significantly (up to a factor of four higher for the same lifetime in a rep-rate system). The device can then be made several times smaller for the same lifetime, appreciably reducing the system inductance. This has considerable advantage for airborne devices, portable devices, or systems where extremely low internal impedance on the order of a few tenths of an ohm is required.

Each system has its own problem areas and performance limitations. When the switch closes, all the stored energy, if there isn't perfect energy transfer, is not dissipated in the load. Then, when the switch tries to open again, at the zero of voltage, the small voltage perturbations from the energy stored in residual inductance and stray capacitance of the system generate very high frequency transients in the system discharge loop and in the energy storage system. These transients must be dampened, otherwise the switch will generally reclose and fault the system power supply. This is the main reason why the early high-rep-rate TEA lasers were so very difficult to operate at rep rates above a kilohertz. There was enough energy stored in the loop inductances and stray capacitances in the system so that, whenever the switch started to open, there was sufficient inverse voltage applied to the thyatron switch (25 kv or so) to immediately fault it. It then acted as its own spark gap, tripping off the power supply. These transients can cause significant internal damage to triode thyatrons. Tetrode tubes are far more immune. Spark gaps, on the other hand, if they turn on again, stay on, and fault the power supply suffering no permanent damage. Ignitrons will also tolerate such treatment if they have proper anode and "keep-alive" auxiliary electrode materials and will then withstand significant current reversals. The ignitron auxiliary electrode is pulsed with a very large trigger pulse and a relatively high energy is dumped into this auxiliary electrode (which keeps the pool ionized during the reversal period) so the mercury pool becomes the anode

instead of the cathode. The only disadvantage is that high rep-rates cause a tendency for the reverse arc to track along the surface of the anode insulator, destroying the tube hold-off capability and reducing lifetime.

2. CHARACTERISTICS OF PULSE SHAPES.

In generating a voltage pulse, there are a number of parameters that are important in engineering, such as the peak value of the voltage and of the current through the load. These and other pulse shape parameters are summarized in Fig. 3, and their determining factors, described in greater detail below, are shown in Fig. 4. One area of considerable interest is the rate of rise of the voltage pulse on the load τ_r ; the second area is the rate of fall of the voltage, called τ_f . The required smoothness of the top of the voltage pulse is determined by the load characteristics. In the case of magnetrons, klystrons, and similar systems, this particular fluctuation of voltage and subsequent fluctuation in current causes a tendency toward frequency pulling in the device, and results in a time-varying RF spectrum from the system.

Now, rates of rise can be defined in a number of ways. For these discussions, it will be defined as the time it takes the voltage to rise from 10% to 90%, rather than the e-folding time (i.e., the time it takes the pulse at an amplitude level of $1/e$ of its peak to fully fold back upon itself to zero initial amplitude). The same applies to the fall time. When the output switch is turned on, the voltage rate of rise is determined not only by the properties of the PCS, but also by those of the load. In the case of a magnetron or laser, until the voltage rises to 50-60% of the peak value, the load is almost an open circuit. The slope of this voltage curve at the beginning can then determine how the current begins to rise through the system and this can have several forms, depending upon the stray capacitances in the system and the load impedance. For a magnetron or laser, when the thyatron is first turned on, the PCS discharges into an equivalent shunt capacity across the laser

FIG. 3: PULSE SHAPE PARAMETERS

PARAMETERS DESCRIBING PULSER CONDITIONING SYSTEMS

A. FUNDAMENTAL

1. PULSE SHAPE:

- A) τ_r : FROM 10-90%: RATE OF RISE
- B) SPIKES AND OVERSHOOT
- C) FLATNESS
- D) τ_f : FROM 90-10%: RATE OF FALL
- E) τ : DURATION AT 90% POINTS
- F) PEAK VALUE: V_{PK} AND I_{PK}

2. PEAK PULSE POWER:

$$P_{PK} \equiv V_{PK} \times I_{PK}$$

3. ENERGY:

ENERGY, W_L , DELIVERED INTO THE LOAD OVER THE PULSE DURATION VS THE TOTAL ENERGY DELIVERED INTO THE LOAD, W , DURING THE INTERPULSE INTERVAL.

4. PULSE REPETITION FREQUENCY:

$PRF = \frac{1}{T}$: PROBLEM IS GENERALLY ONE OF INTERPULSE SWITCH RECOVERY TO PREDISCHARGE, QUIESCENT STATE.

B. DERIVED

5. DUTY FACTOR $\equiv \tau/T \equiv DF$ (DUTY RATIO)

6. AVERAGE POWER:

$$P_{AV} = \frac{\tau}{T} \times P_{PK}$$

7. IMPEDANCE LEVELS:

ESPECIALLY IMPORTANT FOR TIME-VARYING LOADS IN ORDER TO MAINTAIN HIGH EFFICIENCIES.

FIG. 4

electrodes or magnetron, in series with some equivalent series inductance; this forms a series-resonant circuit with some damping in the equivalent inductance, allowing the output voltage to ring up rather quickly to values well in excess of the storage element charging voltage (overshoot). In fact, significant overvoltages can thus occur in these types of systems, which is often of a considerable advantage in the actual operation of a laser system but generally causes additional insulation problems.

The falling characteristics of the pulse really depend upon the way in which the impedance of the load varies with time. The discussion up to this point has generally assumed that there was a well-defined resistive impedance acting as the load for the PCS. As the voltage starts to decrease, for example, in the case of a magnetron, a point is reached at some voltage where suddenly the magnetron ceases to draw any current and the load in the equivalent circuit for the PCS then becomes the shunt stray capacitance in series with the loop inductance, L . This resonant circuit has an energy of $1/2 Li^2$ stored in this L at the moment of current decay that can then give rise to significant voltage ringing across the load. It is the damping of this ringing that is one of the most awkward problems in very high rep-rate PCS design.

The pulse duration, τ , for our applications will be defined as the duration of the pulse at the 90% of peak-pulse amplitude. Note that there is a τ for the current and for the voltage pulse, which are often significantly different.

The various parameters that can be used to describe power conditioning systems can now be summarized. The fundamental parameters for the voltage and current pulses were previously discussed, e.g., the risetime and also the rate of rise of voltage (of interest in the formation of glow discharges in lasers and in the initial turn-on conditions for klystrons and magnetrons) and are unique to each particular application. In the case of magnetrons, for example, the τ_r has to be slower than a critical value or a problem called "mode

hopping" (a shift from one rf frequency to another during the time the current is building up) arises.

The second area of interest is spiking. Spikes are a major problem, even for relatively well-defined loads, such as the magnetrons and klystrons, as well as for laser systems, because these can give rise to arcing in the system when the voltage stresses are too high. If, too, the current oscillations are excessively large, they can cause the discharge in some laser loads to effectively quench as the current goes down through zero. This produces a system in which the switch will try to reclose and will no longer be operating in the proper mode, generally causing the switch or laser to arc and fault the system.

The pulse flatness is of primary consideration in applications requiring accurate voltage pulses for driving loads like Pockel's Cells and other electro-optic light modulators. On the other hand, this flatness isn't terribly important in many laser devices, which generally tend to behave as constant-voltage loads, smoothing out these peaks. As long as the energy is deposited into the resistive phase of the discharge in these systems (where the impedance is roughly constant), there's generally little advantage in efficiency for the applied voltage and current to have perfectly flat pulse shapes, provided the applied voltage is sufficiently above the threshold for efficient excitation laser kinetics. This has been a point of some considerable discussion and analytical work in the case of the CO_2 lasers in the course of the last few years. Work at LASL has shown that for an E-beam laser system with a very carefully controlled rectangular voltage and current pulse shape from the PCS into it, for which the gain is measured in the medium, this gain starts to grow up to some peak value above which deactivation processes begin to dominate. In contrast, take, then, the area under the power curve, which is some energy W deposited in the gas up to the gain peak and then deposit the same energy in the same time, but without a sophisticated, complicated network. Rather, what one uses is an inductor and a series capacitor; so then the energy is deposited in

something like a $\sin^2 \omega t$ fashion, and measurement and calculation show that almost the same peak gain is obtained as for the carefully shaped flat pulse case. It's a very interesting observation for laser systems. As long as the energy is deposited into a constant-resistance system (glow-discharge) and the applied voltage is above some critical value, in general, it appears that one doesn't have to be very sophisticated about the pulse shape. This applies to systems primarily where electron attachment and recombination are the dominant mode in which the electron density decreases during this portion of the pulse. In the case of some of the ultraviolet laser systems, that argument isn't quite valid. This explains why you can get away with so much in driving most laser devices, in terms of pulse shape, and still have a relatively high efficiency. Whereas, in contrast, magnetrons and klystrons are very pulse-shape dependent for efficiency, particularly for very large pulsed klystrons.

There are three derived parameters in pulse shape characteristics. One is the duty factor, the ratio of the duration of the pulse to the interperiod spacing. This is usually written as DF in most of the texts. If you multiply that by 100, this gives the percentage of time during which a system is actively discharging energy. The duty factor can be used to calculate the average power, which is basically the duty factor times the peak power through the load per pulse, P_{PK} . A comment: that does not necessarily mean that the total energy discharged into the load per pulse is useful energy. It says that there is some type of time history of the power flowing into the load and now all the energy that was stored in the energy storage device has been removed from the latter. The characteristic of the load device (laser, magnetron, or whatever) may be such that the input energy, beyond some point, alters the load characteristics and all this energy that is flowing later on into the system (i.e., beyond this point in time) is lost, so far as efficiency is concerned. This is a significant difference from the PCS requirements of yesterday. In the 1950s one had well-defined loads, such as magnetrons and there were few efficiency

problems. The model of a magnetron in this case was a resistor in series with a reverse-biased diode, which worked out quite accurately and efficiencies for these systems were often greater than 70%. Today, energy transfers in lasers are typically 20 to 50%. (That's the ratio of the energy stored in the PCS to the useful energy delivered into the load.) This fact can often be cause for considerable concern. This energy that is being delivered into the load that is useful energy and is producing either photons or RF power or the other output characteristics desired is often much less than the energy that is stored in the energy storage device of the PCS. Normally, this ratio is desired to be one. For a number of new systems of considerable interest today, it's much less than one. Therefore, the overall system efficiency that was formerly around 80% has decreased to 20-35%. When one considers devices where it is desired to put in hundreds of kilowatts of useable average power, this means that the total input power must be on the order of megawatts. Another reason this point is becoming important is that a number of the load interface considerations in previous systems now change because there is a great deal of resistive power (Joule heating) to dissipate in the load. One must transfer that thermal energy to somewhere else. This means either wasting energy in heating the gas laser or load medium or wasting it in the load device generating, say, RF radiation, giving rise to a very severe thermal load problem. If, for example, 70% is lost energy, at a megawatt of average power flowing into the system, there is, then, 700 kW of undesired power in the load, which must be disposed of. It's not useful. Such thermal loading can cause significant problems.

The peak pulse power, P_{pk} , is the product of the voltage and the current at the point at which P_{pk} has a maximum value. More sophisticated definitions can be generated because of load characteristic shifts, but the convention suggested is that we look at P_{pk} , at the voltage peak, then we look at P_{pk} at the current peak, and define the peak power to be the larger number of the two.

The energy W_L is delivered into the load over the pulse duration, but it may not be the same as the total energy delivered into the load, W . That difference significantly affects, then, the efficiency of the overall system.

The Pulse Repetition Frequency (PRF, or sometimes Pulse Repetition Rate - PRR) is a perfectly good parameter to use in systems that are repetitive at a given frequency (i.e., there is a constant period of time from one pulse to the next, say 100 pulses per second). There are some difficulties when one considers, for example, modulators or pulsers with pulse rep-rate agility. These are normally hard-tube (vacuum tube) switched systems, where, upon commands into them a series of output pulses is generated at this certain externally defined interpulse spacing, which can be varying all the time; as well, the pulse width of the pulses can be varying all the time according to some predisposed computer program. In that particular case, the concept of PRF doesn't apply.

The main problem in all of these power conditioning systems is the one of interpulse recovery to the predischage quiescent state. If there is some post-discharge energy left in the PCS, oscillations can arise in the discharge loop, and switches stay on that should not and components often subsequently become overstressed. What is desired is to recharge the energy storage elements after all switches are fully recovered.

The load being reproducible on every pulse means that the system output pulse shapes are perfectly reproduceable on every pulse. For example, the impedance with time might have a specific shape, but in engineering a system to work at even a single shot, you assume that this does not vary from pulse to pulse. The next pulse that comes along, then, varies the same way with time, providing that everything is charged to the same voltage.

Comment: in high-rep-rate systems, particularly lasers and some RF tubes, this is not the case. A residual degree of ionization is left inside the system so the impedance profile per pulse may experience a

different time history until equilibrium is reached. That is an important problem in very high rep-rate systems. In small, high-rep-rate magnetrons, or in klystrons, this does happen, but to a lesser degree. In that particular case this is primarily a result of the space-charge saturation limitation in the cathode region. As you come up to cathode operating temperatures from start-up, there's a small shift of the available charge per pulse available from the cathode. The time-history of impedance, then, changes slightly. It is meaningful to emphasize this point because, in a number of the systems under consideration now, this will become a significant factor in PCS design for high-efficiency of energy transfer. In CO_2 TEA lasers operating at multikilohertz rep-rates, this particular impedance pulse shape is primarily determined by a combination of gas heating combined with the effects of residual, interpulse, ionization left in the system. Then it is necessary to choose which is the most efficient load profile in time to match into for maximum output energy for the specific experiment under consideration. If a very long operating time is not required, one could choose the start-up load impedance, but if long runs are needed at maximum efficiency it will be necessary to utilize a pulse-forming network that will maximize the energy transfer into the system at the equilibrium impedance level.

These, then, are some of the different types of loads often found connected to PCS. A summary of their characteristics is presented in Fig. 5. Pure resistive, capacitive, and inductive loads are relatively straightforward to handle. The diode magnetron, on the other hand, has an equivalent circuit of a reversed biased diode that turns on at some standard voltage. When it turns on, then the voltage-curve is almost flat with time. The dynamic impedance is quite low, basically like that of a glow-discharge, constant-voltage device.

In a magnetron the dynamic (slope) impedance is about a tenth the static value at the operating point. In the case of lasers, there is generally a combination of this load behavior, with a time-varying

LOADS FOR POWER CONDITIONING SYSTEMS

1. A) RESISTIVE: B) INDUCTIVE: C) CAPACITIVE:

2. DIODE MAGNETRON:

Statically

Dynamically

V-I Characteristic

3. LASER:

DIODE MAGNETRON LOAD OFTEN COMBINED WITH TIME VARYING RESISTANCE, R:

4. TIME VARYING LOADS:

DISPERSIVE LOADS AND LOADS WHOSE "L" AND/OR "C" CHANGES WITH TIME:

-- "INDUCTIVE" AND "RESISTIVE" PHASES DURING THE DISCHARGE TIME.

E.G. FERRITE SHOCK LINES, ARCS AND SPARKS, HIGH PRESSURE DISCHARGE LAMPS, EXCIMER LASERS.

Now: THE CHOICE OF THE PCS INTERNAL IMPEDANCE DEPENDS UPON:

- 1) LOAD IMPEDANCE
- 2) PEAK POWER LEVEL
- 3) PRACTICAL CONSIDERATIONS OF CIRCUIT ELEMENTS : (CAN ALSO USE PULSE TRANSFORMERS).

FIG. 5

component in the impedance. Fig. 5 shows a rough sketch of what the impedance with time looks like for an attachment-dominated excimer laser system (e.g., KrF^* , XeCl^* , or XeF^*). Evidently, the impedance decreases dramatically with time, but if one could put most of the energy into the system at times when the field intensity is sufficiently high for efficient pumping, then the laser system would work well and with high efficiency. Unfortunately, when this is attempted, a point is reached later in pumping time where the system arcs internally. This is normally caused by a thermally-induced instability. In terms of load classes, one of the most difficult, then, is the attachment-dominated laser discharge, which also happens to be rather interesting as an optical radiation source. In the copper vapor laser, for example, the impedance-with-time curve is somewhat shallower and alters dramatically as the rep-rate is increased. (The copper vapor laser is a longitudinal, abnormal glow discharge excited with ring electrodes, one at each end of a ceramic tube.) Closing the switch in the PCS generates a voltage pulse that initiates a discharge, dumping the energy stored in a relatively small capacitor into the discharge tube at a rep-rate of up to 25 kHz. By keeping the energy per pulse sufficiently small it is possible to obtain a uniform, longitudinal discharge. Unfortunately, as the rep-rate is increased even farther, thermal instabilities and kinetic effects constrict the discharge, raising the current, and increasing the drop in impedance with time, reducing the system efficiency.

The fourth class is formed by time-varying loads. Dispersive loads change their permeability or dielectric constants with time, as energy is deposited into them. An example is a shock-excited ferrite transmission line (e.g., a coaxial transmission line with the center conductor covered by a ferrite material.) Driving the line could be a capacitor with a switch connected to the line input. Closing the switch starts energy flowing down the line saturating the ferrite and steepening the rising edge of the voltage pulse as the wave front progresses down the line. As the ferrite saturates, a sharpening of the pulse is found. The sharpening is caused by the expenditure of energy in saturating the

ferrite. Once the ferrite is saturated, the permeability is one and, since the dielectric constant is rather low (12-15), the balance of the foreshortened pulse travels down a much higher impedance line. For more details of this, Martin Maley wrote a report on the subject. In a sense, such systems can be represented by lumped-element transmission lines, where the L and the C per unit length are time varying. This is a nonlinear problem and can be numerically solved with iterative solutions to Maxwell's equations. There is no analytical solution to the problem. With these systems, it has been fairly straightforward to generate pulses of 5-10 kV with risetimes of 1-2 ns. With more optimal ferrite materials, one may be able to reach 100 to 200 ps risetimes. Unfortunately, there are serious reflection problems in these systems, making the generation of very accurate pulses a difficult and empirical design matter. This is very good technique for generating a very sharp pulse at the end of a coaxial cable.

Another type of load is the "spark discharge" used as a radiation source for short wave-length UV spectroscopy. Basically, the spark discharge is a point optical source and it is excited by a switch discharging a charged, single-wire transmission line into the spark load. As the line capacitance is discharged, a wave is launched along this line (of about 150 ohms impedance) providing the prompt energy for the spark discharge during the two-way transit time of the line. The spark is not a linear load with time and behaves similarly to the excimer laser loads discussed above. The difference in this case is that the arcing point is at the spark turn-on and it is this arc that provides the optical radiation source. The last area is high-pressure discharge lamps, CW or pulse for driving glass lasers and UV process curing lamps, Hg discharge lamps and other such systems, and these are operated in the glow (cw) or abnormal glow (pulsed) mode, all as optical radiation sources.

Ignitrons are a much maligned switch, generally ideal for crowbar applications. There are modest rep-rate applications (< 100 Hz) at low energies requiring an inexpensive switch that can handle substantial current reversal. This is a good application for ignitrons. One problem with them is the rather sophisticated trigger generators needed for reliable triggering. Normally, about a joule per pulse must be dumped into the igniter, basically through a constant-current source, causing some heating problems at high rep-rates.

The internal PCS impedance selected depends primarily upon three parameters: the load impedance time history, the peak power level (and concomitant useful energy), and, thirdly, the practical considerations in selecting available circuit elements. Capacitors for low rep-rates are readily available. For high rep rates (≥ 1 kHz), low-inductance, high-energy, long-life capacitors are not yet here. For fast discharges (≈ 50 ns) capacitors generally behave as transmission lines. The liquid-impregnated, insulating-film capacitor is made of sections of paper, plastic film, and foil wrapped up with connections on either end. At the self-resonant frequency this is a parallel-plate transmission line with a surge impedance of a fraction of an ohm. Discharging this capacitor into a short circuit, the limiting transmission line discharge time for almost all such capacitors becomes ≈ 150 ns. The only way to make faster discharge capacitors is to develop capacitor geometries or new types of energy storage devices with shorter internal discharge times. Single-shot assemblies with discharge times of 40-50 ns have been developed, but are presently unsuitable for high-repetition-rate operation.

There are two categories of power conditioning systems, as shown in Fig. 6:

1. Ones in which a small amount of energy is dumped from the storage device into the load. These are generally hard-tube power conditioning systems, series vacuum tube switches, crowbar storage devices, and mechanical power conditioning systems, like the compulsator. The advantage of these devices is that they generally utilize an analog switch element, so you can, if desired, modulate the energy flow into fixed and time-varying load impedances as a function of time. Note that the X-ray flux from the high vacuum systems at the 30 kV level and above can be quite considerable.
2. The second class is one in which all the stored energy is transferred into the load every pulse. These are line-type power conditioning systems, Blumlein generators, Marx banks, etc.
3. SWITCHES

Normally, a switch is closed during discharge and opened during the recharge time (Fig. 7). During the recharge period, a closed switch would then short the charging system. The required characteristics of that switch then depend on whether all the stored energy is dumped into the load in each pulse. If one needs to transfer all the energy out of the storage system, at a constant output voltage for a predetermined period of time, a transmission line or a pulse-forming network needs to

POWER CONDITIONING CATEGORIES OR CLASSES

- A) THOSE IN WHICH STORED ENERGY IS PARTIALLY TRANSFERRED EACH PULSE:
E.G., HARD-TUBE PCS, CROWBAR STORAGE BANKS, MECHANICAL PCS.
-- GENERALLY REQUIRE AN ANALOG SWITCH ELEMENT.
- B) THOSE IN WHICH ALL STORED ENERGY IS TRANSFERRED EACH PULSE:
E.G., LINE-TYPE PCS, BLUMLEIN SYSTEMS, MARX BANKS.

SWITCHES

- GENERALLY A SWITCH IS CLOSED DURING THE DISCHARGE AND OPEN DURING THE RECHARGE TIME.

- SWITCH CHARACTERISTICS DEPEND UPON WHETHER OR NOT ALL THE STORED ENERGY IS TRANSFERRED PER PULSE.
NOTE: PULSE SHAPING IS NECESSARY IN MOST DISCHARGE CIRCUITS WHERE ALL THE STORED ENERGY IS TO BE TRANSFERRED PER PULSE.
- NOTE ON CHARGING: SINCE THE INTERPULSE PERIOD IS MUCH LONGER THAN THE DISCHARGE TIME, IN GENERAL THE DISCHARGE CIRCUIT CAN BE LOGICALLY DISCUSSED SEPARATELY FROM THE CHARGING CIRCUIT.

FIG. 7

be employed as the energy-storage device. Since the recharge time is almost always much longer than the discharge time, the discussion of the discharge part can be separated from that of the charging circuit. This approximation is not very good, however, for systems such as the copper vapor laser with its 25-30 kHz operation -- the discharge time is a fraction of a microsecond and the recharge time is a microsecond or so. On that time frame, there are many other problems, particularly keeping the switch turned off during the recharge.

4. PC CONDITIONING SYSTEMS: HARD-TUBE AND LINE-TYPES

Hard-tube power conditioning systems are still alive and well. They are very useful in relatively small, low-voltage (<200 kV) systems. In a hard-tube PCS (Fig. 8), a large energy-storage capacitor is charged from a DC supply through an isolating element. The capacitor is connected to the load by a switch. If the switch is closed, energy is discharged into the load, and the isolating element serves to prevent the DC supply from discharging into the discharging circuit loop as well. The switch, however, opens to see almost the full potential of the DC power supply, since, for good voltage regulation, only a small portion of the stored energy is discharged into the load on each pulse. The switch generally then needs to be a high-vacuum tube or a vacuum interrupter. Hard-tube power conditioning systems are commonly used today in the region of a few kilovolts at subnanosecond risetimes. They have a great advantage in terms of pulse-width agility and rep-rate control. At high rep-rates they are very useful and the drive requirements are not severe. The real advantages of the hard-tube PCS are the variable pulse width and the high PRF capabilities at modest efficiencies. Large hard-tube power conditioning systems (1000 A at 200 kV) exist. At these high voltages, the switching time is rather long and the cost can be very high indeed. They cannot generally be replaced with thyratrons, ignitrons, or spark gaps in these applications. On the other hand, they have limited di/dt capability because the vacuum tubes have a saturation resistance of a few hundred ohms, and that resistance limits the charging

HARD TUBE POWER CONDITIONING SYSTEMS

ENERGY STORAGE:

GENERALLY, ENERGY IS STORED IN A LARGE CAPACITOR AND ONLY A SMALL FRACTION OF THIS ENERGY IS TRANSFERRED TO THE LOAD PER PULSE.

-- THE SWITCH MUST HAVE VERY GOOD RECOVERY CHARACTERISTICS AS WELL AS HOLD-OFF CHARACTERISTICS: E.G., VACUUM TUBE.

NOTE: PULSE TAILORING CAN READILY BE PROVIDED WHEN USING A VACUUM TUBE SINCE THE GRID CONTROLS THE TUBE RESISTANCE.

FIG. 8

rate of whatever stray capacitances there are across the load. Their efficiency is nowhere near that of the line-type pulser (see Fig. 9).

Line-type pulsers are quite different (Fig. 10). The energy is stored in a continuous or lumped-element transmission line, sometimes called an artificial transmission line in the lumped-element case. Since this is the energy-storage element during the pulse discharge as well as the pulse shaper, the general designation of "Pulse-Forming Network," or PFN, was coined by someone during WWII.

There are two main classes of line-type pulsers: voltage fed and current fed. In the case of the voltage-fed pulser, the energy, W_e , is stored in the total capacitance of the transmission line or PFN ($W_e = 1/2 CV^2$, where C is the total RFN capacitance and V is the charge voltage). In the case of current-fed systems, a current is placed into the pulse-forming network and at the peak current value, $1/2 Li_{pk}^2$ of energy is stored in the system. On the charging side is a closed series switch that can be opened at that point. As soon as the switch opens it must withstand the full discharge voltage.

There are two general classes of voltage-fed line-type PCS (Fig. 11). The first is a continuous transmission line that is charged from a voltage source. It has a switch at one end, and, when the line is charged to voltage V , the switch is closed, discharging the energy stored in the transmission line into the load. If the load impedance equals the line impedance then $V_L = 1/2 V$. On the right side of Fig. 11 is sketched a time history of a typical charge and discharge cycle. The energy storage system is charged to the voltage V with a charging network. At the peak voltage, V , the switch is closed and pulse of energy flows out of the system and into the load. The pulse length is twice the one-way transit-time value of the continuous transmission line and all the stored energy is transferred to the load. That's for a matched system.

CHARACTERISTICS OF HARD TUBE POWER CONDITIONING SYSTEMS

ADVANTAGES:

- VERY HIGH PRF AS WELL AS APERIODIC OPERATION POSSIBLE.
- PULSE TAILORING.
- RAPID FAULT CLEARING.

DISADVANTAGES:

- LIMITED DI/DT CAPABILITY BECAUSE OF RATHER HIGH TUBE-ON RESISTANCES ($\approx 100\Omega$).
- COST GENERALLY VERY HIGH COMPARED TO OTHER PCS.

FIG. 9

LINE TYPE POWER CONDITIONING SYSTEMS

1. THE ENERGY IS STORED IN A CONTINUOUS OR LUMPED ELEMENT (I.E., "ARTIFICIAL") TRANSMISSION LINE.
 - SINCE THIS SERVES AS THE ENERGY STORAGE ELEMENT DURING THE DISCHARGE PULSE AS WELL AS THE PULSE SHAPING ELEMENT, THE GENERAL DESIGNATION IS:

"PULSE FORMING NETWORK" OR "PFN"

2. CLASSES OF LINE TYPE SYSTEMS

THERE ARE TWO CLASSES:

VOLTAGE FED

- $W_e = \frac{1}{2} CV^2$
- CLOSING SWITCH USED

CURRENT FED

- $W_e = \frac{1}{2} LI^2$
- OPENING SWITCH USED

FIG. 10

VOLTAGE FED LINE TYPE POWER CONDITIONING SYSTEMS

TYPES

$$W_e = \frac{1}{2} CV^2$$

WHERE C IS TOTAL LINE CAPACITANCE
AND V IS THE PEAK CHARGE VOLTAGE.

NOTES:

1. PFN CONSISTS OF A NUMBER OF CAPACITORS, C , AND INDUCTORS, L , AS LUMPED CIRCUIT ELEMENTS:
2. VALUES AND NUMBER OF ELEMENTS IN PFN DEPEND UPON THE:
 - A) DESIRED PULSE SHAPE
 - B) LOAD IMPEDANCE
 - C) COMPONENT AVAILABILITY
3. WHEN $Z_{LOAD} = Z_{PFN}$ (OR Z_{LINE}) THERE IS MAXIMUM ENERGY TRANSFER AND EFFICIENCY, ASSUMING A "PERFECT" SWITCH IS USED. OTHERWISE SOME ENERGY IS LEFT IN THE PFN AT THE END OF THE DISCHARGE TIME OF THE PFN CAUSING PROBLEMS IF THE MISMATCH IS SEVERE; E.G., SWITCH RECOVERY, FAULTING, LOAD ARCS.

TYPE E PFN:

FIG. 11

The second-class of line-type PCS uses a pulse-forming network comprised of a number of lumped elements of inductances and capacitors and a type E network is shown for illustration in Fig. 11. The designer chooses, basically, the L and C for the given load impedance and a number of sections in the line depending on the pulse fidelity required. With only one section, an L-C discharge network is formed. As more sections are added, the second begins to discharge into the load concurrently with the first section and a more rectangular pulse is formed in the load. With five or six sections, good pulse fidelities with ripples of about 4-5% are possible. Coupling between the inductances is possible and some PFNs, such as the Type E, are built that way. In this case, the different sections communicate in the flow of energy. Some classes of time-varying loads can be accommodated by the addition of additional coupling inductors between elements of the lines. These inductors are generally saturating inductors with ferrite cores. Years ago, there was some work done in far infrared laser driver systems using that particular mode of operation. Such a direct electron pumped far-infrared laser maintains high conversion efficiency as long as the voltage across the laser load exceeds a critical value. To keep the electron temperature high, elements were chosen depending upon the laser impedance time history and component availability to preserve a rectangular voltage pulse shape.

It is intended that we will spend a significant amount of time discussing the current state-of-the-art in components, as they represent the major limiting factors in the technology base of PCS, particularly at high repetition rates and long lifetimes.

Current-fed power conditioning systems will see considerable use in the future as improved opening switches become available. In a current-fed PCS, in contrast to a voltage-fed pulser, the switch closes to allow current to flow through the battery (V_{DC}) to the energy storage (Fig. 12). When the peak current is reached the switch opens and a negative pulse is generated across the load. The switch then experiences a very large voltage across it. Many current-fed power conditioning systems in small modulators are formed with relatively fast switching transistors, and they can be used to generate relatively high-voltage pulses with modest charging voltages. An inductor can be put in series with a transistor with a transmission line or capacitor across it. When the transistor is turned off the energy stored in the inductor collapses and generates a voltage spike, which is useful for voltage multiplication. There are some very useful circuits in Electronics Design for voltage multiplying using this technique.

The pulse length, then, is forced to be determined by the characteristics of the energy-storage device. Either a capacitor or a current-fed transmission line can be used. The latter has been studied extensively by a student of Tom Burkes' over at Texas Tech. Provided that the switch technology can be improved, this particular approach may be of some value in the 10-50 kV region. At peak current, the switch is opened and conservation of energy gives a voltage across the load impedance equal to the peak charge current times the load impedance over 2. But, again, a major problem is switch recovery. There is another problem, in that when analysis of this sort of circuit is done, it is assumed that the switch is an ideal switch. In fact, what is normally used in this type of circuit is either a fast transistor, a series

CURRENT FED POWER CONDITIONING SYSTEMS

NOTES:

- 1) THE CURRENT BUILDS UP IN THE INDUCTANCE OF THE ENERGY STORAGE ELEMENT (TRANSMISSION LINE, PFN, OR TRANSFORMER).
- 2) AT THE PEAK CURRENT, THE SWITCH IS OPENED AND A HIGH VOLTAGE, V_L , APPEARS ACROSS THE LOAD.

$$V_L = \frac{Z_L}{2} \cdot i_{chg}[\text{PEAK VALUE}] : \text{FOR MATCHED IMPEDANCES}$$

- 3) THE SWITCH MUST RECOVER VERY QUICKLY TO HOLD OFF THE PEAK LOAD VOLTAGE.

FIG. 12

high-vacuum tube, or a vacuum interrupter. The equivalent resistance of the device increases as the current through it increases; in some cases, a nonlinear increase is found, increasing the charging losses. One other type of switch technique is to use a forced-commutation thyatron switch or a vacuum current interrupter. An auxiliary circuit can be placed across the switch, then discharged through the switch at the point of peak charging current so that the switch carries no net current and the switch then rapidly recovers. This is a method of turning off a switch that normally only recovers at zero current.

5. IMPEDANCE MATCHING

Matching the load impedance to the pulse-forming-network impedance is done primarily to obtain maximum power transfer (Fig. 13). The disadvantage, of course, is that only half the charging voltage is obtained. If the pulse-forming network is charged to 100 kV, only a 50-kV pulse is generated with typical efficiencies of 80-90%. The other point is that if the load and PFN impedances are not equal, reflections between the load and the PFN occur at the end of the discharge period of the PFN. If the load impedance is lower than the PFN impedance, a current reflection follows and the current flow continues on for a long, long time in an oscillatory manner. If the load impedance is larger than the PFN impedance, the load voltage, V_L is $V/2$ for the discharge time, τ , and, depending upon the reflection coefficient (related to the ratio between the load impedance and the PFN impedance), a number of steps in the voltage pulse as a function of time can occur. Each step is τ seconds long. This points out one of the problems when the load impedance is much higher than the PFN impedance, namely that there's current flowing through the switch for a relatively long period of time. It's a unidirectional current, but takes a long time to die away. In such a case, switches such as spark gaps can revert from an arc to a glow and stay lit like a light bulb with milliamps of current through them. The actual power put into the switch can, operating under these conditions, be very significant. This is a strong argument for trying to

IMPEDANCE MATCHING OF POWER CONDITIONING SYSTEMS TO THEIR LOADS

1. NEARLY MATCHED IMPEDANCES ARE REQUIRED FOR
 - A) MAXIMUM EFFICIENCY
 - B) PULSE SHAPE FIDELITY
 - C) MINIMUM POST-DISCHARGE VOLTAGE STRESSES ON SWITCH AND PFN.
2. IF $Z = Z(t)$ THEN THERE ARE PROBLEMS IN OBTAINING HIGH EFFICIENCIES.
3. IMPEDANCE INCREASES ARE OFTEN HANDLED BY PULSE TRANSFORMERS.
 - PRESENCE OF MANY 50 OHM COAXIAL CABLES TENDED TO MAKE THIS IMPEDANCE AN OFTEN-USED ONE FOR THE PFN.
4. FOR VOLTAGE-FED NETWORKS, THE VOLTAGE, UNDER MATCHED CONDITIONS, DECAYS TO ZERO AT THE END OF THE PULSE.
 - VERY GOOD SWITCH IS ONE THAT RECOVERS AT ZERO CURRENT AND NEAR ZERO VOLTAGE. E.G., SPARK GAP, IGNITRON, THYRATRON, SCR.
 - THE RECOVERY CHARACTERISTICS OF THIS SWITCH ALSO DETERMINE ALMOST EXCLUSIVELY THE NATURE OF THE INTERFACE TO THE CHARGING SYSTEM.

match or slightly undermatch ($Z_L \leq Z_{\text{PFN}}$) the load to the PFN impedance.

If the load impedance is less than the pulse-forming-network impedance, the post-discharge voltage pulse oscillates about zero. Then, the unidirectional switch turns off at the first voltage zero. A problem occurs if the load happens to short, generating a very large voltage reflection, often exceeding the switch-recovery voltage, thus causing it to fault unless a voltage-limiting network immediately shunts the energy away from the switch. A second class of problem arises when the load impedance is variable with time. A time-varying load impedance can move the operating point from a matched regime to unmatched as the time increases. This is particularly true in the case of lasers. It is also true in some high-current density discharge lamps. So, matching isn't really then practical: depositing the maximum amount of energy into the system in the relevant time frame is the most efficient approach. In terms of designing for this regime of operation, it is far easier because of the voltage reversal across the switch, in the latter case. This is in contrast to the case $Z_L \geq Z_{\text{PFN}}$ where a small unidirectional current is flowing through the switch for a very long period of time. The switch starts trying to recover in the zero current and zero voltage regime and any small oscillations in the pulse-charging system will tend to inhibit this recovery. If the switch detects any appreciable positive voltage near the zero of current, it will not recover, faulting on and dumping the high-voltage power supply during recharge. This regime of operation was formerly referred to as "positive mismatch." It was a favorite of many of the WWII radar modulator designers and is totally unnecessary today with the availability of modern inverse-voltage-control networks and considerably increased thyatron inverse-voltage ratings (up to -20 kV).

There were a number of applications, in the mid 1950s, where relatively high voltages in the region of 100 kV or 200 kV were needed at tens to hundreds of amps peak for driving large klystrons. (Klystrons

needed physical length to accelerate the electrons vs the available electrical stress that the system permitted (electrical stress inside the system).) At that time, most switch tubes (thyratrons, ignitrons) were limited to 20-30 kV. The voltage thus had to be increased with a pulse transformer. The transformer technology for 1-5 microsecond pulse lengths is relatively well-established if the risetimes are on the order of fractions of a microsecond. When sub-hundred-nanosecond risetimes at significant average power levels are required, efficient design becomes very difficult. (With a sacrifice in efficiency, 20-30 ns risetimes are possible, but these are very sophisticated transformers with poor energy transfer (about 20-30%).) Few transformers are known that can handle significant energies at such risetimes: one is a special version of a transformer developed by Gerry Rohwein at Sandia. It is a very low leakage tape-wound air-core transformer. The other was proposed by RCA to drive a very large load and to produce a quarter of a megavolt with an extremely fast risetime, at an average power of 30 megawatts. The primaries and secondaries were relatively thin copper foil and water flowed through the copper foil, which was edge graded to prevent significant corona. The intention was to discharge 300 individual PCS into the primary of that transformer.

Pulse transformers were used so much in modulator design because they were straightforward to design at the 50-ohm primary impedance level. Fifty ohms was chosen, so the story goes, because there was 50-ohm cable. This cable existed because it was determined in the 1940s that working at the 50-ohm level gave you the highest available stress in the cable system along with minimal dispersion, and the cable was also easy to make.

Cables normally have rubber, polyethylene, or air-gap dielectrics. For most PCS applications, air-gap dielectrics are not practical. In the case of rubber dielectric cables, there are serious heating problems in short-pulse-length, high-repetition-rate applications. Rubber is not a

good high-frequency insulator as it has considerable loss. In terms of physical flexibility and mechanical lifetime rubber is still the best choice for general-purpose applications, with a design lifetime of many years at a modest PRF. Polyethylene cable, on the other hand, is rather more fragile. It tends to have microbubbles filled with easily ionized water vapor in it. If these bubbles are undisturbed, they cause no damage. If the bubbles crack they propagate and internal treeing and shorts develop.

In the case of the matched load, maximum energy transfer and efficiencies are possible, given a perfect switch (Fig. 13). However, very few switches are anywhere near perfect. An imperfect switch changes a linear circuit to a time-varying one. Spark gaps initially turn-on as high-value resistors, but the resistance decreases dramatically with time, and, as an arc column then forms, it expands or contracts with the current and acts then as an inductor. A switch, then, can be a nonlinear element in the PCS. For short-discharge time, low-inductance systems, switch-limited pulse lengths are on the order of 20-100 ns for single-channel devices (thyratrons, spark gaps, etc.). The switches then become limiting components of the system. The alternate approach is to use a multi-channel spark gap or the very large-discharge area thyratrons currently under development.

If the switch isn't perfect and even if the load impedance is matched to the generator impedance, some energy is left over at the end of the discharge pulse, and this can cause oscillations in the system and give rise to switch-recovery problems. In other words, when you think you have zero current through the switch you often don't.

6. CHARGING SYSTEMS FOR PULSE-FORMING NETWORKS

In the charging system, the switch is open during the charging period and the charging time is much longer than the discharge time (Fig. 14). This is a general assumption applicable to most PCS. When the

CHARGING SYSTEMS

SINCE:

1. SWITCH IS AN OPEN CIRCUIT DURING CHARGING OF THE PFN OR TRANSMISSION LINE
2. THE CHARGING TIME IS MUCH LONGER THAN THE DISCHARGE TIME

THEN:

THE CHARGING SYSTEM CAN BE TREATED AS PLACING CHARGE ONTO THE LINE/PFN
TOTAL TERMINAL-TO-TERMINAL CAPACITANCE, C .

ISOLATING ELEMENTS:

1. RESISTIVE: CHARGING INDUCTOR
2. INDUCTIVE: CHARGING DIODE

FIG 1/1

switch is open, a charge, q , is placed on the transmission line, charging up the total line capacitance, C to voltage V . There are now a number of choices for isolating elements. For a resistive-isolating element, generally used in single shot PCS, the energy-transfer efficiency is always 50% or less. In the case of an inductive isolating element of inductance L , the capacitor and inductor form a resonant circuit, and, at the point of zero current through the charging loop, the charge voltage is equal to twice the DC voltage on the battery. The time for this to occur is $\pi\sqrt{LC}$, and the energy-transfer efficiency is very close to 100%. Most of the losses are resistive losses in the charging inductor and they can be made relatively small, with wall-plug efficiencies of 70-80% possible in the PCS. If the period is chosen to be equal to the above time, this is called the resonant charging configuration, in which the discharge switch is turned on exactly at the zero-charging-current point. To work at other repetition rates, one may insert a blocking or charging diode in the charging loop. Once the voltage has come up to $2V_{DC}$, the current goes to zero, the voltage across the inductor collapses, the diode is reverse biased, and the $2V_{DC}$ remains on the PFM to be discharged at will at any rep rate up to $(\pi\sqrt{LC})^{-1}$. If this rep-rate is exceeded, there will be some DC current flowing through that choke at all times; that causes heating problems in choke design. As well, there is a net DC current through the discharge loop at all times and the rate at which the voltage is reapplied to the switch and transmission line is quite a bit faster than in the resonant case. Switch recovery problems then emerge. Operating at these rep-rates is called linear charging. This had some advantages in early modulator design, but is not very useful today.

Constant current or power charging systems use a high-frequency inverter (usually a push-pull type) with a rectifier bridge (Fig. 15). The DC current, fed through an isolating inductor, charges the pulse-forming network and a closed-loop feedback system is used to

OTHER CHARGING SYSTEMS

A) CONSTANT CURRENT OR POWER

UTILIZES HIGH FREQUENCY INVERTER/RECTIFIER WITH CLOSED LOOP FEEDBACK:

- LEVELS UP TO 1 KJ/SEC CAN BE BOUGHT
- CAN ALSO USE A SERIES REGULATING SATURABLE REACTOR IN THE TRANSFORMER PRIMARY OF A NORMAL DC SUPPLY.

B) COMMAND CHARGING

- USES A SERIES SWITCH, EITHER VACUUM TUBE, IGNITRON, OR THYRATRON, TO CONNECT A RESONANT CHARGING NETWORK TO THE DC SUPPLY RESERVOIR CAPACITOR.
- ALLOWS MAIN PCS SWITCH TO FULLY RECOVER BEFORE CHARGING VOLTAGE IS APPLIED.

C) AC RESONANT CHARGING

- SATISFACTORY IF PRF IS LESS THAN TWICE THE AC FREQUENCY.
- TROUBLESOME FAULT MODES AND LITTLE PRF AGILITY LIMIT APPLICABILITY.
- LOWEST COST CHARGING SYSTEM AS ONLY A TRANSFORMER AND POSSIBLY SOME DIODES ARE REQUIRED.

FIG. 16
COMPARISON OF THE CLASSES OF REPETITIVE POWER CONDITIONING SYSTEMS

Characteristics	Hard-tube PCS	Line-type PCS
Efficiency	Lower; more power lost in tube heating, and in dissipation in the switch tube.	High, >80%, particularly when the pulse-power output is high.
Pulse shape	Better rectangular pulses. Shape control by input pulse in analog fashion.	Poorer rectangular pulses, particularly through pulse transformer.
Impedance-matching	Wide range of mismatch permissible.	Small range of mismatch permissible ($\approx 20\text{-}30\%$). Pulse transformer will match any load, but power input to non-linear load cannot be varied over a wide range.
Interpulse interval	May be very short; as for coding beacons (i.e., ≈ 1 nsec).	Must be several times the deionization time of discharge tube (i.e., $2\text{-}300$ μsec).
Voltage supply	High-voltage supply usually necessary $\approx V_L$.	Lower-voltage supply, particularly with resonant charging.
Change of pulse duration	Determined directly by input pulse shapes.	Requires high-voltage switching to a new network.
Time jitter	Negligible time jitter (≤ 1 ns).	High-power line-type pulsers with a hydrogen thyratron give a time jitter of ≤ 2 ns.
Circuit complexity	Greater, leading to greater difficulty in servicing and high cost.	Less, permitting smaller size and weight.

<1 ns, increasing to around 20 ns risetime in large systems. There are scalability limitations in the hard-tube PCS dictated by component availability for high average power systems.

SUMMARY

Pulse risetimes and the flatness characteristics suffered significantly as a result of long resistive phase turn-ons of early switch tubes. What can be achieved to date in sub-ohm impedance circuits are current risetimes of 20 ns at current rates of rise, circuit-limited, in excess of 1.2 MA/microsecond with new thyratrons under development. Five hundred kA/microsecond thyatron switch tubes are now available. One can now consider designing a number of line-type power conditioning systems for long-life operation at kilohertz rep rates that previously were thought of as impractical. The equivalent inductance of the switch thyratrons has concurrently been reduced by a factor of 20.

In the case of impedance matching one can modulate the equivalent resistance of hard-tube modulators in an analog fashion, so it is possible to accommodate very large impedance mismatches and time-varying loads. This was the reason for the first large-scale efforts to drive high-rep-rate CO₂ systems with hard-tube PCS. (They can operate at Megahertz rep-rates.) In this way the decreasing discharge impedance with time could be accommodated. The mismatch range of 20-30% is one over which the efficiency of the system doesn't vary a lot. Going beyond that, for PFN type PCS, all of the additional stored energy must be disposed of elsewhere if it is not deposited into the load in a time equal to one discharge time of the pulse forming network. It is desired to deposit as much of that energy as possible in the discharge time of the PFN.

Typical recovery times today for small tetrode thyatron devices are one or two microseconds. The WWII-design triode thyratrons that you can buy commonly now have a much longer recovery time. They also suffer

recovery-time degradation with age because of their internal construction. It takes a long time for the plasma from the cathode to lose communication with the anode. Today, one can go down to a microsecond recovery time in modest sized thyratrons discharging millicoulombs/pulse.

In terms of high-power spark gaps, recovery times are hundreds of microseconds and haven't changed. If one wishes very fast recovery in spark gaps it is possible to go to a device that adopts some internal quenching mechanism to get rid of the residual ionization. An example is the work done on Hydrogen quench gaps, which are essentially a pair of electrodes separated by a number of fine meshes. In discharges through the device at a modest pressure of Hydrogen, the presence of the mesh cools the ionized gas in gap very quickly, enhances recombination around each mesh causing little globs of isolated plasma to form. These gaps can then work at a hundred kilohertz rep-rate or above, discharging a few hundred picofarads capacitance per pulse. However, because all the current is passing through the screens it's difficult to flow high average powers through the device.

This covers the introduction to pulse power systems, with an orientation toward higher rep-rate devices.

QUESTION: What dissipating mechanism were you using in the first class of loads in Fig. 5?

ANSWER: None. One can obtain a relatively high Q (>100) in circuits where the characteristic frequencies are on the order of a few megahertz. Normally, if just one discharge cycle is needed and the stored energy requirements are not too large (≈ 100 J), nonlinear thyrites or metal oxide varistors are very effective shunt elements for damping. The alternative choice that works well for inductors is to crowbar the voltage across the inductor with either a thyatron or a spark gap so that as the discharge current starts to reverse through it, the balance of the stored energy is then bypassed into a resistive dump. There is often a problem in timing the crowbar initiation.

Question: Will the course notes be available?

A report with all the diagrams from this course will be out by next Christmas. It will include a considerable bibliography and an expansion of many of the areas from the course. Later, a set of notes, in book form, may become available with a lot of detail in some of the specialty areas that, of time necessity, are briefly discussed in class.

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Power Conditioning Systems for High-Power, Airborne, Pulsed Applications

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Abstract

The power conditioning portion of the high-power study that was performed for the Air Force Aeropropulsion Laboratory by the State University of New York at Buffalo is summarized. This effort defines the power conditioning system and critical component developments which will be required to interface the airborne 10-MW to 50-MW sources defined under separate study efforts with certain loads. Power conditioning systems are considered for use with magnetohydrodynamic generators and turbine driven alternators, both conventional and superconducting. The critical components required for each of the power conditioning systems are identified and then analyzed. The component analyses include estimations of development efforts necessary and of specific weights and volumes for components. The primary components considered are transformers (for alternator as well as for inverter use), switches, capacitors, and inductors. Weight algorithms are developed for each of the components. Following the component analyses, subsystems such as inverters and rectifier and filter packages are considered. The data for the various components and subsystems are then utilized for a comparison of the power conditioning techniques to be used with the various power sources. The weights and volumes of power conditioning systems for 8-point designs (8 variations of power, voltage, duty cycle, and total run time) are derived.

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Introduction

The purpose of this effort is to define the power conditioning system and critical component developments which are required to interface the power sources being defined under separate study efforts with certain loads. The power sources considered were magnetohydrodynamic (MHD) generators and turbine driven alternators, both conventional and superconducting. The load the power conditioning must interface with is not well defined and therefore a range of load parameters were investigated. For purposes of this study, the load was considered to be either continuous during the running time or pulsed at a 5-kHz to 10-kHz rate with a 15-percent duty cycle. The load was assumed to be self-switched. This study provides the Air Force with technical information to make a decision on the type of systems to be considered in a subsequent system analysis, design, and critical component development program.

Two general types of parameters, external and internal, were considered. External parameters are those which must be provided to a designer in numerical form in order for him to design a suitable system. The most important external parameters are shown in Table I. Internal parameters are those which must be selected by a designer in the course of his design activities, and which must be described numerically but are not of essential importance to a system integrator (e.g., conversion frequency, secondary voltages).

The general procedure followed in these studies treats weight and volume as dependent variables and treats external parameters and selected internal parameters as independent variables. Possible power conditioning systems were analyzed to identify all important internal parameters. A simple computer program was written to calculate the weight and volume of the power conditioning system.

Identification of Power Conditioning Techniques

During the course of this study, analyses were performed that permit the evaluation of power conditioning techniques that may be used with ac or with dc power sources. Both continuous loads and loads pulsed at a 5-kHz to 10-kHz rate with a 15-percent duty cycle were considered. In addition, the power conditioning techniques that might be used with an auxiliary load requiring an order of magnitude of less power than the primary load were analyzed. The particular power conditioning techniques that were considered are classified according to the type of power source (alternator or MHD) and are as follows.

A. Alternator Power Source

Shown in Fig. 1(A) are the various power conditioning techniques that were evaluated for use with an alternator power source. The symbols used in Fig. 1 have the meanings listed on the next page:

Fig. 2. Possible inverter configuration.

TABLE II
Critical Component Identification

Component	Where Used
Capacitor	I, F, R&F
Switch	I, R&F, S, LP
Transformer	T, I
Inductor	I, R&F

be required. Also, a switch would probably be required to connect and disconnect the load. If load regulation cannot be accomplished in the MHD source, then an additional regulation device would be required.

In the seventh and eighth techniques direct dc-to-dc conversion of the MHD output is performed. Regulation and switching would be accomplished by controlling the inverter.

Techniques 9 through 13 are those versions of techniques 1 through 8 that would be required for use in conjunction with a pulse-forming network to drive a pulsed load.

Identification of Critical Components

Many of the critical components that may be required for the various power conditioning techniques shown in Fig. 1 are evident from this figure. For example, in addition to the obvious need for transformers, switching devices are required in the rectifiers, and capacitors and inductors may be required in the filters. The inverter may also contain these elements, as is indicated by the circuit diagram in Fig. 2 for one of the inverter configurations that was considered in this study. Of course, many of the elements in the inverter are required to operate under high-frequency ac conditions.

The protection device indicated in Fig. 1, if it is required, will probably contain only one critical component and that is a fast high-current switch. There will be detection and control circuitry; however, this will probably not require a component development.

In Fig. 1(B), a switch is shown connecting the MHD source to the load. If this switch is required to open under loaded conditions, then a switch development effort may well be required.

Table II contains a summary listing of critical components and of the functions in which they are used in Fig. 1.

Critical Component Analysis

A. Transformers

Four classes of transformers were investigated in detail during this high-power study. Two of these were at power levels in the 10- to 50-MW range and the other two were in the 1- to 5-MW range (for use with the auxiliary load). In each power range, three-phase sinusoidal and single-phase square-wave transformers were analyzed in detail. The three-phase sinusoidal transformers would be used to boost the output voltage of the alternator. The single-phase transformers would be used in the inverters.

The transformer study was carried out with the assistance of Thermal Technology Laboratory, Inc. (TTL). Individual transformer designs were performed using the TTL transformer design computer program. This program was used in a mode whereby it automatically minimized the weight of a transformer with any given set of operating parameters. Designs utilizing external cooling (by Freon, for example) and "adiabatic" (wherein the heat capacity of the transformer material is used to absorb the heat generated) were examined in detail. Also, the use of cryogenics was taken into consideration. For the operating periods under consideration (120 s maximum) the adiabatic designs were found to produce substantially lower specific weights than the designs utilizing external cooling. Thus, only the results of the adiabatic design studies are included in this report. An ambient temperature of 160°F (71°C) and a maximum wire temperature of 500°F (260°C) were assumed for these studies. If liquid nitrogen could be used for cooling the transformer prior to the start of a mission, the studies show that a weight reduction by a factor of at least 4 could be realized.

It should be pointed out that, for the transformer designs analyzed for this study, the current density in the conductors was about 13 000 A/in² and it was assumed that all of the conductor losses were stored in the conductors. It was assumed that none of the heat diffused into the insulation. By comparison, it should be noted that in the Garrett, thermal lag alternator designs performed on the Air Force Aeropulsion Laboratory (AFAPL) high-power study, a current density in the conductors of 15 000 to 16 000 A/in² is used. The diffusion of heat into the insulation is taken into account and a temperature rise from 130°F to 400°F is found to occur. It appears, therefore, that if heat diffusion into the insulation were taken into account in the transformer design, the maximum conductor temperature would probably be below 400°F.

Because of the high-operating frequencies anticipated for the transformers, the conductors were assumed to be Litz wire. It was found that if no ac resistance effects occurred and if, as a result, solid conductors could be used, the reduction in transformer weight would be only 6 percent. Thus the use of Litz wire at all frequencies anticipated seems justified to eliminate unnecessary heating.

The use of multiple secondaries on transformers to eliminate the need for rectifier balancing networks was

(A)

(B)

(C)

Fig. 4 Specific weight of inverter transformer as a function of (A) operating power level, (B) dc output voltage of rectifier, and (C) total operating time during mission.

$$S_{TA}(\text{aux}) = 0.1275 (\tau/120)^{0.281} \cdot [0.612 + 0.388 (P/2.5)^{-0.985}] \cdot [0.608 + 0.392 (V_o/200)^{0.71}] \cdot (f_i)^{-0.767} \text{ lb/kW.}$$

The factor 0.1275 gives the specific weight of this transformer when $\tau = 120$ s, $P = 2.5$ MW, $V_o = 200$ kV, and $f_i = 1$ kHz.

2) *Inverter Transformers*: Over 60 single-phase square-wave transformers for use in series inverters at frequencies in the range from 5 kHz to 10 kHz were designed. Fig. 4 contains the specific weights of some of these optimized

Fig. 5. Conceptual design of adiabatic rectifier stack.

Fig. 6. Average forward current as a function of case temperature for International Rectifier 401 PDL series fast-recovery rectifiers.

Fig. 7. Specific weight of three-phase full-wave rectifier as a function of current. Rectifier used is IR 401 PDL fast-recovery type. Voltage is derated by factor of 2.

transformers in the power range from 10 MW to 50 MW. Also included in this figure are curves representing specific weight predictions from the following algorithm:

$$S_{T1}(\text{main}) = 0.00940 (\tau/120)^{0.302} (V_o/100)^{-0.095} \cdot [0.479 + 0.521 (P/25)^{-0.614}] \cdot [0.880 + 0.120 (V_o/100)^{1.33}] \cdot (f_i/10)^{-0.754} (V_{in}/5)^{-0.089} \text{ lb/kW}$$

where τ , P , and V_o are as given for the alternator transformer, f_i is the inversion frequency and V_{in} is the input voltage to the transformer in kilovolts.

Fig. 9. Frequency coefficient of dissipation factor.

for the alternator or the inverter. Except for system startup and shutdown cycles and load fault conditions, these capacitors have a dc voltage with a small (few percent) ripple component applied to them. Capacitors are available (Maxwell and General Electric, for example) which are suitable for this application and which have energy densities in the neighborhood of 100 J/lb.

Capacitors using polyvinylidene fluoride [16] may become available. These have energy densities in the 300 to 500 J/lb range but have a high dissipation factor (on the order of 1 percent). Thus they are suitable for filtering applications at reasonable ripple amplitudes.

Because 100 J/lb capacitors are available at the present time, the use of these will be assumed in the filter analyses sections of this report.

2) *Inverter Capacitors*: An inverter capacitor is used in a series resonant circuit operating at a frequency that may range from 1 kHz to 20 kHz. The most probable frequency range is from 5 kHz to 10 kHz. The total number of cycles of operation that this capacitor may experience is the product of the frequency of operation, the mission duration and the number of missions. This product may be as high as $20 \times 10^3 \text{ Hz} \times 120 \text{ s} \times 100 \text{ missions}$ or 2.4×10^8 cycles. If this capacitor is examined from a pulse capacitor point of view then the number of reversals is extremely high, in fact, beyond the range normally considered for pulse capacitors. If, on the other hand, this capacitor is examined from an ac capacitor point of view, then the life is short compared to that normally considered (several years at 60 Hz or on the order of 2×10^9 cycles).

Discussions were held with many capacitor manufacturers in an effort to determine the characteristics (most importantly, the minimum specific weight and volume) that could be expected for a capacitor being used for this (series inverter) application. No manufacturer was found which has actually built a capacitor for use under these conditions. The information contained in the following paragraphs is a composite of that obtained from several manufacturers and represents the best estimate, at the present time, of the

achievable capacitor characteristics. *It is clear that a research and development program containing a substantial testing task will be required to substantiate and/or correct this estimate.*

The two primary factors that must be considered in designing and fabricating high energy density, high-frequency ac capacitors are 1) heating effects and 2) corona effects. When considering heating effects, the most weight and volume effective way to accommodate heat losses in a capacitor that must operate for a short period (like 120 s) is to utilize the heat capacity of the capacitor material. For a given energy content, externally cooled capacitors (such as those that are manufactured by the Westinghouse Corp., Bloomington, Ind., for induction heater use at 10 kHz) are typically at least an order of magnitude heavier than those utilizing only heat capacity.

The heating of a capacitor can be determined directly from the capacitor dissipation, the energy density of the capacitor and from the number of voltage reversals to which the capacitor is subjected. Thus, if the energy density of the active storage elements of a capacitor is D J/lb and the dissipation factor is δ , then the energy lost per discharge cycle is δD J/lb. The total number of discharge cycles at a frequency of f kHz and a mission duration of τ seconds is $2\tau f \times 10^3$ and so the total energy deposited in the capacitor during a mission is $2\tau f \delta D \times 10^3$ J/lb. The average heat capacity of the active storage elements is about 0.35 cal/gm/°C or 665 J/lb/°C. Thus, the temperature rise ΔT of the active storage elements is

$$\Delta T = 2\tau f \delta D / 0.665 \text{ } ^\circ\text{C}.$$

This is a "worst case" temperature rise. It has been assumed that no cooling occurs during the mission. This assumption is very nearly valid because of the poor thermal conductivity of the dielectric materials used in the active storage elements. If the allowable temperature rise is known, then D can be readily calculated from

$$D = 0.665 \Delta T / 2\tau f \delta \text{ J/lb}.$$

The dissipation factors for the various dielectric systems that might be considered for use are given in Fig. 9. The maximum operating temperature for each of these systems is given below.

dielectric system	maximum temperature (°C)
p-ppl-p-ppl-p castor oil	75
ppl-p-ppl - aroclor	100
ppl - silicone oil	125
ppl	125

Now, Fig. 10 shows the maximum active element energy density possible for each of the above listed dielectric systems based on heating effects only. The ambient temperature and starting temperature of the capacitor has been assumed to be 160°F (71°C) and the operating period has been chosen as 120 s.

In ac capacitors operating at 60 Hz, corona effects and

Fig. 14. Brooks coil [19] configuration for minimizing weight of inductor.

Fig. 15. Allowable current density in copper or aluminum as a function of operating time for ambient temperature of 160° F (71° C) and a final conductor temperature of 500° F (260° C).

The end result of these considerations is that, with the data presently available, the polypropylene-silicone oil dielectric system provides the highest energy density for the inverter capacitor over the range of operating conditions anticipated. This energy density is limited by corona effects rather than by heating effects. When the active elements are packaged to make a capacitor, the energy density in the finished capacitor is, of course, lower than that of the active elements. It will undoubtedly be necessary to use an extended foil construction technique in fabricating these capacitors. In addition, because of the high frequencies and high current levels present in the inverter resonant circuit, it will probably be necessary to use Litz wire leads on the capacitors. It may be necessary to "fan out" the individual stands of the Litz wires so that large-area connections can be made to the active elements. When these factors are taken into account, it appears likely that the overall density of a capacitor may be a factor of 2 lower than the energy density of the active elements. Using this factor of 2, then an empirical relation for the energy density of a polypropylene-silicone oil inverter capacitor can be established. This relation is

$$D = 15.5/f_i^{0.28} \text{ J/lb.}$$

Once more, it is pointed out that this energy density is limited by corona effects rather than by heating elements.

The above relation for the inverter capacitor energy density will be used to determine the specific weights of the inverter capacitors after the capacitor energy storage requirements have been determined.

D. Inductors

Both air-core and magnetic-core inductors have been examined. For the energy levels and operating periods of interest in this study, air core inductors have a significant weight advantage over magnetic core inductors.

For a given inductance, the minimum-weight coil configuration is the Brooks coil configuration shown in Fig. 14. The coil cross section is square with a width or height equal to the inner radius. The inductance of a Brooks coil is given by

$$L = 0.02549 C^2 / \delta^4 \text{ } \mu\text{H}$$

where C is the coil width in centimeters and δ is the center-to-center conductor distance in centimeters. The effect on inductance of conductor insulation is less than 0.1 percent and so is considered to be negligible.

The weight of the coil W is given by

$$\begin{aligned} W &= (3\pi/454) m \rho_m C^3 \text{ lb} \\ &= (3\pi/454) m \rho_m [\delta^4 L \times 10^6 / 0.02549]^{3/5} \text{ lb} \end{aligned}$$

where ρ_m is the conductor density in grams per centimeter cubed and m is the density correction factor to account for insulation.

For a given conductor material, a given operating period and a given temperature rise the maximum conductor current density is

$$J_m = I/A = I/m\delta^2$$

so

$$\delta^2 = I/mJ_m.$$

The weight of the coil can, therefore, be written as

$$W = (3\pi/454) (\rho_m/m^{1/5}) [LI^2 \times 10^6 / 0.02549 J_m^2]^{3/5} \text{ lb.}$$

Now, the energy density for the Brooks coil is

$$\begin{aligned} D_L &= 1/2 LI^2 / (3\pi/454) (\rho_m/m^{1/5}) \\ &\quad \cdot [LI^2 \times 10^6 / 0.02549 J_m^2]^{3/5} \end{aligned}$$

or

$$\begin{aligned} D_L &= 8.83 \times 10^{-4} (m^{1/5} / \rho_m) J_m^{6/5} \\ &\quad (\frac{1}{2} LI^2)^{2/5} \text{ J/lb.} \end{aligned}$$

Fig. 18. LC filter.

$$S_c = 1/124 f_i^{0.72}$$

where f_i is the inversion frequency in kilohertz. The only other element of significance in the transformer inverter shown in Fig. 17 is the transformer.

Next, a comparison between the full bridge transformer-type and transformerless inverters must be made. The transformer in one is replaced by capacitors and inductors in the other. From an operational point of view, there appears to be no substantial difference between the two units. It should be pointed out, however, that the transformer-type inverter has been under development for several years and the analysis of its operation including its ability to achieve precise load regulation has been reported in the literature [1]-[13]. Because of these factors its adaption to high-power levels may be more readily accomplished than the development of the transformerless type.

Judging from the information available concerning the transformerless inverter, the specific weight of this unit is essentially the same as that of the transformer-type unit. This is demonstrated by comparing the specific weight of the capacitors and inductors of the transformerless inverter with the specific weight of the capacitor and transformer in the transformer-type unit. From [23] the peak energy stored in the capacitors of a 10-MW, 10-kHz transformerless inverter would be 786 J. The energy stored in the inductors would be 1130 J. (These levels would vary slightly with voltage level and would scale directly with output power.) Using an energy density for the capacitor of 8 J/lb and for the inductor of 25 J/lb (estimated in a previous section of this paper) then the specific weight of the transformerless inverter reactive elements is 0.014 lb/kW. For a 10-kHz transformer-type inverter, the specific weight of the transformer and the series capacitor would be approximately 0.01 lb/kW. Thus, since the main part of the weight of either inverter is in the thyristor and diode packages, and these are the same for both inverters, the weights of the two inverters would be essentially the same.

In summary, at this point in time, because of development experience and analyses performed, it appears that the transformer-type inverter should be selected for development.

B. Rectifier and Filter

Before it is possible to define the filtering requirements of a power conditioning system it is necessary, of course, to describe the waveform that must be processed by the filter

and to define the load ripple requirement. In this power conditioning study, the load ripple requirement was not defined and so it was left as an input parameter to be supplied by the high-power system designer.

On the other hand, the waveforms that must be processed by the filter can be described reasonably well. The waveforms resulting from the rectification of the outputs of three-phase alternators and of inverters are described in the following sections. Then the specific weights and the energy content of LC and π filters are analyzed. Some attention is paid to the minimization of one or the other.

1) *Alternator Power Rectification*: When the output of a three-phase alternator (or three-phase alternator/transformer package) is full wave rectified the resulting average voltage (divided by the line-to-line rms voltage) can be expressed by the following empirical relations (from the Appendix): when $\alpha < \beta$,

$$V_{avg}/V_{1-lrms} = 1.350 \cos(0.660 \alpha + 2 \times 10^{-5} \alpha^3) \cdot \cos(0.692 \beta + 0.07 \alpha)$$

and when $\alpha \geq \beta$,

$$V_{avg}/V_{1-lrms} = 1.350 \cos(0.660 \alpha + 2 \times 10^{-5} \alpha^3) \cdot \cos(0.699 \alpha)$$

where α is the firing angle of the SCRs in the rectifier and β is the conduction overlap angle.

Similarly, the sixth harmonic ripple factor (the rms value of the sixth harmonic voltage divided by the average voltage) can be expressed by the following empirical relations (from the Appendix). When $\alpha < \beta$,

$$r_6 = 0.01 \{ 0.217 \beta - 0.04 \alpha + [2 + 3.3(\alpha/30)^2] \cdot \sin[(6.32 + \alpha^{0.15})(\beta - \alpha)] + [4.1 + 38.8(\alpha/60)^2 - 0.217 \beta + 1.5 \sin 5\beta + 0.04 \alpha] \cdot \exp[-(\beta - \alpha)/(8.5 + \sqrt{\alpha}/1.5)] \}$$

and when $\alpha \geq \beta$,

$$r_6 = .01 [4.1 + 38.8(\alpha/60)^2 + 1.5 \sin 5\alpha]$$

These empirical relations for V_{avg} and r_6 provide an excellent characterization of the waveform to be processed by the filter used with the bridge.

When the load regulation requirement is limited to a few percent and the conduction overlap angle is 45° , the value of r_6 is approximately 0.08.

At this point it should be mentioned that if two three-phase transformers are used with an alternator, then some reduction in the ripple percentage can be achieved. This is done by using one of the transformers in a wye-wye configuration and the other in a wye-delta configuration. The ripple-voltage waveforms produced by three-phase full-wave

Fig. 20. Full bridge series inverter with bridge rectifier and π filter.

Fig. 21. (A) Current waveform in primary of inverter transformer. (B) Current waveform from bridge rectifier.

and the energy as a function of capacitance written in normalized form is

$$E/E_{\min} = \frac{1}{2} C/C_{\min(E)} + \frac{1}{2} C_{\min(E)}/C.$$

The curves in Fig. 19 are plots of normalized specific weight and energy as functions of capacitance [normalized with respect to $C_{\min(s)}$]. The ratio of $C_{\min(E)}$ to $C_{\min(s)}$ is

$$C_{\min(E)}/C_{\min(s)} = (D_L/D_c)^{1/2}$$

and values of D_L of 25 J/lb and D_c of 100 J/lb have been used. Notice that, for this particular example, if the specific weight is minimized, then the energy is 25 percent above the minimum value. Similarly, if the energy is minimized, the specific weight is 25 percent above its minimum value. If, however, the value of capacitance that causes the two curves to cross is used, then the specific weight and the energy are only about 6 percent above their minimum values.

As an example of the use of the specific weight and energy equations for an LC filter, consider a requirement to reduce the ripple of a rectifier by a factor of eight (for example, from 8 percent for a three-phase full-wave bridge with a 45° conduction overlap to 1 percent required by a load). If the alternator frequency is 0.5 kHz, D_L is 25 J/lb and D_c is 100 J/lb. Then

$$S_{L\min} = (1/2\pi \times 3) (1/0.1 \times 25 \times 100)^{1/2}$$

or

Fig. 22. Voltage waveform appearing on capacitor C_0 (curve a) resulting from charging by rectifier current (curve b) and discharging (curve c).

$$S_{L\min} = 3 \times 10^{-3} \text{ lb/kW.}$$

For an output power of 25 000 kW the minimum energy is

$$E_{\min} = (25\,000/2\pi \times 3)(1/\sqrt{0.1})^{1/2}$$

or

$$E_{\min} = 3750 \text{ J.}$$

For an output voltage of 60 000 V,

$$C_{\min(s)} = 2.08 \mu\text{F}$$

$$C_{\min(E)} = 1.04 \mu\text{F.}$$

3) Inverter Power Rectification: The output of the series inverter may be fullwave rectified with a bridge rectifier and then filtered as shown in Fig. 20. The purpose of this section is to describe the waveform of the ripple voltage at the output of the bridge rectifier. Under fully loaded conditions the approximate current waveform in the primary winding of the transformer in Fig. 20 is as shown in Fig. 21(A) [1]. The approximate current waveform after rectification is shown in Fig. 21(B). During load voltage regulation, the angle α is adjusted to control the average current supplied by the inverter. The angle α may also be modulated to reduce the effects of variations in the voltage supplied to the inverter.

When the rectifier current is fed into a capacitor C_0 and load current is simultaneously drawn from C_0 , the resulting voltage waveform on C_0 is as shown in Fig. 22. The ripple component of this waveform is very nearly sinusoidal with a frequency of twice the operating frequency of the inverter. The magnitude of the ripple voltage can be estimated (the estimated value will be slightly smaller than will actually occur) by assuming that the output waveform of the inverter is a rectified sine wave. The rms value of the ripple factor, r_{in} (the ratio of the rms ripple voltage to the average load voltage) is then

The specific weight as a function of input ripple factor can be written in normalized form as

$$S_{\pi}/S_{\pi \min} = \frac{1}{3} r_{in}(\min s)/r_{in} + \frac{2}{3} (r_{in}/r_{in}(\min s))^{1/2}.$$

The minimum energy condition can be determined by following a procedure identical to that used for the specific weight. The energy content of the π filter is given by

$$E_{\pi} = (\sqrt{2} - 1) P_0 / 8\sqrt{2} f_r r_{in} + n P_0 / 2\pi f_r (r_{in}/r_l)^{1/2}$$

where the factor n is included to account for a situation in which the energy of the LC section is not minimized. (For energy minimization, $n = 1$). The energy is minimum when

$$r_{in}(\min E) = [(\sqrt{2} - 1)^{2/3} / 2] (\pi/n)^{2/3} r_l^{1/3}$$

and the value of the minimum energy is

$$E_{\pi \min} = 3(\sqrt{2} - 1)^{1/3} P_0 / 4\sqrt{2} (\pi/n)^{2/3} f_r r_l^{1/3}.$$

The energy as a function of the input ripple factor written in normalized form is

$$E_{\pi}/E_{\pi \min} = \frac{1}{3} r_{in}(\min E)/r_{in} + \frac{2}{3} (r_{in}/r_{in}(\min E))^{1/2}.$$

The curves in Fig. 23 are plots of normalized specific weight and energy as functions of input ripple factor (normalized with respect to $r_{in}(\min s)$). The ratio of $r_{in}(\min E)$ to $r_{in}(\min s)$ is

$$r_{in}(\min E)/r_{in}(\min s) = (m/n)^{2/3} (D_c/D_L)^{1/3}$$

and values of D_c of 100 and D_L of 25 have been used. Curves for $m = 1.25$, $n = 1$, and for $m = 1$, $n = 1.25$ corresponding, respectively, to energy minimization and to weight minimization in the LC section are given. Also, a curve for m/n corresponding to a compromise situation (equal weight and energy penalties) in the LC section is given. The utilization of these curves is probably best illustrated with an example of their use. Consider a minimum specific weight situation in which f_r is 20 kHz, r_l is 1 percent, D_L is 25 and D_c is 100. Then $S_{\pi \min}$ is calculated to be

$$S_{\pi \min} = 3(\sqrt{2} - 1)^{1/3} / 4\sqrt{2} \pi^{2/3} 20 \times 100^{2/3} (10^{-2} \times 25)^{1/3}$$

or

$$S_{\pi \min} = 6.8 \times 10^{-4} \text{ lb/kW}.$$

(This, by the way, is a factor of 2.7 below the specific weight of a filter consisting only of a capacitor.)

The energy content of this filter is (from the $m = 1$, $n = 1.25$ curve) 2.6 percent above the $E_{\pi \min}$ value. The $E_{\pi \min}$ value contains a factor of $n^{2/3}$ and is therefore 1.160 times greater than it would be for a filter designed

on the basis of minimized energy. Thus the energy content of this filter at 25 000 kW is

$$E_{\pi} = 1.026 \times 1.160 \times 3(\sqrt{2} - 1)^{1/3} 25\,000 /$$

$$4\sqrt{2} \pi^{2/3} 20 \times (10^{-2})^{1/3}$$

or $E_{\pi} = 1270 \text{ J}$. (This is a factor of 3.5 below the energy content of a filter consisting only of a capacitor).

If this were designed for minimum energy rather than minimum weight, then

$$S_{\pi} = 8.3 \times 10^{-4} \text{ lb/kW}$$

and

$$E_{\pi \min} = 1070 \text{ J}.$$

In the equation for the specific weight of the filter, it should be noted that weight varies with the cube root of ripple factor. Thus if a ripple factor of 0.1 percent rather than 1 percent is desired, the specific weight increases by a factor of $\sqrt[3]{10}$, or 2.15. Thus, for 0.1 percent ripple,

$$S_{\pi \min} = 1.46 \times 10^{-3} \text{ lb/kW}$$

and this is a factor of 12.5 below the specific weight of a filter consisting only of a capacitor. The energy content of this filter would be $E_{\pi} = 2740 \text{ J}$, which is a factor of 16.7 below the energy content of a filter consisting only of a capacitor.

C. Load Protection

The load protection requirement was not defined at the outset or during this high-power study. It is likely, considering the nature of the anticipated load, that an extremely fast and reliable load protection circuit will be required. Not only will fast switches be necessary, but also, the amount of energy to be diverted by the switches will probably have to be minimized. This means that the energy considerations previously given for filters may become extremely important. In addition, the use of multiple stagger timed inverters may be warranted to further reduce the filter energy content.

Shown in Fig. 24 are the components of the various algorithms, S_1 through S_{13} , required to determine the specific weights of the corresponding power conditioning techniques shown in Fig. 1. The quantities $S_{3\phi R}$, S_{LC} , etc., are the algorithms of the various components and subsystems presented earlier in this report. The factors W_1 through W_{13} are used to account for the estimated weight penalty incurred in interconnecting and packaging the components and subsystems. Corresponding factors V_1 through V_{13} (not shown in Fig. 24) may be used to determine the

TABLE IV
Specific Volumes ($\text{ft}^3/\text{kW} \times 10^{-4}$) for Various Point Designs

Parameter	Point Design Number					
	1 and 2	3	4	5 and 6	7	8
power (MW)	10	25	25	25	50	50
voltage (kV)	60	60	60	60	200	200
total run time (s)	63	64	120	63	75	120
Specific volume						
high-voltage alternator S_1 (vol)	1.1	1.1	1.7	1.1	1.4	2.0
low-voltage alternator S_2 (vol) and transformer	5.8	4.5	5.8	4.5	4.9	5.9
low-voltage alternator S_3 (vol) and inverter	4.5	4.5	7.0	4.4	5.1	7.1
MHD S_4 (vol)	3.9	3.7	5.7	3.6	4.2	5.9

Note: Add $\sim 0.8 \times 10^{-4} \text{ ft}^3/\text{kW}$ to account for inverter system supplying additional 10 percent power to auxiliary load.

The critical components (transformers, switches, capacitors, and inductors) for these systems have been identified and analyzed in detail and algorithms describing their weights have been developed. The major subsystems envisioned for possible use in the power conditioning systems have been analyzed. Finally, an analysis and comparison of the various power conditioning techniques for the point design operating conditions supplied by the AFAPL has been performed.

Appendix

Full-Wave Three-Phase Rectifier Voltage Waveforms

A circuit for the full-wave rectification of the output of a three-phase alternator (or three-phase alternator/transformer package) is shown in Fig. 25. The line to neutral voltages appearing at terminals A, B, and C are shown in Fig. 26. When the reactance of the alternator or the alternator/transformer package is low, rectifier conduction occurs during the periods indicated in Fig. 26 and the rectified waveform is as shown. When appreciable reactances exist in the alternator or the alternator/transformer package, inductive voltages are generated which extend the conduction periods of the rectifiers. As a result, periods exist in which two rectifiers are conducting simultaneously. The voltage is approximately the average of the two-phase voltages during the conduction overlap period. This conduction overlap situation is shown in Fig. 27 along with the rectified waveform.

It is evident from Figs. 26 and 27 that there is an appreciable increase in the ripple content of the rectified voltage when conduction overlap occurs. In addition, there is a decrease in the average value of the rectified voltage.

When SCRs are used in the three-phase rectifier and their firing angle, α , is controlled to regulate the average voltage, then the rectifier voltages are as shown in Fig. 28. By performing a Fourier analysis of the rectified voltage waveform, the amplitudes of the average output voltage and of the harmonic ripple voltages of the rectifier can be determined. Fig. 29 contains plots of the ratio of the average

Fig. 25. Three-phase full-wave rectifier.

Fig. 26. Line to neutral voltages and rectified voltage for three-phase full-wave rectifier operating from a low reactance alternator or alternator/transformer package.

output voltage to the line-to-line rms voltage. The solid points represent average voltage values resulting from the Fourier analysis. The curves represent calculations performed using the following empirical relations.

When $\alpha \leq \beta$,

$$V_{\text{avg}}/V_{L-L\text{rms}} = 1.350 \cos(0.660\alpha + 2 \times 10^{-5}\alpha^3) \\ \cdot \cos(0.692\beta + 0.07\alpha)$$

and when $\alpha \geq \beta$, then

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TITLE: LECTURE 2: DC POWER SUPPLIES AND HARD-TUBE POWER CONDITIONING SYSTEMS

AUTHOR(S): W(alter) James Sarjeant

SUBMITTED TO: PULSE POWER COURSE

University of California

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LECTURE 2

DC POWER SUPPLIES AND HARD-TUBE POWER CONDITIONING SYSTEMS

by

W. J. Sarjeant

LECTURE 2

DC POWER SUPPLIES AND HARD-TUBE POWER CONDITIONING SYSTEMS

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LECTURE 2:
DC POWER SUPPLIES AND HARD-TUBE POWER CONDITIONING SYSTEMS
by
W. J. Sarjeant

I. INTRODUCTION

In this lecture, we shall discuss some of the major characteristics of DC power supplies and hard-tube pulsers, along with many of their PCS limiting properties and several of the more powerful vacuum tubes that are currently available for larger hard-tube pulsers systems. The initial topic will be elementary DC power supplies (Fig. 1), operating with standard 60 Hz AC input power. The full-wave bridge design will be discussed in detail, illustrating the majority of the significant circuit parameters.

II. DC POWER SUPPLIES

If a simple DC power supply is snap started (suddenly energized to full voltage), and then the diodes, transformers and sometimes the filter choke are damaged, what has caused this damage is often a transient overvoltage effect rather than a short circuit on the output. These types of relatively low-frequency transients, which can cause damage and component breakdown in 60-400 Hz DC supplies, will be the subject of considerable discussion in this section. The standard 60 cycle, or 60 Hz, full-wave bridge geometry (FWB) will serve as the design center, and the worst-case relations for all the design parameters applying to the diode, transformers, and filter design and selection, that apply all the way from full-wave doublers to 6-phase and 12-phase Y-delta DC power supply systems will be compared and related to these FWB parameters. For all of these classes of supplies, the same type of transient analysis will apply to

DC POWER SUPPLIES

THIS SECTION CONSIDERS ELEMENTARY DC POWER SUPPLIES OPERATING FROM 60 Hz AC POWER. THE FULL-WAVE BRIDGE DESIGN WILL SERVE TO ILLUSTRATE IMPORTANT CIRCUIT PARAMETERS. THROUGHOUT THE DISCUSSION, RELATIONS APPLICABLE TO OTHER GROUPS OF DC SUPPLIES WILL BE CORRELATED WITH THE FULL-WAVE BRIDGE (FWB) PARAMETERS.

CLASSES OF DC SUPPLIES ARE ILLUSTRATED IN THE "POWER SUPPLY" SECTION OF Terman (A HANDOUT).

ELEMENTARY TYPES AS SHOWN IN THE NEXT FIGURE

- A) HALF WAVE
- B) FULL WAVE, CENTER-TAP
- C) FULL-WAVE BRIDGE
- D) FULL-WAVE VOLTAGE DOUBLER
- E) VOLTAGE MULTIPLIER

modeling the potential damage to components. A first-cut design of the power supply and the prerequisite damping networks can be obtained from this data and then, returning to the reference data that will be discussed, cost-benefit ratios for the supply design can be effected. In other words, what may come from the worst-case analysis here may be that the diode chosen has a higher current rating than that found necessary when a detailed circuit analysis has been executed. In this simplified, worst-case analysis, the design parameters determined will always err on the conservative side. One exception is the case in which the power line impedance is very low, potentially giving rise to enormous surge currents measuring hundreds of amps. Here for large, high-regulation, solid-state DC supplies of this type, it is necessary to obtain more accurate design data requiring a return to the fundamental relations in the references. The best source for most of the diode and filter design data is the RCA thyristor manual. In the back is an application note that is a summary of the work by Shade in the 1960s and it has almost all the design information needed to select a rectifier diode for the design. The other area discussed in the thyristor manual is how to execute relatively accurate ripple calculations when dealing with fairly precision supply requirements.

The five basic classes of elementary DC supplies are the half wave, the full-wave center tap, the full-wave bridge, the voltage doubler, and the voltage multiplier. These types of supplies are described in detail in the handout from Terman. Although it deals with tubes, this is probably one of the better analyses of the general characteristics of these types of supplies for ripple and the ratings the diodes must have. A summary of these different types of elementary supplies, A through E, is shown in Fig. 2.

A. HALF WAVE: The load is pure resistive. There is no filter capacitor included, giving bursts of voltage, one for each positive going supply cycle .

Half-wave (single-phase) output

(a) Half-wave rectifier circuit

Full-wave (two-phase) output

(b) Full-wave circuit with center-tapped transformer

(c) Full-wave bridge circuit

Voltage multiplying circuits

(d) Conventional voltage doubling circuit

(e) Cascade doubler

(f) Cascade quadrupler

- Terman

FIG. 2: ELEMENTARY DC POWER SUPPLIES

- B. FULL-WAVE WITH CENTER-TAP: There is a diode added to the bottom half of the transformer that gives a ripple frequency of 120 Hz, or twice the input frequency.
- C. FULL-WAVE BRIDGE: The main advantage of this supply is the high overall efficiency through full utilization of all the available transformer KVA or power rating.
- D. VOLTAGE MULTIPLYING CIRCUITS: These are basically charge stacking systems that can be used to provide very low currents at relatively high voltage (We will not discuss them in detail). At 60 Hz, they tend to be very difficult to design for currents above 10-15 milliamps, so they're basically very low average current supplies. For low current requirements, on the order of milliamperes, they're an inexpensive voltage multiplying technique. The slight disadvantage of using them is the relatively high RMS rating that the diodes and capacitors must have.
- E. COCKROFT-WALTON: This is the same type of voltage multiplier with a very high-frequency input. Then the reactances of the filter capacitors becomes very low, so that the amount of capacity needed to achieve very good regulation is small. Very large voltage multiplication factors with low ripples are obtainable.

FULL-WAVE BRIDGE DC SUPPLY - DETAILED ANALYSIS WITH CHOKE INPUT FILTER

Fig. 3 shows a sketch of the full-wave bridge DC supply. On the left side is the main power transformer, fed power through a set of line filters to keep transients from passing through in either direction, and a circuit breaker that is rated to trip on severe overloads. If the power supply is reasonably large, the contactor shown is employed to turn on the supply and it is generally energized by a latching circuit. The transformer feeds the diode full-wave bridge and the output voltage ripple

FIG. 3: FULL-WAVE BRIDGE DC POWER SUPPLY

is reduced by the LC filter. On a positive line cycle, current flows as shown by the arrows into the filter capacitors and back to the transformer. The transformer here is shown as having electrostatic shields, which are normally used in all sensitive circuitry or circuitry where there are relatively high voltages and transients, in order to shield the primary electrostatically from the secondary by a grounded metal structure. For single phase designs the shield cost is at most a few percent of the transformer cost, so there is no excuse not to ask the manufacturer to include them. Generally, for only a single shield, it is referred to the primary because of cost, since if a relatively high voltage is on the secondary, the added insulation for an electrostatic shield on the secondary is often expensive. The shield is generally a single layer of copper or aluminum foil over the primary, slit so the foil does not act as a shorted turn on the transformer, with a wire connecting the shield to the transformer case. That effectively puts an equipotential plane between the primary and the secondary as far as capacitive coupling is concerned, as no matter how the transformer is wound, there is some capacity from the primary to the secondary, and what is desired is that any current that the secondary tries to induce from capacitive coupling to the primary is carried off to the ground via the shield.

On the primary side of the power transformer in the DC supply, when the contactor is energized, the equivalent circuit here has the minimum inductance in the transformer, giving rise to large in-rush currents that must be taken into account in selecting circuit breaker and contactor surge ratings. In addition, a relatively large voltage transient can arise when the contactor is opened, caused by the collapse of the magnetic energy stored in the leakage and magnetizing inductances. The critical damping network, $R'_S - C'_S$, is employed here to dampen that transient when the contactor opens, particularly if one is unlucky enough to have it open at maximum current into the transformer. The damping network, $R_S'' - C_S''$, on the secondary of the transformer is used for protection from both rapid turn-on (if turn-on is at the peak of the primary voltage this

network dampens the oscillations in the secondary loop), and for turn-off of the system when the diode FWB is then decoupled from the load loop as the transformer secondary voltage drops to zero. The diodes then no longer conduct current and are no longer pushing charge into the filter capacitor, so the secondary is just an RLC oscillator of R'_s C'_s and the transformer leakage and magnetizing inductances (referred to the secondary). Selection of R'_s and C'_s values for near critical damping then controls the oscillation amplitude and duration. The same argument applies to damping transients across the filter choke. All these situations use an analogous damping technique.

The output of the supply in this case is shown as two filter capacitors in series, which indicates that, when the voltage is sufficiently high that a single capacitor is unavailable, it is necessary to stack a number of them in series and to use the voltage-grading resistors shown. A grading resistor is generally incorporated to pass on the order of 8 to 10 times the maximum leakage current through the capacitor at the maximum operating temperature of the supply. This ensures no more than a 15-20% imbalance in the voltage division across the 2 capacitors shown.

The overload current sensor in Fig. 2 serves to protect the diode bridge should the supply output be shorted to ground. It's timed to react within about 50-200 ms (typically this opening time can be achieved with fast electromechanical relays or vacuum interrupters). So if there is a fault on the output, the pulse current will energize the sensor, which then trips open the primary contactor (in this case, the important cost parameter is the number of cycles the diodes can carry the fault current before the contactor opens).

Next, let us discuss the characteristics of each of the individual sub-components in the DC supply. There is an efficiency advantage of a full-wave bridge over the center-tapped geometry because the full-wave bridge constantly loads the secondary of the transformer, whereas the center-tapped geometry unloads the transformer on alternate half cycles

when that particular portion of the winding is not conducting current. Also, during the recovery time of the diodes, it is possible for shock-excited oscillations to arise, damaging the transformer insulation or the reverse-biased diodes in the center-tapped geometry.

Fig. 4 shows a summary from Mlynar of various design terminologies for diodes and transformers that is presented for general information. The key parameters of concern in DC supply design are (* in Fig. 4) the repetitive peak reverse voltage applied to the the diode, the worst-case asymmetrical fault current that the diode string can experience under start-up conditions or short-circuit faults in the load, and the KVA or power rating of the single-phase and three-phase power transformers. In general, most of the needed design information can be obtained from the table from Mlynar that is shown in Fig. 5. However, the question of transients and some of the simplifications possible in designing for low ripple aren't in there. These will be discussed presently.

In multiphase power supplies, each phase has a unique role to play. For very high powers (megawatts at hundreds of kV) the cheapest alternative is the six-phase series bridge (ref. 12-1-1-B in Fig. 5) because the voltage stresses on the transformer can be kept quite low (essentially half) keeping the corona level low. Interphase transformer power supplies in three-phase configurations can suffer from transient saturation effects when a pulse load is used (like a resonant-charging circuit or discharge networks).

Six-phase parallel operation, without interphase transformers is acceptable, except that fairly fast recovery diodes are needed to avoid one diode starting to conduct while the other one is trying to turn off, producing a high reactive kilovar loading of the secondary of the transformer. The reactive power that flows through the winding and the ohmage loss in the winding create significant transformer heating. The three-phase full-wave bridge is the best general-purpose three-phase supply, with either a Y or a Delta secondary configuration.

FIG. 4: TERMINOLOGY - DIODES AND TRANSFORMER

VOLTAGE

1. Design PRV - the arithmetic sum of the individual peak reverse voltages of all seriesed rectifiers; the absolute maximum reverse voltage that the assembly can withstand without catastrophic failure.
2. Rated PRV - the maximum reverse voltage normally expected based on the actual circuit operating conditions; the design peak minus any safety factor.
3. Transient PRV - the maximum allowable transient reverse voltage that might be expected under normal operating conditions.
- 4.* Blocking PRV - the maximum value of repetitive peak reverse voltage rating permitted by the manufacturer under stated conditions.
5. Output voltage - normally applies to the DC voltage output of a complete rectifier assembly.
6. AC voltage - the RMS input value, either line to line or line to neutral, of voltage applied to the rectifier assembly or bridge.

CURRENT

1. Rated current - the average value of DC current, generally the output current of a rectifier assembly; however, can be the per-leg average output current.
2. Short circuit current - the value of current that the output load could experience under continued short circuit conditions, usually limited by circuit impedance.
- 3.* Asymmetrical fault current - the peak value of current delivered to the output load caused by overshoot of the short circuit current; typically 1.5 to 1.8 times the short circuit current allowed by circuit conditions.

KVA

- 1.* Three-phase KVA using three single-phase transformers - system KVA is equal to three times the individual KVA of the single-phase transformers; however the system impedance is 3 times the individual single-phase impedance value.
2. Short circuit KVA - maximum KVA available under short circuit conditions.

- Mlynar

FIG. 5: DC POWER SUPPLY DESIGN DATA FOR RESISTIVE LOADS

CONFIGURATION	SINGLE PHASE HALF-WAVE	SINGLE PHASE FULL-WAVE CENTER TOP	SINGLE PHASE FULL-WAVE BRIDGE	THREE PHASE HALF-WAVE WYE	THREE PHASE FULL-WAVE BRIDGE
SCHEMATIC					
SYMBOLIC NOTATION	1-1-1-H	2-1-1-C	4-1-1-B	3-1-1-Y	6-1-1-B
OUTPUT WAVEFORM					
PEAK REVERSE VOLTS PER RECTIFIER LEG (WORKING PRV)	$1.414 V_{AC}$ <i>X 2.23</i>	$2.828 V_{AC}$	$1.414 V_{AC}$	$2.45 V_{AC}$	$1.414 V_{AC}$
PEAK REVERSE VOLTS PER RECTIFIER LEG (WORKING PRV) RESISTIVE OR INDUCTIVE LOAD ONLY	$314 V_{DC}$	$314 V_{DC}$	$187 V_{DC}$	$209 V_{DC}$	$105 V_{DC}$
RMS VOLTS OUTPUT RESISTIVE OR INDUCTIVE LOAD ONLY	$187 V_{DC}$	$111 V_{DC}$	$111 V_{DC}$	$102 V_{DC}$	$100 V_{DC}$
PEAK VOLTS OUTPUT RESISTIVE OR INDUCTIVE LOAD ONLY	$314 V_{DC}$	$187 V_{DC}$	$157 V_{DC}$	$121 V_{DC}$	$105 V_{DC}$
NO LOAD RMS A.C. VOLTAGE $[V_{AC} (NL)]$	$2.22 V_{DC} (NL)$	$2.22 V_{DC} (NL)$	$1.11 V_{DC} (NL)$	$1.48 V_{DC} (NL)$	$.74 V_{DC} (NL)$
AVERAGE D.C. OUTPUT CURRENT PER RECTIFIER LEG	$1.00 I_{DC}$	$.5 I_{DC}$	$.5 I_{DC}$	$.333 I_{DC}$	$.333 I_{DC}$
RMS CURRENT PER RECTIFIER LEG	$1.57 I_{DC}$	$.707 I_{DC}$	$.707 I_{DC}$	$.577 I_{DC}$	$.577 I_{DC}$
I_{AC} RMS CURRENT IN THE LINE BETWEEN THE TRANSFORMER AND THE RECTIFIER	$1.57 I_{DC}$	$.707 I_{DC}$	$1.00 I_{DC}$	$.577 I_{DC}$	$.816 I_{DC}$
RATED POWER OF THE TRANSFORMER PRIMARY	$2.47 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.21 I_{DC} V_{DC} (NL)$	$1.05 I_{DC} V_{DC} (NL)$
RATED POWER OF THE TRANSFORMER SECONDARY	$3.53 I_{DC} V_{DC} (NL)$	$1.57 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.48 I_{DC} V_{DC} (NL)$	$1.05 I_{DC} V_{DC} (NL)$
INTERPHASE TRANSFORMER RMS VOLTAGE (LINE TO NEUTRAL)	—	—	—	—	—
EQUIVALENT POWER RATING OF INTERPHASE TRANSFORMER	—	—	—	—	—

V_{AC} - RMS A.C. SECONDARY VOLTAGE
 $V_{DC} (NL)$ - AVERAGE NO. LOAD D.C. OUTPUT VOLTAGE
 V_{DC} - AVERAGE D.C. OUTPUT VOLTAGE OF RECTIFIER $PRV = \sqrt{2} V_{AC} (NL)$
 I_{DC} - AVERAGE D.C. OUTPUT CURRENT OF RECTIFIER $I_{DC} V_{DC} (NL)$ - IDEAL OUTPUT POWER OF THE RECTIFIER

THREE PHASE DOUBLE WYE	FOUR PHASE FULL-WAVE CENTER TAP	FOUR PHASE FULL-WAVE PARALLEL BRIDGE	FOUR PHASE FULL-WAVE SERIES BRIDGE	SIX PHASE STAR (THREE PHASE DIAMETRIC)	SIX PHASE PARALLEL BRIDGE WITHOUT INTERPHASE TRANSFORMER	SIX PHASE PARALLEL BRIDGE WITH INTERPHASE TRANSFORMER	SIX PHASE SERIES BRIDGE
6-1-1-Y	4-1-1-C	2(4-1-1-B)	6-1-1-B	6-1-1-B	2(6-1-1-B)	12-1-1-B	12-1-1-B
$2.45 V_{AC}$	$2.828 V_{AC}$	$1.414 V_{AC}$	$1.414 V_{AC}$	$2.828 V_{AC}$	$1.36 V_{AC}$	$1.414 V_{AC}$	$1.414 V_{AC}$
$2.09 V_{DC}$	$2.22 V_{DC}$	$1.11 V_{DC}$	$.785 V_{DC}$	$2.09 V_{DC}$	$1.01 V_{DC}$	$1.05 V_{DC}$	$.524 V_{DC}$
$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$	$1.00 V_{DC}$
$1.05 V_{DC}$	$1.11 V_{DC}$	$1.11 V_{DC}$	$1.11 V_{DC}$	$1.05 V_{DC}$	$1.05 V_{DC}$	$1.05 V_{DC}$	$1.05 V_{DC}$
$1.71 V_{DC} (NL)$	$1.57 V_{DC} (NL)$	$.785 V_{DC} (NL)$	$.555 V_{DC} (NL)$	$1.48 V_{DC} (NL)$	$.716 V_{DC} (NL)$	$.74 V_{DC} (NL)$	$.37 V_{DC} (NL)$
$.167 I_{DC}$	$.25 I_{DC}$	$.25 I_{DC}$	$.5 I_{DC}$	$.167 I_{DC}$	$.167 I_{DC}$	$.167 I_{DC}$	$.333 I_{DC}$
$.289 I_{DC}$	$.50 I_{DC}$	$.50 I_{DC}$	$.707 I_{DC}$	$.408 I_{DC}$	$.408 I_{DC}$	$.289 I_{DC}$	$.577 I_{DC}$
$.289 I_{DC}$	$.50 I_{DC}$	$.707 I_{DC}$	$1.00 I_{DC}$	$.408 I_{DC}$	$.577 I_{DC}$	$.408 I_{DC}$	$.816 I_{DC}$
$1.05 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.11 I_{DC} V_{DC} (NL)$	$1.28 I_{DC} V_{DC} (NL)$	$1.01 I_{DC} V_{DC} (NL)$	$1.01 I_{DC} V_{DC} (NL)$	$1.01 I_{DC} V_{DC} (NL)$
$1.48 I_{DC} V_{DC} (NL)$	$1.57 I_{DC} V_{DC} (NL)$	$1.57 I_{DC} V_{DC} (NL)$	$1.57 I_{DC} V_{DC} (NL)$	$1.81 I_{DC} V_{DC} (NL)$	$1.43 I_{DC} V_{DC} (NL)$	$1.05 I_{DC} V_{DC} (NL)$	$1.05 I_{DC} V_{DC} (NL)$
$252 V_{DC} (NL)$	—	—	—	—	—	$.085 V_{DC} (NL)$	—
$.162 I_{DC} V_{DC} (NL)$	—	—	—	—	—	$.039 I_{DC} V_{DC} (NL)$	—

Fig. 6 presents a tabulation from Terman for the choke-input DC power supply parameters, and Fig. 7 illustrates these rectifier circuits. In this case, there is a large inductor in series with the rectifier assembly, and one chooses this inductance to be sufficiently large so that at all times there is a current flow through that inductor. We'll see later that there is a critical value of the inductance, depending upon the minimum load current. If the current in the load becomes sufficiently small, a point is reached at which the inductor no longer conducts current continuously into the capacitance; there is thus no longer a unidirectional current flow through the circuit filter network so that the stored energy in the inductor can start impressing current into the filter capacitor if the net current flow is small enough, and the output voltage then rises to the peak voltage of the transformer. This is the turnover point in the regulation performance of the system, going from the choke input to capacitor input performance. This means that when a choke input supply is designed, there is a minimum load current that must flow through the output loop at all times. This is normally not a particularly difficult problem if one selects a type of choke whose inductance is a function of the current passing through it (swinging choke). The range in inductance from the minimum current to full-load current can normally exceed five to one, falling at the higher current as the choke core approaches saturation.

This summary, then, has indicated some of the salient aspects of single and multiphase DC power supply design. The tables allow selection of the RMS value of the transformer secondary voltage and current ratings. The maximum diode inverse voltage, for example, for the double three-phase series connection (Fig. 6) was 1.05 and for the single phase full-wave center-tapped connection it was 3.14. Analysis by Greenwood discloses that the peak overvoltage in any of these systems under stop-and-start conditions is about two. On rare occasions, peak overvoltages of three have been noticed. The usual engineering practice is to multiply diode inverse voltage requirements by two. This factor of two (a safety factor) almost guarantees that the device will not break

Rectifier Circuits

CHOKE INPUT DC POWER SUPPLY	Single- Phase Full- Wave Center- Tapped	Single Phase Full- Wave Bridge	Three- Phase Star	Three- Phase Broken Star	Six Phase Single Y	Double- three- Phase With Balance Coil	Double- three- Phase Series Convec- tion	Six Phase Star
	1b	1c	2a	2b	2c	2d	2e	2f
<i>Relations</i>								
Voltage Regulation (d-c component of output voltage taken as 1.0):								
a. Rms value of transformer secondary voltage (per leg).....	1.11*	1.11	0.855	0.985	0.428	0.855	0.428	0.740
b. Maximum inverse voltage.....	3.14	1.57	2.09	2.09	1.05	2.42	1.05	2.09
c. Lowest frequency in rectifier output (F = frequency of power supply).....	2F	2F	3F	3F	6F	6F	6F	6F
d. Peak value of first three a-c components of rectifier output:								
Ripple frequency.....	0.667	0.667	0.250	0.250	0.057	0.057	0.057	0.057
Second harmonic of ripple frequency.....	0.133	0.133	0.057	0.057	0.014	0.014	0.014	0.014
Third harmonic of ripple frequency.....	0.057	0.057	0.025	0.025	0.006	0.006	0.006	0.006
e. Ripple peaks with reference to d-c axis:								
Positive peak.....	0.363	0.363	0.209	0.209	0.0472	0.0472	0.0472	0.0472
Negative peak.....	0.637	0.637	0.395	0.395	0.0930	0.0930	0.0930	0.0930
Current relations:								
f. <u>Average current per anode</u>	0.500	0.500	0.333	0.333	0.333	0.333	0.333	0.167
Peak anode current.....								
g. <u>Average current per anode</u>	0.500	0.500	0.333	0.333	0.333	0.167	0.333	0.167
Direct-current in load.....								
h. <u>Peak current per anode</u>	1.000	1.000	1.000	1.000	1.000	0.500	1.000	1.000
Direct-current in load.....								
Transformer requirements (d-c output power = 1.0):								
i. Primary kva.....	1.11	1.11	1.21	1.05	1.05	1.05	1.05	1.28
j. Secondary kva.....	1.57	1.11	1.48	1.71	1.05	1.48	1.48	1.81
k. Average of primary and secondary kva...	1.34	1.11	1.35	1.38	1.05	1.26	1.26	1.55

NOTE: This table assumes that the input inductance is sufficiently large to maintain the output current of the rectifier substantially constant, and neglects the effects of voltage drop in the rectifier and the transformers.

- Secondary voltage of one side of center tap.

The principal component of voltage across the balance coil has a frequency of 3F and a peak amplitude of 0.500. The peak balance coil voltage, including the smaller higher harmonics, is 0.605.

Refer to Fig. 7 for schematic diagrams of each rectifier circuit.

- Terman

FIG. 6: RECTIFIER CIRCUIT PARAMETERS FOR RESISTIVE LOADS OR VERY LARGE INPUT INDUCTANCE VALUES.

Circuits giving three-phase output wave

Circuits giving six-phase output wave

FIG. 7: RESISTIVE-LOAD OR CHOKE-INPUT RECTIFIER CIRCUITS

down. If, however, one deals with very high voltages, for example with 200-500 kV diodes, the ratio of cost to kV becomes larger and larger, making high kV ratings ever more expensive. In that particular case, the design is chosen to be as close to the safety margin as possible but, in general, not below 1.5 times the anticipated inverse voltage per leg is used. For example, for the above two cases, the inverse voltage ratings for the diode strings would be 1.6 and 4.7 times, respectively. In Fig. 6, Part d indicates that the ripple frequency amplitude components are expressed as a factor of the DC output voltage. For example, given a double three-phase series connection, the peak-to-peak ripple is .057 times the average output voltage. In Fig. 6, parts f to h, the peak diode current is compared to the direct-current in the load. The output voltage then rises to the peak of the AC input voltage. With the peak and average current through the diode known, one can go back to the RCA monograph and, after a brief analysis, determine the safety factors to apply and the diodes to choose. The last entries in Fig. 6, parts i-k, pertain to the transformer kVA relative to the DC output power as 1.0. Transformer kVAs up to a few hundred are not hard to buy, but diode costs rapidly escalate above 3A (average current).

Returning now to the full-wave bridge, its principal advantage is the superior transient suppression (Figs. 8 & 9) and more efficient use of transformer KVA. The center-tap design has a pair of solid-state diodes connected to a filter capacitor. During the transition when this diode becomes nonconducting, a small recombination charge current flows through it leading to a step in the recovery response of the diode. When that happens, this current shock excites this part of the transformer into oscillation. An R-C shunt network across the secondary will tend to dampen these oscillations. In the case of the full-wave bridge, the transformer is fully loaded for both cycles, minimizing the formation of such transients.

In the choke input case for the FWB (Ref. Fig. 6), if V_{DC} is the output DC voltage at I_{DC} , then if regulation is defined this way: the

FULL WAVE BRIDGE (FWB) POWER SUPPLY

THE MAIN ADVANTAGE OVER THE CENTER TAP DESIGN IS THE FAR SUPERIOR TRANSIENT SUPPRESSION OF FWB DESIGN DURING RECTIFIER COMMUTATION. ALSO, THE MAXIMUM INVERSE VOLTAGE IN THE FWB CASE IS ONE-HALF THE CENTER TAP DESIGN.

CHOKE INPUT

LET V_{DC} BE THE (FULL LOAD) OUTPUT DC VOLTAGE AT CURRENT I_{DC} . DEFINE % REGULATION $= \frac{V_{NL} - V_{DC}}{V_{NL}}$

FOR A CHOKE INPUT FILTER THE MINIMUM INDUCTANCE AT THE MINIMUM LOAD CURRENT IS: $\frac{\omega L}{R_L^{max}} \geq \frac{E_{ripple}}{V_{DC}}$ $\omega = 2\pi \times \text{ripple frequency}$
 $E_{ripple} = \text{peak ripple voltage}$

IF A LOWER CURRENT IS DRAWN, THEN THE CHOKE BECOMES INEFFECTIVE AND THE OUTPUT VOLTAGE RISES TO THE PEAK VOLTAGE OF THE TRANSFORMER SECONDARY.

FOR FWB DESIGN

$$\frac{\omega L}{R_L^{max}} > 0.7 \quad \therefore \quad \frac{L}{R_L^{max}} > 10^{-3} \quad \text{AND} \quad R_L = \frac{V_{DC}}{I_{DC}}$$

RIPPLE

$$\text{PEAK VALUE} \approx \frac{0.7 V_{DC}}{\omega^2 LC} \quad \text{CF FOR RC FILTER} \approx \frac{0.7 V_{DC}}{\omega RC}$$

$$\text{WHERE } \omega = 2\pi (2 \times 60)$$

FIG. 8

SINCE: $\frac{R_L^{\max}}{L} \leq 10^{-3}$

$E_{\text{output}}^{\text{ripple}} \approx 10^{-6} \cdot \frac{V_{\text{DC}}}{(LC)}$ FOR EXAMPLE, $C = \frac{C_0}{2}$ IN FIG. 3

COMPARE TO THREE-PHASE FWD WHERE:

$\frac{R_L^{\max}}{L} < 40,000$ AND $E_{\text{ripple}}^{\text{output}} \approx 10^{-8} \cdot \frac{V_{\text{DC}}}{(LC)}$

THUS THE DESIRED RIPPLE AND LOAD RANGE DEFINES THE VALUE OF L.
SWINGING CHOKES OFFER IMPROVED REGULATION FOR A LARGE RANGE IN
 R_L VALUES: A $\frac{\Delta L}{L}$ OF FIVE TIMES IS OBTAINABLE.

THE VOLTAGE RATING OF CHOKE = $2 \times V_{\text{DC}}$.

TRANSFORMER PARAMETERS IN THE FWD SUPPLY

1. $V_{\text{RMS}} = 1.11 V_{\text{DC}}$	}	NEGLECTING CORE AND
WINDING LOSSES
2. $\text{KVA} = 1.11 \text{ DC OUTPUT POWER}$		

DIODE PARAMETERS IN THE FWD SUPPLY

$\text{PRV} \geq 2 V_{\text{DC}}$ (X2 SAFETY FACTOR)

$I_{\text{AV}} = \frac{1}{2} I_{\text{DC}}$ AND $I_{\text{PK}} = I_{\text{DC}}$

R:
—

SELECT TO BE LESS THAN 10% OF THE LEAKAGE RESISTANCE OF
CAPACITOR C_0 AT THE MAXIMUM TEMPERATURE. THIS ENSURES AN EQUAL
DIVISION OF VOLTAGE TO WITHIN $\pm 20\%$.

FIG. 9

no-load voltage minus the DC voltage over the no-load voltage, then the choke input filter minimum inductance for a given R_L is:

$$\frac{\omega L}{R_L^{\max}} \geq \frac{E_{\text{ripple}}}{V_{\text{DC}}}$$

The angular frequency, ω , (2π times the ripple frequency) times L (at the current $V_{\text{DC}}/R_L^{\max}$) over the maximum value of the load resistor has then to be larger than the peak ripple voltage over the DC voltage. If a lower current is drawn, there is a point at which the choke is no longer effective (Fig. 10). A measure of the ripple on the output of the filter is obtained from Fig. 6. If instead of an LC filter an RC filter is used, the ripple at ω , the angular ripple frequency, would be considerably larger. From the calculation of the specified ripple required one can work back and calculate the value of the LC product, under the constraint of the relations in Fig. 9. That capacity, C , is the total capacity on the output of the LC filter, but in the case of the full-wave bridge we had two capacitors, each of value C_0 , in series so that:

$$C = \frac{C_0}{2}$$

If a broad range in output current is required, one can derive I_{max} from the amount of tolerable ripple, the minimum load current, and the swinging choke inductance ranges available. Given L and C , all the remaining design parameters in the system can be calculated.

In the full-wave bridge, the peak output ripple is:

$$10^{-6} \cdot \frac{V_{\text{DC}}}{LC}$$

In this case, V_{DC} is the actual DC output voltage. This is to be compared to the three-phase power supply, where the increased ripple frequency results in a factor of 100 times reduction in the ripple. That's one of the real advantages of three-phase over single-phase operation, other than regulation. It's either easier to filter to a given

(a) Circuit of rectifier and filter-
two-phase case

(b) Circuit of rectifier and filter-
three-phase case

- Terman

FIG. 10: EFFECT OF A FINITE INPUT INDUCTANCE
ON CURRENT WAVESHAPES

ripple level or, if a much smaller ripple is desired, the same size of inductors and capacitors can be used to gain several orders of magnitude reduction in the ripple. The major point about a swinging choke is that if there is a large range of load resistance to deal with, a choke whose inductance increases as the current decreases can keep the above relationships relatively constant. Now, a typical range in inductance is 5:1. Normally, in the design of filter choke one assumes that at turn-on there is zero voltage on the output, while peak volts are on the input. The insulation stress on the choke at 60 Hz is normally taken as twice that peak voltage -- the safety margin then being a factor of two. Recall that the safety margin in the diodes is also a factor of two. The rationale for that is in the economics of manufacture. It's relatively cost-effective to design, up to voltages of 100 kV, with such an insulation and diode safety factor. The damping network then controls the transient voltages to a factor of twice the peak voltage. This all fits in rather nicely with the design factor of two for the insulation level and defines the peak overvoltages to be experienced under worst-case conditions.

Where substantial high-voltage insulation is necessary in high-voltage chokes, the choke can then cost as much as or more than a modest-sized power transformer. A typical choke, for example, that might cost several hundred dollars to buy in an open mounting configuration could cost a factor of 2 to 3 more if hermetically sealed with a pair of high-voltage feed-through bushings. So the optimal solution is generally to put all power supply components in one container, fill it with transformer oil, and put a top on it.

A shunt resistance, R , is needed across each of the filter capacitors, C_0 . Normally if R is chosen to be 10% of the leakage resistance at the maximum operating temperature, then the percent variation of the voltage across C_0 under all conditions caused by temperature changes is $\pm 20\%$. The capacitor heats up and cools down (from the environment and ripple current heating), and the leakage

resistance can vary over a wide range. If the resistors were not there, it's conceivable that one of the capacitors could have a leakage resistance 5-6 times smaller than the other, causing the voltage rating of the latter to be dangerously exceeded.

The third option in regulation control for the choke input DC supply when no-load to full-load operation is needed is to use a bleeder resistor. If the power supply is going to operate effectively from no load to some average current, this resistance can be chosen to draw sufficient current to ensure that the swinging choke is operating within the constraint of the relations in Fig. 9. The bleeder resistor can be combined with the shunt resistance, R , to simplify construction.

We now address the question of voltage division across the diode stacks as shown in Fig. 11. Consider a number of diodes in series (say N) and a very sharp transient applied across them (the typical capacitances of each diode being 100-300 pF). The transient wavefront appears across the input diodes in proportion to the diode to distributed (C_p) capacitance ratio. It's quite conceivable that a particular diode experiences a transient voltage well in excess of its reverse voltage holdoff capability. What normally happens in a high-voltage stack is that this diode burns out, and with the next pulse the next diode burns out, then the next, and so forth all the way down the stack. The voltage across each of those diodes can be kept close to the peak voltage divided by N , by using relatively large capacitors across the diode (capacitors, in fact, that are much larger than any of the distributed capacitances or diode capacitances in the circuit). Under these conditions, the voltage grading becomes quite uniform and the transient voltage jumps across each of the divides are quite small. Fig. 11 (2-3) is an illustration of what happens when no shunting components are included in the system. There are very sharp spikes at the beginning and the ending of the commutation intervals. At the end of the interval where the diode becomes non-conducting, the energy stored in the diode and distributed capacitances has to discharge. Where does it go? Since the peak voltage

FIG. 11: TRANSIENT VOLTAGE DIVISION ACROSS DIODE STACK ELEMENTS

FIG. 2-3 Voltage Division with no Shunting Components

FIG. 2-4 Voltage Division with a Shunt Capacitor Equal to Ten Times the Junction Capacitance

FIG. 2-5 Voltage Division with a Shunt Capacitor Equal to 100 Times the Junction Capacitance

FIG. 2-6 Voltage Division with a Shunt Resistor Equal to the Reverse Resistance

FIG. 2-7 Voltage Division with a Shunt Resistor Equal to the Reverse Resistance and a Shunt Capacitor Equal to 100 Times the Junction Capacitance

(A) Diode A

(B) Diode B

can then exceed the diode PRV, the diode is forced back into conduction in the reverse direction, avalanches, and the current pulse travels up through the stack. That little current bump can cause significant damage by destroying the diodes during the avalanche breakdown. If an appropriate shunt resistor-capacitor network is inserted across each diode, the input and output become virtually identical. The shunt capacitances are normally in the range of 1-3 nanofarads. Most of the modest-sized diode structures (1 to 5A) can be fabricated up to the several-hundred-kV level.

Experimental diode stacks can be built in the lab quite inexpensively. Avalanche-protected diodes are unnecessary. An avalanche-protected diode is one that behaves like a Zener diode if a voltage in excess of its avalanche-rated inverse voltage is applied. That's supposed to protect it. There are a number of manufacturers who do not make RC-compensated diode stacks. Virtually everything they make is avalanche protected. The diode leakage is matched to within a few percent and many of these diodes are stacked in series. A number of people who are using them end up incorporating RC compensation resistors and capacitors across each of those avalanche stacks. The point here is that the RC compensated stack guarantees equal voltage division even for very fast transients, particularly as experienced in laser systems.

Fig. 12 shows several high-voltage assemblies made by Westinghouse illustrating individually RC diode compensation and the heat sink is a large, thick aluminum plate between each of the diode assemblies. At the top is an illustration of one of the capacitor-resistor shunt assemblies for each of the 35A diode elements. Westinghouse does make one now that uses this geometry but has a fully screened heat-sink arrangement so that they can be used in circuits where there are very steep transients on the leading edge. The reason that's a problem is that if a very fast transient is applied that rises more quickly than the propagation time of the stack, large transient overvoltages can occur and LC resonances can be shock excited causing diode elements to avalanche and burn out. If the

FIG. 1-19b Back side of a module showing shunting components for four 35 ampere rectifiers

FIG. 1-20 70 ampere high voltage assembly utilizing circular aluminum cooling fins

FIG. 12: HIGH VOLTAGE, RC-COMPENSATED DIODE STACKS

diode assembly is encased in a metal box with only a little slot in it, one side being the input side and the other side being the output side of the diode, two equipotential planes are formed. What happens inside is screened from the outside world. These devices are made up to 150 A at the 500 kV level.

Most of the small stacks that can be bought (average currents up to 1A) generally use avalanche diodes. The solid state world being what it is, it's fairly straightforward at this current level; automated machines are available that select out and grade according to reverse current leakage factors and take the diodes in each group and stack them all together. To use one of these stacks in a fast circuit, the best thing to do is put a capacitive shunt across the total circuit, whose value can be estimated from the number of diode elements in the stack and the assumption that a shunt network of 3 nanofarads would be correct for each individual diode.

Individual RC-compensated diodes can be bought from IRC and EDI, among others. There's a curve in Fig. 13 giving an indication of diode reliability as a function of junction temperature, showing a very strong argument, then, in favor of buying as much average current capability as one can afford in a diode stack for maximum reliability. There's been a tendency in the high-voltage supply business over the years to use 1N4000 series diodes for all 1 A power supply stacks. Unfortunately the junction temperatures tend to run relatively high and the percent failure goes up. For a given known probability of failure of the device the weakest link in the system tends to be a solid-state diode running at a high junction temperature and the second weakest link tends to be transient suppression. If you compensate them properly and carefully, the devices still most likely to fail at high temperatures in the system, statistically, are the solid-state diodes. They're very good, but the hotter they run the more problems they're likely to have. The power dissipated in the diode in watts (Fig. 13) is given in the manufacturer's data as a function of average current. The case temperature also goes up

FIG. 8-1 Junction Temperature versus Percent Failure per 1000 Hours

FIG. 13: SOLID STATE RECTIFIER RELIABILITY

$$\theta_{JC} \approx 5^{\circ}\text{C/watt}$$

Given T_A , we can obtain T_j

$$T_j \approx (5 + T_A) / (1 \text{ watt})$$

for T_j at one amp I_{AV}

so many degrees for every watt of average power dissipated. That number is available from the manufacturer. Then, to specify a certain failure probability, Fig. 13 gives the junction temperature, and one can then calculate the corresponding case temperature, defining what the heat sink thermal resistance ($^{\circ}\text{C}/\text{watt}$) rise above ambient has to be for that given case and ambient temperature. RCA has a set of notes out in their audio amplifier applications brochure that does this calculation rather nicely. This is a representative diode curve in Fig. 13 for a 10 A silicon diode that Westinghouse made in the 1960s. It might be a lot better today. There is a reliability problem if the junction temperatures become too high, and if it doesn't cost a great deal, it's advantageous from a reliability point of view to keep the junction temperature down. When it begins to cost a lot, it is necessary to make the heat sink capacity larger, reducing the junction temperature.

The manufacturer will generally provide his lifetime expectancy at least at the 99.5% confidence level. (There are some other applicable military standards for diodes.) Operating at the rated junction temperature, and therefore at a specific average current for the given thermal resistance, for a failure rate of 0.5%, only five of a thousand of these devices will then fail in a thousand hours. That assumes that the distribution is normal. Now, there is another way to make considerable improvement to this reliability, and that's to fold into that the number of hours the device is actually on vs on-off cycling. That's another component of reliability previously unmentioned. There is a report put out by the Boeing Corp. that provides additional information in this area. Heretofore, it was assumed that the power supply will work for so many hours and the diode will be on and under constant load. Cycling it on and off can buy lifetime. If it's on for a short period compared to thermal equilibrium time (adiabatic operation) and off for a couple of days, this curve is weighted enabling the use of smaller devices. Experimentally, this appears to work, although the data base is sparse for high-power devices.

CAPACITOR INPUT FWB DC POWER SUPPLY

The other interesting power supply is the small FWB built in the lab. It serves as a full-wave bridge power supply, but uses a capacitor input instead of an inductor input filter at the output of the FWB. The capacitor charges to the peak secondary voltage. Fig. 14 illustrates the characteristics of the full-wave capacitor-input power supply. In the FWB case, the ripple frequency is doubled. There are some changes in the transformer requirements and the diode requirements. What will be discussed here are worst cases based upon very good transformer performance -- the lowest core losses and the lowest winding losses practical -- and then some relations can be derived that will give an upper bound to device requirements. This will allow determination, for a diode with specific characteristics and a transformer with a given set of characteristics, whether or not any of the components are going to burn out. In the case of a transformer, the kVA or power requirement is as reported for the choke input plus the three loss terms (which the manufacturers provide) shown in Fig. 15. When dealing with a relatively low-current supply (an amp or two at a few hundred volts), these losses matter little. These factors really become important when dealing with high currents or fairly high-voltage supplies where the transformers must be specified carefully for the best cost/performance tradeoff. During iteration with the manufacturer there will be a cost tradeoff. He can provide low copper losses that tend to improve regulation, but it will cost considerably more. As a rule of thumb, 2-10% is a reasonable regulation range to consider. Asking for 1% is very expensive for large supplies. These are manufacturing restrictions, not fundamental engineering or physics restrictions. Now the diode losses were estimated from the curve on the reliability figure. The diode conducts such an average current and dissipates so many watts, and those are lost watts. Those must be compensated for by increasing the power capability of the power transformer. From the diode curves in the RCA handbook the worst-case RMS current rating for the transformer is 5 times I_{DC} , and I_{PK} is 40 I_{DC} . This indicates that one should go back and have another close look, because RMS current capability in a transformer has a large copper cost for a given percentage loss and this power loss must be folded into the total kVA lost in the transformer.

FIG. 14: ACTUAL AND EQUIVALENT CIRCUITS OF A FULL-WAVE CAPACITOR-INPUT POWER SUPPLY.

In designing supplies where the transformer is to be bought, it is suggested to spend a considerable amount of time interfacing with the transformer manufacturer. He can build into his transformer a certain amount of reactive component that doesn't represent a real power loss but one that does allow the limiting of the fault current in the secondary when it is shorted. Effectively this puts an inductor in series with the transformer primary as a current-limiting element, physically, in the way the transformer is built. In addition, transients can be significantly reduced by the different ways in which the transformer windings are configured. A great deal depends on whether one winds the primary directly on the secondary, beside it, or on another leg of the core. By interfacing with the manufacturer considerable money can often be saved for a given performance level requirement. A typical transformer cost saving on a large supply can be a factor of 2 or 3, and for a transformer costing in the \$50k region, this can be worthwhile.

The most expensive approach is to prepare a set of design specifications, send them out for quotes, and wait for everybody to write back with the costs. It costs time and money. The more effective route is to call the various people in the transformer-manufacturing field and talk with them. It might take a morning. When all that information is available, a more cost-effective design is achieved. In most cases the cost of the telephone calls is more than recuperated. This is particularly true in cases of having to build very high kVA supplies at modest voltages (say, 5-10 kV).

Diode costs can be staggering for the larger kVA high-current supplies. The parameter estimates given in Fig. 15 are a worst case and are inappropriate for very large (hundreds of kilowatts to megawatts) supplies. The analysis done in the RCA note advocates the selection of a PRV twice the peak voltage. This factor of 2 times is a reasonable approach to use for up to 100 kV supplies. Above 100 kV it's probably too expensive and 1.3 to 1.5 times can be used, provided very careful fault-mode analysis modeling is executed. Westinghouse assures you with

FIG. 15: FWB POWER SUPPLY

CAPACITOR INPUT

- SHORT OUT INPUT INDUCTOR L IN FIG. 3.
- $C = \frac{C_0}{2}$ NOW IS CHARGED TO THE PEAK SECONDARY VOLTAGE.

(1) TRANSFORMER PARAMETERS

A. $V_{RMS} = 0.7 V_{DC}$

B. $KVA = 1.11 \times P_{DC} + \text{WINDING LOSSES} + \text{CORE LOSSES}$
 $+ \text{DIODE LOSSES.}$

CALCULATE: $I_{RMS} \approx 5 I_{DC}$ } FROM THE DIODE CURVES THE
 $I_{PK} \approx 40 I_{DC}$ } POWER LOSS IS OBTAINED.
 (THIS CAN BE ESTIMATED AS
 $\approx 1 \text{ WATT PER 2-A PEAK.})$

(2) DIODE PARAMETERS

A. $PRV = 2\sqrt{2} V_{RMS}$ WITH A SAFETY FACTOR OF 2

- AT VERY HIGH VOLTAGES THIS IS USUALLY REDUCED TO:

$PRV = 1.5 \sqrt{2} V_{RMS}.$

B. $I_{RMS} = 5 I_{DC}$ $I_{PK} = 40 I_{DC}$

C. THE RIPPLE SPECIFIED DETERMINES THE FILTERING NEEDED WHICH THEN DEFINES C. REGULATION DEFINES THE ALLOWABLE SERIES CURRENT LIMITING RESISTANCE:

FOR FWB: GOOD REGULATION AND LOW ($\approx 20\%$) RIPPLE DEMANDS

$v \omega C R_L = 50$ $v = 2$ $F = 60 \text{ Hz}$

NOTE: FOR HALF-WAVE $v = 1$, VOLTAGE DOUBLER $v = 1/2$:

THE FOLLOWING APPLIES IN GENERAL:

THE RIPPLE RANGE IS 2% (FWB) TO 5% (FW DOUBLER).

- CAPACITOR RMS RIPPLE CURRENT $\approx 5 I_{DC}$
- AT 120 HZ RIPPLE FREQUENCY FOR FWB
- AT 60 HZ RIPPLE FREQUENCY FOR FW DOUBLER.

90% confidence that 1.5 is sufficient PRV margin if a snubber is placed in the transformer primary and the secondary, as well as the filter choke. The worst-case RMS current rating of the diode should be 5 times the average current and the peak current is 40 times the average current (Fig. 16). This can then be compared to detailed model predictions (using, for example, NET-2) in order to reduce the current requirements of the components to a safe minimum value.

To obtain a degree of ripple reduction to 2%, find C from Fig. 15. Then the regulation-allowable voltage drop in the diode stack depends upon how much equivalent series resistance is present in each diode leg. Selecting, for 1A DC, a 40 A peak diode, the output is faulted and the peak current is obtained from the model (Fig. 16 and 17). For example, with a hundred ohm load and a 1 A supply, faulting the output for a perfect transformer, diode, and zero input line impedance, there would be infinite fault current. There is, however, some internal resistance in the supply. What this does is if the diode has a 40 A rating this says that for a 100 ohm load the internal supply resistance should be 100 over 40, which is 2.5 ohms. With a 2.5 ohm equivalent resistance inside the power supply, shorting the output of the power supply will keep the fault current to 40 A. Normally the transformer manufacturer can tell you the equivalent resistance of the transformer. This resistance is normally the DC copper resistance referred to the secondary of the transformer. There is also a small contribution from the diode stack. In this particular case for a 2.5 ohm internal resistance, faulting the output using a 40 A diode, the circuit breaker would open or the fuse would blow, and the diode would survive.

These calculations can become more sophisticated for three-phase or more multi-phase supplies or for complicated, very low ripple systems with a very high-degree of regulation. In the case of the full-wave bridge, if $v CR_L = 50$ is chosen, and $v=2$ in this case, for good regulation the ripple is also low ($\approx 2\%$). In the half-wave supply $v=1$, and for the case of a voltage doubler it is $1/2$. This relationship allows the calculation

FIG. 16

NOW THE DIODE SURGE RATINGS CAN BE ASSESSED:

LET $\underline{R}$ BE THE TOTAL SERIES RESISTANCE =
WINDING + EQUIVALENT DIODE RESISTANCE + RESISTIVE
COMPONENT OF FILTER CAPACITOR REACTANCE.

$\underline{R} \underline{C}$ = SURGE TIME CONSTANT

IF $\underline{R}$ IS NOT KNOWN THEN THE ITERATIVE TECHNIQUE MUST BE
USED: ASSUME $\underline{R} \approx 0.01 R_L$ (A WORST CASE). THEN, KNOWING FROM
RCA APPLICATION NOTES:

$$I_{RMS} \cdot (\underline{R} \cdot C) = 0.7 \cdot V_{PEAK} \cdot C$$

CALCULATE I_{RMS} AND CHECK ON DIODE GRAPH. IF ABOVE THE LIMIT
LINE, INCREASE $\underline{R}$. FOR A WORST-CASE ESTIMATE FOR $\nu \omega C R_L = 50$,
 $\underline{R} = 0.01 R_L$ $I_{RMS} = 5 I_{AV}$ $I_{PK} = 40 I_{AV}$ (GENERALLY APPLICABLE)

FIG. 17: FAULT CURRENT AT TURN-ON OR OUTPUT SHORT CIRCUIT

FOR A TRANSFORMER:

$$\% \text{ REGULATION} = \frac{V_{NL} - V_{FL}}{V_{FL}}$$

$$\text{SINCE } V_{NL} - V_{FL} = I_{FL} R_S = \frac{V_{FL}}{R_L} R_S$$

$$\text{THEN } R_S = (\% \text{ REG}) R_L$$

WHERE THE TRANSFORMER IS LOADED WITH A PURE RESISTANCE
SO THAT SECONDARY POWER IS EQUAL TO DC POWER DESIRED
PLUS LOSSES.

NOTE: GENERALLY $R_S \approx R_L$ EXCEPT FOR LOW-POWER, HIGH-VOLTAGE
UNITS.

FAULT CURRENT:

FOR THE WHOLE SUPPLY, MLYNAR HAS SHOWN:

$$I_{PEAK} \approx \frac{I_{DC} \times 1.8}{(\% \text{ REG})} \quad \text{ASYMMETRIC FAULT CURRENT}$$

- AVAILABLE ON TURN-ON INTO A SHORT CIRCUIT
AT PEAK LINE VOLTAGE

of C and determination of the ripple range. The ripple range is 2-5% (peak-to-peak), in going from the FWB to voltage doubler geometries. In attempts to reduce the amount of ripple further, the RCA curves reveal that peak current begins to rise enormously posing a clear trade-off situation. Should you put another LC filter in the supply? The answer is if one designs to remain near the fault current levels of Fig. 16 for these three classes of supplies, it's almost always more cost effective to put another LC filter in the circuit. In fact, on very high-voltage supplies, the additional filters can be inserted into the negative line. This is a very inexpensive way to make a low-ripple high-voltage supply with relatively inexpensive filter chokes. Most such chokes are rated for 1000 V RMS. The inexpensive way to protect them is to use a small spark gap (e. g. Centralab "Gap-Cap") that costs less than one dollar. It can be joined to a current-limiting resistor if desired. These gaps can handle about one joule of energy. For lab applications they're good for a couple of hundred faults. They are not quite as effective as a nonlinear varistor that has a softer damping curve (so that as the voltage across it is increased, it gradually increases the current) but they are inexpensive. To stack 4-5 chokes like this in series (which, although not recommended, will work) the choke cores are floated and the core is tied to the next lower choke output lead (if two or more of them are in series) and the Gap Cap placed across each set of choke leads.

During a fault in the output the chokes tend to develop the peak secondary voltage across them as they try to act as current regulators. The "Gap-Cap" shorts, essentially bypassing the fault current through the limiting internal resistance of the supply. There is another type of "Gap-Cap" that has a 0.01 microfarad capacitor inside it. These were extremely common in the days when radios had tubes where they would be used as protectors across the output transformer, particularly in radios that had remote extension speakers. Thyrites and metal oxide varistors are much better, but also far more expensive. For general laboratory applications, "Gap-Caps" (Centralab) are most effective.

These relations in Fig. 15 allow one to select a value for C in the capacitor input case. Keeping $v_{wCR_L} = 50$, in this ripple range of 2-5%, will cause the RMS and peak currents to obey the relations in Fig. 16. That's true for all types of supplies except in multi-phase units. In general, in multi-phase supplies, these would be very conservative design ratings. Additional design information is available in Mlynar and Terman.

Let us next show the effects of leakage inductance with reference to Fig. 18. Leakage inductance just means that, between the primary and the secondary of the transformer, not all of the primary magnetic flux is coupled to the secondary. The loss arises primarily from geometrical factors. A small equivalent inductance is put in series with the AC driving voltage in the transformer equivalent circuit (Fig. 18). There is the resistance of the diode, R_D , and that of the winding R_S , of the transformer. Consider each as an ideal diode switch closing once every $1/180$ of a second. During each diode cycle, a large current pulse flows and puts a voltage on the load. Each diode is then sequentially reverse-biased and it's at these points during turn-on and turn-off that transient oscillations in the system can arise. The diode has to recover, it has stored charge in it, and the equivalent of an LRC generator in the supply circuit can then arise. The details of this are complicated and we will not address them. Practically, all that is required is to add damping networks to the primary and secondary of the transformer. If the circuit does oscillate, nowhere in this voltage ringing will the voltage exceed twice the AC peak value from the transformer.

As capacitors become large, they can have a significant RMS ripple currents passing through them. In the case of the full-wave bridge the RMS current is about $5 \cdot I_{DC}$ at 120 Hz. It's just a point to be checked in the capacitor specifications to ensure that the capacitor will tolerate

(a) Three-phase half-wave rectifier circuit

(b) Current of individual anodes

(c) Voltage waves

Spike shock
excites resonance
in filter choke
and power trans-
former.

FIG. 18
EFFECTS OF LEAKAGE INDUCTANCE ON OUTPUT VOLTAGE

that amount of current heating. In the case of the surge ratings, this aspect is covered in more detail in the RCA notes. For a given filter capacitance C and total internal resistance R of the power supply, the peak voltage instantaneously applied to this series RC network, through the diodes is N times the primary voltage of the transformer. Assuming an ideal diode, at the peak voltage the current flow is through an RC type of network. R in this case is this R . If the R is known, the peak current can be calculated, and the proper diode selected. If the current is not known, one may estimate for practical transformers, assuming that R_{eq} is 1% of R_L (quite a good assumption) and, calculating from the application notes, this means the transformer regulation is 1%, which is a firm minimum number, (Fig. 16) so that:

$$I_{rms} \cdot R \cdot C = 0.7 \cdot V_{peak} \cdot C$$

I_{rms} is calculated and checked against the diode curves available from the manufacturer. (Time is shown in RC units in the insert in Fig. 16.) If the number on the left side is above the limit line, the diode will not survive. What this means basically is that if the calculated RMS current is above the limit line the diode will melt at turn-on. This is really an energy limit above which the chip melts.

Percent regulation has already been defined (Fig. 17). The voltage difference between the no-load and the full-load voltage is the full-load current times R_s , and since:

$$I_{FL} = \frac{V_{FL}}{R_L}$$

then the internal supply resistance, R_s is:

$$R_s = (\% \text{ Regulation}) \cdot R_L$$

For a given percentage regulation the equivalent resistance can thus be estimated.

In this case, generally the equivalent loss resistive R_s is the DC (copper) resistive loss in the transformer plus the core loss in the transformer. The exception to this consideration arises in very large polyphase transformers with many amps through the diodes where asymmetric effects in the multiphase transformer can predominate. In this case, the peak current is the DC current divided by the percent regulation. There's an asymmetric fault current factor ratio of 1.8 to be applied for the peak fault current as discussed in Mlynar. This is the peak current available when the whole supply is turned on to a short circuit at peak line voltage. In conjunction with the transformer regulation and the average DC current, that will then indicate the absolute peak current to be expected, so far as the transformer or diode are concerned.

The last part of this lecture will discuss damping networks, what they do, transformer-equivalent circuits, oil insulation, and an introduction to hard-tube pulsers.

QUESTION: Is there a general text concerning high-voltage insulation and measurement techniques?

"High Voltage Engineering," by E. Kuffel and M. Abdullah (Pergamon Press 1970) covers many interesting areas such as these. It's probably one of the best practical books published on spark breakdown, ionization, decay processes, electric breakdown of gases, and a most important area -- measurement of high voltages. It introduces the reader to the literature of applied measurement techniques and forms a very valuable resource book. It actually covers practical areas, such as switch triggering techniques, many types of high-voltage generators, and the measurement section is comprehensive, starting at electrostatic volt meters and finishing with compensated probes.

TRANSIENT DAMPING

Throughout all pulse-power systems there are potential transient problems. It's usually the transient phenomena that cause device

breakdowns. Rarely in a well-engineered system, where these transients are under control, will the components fail. Where systems usually fail is when somebody turns on the power supply without the output load, or with the output load shorted, a transient is generated, a large peak voltage is generated in some series-resonant circuit and a capacitor, choke, diode, or switch is lost. What will be discussed in this section is the problem of damping networks (Fig. 19) to control these transients. Essentially the problem is one of voltage damping in the series resonant circuit. When the power is initially applied, there is some inductance, L , there's a surge resistance, R , and the filter capacitance, C . If the power sine-wave is applied at $t = 0$, the voltages will build up in the circuit until the equilibrium value is reached, generally requiring 10 to 30 periods at 60 Hz (i.e., ≈ 170 to ≈ 500 ms).

For the purposes of our discussion, the fundamental concern is with the absolute maximum peak-to-peak secondary voltages across each high-voltage component, not with their growth or decay. Given that, the next question is how to ensure that this voltage can be kept within controlled limits so that the diodes, inductors, resistors, or capacitors don't break down.

For a full-wave bridge, when the power supply is turned on, the magnetic core of the transformer must be magnetized: the amount of current needed to do so is called the magnetizing current. The area inside the B-H curve is related to the core loss per cycle (it takes one full period to retrace the curve). The magnetizing current is essentially the amount of current applied that generates the magnetic field for zero secondary load power. In general terms, we can call it I_E . It is required to start the motion on this curve. The smaller this curve area, the smaller the core losses will be. In traversing the curve, say counter-clockwise, work is being done in moving the orientation of the ferromagnetic domains of the material as the coercive force, $H(I)$, varies. The thermal losses arising from this motion (core losses) change as the frequency (usually drastically increasing) increases, and with the core material.

DAMPING NETWORKS

FOR THE SINGLE-PHASE FWD DC SUPPLY:

WHEN THE SUPPLY IS TURNED OFF AT THE PEAK OF THE LINE VOLTAGE AND THE LOAD IS NOT CONNECTED, THE PRIMARY CURRENT THROUGH THE MAGNETIZING INDUCTANCE " L_E " RAPIDLY COLLAPSES, GENERATING A HIGH VOLTAGE:

$$V_{\text{pri}}^{\text{peak}} \approx \left(\frac{L_E}{C_{\text{shunt}}^p} \right)^{\frac{1}{2}} I_{\text{magnetizing}}^{\text{peak}} \quad \begin{array}{l} \text{:FROM CONSERVATION} \\ \text{OF ENERGY} \end{array}$$

UNLESS C_{shunt}^p IS SUFFICIENTLY LARGE, $V_{\text{pri}}^{\text{peak}}$ CAN RISE TO LARGE VALUES, CAUSING PRIMARY INSULATION BREAKDOWN. TO CONTROL THIS, ADDITIONAL SHUNT CAPACITANCE IS ADDED AND, IF NECESSARY, SOME RESISTANCE IS PLACED IN SERIES WITH THE ADDED CAPACITANCE TO PROVIDE RAPID DAMPING OF THE TRANSIENT.

When the power supply is turned on, current flows in the transformer primary even if the secondary is not connected. To measure I_E an ammeter is put in series with the primary of the power transformer, the meter is shunted, and with the secondary open, power is applied. After a few seconds the shunt is opened. The average value is measured with the milliampmeter and that can be converted to the RMS equivalent. Given that, the energy stored in the equivalent magnetizing inductance L_E can be measured in the following way: for an ideal transformer with a magnetizing inductance L_E , then the shunt damping capacitance C_S can be determined from conservation of energy

$$\frac{1}{2} C_S V_{pk}^2 = \frac{1}{2} L_E I_{pk}^2$$

As before, V_{pk} is set equal to twice the peak line voltage and L_E obtained from $V_{pri} = \omega L_E I_{pri}$. Then, since this is an oscillatory RLC circuit, this energy $\frac{1}{2} L_E I_{pk}^2$ is dumped into the capacitor and oscillates repeatedly. This calculation can then give an estimate of the peak voltage that can be expected. Depending upon C_S and L_E , V_{pk} on the secondary can exceed the power supply peak voltage by a factor of 3-4. To dampen this voltage to twice the AC peak voltage applied, we will show later on that the:

$$Q = \frac{\omega L_E}{R_S} = 2$$

Now C_S and R_S in series with C_S are determined.

This magnetizing energy is not lost. It's in the iron, causing magnetic energy to be stored in the lumped equivalent inductance of the transformer primary. A current meter put into the circuit will measure a current flowing through the system with no load on the secondary. The transformer there is thus not ideal. If the current is measured, the value of L_E with no applied secondary load is obtained. It may not be a real inductor, but L_E is represented as a lumped equivalent inductance.

The problem, depending upon the transformer parameters, is that there can be significant transient voltages developed well above the line voltage. What one wants to do is to have a way of damping this large peak, irrespective of initial conditions. The same sort of argument applies to the damping of the transients across the choke in the filter (Fig. 20). If this is an ideal transformer, it has a turns ratio of one to N so that if you apply one volt AC to the input, N volts AC are at the output. This is what everybody would like to be the case, but alas the equivalent circuit is more complex and is sketched in Fig. 21. There's a magnetizing inductance, $L_{\text{magnetizing}}^P = L_E$, and a shunt resistance

$R_{\text{core loss}}^P$ representing the core loss (the work done in moving the ferromagnetic domains around on the B-H curve). This series resistance, R_{winding}^P is equal essentially to the resistance of the copper primary wire. The leakage inductance, L_{leakage}^P , which is shown lumped in the primary, represents essentially the percentage of the magnetic flux generated in a primary that is not fully coupled to the secondary. A rough number for that leakage inductance is 1-5% of the primary inductance. The transformer manufacturer can provide a more accurate number for a given design.

The equivalent elements on the secondary side are complimentary. One small element that can become important is the primary to secondary capacity C_{ps} . It's the real physical capacity from primary to secondary that one would measure with a capacitance bridge. In most circuits it's a relatively small number. The only time it becomes sufficiently large to be a problem is in multilayer, interleaved (multifilar) transformer designs popular in wideband pulse and audio transformers. For conventional pulse transformers in the microsecond region, it's preferable not to wind them with these interleaved structures to minimize C_{ps} and reduce the coupling of transients through the transformer.

FIG. 7-50 Swinging Choke

The main factor in this arrangement is the distributed capacitances which is present from the coil to ground. This key parameter causes an oscillatory effect of this regenerative voltage and will produce a voltage of reverse polarity across the rectifier assemblies after the rectifiers have commutated the forward bias pulse. Refer to Figure 7-51.

- Mlynar

FIG. 20: TRANSIENT VOLTAGES GENERATED ACROSS THE INPUT INDUCTOR DURING COMMUTATION OF THE DIODES.

FIG. 21: GENERAL TRANSFORMER EQUIVALENT CIRCUIT

In this equivalent circuit in Fig. 21, all these lumped equivalent elements can be measured. For example, R_{gen} is the equivalent AC resistance in series with the generator; C_{shunt}^S is the shunt capacity of the secondary -- it's a measure of the lumped shunt capacity across the secondary, and it is ascertained in the same way as for the primary (i.e., by injecting a small current into the secondary and measuring the natural resonant frequency in resonance with the secondary leakage inductance with the primary shorted). Manufacturers of transformers for pulse applications can almost always give all these values to within 10-15%, which is generally satisfactory. If they cannot estimate them, then one should really consider whether he wants to buy from that source. One very important point to note is that there is no electrostatic shield shown in Fig. 21. Since this represents an ideal transformer, there is no worry about that. It is suggested to put electrostatic shields on the primaries of all transformers. There's no reason the manufacturer can't do it, but for separate primary and secondary bobbins, the individual can do it if he wishes. If the transformer primary is being referred to someplace other than ground, that electrostatic shield could quite legitimately be referred to that same place, recognizing that any current passing into it is going to traverse whatever other components are in that return loop to ground.

Now is the time to simplify this complex circuit. In the high voltage case, shielding is really intended to isolate the primary from fast transients that are being coupled back from the load, across the stray capacity of the filter inductor, back through the diodes (because they're shunted with capacitors), back through the transformer and into its primary. The advantage of the electrostatic shield on the transformer is that these transient currents travel into the shield and are harmlessly directed off to the ground of the transformer assembly. This shield then protects the primary from being exposed to such fast transients. Similarly, a fast transient coming into the primary side from the line cannot couple to the transformer secondary where it may become large enough to cause damage.

In the case of the ideal transformer, because there is conservation of power, the output current is the input current over N (Fig. 22). In the real world, referred to the primary, since dissipated power is conserved when it is reflected from the secondary loss elements, all losses can be expressed as equivalent circuit elements in the primary, as illustrated in Figs. 22 and 23.

If the primary current is i_1 and the secondary current is i_o , the same equivalent primary loss for power $i_1^2 R'$ equivalent is sustained for both of these. So the overall equivalent resistance in the primary, taking into account the resistive loss in the secondary as well as the primary loss resistance gives:

$$R = R_{\text{winding}}^P + \frac{1}{N^2} R_{\text{winding}}^S$$

Whatever loss resistance is in the secondary can be referred back to the primary side, so lumped loss elements can be used for energy-dissipation calculations. This lumped equivalent model is a way of moving dissipative terms, capacitances, and inductances from one side to the other through ideal transformers to make calculations a little more straightforward. The power loss terms can be referred to the secondary or to the primary. This is an elementary way of referring the lumped elements to the primary side from a loss point of view, an equivalent shunt capacity point of view, and from an equivalent inductance point of view. To reflect the other elements back into the primary, refer to Fig. 22 for the equivalents. This circuit comes from Terman and there is a very good discussion in Fink's Radio Electronic Engineering.

When all these lumped elements are reflected to the primary at low frequencies, the circuit of Fig. 23 results. At low frequencies the capacitances are not required. R is the winding loss term for both primary and secondary losses and R_{core}^P loss is the core loss term. The total magnetizing inductance is $L_{\text{magnetizing}}^P$, the load is Z_{load}/N^2 , and this circuit equivalent is applicable from about 60 Hz to 5 kHz. It's a good circuit for the transformer and can be used for most power supply design.

FIG. 22: SIMPLIFIED TRANSFORMER EQUIVALENT CIRCUITS:
REFERRED TO PRIMARY

IDEAL

$$E_i I_i = E_o I_o \quad \therefore \text{POWER CONSERVED}$$

$$E_o = N E_i$$

$$I_o = \frac{I_i}{N}$$

$$1:N = \text{TURNS RATIO}$$

REAL: REFERRED TO PRIMARY

CONSERVING POWER DISSIPATED:

$$A. \quad I_i^2 R'_{\text{equivalent}} = I_o^2 R_{\text{winding}}^s$$

$$\therefore \underline{R'_{\text{equivalent}}} = \frac{1}{N^2} R_{\text{winding}}^s$$

$$B. \quad \frac{1}{2} E_i^2 C'_{\text{equivalent}} = \frac{1}{2} E_o^2 C_{\text{shunt}}^s$$

$$\therefore \underline{C'_{\text{equivalent}}} = N^2 C_{\text{shunt}}^s$$

$$C. \quad \frac{1}{2} I_i^2 L'_{\text{equivalent}} = \frac{1}{2} I_o^2 L_{\text{magnetizing}}^s$$

$$\therefore \underline{L'_{\text{equivalent}}} = \frac{1}{N^2} L_{\text{magnetizing}}^s$$

$$D. \quad \frac{E_i^2}{Z'_{\text{equiv}}} = \frac{E_o^2}{Z_{\text{load}}}$$

$$\therefore \underline{Z'_{\text{equiv}}} = \frac{1}{N^2} Z_{\text{load}}$$

FIG. 23: SIMPLIFIED LOW-FREQUENCY TRANSFORMER EQUIVALENT CIRCUIT REFERRED TO THE PRIMARY.

At high frequencies the second equivalent circuit (Fig. 24) is obtained. Shunt resistive and inductive components are no longer of concern, so further simplifications arise. The magnetizing inductance term was eliminated (the core loss still exists, but is relatively insignificant in terms of the energy stored in various shunt capacitances and leakage inductances). The L is the sum of all the leakage inductances. The winding loss and generator resistance remain although they may now be frequency dependent. The secondary shunt capacity is referred back to the right side of the equivalent leakage inductance as $N^2 C_{shunt}^s$. That's important as now we have the potential for transient excitation of this dampened series RLC oscillator. The idea now is to dampen these oscillations and try to keep them under control so that the voltage nowhere exceeds twice the peak applied voltage. This amplitude limit is selected primarily because it gives a good cost/performance trade off.

There is now a resonant circuit as illustrated in Figs. 25 and 26, where the the generator voltage is E_{GEN} . Solving the circuit equations at the resonant frequency $\omega^2 LC = 1$ and $Q = \omega L/R$. At resonance, the circuit is pure resistive, the current is a maximum, so the voltage across the resistor is E_{GEN} .

What about the voltage across the inductor? If this voltage is desired to be twice E_{GEN} , then:

$$V_L = j Q E_{GEN} = j 2 E_{GEN}$$

Now, for the capacitor, since $V_C = -j \cdot Q \cdot E_{GEN}$ or $= -j \cdot 2 \cdot E_{GEN}$ (for a $Q = 2$ circuit), the peak voltage across any element is never greater than twice the peak generator (or transient) voltage. That's all there is to snubbers. No matter what the initial conditions are, no matter what is done, there's no way for more voltage to appear across the inductor than twice the peak value of the AC voltage that is in the circuit. There is, however, one rare exception: there's a very special category of circuits called "two-frequency oscillatory discharge networks" and the one time they

NOTE: R_{GEN} AND R_W MAY BE FREQUENCY DEPENDENT.

FIG. 24: HIGH-FREQUENCY TRANSFORMER EQUIVALENT CIRCUIT

FIG. 25: THE SERIES RESONANT CIRCUIT

FIG. 26

$$E_{\text{GEN}} = E_0 e^{j\omega t}$$

$$\text{SET } I = I_0 e^{j[\omega t + \phi]}$$

THEN, AT TIME T FOR $I = 0$ AND NO "Q" ON C:

$$E_{\text{GEN}} = I R + L \frac{dI}{dt} + \frac{1}{C} \int I dT$$

$$= I_0 e^{j[\omega t + \phi]} R + L I_0 e^{j[\omega t + \phi]} j\omega +$$

$$\frac{1}{C} \left[I_0 \int e^{j[\omega t + \phi]} dT \right]$$

$$\therefore \int e^{j[\omega t + \phi]} dT = \frac{1}{j\omega} e^{j[\omega t + \phi]} \Big| = \frac{1}{j\omega} \left[e^{j[\omega t + \phi]} - e^{j\phi} \right]$$

$$\therefore E_{\text{GEN}} = I R + j\omega L I + \frac{1}{j\omega C} \left\{ I - e^{j\phi} \right\}$$

SINCE AT $T = 0$, $Q = 0 \therefore \phi = 0$:

$$\therefore E_{\text{GEN}} = Z I \text{ WHERE } Z = \left\{ R + j\left(\omega L - \frac{1}{\omega C}\right) \right\}$$

$$\text{SET } \omega^2 L C = 1 \longrightarrow Z = R \longrightarrow E_{\text{GEN}} = R I$$

$\therefore$ CIRCUIT PURE RESISTIVE

$$\text{NOW } V_L = j\omega L I = j\omega L \frac{E_{\text{GEN}}}{R} = j\frac{\omega L}{R} E_{\text{GEN}}$$

$$\text{DEFINE } Q = \frac{\omega L}{R} \therefore V_L = Q E_{\text{GEN}}$$

I.E. V_L ACROSS THE INDUCTOR
LEADS E_{GEN} BY 90 DEGREES AND
IS "Q" TIMES LARGER.

FIG. 26A

NOW LET US RESTRICT THE PEAK VALUE OF THE VOLTAGE ACROSS THE INDUCTANCE TO $2E_0$.

$$\text{I.E. } (V_L)_{\text{PEAK}} = j Q E_0 = j 2 E_0 \rightarrow Q = 2$$

IN THIS CASE, THEN, $\omega L = 2R$

AND ACROSS CAPACITOR:

$$V_C = \frac{1}{j\omega C} I = \frac{1}{j\omega C} \frac{E_{\text{GEN}}}{R} = -j \frac{\omega L}{R} \cdot E_{\text{GEN}}$$

$$V_C = -jQ \cdot E_{\text{GEN}}$$

THUS THE PEAK VALUE HERE IS ALSO $2E_0$.

will be a problem is in the case of ferroresonances in power transformers. With most of the problem areas of interest in this course, there'll never be a worry about this. It's a problem that comes about when circuit breakers don't always open simultaneously or they don't close simultaneously, or when people insist upon building power supplies that are three phase and do not put phase drop-out detectors in them. When ferroresonance like that occurs, enormous voltages can be generated in the primary of the transformer. That's the only time not to believe the above analysis. Greenwood has an excellent discussion of this topic. To avoid that, just ask every power-supply supplier who's supplying three-phase supplies to put in a phase-drop or voltage detector. Then the problem goes away.

The damping network to be used for primary damping is shown in Fig. 27. The leakage inductance, L_E , is measured this way; $(E_i)_{rms}$ is applied and $(I_i)_{rms}$ measured for the secondary open circuit. From the previous analysis it can be shown that L_E can be calculated (Fig. 19). Just at turn-on there is no load on the power supply and the main inductance of the circuit is the magnetizing inductance. A series resonant circuit is then being excited, there is a shunt capacity C_{shunt}^P in series with the leakage and magnetizing inductances. The resonance can be dampened by inserting a shunt-loss term across the C_{shunt}^P . The resonant frequency of the circuit is 60 Hz. Now from $Q = 2$, the value of the shunt capacitance C_S' can be derived (Fig. 27) from $\omega^2 L_E C_S' = 1$ (generally $L_E \gg L_{leakage}^P$). The core loss, $R_{core\ loss}^P$ is known, so from Fig. 27 the series damping resistance R_S' can be calculated. It turns out that C_S' is generally about 100 times bigger than C_{shunt}^P . So there is how a resonant network here, $C_S' - L_E - (R_{core\ loss}^P + R_S')$ resonating at 60 Hz, dampened with a Q of two so it cannot oscillate with a higher Q at the natural resonant frequency of $C_{shunt}^P - L_E$, which could be several tens of kilohertz. Capacitors in the primary side, for C_S' are quite inexpensive. SCR snubber capacitors are generally used here because, although they cost about 20% more, they're of extended foil or multi-tab construction and their design lifetime is an order of magnitude longer than the less

FIG. 27: PRIMARY DAMPING FOR A REAL TRANSFORMER

1. DETERMINE $L_E = \frac{(E_i)_{rms}}{\omega(I_i)_{rms}}$ FOR NO SECONDARY LOAD. NOW $L_E \gg L'_{leakage}$

2. RESONATE C'_S WITH $L_E \Rightarrow \omega^2 L_E C'_S = 1$ $f = 60$ Hz

FOR $V_{peak} = 2 (E_i)_{peak}$ $Q = 2 \therefore \frac{\omega L^P}{R^P + R'_S} = 2$ WHERE $R^P = R^P_{winding} + R^P_{core loss}$

SINCE $R^P_{core loss}$ IS KNOWN, THEN R'_S CAN BE CALCULATED.

expensive ones. An SCR (Silicon Controlled Rectifier) snubber capacitor is a pulse discharge capacitor made by the thousands. The people who make cycloinverters, for example, would be in serious difficulty if one of those failed. If one fails, this can destroy the whole SCR stack very quickly. Capacitor manufacturers will sell them at a modest premium over the cheaper filter capacitor design.

The secondary, depending upon the conduction angle of the diodes, for a very short time has a chance of being unloaded as the supply is turned on. That's one way of generating secondary transients, so a secondary snubber across the transformer is highly recommended. The same $Q=2$ constraint will be applied to the secondary damping case. The secondary damping is described in Fig. 28. Fig. 29 shows the equivalent circuit and an analysis of damping characteristics by frequency shifting.

If the line voltage is turned on faster than the diode turns on (and there is a good possibility of that happening) -- the coupling from primary to secondary via $L_{leakage}^S$ and C_{ps} tend to do that -- oscillations in the secondary circuit can be shock excited at the resonant frequency of the transformer secondary with its shunt capacity. As an observation, experience has generally been that damping the secondary is far more important than damping the primary. However, it's more expensive. The difficulty is with the physical size of the capacitors needed in relatively high-voltage power supplies. For example, consider a 250-kW, 100-kV transformer system, that would probably be a six-phase Y-delta stacked configuration. (There is a Y secondary and FWB and delta secondary and FWB and the two are added in series.) In that type of circuit C_s' of values up to 50 nanofarads for the snubber are common. This now represents a significant reactive power flow through the snubbers and, at these voltages, the damping networks become quite expensive.

The secondary resonant frequencies tend to be in the 20 to 100-kHz range. The resonance is between the leakage inductance and C_{shunt}^S , with resistive damping from $R_{winding}^S + R_{core}^S$ loss. There can be a large peak voltage -- Q times the peak voltage in steady state at turn-on. Snubbers should be added to control these voltages and then

FIG. 28: SECONDARY DAMPING

THE APPLICATION OF PEAK LINE VOLTAGE AT A RATE FASTER THAN DIODE TURN-ON (I.E. SUPPLY IS OPEN CIRCUITED) CAN GENERATE AN OSCILLATORY DISCHARGE IN THE SECONDARY CIRCUIT, ESSENTIALLY AT THE NATURAL RESONANT FREQUENCY OF THE TRANSFORMER (FROM 20,000 TO 100,000 Hz). THE FREQUENCY DEPENDS UPON THE SECONDARY "LEAKAGE" INDUCTANCE IN SERIES WITH INTERNAL SHUNT CAPACITANCE. THE CIRCUIT IS OF A HIGH Q AND CAN GENERATE VERY HIGH PEAK VOLTAGES ($\approx Q \cdot V_{SEC}^{PEAK}$). THESE TRANSIENT PEAKS CAN BE REDUCED TO REASONABLE AND CONTROLLABLE DESIGN LIMITS, WITHIN INSULATION RATINGS OF TRANSFORMERS (AND DIODES) TO, SAY, $2 V_{SEC}^{PEAK}$, BY ADDING AN RC SNUBBER ACROSS THE SECONDARY OF THE TRANSFORMER. THE VALUES ARE OBTAINED FROM THE FOLLOWING MODEL, DERIVED FROM THE SERIES RESONANCE CIRCUIT THAT WAS BRIEFLY DISCUSSED BEFORE.

FIG. 29: HIGH FREQUENCY SECONDARY DAMPING

1. NATURAL SELF RESONANT FREQUENCY: $\omega_R L^S C_T = 1$ WHERE $C''_S = 0$; $C_T^{-1} = (C^S)^{-1} + \frac{C'^S}{N^2}$
2. NOW: FOR A PEAK OVERVOLTAGE OF 2X WHEN SWITCH OPENED THEN SHIFT THE RESONANCE BY N TIMES IN FREQUENCY (5X ABSOLUTE MINIMUM). IN THIS CASE IT CAN BE SHOWN THAT THE PEAK VOLTAGE AT ω_R REMAINS BELOW $(N E_i)$ =

$$\left\{ \frac{1}{\frac{1}{C^S_{\text{SHUNT}} + C''_S} + \frac{N^2}{C'_S}} \right\} \div C^S_{\text{SHUNT}} = 100 \quad \text{ALSO, } 2 R_T = \sqrt{\frac{L^S}{C_{T0}}}; \therefore Q = \frac{\omega_0 L^S}{R_T} = 2; \omega_0^2 L^S C_{T0} = 1$$

$$R_T = R''_S + N^2 R'_S + R, \text{ WHICH DEFINES } R''_S.$$

values are derived from the model of the series resonant circuit, which was briefly discussed before (Fig. 25).

This is interesting because this particular general circuit, with its damping snubbers, appears everywhere throughout PCS design. Snubbers are used on some laser systems, magnetrons (often for mode control), or klystrons, and it's always the same argument. What we're going to try to do here is come up with a general way of deriving the values for C'_S and R'_S (C'_S and R'_S having been determined in the preceding (Fig. 30) analysis). There are a lot of cookbook prescriptions for these snubbers that are found in the various diode manuals or the transformer literature, but they tend to overcompensation techniques, using large capacitors and modest values of resistors. This technique here allows considerable economy, because capacity in high-voltage capacitors costs money. If there's any significant RMS current (more than an amp or two) it does cost a lot.

It can be shown that, starting from the $Q(\omega)$ analysis in Terman, if a snubber network is designed whose resonant frequency is 1/10 the natural resonant frequency, there is no way that for any Fourier components of any pulse of any shape that is applied to the circuit to generate a voltage across any component of more than twice the peak voltage of that Fourier component. The circuit lumped equivalent elements are now all referred to the secondary to simplify the circuit in Fig. 29. We've referred the primary damping capacitor, the primary damping resistor into the secondary. $L_{leakage}^S$ is the leakage inductance, R is the total winding loss, C_{shunt}^S is the shunt capacitance of the transformer. There's also an output switch: it could in fact be a diode. The power supply is turned on, the diode closes, and it conducts current into the load. The transformer load is in this case the filter inductor, the filter capacitor, and the load resistor that is attached across the supply.

The natural resonant frequency occurs when this $L_{leakage}^S$ is in resonance with the C_{T_0} , which is $(C_{shunt}^S + C'_S)$ in series with C'_S/N^2 .

Initially, the resonant frequency ω_R is calculated for zero snubber (C_S'') capacitance. The analysis is shown in Fig. 29. For the peak overvoltage of twice the maximum secondary voltage when the switch is opened, we proceed as before and shift the resonant frequency downwards by a factor of ten. Then the peak voltage at ω_0 , the new resonant frequency, always remains below the peak secondary voltage. Shifting ω_R by only a factor of five means that the peak voltage at this new resonant frequency ω_0 could be twice the peak secondary voltage. For a shift in resonant frequency of ten times, C_S'' and R_S'' are determined from the relations in Fig. 29. If the resonant frequency in the secondary is shifted by ten times, then it can be shown that nowhere can there be more peak volts than the peak volts from this new resonant frequency, even though as higher frequencies are reached higher Qs appear. Terman provides a lucid proof of this. This is important because it means no matter what happens there's no way the total peak voltage (the sum of these transformer and transient peak voltages) can then be more than twice the peak secondary voltage across any component. There are different self-resonant frequencies in the primary and the secondary of the transformer and normally they differ by a significant amount. They are determined primarily by the shunt capacities and the leakage inductances.

Everything we've discussed so far can also be used for designing pulse transformers. The same arguments apply for snubbing and control of oscillations in pulse transformers. Rise and fall-time aspects will be considered later. If the resonant frequency is shifted by ten times, and if Q is equalled to 2, there is a new ω_0 . C_{T0} is known, then if that is divided by C_{shunt}^S and equated to 100 (this ensures that the resonant frequency is shifted by 10 times), then the resonances are going to be well-behaved and there won't be more than twice the peak applied volts. For $Q = 2$, then:

$$2R_T = \sqrt{\frac{L_S}{C_{T0}}}$$

With R_T known, then R_S'' can be calculated. C_{T0} provides C_S'' and the snubber values are now known. Typical values are kilohms

and fractions of a nanofarad. Analysis reveals that the added KVAR loading from the snubber is around 5% for $Q = 2$.

If it is desired to eliminate the snubber capacitors entirely, they can be replaced with nonlinear elements called thyrites or metal-oxide varistors (MOV). Basically, what these are are non-linear resistors that have a characteristic where above a certain critical voltage the current increases rapidly. They're essentially similar to symmetrical Zener diodes, with a finite lifetime as a percentage of pulse overloads. The lifetime shortens drastically at high-peak currents, making it necessary to control the peak fault currents through them very carefully in order to obtain reliable, long-life operation. The cheapest way to do this and the one that works very well is to put them across the snubber capacitors so you can reduce their capacitance value. Even if more voltage appears across that capacitor (as the Q is higher then) the varistors protect it by clamping the voltage level. They are difficult to use in large power supplies. They can only dissipate a few watts (up to 50 W), have significant capacitance, and are quite expensive. At higher voltages they're as expensive as the snubber capacitors. The normal MOV acts like a 0.01 to 0.1 microfarad capacitor, so there is a free damping capacitor. One type that is rather useful is a zero-capacity MOV. Zero-capacity MOVs are valuable for pulse sharpening applications and have nanosecond risetimes. They're made by a different process and have a 6th power law increase in current with voltage above the knee value, at a very low capacity, making them extremely useful for shunting transients and RFI in circuits when there are large peak-to-peak voltage problems at high frequencies. They can carry a fair amount of RMS current at high frequencies.

This, then, is how the transformer secondary can be dampened. The capacitor C_s'' resonates with the series RLC circuit at $1/10$ the natural resonant frequency, and the resistor R_s'' is selected so the Q is 2 (arbitrary choice to control the voltage to within a factor of 2 everywhere). The same argument applies to the filter choke damping.

Typical values for the R_s'' of the right side are kilohms and typical values for the C_s'' are fractions of a nanofarad. For interest sake, at the 100 kV level a 1-nF capacitor is a value above which costs escalate rapidly. If 10 nF is desired, the price goes up very quickly. There's clearly a tradeoff required. Fig. 18 illustrates the effect of leakage inductance in generating voltage transients in the secondary circuit.

For 60 Hz to 360 Hz, kilovolt power supplies, thyrites and MOV are quite useful devices. They can be used to build diode stacks in disaster situations to replace the capacitor/resistor combination in the compensative diode stacks just discussed. For 1 kV diodes, the 750 V standard MOV units can be used although they are rather expensive.

OIL INSULATION

The Fig. 30 is a simplification of much of the insulation data in Mlynar's book. A study of these points may save a great deal of agony and lost time. Note that if there are even a few parts per million of water in oil and the water stays suspended, the dielectric strength in kV per 0.1 inch drops drastically. People who make single-shot generators (e.g., large Marx banks) really don't have to worry about that. When these large devices are assembled the water sinks to the bottom. When dealing with small-size assemblies, particularly at high rep-rates, the AC corona component is significantly enhanced by the presence of water in the oil. Anyone contemplating any high-voltage work is advised to acquire a small oil purifier. If at all possible it is advisable to purify the oil whenever it is taken out. It's worth the trouble and the small investment in time. The typical oil sitting around the lab or outside, even in "sealed" containers rapidly ($\approx$ 6 months) can become contaminated to such an extent that the dielectric strength is seriously degraded. The oil purifier is a small centrifuge device that separates impurities from the oil, possibly followed by an addition of a particulate and water filter.

Antioxidants put into high voltage transformer oils react readily with a large number of components of which high-voltage assemblies are made, such as resistor varnishes and the wax coatings on cheap

FIG. 30: OIL INSULATION DATA

1. DESIGN FOR 30,000 VOLTS PEAK PER INCH.
2. CLOSER SPACINGS REQUIRE INSULATION BARRIERS BETWEEN HIGH-VOLTAGE POINT AND GROUND.
3. OIL EXPANSION WITH TEMPERATURE IS SIGNIFICANT.
4. FOREIGN MATTER CAN CONTAMINATE THE OIL (E.G. WAX-COVERED COMPONENTS, DIRT, PAINT, ETC.)
 - CAN REDUCE DIELECTRIC STRENGTH (AC, DC, OR PULSE)
 - CAN INCREASE LOSSES AT HIGH FREQUENCIES (E.G. RF CIRCUITS)
5. DISPLACE AIR IN TANKS WITH DRY NITROGEN OR ARGON. FREONS CAN BE USED IN EXPERIMENTAL SET-UPS (BUT FAR, FAR AWAY FROM CO₂ LASERS.)
6. OVERSTRESSES CAUSE WHIRLPOOLING AND LEAD TO ARCS.

capacitors. These antioxidants and other materials dissolved in the oil can polymerize, allow tracking, and carbonize insulating surfaces. If components are to be used inside high-voltage assemblies, wax-covered components are not recommended unless specifically designed to work in oil (the manufacturer provides this information). There are components designed for use in the standard high-voltage oils.

The recommended stress level near 60 Hz is 30,000 volts peak per inch. At high frequencies, this decreases by a factor of 10 (at a few megahertz). With Shell Dialax, above a megahertz in RF systems becomes a problem. In dealing with high-rep-rate circuits with very high-frequency components in them, consider using the new silicone liquid dielectric insulating fluids from Dow Corning: DC 200, which has been available for years, and a new one, DC 561, that is about a factor of four more expensive than Shell Dialax and is quite impressive. It has a low corona inception point and a very low loss factor at high frequencies. Some people are sensitive to oils, so if you get them on your hands, it's recommended to wash often with pumice soap.

The antioxidants in high-voltage oils like Shell Dialax can degrade over a period of years under relatively high electric field stresses. Therefore, old oil should be rejuvenated with an additive before it is used. Over long periods of time, the hydrocarbons cross-link, combining to lose their ability to function as a high-quality insulator. They all depend upon molecular structure to absorb the gas molecules from corona areas and this structure changes with time. If the oil is filtered and the antioxidants replenished, the oil can be rejuvenated. Oil should not be heated above 120⁰ F. If it is, and then new transformers are impregnated in it, it is changed into something else that isn't as good (its long-term antioxidation properties, its long-term breakdown, and its long-term corona resistance are all poor) and the system can then fault. Silicone fluid can be heated (although there's no need to) to 240⁰ F without damage; there's no damage until very high temperatures are reached. Mineral-based oils are also very stable and can be highly

abused. The best oils for lab applications are Dow Corning 200 and 561. Given a resistor about two inches long that is normally rated for 10 kV dc, when it was put in Dow Corning 200, the resistor sustained 110 kV dc indefinitely. With Shell Dialax the resistor tracked at about 30 kV. Dow Corning 200 is very useful material for lab applications, but may be too expensive for commercial applications. There are some insulating dielectrics better than the silicone fluids. They're very expensive and for a given geometry they can sustain a very high DC voltage before any significant corona occurs. That's what's really good about them. Their loss is generally high at high frequencies. The real advantage of silicone fluids is that they are very low loss up to 200-300 megahertz, and even then there's a transition to only a slightly higher loss. Silicone fluids (e.g., diffusion pump oils) can be rerefined by the manufacturer and will work well afterwards.

Closer spacings and higher stresses than the above require insulation barriers between the high voltage point and ground. If this is to be done, be sure that the insulating spacers do not suffer corona degradation or polymerize. One of the best materials known for this use, with high mechanical strength, is made by Permali. The possible exception to Permali might be in multimegahertz generators in which there are a large RF voltages. Then it might tend to break down and Teflon or Kapton may be preferable.

Oil, unfortunately, expands with temperature. This is obvious, but I once made a thyatron driver module of 1/2-inch thick brass in a fully-sealed geometry. The leads blew off the top. Silicones are considerably better in this respect than standard transformer fluids.

Garbage can dilute the dielectric strength of oils. Rosin in solders can polymerize and cause tracking. If possible, clean soldered items before putting them into oil (e.g. in Chlorothene). Rosin solder is a very odd material. The nonactivated flux that Kester and others sell for soldering printed circuit boards, a very low viscosity material, is very

good in terms of DC leakage and corona damage resistance. The standard rosin-core solder, particularly the activated types are to be avoided if fairly high voltage stresses abound.

To minimize water vapor from the air passing into insulating fluids, the air in the high-voltage assembly is often displaced with dry nitrogen or argon (or Freon if there's no CO₂ laser around). Freon-12 dramatically suppresses surface tracking, keeps the water vapor out, displaces all the air, and won't allow air into the transformer.

Electrical overstressing causes whirlpooling in insulating fluids. Above certain stress points, the oil forms a whirlpool. For the lab application it generally doesn't matter, but this level of stress will polymerize the oil and cause degradation in hold-off capability and is not a recommended approach for long-life systems. This whirlpooling is caused by electron or ion field emission from sharp edges where the ions push the molecules into a whirlpool. The high emission currents and fields also polymerize the oil. The whirlpool is simply an indication of other factors that are not obvious but are occurring simultaneously. A typical number for time limits in a test system at 100 kV was days. After weeks of operation deposits occurred causing numerous arcs. The other thing whirlpooling does is pushes the oil up the container surface and out onto the floor.

III. HARD TUBE POWER CONDITIONING SYSTEMS

A hard tube pulser can be represented as an analog switch in series with a battery. To a very good approximation, (Fig. 31) the capacitive storage hard-tube pulser is a large reservoir capacitor with a series switch, some stray capacitance across the load, an isolating inductance for recharging, and some type of load. The inductive storage type is illustrated in Fig. 32. The balance of this discussion talks about what happens when the switch (1) in Fig. 31 is opened and how to

FIG. 31: HARD TUBE PULSERS

1. CLOSE SWITCH 1 TO CHARGE C;
SWITCH 2 OPEN:

$$V_C(\tau) = V_{DC} (1 - e^{-\tau/R_L C}) - V_C(0)$$

2. OPEN SWITCH 1 AND CLOSE SWITCH 2:

$$V_L = V_{DC} e^{-t/RC} - V_P$$

SINCE $R = R_L + R_P$ AND $V_P = I_P R_P$, SWITCH VOLTAGE DROP = CONSTANT

NOW, LET SWITCH 2 BE CLOSED FOR TIME $\tau \ll RC$. $\Delta V_L \approx \frac{V_{DC}}{RC} \cdot \tau$

DURING WHICH TIME SWITCH 2 DISSIPATES POWER.

FOR THE TUBE SWITCHED FULLY ON, THE TUBE DROP IS V_P AT PEAK CURRENT I_P . THE ENERGY LOSS PER PULSE IS $V_P \cdot I_P \cdot \tau$. AVERAGE POWER LOST PER PULSE IS THEN $P_{AV} = W_P \cdot \text{PRF}$.

FIG. 32: POWER CONDITIONING SYSTEMS USING
INDUCTIVE ENERGY STORAGE

CLOSE THE SWITCH, THEN:

$$I_L(t) = \frac{V_{DC}}{R_P} \left(1 - e^{-\frac{R_P}{L}t} \right)$$

ASSUMING $R_P \ll R_L$ AND ZERO INDUCTOR RESISTANCE, AND OPENING THE SWITCH:

$$I_L(t) = \frac{V_{DC}}{R_P} \cdot e^{-\frac{R_L}{L}t} \quad (\text{ASSUMING THE SWITCH WAS OPENED AT PEAK CURRENT}).$$

THE PEAK LOAD VOLTAGE,

$$V_L = \frac{V_{DC}}{R_P} \cdot R_L, \text{ WHICH CAN BE VERY LARGE FOR } R_P \ll R_L \text{ AND THE}$$

SWITCH MUST HOLD OFF THIS VOLTAGE AT THE INSTANT OF OPENING.

$$\text{I.E. } V_L = -L \frac{di}{dt} = - \frac{V_{DC}}{R_P} \cdot R_L \cdot e^{-\frac{R_L}{L}t}. \text{ AT } t = 0,$$

$$V_L = -V_{DC} \left(\frac{R_L}{R_P} \right)$$

NOTE THAT THE SWITCH CURRENT REMAINS ON DURING THE CHARGE TIME SO THE POWER DISSIPATION IS HIGH.

keep switch (2) dissipation at a minimum. Typical switch resistance is several hundred ohms. That means a low-impedance Blumlein can't be driven with a hard-tube pulser, so that hard tube pulsers are very good to drive a high-impedance load. In the kilovolt range, small planar triode tubes achieve only 50 Ohms when saturated, and even the kiloamp tubes that cost thousands of dollars are 100 Ohms or so. That's basically the space-charge limited current the cathode can produce.

There are two switches in Fig. 31. If switch 1 is closed and C is charged up, the RC charging time of that is limited by R_L . Opening switch 1 and closing switch 2, then the series resistance in the circuit is $R_L + R_p$. If switch 2 is closed for a short time, τ , the change in the voltage across R_L is $V_{DC} \tau / RC$. The voltage is almost V_{DC} , but charge is being pulled out of this capacitor in an RC discharge. That's the prime switching cycle in a hard tube pulser.

When switch 1 or 2 is closed it is dissipating power. The power lost can be calculated. Given a voltage drop across the tube, V_p at the peak current I_p , then the energy lost in the switch per pulse is approximately $V_p I_p$. This times the PRF gives the average power dissipated, which can be a very large number. In a megawatt modulator, many kilowatts of average power can easily be lost into the switch.

Risetime is determined by the RC_{stray} time constant (Fig. 33) where R is a total equivalent resistance in series with C_{stray} , giving a risetime of that R times C_{stray} . Since generally for high-efficiency $R_p \ll R_L$, then $R \ll R_p$, which is the switch resistance. The high-power tubes are very large and have inductances of many microHenries. Risetimes for hard tube pulsers for such systems are several microseconds. This inductance does enter when the switch is opened in some circuits, sometimes giving rise to oscillations. If there is a switch with RC snubbers across it (added because it's misbehaving itself), there can be a RLC loop, in which the stored energy in that gives rise to large oscillations, causing the tube to arc and fault the DC supply. On

$$V_{\text{LEADING EDGE}}(T) = \frac{V_{CHG}}{R_P} \cdot R \cdot (1 - e^{-\frac{t}{RC_S}})$$

$$\text{WHERE } \frac{1}{R} = \frac{1}{R_P} + \frac{1}{R_L} \quad \text{RISETIME} = RC_{STRAY} = \tau_r$$

$$\text{FOR } R_P \ll R_L \text{ THEN } \tau \approx R_P \cdot C_{STRAY}$$

$$\text{FOR EXAMPLE, IF } R_P = 100 \, \Omega \quad C_{STRAY} = 20 \, \text{PF}$$

$$\text{THEN } \tau_r = 2 \, \text{NS.}$$

$$\text{B) } V_{\text{FALLING EDGE}}(T) = V(\tau) e^{-\frac{(t-\tau)}{R_L \cdot C_S}}$$

$$\text{SINCE } R_L \gg R_P, \text{ THEN } \tau_f = R_L C_S \gg \tau_r \text{ FOR EXAMPLE IF } R_L = 1000 \, \Omega$$

$$\text{THEN } \tau_f = 20 \, \text{NS IF } C_{STRAY} = 20 \, \text{PF.}$$

$$\text{NOTE: IF } T > 3 \cdot R \cdot C_{STRAY}, \text{ THEN } V(\tau) \approx \frac{V_{CHG}}{R_P} \cdot R. \text{ ASSUMING THAT}$$

THERE IS SMALL VOLTAGE DROP ON "C".

NOTE: WHEN CONNECTED TO A CHARGING SUPPLY AT POINT A THROUGH A RESISTOR R_{CHG} , THEN ON CLOSURE, IF $R_{CHG} \gg R_P$, IT HAS NO EFFECT ON τ_r . NOW, WHEN THE SWITCH IS OPENED, R_{CHG} IS IN PARALLEL WITH R_L AND CAN BE USED TO REDUCE THE FALLTIME.

the falling edge, the switch is opened up and R_L is parallel with C_{stray} , revealing the falling-time constant.

When the power supply is added there is another resistive term to take into account. In inductive charging (Fig. 34) the inductor is placed across the load in series with switch 2, forming the DC return. The inductor is added to reduce the recharging losses and the output pulse appears on the load when the voltage exceeds the switch 2 closure voltage. The second switch (2) is closed (switch 2 could represent a laser load, for example, that stays on while there are so many volts across it and if the voltage on the load falls below the holding voltage suddenly switch 2 opens and the laser turns itself off. Magnetrons also behave that way.) When switch 2 opens there is another resonant circuit. If the Q of that is controlled correctly, there is no problem. If the Q is not low enough, a diode can be inserted across C_s as a shunt diode, clamping the inverse voltage, as shown in Fig. 34. The conditions in here can be chosen for critical damping, as was previously described. A Q of 0.5 is chosen, and the critical damping is determined by resistor $R_s + R_p$ in series with L_s .

The other possibility for fall-time control is to use a low-Q inductive resistor connected to the DC charging supply. The same argument then applies here. That R that we looked at before can be an inductive resistor. A lumped inductor and a resistor can be used to make an inductive resistor. The relation for the damping time is given in Fig. 35 as:

$$\text{DAMPING TIME} = \frac{6}{\frac{R_s}{L_s} + \frac{1}{R_p C_s}}$$

Experience has shown that one of the best wire-wound resistors available is made by Ward-Leonard if you can find a source of them. They take an unusually high surface stress. Rod Carborundum resistors are capable of absorbing even higher energies and are available for operation up to 100 kV in air (and considerably more in oil).

No shunt diode.

Good shunt diode.

1. CLOSE SWITCHES 1 AND 2.
2. AT THE END OF PULSETIME, SWITCH 1 IS OPENED.
3. VOLTAGE ACROSS R_L DROPS UNTIL SWITCH 2 OPENS
(E.G. LASER LOAD OR MAGNETRON)

THEN: SINCE CURRENT IS FLOWING IN L_S A SERIES RESONANT CIRCUIT OF C_S , R_S , L_S IS EXCITED:

OSCILLATIONS ARISE IF $\frac{R_S}{L_S} < \frac{2}{\sqrt{L_S C_S}}$ AT THE FREQUENCY $\omega^2 = \frac{1}{C_S L_S} - \frac{R_S^2}{4L_S^2}$

THESE OSCILLATIONS ARE DAMPENED AND APERIODIC IF

$$\frac{R_S}{L_S} > \frac{2}{\sqrt{L_S C_S}}$$

FOR EXAMPLE, IF $R_S = 50 \, \Omega$ $L_S = 500 \, \mu\text{H}$ $C_S = 50 \, \text{PF}$

$$\frac{R_S}{L_S} = 10^5 \quad \frac{2}{L_S C_S} = 4 \times 10^8 \Rightarrow \text{OSCILLATIONS}$$

FIG. 35: INDUCTIVE RESISTOR USED TO CHARGE ENERGY STORAGE CAPACITOR AND PROVIDE FALLTIME DAMPING

AN INDUCTIVE RESISTOR WITH $\frac{L_S}{R_S} \approx \frac{1}{2} \sqrt{L_S C_S}$ CAN BE USED TO

TO OBTAIN AN APERIODIC FALLTIME $\propto e^{-At}$ THEN:

$$2A = \frac{R_S}{L_S} + \frac{1}{R C_S} \quad \text{AND} \quad \frac{1}{R} = \frac{1}{R_P} + \frac{1}{R_L}$$

GENERALLY, $R_P \ll R_L$, SO $R = R_P$ MEANING

$$2A \approx \frac{R_S}{L_S} + \frac{1}{R_P C_S}$$

NOTE: DAMPING IS THE SAME FORM FOR APERIODIC OR OSCILLATORY CASE. HERE THE VOLTAGE ≈ 0 IN A TIME $T_D = 3/A$ OR "DAMPING TIME"

T_D WHERE

$$T_D = \frac{6}{\frac{R_S}{L_S} + \frac{1}{R_P C_S}}$$

Resistive charging is generally not used because the charging efficiency is only 50%, so inductive charging (Fig. 36) is required. If the damping components are chosen correctly, one can have critical damping. The capacitor C can be charged very quickly back to the DC voltage with a very small amount of overshoot. It can be shown that the overshoot on recharge is within 1% of the final voltage in the time $\pi/2 \sqrt{L_C C}$. If any significant overshoot on the pulse there is undesirable, a shunt diode can be put across R_L . It ensures that on the recharge, R_L doesn't carry any significant current. This is basically how high-efficiency, high-rep rate hard tube pulsers are charged. This L_C is chosen to recharge C in a fraction of the PRF period.

APPLICATION OF PULSE SHAPING NETWORKS TO HARD-TUBE PCS

This is included because it has a significant potential for constant-current drive circuits (Figs. 37 and 38). The same arguments apply to using a thyratron or any type of pulser switch imaginable. For the shunt LR network, if everything is correctly selected, and if $L = R^2 C$, it can be shown that the current under certain conditions remains relatively constant during the discharge interval. It's a great driver, for example, for flash tubes or for certain classes of lasers (not KrF lasers but a lot of other interesting glow-mode lasers). There's one important thing here, which impacts on the design of modulators. The current in this network can remain monotonic (i.e., it won't reverse) for $(\pi/2) \sqrt{LC}$ seconds. This is an important result for solid-state diode recovery. The current should remain monotonic through a diode while it's recovering. If this condition is chosen during snubber design, the result is the correct element to put in series with the diode so the diode cannot suffer a very rapid current reversal. This is a vital element in a number of high-rep-rate modulator designs. Decoupling networks are needed to protect diodes in large modulators as without them oscillations can ensue. This network will hold the current constant for that period of time, $(\pi/2) \sqrt{LC}$. As long as this time is longer than the diode recovery

FIG. 36: INDUCTIVE CHARGING ELEMENTS

1. OPEN THE SWITCH.

RECHARGE IS CRITICALLY DAMPENED IF $Q = 0.5$, I.E.

$$\frac{\omega L_C}{R} = 0.5 \text{ AND } \omega^2 L_C \cdot C = 1 \text{ AND } R \equiv \{R_C + R_L\}$$

THEREFORE: $\frac{L_C}{\sqrt{L_C C} \cdot R} = \frac{1}{2} \quad \text{OR} \quad R = \frac{1}{2} \sqrt{\frac{L_C}{C}}$

IN THIS CASE, OVERSHOOT ON RECHARGE IS NEGLIGIBLE AND RECHARGE TIME TO WITHIN 1% OF FINAL VOLTAGE IS

$$\frac{\pi}{2} \sqrt{L_C C}$$

2. OSCILLATIONS MAY BE DAMPENED USING SHUNT DIODES ACROSS R_L TO CONTROL THE OVERSHOOT.

FIG. 37: APPLICATION OF PULSE SHAPING NETWORKS
TO HARD TUBE POWER CONDITIONING SYSTEMS

When $L = R^2 C$, the load current has zero slope at $t = 0$,
implying a constant current source independent of circuit Q .

NOTE:

THE CURRENT REMAINS MONOTONIC FOR A TIME $\tau \approx \frac{\pi}{2} \sqrt{LC}$

IMPORTANT FOR SOLID STATE DIODE RECOVERY, SINCE CURRENT
FLOW THROUGH THEM MUST BE UNIDIRECTIONAL UNTIL THE DIODE
RECOVERS ($\approx 2-3 \mu s$).

FIG. 38: CONSTANT CURRENT PULSE SHAPING NETWORKS

Output circuit of a hard-tube pulser with a network of passive elements to maintain approximately constant load current during the pulse.

time, its precise value doesn't matter. If the diode can recover in three microseconds, the time is selected to be 5 to 10 microseconds. This is a very important point. One of these in series with diode bridge elements and recovery problems are negligible. One of these in series with the charging diode will keep the diodes from being damaged under any fault conditions except a prolonged DC fault. This network forces the current to remain monotonic during that short period of time.

CHARACTERISTICS OF SWITCH TUBES

A sketch of a vacuum switch tube is shown in Fig. 39. All of the rest of this discussion is based to a large extent on Tom Burkes' switch report. This sketch shows a profile of what tubes can be purchased (Fig 40), to about 400 kV can be bought at low peak currents. Three hundred megawatts peak power capability is available at just under 100 kV, although expensive to buy. Response time is generally in the area of microseconds because of the tube's size. The peak current is space-charge-limited, as it is cathode phenomena that limit all peak current.

Several points of interest are noted in Figs. 41-43.

1. Peak Current: Limited to 1-2 kiloamp by available cathode-emission structures. Improving cathode emission would make more peak current become available.
2. Lifetime observations: After 10,000 hours, the thoriated tungsten cathode emission drops. Oxide cathodes have shorter lifetimes than this by a factor of ten.
3. Anode voltage: The basic limit is vacuum breakdown from induced field emission.
4. X-Ray radiation: Especially troublesome for long pulse durations at high voltages and rep rates.

FIG. 39: SWITCH TUBE MATERIALS

FIG. 40: CHARACTERISTICS OF HIGH VACUUM SWITCH TUBES

PEAK CURRENT IS SPACE CHARGE LIMITED.

FIG. 41: CHARACTERISTICS OF SWITCH TUBES

1. PEAK CURRENT: LIMITED BY CATHODE EMISSION AND GRID INTERCEPTION (AND CONSEQUENT HEATING) ≈ 1000 A.
2. LIFETIME: OXIDE CATHODES HAVE SHORT LIVES COMPARED TO THORIATED TUNGSTEN. I.E. 10,000 VS 1000 HOURS AS THE EMISSION DROPS.
3. ANODE VOLTAGE: LIMITED BY VACUUM BREAKDOWN FROM INDUCED FIELD EMISSION AND/OR POSITIVE ION BOMBARDMENT
 - SPACING/SURFACE EFFECTS
4. X-RAY RADIATION: A SIGNIFICANT PROBLEM AT HIGH REPETITION RATES, VOLTAGES, AND LONG PULSE DURATIONS.
 - ANODE PHENOMENON.
5. PRF: LIMITED PRIMARILY BY ANODE AND GRID HEATING:

$$I_{\text{RMS}}^2 = I_{\text{PK}}^2 (\text{DF}) \quad \text{DF} = \text{DUTY FACTOR}$$
6. CURRENT RATE OF RISE: IF C_S IS STRAY CAPACITANCE, THEN $R_S C_S$ IS FASTEST RISETIME WHERE R_S IS TUBE (SATURATED) ON RESISTANCE.

TUBE CHARACTERISTICS

FIG. 42

6. RATE OF RISE OF CURRENT: NOTE THAT RESONANT CIRCUITS HAVE BEEN USED TO IMPROVE RISE TIMES, BUT GENERALLY AT THE EXPENSE OF AVAILABLE PEAK PULSE VOLTAGE

7. PULSE WIDTH: LIMITED BY GRID HEATING FOR ALL CATHODES; FOR OXIDE CATHODES SUBLIMATION OF OXIDE COATS GRIDS AND HENCE GRID EMISSION ENHANCED AT LOWER GRID OPERATING TEMPERATURES.
- CURRENT LIMITS ARE SEVERAL SECONDS ON AT $\approx 1 \text{ MW}$ LEVELS.

FIG. 43

8. DELAY: LIMITED BY LRC RESONANCE AND TRANSMISSION LINE EFFECTS IN THE CASE OF CABLE STORAGE OR PFN SYSTEMS.
9. JITTER: USING DC ON HEATERS, JITTERS $\ll 1$ NS ACHIEVABLE.

5. PRF: Basically limited by anode and grid heating. The electrons are accelerated through the grids, each having a scattering cross-section, and some strike each grid, then the anode, heating them all up.
6. Current Rate of Rise: Fundamentally stray-capacitance and tube saturation-resistance limited. To increase this a saturating inductor can be used in the anode. Turning the tube on starts a little current flow through the large inductance. The ferrite then saturates, decreases the inductance, and a fast-rising output pulse is obtained. Unfortunately, when the tube is reopened, a long voltage decay ensues because the magnetic energy stored in the ferrite feeds energy back into the load.
7. Pulse Width Limit: Grid heating, a thermal effect limits the average power, except for oxide cathodes, where material sublimation limits temperatures because of enhanced field emission. Present limits are several seconds on time at the megawatt average power level.
8. Delay: Limited by RLC resonance effects in the external circuit and tube electron transit time.
9. Jitter: A couple of hundred picoseconds for planar triodes with DC heater power.

TYPICAL HARD-TUBE PCS

Last but not least, Fig. 44 shows what a hard-tube pulser looks like. Believe it or not they haven't fundamentally changed significantly since 1945. In this circuit there are a number of 6D21 tubes in parallel connected to a magnetron load through the two 0.01 microfarad storage capacitors. Turning the tubes on shorts the capacitors to ground, placing a negative pulse on the cathode of the magnetron. The pulse length is

FIG. 44: HIGH-POWER, SHORT-PULSE HARD-TUBE PULSER

The pulser was designed for a ...

maximum voltage-pulse amplitude of about 30 kv, and a pulse current of about 35 amp. Since it was desired that the short pulses be nearly rectangular, the output circuit had to be designed for a very high rate of rise of voltage. The inductance and the shunt capacitance in the pulser circuit therefore had to be kept as small as possible. In order to minimize the inductance introduced by the circuit connections, the components were mounted as close together as their physical size would permit, allowing for the spacing necessary to prevent high-voltage flash-over. The importance of keeping the stray capacitance small is evident when

FIG. 5-12.—Schematic diagram of the circuit for a high-power short-pulse hard-tube pulser.

the charging current, $C, dv/dt$, for this capacitance is considered. For example, if the capacitance across the output terminals of the pulser is $20 \mu f$ and the voltage pulse at the load is to rise to 30 kv in $0.01 \mu sec$, the condenser-charging current is 60 amp. Since this current must flow through the switch tube, the maximum plate current needed to obtain a large value of dv/dt may be considerably greater than that needed to deliver the required pulse power to the load. The magnetron may have a capacitance of 10 to $15 \mu f$, between cathode and anode, so it is obvious that the additional capacitance introduced by the pulser circuit must be small.

6D21

CATHODE: Thoriated Tungsten, 150 Watts Heating Power
 $V_p(\max)$: 38 kv
 $I_p(\max)$: 15 A

determined by the two-way transit time of the pulse cable connected to the triggering 4C35 thyratron. The pulse from this cable drives all the 6D21 grids positive, forming a fast driver that is contemporary today. The modulator produced 30-kV pulses at a 4 ns risetime.

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Application of RCA Silicon Rectifiers To Capacitive Loads

by

B. J. Roman and J. M. S. Neilson

When rectifiers are used in capacitive-load circuits, the rectifier current waveforms may deviate considerably from their true sinusoidal shape. This deviation is most evident for the peak-to-average current ratio, which is somewhat higher than that for a resistive load. Because of the variation in current waveshapes, calculations of ratings for capacitive-load circuits are generally more complicated and time-consuming than those for resistive-load rectifier circuits.

This Note describes a simplified rating system which allows designers to calculate the characteristics of capacitive-load rectifier circuits quickly and accurately. The effect of the addition of a series limiting resistance to such circuits and the importance of the ratio of the limiting resistance to capacitive reactance are described, and curves of rectifier current ratios are presented as functions of the effective ratio. Typical design examples are given, and output-ripple considerations are discussed. Table I defines the symbols used in the equations and calculations.

Design of Capacitor-Input Circuits

In the design of a rectifier circuit, the output voltage and current, the input voltage, and the ripple and regulation requirements are usually specified. The transformer and the type of rectifier to be used are selected by the designer, and the load resistance is determined on the basis of the output voltage and current requirements. The ripple requirements are satisfied by use of a capacitor to shunt the load R_L , as shown in Fig. 1. The waveforms for this circuit indicate that the voltage across the capacitor E_C coincides with the supply voltage E when the rectifier is conducting in the forward direction. A high initial diode surge current I_S occurs because the capacitor acts as a short circuit when power is first applied. The diode turns off at the peak

Table I - Definition of Symbols

E	= sinusoidal input voltage ($E = E_0 \sin \omega t$)
E_0	= peak input voltage
E_{avg}	= average output voltage
f	= input frequency (Hz)
ω	= angular frequency of input ($\omega = 2\pi f$ radians per second)
t	= time counted from beginning of cycle
R_S	= limiting resistance
R_L	= load resistance
C	= load capacitance
I_0	= absolute peak current through rectifier
I_{pk}	= actual peak current through rectifier
I_{rms}	= root-mean-square current through rectifier
I_{avg}	= average current through rectifier
n	= charge factor; 1 for half-wave circuit, $\frac{1}{2}$ for doubler circuit, 2 for full-wave circuit

of the curve (point D), and remains off until E_C is again equal to E (point A). The turn on point t_{on} is determined by the time constant $R_L C$, and affects the average, peak, and rms currents through the device.

As stated above, the low forward voltage drop of silicon rectifiers may result in a very high surge of current when the capacitive load is first energized. Although the generator or source impedance may be high

Fig. 1 - Circuit showing use of capacitor to shunt the load, and resulting waveforms.

Fig. 2 - Circuit showing addition of limiting resistance, and resulting waveforms.

enough to protect the rectifier, in some cases additional resistance must be added to the generator-rectifier-capacitor loop, as shown in Fig. 2, to keep the surge within device ratings. The waveforms in Fig. 2 show that the capacitor voltage E_C is no longer coincident with the steady state supply voltage E during any part of the cycle. The sum of the additional limiting resistance plus the source resistance is referred to as the total limiting resistance R_S . The ratio of R_S to capacitive reactance $1/\omega C$ is an important consideration in capacitor-input rectifier circuits; ideally, R_S should be much smaller than $1/\omega C$. The magnitude of R_S required in a particular circuit is calculated as described below.

Calculation of Limiting Resistance

The value of resistance required to protect the rectifier is calculated from the surge rating chart for the particular device used. Fig. 3 shows surge rating charts for the RCA CR100 and CR200 series of diffused junction stack rectifiers. Each point on the curves defines a surge rating by indicating the maximum time for which the device can safely carry a specific value of rms current.

With a capacitive load, maximum surge current occurs if the circuit is switched on when the input voltage is near its peak value. When the time constant $R_S C$ of the surge loop is much smaller than the period of the

Fig. 3 - Surge-rating chart used for calculation of limiting resistance.

input voltage, the peak current is equal to the peak input voltage E_0 divided by the limiting resistance R_S , and the resulting surge I_S approximates an exponentially decaying current with the time constant $R_S C$, as follows:

$$I_S = (E_0/R_S) \exp(-t/R_S C) \quad (1)$$

Surge-current ratings for rectifiers are often given in terms of the rms value of the surge current and the time duration t of the surge, as shown in Fig. 3. For rating purposes, the surge duration t is defined as the time constant $R_S C$. The rms surge current is then approximated by the following equations:

$$I_{rms} = 0.7 (E_0 / R_S C) = 0.7 (E_0 C / t) \quad (2)$$

and

$$I_{rms} t = 0.7 E_0 C \quad (3)$$

The values for E_0 and C specified by the circuit design are used in Eq. (3) to obtain an equation which relates the rms surge current I_{rms} to surge duration t . This equation may then be plotted on the surge rating chart. Because $R_S C$ is equal to t , any given value of R_S defines a specific time t , and hence a specific point on the plot of Eq. (3). However, R_S must be large enough to make this point fall below the rating curve.

The following examples illustrate the procedure described for calculating the limiting resistance required in a particular circuit.

Example No. 1: Fig. 4 shows a half-wave rectifier circuit that has a 60-Hz frequency and a peak input voltage E_0 of 4950 volts. The values of E_0 and C are substituted in Eq. (3) to obtain the value of $I_{rms} t$, as follows:

$$I_{rms} t = 0.7 (4950) (2.5 \times 10^{-6})$$

$$I_{rms} t = 0.0086$$

This value is then plotted on the surge-rating chart of Fig. 3 and is found to intersect the CR210 rating curve at 2.7×10^{-4} second. The minimum limiting resistance which affords adequate surge protection is then calculated as follows:

$$R_S C \geq 2.7 \times 10^{-4}$$

$$R_S \geq \frac{2.7 \times 10^{-4}}{2.5 \times 10^{-6}} = 108 \text{ ohms}$$

Because the value given for R_S is 150 ohms, the circuit has adequate surge-current protection for the rectifiers.

Fig. 4 - Half-wave rectifier circuit ($E = 3500 \text{ V rms}$, $E_0 = 3820 \text{ V}$, $f = 60 \text{ Hz}$).

Example No. 2: The doubler circuit shown in Fig. 5 has a peak input voltage of 3800 volts and a load capacitance of 10 microfarads. These values are substituted into Eq. (3), as follows:

$$I_{rms} t = (0.7) (3800) (10^{-5})$$

$$I_{rms} t = 0.0266$$

This value is then plotted on Fig. 3 and intersects the CR108 rating curve at 5.4×10^{-4} second. Therefore, the equation for the time constant is given by

$$R_S C \geq 5.4 \times 10^{-4}$$

$$R_S \geq \frac{5.4 \times 10^{-4}}{10^{-5}} = 54 \text{ ohms}$$

Fig. 5 - Voltage-doubler rectifier circuit ($E = 2700 \text{ V rms}$, $E_0 = 3820 \text{ V}$, $f = 60 \text{ Hz}$).

Calculation of Rectifier Current

The design of rectifier circuits using capacitive loads often requires the determination of rectifier current waveforms in terms of average, rms, and peak currents. These waveforms are needed for calculations of circuit parameters, selection of components, and matching of circuit parameters with rectifier ratings. Actual calculation of rectifier current is a rather lengthy process. A much more direct process is to use the current-relationship charts shown in Figs. 6 and 7. These curves can be readily used to find peak or rms current if the average current is known, or vice versa.

The ratios of peak-to-average current and rms-to-average current are shown in Fig. 6 as functions of the circuit constants $n\omega CR_L$ and R_S/nR_L . The quantity ωCR_L is the ratio of resistive-to-capacitive reactance in the load, and the quantity R_S/R_L is the ratio of limiting resistance to load resistance. The factor n is referred to as the "charge factor" and is simply a multiplier which allows the chart to be used for various circuit configurations. It is equal to unity for half-wave circuits, $1/2$ for doubler circuits, and 2 for full-wave circuits. (These values actually represent the relative quantity of charge delivered to the capacitor on each cycle).

Fig. 6 - Relation of peak, average, and rms rectifier currents in capacitor-input circuits.

In many silicon rectifier circuits, R_g may be completely neglected when compared with the magnitude of R_L . In such circuits, the calculation of rectifier current is even more simplified by the use of Fig. 7, which is current ratios under the limitation that R_g/R_L approaches zero. Even if this condition is not fully satisfied, the use of Fig. 7 merely indicates a higher peak and higher rms current than will actually flow in the circuit; as a result, the rectifiers will operate more conservatively than calculated. This simplified solution can be used whenever a rough approximation or a quick check is needed on whether a rectifier will fit the application. When more exact information is needed, Fig. 6 should be used.

Fig. 7 - Forward-current ratios for rectifiers in capacitor-input circuits in which the limiting resistance is much less than $1/\omega C$.

Average output voltage E_{av} is another important quantity because it can be used to find average output current. The relations between input and output voltages for half-wave, voltage-doubler, and full-wave circuits

are given in Figs. 8, 9, and 10, respectively. Output ripple is shown in Fig. 11 for all three circuits. Although these curves were originally calculated for vacuum-tube rectifiers, they are equally applicable to silicon rectifier circuits.

Fig. 8 - Relation of applied alternating peak voltage to direct output voltage in half-wave capacitor-input circuits.

Fig. 9 - Relation of applied alternating peak voltage to direct output voltage in capacitor-input voltage doubler circuits.

Fig. 10 - Relation of applied alternating peak voltage to direct output voltage in full-wave capacitor-input circuits.

Fig. 11 - RMS ripple voltage of capacitor-input circuits.

The following examples illustrate the use of Figs. 8 through 11 in rectifier-current calculations. Both exact and approximate solutions are given for each example.

Example No. 3: For the half-wave circuit of Fig. 4, the resistive-to-capacitive reactance is found to be:

$$\omega CR_L = (2\pi)(60)(2.5 \times 10^{-6})(200,000)$$

$$\omega CR_L = 189$$

Exact solution using Fig. 6: The ratio of R_S to R_L must first be calculated as follows:

$$\% \frac{R_S}{R_L} = \frac{150 \times 100\%}{200,000} = 0.075\%$$

The values given above are then plotted in Fig. 8 to determine average output voltage and average output current, as follows:

$$E_{avg}/E_o = 98\%$$

$$E_{avg} = (0.98)(4950) = 4850 \text{ volts}$$

$$I_{avg} = E_{avg}/R_L$$

$$I_{avg} = 4850/200,000 = 24.2 \text{ milliamperes}$$

This value of I_{avg} is then substituted in the ratio of I_{rms}/I_{avg} obtained from Fig. 6, and the exact value of rms current in the rectifier is determined, as follows:

$$I_{rms}/I_{avg} = 4.4$$

$$I_{rms} = (4.4)(24.2) = 107 \text{ milliamperes}$$

Simplified solution using Fig. 7: Average output current is approximately equal to peak input voltage divided by load resistance, as given by

$$I_{avg} = E_o/R_L$$

$$I_{avg} = 4950/200,000 = 24.7 \text{ milliamperes}$$

This value of I_{avg} is then substituted in the ratio of I_{rms}/I_{avg} obtained from Fig. 7 and the approximate rms current is determined, as follows:

$$I_{rms}/I_{avg} = 5.7$$

$$I_{rms} = (5.7)(24.7) = 141 \text{ milliamperes}$$

Example No. 4: For the doubler circuit of Fig. 5, the resistive-to-capacitive reactance is determined as follows:

$$\omega CR_L = (2\pi)(60)(10^{-5})(50,000)$$

$$\omega CR_L = 189$$

$$\pi \omega CR_L = 94$$

Exact solution: The ratio of R_S to R_L is determined as follows:

$$\% \frac{R_S}{R_L} = \frac{100 \times 100\%}{50,000} = 0.2\%$$

This percentage is then used in conjunction with Fig. 9, and E_{avg} and I_{avg} are determined as follows:

$$E_{avg}/E_o = 186\%$$

$$E_{avg} = (1.86)(3820) = 7100 \text{ volts}$$

$$I_{avg} = E_{avg}/R_L$$

$$I_{avg} = 7100/50,000 = 142 \text{ milliamperes}$$

The values given above are then plotted in Fig. 6, and the rms current is calculated as follows:

$$I_{rms}/I_{avg} = 3.7$$

$$I_{rms} = (3.7)(142) = 525 \text{ milliamperes}$$

Simplified solution: The average output current is given by

$$I_{avg} = 2E_o/R_L$$

$$I_{avg} = (2 \times 3820)/50,000 = 153 \text{ milliamperes}$$

This value is then plotted in Fig. 7, and the rms current is determined as follows:

$$I_{rms}/I_{avg} = 4.8$$

$$I_{rms} = (4.8)(153) = 734 \text{ milliamperes}$$

As previously noted, the simplified solution in both examples predicted a higher rms current than the actual value: about 32 per cent higher in Example No. 3 and 40 per cent higher in Example No. 4. The amount of error involved depends on both ωCR_L and R_S/R_L .

Rating Curves for RMS Current Versus Temperature

In most technical data for rectifiers, the current-versus-temperature ratings are given in terms of average current for a resistive load with 60-Hz sinusoidal input voltage. However, when the ratio of peak-to-average current becomes higher (as with capacitive loads), junction heating effects become more and more dependent on rms current rather than average current. Therefore, the capacitive-load ratings should be obtained on a curve of rms current as a function of temperature. The average current-rating curves for a sinusoidal source and resistive load may be converted to rms-rating curves simply by multiplying the current axis by

1.77 because this value is the ratio of rms-to-average current for such service (as shown by I_{rms}/I_{avg} at low ωCR_L in Figs. 6 and 7). An example of this conversion is shown in Fig. 12 for the CR100- and CR200-series rating curves.

The following examples illustrate the use of the rms current ratings.

Example No. 5: For the half-wave circuit of Fig. 4, it was found in Example No. 3 that the actual rms current in the rectifier is 107 milliamperes. The rms rating curve in Fig. 12 shows that the CR210 may carry up to 107 milliamperes at ambient temperatures up to 115°C.

Example No. 6: For the doubler circuit of Fig. 5, the actual rms current was determined to be 525 milliamperes. The rms rating curve for the CR108 in Fig. 12 shows that the circuit may be operated up to 88°C ambient temperature.

Example No. 7: If the higher values of rms current given by the simplified solution are used instead of the actual currents, the rms rating curves of Fig. 12 also give more conservative ratings because they predict a lower value for the maximum permissible ambient temperature. For example, for the half-wave circuit the exact rms current was found to be 107 milliamperes, and the approximate value was 141 milliamperes. These current values correspond to a maximum ambient temperature rating of 115°C by the exact solution and 110°C by the approximate solution.

Fig. 12 - Current as a function of temperature for silicon rectifier stacks.

APPLICATIONS CORNER

What is a Fast Recovery Rectifier ... Where and Why is it Used?

If you have asked yourself these questions, it may be because of a lack of understanding of the nature of fast recovery rectifiers or what the terms used to specify them mean. Or you may not fully understand where or why they are used. In either case, this applications corner will attempt to answer these questions for you.

Fig. 1

Fig. 1 illustrates the current and voltage waveforms of a diode in a typical application. As can be noted, when we refer to recovery of a diode, we are discussing the period in time between which the forward current decreases to zero, reaches its peak reverse value and again returns to zero.

The basic defining parameter of this recovery period is the reverse recovery time, T_{rr} , as shown in Fig. 1. This time period is generally measured from the point that forward current first reaches zero until it returns to 10% of its peak reverse value.

Another parameter often specified when devices are to be applied in series is the magnitude of reverse recovered charge, Q_{rr} , shown in Fig. 1 also. The magnitude and differential value of this charge is used to design RC snubber networks which are used to balance the reverse voltage across the devices in a series stack.

Shown in the T_{rr} matrix in Fig. 1 is the major difference between standard, general purpose rectifiers and fast recovery rectifiers. This difference is simply in the length of the reverse recovery time period.

In general, a fast recovery rectifier is anywhere from 4 to 10 times faster than its general purpose counterpart.

In the past and in many cases today, it is enough to specify a fast recovery rectifier

simply by the maximum recovery time. However, for a number of applications, this simple definition is no longer adequate. Newer, more stringent applications have necessitated the extension of fast recovery rectifier specifications. Some of these specifications and the reasons for them are shown in Fig. 2.

Primarily the reverse recovery time has been divided into two regions termed t_a and t_b . The ratio of these two time periods is termed the shape factor, "S." This is usually defined as the ratio of t_b to t_a , but in some cases the inverse may be specified. This ratio is critical in that it is used to differentiate between the soft and abrupt recovery rectifier as shown in Fig. 2. The abrupt rectifier with the lower "S" factor value displays a higher value of reverse transient voltage and dv/dt which can be seen in Fig. 2.

Fig. 2

Limiting the value of this reverse transient voltage by specifying a soft recovery diode allows the designer to specify other less costly, lower voltage components for his equipment.

It would appear then that the best way to specify a fast recovery diode would be simply to specify a soft recovery diode. However, it is not that simple. (See Fig. 3).

Fast Recovery/Soft Recovery Determining Factors

- T_{rr} time
- Voltage rating
- Circuit design
- Snubber circuit design
- Operating frequency

Fig. 3

Fast recovery does not necessarily mean soft recovery. There are, in fact, a number of factors which determine whether a rectifier will be a soft or abrupt recovery device and whether it will be usable in an application.

In general, the shorter the recovery time of a given device, the more abrupt the recovery. Higher voltage rated rectifiers will display more abrupt recovery than their lower voltage equivalents. And circuit and snubber designs are a major factor in determining how a given rectifier will behave. Highly inductive circuits or poorly designed snubbers will make even the best diode recover abruptly.

Lastly, frequency is an important factor. It is generally possible to use a snubber to compensate for an abrupt recovery rectifier. However, in high frequency circuits, such as switching regulators or induction heating inverters, the reduced efficiency often makes this approach impractical.

Now let's look at where these devices are typically used in Figs. 4, 5, and 6.

High Frequency Rectification

Fig. 4

High Frequency Rectification

High power, high efficiency switching regulators require fast recovery rectifiers to achieve maximum efficiency. Schottky and ion implanted rectifiers are the big factor in the low voltage end of this market.

Fig. 5

Freewheeling Diodes in DC Motor Controls

The primary requirements for fast recovery rectifiers for these applications are high voltage and high current capability. These rectifiers are used to reduce voltage transients and di/dt stress on other circuit components. Soft recovery characteristics are desirable, but abrupt characteristics can be compensated by properly designed snubbers.

Feedback or Clamp Diodes in Inverters or Choppers

Fig. 6

Feedback or Clamp Diode in Inverters or Choppers

The key requirement here is soft recovery which limits the dv/dt rate and peak voltage reapplied to the SCR during turn-off. Snubbers can often compensate for abrupt recovery rectifiers in low frequency choppers, but this approach is usually impractical in higher frequency inverters.

Westinghouse has a broad product line of fast recovery rectifiers extending from 50-600 volt, 6 amp .2μs DO-4 devices to the 100-3200 volt, 900 ampere 3μs R9G2 device. See your Power Semiconductor User's Manual & Data Book for detailed technical data or consult the factory.

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Characteristics of Common Rectifier Circuits

If you have a circuit design problem, need further data, or exact information on rectifiers for your specific needs, contact your G-E Electronic Components Sales Representative. You'll be glad you did.

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Characteristics of Common Rectifier Circuits*

TYPE OF CIRCUIT		SINGLE PHASE HALF WAVE (1-1-1-H)	SINGLE PHASE CENTER-TAP (2-1-1-C)	SINGLE PHASE BRIDGE (4-1-1-B)	THREE PHASE STAR (WYE) (3-1-1-Y)	THREE PHASE BRIDGE (6-1-1-B)	SIX PHASE STAR (THREE PHASE DIAMETRIC) (6-1-1-S)	THREE PHASE DOUBLE WYE WITH INTERPHASE TRANSFORMER (6-1-1-Y)	* Assumes zero forward drop and zero reverse current in rectifiers and no AC line or source reactance.
PRIMARY									TO DETERMINE ACTUAL VALUE OF PARAMETER IN FIRST COLUMN, MULTIPLY FACTOR SHOWN BY VALUE OF:
SECONDARY									
ONE CYCLE WAVE OF RECTIFIER OUTPUT VOLTAGE (NO OVERLAP)		0	0	0	0	0	0	0	
NUMBER OF RECTIFIER ELEMENTS IN CIRCUIT		1	2	4	3	6	6	6	
AVERAGE D.C. VOLTS OUTPUT		1.00	1.00	1.00	1.00	1.00	1.00	1.00	X AVERAGE D.C. VOLTAGE OUTPUT
RMS D.C. VOLTS OUTPUT		1.57	1.11	1.11	1.02	1.00	1.00	1.00	X AVERAGE D.C. VOLTAGE OUTPUT
PEAK D.C. VOLTS OUTPUT		3.14	1.57	1.57	1.21	1.05	1.05	1.05	X AVERAGE D.C. VOLTAGE OUTPUT
PEAK REVERSE VOLTS PER RECTIFIER ELEMENT		3.14	3.14	1.57	2.09	1.05	2.09	2.42	X AVERAGE D.C. VOLTAGE OUTPUT
PEAK REVERSE VOLTS PER RECTIFIER ELEMENT		1.41	2.82	1.41	2.45	2.45	2.83	2.83	X RMS SECONDARY VOLTS PER TRANSFORMER T.T.
PEAK REVERSE VOLTS PER RECTIFIER ELEMENT		1.41	1.41	1.41	1.41	1.41	1.41	1.41 (DIAMETRIC)	X RMS SECONDARY VOLTS LINE TO LINE
AVERAGE D.C. OUTPUT CURRENT		1.00	1.00	1.00	1.00	1.00	1.00	1.00	X AVERAGE D.C. OUTPUT CURRENT
AVERAGE D.C. OUTPUT CURRENT PER RECTIFIER ELEMENT		1.00	0.500	0.500	0.333	0.333	0.167	0.167	X AVERAGE D.C. OUTPUT CURRENT
RMS CURRENT PER RECTIFIER ELEMENT	RESISTIVE LOAD	1.57	0.785	0.785	0.587	0.579	0.409	0.293	X AVERAGE D.C. OUTPUT CURRENT
	INDUCTIVE LOAD	----	0.707	0.707	0.578	0.578	0.408	0.289	X AVERAGE D.C. OUTPUT CURRENT
PEAK CURRENT PER RECTIFIER ELEMENT	RESISTIVE LOAD	3.14	1.57	1.57	1.21	1.05	1.05	0.525	X AVERAGE D.C. OUTPUT CURRENT
	INDUCTIVE LOAD	----	1.00	1.00	1.00	1.00	1.00	0.500	X AVERAGE D.C. OUTPUT CURRENT
RATIO: PEAK TO AVERAGE CURRENT PER ELEMENT	RESISTIVE LOAD	3.14	3.14	3.14	3.63	3.15	6.30	3.15	X AVERAGE D.C. OUTPUT CURRENT
	INDUCTIVE LOAD	----	2.00	2.00	3.00	3.00	6.00	3.00	X AVERAGE D.C. OUTPUT CURRENT
% RIPPLE (RMS OF RIPPLE AVERAGE OUTPUT VOLTAGE)		121%	48%	48%	18.3%	4.2%	4.2%	4.2%	
Resistive Load									
TRANSFORMER SECONDARY RMS VOLTS PER LEG		2.22	1.11 (TO CENTER-TAP)	1.11 (TOTAL)	0.855 (TO NEUTRAL)	0.428 (TO NEUTRAL)	0.740 (TO NEUTRAL)	0.855 (TO NEUTRAL)	X AVERAGE D.C. VOLTAGE OUTPUT
TRANSFORMER SECONDARY RMS VOLTS LINE-TO-LINE		2.22	2.22	1.11	1.48	0.740	1.48 (MAX.)	1.71 (MAX.—NO LOAD)	X AVERAGE D.C. VOLTAGE OUTPUT
SECONDARY LINE CURRENT		1.57	0.707	1.00	0.578	0.816	0.408	0.289	X AVERAGE D.C. OUTPUT CURRENT
TRANSFORMER SECONDARY VOLT-AMPERES PER LEG		3.49	1.57	1.11	1.48	1.05	1.81	1.48	X D.C. WATTS OUTPUT
TRANSFORMER PRIMARY RMS AMPERES PER LEG		1.57	1.00	1.00	0.471	0.816	0.577	0.408	X AVERAGE D.C. OUTPUT CURRENT
TRANSFORMER PRIMARY VOLT-AMPERES PER LEG		3.49	1.11	1.11	1.21	1.05	1.28	1.05	X D.C. WATTS OUTPUT
AVERAGE OF PRIMARY AND SECONDARY VOLT-AMPERES		3.49	1.34	1.11	1.35	1.05	1.55	1.26	X D.C. WATTS OUTPUT
PRIMARY LINE CURRENT		1.57	1.00	1.00	0.817	1.41	0.817	0.707	X AVE. LOAD CURRENT X SEC. LEG VOLT. PRI. LINE VOLT.
LINE POWER FACTOR		----	0.900	0.900	0.826	0.955	0.955	0.955	

Use this chart in designing circuits . . . then use **GENERAL ELECTRIC RECTIFIERS** in building them

Figure 13-5 Klystron beam voltage pulse.
(Time scale: 1 μ sec/large division.)

Modulator power supplies (PCE)

The three-phase ac input to the modulator can be any level from 258 to 595 V, depending on the setting of the external induction voltage regulator. This regulator furnishes the ac power to a total of sixteen modulators. The input ac enters the modulator through three 125-A line filters which decouple high-frequency noise from the ac lines, thus preventing possible RFI.

The power supply used in the modulator is a conventional, three-phase, double-way, dc power supply. The three-phase variable voltage is delivered through a circuit breaker and contactor to a step-up rectifier transformer (see Fig. 13-6). The secondary of this transformer feeds power to a semiconductor rectifier assembly. The rectified output, proportional to the ac input, is delivered to the pulse-forming network via the inductance-capacitance (LC) rectifier filter and charging transformer.

Figure 13-6 Schematic diagram of modulator power supply.

The rectifier transformer is an oil-filled, hermetically sealed, floor-mounted unit. It is rated to provide 3.8 A (rms) at 17.3 kV with 595 V in the primary. The primary has taps brought out to external terminals to allow reduction of dc output voltage in steps of 2.5, 5, 7.5, and 10%. The dc regulation is 8% (maximum) for a range from 5 to 100% full load. The reactance of the transformer was selected to be $10 \pm 0.75\%$. This fairly high leakage reactance was chosen to limit the load current during short circuit faults in order to protect rectifier units from damage. The transformer excitation does not exceed 5% at 595 V.

The secondary of the rectifier transformer is connected to a three-phase, double-way, semiconductor rectifier assembly. This assembly consists of six stacks of 100 silicon diodes, each capable of delivering 3 A average at 600 V. Each diode is paralleled with a resistance and a capacitor to equalize voltage distribution across the series string. The entire rectifier assembly is rated to deliver 23 kV at 4.5 A dc when the modulator is running at the maximum repetition rate of 360 pulses/sec.

The rectifier transformers for half of the 240 modulators are connected with a delta primary and a delta secondary while the remainder are connected delta-Y. The result is a phase shift in the harmonic currents of one group with respect to the other. The characteristic fifth and seventh harmonics of a six-phase rectifier effectively cancel one another.

Low-frequency transient suppressor networks are connected from line to line in the primary and secondary of the rectifier transformer. These suppressors not only reduce transients resulting from rectifier commutation action in the secondary but also prevent the occurrence of overvoltage caused by the interruption of exciting current in the primary.

If these transients are not suppressed, damage to the solid-state rectifier assembly can occur, and dielectric insulation failure can result from the repetition of transient overvoltages caused by the action of rectifier commutation. The capacitors in the primary suppressor are sized to maintain the transformer exciting current magnitude by resonating with the transformer's inductance at a frequency not greater than 60 cycles/sec for a short period after the primary voltage is disconnected from the transformer by opening the line circuit breaker. The transformer excitation and primary transient suppressor capacitor circuits are analyzed as follows:

1. The transformer excitation equivalent inductance for Δ winding connection is calculated from

$$L_e = \sqrt{3} V' / 2\pi f I_e$$

where

L_e = the equivalent inductance of the transformer excitation circuit in henrys

I_e = the exciting current measured in the supply lines, rms amperes

V' = the supply line to line voltage, rms volts

f = the frequency (= 60 Hz)

$$I_e = \int$$

2. The primary line to line suppressor capacitance, C'_s , is calculated from

$$C'_s = [(2\pi f)^2 L'_L]^{-1} \text{ farads}$$

The primary side suppressor circuit is connected in Y (with the neutral isolated). Therefore each capacitance has to be twice the value calculated from (2) above. The voltage rating of the capacitors is 660 V ac in order to provide sufficient safety factor. The transformer core loss serves as a damping feature for $L'_L C_s$, so that only a nominal value of resistance is needed in series with the suppressor capacitance to provide proper damping.

Transients due to rectifier commutation tend to cause ringing in the secondary circuit comprising the transformer winding leakage inductance and the coil inherent capacitance. The purpose of the secondary suppressors is not only to reduce this ringing frequency but also to damp the transient oscillation by the insertion of series resistance. Because resistance cannot be placed in a load current circuit, it is included in series with the capacitor. The secondary circuit is connected in Y with the neutral isolated. Thus, the suppressor is a series resistance-capacitance network shunted across the 17-kV lines. Analysis of the secondary circuit includes the following steps:

1. The capacitance C_2 , reflected into the secondary from the suppressor capacitance C'_s in the primary, is given by

$$C_2 = \frac{2C'_s}{N^2}$$

where N = the secondary-to-primary turns ratio.

2. The secondary suppressor capacitance C''_s is calculated from

$$C''_s = [(2\pi f_1)^2 L''_L]^{-1}$$

and

$$C_s = [(2\pi f_0)^2 L''_L]^{-1}$$

where

C''_s = the total capacitance formed by C_2 in series with the parallel combination of the suppressor capacitance C'_s and C_s

L''_L = the equivalent transformer leakage inductance in henrys

C_s = the coil inherent equivalent capacitance in farads

f_1 = the desired ringing frequency in hertz

f_0 = the natural frequency of the coil in hertz

Note that the transformer inherent frequency is typically 10–50 kHz, and $C''/C_s = (f_0/f_1)^2$. In order to reduce f_1 by one order of magnitude to avoid frequencies within the $L''_L C_s$ frequency bandwidth, the ratio C''/C_s has to be 100:1. The value of the damping resistance is calculated using $R_s = 2(L''_L/C'')^{1/2}$ ohms, where R_s is the series damping resistance including the transformer equivalent copper load loss resistance.

The LC filter in the dc circuit performs two functions. It attenuates the 360-Hz rectifier ripple to an acceptable value and reduces the ac power line

Table 13-2 Alternating current power-line harmonic currents at master substation

Pulse repetition rate (pulses/sec)	For harmonic ^a frequency (Hz):									
	120	180	240	300	420	660	780	1380	1500	Total rms
360	0.3	0.5	—	0.7	0.5	0.2	0.2	0.8	0.6	1.5
180	0.4	0.4	0.3	0.4	0.2	0.1	0.1	0.4	0.3	1.2
60	0.8	0.3	0.2	0.3	0.2	—	—	0.2	0.2	1.1

^a Harmonic current expressed as percent of 60-Hz full load current at 360 pulses/sec.

harmonics, which result from resonant charging of the modulator PFN to less than 3% (total rms) of full modulator line current. The largest harmonic current occurs at 120 Hz when the modulator is operating at 60 pulses/sec.

The ac power line harmonic currents associated with operation of the accelerator are individually less than 1% of the fundamental (60-Hz) line current when the modulator is operating at full load. Table 13-2 lists the harmonics resulting from accelerator runs at 360, 180, and 60 pulses/sec. The maximum of all the harmonics is 1.5% rms and occurs at 360 pulses/sec. These harmonics flowing in the power system source impedance produce a 3% distortion of the line frequency sine wave. This amount of distortion is acceptable from the standpoint of interference with communications and control systems.

Typical harmonic current characteristics of an individual modulator are shown in Fig. 13-7. The result of using delta-delta and delta-Y transformer connections together with a phase-shifting transformer in the variable voltage substation is shown in Figs. 13-8 and 13-9. These circuit arrangements and the 12.47-kV cable itself attenuate the harmonic currents so that the master substation carries only a small fraction of the harmonics generated by the individual rectifiers in each modulator. This can be seen by comparing Table 13-2 with Fig. 13-7. Typical harmonic voltages in the 12.47-kV distribution system that result from operating the modulators at 360 pulses/sec are shown in Fig. 13-10.

The total power demand of the modulator system when operating at a level which produces 20-GeV beam energy at 360 pulses/sec is 20.4 MW as listed in Table 13-3. The wattless reactive component required to supply core excitation for the VVS induction voltage regulators as well as that resulting from the rectifier commutation phase angle lag is 10.4 MVAR. Table 13-3 lists the overall accelerator power loads typical for various repetition rates. The power demand load for klystron gallery conventional facilities and auxiliaries is less when the klystrons are operating at 360 pulses/sec than it is when they are being pulsed at 180 or 60 pulses/sec because the waveguide heaters shut off when the heating from RF system losses is sufficient to

Figure 13-7 Individual modulator ac line-current harmonics.

Figure 13-8 Plots of 12.47-kV line current harmonics at VVS input for 360 pulses/sec operation.

Figure 13-9 Plots of 12.47-kV line current harmonics at VVS input for 60 pulses/sec operation.

Figure 13-10 Plots of 12.47-kV line voltage harmonics at variable voltage substation input for 360 pps operation.

**Table 13-3 Alternating current power requirements^a
at master substation for 20 GeV**

<i>Pulse repetition rate (pulses/sec)</i>		<i>Power load (kW)</i>	<i>Reactive load (KVAR)</i>
360	Modulators (240) (ref. 105 V) plus injector	20,400	10,400
	Klystron gallery conventional facilities	4,141	1,900
	Total	24,541	12,300
180	Modulators (240) (ref. 105 V) plus injector	10,600	6,530
	Klystron gallery conventional facilities	5,128	1,870
	Total	15,728	8,400
60	Modulators (240) (ref. 105 V) plus injector	3,700	3,300
	Klystron gallery conventional facilities	5,128	1,870
	Total	8,828	5,170

^a Not including cable or power factor correction capacitance.

hold the temperature. At 360 pulses/sec and 105 V reference the VVS output voltage is 502 V, the current is 92 A, and the power demand is 75 kW per modulator.

Pulse-forming network (ALW)

The pulse applied to the high-power klystron is specified to be a maximum of 250 kV at 262 A, with a rise time of 0.7 μsec , a fall time of 1.2 μsec , and a duration of 2.5 μsec , during which time the voltage is to be constant within $\pm 0.5\%$. The pulse is supplied by a pulse transformer, which is driven by the modulator.

To drive the primary of the pulse transformer, a two-terminal device, the PFN, is used. The PFN simulates the characteristics of a transmission line by the use of lumped capacitive and inductive elements. The PFN used in this modulator is a modified type-E circuit consisting of two parallel networks of ten fixed-value capacitors and ten essentially uncoupled slug-tuned inductors (see Fig. 13-11).

Figure 13-11 Schematic diagram of simplified modulator.

SECTION 8

POWER-SUPPLY SYSTEMS

RECTIFIER-FILTER SYSTEMS FOR ANODE POWER

1. Rectifiers.—Most rectifiers used for anode-power-supply systems are either high-vacuum diodes or mercury-vapor hot-cathode types. In addition, mercury-arc and ignitron rectifiers are sometimes used for high power, and copper oxide, selenium, and cold-cathode gaseous rectifiers are occasionally employed for developing small amounts of anode power.

High-vacuum Rectifiers.—The high-vacuum thermionic rectifier is a diode tube consisting of cathode and anode electrodes. The rectifying action results from the fact that when the anode is positive with respect to the cathode, electrons flow to the anode, while if the anode is negative the tube becomes nonconducting. Further details relative to diode rectifier tubes are given in Par. 6, Sec. 4.

The important characteristics of a high-vacuum diode rectifier are the allowable peak and average plate currents, the allowable inverse voltage, and the voltage drop in the tube. The peak plate current represents the maximum electron emission that the cathode can be counted upon to supply during the entire useful life of the tube and still maintain a complete space charge around the cathode. The allowable average current represents the d-c output current that can be continuously carried without overheating the tube or in other ways reducing the life below the normal expectancy. Since the rectifier never allows the current to flow more than half the time, the average plate current will never exceed one-half of the peak current, and tubes are often designed for a still lower ratio of average to peak current in order to provide for transient peaks.

The maximum allowable inverse plate voltage is the largest negative voltage that may be applied to the plate with safety, and determines the direct-current voltage that can be obtained from the rectifier tube. The exact relationship between d-c output voltage and allowable inverse voltage depends upon the rectifier circuit used. The d-c potential will always be less than the inverse voltage (except for voltage-multiplying circuits), and in certain rectifier connections is only $1/\sqrt{2}$ times as great.

The voltage drop is an important factor in determining the voltage regulation of the rectifier-filter system, and also fixes the amount of plate dissipation that must be provided in the tube design. The voltage drop is determined by the current, the cathode area, and the cathode-anode spacing. In tubes where the inverse voltage rating is of the order of 2,000 volts and less, very small spacings can be employed, with a resulting voltage drop that is remarkably low in proportion to the current.

High-vacuum rectifiers are used almost to the complete exclusion of other types for developing d-c output voltages up to about 400 volts at currents up to several hundred milliamperes. Compared with the hot-cathode mercury-vapor type, they have the advantage in low-power applications of being able to stand more abuse, of having voltage drops that are little if any greater, and of not producing radio-frequency transients.

Hot-cathode Mercury-vapor Rectifier Tubes.—The hot-cathode mercury-vapor rectifier is a diode tube containing mercury vapor in equilibrium with liquid mercury.

¹ A discussion of hot-cathode mercury-vapor rectifier tubes used in radio work is given by H. C. Steiner and H. T. Maser, *Hot-cathode Mercury-vapor Rectifier Tubes*, *Proc. I.R.E.*, p. 67, January, 1930.

When the plate of such a tube is positive, the mercury vapor is ionized by collision with the electrons, producing positive ions that neutralize the space charge of the electrons emitted from the cathode. The result is that the full cathode emission current can be drawn to the anode with a potential drop in the tube of the order of only 10 to 15 volts, even when the anode-cathode spacing is large, and some of the electrons are emitted from recesses in a heater-type cathode. The low voltage drop existing in a hot-cathode mercury-vapor tube permits the use of oxide-coated cathodes with their high thermionic efficiency, since positive-ion bombardment of the cathode will not cause serious deterioration until the drop in the tube is of the order of 22 volts. Further details relative to the characteristics of hot-cathode mercury-vapor tubes are given in Par. 17, Sec. 4.

The important characteristics of hot-cathode mercury-vapor rectifier tubes are the allowable peak and average plate currents, and the maximum safe inverse plate voltage.¹ The allowable peak plate current represents the maximum instantaneous current that it is permissible for the tube to carry in normal operation. In order to ensure satisfactory operation during the entire life of the tube, the total electron emission provided by the cathode must be much greater than the rated peak current. The safe average current is determined by the allowable heating of the plate and the current that can be drawn continuously from the cathode, and still realize a reasonable life expectancy. The maximum allowable inverse plate voltage is the greatest negative potential that can exist on the plate while substantially complete freedom from arc-back within the tube is maintained. The value of this voltage is influenced greatly by the construction of the tube, tends to be greater with filament as compared with heater cathodes, and is less as the pressure of the mercury vapor is increased (i.e., as the temperature of the condensed mercury is increased).

Hot-cathode mercury-vapor rectifier tubes require considerably more care in operation than do high-vacuum rectifiers. The cathode must be brought to normal operating temperature before plate voltage is applied. The temperature of the condensed mercury must also lie within the proper range, for if the temperature is too great there is danger of arc-back, and if it is too small, there will not be sufficient ionization to give space-charge neutralization. It is also necessary that hot-cathode mercury-vapor tubes be carefully protected from even momentary short circuits and overloads, since if the peak current ratings are exceeded for even a few seconds, the cathode of the tube may be permanently damaged.

Hot-cathode mercury-vapor rectifier tubes are almost universally used where high voltage and high currents are required. In such applications they have the advantage over high-vacuum rectifier tubes of much greater efficiency, lower first cost, better voltage regulation, and lower filament power. The chief disadvantage is the greater care that must be employed in their use.

Miscellaneous Types of Rectifiers.—Mercury-arc rectifiers² have been used to a limited extent in high-power radio transmitters, particularly in Europe. The types used ordinarily are provided with grids that enable the output voltage to be controlled and regulated, and give means for high-speed protection in case of trouble.

Mercury-pool rectifiers of the ignitron type have been proposed for high-power radio uses.³ As compared with mercury-arc rectifiers, they have the advantage of being less expensive in first cost and maintenance, are more flexible, and are adapted

¹ In some cases, there is also specified a maximum allowable instantaneous current. This is the maximum current that can be allowed to flow momentarily under infrequent special circumstances without permanently damaging the cathode. It is useful in indicating the circuit impedance that is required to protect the rectifier against faults, such as short circuits.

² For further information on such tubes, see S. R. Durand and O. Keller, Grid Control of Radio Rectifiers, *Proc. I.R.E.*, Vol. 25, p. 570, May, 1937.

³ C. B. Foos and W. Lattemann, High Voltage Mercury-pool Tube Rectifiers, *Proc. I.R.E.*, Vol. 24, p. 977, July, 1936.

to circuits that require the cathodes associated with different anodes to be at different potentials.

The cold-cathode gaseous rectifier makes use of the fact that the current that ionization causes to flow between two electrodes immersed in a low-pressure gas is roughly proportional to the cathode area. Hence if one electrode has a very small, and the other a very large area, a rectifying action takes place. Such rectifiers are used to a limited extent in connection with vibrator power supplies. They have the advantage of needing no filament heating power, but do require a relatively high voltage drop to start the flow of current, and introduce high-frequency transients.

Copper oxide and, more recently, selenium rectifiers are used for a variety of miscellaneous applications such as rectifiers for producing bias voltages, for exciting the fields of loud-speakers, for battery charging, and in a few cases for producing plate power. The copper oxide rectifier makes use of the fact that when a thin film of cuprous oxide is formed upon a metallic copper surface, the resistance that this film offers is small for currents flowing in one direction and high for currents going the opposite way.⁴ This rectifying action is very stable, and the only effect of long use is a slight increase of the resistance in the conducting direction during the first 10,000 hours of use.

Selenium rectifiers utilize the rectifying action obtained when a properly processed selenium film is formed on a metallic surface such as iron.⁴ Selenium rectifiers have characteristics similar to those of copper oxide rectifiers with respect to stability and long life. The resistance in the forward direction is less, however, so that the rectifier efficiency and the current carrying capacity for a given physical size are greater.

Parallel Operation of Rectifiers.—High-vacuum, copper oxide, and selenium rectifiers may be operated in parallel without any particular precautions other than ensuring that the units in parallel are reasonably similar in characteristics.

Rectifiers involving ionization, however, give difficulty in parallel operation, because if one tube ionizes first, the voltage across that tube then drops and the other tube will fail to ionize. In the case of hot-cathode mercury-vapor tubes this situation can be remedied by placing in series with the plate of each tube a resistance of such a size as to produce a voltage drop of the order of 10 to 30 volts with the peak current being carried by the tubes. Cold-cathode gaseous rectifiers will not operate satisfactorily in parallel even with series resistance. Where more current is required than can be handled by one tube, it is necessary to operate each tube from a separate transformer winding or to deliver the output of each tube to separate filters having a common output.

2. Rectifier Circuits.—The circuits in which rectifiers are used can be conveniently classified according to whether the power source is single-phase or polyphase. Single-phase rectifier circuits are used to develop anode power for radio receivers, public-address systems, small radio transmitters, etc., whenever the power required is small, i.e., of the order of 1 kw or less. Polyphase rectifiers are used for higher powers. They have the advantage of utilizing the possibilities of the transformers and rectifier tubes more completely than the single-phase arrangements but the disadvantage of being more complicated and requiring a polyphase source.

Single-phase Rectifier Circuits.—The various types of rectifier connections that may be employed with a single-phase power source are shown in Fig. 1, together with the

⁴ For further information, see L. O. Grondahl and P. H. Geiger, A New Electronic Rectifier, *Trans. A.I.E.E.*, Vol. 46, p. 367, 1927.

⁵ Detailed information on selenium rectifiers is given by Carole A. Clarke, Selenium Rectifier Characteristics, Application and Design Factors, *Elec. Comm.*, Vol. 20, No. 1, p. 47, 1941; J. E. Yarmack, Selenium Rectifiers and Their Design, *Trans. A.I.E.E.*, Vol. 61, p. 488, July, 1942; Selenium Rectifiers for Closely Regulated Voltages, *Elec. Comm.*, Vol. 20, No. 2, p. 124, 1941; E. A. Richards, The Characteristics and Applications of the Selenium Rectifier, *Jour. I.E.E.*, Vol. 88, Part III, p. 238, December, 1941, and Vol. 89, Part III, p. 73, March, 1942.

Half-wave (single-phase) output
(a) Half-wave rectifier circuit

Full-wave (two-phase) output
(b) Full-wave circuit with center-tapped transformer

(c) Full-wave bridge circuit

(d) Conventional voltage doubling circuit

(e) Cascade doubler

(f) Cascade quadrupler

FIG. 1.—Rectifier circuits operating from a single-phase power source.

wave form of the voltage that is developed across a resistance load. Of these, the full-wave circuit with center-tapped transformer is by far the most widely used with tube rectifiers, and the full-wave bridge circuit is generally preferred with copper oxide and selenium rectifiers. As compared with the center-tapped circuit, the bridge arrangement requires twice as many tubes operated at half the inverse voltage, gives greater d-c power output in proportion to the transformer kva rating, but requires a filament transformer having three separate well-insulated windings instead of a single winding. The half-wave circuit is used only when very small currents are required because the secondary current always flows through the transformer secondary in the same direction, thereby tending to saturate the iron.

The voltage-multiplying circuits provide the possibility of developing a d-c voltage greater than the crest alternating voltage of the source. Such circuits are used when such high voltages are to be developed that it is not convenient to build a transformer for the full voltage, or when it is desired to obtain sufficient anode voltage to operate properly small tubes from a 110-volt a-c source without using transformers. In the conventional voltage-doubling circuit of Fig. 1d, the condensers C_1 and C_2 are charged on alternate half cycles, and are then arranged so that the voltages of these two condensers add, thereby giving the possibility of an output voltage that is twice the peak voltage of the a-c supply. The cascade doubler of Fig. 1e produces a result equivalent to that of the conventional voltage-doubling circuit, but has the advantage that there is a common terminal between the supply and the output, so that both may be grounded simultaneously. The operation of this circuit can be explained as follows: If tube 2 is disconnected, then condenser C_1 would charge to the peak supply voltage and the potential at the point A would fluctuate between zero and twice the peak supply voltage. If now tube 2 is connected, the voltage between A and ground will tend to charge condenser C_2 to the peak voltage existing at A, thereby giving the possibility of twice the peak supply voltage being obtained in the output. Such a cascading arrangement can be extended indefinitely.¹ A four-stage system developing a d-c output potential of four times the peak supply voltage with small load currents is shown in Fig. 1f.²

Polyphase Rectifier Connections.—Polyphase rectifiers ordinarily operate from a three-phase power system.³ Although an unlimited number of circuit combinations are possible, the circuits of Fig. 2 include all the arrangements ordinarily encountered in radio work. They can be conveniently subdivided into systems giving three- and six-phase output waves, i.e., ripple frequencies of three and six times the supply, respectively.

The three-phase output developed by connections a and b of Fig. 2 is much more nearly constant than is the output of the full-wave single-phase circuits of Fig. 1, and the ripple frequency is three times the supply frequency instead of only twice as great. The star connection has the advantage over the broken star of slightly better utilization of the transformer capacity, but the disadvantage that in order to avoid d-c saturation in the core, it is necessary to use a single three-phase transformer, whereas with the broken star it is possible to employ three single-phase transformers, provided that each transformer has two secondary windings. With the broken star, it is also permissible to use a Y connection in the primary.

¹ Such cascade systems are described by J. D. Cockcroft and E. T. Walton, Experiments with High Velocity Positive Ions: (1) Further Developments in the Method of Obtaining High Velocity Positive Ions, *Proc. Roy. Soc. (London)*, A, Vol. 136, p. 619, 1932. See also E. B. Muir, External Characteristic of Thermionic Rectifier, *Jour. I.E.E.*, Vol. 80, p. 553, 1937; also, *Wireless Section, I.E.E.*, Vol. 12, p. 156, June, 1937.

² Another type of voltage quadrupler is described by William W. Garstang, A New Voltage Doubler, *Electronics*, Vol. 4, p. 50, February, 1932.

³ Information on rectifiers operating from a two-phase supply is given by R. W. Armstrong, Polyphase Rectification Special Connections, *Proc. I.R.E.*, Vol. 19, p. 78, January, 1931.

The six-phase output wave obtained with connections *c* to *f* in Fig. 2 is even more nearly constant than the three-phase output wave, and the ripple is now six times the supply frequency. The six-phase single Y circuit makes the most effective use of the transformer and tube of any of the polyphase circuits shown, and also has the lowest inverse voltage of any of the circuits. The only disadvantage of this circuit is that it requires four separate well-insulated windings on the filament transformer.

FIG. 2.—Rectifier circuits operating from a three-phase power source and developing three-phase and six-phase output waves.

Although a delta primary connection is shown in Fig. 2c, a Y connection is also permissible.

The double three-phase arrangement with balance coil requires a somewhat higher kva transformer rating than does the six-phase Y, but is decidedly superior in this respect to the circuits giving a three-phase output. If the balance coil is short-circuited, this arrangement reduces to a six-phase star, an undesirable arrangement from the point of view of the effectiveness with which the transformer and tubes are utilized. In order that the balance coil may operate satisfactorily, it must have sufficient inductance to maintain current flowing continuously through each half of the

coil.¹ The double three-phase arrangement with series connection gives twice as much voltage and half the current as the same transformers and tubes arranged with the balance coil, while the effectiveness with which transformers and tubes are utilized, the inverse voltage, etc., are the same in the two cases.

The permissible transformer arrangements for the systems giving a six-phase output vary with the rectifier connection. In the case of the six-phase single-Y rectifier circuit, either three separate single-phase transformers or one three-phase transformer can be used. With the double three-phase arrangements, however, it is not permissible to use single-phase transformers unless these are arranged so that each transformer has two secondaries divided between the two stars. Six single-phase transformers cannot be used because d-c saturation of the core would result. Delta primary connections are shown in Figs. 2c to 2f, but it is permissible to use a star primary for the six-phase single Y, and the same is true of the double three-phase circuits provided that a single three-phase transformer or three single-phase transformers are used. If separate three-phase transformers are used to supply each secondary star of a double three-phase system, then a delta primary is necessary.

It is possible to operate three- and six-phase rectifier systems from two special transformers arranged in a T connection. When this is done, slightly greater transformer kva must be provided, but since two large transformers are often cheaper than three small transformers of slightly less total rating, there is economy in the T connection.²

Rectifier circuits providing output waves of twelve and more phases can be readily obtained from a three-phase supply, and are widely used in mercury-arc rectifiers. They have the advantage of much less ripple voltage and a higher ripple frequency than the six-phase circuits, but are seldom employed in radio work because of the greater number of rectifier tubes required.

Transformer Insulation Requirements.—The insulation that must be provided for the secondary of the power transformer in the circuits of Fig. 1 depends upon the rectifier connections involved. On the assumption that the negative side of the rectified output is grounded, then in the center-tapped arrangement, the middle of the transformer secondary requires negligible insulation, and the ends must withstand half of the total secondary voltage. In contrast with this, the bridge circuit requires that the center of the transformer withstand half and the terminals the full secondary voltage, whereas in the conventional voltage-doubling circuit, the lower end of the transformer must be insulated to stand the full secondary voltage to ground while the other end must be capable of handling twice the secondary voltage.

In the polyphase circuits of Fig. 2 the insulation that is required also varies with the connection. In all cases except the three-phase star and the double three-phase arrangement with balance coil, there is a winding or a portion of a winding subjected to a greater voltage to ground than the crest alternating voltage developed between the terminals of that particular coil. Polyphase rectifier circuits employing single-phase transformers designed for power-line service can accordingly be expected to overstress the insulation in many cases.

The secondaries of the filament transformers must be well insulated from ground, and from each other. Thus in Fig. 1b, the secondary is subjected to half the transformer voltage, and in Figs. 1a and 1c, each filament winding must stand a voltage to ground equal to the full secondary voltage of the power transformer. In Fig. 1c, this same insulation is also required between individual filament windings. In the polyphase circuits of Figs. 2a, b, d, and f, the filament winding is subjected to the full

¹ Further discussion of the requirements that a balance coil must meet in order to operate satisfactorily are given in Par. 3.
² For further information see R. W. Armstrong, Polyphase Rectification Special Connections, *Proc. I.R.E.*, Vol. 19, p. 78, January, 1931.

voltage to neutral of the power transformer, and in Fig. 2c the voltage stress is twice that developed to neutral by one set of transformers. With the six-phase single Y, the filament windings must all be insulated to stand the secondary delta voltage of the power transformer both to ground and to each other.

Combination Rectifier Circuits.—Two different types of rectifier connections are sometimes combined in a single arrangement to provide different output voltages. Examples are shown in Fig. 3. The arrangement at a combines the full-wave bridge rectifier circuit of Fig. 1c with the center-tapped circuit of Fig. 1b to give two d-c outputs developing voltages in the ratio of 2:1.¹ Analogously, the circuit of Fig. 3b combines the six-phase single-Y circuit of Fig. 2c with the three-phase star of Fig. 2b to give two output voltages in the ratio of 2:1. Such arrangements provide a simple

FIG. 3.—Examples of combination rectifier circuits.

means of obtaining a lower voltage output of comparatively small current rating without the addition of appreciable accessory equipment.

3. Performance of Rectifiers When Used with Filter Systems Having Series-Inductance Inputs.—The output of a rectifier is normally delivered to a filter that smooths out the pulsations and produces a substantially constant d-c voltage. The filters used for this purpose can be divided into two types, i.e., those which present a series inductance to the rectifier output and those which present a shunt condenser to the output terminals of the rectifier. Either arrangement can be used with any of the rectifier circuits of Figs. 1 and 2 except that the voltage-multiplying circuits and the half-wave connection of Fig. 1 require condenser input. Inductance-input systems utilize the tubes and transformers more effectively than condenser-input arrangements and so are used wherever large powers are involved. They also have better voltage regulation and so are employed where the load on the system varies, as in Class B amplifiers.

¹ For further details of a practical character see G. E. M. Bertram and R. S. Quimby, A Duplex Plate Supply Using Type 83 Tubes, Q.S.T., Vol. 17, p. 31, March, 1933.

Voltage and Current Relations in Rectifier Tubes and Transformers.—The basic voltage and current relations existing with an inductance-input filter can be understood by considering the ideal case of an infinite input inductance combined with a transformer having zero leakage inductance and resistance. Under these conditions the output current delivered by the rectifier system is constant, the current simply shifting from anode to anode during the cycle. The resulting current waves in typical cases are shown in Fig. 4. The fact that the current waves in the transformer are not sinusoidal causes the heating of the transformer windings to be greater for a given d-c power output from the rectifier than would be the case if the same amount of a-c power were

FIG. 4.—Typical current waves in primary and secondary windings of transformers for ideal inductance-input system.

drawn by a resistance load. This makes it necessary to design the transformer windings more generously for rectifier applications than would otherwise be the case. The ratio of d-c output to required kva capacity is termed the transformer utilization factor, or *utility factor*. Its value depends upon the rectifier connection, and is also commonly not the same for the primary and secondary windings.

The voltage developed across the input terminals of the filter system is the same as the voltage developed across a resistance load. The character of the output voltage therefore depends upon the number of phases in the output wave, as shown in Figs. 1 and 2, and is not affected by the particular way in which the rectifier is connected to develop this number of phases.

Quantitative Behavior of Ideal Rectifier System with Inductance-input Filter.—The quantitative conditions existing in ideal rectifier-filter systems, assuming zero trans-

former resistance and leakage inductance, zero tube resistance, and an infinite input inductance are given in Table 1.¹ This table gives all the information essential for determining the voltage and kva capacity required of each transformer winding, the peak and average currents through the rectifier tubes, the maximum inverse voltage that the rectifier tube must stand, and the harmonic components contained in the output voltage. A simple calculation showing the use of the table in a practical case is given in Par. 6.

TABLE 1

	Rectifier circuit							
	Single-phase, full-wave center-tapped connection (Fig. 1b)	Single-phase, full-wave bridge (Fig. 1c)	Three-phase star (Fig. 2a)	Three-phase broken star (Fig. 2b)†	Six-phase single Y (Fig. 2c)	Double three-phase with balance coil (Fig. 2d)	Double three-phase series connection (Fig. 2e)	Six-phase star (Fig. 2f)
Voltage relations (d-c component of output voltage taken as 1.0):								
a. Rms value of transformer secondary voltage (per leg).....	1.11*	1.11	0.855	0.985	0.428	0.855	0.428	0.740
b. Maximum inverse voltage.....	3.14	1.57	2.09	2.09	1.05	2.42	1.05	2.09
c. Lowest frequency in rectifier output (P = frequency of power supply)....	2F	2F	3F	3F	6F	6F	6F	6F
d. Peak value of first three a-c components of rectifier output:								
Ripple frequency.....	0.667	0.667	0.250	0.250	0.057	0.057	0.057	0.057
Second harmonic of ripple frequency.....	0.133	0.133	0.057	0.057	0.014	0.014	0.014	0.014
Third harmonic of ripple frequency.....	0.057	0.057	0.025	0.025	0.006	0.006	0.006	0.006
e. Ripple peaks with reference to d-c axis:								
Positive peak.....	0.363	0.363	0.209	0.209	0.0472	0.0472	0.0472	0.0472
Negative peak.....	0.637	0.637	0.395	0.395	0.0930	0.0930	0.0930	0.0930
Current relations:								
f. Average current per anode.....	0.500	0.500	0.333	0.333	0.333	0.333	0.333	0.167
Peak anode current.....	0.500	0.500	0.333	0.333	0.333	0.167	0.333	0.167
g. Direct-current in load.....	0.500	0.500	0.333	0.333	0.333	0.167	0.333	0.167
h. Peak current per anode.....	1.000	1.000	1.000	1.000	1.000	0.500	1.000	1.000
i. Direct-current in load.....	1.000	1.000	1.000	1.000	1.000	0.500	1.000	1.000
Transformer requirements (d-c output power = 1.0):								
j. Primary kva.....	1.11	1.11	1.21	1.05	1.05	1.05	1.05	1.28
k. Secondary kva.....	1.57	1.11	1.48	1.71	1.05	1.48	1.48	1.81
l. Average of primary and secondary kva.....	1.34	1.11	1.35	1.38	1.05	1.26	1.26	1.55

Note: This table assumes that the input inductance is sufficiently large to maintain the output current of the rectifier substantially constant, and neglects the effects of voltage drop in the rectifier and the transformer.

* Secondary voltage one side of center tap.

† The principal component of voltage across the balance coil has a frequency of 3F and a peak amplitude of 0.500. The peak balance coil voltage, including the smaller higher harmonics, is 0.605.

Voltage Regulation in Input-inductance Systems.—The output voltage of a rectifier-filter system employing a properly designed input inductance falls off with increasing load as a result of (1) resistance in the filter; (2) resistance of the rectifier tubes;²

¹ Most of the data in this table are from Armstrong, *loc. cit.* For corresponding information on additional connections see Armstrong, *loc. cit.*; D. C. Prince and F. B. Vodge, "Mercury Arc Rectifiers and Circuits," McGraw-Hill, New York, 1927; O. K. Marti and H. Winograd, "Mercury Arc Power Rectifiers," McGraw-Hill, New York, 1930.

² With hot-cathode mercury-vapor tubes, the voltage drop is constant irrespective of the current. It is accordingly permissible to assume here that the tube has zero resistance and that the output potential is always 10 to 15 volts less than would be calculated on the basis of an ideal tube.

In grid-controlled and ignition rectifiers, the starting voltage, at which the tube fires may be much

(3) resistance in the transformer; (4) leakage reactance of the supply transformer. The various resistances in the circuit reduce the output voltage without affecting the wave shape in the system. In calculating the effect of the resistances in the circuit, the tube drop is determined by the number of tubes in series and the tube drop required to carry the anode currents. The voltage loss in the transformer resistance is calculated on the basis of the primary and secondary currents in the transformer at any one time and the transformer resistances through which this current must flow. With

(a) Three-phase half-wave rectifier circuit

(b) Current of individual anodes

(c) Voltage waves

FIG. 5.—Effect of leakage inductance in the transformer on the behavior of a polyphase rectifier.

choke-input systems, the tube and transformer resistance drop is constant throughout the cycle, even though the current shifts between tubes and transformer windings.

The leakage reactance of the transformer prevents the current from shifting instantly from one winding to another as in the ideal case in Fig. 4. The situation in a typical case is shown in Fig. 5b, where μ represents the interval required to transfer the current. During this transition period, the output voltage, instead of following the open-circuit potential of the more positive anode, as in the case of an ideal rectifier,

greater than the voltage drop required to maintain the current, once the flow has started. This causes the time at which the tube fires to be later in the cycle than in the case of an ideal tube, with a resultant lowering of the output voltage by an amount that depends upon the firing voltage, i.e., upon the delay in firing time.

assumes a value that is the average of the open-circuit voltages of the two windings that are simultaneously carrying current, as shown in Fig. 5c. This causes the average voltage of the output to be less than if there were no leakage inductance by an amount indicated by the shaded areas of Fig. 5c. The calculation of this voltage loss is complicated, and depends both upon the rectifier and transformer connections. The reader wishing to make such computations can find the details in books devoted solely to the subject of rectifiers.¹

FIG. 6.—Effect of a finite input inductance on current waves.

Effect of Finite Input Inductance.—When the input inductance is large but not infinite, the output current of the rectifier will vary somewhat, increasing when the output voltage developed by the rectifier is greater than the d-c component of the output, and dropping when it is less. The behavior in two typical cases is shown in Fig. 6, and it is apparent that the peak current that the tubes must now carry to develop a given d-c output current is greater than when an infinitely large choke is used.

Most of the fluctuation in output current with finite inductance is composed of current of the lowest ripple frequency. To the extent that this is true, the increase in peak rectifier current resulting from a finite input inductance is given by the equation

$$\frac{\text{Peak current with finite input inductance}}{\text{Peak current with infinite input inductance}} = 1 + \frac{E_1 R_{eff}}{E_0 \omega L_1} \quad (1)$$

¹ Prince and Vodge, *op. cit.*; Marti and Winograd, *op. cit.*

where E_1/E_0 is the ratio of lowest-frequency ripple component to the direct-current voltage in rectifier output as given by Table 1, R_{eff} is the effective load resistance (actual load plus filter resistance plus equivalent tube and transformer resistances), and ωL_1 is the reactance of the input inductance at the lowest ripple frequency. The fact that the input inductance is finite instead of infinite has practically no effect upon the output voltage, provided that the input inductance exceeds the critical value discussed below.

The minimum input inductance that will give normal input-inductance operation of the rectifier system is the smallest inductance that will maintain a continuous flow of current from the rectifier output. The requirement for continuous current flow is given by the approximate relation¹

$$\frac{\omega L_1}{R_{eff}} \geq \frac{E_1}{E_0} \quad (2)$$

where the notation is the same as in Eq. (1). In the important practical case of a 60-cycle single-phase full-wave circuit, Eq. (2) becomes

$$L_1 = \frac{R_{eff}}{1,130}$$

FIG. 7.—Modified swinging-choke arrangement.

The minimum allowable input inductance as given by Eq. (2) is termed the **critical inductance**.² When the inductance is less than the critical value, the system acts as a condenser-input system. The d-c voltage developed across the filter input is then greater than given by Table 1, and the regulation is poorer than would be the case with adequate input inductance.

When the direct current drawn from the system varies, it is necessary to satisfy Eq. (2) or Eq. (3) at all times if good voltage regulation is to be maintained. In order to do this with very small load current without excessive inductance, it is necessary to place a bleeder resistance across the filter output to ensure that R_{eff} in Eq. (2) will never exceed a reasonable value.

With single-phase full-wave systems, the input inductance required for light loads, even with a reasonable bleeder resistance, is still greater than desirable. One way of meeting this situation in an economical manner is to use a moderate-sized input inductance and then to make the air gap so small that the required incremental inductance is obtained with a reasonable bleeder current. With large currents, the core saturates, and the inductance will drop, but Eq. (2) is still satisfied. An input inductance adjusted to operate in this way is sometimes referred to as a **swinging choke**. Another method³ of maintaining a continuous flow of current from the rectifier without employing an excessively large inductance is shown in Fig. 7, and consists of a swinging choke L_1 so small as to be inadequate when operating alone at small currents, in series with another small choke L_2 that is tuned to the fundamental ripple frequency.⁴

¹ This assumes that the entire fluctuation of current in the input inductance is a result of the lowest ripple voltage contained in the rectifier output acting against the impedance of the input inductance. This assumption is very closely realized because the higher frequency components of current in the input inductance are quite small and because, in practical filters, the shunting condenser following the input inductance has very small impedance to the ripple frequency compared with the reactance of the input inductance. A more complete analysis is given by C. R. Dunham, *Some Considerations in the Design of Hot-cathode Mercury-vapor Rectifier Circuits*, *Jour. I.E.E.*, Vol. 75, p. 278, 1934, Vol. 76, p. 421, 1935 (also *Wireless Section, I.E.E.*, Vol. 9, p. 275, September, 1934, and Vol. 10, p. 108, June, 1935).

² For further discussion, see F. S. Dellenbaugh, Jr., and R. S. Quimby, *The First Filter Choke—Its Effect on Regulation and Smoothing*, *QST*, Vol. 16, p. 26, March, 1932.

³ See Reuben Lee, *Solving a Rectifier Problem*, *Electronics*, Vol. 11, p. 39, April, 1938.

⁴ If the inductance L_2 is not too small, the swinging choke L_1 in Fig. 7 can be eliminated.

In rectifiers where the voltage required to initiate a discharge between cathode and anode is greater than the subsequent voltage drop when carrying current, as is the case in all grid-controlled and ignition mercury-pool arrangements when the adjustment is such as to give delayed firing, the input inductance required to maintain a continuous flow of current and give proper operation is greater than called for by Eq. (2).¹ Failure to employ sufficient input inductance in such cases not only will make the voltage regulation poorer but also may cause erratic behavior, such as irregular firing of the tubes.

4. Performance of Rectifiers When Used with Filter Systems Having Shunt-condenser Input. Mechanism of Operation.

FIG. 8.—Actual and equivalent circuits of condenser-input rectifier-filter system, together with oscillograms showing operation under typical conditions.

and the characteristics of the rectifier tube. For purposes of analysis, the actual rectifier circuit, such as that of Fig. 8a, can be replaced by the equivalent circuit of Fig. 8b. Here the rectifying action of the tube is replaced by a suitable switch S that is assumed to be closed whenever one of the rectifier anodes is conducting current and that is open the remainder of the time. The inductance L of the equivalent circuit is the leakage inductance of the transformer, measured in Fig. 8 across one-half the secondary winding with the primary short-circuited, and the equivalent resistance R is the corresponding transformer resistance R_s measured in the same way, plus a fixed resistance R_r that takes into account the voltage drop in the rectifier tube. The input condenser of the

When the input to the filter presents to the rectifier a shunt condenser instead of a series inductance, the behavior is greatly modified. Each time the crest alternating voltage of the transformer is applied to one of the rectifier anodes, the input condenser charges up to just slightly less than this peak voltage. The rectifier then ceases to deliver current to the filter until another anode approaches its peak positive potential, when the condenser is charged again. During the interval when the voltage across the input condenser is greater than the potential of any of the anodes, the voltage across the input condenser drops off nearly linearly with time, because the first filter inductance draws a substantially constant current from the input condenser. A typical set of voltage and current waves is shown in Fig. 8.

Quantitative Relations.—The detailed action that takes place in a condenser-input system depends in a relatively complicated way upon the load resistance in the rectifier output, the input capacity, the leakage reactance and resistance of the transformer, and the characteristics of the rectifier tube. For purposes of analysis, the actual rectifier circuit, such as that of Fig. 8a, can be replaced by the equivalent circuit of Fig. 8b. Here the rectifying action of the tube is replaced by a suitable switch S that is assumed to be closed whenever one of the rectifier anodes is conducting current and that is open the remainder of the time. The inductance L of the equivalent circuit is the leakage inductance of the transformer, measured in Fig. 8 across one-half the secondary winding with the primary short-circuited, and the equivalent resistance R is the corresponding transformer resistance R_s measured in the same way, plus a fixed resistance R_r that takes into account the voltage drop in the rectifier tube. The input condenser of the

¹ A comprehensive discussion of the input inductance required in such cases, including formulas for calculating the critical inductance, is given by W. P. Overbeck, Critical Inductance and Control Rectifiers. *Proc. I.R.E.*, Vol. 10, p. 655, October, 1939.

filter system is C , and the first inductance L_1 is assumed to draw a constant current I_a equal to the d-c voltage E_a developed across the input condenser divided by a resistance equal to the sum of the actual load resistance plus the resistance of the filter inductances.

The most important use of condenser-input filter systems is with single-phase full-wave rectifier circuits supplying power to radio receivers, small public-address systems, etc. The performance of such rectifier systems can be determined by a relatively

FIG. 9.—Charts giving performance of shunt-condenser filter system for 60-cycle supply source and full-wave rectifier.

complicated analysis based on the equivalent circuit of Fig. 8b and leads to the results given in Fig. 9.¹ The approximations involved in Fig. 9 are as follows: (1) The rectifier

¹ For details of the analysis, see F. E. Terman, "Radio Engineering," 2d ed., pp. 492-498, McGraw-Hill, New York, 1937, or the thesis by Dr. R. L. Freeman, Analysis of Rectifier-filter Circuits, Stanford University Library, 1934.

Analyses for a variety of special cases of condenser-input systems have been developed. Although none of these take into account all of the factors involved and at the same time present the results in convenient form for practical use, they are helpful in estimating the behavior that can be expected. The following references, with a brief summary of the ground covered by each, will be helpful to the reader wishing to go into the subject more deeply:

E. B. Moulton, The External Characteristic of a Diode Rectifier, *Jour. I.E.E.*, Vol. 80, p. 553, 1937; also, Wireless Section, *I.E.E.*, Vol. 12, p. 156, June, 1937. Analysis is made of single-phase half-wave and full-wave circuits, assuming a linear tube characteristic, zero generator impedance, and a resistance load connected directly in shunt with the input capacity. Equations are derived for power output, power loss in diode, effective value of diode current, and voltage regulation characteristic. The results are extended to voltage multiplier systems. N. H. Roberts, the Diode as Half-wave, Full-wave and Voltage-doubling Rectifiers, *Wireless Eng.*, Vol. 13, p. 351, July, 1936, which covers the same

tube is assumed to begin passing current as soon as the anode becomes positive with respect to the cathode; (2) the rectifier is replaced by a fixed resistance during its conducting period; (3) the current flowing into the first inductance is assumed to be constant throughout the cycle; (4) the voltage at the primary terminals of the power transformer is assumed to have perfect regulation. The errors that result from the last three of these approximations are small under practical circumstances, and the first approximation is also satisfied in the case of high-vacuum, copper oxide, and selenium rectifiers. The case of hot-cathode mercury-vapor tubes can be handled by assuming that the tubes have zero resistance and then subtracting from the calculated output 15 volts drop for each tube that is in series. An example of the use of the curves in Fig. 9 is given in Par. 6.

Discussion of Shunt-condenser Systems.—Examination of Fig. 9 shows that the d-c voltage developed by the rectifier operating with a shunt-condenser filter system depends primarily upon the load resistance and the internal impedance of the power source, as represented by R and L in the equivalent circuit of Fig. 8b. The ratio of peak to average current through the rectifier tubes and transformer secondaries also depends upon these same quantities, with the peaks being smaller in proportion to the d-c current when the internal impedance is high and a large load current is drawn from the rectifier. The regulation of the d-c voltage is controlled largely by the internal impedance of the source, which should be small if the regulation is to be good. The ripple voltage is greater the lower the load resistance (i.e., the greater the current drawn from the rectifier), and the smaller the input condenser, but is roughly independent of the source impedance. Oscillograms showing the effects of circuit parameters on the detailed behavior of a typical shunt-condenser input system are shown in Fig. 10.

Compared with filters having a series input inductance, the use of a shunt condenser results in the development of a higher d-c voltage across the rectifier output, together with a smaller ripple in this voltage. At the same time the shunt-condenser arrangement results in poorer voltage regulation, a much poorer utilization factor for the transformer, and a relatively high peak current through the rectifier tube in proportion to average current.

Condenser input systems find their chief use in single-phase full-wave rectifier systems developing small amounts of power at reasonably high output voltages under conditions where voltage regulation is not important. In particular, shunt-condenser input systems are almost universally used for supplying anode power to radio receivers and small public-address amplifiers where the total power involved is of the order of 50 watts or less, at voltages of 250 to 400 volts.

High-vacuum rectifiers are usually preferred to hot-cathode mercury-vapor tubes in condenser-input systems, because the life of the high-vacuum tube is not so critical

ground as Moullin and likewise assumes a linear tube characteristic, but extends the analysis to include the calculation of ripple voltages for the idealized case of zero reactance in the source. W. H. Aldous, *The Characteristics of Thermionic Rectifiers*, *Wireless Eng.*, Vol. 13, p. 576, November, 1936. Calculation of anode dissipation, output voltage, and peak current, on the assumption of a three-halves power-tube characteristic, an infinite input capacity, and zero resistance in the source of alternating voltage. C. M. Wallis, *Half-wave Rectifier Circuits*, *Electronics*, Vol. 11, p. 12, October, 1938. Analysis of voltage and current wave forms for half-wave rectifier of gaseous conduction type, assuming that the voltage drop in the tube is independent of current and that the voltage required to initiate the discharge is no greater than this drop. It is also assumed that the power transformer has zero resistance and leakage inductance. Analysis is given for the load resistance in shunt with input condenser (including the case where the shunt condenser is reduced to zero) and for the load resistance in series with an inductance (but no condensers in system). M. B. Stout, *Analysis of Rectifier Filter Circuits*, *Elec. Eng.*, Vol. 54, p. 977, September, 1935. Methods of calculating current wave forms are given for a variety of conditions where the current is both continuous and discontinuous. In the latter case, the tube drop and impedance of the source are neglected. D. L. Waiselich, *The Full-wave Voltage-doubling Rectifier Circuit*, *Proc. I.R.E.*, Vol. 29, p. 854, October, 1941. This article gives an analysis of the full-wave voltage-doubling circuit with a resistance load connected directly across the input condensers and with zero tube drop and source impedance assumed.

with respect to peak current. When hot-cathode mercury-vapor tubes are used, it is often desirable to place between the rectifier output and the shunt condenser a small series inductance that is not large enough to satisfy Eq. (2). Such a choke is seen from the equivalent circuit of Fig. 8 to have the same effect as increasing the leakage inductance of the transformer. It will therefore reduce the peak current passing through the rectifier tubes at the cost of poorer voltage regulation and reduced output voltage. Cold-cathode gaseous rectifiers are always used with shunt-condenser filter systems. No special precautions need be taken with regard to such tubes, since the gaseous ionization permits the momentary passage of very large currents.

Inductance-input systems in which the input inductance is less than the critical value act as condenser-input systems in which the effective source inductance L assumed acting in the equivalent circuit of Fig. 8b consists of the actual transformer

FIG. 10.—Effect of circuit parameters on behavior of rectifier operated with condenser-input filter.

inductance plus the input inductance of the filter. The essential distinction between series-inductance and shunt-condenser systems is that the former refers to operation in which the current flows continuously from the rectifier output into the filter system, and the latter refers to systems in which the current flows intermittently from rectifier into the filter. If intermittent action is present, then the system is classified as a shunt-condenser arrangement even though there may be a series inductance of less than the critical value present.

Shunt-condenser systems are sometimes operated with the load resistance connected directly across the condenser. Such an arrangement is practical when the load current is so small in proportion to the condenser capacity that the variation in voltage across the input condenser is small. The performance of such cases for a single-phase full-wave circuit can be obtained with reasonable accuracy with the aid of Fig. 9.

The performance curves of Fig. 9 can be extended to cover the single-phase half-wave rectifier of Fig. 1a by reading the curve on the basis of a leakage inductance that is half the actual inductance and a shunt capacity half the value really present. The ripple voltage in such cases has the same frequency as the supply.

3. Filters.—A filter is ordinarily placed between the output of a rectifier and the load resistance for the purpose of smoothing out the voltage fluctuations of the rectifier output. Such filters consist of alternate series impedances that oppose the flow of alternating currents and shunt condensers that by-pass these currents. Typical filter arrangements are given in Fig. 11. The side of the circuit with the series impedances may be placed in either lead of the filter. However, if one output terminal is grounded, and it is essential that the hum voltage in the output be extremely small, then the filter chokes must be placed in the ungrounded lead.¹

For purposes of discussion and analysis, filters are ordinarily divided into sections, each of which consists of a series impedance associated with a shunt impedance as indicated in Fig. 11. In making this division, a series inductance is considered part of the first section, and a shunt capacity across the filter input is not included in the first section.

FIG. 11.—Typical inductance-capacity filters.

Inductance-capacity Filters.—Most filter sections are composed of a series inductance and a shunt capacity. In practical filters of this type, the reactance of the shunt capacity is much smaller than the reactance of the series inductance and of the load resistance. To the extent that this is true, such a filter section reduces each frequency component of voltage applied to the input side according to the relation

$$\frac{\left\{ \begin{array}{l} \text{Alternating voltage} \\ \text{across output of section} \end{array} \right\}}{\left\{ \begin{array}{l} \text{Alternating voltage applied} \\ \text{to input of section} \end{array} \right\}} = \frac{1}{(\omega^2 LC - 1)} \quad (4)$$

where L = series inductance of filter section.

C = shunt capacity of section.

$\omega/2\pi$ = frequency of ripple voltage involved.

¹ This is because of electrostatic capacity of the transformer secondary to ground. See F. E. Terman and S. B. Pickles, Note on a Cause of Residual Hum in Rectifier Filter Systems. *Proc. I.R.E.*, Vol. 22, p. 1040, August, 1934.

Results of Eq. (4) are given in Fig. 12 for the usual case where the ripple components are harmonics of 60 cycles.

When a filter consists of more than one section, the total reduction in ripple voltage produced by the combination is very nearly the product of the voltage-reduction factors of the individual sections, or what is the same thing, the sum of the db attenuations.

FIG. 12.—Reduction factor in ripple voltage produced by a single-section inductance-capacity filter having inductance L and capacity C .

tions of the individual sections. In the case of multisection filters, the most efficient way to divide a given amount of inductance (or capacity) between the sections is to do so equally.¹

Resistance-capacity Filters.—A resistance-capacity filter section is shown in Fig. 13a and consists of a series resistance followed by a shunt condenser. Such sections

¹ L. B. Hallinan, Jr., A Note on the Simple Two-element Low-pass Filter of Two and Three Sections. *Proc. I.R.E.*, Vol. 21, p. 1003, November, 1933.

are frequently used where the current is so small that the loss in voltage in a large resistance will not be excessive. Resistance-capacity sections also find use where it is necessary to drop the voltage, as, for example, in obtaining screen voltage from the same power supply that operates the plate electrodes of the tube. In practical resistance-capacity filters, the reactance of the shunting condenser is always made small compared with the series resistance of the filter and the load resistance to which the

FIG. 13.—Miscellaneous filter circuits.

filter output is delivered. Under these conditions, the resistance-capacity filter produces a reduction in a-c voltage according to the relation

$$\frac{\left\{ \begin{array}{l} \text{Alternating voltage} \\ \text{across output of section} \end{array} \right\}}{\left\{ \begin{array}{l} \text{Alternating voltage applied} \\ \text{to input of section} \end{array} \right\}} = \frac{1}{R\omega C} \quad (5)$$

where R is the series resistance and C is the shunt condenser (see Fig. 13a). The results of Eq. (5) are given in convenient form in Fig. 14 for the usual case where the ripple frequencies involved are harmonics of 60 cycles.

Filters Employing Resonant Elements or Involving Balancing Arrangements.—The effectiveness with which a filter suppresses a particular ripple frequency can be greatly increased by using resonant impedance elements or by balancing arrangements. Thus tuning the series inductance to parallel resonance with the undesired ripple frequency or the shunt condenser to series resonance, as shown in Fig. 13, will greatly increase the attenuation of this frequency. Such arrangements have the disadvantage, however, of requiring a careful adjustment of circuit constants in order to be fully effective, and also offer less attenuation to the higher ripple frequencies.

Balancing systems are shown in Figs. 13d, 13e, and 13f. In the tapped-choke arrangement of Fig. 13d, the capacity of the condenser C_2 and the location of the tap on

FIG. 14.—Reduction factor in ripple voltage produced by a single-section resistance-capacity filter having a resistance R and a capacity C .

inductance L_1 should be such as to satisfy the relation¹

$$X_c = (n + 1)X_L \quad (6)$$

where n = ratio of turns in section BC to number of turns in section AB of coil L_1 .

X_c = reactance of condenser C_2 at the lowest ripple frequency.

X_L = reactance of the choke between points A and B at the lowest ripple frequency.

¹ This formula assumes that there is zero leakage inductance between the two parts of the choke. Experimental data showing the hum reduction that can be achieved in practice is given by B. F. Miesner, Hum in All-electric Radio Receivers, *Proc. I.R.E.*, Vol. 18, p. 137, January, 1930.

If the direct-current resistance of the filter inductances is neglected, the rectifier must deliver a direct-current output voltage of 2,600 volts, and Table I shows that the rms voltage that each secondary leg must develop is $2,600 \times 0.855 = 2,135$ volts. Since the utilization factors of the primary and secondary, as given by Table I, are 0.827 and 0.875, respectively, each leg of the primary must have a rating of

$$\frac{2,600 \times 0.4}{(3 \times 0.827)} = 403 \text{ watts}$$

and each leg of the secondary, a rating of $2,600 \times 0.4 / (3 \times 0.875) = 493$ watts. Tentative calculation based on Eq. (4) shows that the filter of Fig. 11a with $C_1 = 1.0 \mu\text{f}$ and $L_1 = 9.8$ henrys will keep the ripple voltage down to 2 per cent and will be generally satisfactory. Reference to Table I shows that the peak anode current would be 0.4 amp for infinite input inductance, and the maximum inverse voltage that each rectifier must stand is $2,600 \times 2.09 = 5,225$ volts. Type 866 mercury-vapor tubes will meet these requirements. In actual practice, the secondary voltage of the transformer would be made greater than 2,135 volts by perhaps 10 per cent to compensate for the loss of voltage caused by the resistance of the filter inductance, the voltage drop in the rectifier, the leakage reactance of the transformer, and the transformer resistance.

Example 2.—A load of 4,000 ohms is to be supplied by direct current from a condenser-input rectifier-filter system. The power transformer develops a potential of 300 volts rms on each half of the secondary. The effective leakage inductance and resistance are 40 mh and 125 ohms, respectively, and a Type 80 rectifier tube is used. The input condenser is $4 \mu\text{f}$, and is followed by a single-section filter consisting of a 10-henry choke with 400 ohms resistance and an 8- μf condenser across the output. Determine the performance of the system.

The first step in the solution is the determination of the effective load and source resistances. The former is the actual load resistance plus the choke resistance, or $4,000 + 400 = 4,400$ ohms. The average resistance of the rectifier tube is (from tube manual) approximately $62/0.125 = 498$ ohms, so that the source resistance is $125 + 498 = 623$ ohms. Reference to Fig. 9 shows that for three values of source and load resistances and input capacity one has

	For $L = 0$	For $L = 0.1$	For $L = 0.040$
K_0/K_1	0.73	0.73	0.73
I_{max}/I_0	2.9	3.00	2.05
K_0/K_1	0.12	0.115	0.117

The last column is obtained by interpolation between the first two. From this column one has

$$\text{D-c voltage across input condenser} = 0.73 \times 300 \sqrt{2} = 309$$

$$\text{D-c voltage across actual load} = 309 \times 4,000/4,400 = 281$$

$$\text{D-c current in load} = 309/4,400 = 0.0703 \text{ amp}$$

$$\text{Peak anode current} = 0.0703 \times 2.95 = 0.207 \text{ amp}$$

$$\text{Ripple voltage across input condenser} = 0.117 \times 309 = 36 \text{ volts crest}$$

$$\text{Ripple voltage across load [from Eq. (4)]} = \frac{36}{(754)^2 \times 10 \times 8 \times 10^{-7} - 1} = 0.79 \text{ volt crest}$$

MISCELLANEOUS

7. Anode Power from Low-voltage Direct-current Sources. Vibrator Systems.¹—Direct current at high voltage can be obtained from a storage battery by using a vibrating contact to change the direct current delivered by the battery into alternating current that can be stepped up in voltage by a transformer and rectified. A typical arrangement of this type is shown in Fig. 16. Here a vibrating reed is provided, with contacts connected in the circuit in such a manner that the storage-battery voltage is first applied across one-half of the primary winding of a transformer, and then in the opposite direction across the other half. This induces an alternating voltage in the secondary having a value determined by the battery voltage and the transformer ratio. The reed is kept in vibration at its frequency of mechanical resonance by the magnet M , which is so arranged that when the reed is drawn to the magnet, the terminals of the latter are short-circuited, and the reed is allowed to spring back. The resistances R and capacity C_1 shown in Fig. 16 are for the purpose of minimizing sparking at the contacts, and the buffer condenser C associated with the secondary, as well as the radio-

¹ An excellent discussion of commercial types of vibrators is given in *Vibrators, Electronics*, Vol. 9, p. 25, February, 1936.

frequency chokes, are essential to control transient voltages and to prevent radio interference.

The secondary voltage developed by the transformer associated with the vibrator can be rectified by a rectifier or by an extra set of contacts on the reed, as in Fig. 17. The latter arrangement is termed a *synchronous vibrator*, and eliminates the cost of the rectifier tube and the voltage drop in the tube, but is more expensive and more likely to get out of order. Both high-vacuum and cold-cathode rectifiers are used in association with vibrator systems that require a tube.

Fig. 16.—Circuit of nonsynchronous vibrator power supply.

The vibrating reed normally has a resonant frequency of the order of 90 to 150 cycles. The associated transformer and filter system are designed accordingly, instead of for a 60-cycle fundamental. In the design, it is to be kept in mind that the wave shapes of the voltages involved are square rather than sinusoidal. It is important that the transformer have the lowest possible leakage inductance in order to minimize sparking at the contacts, and it is desirable that the transformer be designed to operate at a low flux density.

Vibrator power supplies are used to operate radio receivers from low-voltage storage batteries, particularly automobile and aircraft equipment, and are also employed for small transmitters. Compared with the dynamotor, a vibrator power supply is less expensive but has shorter life, and tends to produce radio interference (often referred to as "hash").

Dynamotor.—A dynamotor is a combined motor generator having two or more separate armature windings and a common field. One of the armature windings is operated from a low-voltage direct-current power source, commonly a storage battery, to give motor operation, while the other windings then serve as generators to produce the desired direct-current voltages.

Dynamotors are built in various sizes and ratings, ranging from small permanent magnet types intended for six-volt storage batteries, developing plate power for radio receivers and very small transmitters, to large machines supplying plate power for good-sized radio transmitters.

When plate power is obtained from a dynamotor or other equivalent generator of the commutator type, it is necessary to place a filter in the output of the machine to eliminate ripple voltages arising from the finite number of commutator bars and from miscellaneous irregularities.

Fig. 17.—Circuit of synchronous vibrator power supply.

8. Self-rectifying Circuits.—In a self-rectifying circuit, alternating voltage is applied directly to the plate of an oscillator or power amplifier. On the positive half of the cycle, output voltage will be developed, while on the negative half cycle, the tube will be inoperative. The usual self-rectifying circuit employs two tubes in each stage, so arranged that the two plates are supplied with alternating voltages that are 180° out of phase, and a large inductance L is inserted in the common filament return lead, as shown in Fig. 18. When the inductance is large, the total current drawn by the two tubes will be approximately constant, and the amplitude of the generated oscillations will be reasonably constant.

The radio-frequency output from a self-rectifying arrangement normally possesses both amplitude and phase modulation. The amount of amplitude modulation depends upon the size of the filter choke and is given approximately by the equation

$$\text{Amplitude modulation} = \frac{2}{3 \sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} \quad (9)$$

where ωL = reactance of filter inductance at twice the supply frequency.

$$R = \text{equivalent d-c load resistance} \\ = \frac{\{1.11 \times \text{rms voltage on each} \\ \text{half of transformer secondary}\}}{\text{d-c current through } L}$$

The phase modulation arises from the fact that the tubes operate alternately, and is less the more closely the two sides of the circuit are balanced.

Self-rectifying circuits find use where the complications of a rectifier and filter are to be avoided. They are also used to some extent in radio telegraphy for the purpose of creating

a number of side bands in the radiated wave in order to minimize fading.

9. Voltage-regulating Systems.—The voltage developed by an anode-supply system may be made approximately independent of the a-c line voltage by (1) an electronic voltage stabilizer employing a series regulating tube, (2) magnetic voltage regulator utilizing magnetic saturation, (3) a neon voltage-regulating tube, (4) a ballast lamp. The last three of these arrangements can also be utilized to regulate the filament voltage obtained from a transformer.

Electronic Voltage Regulators.—A typical regulator of this type is shown in Fig. 19, which also gives constants suitable for a small power-supply system.¹ The system operates in such a way as to cause the output voltage E_o of the system to be substantially independent of the load impedance connected across E_o , or of the d-c voltage E developed by the filter output.² This is because any fluctuation in the output voltage

¹ An extensive discussion of electronic voltage regulators of various types is given by F. V. Hunt and R. W. Hickman, *On Electronic Voltage Stabilizers*, *Rev. Sci. Instruments*, Vol. 10, p. 6, January, 1939. Special types of electronic voltage regulators having extremely high stability in order to meet the severe requirements of electron microscopes are described by A. W. Vance, *Stable Power Supplies for Electron Microscopes*, *R.C.A. Rev.*, Vol. 5, p. 293, January, 1941.

² This design is given in *A Voltage Regulator for D-C Power Supplies*, *R.C.A. Mfg. Co. Application Note 96*, Aug. 24, 1938.

³ The regulating system accordingly acts as a filter section by causing the hum voltage in the output E_o to be much less than in the input E to the regulator system.

Fig. 18.—Typical self-rectifying oscillator circuits.

will vary the potential E_1 applied to the grid of T_2 , and hence the grid-cathode potential, since the regulating action of neon tube T_2 keeps the cathode potential E_3 of T_2 constant. Any variation in output voltage is accordingly applied to T_2 , and after amplification by this tube affects the grid potential of T_1 in such a way as to produce a change in voltage drop in this tube that opposes the change in output voltage. The value of output voltage that the system attempts to maintain is determined by voltage E_1 and hence can be readily controlled by the setting of the potentiometer R_2 .

The fact that the output potential E_o obtained in Fig. 19 tends to be independent of the load placed on the system causes E_o to act as though it had very low internal impedance.¹ This property is very helpful in eliminating regeneration and motorboating in multistage audio amplifiers and in preventing coupling between different types of equipment operating from the same supply system.

Fig. 19.—Circuit diagram of typical voltage regulator of the electronic type.

Voltage Regulators Using Magnetic Saturation.²—The fundamental principles involved in this type of voltage regulator can be explained with the aid of Fig. 20. Here T_1 is an ordinary transformer, and T_2 is a transformer designed so that when the primary winding is placed in series with T_1 , the core of T_2 will be at least moderately saturated over the range of applied voltage for which constant output voltage is desired. The secondary windings are so poled as to oppose each other, with the saturated transformer developing most of the secondary voltage. When the alternating voltage applied to such an arrangement increases in amplitude, the division of voltage between the two transformers changes, with the saturated transformer T_2 taking proportionately less of the total, as shown in Fig. 20b. This causes the secondary voltages to become more nearly equal, and by properly proportioning the system, the output will remain substantially constant over an appreciable range of applied voltage, as shown.

The actual details of magnetic voltage regulators can be varied in many respects. Thus it is possible to combine the two transformers in Fig. 20a into a single three-

¹ This low internal impedance is effective only at low and moderate frequencies, such that the neon lamp is able to follow the cyclic variations in current. A low impedance can be assured at higher frequencies by shunting the output with a large condenser C , as shown in Fig. 19.

² An excellent survey of regulators of this type is given by K. J. Way, *Voltage Regulators Using Magnetic Saturation*, *Electronics*, Vol. 10, p. 14, July, 1937.

legged transformer. The windings can also be arranged in various combinations, including auto-transformer connections. A condenser is sometimes placed across the saturated transformer, as shown dotted in Fig. 20a, for the purpose of improving the voltage regulation with variation in the applied voltage and with load changes and for improving the wave shape of the output voltage.

FIG. 20.—Circuit and relations existing in a typical voltage regulator of the magnetic-saturation type.

FIG. 21.—Methods of utilizing a neon tube for voltage regulation.

Voltage regulators of the magnetic-saturation type are capable of maintaining the output voltage constant to ± 1 per cent, with variations in applied voltage of ± 15 per cent. Such regulators are widely used in conjunction with filament and plate-supply transformers, and are commercially available in ratings from a few watts to 25 kilowatts.

Neon-tube Voltage Regulators.—The fact that neon and similar cold-cathode gaseous tubes give a substantially constant voltage drop over a wide range of current, as shown

in Fig. 21a can be utilized in a variety of ways to regulate voltage. Thus in Fig. 21b, a series of neon tubes are placed across terminals *AB* where the d-c voltage is to be maintained constant, and the arrangement is then fed through a series resistance *R* that absorbs any fluctuation in supply voltage. Such an arrangement not only causes the output voltage to be substantially independent of the supply voltage but also tends to make the output independent of the load impedance placed upon the system. It accordingly gives the output of the system a very low internal impedance, and reduces hum.

Another type of voltage-regulating system is shown in Fig. 21c.¹ Here neon tubes are placed back to back across the 110-volt supply to a transformer, and a resistance *R* is placed in series with the line. This limits the peak voltage applied to the transformer to a fixed value, irrespective of the supply voltage, and so tends to keep the wave at the terminals of the transformer constant, as shown at Fig. 21d.

Voltage Regulators Employing Ballast Resistances.²—A ballast resistance is a resistance element having a temperature coefficient of resistivity such that over an appreciable range of voltage, the current through the resistance is independent of voltage. Such a characteristic is illustrated in Fig. 22. A ballast resistance placed in series with the filament of a tube or the primary of a transformer will accordingly absorb moderate voltage fluctuations and maintain the current through the tube constant.

10. Problems Arising from Use of Alternating Current to Heat Cathodes.—When alternating current is used to heat the cathode of a vacuum tube, it is to be expected that the amplified output of the tube will contain a certain amount of hum. This hum is either in the form of alternating components in the plate current that are in harmonic relationship to the heating voltage or in the form of a corresponding modulation of the voltages and currents being amplified by the tube.

Hum from Alternating Filament Current.³—The chief causes of hum in filament tubes are (1) alternating voltage drop in the filament; (2) magnetic effect of the alternating filament current on the space current; (3) temperature variations of the cathode.

The effect of an alternating voltage drop in a filament cathode upon the plate current can be minimized by connecting the grid- and plate-return leads to a point having the same potential as the center of the filament. This can be done by the use of either a center-tapped resistance across the filament or a center-tapped winding on the filament transformer. Connecting the grid- and plate-return leads in this way eliminates the principal cause of hum having the same frequency as the filament heating current. However, even with the center-tapped filament connection, there still remains a residual component of hum having twice the supply frequency. This results from the fact that with a center-tapped connection, the half of the filament that is negative at the moment supplies more electrons to the anode than does the positive half, and since the electron flow is proportional to the three-halves power of the electrostatic field strength, the current from the negative end of the field is increased more by the voltage

FIG. 22.—Characteristics of a typical ballast tube, showing how the tube operates as a constant-current device over an appreciable voltage range.

¹ G. F. Lampkin, A Simple A-C Voltage Regulator, *Electronics*, Vol. 10, p. 30, August, 1937.

² For further information see S. G. Gordon, Ballast Tubes as Automatic Voltage Regulators, *Electronics*, Vol. 15, p. 26, January, 1942.

³ W. J. Kimmell, The Cause and Prevention of Hum in Receiving Tubes Employing Alternating Current Direct on the Filament, *Proc. I.R.E.*, Vol. 16, p. 1089, August, 1928.

drop than the current drawn from the positive end is decreased. Hence each time there is a voltage drop, which is twice per cycle, the space current increases.

The magnetic field produced by the filament current introduces a double-frequency hum by virtue of the fact that the electrostatic field deflects the electrons from the paths that they would otherwise follow, and so tends to decrease the plate current slightly. This effect is exactly out of phase with that arising from voltage drop in the filament, and by proper design the two can be made to cancel each other for the normal operating condition.

Alternating filament current produces its maximum heating effect at the peaks of the cycle. The filament temperature and hence the emission accordingly fluctuate at twice the supply frequency. However, with ordinary filaments, the heat capacity is such that there is extremely small change in temperature with a 60-cycle supply. Temperature fluctuations are therefore usually unimportant as a source of hum, particularly if a full space charge is present.

Factors favoring a low double-frequency hum with a-c heating of a filamentary cathode are (1) filament designed for relatively low voltage and high current; (2) a V or W arrangement of the filament wires to reduce the magnetic effect and to permit exchange of electrons between opposite ends of the filament in order to minimize the effect of voltage drop in the filament; (3) tube design such that the magnetic and voltage drop components of hum are approximately equal at the usual operating point.

Filamentary cathodes are always used in large high-vacuum tubes, in many hot-cathode mercury-vapor rectifiers, and to some extent in small power tubes. They are seldom used in radio-frequency or audio-frequency voltage amplifiers or in detector tubes, except low-filament power tubes intended for battery operation.

Polyphase and Quadrature Arrangements for Filament Heating.—Large transmitting tubes having a multistrand filament are frequently arranged so that the different strands can be excited from different phases of a polyphase source. This results in a material reduction in hum.

In large hot-cathode mercury-vapor tubes employing filamentary cathodes, it has been found desirable to make the filament current 90° out of phase with the voltage applied to the anode of the rectifier tube. In this way, the plate current flows when the voltage drop in the filament is minimum. The tube then acts more nearly as though it had an equipotential cathode, and so gives more uniform behavior along the length of the filament than would otherwise be the case. This is important in the case of hot-cathode mercury-vapor tubes, because the average anode drop is only 10 to 15 volts, and a few volts' difference in potential between the ends of the filament will greatly influence the behavior.

Hum with Heater Cathodes.¹—The use of a heater-type cathode removes most of the factors that produce hum in filament tubes, but there is still present a residual hum as a result of the following causes: (1) magnetic field produced by the heater current; (2) leakage resistances from heater to other electrodes; (3) electrostatic fields within the tube arising from unshielded portions of the heater or heater leads; (4) electron emission from the heater.

The magnetic field of the heater introduces a double-frequency hum, just as in filament tubes. This can be minimized by using a high-voltage, low-current heater and employing proper geometry in the heater so that a minimum of external field is produced.

¹ J. O. McNally, Analysis and Reduction of Output Disturbances Resulting from the Alternating-current Operation of the Heaters of Indirectly Heated Cathodes, *Proc. I.R.E.*, Vol. 20, p. 1263, August, 1932.

Leakage paths from the heater to other electrodes cause voltages of the power-supply frequency to be developed between the electrodes and ground.² The most important leakage of this character is that from heater to cathode. Any leakage current of this character must flow through the bias resistance between cathode and ground, and will produce a voltage that is effectively applied between grid and cathode. Hum from this source can be eliminated by more adequate by-passing of the bias resistance, by grounding the cathode, and in some cases by properly biasing the heater with respect to the cathode. Leakage from the heater to other electrodes, notably the grid, is usually the result of getter material deposited between the leads of the tube. The remedy in such cases is either to try another tube, lower the impedance between the affected electrode and ground, or bias the heater with respect to ground.

Unshielded portions of the heater produce electrostatic fields that may influence the space current and result in 60-cycle hum. Electrostatic fields from the heater leads may also have direct capacitive coupling to other electrodes, and result in the production of hum voltages that are not negligible if the electrode impedance to ground is large.

When low hum is desired and the heater is to be operated with alternating current, it is desirable to select the tube type carefully. Special low-hum tubes are available to meet very severe requirements, and commercial tubes used in broadcast receivers vary greatly from type to type as a result of differences in heater geometry, shielding, etc. Some differences in hum between individual tubes of the same type and make can be expected as a result of chance differences in leakage films. Where complete absence of hum is essential, as in very high-gain audio amplifiers, the use of direct current for the heater is recommended.

11. Bias Voltage.—The grid bias for voltage amplifiers and small power amplifiers is nearly always derived from the plate-supply system. The commonest method of doing this is to use a resistance between cathode and ground that makes the cathode positive by the resistance drop produced by the total space current. This arrangement is satisfactory wherever the total space current is substantially constant during operation, as in Class C, Class A, linear, and most types of modulated amplifiers. It is not suitable for Class B and Class AB audio amplifiers, however. To avoid negative feedback type of regeneration, the cathode biasing resistor must be shunted by a condenser such that the impedance between cathode and ground to the alternating currents being amplified satisfies the relation³

$$g_m Z_f \ll 1 \quad (10)$$

where g_m is the transconductance of the tube and Z_f is the impedance between cathode and ground to the currents being amplified. In the case where the tube under question is excited by transformer coupling, negative feedback can be avoided by the use of a decoupling network RC , as illustrated in Fig. 23. In this arrangement, the resistance R corresponds to a grid leak and can be quite high. The capacity C must be large enough that its reactance at the lowest frequency for which feedback is to be avoided is appreciably less than the resistance R . Since R is much greater than the bias resistance, a much smaller condenser is required in this circuit than in the usual arrangement consisting of a bias resistor shunted by a condenser.

² See Heater Cathode Leakage as a Source of Hum, *Electronics*, Vol. 13, p. 48, February, 1940; also, Notes on Hum in Heater-type Tubes, *R.C.A. Application Note 88*.

³ This applies directly to the case of a Class A pentode tube, and is based upon the analysis given in Par. 4, Sec. 5. In the more general cases, the requirement is that the alternating voltage developed across the bias impedance by the space current be small compared with the signal voltage applied to the grid of the tube.

When the bias is obtained from a cathode resistor, it is necessary that the power-supply system be capable of developing an output potential greater than the desired anode-cathode voltage by the value of the bias potential.

The circuit of Fig. 24 employs the resistance drop in the filter choke to develop the bias. In this way, use is made of a potential that is otherwise wasted, and the total voltage that must be developed by the output of the filter is only that required by the anode of the tube. With this circuit, it is necessary to employ a resistance-capacity filter RC to prevent the hum voltage developed across the filter chokes from reaching the grid of the tube.

FIG. 23.—Self-bias circuit with decoupling network.

FIG. 24.—Method of utilizing the voltage drop in the filter choke to produce grid bias.

Small batteries are sometimes used for bias purposes, particularly in portable equipment and in laboratory apparatus. These may be either small dry cells or specially constructed bias cells having an extremely low current capacity and high internal resistance but very long shelf life.

Bias voltages for large power tubes can be obtained from a self-bias arrangement, direct-current generators, rectifier-filter systems, and in the case of Class C amplifiers and oscillators, by grid-leak grid-condenser combinations. When rectifier-filter systems are employed, it is essential that a load resistance be connected across the output of the filter to provide a path by which d-c grid current can return to the cathode. With direct-current generators, a filter must be employed between the generator and tube to eliminate commutator ripple.

W. J. SARJEANT
PHYSICS

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Westinghouse

HIGH-VOLTAGE SILICON RECTIFIER DESIGNERS' HANDBOOK

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2. DESIGN PHILOSOPHY OF HIGH VOLTAGE SILICON RECTIFIER ASSEMBLIES

The philosophy of high voltage "Stack" design differs from conventional rectifier designs in the number of rectifier diodes in series. Additional shunting components in parallel with each diode must be provided when greater than ten diodes are series connected.

In the common application where ten diodes or less are connected in series to obtain the voltage capability necessary to withstand the peak voltage in the system, it becomes necessary to supply some means of assuring a uniform voltage distribution among the series diodes. The reason for using forced voltage distribution is that the basic diodes may vary widely in their reverse characteristics. If the diodes that are used in these applications were matched with sharp knee characteristics, forced voltage distribution would not be of paramount importance. On the other hand, if "non-zener" type diodes are connected in series and no matching is employed, it is very possible for one or more diodes to "hog" most of the voltage to such an extent that the reverse voltage may be in excess of the dielectric property of the basic silicon, (between 1000 and 2000 volts). To prevent this occurrence, resistors are used to shunt the basic diodes.

By selecting the proper resistance value, it is possible to force voltage balancing to within $\pm 10\%$. The maximum value of resistance that should be used to shunt Westinghouse diodes can be simply computed by dividing the PRV rating of the individual diode by the maximum allowable reverse current as specified on the applicable technical data sheets. A resistor of one-half this value would normally be used in an application; however, in applications where the heat generated by this reverse power loss becomes intolerable, the maximum resistor value, equal to that computed above, should be used.

The rudimentary method of computing the shunting resistance value given above results in a "ball-park" figure. A resistor value higher than our recommended maximum could be used; however, the system designer must be willing to tolerate a higher percentage of unbalance which would be greater than the normal $\pm 20\%$.

Shunting capacitors may also be used to supply similar protection; however, their physical size, due to the necessary capacitance value and voltage rating, makes their use impractical. The capacitance value is based on the 60 cps capacitive reactance equal to one-half the minimum reverse resistance of the diode.

In high-voltage applications it is very rare for the diodes to operate at the maximum designed junction temperature and maximum PRV; therefore this maximum reverse current is seldom encountered. Since silicon diodes have a negative temperature coefficient of reverse resistance, the diodes will have a much lower reverse current than that specified on the data sheets because the reverse resistance will be higher than that computed above.

In high voltage applications the problem of distributed capacitance between the high voltage assembly and ground, is encountered. The diodes that are located near the leading end of the steep rising voltage wave front will experience a greater percentage of reverse voltage than it would normally experience if the voltages were distributed uniformly throughout the series string.

A series chain of diodes may be represented by the circuit in Figures 2-1 and 2-2, where C_d is the diode junction capacitance and C_g is the stray capacitance from diode to ground. The steep wavefront voltage between point "A" and ground will not distribute uniformly across the diodes in the series chain but will tend to pile up

at end "A". The diodes at the top end of the chain may therefore be stressed beyond their rated peak inverse voltage even though the average transient voltage per diode is well below the individual diode rating.

FIG. 2-1 "Series Connected Diodes"

As can be seen in Figures 2-1 and 2-2, if the capacitance value of C_p is equal to $10 \mu\text{fd}$ and if the diode shunt capacitance, C_d , also equals $10 \mu\text{fd}$, we would have essentially an equal voltage divider which would divide the steep rising voltage wavefront, assuming it is of a high frequency nature, uniformly across the diode and the distributed capacitance to ground. This would make point "B" equal to one-half of the applied rising wavefront which would make the differential voltage across the first diode equal to one-half of the applied voltage. Thus, it can be seen from this illustration that if the diode is shunted with a much higher capacitance value, which would make the shunting impedance a very low value, there would be a very small differential voltage between points "A" and "B", thereby causing the applied steep voltage wavefront to distribute itself uniformly throughout the series string.

Since it is rather impractical to make this shunting impedance extremely low, such as 1 ohm, a compromise must be made so that the shunting impedance is within the design rating of the particular diode in question. It is not feasible to make C_d (external diode shunting capacitor) extremely large, because the size and cost of the capacitor would be prohibitive. However, if the capacitor values were graduated, practical capacitor sizes could be used. If the capacitors were continuously variable in such a manner as to make C_{d1} slightly greater than C_{d2} , which in turn is slightly greater than C_{d3} , etc., the values could be chosen so that the voltage distribution would be perfectly uniform. This condition may be approximated, using commercially available fixed capacitors, by graduating their values in steps along the length of the diode chain. The D-C analog of the circuit shown in Figure 2 may be used as an aid in designing such graduation. The resistors are made proportional to the capacitive reactances in the diode circuit at high frequencies. A D-C voltage is impressed across the resistor chain. Voltage measurements may then be made across each series resistor (R_s) using a D-C voltmeter without the complications which would

arise with transient voltage measurements in the capacitive circuit. Given a value of peak transient voltage, a capacitor combination must be chosen so that the peak voltage across any diode is less than the rated PRV.

SERIES DIODE CHAIN

RESISTANCE ANALOG

FIG. 2-2 "Series Diode Chain and Resistance Analog"

Distributed capacitance in a high voltage system is a result of the exposed metallic surface coupled to ground through a dielectric medium. This capacitance represents itself schematically as a fixed capacitance value between points of finite potential. This distributed capacitance is of a uniform nature once a system is designed and contained in an ambient in which the metal in the system is fixed relative to the situation in which it is applied. Early philosophies on this subject have represented the impedance in diode strings as equivalent to those normally associated with transmission lines. This is partially true; however, using the expression, normally employed from other references,

$$E_N = E_a N \sqrt{\frac{C_p}{C_d}} \coth N \sqrt{\frac{C_p}{C_d}}$$

where: N = number of diodes in series, E_a = voltage across any diode, assuming uniform voltage division, E_N = voltage across the leading diode, C_p = distributed capacitance and C_d = diode shunt capacitance, would result in higher shunting capacitor

values than necessary. Westinghouse series string diode assemblies are compensated with sufficient capacitance necessary to off-set the effect of distributed capacitance in systems where the maximum value of distributed capacitance exists. The capacitor values are directly associated with the number of diode stack assemblies rather than the number of diodes themselves.

The resistor which is added in parallel with the rectifier diodes is chosen to equalize the steady-state voltage division. It eliminates the need for the expensive process of matching diodes over the operating temperature range. Although the diodes may withstand certain unbalance, the steady-state voltage division across the capacitor must be controlled within specified limits to give the greatest reliability.

To illustrate the need for shunting components, tests were performed on two diodes connected in series with and without shunting components. These diodes were operating under load in a three phase bridge circuit and the reverse voltage appearing across each diode was observed on an oscilloscope. The time scale shown on the photographs in Figures 2-3 through 2-7, moves from right to left, having a sweep of 2 milliseconds per major division.

Figure 2-3 shows the voltage across diodes "A" and "B" with no shunting components. The leading edge of the voltage wave across diode "A" shows a spike of voltage exceeding the voltage across diode "B" while the remaining portion of the reverse voltage across diode "A" is only one-sixth of that across diode "B". This spike of voltage on the leading edge of the reverse voltage across diode "A" is an effect of recovery time differences, while the difference in the remaining portion of the reverse voltages is due to the differences in the reverse resistances of the diodes.

In Figure 2-4 a capacitor equal to ten times the junction capacitance has been added in parallel across each diode. This has reduced the amplitude of the voltage spike at the leading edge of diode "A". To eliminate the spike completely, it is necessary to add a capacitor approximately 100 times the junction capacitance. This effect is shown in Figure 2-5. The voltages at the leading edges of the waveforms of diodes "A" and "B" are equalized, but as the capacitors will be charged to a value depending upon the reverse resistance of the cells, it is observed that the trailing edges of the voltage waveforms still have different amplitudes.

The effect of the addition of only a shunting resistor approximately equal to the reverse resistance is shown in Figure 2-6. Except for the voltage spike on the leading edge of the voltage waveform across diode "A", the voltages are more evenly divided. By adding both the resistance and capacitance in parallel with the diodes, optimum voltage division is obtained as shown in Figure 2-7.

In high-voltage silicon rectifier diode applications which require seriesing of many rectifier diodes, it is necessary that each rectifier diode be shunted with a capacitor and resistor to compensate for the system parameters which would eventually destroy the rectifier assembly if left uncompensated. The value of the shunting capacitor required is a function of the number of assemblies connected in series. Shunting each rectifier diode with a low value of capacitance and then shunting groups of series connected rectifier diodes with a smaller capacitor, it is possible to obtain the same effect as if each diode were shunted with a large capacitor. For example, if 400 rectifier diodes are each shunted with a $.05 \mu\text{fd}$ capacitor and if each group of 50 diodes are shunted with a $.001 \mu\text{fd}$ capacitor, the same effect will be achieved as if each diode were shunted with a $.1 \mu\text{fd}$ capacitor. The reason for this is that when capacitors are connected in series, the effective capacitance between the end points is equal to the reciprocal of the sum of the reciprocals of the individual capacitors.

In the example above, the effective capacitance of 50 diodes individually shunted with a $.1 \mu\text{fd}$ is equal to $.002 \mu\text{fd}$ and the effective capacitance of 50 diodes shunted with $.001 \mu\text{fd}$ and each diode shunted with $.05 \mu\text{fd}$ also equals $.002 \mu\text{fd}$. The internal diode capacitance, which is non-linear, will vary up to approximately $100 \mu\text{fd}$ for a medium power rectifier cell. This non-linear internal capacitance will

cause a slight unbalance in capacitance distribution; however, its effect is almost negligible when each diode is shunted with a capacitor approximately 100 times the maximum internal capacitance value. For a 35-ampere rectifier diode (Type 302), this value of individual shunting capacitor should be $.01 \mu\text{fd}$ minimum. This individual rectifier diode shunting must be employed to make group shunting effective. Experimental D-C analogue techniques were employed in arriving at the feasibility of group capacitor-type shunting. The first rectifier bridge assembly (100 kv) using this method of shunting was built by Westinghouse for Stauffer Chemical Company. This assembly successfully passed all tests and was shipped on September 18, 1959. Since this time hundreds of rectifier stack assemblies, with group shunting capacitors have been built for use in high voltage rectifier applications.

FIG. 2-3 Voltage Division with no Shunting Components

FIG. 2-4 Voltage Division with a Shunt Capacitor Equal to Ten Times the Junction Capacitance

FIG. 2-5 Voltage Division with a Shunt Capacitor Equal to 100 Times the Junction Capacitance

FIG. 2-6 Voltage Division with a Shunt Resistor Equal to the Reverse Resistance

FIG. 2-7 Voltage Division with a Shunt Resistor Equal to the Reverse Resistance and a Shunt Capacitor Equal to 100 Times the Junction Capacitance

(A) Diode A

(B) Diode B

3.0 OPERATING MEDIA FOR HIGH-VOLTAGE ASSEMBLIES

3.1 GENERAL

Operating media of a high voltage application is a parameter over which a design engineer may or may not have control. Certain conditions of an application may dictate or impose various restrictions on the designer which more or less force him into a specific type of operation media. The following discussion will point out various types of operating media, and some of the advantages and disadvantages of each of them as pertaining to Westinghouse high-voltage silicon rectifier assemblies. The types of operating media available to a designer are: air, oil, askarel (such as Inerteen), SF₆ (a vaporous atmosphere), and fluorocarbons.

3.2 AIR COOLING

Using air as a media, the designer is forced to live with the environments of nature unless other precautions are taken such as specially designed enclosures, baffling, filtering, de-humidifying and the like. With normal air, the design engineer has to contend with salt, humidity, temperature, air velocity, dirt, and fungi. Westinghouse has established electrical ratings for its complete line of silicon high-voltage assemblies when operating in air. First for discussion is the natural convection rating of our assemblies.

Here Westinghouse has calculated the natural convection ratings based on a completely solid enclosure in which no air movement was permitted to enter from the outside, thereby cutting off even the effect of chimney convection which is normally inherent in convection cooled applications. By taking this pessimistic approach to natural convection rating, Westinghouse can apply their silicon rectifier high-voltage assemblies right up to the maximum allowable permissible current shown on the data sheets. The natural convection rating takes into consideration the limitations of the auxiliary components. For example, the capacitors that are used in shunting the diodes vary from 85°C to 125°C components. In establishing permissible electrical output ratings Westinghouse has fully considered the limitations of the auxiliary components and thereby imposes a restriction on the ambients in which the components are contained, which in turn restricts the amount of power dissipated in the assemblies. This maximizes the reliability of the components and thereby the complete reliability of the application.

As one can observe by looking at the rating curves, as forced air cooling is applied, the permissible output current from the various assemblies is drastically increased. This is primarily due to the fact that under the conditions at which the natural convection rating is established, a slight amount of air movement through the assembly will greatly alleviate the extreme temperature conditions that are built up within the assembly. This will allow the designer greater current outputs from small assemblies. The forced air rating is normally designated by linear feet per minute. This air flow measurement is established by a velometer which monitors the air flow through the rectifier assembly. If a designer is employing forced air cooling and has a fan with a specified CFM (cubic feet per minute), he should be able to predict his linear feet of air flow by dividing the CFM rating by the total area of the high voltage stack openings that are contained across the baffled fan enclosure. The pressure drop through a standard Westinghouse high-voltage assembly is approximately .02 inches of water head. This places very little limitation on the designer in using convection blowers in his forced air cooling system. It is recommended for uniform air flow through the assemblies, that the designer employ

the exhaust principle rather than the blower principle in constructing his forced air cooled assembly. The exhaust principle eliminates air turbulence which may be encountered in conventional baffling. An example of an air cooled assembly is illustrated in Figure 3-1.

FIG. 3-1 Westinghouse High-Voltage Assemblies using Forced Air Cooling

When forced air cooling is employed, an interlock should be provided to prevent catastrophic system failure in the event of a blower failure. This interlock may be easily undertaken by the use of an air-switch similar to that shown in the illustration of Figure 3-1. This air-switch is comprised of a moving vane which is positioned in the air flow from the blower motor and is mechanically coupled to a micro-switch which can be used to interlock the electrical system. For air flows up to 300 LFM (linear feet per minute), baffling would not necessarily be required. However, good engineering judgment would normally dictate placing the higher voltage stacks such that the air would be forced through them in a perpendicular plane as illustrated in Figure 3-2. For air flow requirements as dictated by the electrical ratings of the Westinghouse assemblies from 500 to 1000 LFM of air, it is suggested that the designer employ restricted baffling. This should restrict the air to enter primarily through the stack openings and be exhausted through the fan itself, similar to the illustration in Figure 3-1.

FIG. 3-2 Westinghouse High-Voltage Assemblies mounted to provide flow of cooling air perpendicular to the channels

3.3 LIQUID COOLING

3.3.1 Oil Cooling

Many designers are not fully aware of the limitations placed upon them when oil coolant is employed. Many are of the opinion that all that is necessary for oil cooling is that the assembly be contained in an oil media of sufficient volume to completely cover the assembly. This sounds a lot easier than it actually is.

The oil provides an electrical insulating medium which also carries the heat away from the diodes. The insulating oil is just as much a part of the apparatus in which it is used as any of the other materials which are built into the apparatus.

The oil used with a rectifier should be either that originally supplied or an oil specifically approved by the manufacturer. The primary requirements are:

- (1) High dielectric strength.
- (2) Freedom from inorganic acid, alkali and corrosive sulphur to prevent injury to insulation or conductors.
- (3) Low viscosity to provide good heat transfer.
- (4) Good resistance to emulsion so that the oil will throw down any moisture entering the apparatus instead of holding it in suspension. (Water in suspension is a menace to safe operation.)
- (5) Freedom from sludging under normal operating conditions.

The principal causes of deterioration of insulating oil in service are water and oxidation. The oil may be exposed to moisture through condensation from air due to "breathing" of the tank especially when the rectifier is not continuously in service. The moist air drawn into the tank condenses moisture on the surface of the oil, and on the inside of the tank.

Oxidation causes sludging, the amount of sludge formed in a given oil being dependent upon the temperature and the time of exposure of the oil to the air. Excessive temperatures may cause sludging of any oil.

An oil immersed assembly does have the advantage of being completely isolated from the elements of nature to the extent that the unit is completely immersed in a high dielectric medium and completely sealed. Oil immersion has the advantages of eliminating maintenance problems such as air filtering, cleaning, humidity control, fungi and many other elements normally encountered in industry. However, the designer pays for these advantages in many ways. An oil design is first undertaken by the diode selection. The factors to be considered are the current ratings anticipated, the ambient in which the oil tank will operate, and the permissible oil ambient in which the diodes are designed to operate. Westinghouse has solved the first major step for the high voltage designer by publishing electrical load current ratings for the various configurations designed for an oil operating ambient. To fully appreciate what these electrical ratings mean requires a little insight into the limitations placed on the ratings that govern the load currents actually specified. Westinghouse has computed in the thermal load ratings a limitation on the maximum permissible case temperature or "hot spot" temperature for the oil. Clean dry Wemco "C" oil, for example, has a flash point of 135°C. Westinghouse in computing the load current ratings has placed a restriction on the upper limit to which the diode case temperatures can be operated. This limit was arbitrarily set at 125°C. The junctions of the individual diodes operating at this case temperature are well below their maximum design values; however, for safe reliable operation, a "hot spot" limitation has been a prime factor in dictating the permissible output load current that one can obtain from the Westinghouse assemblies. In conjunction with the "hot spot" limitation, Westinghouse has also recognized the limitations of the ambient oil in which the shunting components are contained. If one observes the output current rating versus ambient oil (measured 5 inches from the top of the liquid), he would note that the maximum ambient oil is not permitted to exceed 110°C. This upper limit on the oil ambient is dictated by the limitations on the auxiliary shunting capacitors. The capacitors used in oil are rated for 125°C. Westinghouse has lowered the upper limit to allow an extra margin of safety in their applications.

After a designer determines the particular diode which can carry the load current in his particular application, and the number of diodes that would be required for his peak operating voltage, he should then determine the power losses on the assem-

blies that he decides to use. Westinghouse has tabulated the power loss for various diode configurations and assemblies. This permits the tank designer to specify the geometry of the tank enclosure for the silicon rectifier assemblies.

In order to maintain the ambient of the oil coolant, the designer would have to supply a very large container, many times the cubage of the rectifier assemblies. Many people lose sight of the fact that the heat must be dissipated from the enclosed assembly in order to maintain the operating environment necessary for reliable operation of the diodes. People have a tendency to have a mis-conception of the cubage required to provide adequate cooling for the assemblies. Westinghouse has never had any problem in which the cubage of the stack assemblies was out of proportion to the tank that was necessary for cooling. For a designer to satisfy the temperature limitations placed upon the assemblies, with the power losses that were secured from the data sheet, the tank was, in all cases, many times the volume necessary for the cubage of the rectifier stack assemblies themselves. Standard steel rectifier tank enclosures average a dissipation level of approximately .0075 watts/°C per square inch¹. This can be expressed in an equation similar to that illustrated below:

$$P(\text{watts}) = (.0075 \text{ watts/}^\circ\text{C in.}^2) (\text{tank periphery in inches}) (\text{oil height in inches}) (\Delta T \text{ }^\circ\text{C})$$

$$\text{where } \Delta T = T_{\text{oil}} - T_{\text{ambient}}$$

Additional dissipation can be afforded the cooling system by adding fins welded to the external surface of the tank. These fins will average approximately .005 watts/°C per square inch dissipation. The length of the fins should not exceed eight inches. As the fin length is extended beyond 8 inches, the turbulent air pockets that were normally generated in the smaller length will smooth out and run parallel to the fin itself. This laminar flow will not effectively convey the heat away from the fin, but will pass over the fin surface leaving a stagnant air barrier in between.

Many of the disadvantages of liquid cooled systems are a result of the dielectric properties of the media itself. The dielectric strength for clean dry Wemco "C" insulating oil should not be less than 22,000 volts when tested in a testing cup fitted with one inch diameter flat disc electrodes with flat edges, and the surfaces in parallel planes 0.1 inch apart. The axes should be on the same horizontal line. This is only a test voltage, but in practice, if one considers the flash over from point-to-point or point-to-plane, it is good engineering design practice to use approximate 30,000 volts peak per inch. Closer spacing can be used, provided the designer employs a more sophisticated termination of his voltage gradients, or by the insertion of press-board insulation barriers between the rectifier assemblies and the grounded tank.

Another important point that a designer must take into consideration when working with liquid filled tank enclosures is the expansion of the liquid upon being heated. Provisions for this expansion must be made in the basic design of the tank structure itself. On large volume enclosures sufficient air clearance should be provided to accommodate the expansion of the liquid media. Where tight, compact enclosures are required, the designer should employ a flexible bellows to displace a sufficient volume to permit the oil its full margin of expansion. Westinghouse has properly engineered and designed their high-voltage assemblies to work in the oil media specified. These assemblies may differ slightly from the assemblies that would be supplied for air cooled applications. For example, disk capacitors employed for shunting purposes in oil media are furnished with the wax pregnation removed. This wax coating, if not removed, could contaminate the liquid, which in turn would detract from the dielectric properties of the oil. One should keep in mind that any foreign matter such as lint, dirt adhesives, or even paint on the resistors can contaminate the liquid thereby decreasing its dielectric property. Figure 3-3 which

1. This is based on convection oil. The physical size of the package can be reduced considerably by the use of forced oil cooling or heat exchangers.

illustrates the dielectric degradation with moisture shows that a fraction of a teaspoonful of water in suspension in a barrel of oil will make it necessary to retreat the oil. However, relatively large quantities of water may be present in a cooling tank without disastrous results. The water may be removed by filter or by centrifuge.

Oil filled enclosures should be evacuated to remove the air, which could cause serious arc-over problems, and should be filled with an inert gas or a combination of gas and air.

FIG. 3-3 Relation between Dielectric Strength and Amount of Water in Insulating Oil

If the voltage gradients are not properly terminated in the oil coolant, the overstressed oil could move in such a fashion as to create whirlpools which could eventually lead to internal arcing. If a designer has intentions on getting a small compact oil cooled package, he may be very disappointed to find out that the physical package is rather large and also rather heavy. It would be very impractical for a man to handle one without the aid of a hoist or other auxiliary equipment.

One fact that may not be very apparent to the average layman is that when working around an oil tank enclosure, extra precautions must be taken to prevent accidentally falling into the liquid since oil (only 7.5 pounds per gallon, whereas water is 8.39 pounds per gallon) will not support the body weight. Therefore, since one is unable to swim in oil, an accidental fall into an oil tank could prove fatal to an individual if he is not immediately rescued or if he does not propell himself immediately to the surface on the first try.

A large 270 kv power supply cubicle is illustrated in Figure 3-4. This figure shows the rectifier enclosures made of press board material. The entire set of rectifier column enclosures, which look like vertical coffins, are completely oil immersed when the power supply is in operation.

FIG. 3-4 Rectifier Tank for 270 kv 2 amp Silicon Rectifier

3.3.2 Askarel Cooling

Reviewing the fact that oil has limitations and that the flash point of Wemco "C" oil is 135°C and the fire point of Wemco "C" is 152°C, underwriters place restrictions on the usage of such a media when used internally or within certain building areas. Indoor oil-filled tanks must be placed in vaults for fire protection and safety. The use of a cooling and insulating medium which is at the same time non-inflammable and non-explosive, would do away with the expense of building vaults, and allow more latitude in locating the tanks. The only alternative for a designer, if liquid coolant is still his prime concern, is to resort to an askarel, such as Inerteen. Inerteen is a synthetic, non-inflammable and non-explosive insulating and cooling liquid, straw-yellow in color, with a specific gravity of 1.560 (at 60°F) and a Saybolt viscosity of 54 seconds (at 100°F). Askarels are not introduced here as the panacea for liquid cooled designs. Askarels have both advantages and disadvantages. One of the advantages is that it will not burn. However, when heated to high temperatures, askarels will boil (for example, Westinghouse's Inerteen will boil at approximately 240°C), and in doing so, they emit poisonous phosgene gas.

Askarel cannot be used in D-C supplies above 85,000 volts. Voltages higher than this level will cause polarization of the media which could result in dielectric breakdown. The dielectric property of Inerteen is 30,000 volts per 0.1 inch when tested in a similar set-up described for oil.

The tank required to support the askarel must be constructed with more rigid material than used for oil since the density of Inerteen is approximately twice that of oil (Inerteen is 12.9 pounds per gallon where Wemco "C" is 7.5 pounds per gallon). At low temperatures, askarel is thicker than molasses. Tanking enclosures for askarel must be superior to those designed for oil in that a slight amount of moisture can seriously disrupt the dielectric properties.

Special attention must be given to the inherent components when working in this media. Askarels can dissolve or decompose a great many resins and insulating

material. Westinghouse silicon high-voltage assemblies are designed for operation in Inerteen or other askarels; however, the designer should specify the operating media when ordering high voltage assemblies to be assured of getting the properly designed assemblies. The data published in Westinghouse technical data sheets for oil cooling can be used for askarel and in addition the designer can expect to get more output current than that specified for oil at the same ambient temperature.

Inerteen has an irritating effect upon the skin. This is more pronounced to some persons than to others. Especially the eyes, nose, and lips are affected when coming in contact with Inerteen, so certain safety precautions must be observed when handling it. When working with the hands in Inerteen, it will irritate skin abrasions or the tender parts between the fingers. Continued exposure may cause skin eruptions with certain individuals due to absorption of the Inerteen through the pores of the skin. Cleanliness among workmen handling Inerteen is essential, and is a very good safeguard against such effects. An application of castor oil is recommended for the eyes and castor oil or cold cream for the nose and lips. In case Inerteen comes in contact with the skin, the part should be thoroughly washed and cleansed. A supply of these materials should be kept available at all times where men are working with Inerteen. Special gloves are available for handling Inerteen.

3.3.3 Fluorochemical Cooling

Westinghouse silicon high-voltage rectifier assemblies will perform satisfactorily in fluorochemical coolants (vaporous atmospheres) such as SF₆. The insulation value of vaporized fluorochemical depends upon the density of the vapor blanketing the electrical assemblies. The breakdown potential of the vapor above the liquid reservoir is increased by raising the vapor pressure in the enclosure. Obviously, the vapor pressure will depend upon the temperature and geometry of the enclosure. Rapid heat transfer through the walls of the enclosure will tend to condense vapors and therefore drop the vapor pressure, which in turn lowers the breakdown potential of the protective atmosphere. Equilibrium must, therefore, be established between the rates of vaporization and condensation to provide adequate vapor density for a given insulation gap.

Initial vaporization can be accomplished by heating the liquid pool of fluorochemical with the heat generated by rectifiers conducting in the forward direction.

3.3.4 Gaskets

It is very important that all openings in the tank be tightly closed before putting a rectifier into operation. This is necessary whether the rectifier is for indoor or outdoor operation. Careful work in preparing and placing all gaskets, and in uniformly tightening all nuts and bolts is required.

The best insurance against leaks after opening a tank is the use of a new gasket, properly cemented to thoroughly cleaned surfaces. Gaskets for liquid rectifiers are cut from either corkneoprene (Corprene) or cork-Buna-N (Cortite). Both are suitable for sealing oil, but only Cortite is suitable for sealing Inerteen. Corprene is dissolved by Inerteen.

Corprene should be compressed to from 60 to 70% of its original thickness depending on the grade of material. Cortite should be compressed to 50%.

Gaskets are of both retained and non-retained types. Retained gaskets should be approximately 80% of the recess width. Non-retained gaskets should be the full flange width with holes on the centerline.

For many years it has been quite a problem to get a good job of installing gaskets on bushings, manhole and main covers, etc., on outdoor tanks, but with the development of the new Westinghouse #7386 cement, a reliable job can be done. This is now considered to be the most reliable gasket cement available on the market.

Number 7386 can be used for sealing pipe thread joints as well as gasketed joints, and is suitable for both oil and Inerteen cooled rectifiers, cork and neoprene cork gaskets.

3.4 INSULATING MATERIAL

There are many suitable insulating materials available for mechanically supporting Westinghouse silicon high-voltage assemblies. These are Micarta, press board, wood (mainly hard maple), Permalite (a special treated wood with very good dielectric properties), porcelain insulators and many others. One shouldn't mistake the insulation level of the various insulating materials listed above for the tracking level of these materials. In high voltage mounting, we are mainly concerned with tracking or voltage arc-over across the insulating surfaces to ground or to a different potential level. While many of the materials mentioned above have high dielectric properties through the thickness of the material, one should not confuse this with surface tracking. Many insulators under clean, dry operating conditions exhibit very good properties against tracking, however, under moisture, dirt, or sludge accumulation, these various insulators can become good surface conductors. A general rule of thumb for insulating surfaces is to use approximately 2000 to 4000 volts per inch in air and 20,000 volts per inch in oil. For air dielectric (clearance) one should use approximately 13,000 to 15,000 volts per inch. These voltage levels are crude approximations applicable for industrial ambients in which these materials may be employed. The designer should be aware of certain limitations mentioned above and should consult with the supplier of the material so as to properly insure himself against insulation failure. Westinghouse high-voltage stacks have built-in compensation for the voltage gradients mentioned above; however, the designer must apply proper mounting insulation within his particular enclosure.

5.0 CORONA

As the application of silicon diodes grows in magnitude to the point where the operating PRV exceeds 30,000 volts, one is likely to encounter an old phenomena known as corona. Corona is rather mysterious and not fully understood; however, it can be properly diagnosed and remedies are available. Just what is corona? Corona is an ionization of the surrounding media which results from over voltage stress caused by high potential gradients emanating from sharp edges. For one to say that an assembly or rectifier stack is corona free is very ambiguous. Corona is relative—relative to position (physical mounting) in an application or system in which it is used. For example, a sharp point having a $\frac{1}{8}$ " radius, connected to 1,000,000 volts potential which is located 100 feet from ground in air ambient having a relative humidity equal to 50%, will exhibit less than 5 microamperes corona current.

If the mechanical and the environmental conditions of the above test were not specified, one could be misguided in interpreting the results. To establish the properties of particular devices with regard to corona, one must specify the conditions under which such measurements are made. Voltages under 30 kv exhibit very little corona, essentially negligible in the environments normally encountered in circuit applications. However, beyond 30,000 volts, such assemblies manufactured by Westinghouse do exhibit a radical change in corona. To off-set this situation, Westinghouse has designed a mechanical attachment, known as a corona ring, which is readily available and adaptable to the existing rectifier stacks. Corona rings are mechanical devices that are physically and electrically connected to the terminating ends of the high voltage assemblies. (See Figure 5-1.) These corona rings allow corona-free operation under normal environmental conditions.

Up to 60,000 volts PRV rating, Westinghouse standard rectifier high voltage assemblies can operate without corona compensation, provided that the user employs heavy bus connections to the terminating points at which the high voltage gradient stresses are normally encountered. The exposed surface of the standard Westinghouse assembly is represented in a high voltage system as a planular surface rather than sharp emanating points which normally would be conducive to voltage stress or corona. The close proximity of the modular insert diode boards are of a designed separation so as to minimize the effect of corona normally encountered in such applications. (See Figure 5-2.) This may be difficult for one to appreciate from normal appearance; however, electrical tests have shown this statement to be true.

Tests were conducted by Westinghouse in which corona currents were measured, with reference to fixed distances from ground planes, at various voltage levels. For corona currents of less than 20 microamperes, the assembly was found to be essentially "corona free."¹ Corona became audible when the corona currents exceeded 100 microamperes. Flash-over usually occurred when the corona currents exceeded 300 microamperes.

The corona tests were conducted in two planes as illustrated in Figures 5-3a and 5-3b.

In Figure 5-3a, the open side of the high voltage channel was fixed twelve inches from the ground plane, and the end of the channel, containing a 10-32 fillister screw, was varied between one and twelve inches from an adjacent ground plane.

In Figure 5-3b, the end of the channel containing the 10-32 fillister head screw was mounted twelve inches from the ground plane, and the open side of the channel

¹ The most sensitive range on the oscilloscope that was used for these measurements was 100 microwolts per division which corresponded to 20 microamperes per division due to the fixed shunt impedance of 50 ohms.

FIG. 5-1 Westinghouse High-Voltage Stack Employing Corona Rings

was varied between one and twelve inches from an adjacent ground plane. A photograph of this test installation is illustrated in Figure 5-4.

Figure 5-5 illustrates an example of the corona current as witnessed under test.

Results of these tests are illustrated in Figures 5-6, 7. These curves indicate that corona becomes evident at 14 kv when a spacing of one inch is employed between the assembly and ground. Corona is not evident in the Militarized Plastic assembly until approximately 20 kv at the one inch spacing from ground. As evidenced from these curves, the unencapsulated assemblies exhibit very little difference between side mounting or end mounting with regard to corona. There is a noticeable difference between end mounting and side mounting with the Militarized Plastic assembly. A significant improvement in the kv level with regard to corona has been achieved by use of the special designed corona shield mentioned previously. With corona rings connected to the high voltage assemblies, corona was not detectable until approximately 40 kv. Militarized Plastic assemblies modified with corona shields exhibit a more superior corona-free operation than the standard assemblies.

FIG. 5-2 Typical Westinghouse High-Voltage Stacks

FIG. 5-3 Arrangement for Corona Testing a High-Voltage Assembly

Prior to the introduction of Westinghouse's high density packaging, the assemblies that were available to the public operated corona free within the level of voltages indicated. With the advent of the high density packaging, Westinghouse

FIG. 5-4 Westinghouse High-Voltage Assembly Undergoing a Corona Test

exerted special precautions to assure the user the same corona-free operation. As evidenced from the published curves, if one employs the safe electrical insulation level normally required in good design practice, the user can assure himself of a corona-free installation. If tight conditions prevail, corona suppression can be used and is equally as effective in assuring corona-free operation.

In designing extra high voltage power supplies, the designer may incorporate his own corona shielding to minimize the effects of corona in his applications. Instead of individual corona rings, for example, the designer may use large toroidal doughnuts in which the Westinghouse high voltage stack assemblies could be contained. A picture of a typical illustration is drawn in Figure 5-8. This type of construction is imperative when oil ambients are employed, because the huge corona shielding will help uniformly divide the potential gradient within the oil thereby minimizing excessive voltage stress in the dielectric medium. High voltage stress in oil can cause an oil movement or a circular whirlpool which could cause high voltage flash-over and result in catastrophic system failure.

A typical installation exhibiting corona-free operation is illustrated in Figure 5-9.

For 100 kv potential per voltage level, four-foot corona rings, having a toroidal diameter of three inches, are used. The rings are mechanically spaced with press-board, or suitable insulating material, equal to six inches in height. This stack or column will permit the free flow of convection oil which will in turn provide

FIG. 5-5 Oscilloscope trace during corona testing. The time scale is represented by the horizontal axis with each major division being 2 microseconds. The vertical scale represents the corona current with each major division being equal to 20 microamperes.

adequate cooling for the voltage assemblies and will minimize the effects of voltage gradients. This type of stacking is very easily adaptable to voltage doubler configurations where the A-C leads may be attached midway in the column thereby providing two legs of a bridge assembly. This design lends itself very readily for various kv levels of output voltage. The corona rings themselves offer a built-in capacitive divider for the cascaded series rectifier stacks. This additional shunting capacitance will offer additional voltage division for the rectifier assemblies under transient and operating voltage wavefronts. If a designer has any doubts as to means or methods of mounting Westinghouse high voltage rectifier assemblies in any particular configuration, additional engineering assistance is available at Westinghouse. The Application Engineering Department will freely work with any user of Westinghouse high voltage stack assemblies.

Corona may also be minimized by placing dielectric media between the rectifier assemblies and ground. In oil tanking, for example, corona can be minimized by placing layers of press-board between the rectifier assemblies and the transformer tank. Corona is minimized by multiple layers of dielectric media rather than one thick layer. The test measurements of corona in an oil dielectric occur at a much higher kv than they do in air. For example, with one inch spacing in oil, corona does not appear until 42 kv. From our oil tests, it has been concluded that if a designer used adequate voltage insulation in the oil (approximately 30 kv peak volts per inch), he can be assured of corona-free operation.

The Militarized Plastic design was intended for high voltage operation in an air dielectric. The designer can see from the plotted data that such high voltages are

possible with Westinghouse's standard Militarized Plastic encapsulated assemblies in an air medium with reasonable insulation levels or clearances between the assemblies and ground.

Designs based on theoretical assumptions without empirical data to substantiate them, may or may not work. Westinghouse has demonstrated the practicality of seriesing silicon diodes for high voltage applications by attaining a high degree of reliability which is exemplified by the record of over 50,000,000 failure-free stack hours of operation. Westinghouse pioneered the field of high voltage silicon rectifier assemblies and established the criteria which was necessary to make them work.

After reviewing the conditions of seriesing silicon diodes, i.e., the junction phenomena associated with recovery spikes, the voltage unbalance normally associated with reverse characteristics, need for shunting resistors, shunting diodes to compensate for distributed capacitance, corona rings, properly terminating points of high voltage stress, we are now able to proceed to conditions normally encountered in applications where such rectifier assemblies would be employed.

FIG. 5-6 Corona Shielding Involving Large Toroidal Shields

FIG. 5-7 Results of Corona Testing of Shielded Assemblies

FIG. 5-9 A Typical Corona-Free Installation Utilizing the Toroidal Corona Shields Shown in Figure 5-8

6.0 DESIGN CRITERIA IN THE SELECTION OF HIGH VOLTAGE STACKS

6.1 GENERAL

There are many considerations in the design and application of high voltage rectifier stacks. The three most important design criteria are:

- (1) The overcurrents, due to shorts or abnormal load conditions, which the rectifier stacks must conduct.
- (2) The average current, or normal load current, that must be conducted, and the ambient conditions.
- (3) The peak recurrent voltage (and, if known, the transient voltages) the rectifier stack must block.

In this chapter rectifier stacks for high voltage power supplies will be considered; Chapter 7 considers special applications. In addition, this section will cover the basic design philosophy for determining stack ratings.

6.2 CHOOSING THE CIRCUIT

Many times the circuit designer has flexibility in his choice of circuit; that is, single or three phase, in half or full wave configurations. Normally this choice is an economic one and generally resolves itself in the use of the minimum number of stacks or rectifier diodes.

Figure 6-1 is intended to help in the choice of a particular circuit and is also used to determine the average current that a stack will conduct plus the peak voltage that the stack must block during normal operation. The values shown are based on ideal conditions; assuming perfect rectifier diodes and a perfect source.

6.2.1 Single and Three Phase Circuit Comparisons

The choice between single-phase and three-phase operation, the most standard configurations, is usually based on the following considerations:

- (1) Three-phase circuits have lower ripple.
- (2) Three-phase circuits usually require less transformer kva, for a given D-C volt-amperes.
- (3) Three-phase circuits deliver 1.5 times as much load current for a particular rectifier assembly.

The choice between full-wave (FW) and half-wave (HW) circuits is based on the following considerations:

- (1) FW circuits require less transformer kva, see Figure 6-1.
- (2) FW circuits use at least two rectifier diodes or stacks in each forward conduction path (from the transformer to the load and back to the transformer) while HW circuits have as little as one.
- (3) The working peak reverse voltage (PRV)¹ in a FW bridge is approximately equal to the peak load voltage in a three-phase circuit and approximately 1.5 times the peak load voltage in a single phase circuit. The HW circuit has recurring PRV's that are approximately 2 and 3 times the load voltage, respectively.

Based on the above comparisons, the three-phase full-wave circuit (3-1-1-B) for a given output current, has the disadvantage of needing twice as many rectifier diodes or stacks as a three-phase HW circuit (3-1-1-Y), while the HW circuit has the disadvantage of needing twice the rated PRV of those in the FW circuit. Where

1. Also known as peak inverse voltage (PIV.)

CONFIGURATION	SINGLE PHASE HALF-WAVE	SINGLE PHASE FULL-WAVE CENTER TOP	SINGLE PHASE FULL-WAVE BRIDGE	THREE PHASE HALF-WAVE WYE	THREE PHASE FULL-WAVE BRIDGE
SCHEMATIC					
SYMBOLIC NOTATION	1-1-1-H	2-1-1-C	4-1-1-B	3-1-1-Y	6-1-1-B
OUTPUT WAVEFORM					
PEAK REVERSE VOLTS PER RECTIFIER LEG (WORKING PRV)	1.414 V _{AC}	2.828 V _{AC}	1.414 V _{AC}	2.45 V _{AC}	1.414 V _{AC}
PEAK REVERSE VOLTS PER RECTIFIER LEG (WORKING PRV) RESISTIVE OR INDUCTIVE LOAD ONLY	314 V _{DC}	314 V _{DC}	157 V _{DC}	209 V _{DC}	105 V _{DC}
RMS VOLTS OUTPUT RESISTIVE OR INDUCTIVE LOAD ONLY	157 V _{DC}	111 V _{DC}	111 V _{DC}	102 V _{DC}	100 V _{DC}
PEAK VOLTS OUTPUT RESISTIVE OR INDUCTIVE LOAD ONLY	314 V _{DC}	157 V _{DC}	157 V _{DC}	121 V _{DC}	105 V _{DC}
NO LOAD RMS AC VOLTAGE [V _{AC} (NL)]	2.22 V _{DC} (NL)	2.22 V _{DC} (NL)	1.11 V _{DC} (NL)	1.48 V _{DC} (NL)	.74 V _{DC} (NL)
AVERAGE DC OUTPUT CURRENT PER RECTIFIER LEG	1.00 I _{DC}	.5 I _{DC}	.5 I _{DC}	.333 I _{DC}	.333 I _{DC}
RMS CURRENT PER RECTIFIER LEG	1.57 I _{DC}	.707 I _{DC}	.707 I _{DC}	.577 I _{DC}	.577 I _{DC}
I _{AC} RMS CURRENT IN THE LINE BETWEEN THE TRANSFORMER AND THE RECTIFIER	1.57 I _{DC}	.707 I _{DC}	1.00 I _{DC}	.577 I _{DC}	.816 I _{DC}
RATED POWER OF THE TRANSFORMER PRIMARY	2.47 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.21 I _{DC} V _{DC} (NL)	1.05 I _{DC} V _{DC} (NL)
RATED POWER OF THE TRANSFORMER SECONDARY	5.5 I _{DC} V _{DC} (NL)	1.57 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.48 I _{DC} V _{DC} (NL)	1.05 I _{DC} V _{DC} (NL)
INTERPHASE TRANSFORMER RMS VOLTAGE (LINE TO NEUTRAL)	-	-	-	-	-
EQUIVALENT POWER RATING OF INTERPHASE TRANSFORMER	-	-	-	-	-

V_{AC} = RMS AC SECONDARY VOLTAGE
 V_{DC} = AVERAGE DC OUTPUT VOLTAGE OF RECTIFIER $PRV = \sqrt{2} V_{AC}(NL)$
 I_{DC} = AVERAGE DC OUTPUT CURRENT OF RECTIFIER $I_{DC} V_{DC}(NL)$ = IDEAL OUTPUT POWER OF THE RECTIFIER

FIG. 6-1 Design Chart

THREE PHASE DOUBLE WYE	FOUR PHASE FULL-WAVE CENTER TAP	FOUR PHASE FULL-WAVE PARALLEL BRIDGE	FOUR PHASE FULL-WAVE SERIES BRIDGE	SIX PHASE STAR (THREE PHASE DIAMETRIC)	SIX PHASE PARALLEL BRIDGE WITHOUT INTERPHASE TRANSFORMER	SIX PHASE PARALLEL BRIDGE WITH INTERPHASE TRANSFORMER	SEVEN PHASE BRIDGE
6-1-1-Y	4-1-1-C	2(4-1-1-B)	8-1-1-B	6-1-1-S	2(6-1-1-B)	12-1-1-B	12-1-1-B
2.45 V _{AC}	2.828 V _{AC}	1.414 V _{AC}	1.414 V _{AC}	2.828 V _{AC}	1.56 V _{AC}	1.414 V _{AC}	1.414 V _{AC}
2.09 V _{DC}	2.22 V _{DC}	1.11 V _{DC}	.785 V _{DC}	2.09 V _{DC}	1.01 V _{DC}	1.05 V _{DC}	.524 V _{DC}
1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}	1.00 V _{DC}
1.05 V _{DC}	1.1 V _{DC}	1.11 V _{DC}	1.11 V _{DC}	1.05 V _{DC}	1.05 V _{DC}	1.05 V _{DC}	1.05 V _{DC}
1.71 V _{DC} (NL)	1.57 V _{DC} (NL)	.785 V _{DC} (NL)	.555 V _{DC} (NL)	1.48 V _{DC} (NL)	.715 V _{DC} (NL)	.74 V _{DC} (NL)	.37 V _{DC} (NL)
.167 I _{DC}	.25 I _{DC}	.25 I _{DC}	.51 I _{DC}	.167 I _{DC}	.167 I _{DC}	.167 I _{DC}	.333 I _{DC}
.289 I _{DC}	.50 I _{DC}	.50 I _{DC}	.707 I _{DC}	.408 I _{DC}	.408 I _{DC}	.289 I _{DC}	.577 I _{DC}
.289 I _{DC}	.50 I _{DC}	.707 I _{DC}	1.00 I _{DC}	.408 I _{DC}	.577 I _{DC}	.408 I _{DC}	.816 I _{DC}
1.05 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.11 I _{DC} V _{DC} (NL)	1.29 I _{DC} V _{DC} (NL)	1.01 I _{DC} V _{DC} (NL)	1.01 I _{DC} V _{DC} (NL)	1.01 I _{DC} V _{DC} (NL)
1.48 I _{DC} V _{DC} (NL)	1.57 I _{DC} V _{DC} (NL)	1.57 I _{DC} V _{DC} (NL)	1.57 I _{DC} V _{DC} (NL)	1.81 I _{DC} V _{DC} (NL)	1.43 I _{DC} V _{DC} (NL)	1.05 I _{DC} V _{DC} (NL)	1.05 I _{DC} V _{DC} (NL)
.252 V _{DC} (NL)	-	-	-	-	-	.085 V _{DC} (NL)	-
.162 I _{DC} V _{DC} (NL)	-	-	-	-	-	.059 I _{DC} V _{DC} (NL)	-

FIG. 6-1 Design Chart

twice the number diodes is needed to satisfy the double voltage requirement the two disadvantages cancel each other, and the FW circuit has the obvious advantage of lower transformer kva.

6.3 FAULT OR OVERCURRENTS

The rectifier diodes used in high voltage stacks may be subjected to current overloads from several sources.

- (1) Accidental D-C shorts on the output of the power supply.
- (2) Arcing or intermittent loads; i.e., gaseous thyatrons and power amplifier tubes.
- (3) A shorted high voltage assembly elsewhere in the rectifier.

In general, the designer controls the problem of overload currents by his choice of:

- (1) Fuses
- (2) Circuit breakers
- (3) Current rating of the rectifier diodes stacks

The most positive protection is to insure that the high voltage stack applied is capable of conducting the maximum overload current the transformer can deliver during a shorted condition, until the protective breaker or breakers clear. During this shorted load condition the only current limiting afforded is that due to the system (transformer, regulators, and reactors) and is generally reactive.

A quick approximation to the short circuit current in any single or three-phase circuit, with the exception of the star connection with interphase (6-1-1-S) is:

$$I_{sc(pk)} = \frac{I_{dc}}{\%I_t} \text{ for three phase}$$

$$\text{and } I_{sc(pk)} = \frac{I_{dc} \times 1.57}{\%I_t} \text{ for single phase}$$

where $I_{sc(pk)}$ is the peak short circuit current
 I_{dc} is the total D-C average output current
 $\%I_t$ is the transformer impedance expressed in decimal form; e.g.,
 15% = 0.15 in the calculations

For the six phase star configuration this approximation is

$$I_{sc(pk)} = \frac{I_{dc}}{2\%I_t}$$

This is only a first approximation as to the type rectifier stack that might be used, see Figure 10-1.

As an example assume the I_{dc} output of a three phase power supply is

$$I_{dc} = 50 \text{ amperes}$$

$$\%I_t = 0.10 \text{ (10\%)}$$

$$\text{the peak current } I_{sc(pk)} = \frac{50}{0.10} = 500 \text{ amperes}$$

From Figure 10-1 it can be seen that the single cycle surge rating of the type 302 rectifier diode is 560 amperes; therefore, as a first approximation the type 302 rectifier stack can be used.

Because of the economic advantages in using a lower current rated stack, more exact calculations of the available short circuit current are necessary. Section 6.3.1 contains, in detail, methods of calculating short circuit currents in single and three-phase circuits plus examples of same.

6.3.1 Short Circuit Calculations

6.3.1.1 GENERAL

- (1) Wherever possible, all system reactances should be considered in the calculations in order to arrive at the most realistic short circuit current and thus avoid over specifying surge capacity.
- (2) In high voltage transformers (above 10 kv), regardless of specified impedance, 5% is about the practical minimum which can be attained based on maximum transformer utilization.
- (3) In three-phase systems, total kva = sum of phase kva's.
- (4) Per cent impedance of three-phase = $\sqrt{3} \times \% \text{ impedance of one-phase (or \% impedance per line)}$.
- (5) The series impedances of transformers and similar elements are predominately reactive so that they may be added arithmetically without gross error.

6.3.1.2 SHORT CIRCUIT KVA

- (1) Per cent impedance is the % of rated voltage dropped in the series impedance when rated current flows.
- (2) Short circuit kva is the maximum kva which can be delivered by the system under short circuit conditions. Thus:

$$\text{Short circuit kva} = \frac{\text{rated kva}}{\% \text{ impedance}}$$

with % impedance expressed in decimal form, i.e.,
 15% impedance = 0.15 in calculations

6.3.1.3 SERIES CIRCUIT ELEMENTS

- (1) In order to conveniently add the effect of the series system elements, such as feeder transformers, regulators, etc., a convenient kva base should be selected and the % impedance of these series elements converted to the equivalent % impedance for the base kva.
- (2) It is usually most convenient to use the kva rating of the output transformer as the base of calculations. The % impedance of each other element is then converted by multiplying its % impedance by the ratio of base kva to the series element kva.
- (3) All the referred % impedances are added to the % impedance of the output transformer to obtain a total system % impedance. For example, the following circuit arrangement may be used for a high voltage rectifier supply:

The individual referred impedances will be:

$$(4) \left(\frac{30}{300} \right) + (5) \left(\frac{30}{50} \right) + 20 = 23.4\%$$

based on output transformer rating (30 kva).

- (4) The net short circuit kva is then (using total % impedance in decimal form):

$$\frac{(30 \text{ kva})}{0.234} = 128 \text{ kva}$$

6.3.1.4 SHORT CIRCUIT CURRENT

(1) For Single-Phase System

$$kva = I_{L(rms)} V_{L(rms)} \times 10^{-3}$$

and short circuit rms current is:

$$I_{sc(rms)} = \frac{kva_{sc}}{V_{L(rms)}} 10^3$$

The peak steady-state short circuit current is then:

$$I_{sc(pk)} = 1.41 \frac{kva_{sc}}{V_{L(rms)}} 10^3$$

(2) For Three Phase Systems

$$kva = \sqrt{3} I_{L(rms)} V_{L(rms)} 10^{-3}$$

and the short circuit current per line (and thus the short circuit current seen by the rectifier) is:

$$I_{sc(rms)} = \frac{kva_{sc} 10^3}{\sqrt{3} V_{L-L(rms)}}$$

The peak steady-state short circuit current is:

$$I_{sc(pk)} = \frac{\sqrt{2} kva_{sc} 10^3}{\sqrt{3} V_{L-L(rms)}}$$

(3) Asymmetric offset factor

To each of the above should be applied an asymmetric offset multiplying factor of 1.5 due to initial oscillatory action. The current due to this offset factor decays in less than four cycles leaving only the steady-state short circuit current.

Westinghouse rectifier diode surge current curves decline by about the same ratio in 5 to 6 cycles. Therefore, if the peak asymmetric current is less than the rated one-cycle surge current, the overload current will be within the device rating if the source is disconnected by a breaker or fuse within 5 to 6 cycles.

(4) Design Limit Criterion

The surge capability of a given rectifier in a given circuit can then be evaluated as follows:

The peak asymmetric current, as described in (3) above, must be less than the one-cycle surge rating of the rectifier.

(5) Example

Assume 275 kva, 20% transformer furnishing 20A, 10 kv D-C through a three-phase, full-wave bridge,

$$\text{the line to line voltage} = \frac{10 \times 10^3}{1.35} = 7400 V_{rms}$$

$$kva_{sc} = \frac{275}{0.20} = 1375 \text{ kva}$$

$$I_{sc(pk)} = \frac{\sqrt{2} (1375) 10^3}{\sqrt{3} (7400)} = 152A$$

with offset factor:

$$I_{sc(pk \text{ with offset})} = (152) (1.5) = 228A \text{ (A type 304 rectifier diode can be used)}$$

6.3.1.5 ADDITIONAL CIRCUIT IMPEDANCES

(1) Series Line Reactors

Where additional limiting reactance is necessary, this can be added as series reactors in the primary or secondary A-C lines.

(a) Three-Phase System

- (1) The total required system reactance (%_T) to limit the short circuit current to the proper value for the single-cycle surge rating of the rectifier can be calculated from the equation:

$$\%_T \text{ Impedance} = \frac{1.5 \sqrt{2} va (\text{rated})}{I_M \sqrt{3} V_{L-L(rms)}}$$

where:

%_T Impedance is in decimal form.

va (rated) is the va base on which the % impedance is specified (usually the rectifier transformer rating).

I_M is the single-cycle surge rating of the rectifier diode.

- (2) The existing total % impedance calculated per 6.3.1.3 is then subtracted from the required % impedance calculated above and the remainder is the % impedance which must be added to the system (%_{OA}).
- (3) The ohms reactance corresponding to this added % impedance may be calculated by:

$$X_{3\text{-phase}} = \frac{\%_{OA} V_{L-L(rms)}^2}{va (\text{rated})}$$

Since the added reactance must be added in the lines, this must be converted to the per-phase value or:

$$X_{\text{phase}} = \frac{\%_{OA} V_{L-L(rms)}^2}{\sqrt{3} va (\text{rated})}$$

where %_{OA} is the added % impedance in decimal form and va (rated) is as defined above.

(b) Single-Phase System

The same procedure is used for a single-phase system except that only one series reactor is required and:

$$X = \frac{\%_{OA} V_{L-L(rms)}^2}{va (\text{rated})}$$

(c) Inductance of reactor

The inductance of the required reactor is then:

$$L = \frac{X}{2\pi f} \text{ or for 60 cycle supply}$$

$$L = \frac{X}{377}$$

(d) Secondary-Primary Conversion

Where high step-up voltage ratios are used in the transformer, much smaller inductances may be used if the reactors are installed in the primary line wires. The values for X or L referred to the primary windings can be calculated by:

$$X \text{ (or L)}_{\text{primary}} = \left(\frac{V_{L-L(\text{primary})}}{V_{L-L(\text{secondary})}} \right)^2 X \text{ (or L)}_{\text{secondary}}$$

(2) Effect of Filter Chokes

(a) Where choke input filters are used, the choke can provide surge current limiting.* Its effect can be evaluated as follows:

- (1) It may be assumed that the effective inductance of the filter under short circuit conditions will be at least 0.10 of its normal inductance. Thus, its 60-cycle short circuit reactance will be:

$$X_c \approx 37 \text{ L}$$

- (2) The equivalent % impedance contributed by the choke will be:

$$\%_{oc} = \frac{(X_c) \text{ va (rated)}}{V_{L-L(rms)}^2}$$

Where $\%_{oc}$ is the three-phase % impedance (in decimal form) contributed by the choke and is based on rate va.

- (3) $\%_{oc}$ can be added arithmetically to the % impedance of the rest of the system and the sum used to calculate short circuit kva and short circuit current.

6.4 CURRENT RATINGS OF HIGH VOLTAGE STACKS

6.4.1 Discussion

The ratings are computed considering the following:

- (1) Forward power losses of the rectifier diodes
- (2) Reverse power losses
 - (a) Rectifier diodes
 - (b) Shunting resistors
 - (c) Shunting capacitors, (this wattage is negligible and is neglected.)
- (3) A form factor (I_{rms}/I_{ave}) of 1.76 is used in determining the forward power losses. This allows applying the stacks to any single or three phase circuit. The only configuration to which the ratings do not apply is the six-phase star circuit (6-1-1-S), which has a form factor of 2.45.
- (4) The ratings as they are determined apply to high voltage stacks of any PRV rating.

As an example of the scope of the average current ratings, a single-phase FW bridge output of 10 amperes and a three-phase FW bridge output of 15 amperes can use the same rectifier stack, provided the short circuit requirements are within limits. In both cases the average current per leg is 5 amperes.

The cooling mediums also affect the current output ratings of the stacks. Westinghouse high voltage stack rating curves are given in three cooling mediums; natural convection air, forced cooling air at 1000 linear feet per minute (LFM), and natural convection oil. In addition, a conversion curve for air velocities other than a 1000 LFM is given in Figure 10-8. For each particular method of cooling the ratings are established with the following considerations:

Cooling Method	Limiting Factors
Natural and forced air convection	<ol style="list-style-type: none"> 1. The maximum ambient temperature is assumed to be 110°C. 2. The maximum allowable operating temperature of the shunting capacitors must not be exceeded. 3. The maximum operating junction temperature of the rectifier diodes in the stack must not be exceeded.

* The manufacturers data of the choke (saturated inductance and d-c resistance) must be known in order to determine whether current limiting is afforded.

- Natural Convection Oil
1. The maximum allowable operating temperature of the shunting capacitors must not be exceeded.
 2. The operating diode case and component temperatures must not exceed the "Flash Point" temperature of the coolant.
 3. The above limitations insure that the maximum junction temperature of the rectifier diodes is not exceeded.

6.5 VOLTAGE RATINGS

6.5.1 Discussion

In addition to the rectifier diodes, the proper application of the shunting resistors and capacitors is necessary to insure maximum reliability. To properly select a resistor, the wattage rating must be chosen carefully. Generally the ratings are determined using the rated PRV of the stack.

The capacitor applied depends primarily on its dissipation factor. This parameter of the capacitor determines the maximum working PRV and frequency at which a high voltage stack can safely be operated without jeopardizing capacitor and subsequently the stack reliability.

6.5.2 Voltage Safety Factor

The recommended PRV ratings of Westinghouse high voltage stacks are given in Figure 6-2. As previously indicated, the working PRV for the various circuit configurations can be determined from Figure 6-1. To utilize the reduced safety factors, transient suppression networks are required. Section 6.5.4 gives design aids for determining the values and ratings of the RC suppression networks.

Diode Type	PRV	With Transient Voltage Surge Suppressors*		Without Transient Voltage Surge Suppressors*
		Air Cooled	Oil Cooled	All Types of Cooling
320-H	400	1.33	1.33	2.0
320-K	500	1.4	1.33	2.0
320-M	600	1.5	1.33	2.0
320-P	700	1.5	1.33	2.0
366	All PRV's	1.5	1.5	2.0
341	All PRV's	1.5	1.5	2.0
368	All PRV's	1.5	1.5	2.0
304	All PRV's	1.5	1.5	2.0
404	All PRV's	1.5	1.5	2.0
302	All PRV's	1.5	1.5	2.0
300	All PRV's	1.5	1.5	2.0
439	All PRV's	1.7	1.7	2.0
770	All PRV's	1.7	1.7	2.0

FIG. 6-2 Voltage Safety Factors

6.5.3 Distributed Capacitance Effects

Distributed capacitance, in a high voltage system, is a result of the exposed metallic surface coupled to ground through a dielectric medium. This capacitance represents itself schematically in a circuit as a fixed capacitance value between points of finite potential, or where a potential difference exists. This capacitance is of a uniform nature, once a system is designed, and is more pronounced in systems utilizing liquid coolants.

Westinghouse high voltage stacks may require additional external shunting capacitance in order to offset the distributed capacitance which may exist in appli-

* The safety factors are expressed as the ratio of $\frac{\text{Rated PRV}}{\text{Working PRV}}$

cations requiring the seriesing of a large number of rectifier diodes. It is recommended that when more than the number of series rectifier diodes is greater than the indicated number, see Figure 6-3, external shunting capacitance must be applied. These additional shunting capacitors are applied across groups instead of individual rectifier diodes.

Westinghouse High Voltage Stack	External Capacitors must be used when the number of series rectifier diodes exceeds:
all 320 designs	312 +
all 304, 341, 366, 368, and 404 designs	312 +
302 heat sink design	180 +
302 open design	156 +
300 design	56 +
439 and 770 spiral design	80 +

FIG. 6-3 External Shunting Capacitor Requirements

6.5.4 RC Surge Suppression Networks

- (1) A series RC network should be connected either from line to line or line to neutral of a three-phase system. In single-phase, connect across secondary of transformer.
- (2) In very high voltage applications where external stack shunting capacitors are used, transformer secondary suppression may not be required.
- (3) Recommendations are based on maximum system per cent impedance of 25%.

6.5.4.1 CAPACITY CALCULATIONS

$$(1) C = \frac{(10) v_a (60)}{V_{rms}^2 (f)} \text{ microfarads}$$

f = supply line frequency

(a) Single-phase

v_a = total transformer va capacity

V_{rms} = secondary rms voltage across which network is connected

(b) Polyphase "Y" secondary with networks connected line to neutral

$$v_a = \frac{\text{total transformer va}}{\text{no. of phases}}$$

V_{rms} = line-to-neutral rms voltage

(c) Three-phase system with networks connected line-to-line

$$v_a = \frac{\text{total transformer va}}{\sqrt{3}}$$

V_{rms} = line-to-line rms voltage

(d) Where v_a of transformer is not specified, use:

(1) For three-phase with networks connected line-to-line

$$C = \frac{10 W_{dc} (60)}{V_{rms}^2 (f)}$$

where:

V_{rms} = line-to-line rms volts

$W_{dc} = V_{dc} I_{do}$

(2) For single-phase with networks connected across secondary winding

$$C = \frac{15 W_{dc} (60)}{V_{rms}^2 (f)}$$

6.5.4.2 CAPACITOR VOLTAGE RATINGS

(1) General

Oil or Askarel filled capacitors should be used. (Inerteen and Pyranol are Westinghouse and G. E. trade names for Askarel.)

(2) Capacitors are usually rated in WVDC (working volts D-C).

For these applications:

$$WVDC \geq 3 \times V_{rms} @ f \leq 400 \text{ cps.}$$

6.5.4.3. RESISTOR WATTAGE RATINGS

(1) $R \approx 5 \frac{V_{do}}{I_{do}}$ except where wattage becomes impractically large. In this case, R

may be reduced to not less than to $R = \frac{V_{do}}{I_{do}}$.

(2) Wattage Rating of R

$$P_{res} \geq 5 I_{AC}^2 R$$

where:

$$I_{AC} = V_{rms} (2\pi f C)$$

where C is in farads:

f = supply line frequency

6.6 TYPICAL DESIGN PROBLEM

To illustrate the use of the design criteria discussed in this chapter, a typical application is considered.

Given: Circuit — three-phase full-wave bridge (6-N-1-B), 60 cps
 Load requirements — $V_{do} = 15 \text{ kv}$
 $I_{do} = 4 \text{ amperes}$
 LC filter on output of power supply
 $L = 4 \text{ henries}$ (discount any short circuit limiting by this choke for the 5 cycle overload)
 Transformer — 64 kva
 Delta-Wye connection
 10% impedance per phase
 Overload protection — primary breaker, clearing time 5 cycles
 Ambient conditions — natural convection or forced air (specify) ambient temperature 50°C maximum

Solution: Short circuit calculations (all equations from section 6.3)

The percent impedance is

$$\%Z = \sqrt{3} \% \text{ phase}$$

$$\%Z = \sqrt{3} \times 0.10 = .1732$$

NOTE: The effect of the filter choke is neglected; however, it will reduce the asymmetry or offset conditions during the overload.

The net short circuit kva is

$$kva_{sc} = \frac{kva}{.1732}$$

$$kva_{sc} = 370 \text{ kilovolt amperes}$$

The peak steady state short circuit current per rectifier leg is then

$$I_{sc(pk)} = \frac{\sqrt{2} kva_{sc} \times 10^3}{\sqrt{3} V_{L-L(rms)}}$$

$$I_{sc(pk)} = \frac{\sqrt{2}}{\sqrt{3}} \frac{370 \times 10^3}{11.1 \times 10^3} = 27 \text{ amperes}$$

where:

$$V_{L-L(rms)} \approx \frac{V_{do}}{1.35}$$

$$V_{L-L(rms)} \approx 11.1 \times 10^4 \text{ volts}$$

The asymmetric offset factor is discounted because of the limiting action of the choke.

From Figure 10-1 the type 366 or 368 will satisfy the short circuit requirement at 5 cycles.

An increase in the source impedance will permit the use of a smaller device; i.e., the type 320. The amount of additional impedance (%_{OA}) required is calculated below:

$$\%_{OT} \text{ impedance} = \frac{\sqrt{2}}{\sqrt{3}} \frac{v_a \text{ (rated)}}{I_m V_{L-L(rms)}}$$

where: I_m is the 5 cycle rating of the type 320, 22.5 amperes.

$$\%_{OT} \text{ impedance} = \frac{\sqrt{2}}{\sqrt{3}} \frac{64 \times 10^4}{22.5 \times 11.1 \times 10^4} = .209$$

The additional impedance required is

$$\%_{OA} = .209 - .1732 = \sqrt{3} \% \text{ phase}$$

$$\% \text{ phase} = .0207$$

Average current rating of the stack must be

$$I_{ave} = \frac{I_{do}}{3}$$

$$I_{ave} = \frac{4}{3} = 1.33 \text{ amperes.}$$

For the natural convection rating, from Figure 10-2 the type 368 will conduct 1.4 amperes at 50 C.

For forced cooling the following will satisfy the current requirements

Cooling 1000 LFM — type 320

The PRV rating of the stack is

$$PRV = V_{do} \times 1.05$$

$$PRV \approx 16 \text{ kv}$$

In order to utilize the minimum safety factors, see Figure 6-2, RC suppression networks line-to-line must be applied.

The values and ratings of the RC networks are (equations from Section 6.5)

$$C = \frac{10 \text{ VA } 60}{V_{rms}^2 f} \text{ microfarads}$$

$$C = \frac{10 \times 64 \times 60 \times 10^3}{(11.1 \times 10^4)^2 \sqrt{3}} = 3000 \text{ pefarads}$$

The capacitor voltage rating is

$$WVDC \geq 3 \times V_{rms}$$

$$WVDC \geq 33.3 \text{ kv}$$

The resistor value is

$$R \approx 5 \frac{V_{do}}{I_{do}}$$

$$R \approx 18,000 \Omega$$

The wattage rating of the resistor is

$$P_{res} \geq 5 I_{RC}^2 R$$

$$P_{res} \geq 15 \text{ watts}$$

where:

$$I_{RC} = V_{rms} (2\pi f C)$$

For this application the following configurations and types can be used.

(1) If the additional reactance, suppression networks, and 1000 LFM cooling air are provided

Type rectifier diode	—320P
Configuration	—6-24-1-H
PRV rating	—22.4 kv
Safety factor	—1.4
Number of boards	—24
Size of stack (see Figure 10-13)	— $I_{max} = 21.56''$
	W = 3.88"
	H = 2.06"

(2) Natural convection, suppression networks provided

Type rectifier diode	—368K
Configuration	—3(2-48-1-D)
PRV rating	—24 kv
Safety factor	—1.5
Number of boards	—24 in each stack
Size of each stack	— $I_{max} = 21.56''$
	W = 3.88"
	H = 2.06"

RADIO ENGINEERS' HANDBOOK

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type of resistor immersed in cooling water.¹ In this way the rating can be increased to 50 to 80 times that for air, and is of the order of 100 watts per square inch of surface. The power being dissipated can be determined from the rate of flow and temperature rise of the cooling water. By making the resistor the central conductor of a concentric line shorted at the receiving end, the reactance at the input terminals can by suitable design proportions be made zero.

INDUCTANCE AND MUTUAL INDUCTANCE

7. Self-inductance.—Self-inductance is that property of electrical circuits which causes them to oppose a change in the current flowing through them. The opposition to the change in current manifests itself in the form of a back emf that is developed when the current is changed. A circuit is said to have an inductance of one henry if it develops a back emf of one volt when the current through it changes at the rate of one ampere per second.

By virtue of the way in which the back emf in a circuit is related to the time rate of change of the magnetic field, inductance is related to the number of flux linkages per unit current in the circuit. A flux linkage represents one flux line encircling the circuit current once. Flux linkages may be fractional as well as integral. The relation between inductance, total flux linkages, and current is given by

$$\text{Inductance } L, \text{ henrys} = \frac{\text{flux linkages}}{\left\{ \begin{array}{l} \text{current producing} \\ \text{the flux, amperes} \end{array} \right\}} \times 10^{-9} \quad (12)$$

Inductance is also related to the energy stored in the magnetic field by the relation

$$L = 2 \frac{W}{I^2} \text{ henrys} \quad (13)$$

in which W is the energy stored in the field in watt-seconds.

Units and Dimensions.—The practical unit of inductance is the *henry*. This is such a large unit of inductance that it is often more convenient to express inductance in terms of millihenrys (10^{-3} henrys) or microhenrys (10^{-6} henrys).

Inductance has the dimensions of length. It will be found that in all inductance formulas the inductance is proportional to a linear dimension and to the square of the number of turns when the shape of the inductance is kept constant.

Inductance Formulas.—For various geometrical configurations it is possible to calculate the number of flux lines set up or the energy stored in the magnetic field for an assumed current. The inductance formulas that follow have been calculated from Eqs. (12) or (13) in this manner. The collection of inductance formulas given here will be found complete enough to satisfy the ordinary needs of engineers and physicists. In general the formulas are accurate to better than 0.5 per cent. For more extensive collections, the reader is referred to treatises on the subject.²

Because of the way in which the current distribution within a wire changes with frequency, the inductance of a circuit changes somewhat with frequency. Of most interest are the values that the inductance assumes at very low frequencies when the current distribution through the wire cross section is uniform, and the limiting value that the inductance assumes as the frequency approaches infinity. This latter inductance is always less than the low-frequency inductance, since the redistribution

¹ See G. H. Brown and J. W. Conklin, Water-cooled Resistors for Ultrahigh Frequencies, *Electronics*, Vol. 14, p. 24, April, 1941.

² E. B. Rosa and F. W. Grover, Formulas and Tables for the Calculation of Mutual and Self Inductance, *Bur. Standards Bull.*, Vol. 8, No. 1, pp. 1-237, Jan. 1, 1912.

Radio Instruments and Measurements, *Bur. Standards Circ.* 74, pp. 242-296.

J. Hak, "Eisenlose Drosselspulen [Air-core Inductances]," Verlag K. F. Koehler, Leipzig, 1938.

of current at high frequencies is always of such a character as to reduce the flux linkages. The difference between the low-frequency and high-frequency inductance is usually not great, however.

8. Self-inductance of Straight Conductors.—The self-inductance of an unclosed circuit is here taken as the inductance of the unclosed circuit considered as a part of a closed circuit. The total self-inductance of the closed circuit is equal to the sum of the self-inductances of all of its parts plus the sum of the mutual inductances of each one of the component parts on every other part.

FIG. 15.—Self-inductance of a straight round wire at high frequencies.

Straight Round Wires.—The self-inductance of a round straight wire of length l , diameter d , and permeability μ is

$$L_0 = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 1 + \frac{\mu}{4} \right) \text{ microhenrys} \quad (14)$$

where L_0 is the low-frequency inductance and the dimensions are in inches. For wires of nonmagnetic materials such as copper, this becomes

$$L_0 = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 0.75 \right) \text{ microhenrys} \quad (15)$$

For such very short lengths of wire that $l < 100d$, a term $d/2l$ should be inserted within the parentheses above.

As frequency increases, the inductance of a straight round wire assumes a limiting value of

$$L = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 1 \right) \text{ microhenrys} \quad (16)$$

Limiting values of inductance for various wire lengths and diameters are shown in

FIG. 16.—Skin-effect correction factor δ as a function of wire diameter and frequency.

Fig. 15. For intermediate frequencies, the inductance is

$$L = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - 1 + \mu\delta \right) \text{ microhenrys} \quad (17)$$

where δ is a skin-effect factor that depends upon the same quantity x defined in Eq. (3).

$$x = \left(d \sqrt{\frac{\mu f}{\rho}} \right) \times \text{constant} \quad (18)$$

where f is the frequency, ρ is the volume resistivity, and μ the magnetic permeability. Values of δ for copper wire, as determined by d and f , may be obtained from the nomogram of Fig. 16. To obtain from this chart the value of δ for materials other than copper, an equivalent value of frequency, $f' = f\rho_c/\rho$ should be used instead of the actual value of frequency f , where ρ_c is the resistivity of copper. The factor δ approaches the value of 0.25 for low frequencies. For high frequencies, δ is inversely proportional to the square root of frequency, and approaches the value zero as the frequency becomes very large.

The changes of inductance with frequency are comparatively small. Skin effect at very high frequencies decreases inductance from about 6 per cent for short wires to about 2 per cent for long wires.

Return Circuit of Two Parallel Round Wires.—The self-inductance of a pair of parallel round wires carrying current in opposite directions, each having a length l , a diameter d , and spaced a distance D from one another (dimensions in inches), is

$$L = 0.01016l \left(2.303 \log_{10} \frac{2D}{d} - \frac{D}{l} + \mu\delta \right) \text{ microhenrys} \quad (19)$$

This neglects the inductance of the connecting link between the ends of the wires. As before, δ is a skin-effect factor that may be determined from Fig. 16.

Grounded Horizontal Round Wire.—The self-inductance of a wire of diameter d , length l , height above ground h , placed horizontally, with the earth used as a return circuit, is

$$L = 0.001170l \left[\log_{10} \left(\frac{l + \sqrt{l^2 + d^2/4}}{l + \sqrt{l^2 + 4h^2}} \right) + \log_{10} \frac{4h}{d} \right] + 0.00508 \left[\sqrt{l^2 + 4h^2} - \sqrt{l^2 + d^2/4} + \mu\delta - 2h + \frac{d}{2} \right] \text{ microhenrys} \quad (20)$$

The dimensions are in inches, and δ has the same meaning as above. In most applications the diameter is very small compared with the length of the wire, and the preceding expression may be simplified to the following forms:

For $2h/l \leq 1$:

$$L = 0.00508l \left(2.303 \log_{10} \frac{4h}{d} - P + \mu\delta \right) \text{ microhenrys} \quad (21)$$

For $l/2h \leq 1$:

$$L = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - Q + \mu\delta \right) \text{ microhenrys} \quad (22)$$

Here P and Q are functions of $2h/l$ and $l/2h$ respectively, which may be determined with sufficient accuracy from the values in Table 9.

Further simplification is possible when the length is much greater than the height. The inductance per foot for this case is

$$L = 0.1404 \log_{10} \frac{4h}{d} + 0.061 \mu\delta \text{ microhenrys per foot} \quad (22a)$$

Grounded Round Wires in Parallel.—The formulas given here are for n parallel wires evenly spaced in a plane. The resultant expression depends upon the self-inductance of each wire and the mutual inductances between the wires. Exact expressions for this case are quite complicated. The following are approximate expressions accurate to about 1 per cent.

$$L = l \left[\frac{L_1 + (n-1)M_1}{n} - 0.00254k \right] \text{ microhenrys} \quad (23)$$

where l is the length in inches, k is a function of the number of wires n , as tabulated in Table 10, L_1 is the inductance per inch of a single grounded wire, as determined from the Eqs. (21) and (22) in the previous section, and M_1 is the mutual inductance per unit length between two adjacent wires, as determined from the following:

For $2h/l \leq 1$:

$$M_1 = \frac{M}{l} = 0.00508 \left(2.303 \log_{10} \frac{2h}{D} - P + \frac{D}{l} \right) \quad (24)$$

For $l/2h \leq 1$:

$$M_1 = \frac{M}{l} = 0.00508 \left(2.303 \log_{10} \frac{2l}{D} - Q + \frac{D}{l} \right) \quad (25)$$

in which P and Q may be found from the values in Table 9.

TABLE 9.—VALUES OF P AND Q IN EQS. (21), (22), (24), AND (25)

$2h/l$	P	$l/2h$	Q
0.0	0.0000	0.0	1.0000
0.1	0.0975	0.1	1.0499
0.2	0.1900	0.2	1.0997
0.3	0.2778	0.3	1.1489
0.4	0.3608	0.4	1.1975
0.5	0.4393	0.5	1.2452
0.6	0.5136	0.6	1.2918
0.7	0.5840	0.7	1.3373
0.8	0.6507	0.8	1.3819
0.9	0.7139	0.9	1.4251
1.0	0.7740	1.0	1.4672

Straight Rectangular Bar.—The inductance at low frequencies of a straight rectangular bar of length l , width b , and thickness c inches is

$$L_0 = 0.00508l \left(2.303 \log_{10} \frac{2l}{b+c} + 0.5 + 0.2235 \frac{b+c}{l} \right) \quad (26)$$

The last term may be neglected for $l > 50(b+c)$.

TABLE 10.—VALUES OF k IN EQ. (23)

n	k
2	0
3	0.308
4	0.621
5	0.906
6	1.18
7	1.43
8	1.66
9	1.86
10	2.05
11	2.22
12	2.37
13	2.51
14	2.63
15	2.74
16	2.85
17	2.95
18	3.04
19	3.14
20	3.24

Inductance of Concentric Cable.—The low-frequency inductance is

$$L_0 = 0.140 \log_{10} \frac{r_2}{r_1} + 0.015 \text{ microhenrys per foot} \quad (27)$$

where r_1 is the radius of the outside of the inner conductor and r_2 is the radius of the inner side of the outer conductor.

Return Circuit of Two Parallel Rectangular Bars.—The low-frequency inductance of two parallel rectangular bars of length l , width b , breadth c , and center spacing D inches is

$$L_0 = 0.01016l \left(2.303 \log_{10} \frac{D}{b+c} + 1.5 - \frac{D}{l} + 0.2235 \frac{b+c}{l} \right) \text{ microhenrys} \quad (28)$$

The accuracy can be increased slightly by taking D as the geometrical mean distance between the wires.¹

9. Self-inductance of Single-turn Loops. Circular Ring of Circular Section. The inductance of a circular loop of diameter D of wire of diameter d inches is

$$L = 0.01595D \left(2.303 \log_{10} \frac{8D}{d} - 2 + \mu_s \right) \text{ microhenrys} \quad (29)$$

In Fig. 17 is given a nomogram from which the limiting value of inductance at high frequencies may be obtained for any loop and wire diameter. The nomogram may be extended by making use of the fact that inductance is proportional to linear dimension.

FIG. 17.—High-frequency inductance of a circular loop.

Circular Ring of Tubular Section.—The low-frequency inductance of a tube of inner diameter d_1 and outer diameter d_2 , bent into a circle of mean diameter D inches, is

$$L_0 = 0.01595D \left[2.303 \log_{10} \frac{8D}{d_2} - 1.75 - \frac{d_1^2}{2(d_2^2 - d_1^2)} + 2.303 \frac{d_1^4}{(d_2^2 - d_1^2)^2} \log_{10} \frac{d_2}{d_1} \right] \text{ microhenrys} \quad (30)$$

For infinite frequencies this assumes the value

$$L_\infty = 0.01595D \left(2.303 \log_{10} \frac{8D}{d_2} - 2 \right) \text{ microhenrys} \quad (31)$$

Square of Round Wire.—The inductance of a square of side s of wire of diameter d inches is

$$L = 0.02032s \left(2.303 \log_{10} \frac{2s}{d} + \frac{d}{2s} - 0.774 + \mu_s \right) \text{ microhenrys} \quad (32)$$

¹ For a discussion of geometrical mean distance, see E. B. Rosa and F. W. Grover, *Bur. Standards Bull.*, Vol. 8, p. 166, 1912. For parallel rectangular bars, the geometrical mean distance is slightly greater than the center distance. The difference is inappreciable when the cross-sectional dimensions are small compared to the center spacing.

Rectangle of Round Wire.—The inductance of a rectangle of sides s_1 and s_2 and diagonal g of wire of diameter d inches is

$$L = 0.02339 \left[(s_1 + s_2) \log_{10} \frac{4s_1s_2}{d} - s_1 \log_{10} (s_1 + g) - s_2 \log_{10} (s_2 + g) \right] + 0.01016 \left[\mu \delta (s_1 + s_2) + 2 \left(g + \frac{d}{2} \right) - 2(s_1 + s_2) \right] \text{ microhenrys} \quad (33)$$

Rectangle of Rectangular Wire.—The low-frequency inductance of a rectangle of sides s_1 and s_2 and diagonal g of rectangular wire of thickness b and width c , as shown in Fig. 18, with dimensions in inches, is

$$L_0 = 0.02339 \left[(s_1 + s_2) \log_{10} \frac{2s_1s_2}{b+c} - s_1 \log_{10} (s_1 + g) - s_2 \log_{10} (s_2 + g) \right] + 0.01016 \left[2g - \frac{s_1 + s_2}{2} + 0.447(b + c) \right] \text{ microhenrys} \quad (34)$$

Simplified Loop Formula for Regular Figures.—Examination of the foregoing loop-inductance equations shows them to be of the same general form regardless of the shape. If l is the total perimeter of the figure, then the inductance at high frequencies can be written as

$$L_\infty = 0.00508l \left(2.303 \log_{10} \frac{4l}{d} - \theta \right) \text{ microhenrys} \quad (35)$$

The dimensions are in inches. The quantity θ is a constant that depends upon the shape of the loop.¹ Values of θ for the commonest regular figures are given in Table 11.

The above formula is fairly accurate when the perimeter of the figure is large compared to the wire diameter. For ordinary loop shapes it is correct to within 0.5 per cent.

FIG. 18.—Rectangular loop of rectangular cross section.

TABLE 11.—VALUES OF θ FOR USE IN EQ. (35)

Circle.....	$\theta = 2.451$
Regular octagon.....	$\theta = 2.561$
Regular hexagon.....	$\theta = 2.636$
Regular pentagon.....	$\theta = 2.712$
Square.....	$\theta = 2.853$
Equilateral triangle.....	$\theta = 3.197$
Isosceles right-angled triangle.....	$\theta = 3.332$

10. Inductance of Single-layer Coils. *Single-layer Solenoid of Round Wire.*—The low-frequency inductance of a single-layer solenoid is given by¹

$$L_0 = Fn^2d \text{ microhenrys} \quad (36)$$

¹ For the development of the preceding expression, see V. I. Bashenoff, *Inductance of Oddly Shaped Loops*, *Proc. I.R.E.*, Vol. 15, p. 1013, December, 1927. For an extension of Bashenoff's work, see P. L. Kalantaroff and V. I. Worobieff, *Self Inductance of Polygonal Circuits*, *Proc. I.R.E.*, Vol. 24, p. 1585, December, 1936, which tabulates constants for 51 polygonal shapes, including many with reentrant angles.

² This is a rearrangement of the well-known Nagaoka formula, which is frequently written in the form $L_0 = \frac{2.54 \times 0.03948 \left(\frac{d}{2} \right)^2 n^2}{l} K$, where K is a constant depending upon the ratio of diameter d to

length l . Factor F in Eq. (36) is $\left(2.54 \times 0.03948 \times \frac{d}{4l} K \right)$, which is a function only of d/l . See E. B. Ross and F. W. Grover, *Bur. Standards Bull.*, Vol. 8, p. 119, 1912.

TABLE 12.—VALUES OF F FOR USE IN EQ. (36)

Diameter Length	F	Difference	Diameter Length	F	Difference
0.00	0.0000000	+0.0004972	3.00	0.03228	+0.00049
0.02	0.0004972	+0.0004888	3.10	0.03277	+0.00048
0.04	0.000986	+0.000481	3.20	0.03325	+0.00047
0.06	0.001467	+0.000472	3.30	0.03372	+0.00045
0.08	0.001939	+0.000465	3.40	0.03417	+0.00044
0.10	0.002404	+0.000457	3.50	0.03461	+0.00042
0.12	0.002861	+0.000449	3.60	0.03503	+0.00042
0.14	0.003310	+0.000442	3.70	0.03545	+0.00041
0.16	0.003752	+0.000434	3.80	0.03586	+0.00040
0.18	0.004186	+0.000428	3.90	0.03626	+0.00039
0.20	0.004614	+0.000420	4.00	0.03665	+0.00038
0.22	0.005034	+0.000413	4.10	0.03703	+0.00037
0.24	0.005447	+0.000407	4.20	0.03740	+0.00036
0.26	0.005854	+0.000400	4.30	0.03776	+0.00035
0.28	0.006254	+0.000393	4.40	0.03811	+0.00035
0.30	0.006647	+0.000387	4.50	0.03846	+0.00034
0.32	0.007034	+0.000381	4.60	0.03880	+0.00033
0.34	0.007415	+0.000375	4.70	0.03913	+0.00033
0.36	0.007790	+0.000369	4.80	0.03946	+0.00032
0.38	0.008159	+0.000363	4.90	0.03978	+0.00031
0.40	0.008522	+0.000358	5.00	0.04009	+0.00031
0.42	0.008883	+0.000352	5.20	0.04070	+0.00038
0.44	0.009232	+0.000346	5.40	0.04128	+0.00037
0.46	0.009578	+0.000341	5.60	0.04185	+0.00035
0.48	0.009919	+0.000336	5.80	0.04240	+0.00033
0.50	0.01026	+0.000331	6.00	0.04293	+0.00031
0.55	0.01107	+0.000320	6.20	0.04344	+0.00030
0.60	0.01186	+0.000308	6.40	0.04394	+0.00029
0.65	0.01262	+0.000297	6.60	0.04443	+0.00028
0.70	0.01335	+0.000286	6.80	0.04489	+0.00027
0.75	0.01406	+0.000275	7.00	0.04534	+0.00026
0.80	0.01474	+0.000264	7.20	0.04578	+0.00025
0.85	0.01540	+0.000253	7.40	0.04621	+0.00024
0.90	0.01604	+0.000242	7.60	0.04664	+0.00023
0.95	0.01666	+0.000231	7.80	0.04705	+0.00022
1.00	0.01726	+0.000220	8.00	0.04745	+0.00021
1.05	0.01784	+0.000209	8.50	0.04840	+0.00021
1.10	0.01840	+0.000198	9.00	0.04931	+0.00020
1.15	0.01895	+0.000187	9.50	0.05016	+0.00019
1.20	0.01948	+0.000176	10.00	0.05097	+0.00018
1.25	0.02000	+0.000165	10.0	0.05097	+0.00151
1.30	0.02050	+0.000154	11.0	0.05248	+0.00138
1.35	0.02099	+0.000143	12.0	0.05386	+0.00128
1.40	0.02147	+0.000132	13.0	0.05514	+0.00119
1.45	0.02193	+0.000121	14.0	0.05632	+0.00110
1.50	0.02238	+0.000110	15.0	0.05743	+0.00102
1.55	0.02282	+0.000099	16.0	0.05845	+0.00095
1.60	0.02325	+0.000088	17.0	0.05940	+0.00090
1.65	0.02367	+0.000077	18.0	0.06030	+0.00086
1.70	0.02408	+0.000066	19.0	0.06116	+0.00081
1.75	0.02448	+0.000055	20.0	0.06197	+0.00151
1.80	0.02487	+0.000044	22.0	0.06348	+0.00138
1.85	0.02525	+0.000033	24.0	0.06486	+0.00127
1.90	0.02562	+0.000022	26.0	0.06613	+0.00119
1.95	0.02599	+0.000011	28.0	0.06732	+0.00112
2.00	0.02635	+0.000000	30.0	0.0684	+0.0024
2.10	0.02704	+0.000067	35.0	0.0708	+0.0022
2.20	0.02771	+0.000065	40.0	0.0730	+0.0020
2.30	0.02836	+0.000062	45.0	0.0750	+0.0016
2.40	0.02898	+0.000059	50.0	0.0766	+0.0028
2.50	0.02957	+0.000058	60.0	0.0794	+0.0025
2.60	0.03015	+0.000056	70.0	0.0819	+0.0022
2.70	0.03071	+0.000054	80.0	0.0841	+0.0019
2.80	0.03125	+0.000052	90.0	0.0850	+0.0017
2.90	0.03177	+0.000051	100.0	0.0877	

where n is the number of turns, F a quantity that depends upon the ratio of the diameter to the length of the coil, and d is the diameter of the coil in inches. The quantity F is tabulated for a wide range of values of the ratio of (diameter/length) in Table 12 and plotted in Fig. 19.

Where great accuracy is required, a correction factor may be applied to Eq. (36) to take account of the fact that the coil is wound of spaced round wires rather than with a uniform current sheet.¹ This correction rarely exceeds 0.5 per cent, is greatest for widely spaced turns, and increases with the number of turns.

FIG. 19.—Factor F for single-layer solenoid in Eq. (36).

A simple approximate formula for the low-frequency inductance of a single-layer solenoid is²

$$L_0 = \frac{r^2 n^2}{9r + 10l} \text{ microhenrys} \quad (37)$$

where r is the radius of the coil and l its length in inches. This formula is accurate to within one per cent for $l > 0.8r$, i.e., if the coil is not too short.

Single-layer Solenoid of Wire of Rectangular Cross Section.—The low-frequency inductance is most conveniently given by

$$L_0 = F n^2 d - \frac{0.01596 n^2 d c}{l} (0.693 + B_s) \quad (38)$$

where n = number of turns of the winding.

d = diameter of the coil to center of winding, in.

l = length of the coil, in.

c = radial thickness of the wire strip, in.

F = function of $\frac{\text{diameter}}{\text{length}}$ given in Table 12 and Fig. 19.

B_s = function of $\frac{\text{length}}{\text{radial thickness } c \text{ of wire}}$ given in Table 13.

¹ Radio Instruments and Measurements, *Bur. Standards Circ. 74*, p. 253.

² H. A. Wheeler, Simple Inductance Formulas for Radio Coils, *Proc. I.R.E.*, Vol 16, p. 1398, October, 1928.

It will be recognized that the first term gives the inductance of the equivalent solenoid of round wire and that the second is a correction for the fact that the wires are not round.

TABLE 13.—VALUES OF B_s FOR USE IN EQ. (38)

l/c	B_s	l/c	B_s	l/c	B_s
1	0.0000	11	0.2844	21	0.3116
2	0.1202	12	0.2888	22	0.3131
3	0.1753	13	0.2927	23	0.3145
4	0.2076	14	0.2961	24	0.3157
5	0.2292	15	0.2991	25	0.3169
6	0.2446	16	0.3017	26	0.3180
7	0.2563	17	0.3041	27	0.3190
8	0.2656	18	0.3062	28	0.3200
9	0.2730	19	0.3082	29	0.3209
10	0.2792	20	0.3099	30	0.3218

FIG. 20.—Single-layer rectangular coil.

The use of this equivalent diameter generally gives results accurate to within one per cent. The results become more accurate as the number of sides of the polygon increases.

Single-layer Rectangular Coil.—The low-frequency inductance is

$$L_0 = pn^2(G + H) \text{ microhenrys} \quad (40)$$

where p = perimeter, $2(s_1 + s_2)$.

s_1 = length of short side, in.

s_2 = length of long side, in.

b = axial length of the coil, in.

n = number of turns.

G = factor determined by s_1/s_2 and b/s_2 given in Fig. 21.

H = factor determined by n and $\left\{ \begin{array}{l} \text{diameter of wire} \\ \text{spacing between} \\ \text{adjacent turns} \end{array} \right\}$ and given in Fig. 21.

The foregoing expression is accurate only when the distance between turns is small compared to the short side of the coil.

Single-layer Square Coil.—This is conveniently obtained by setting $s_1 = s_2$ in the formula, Eq. (40), of the previous section.

Winding Very Long Compared with Smallest Cross-sectional Dimension.—The low-frequency inductance of such a winding, irrespective of the shape of the cross section, is

$$L_0 = 0.0319 \frac{An^2}{l} \text{ microhenrys} \quad (41)$$

where A is the area of the cross section in square inches, l is the length of the coil in inches, and n is the number of turns.

Toroidal Coil of Circular Cross Section.—The low-frequency inductance of a toroidal coil of n turns with a winding of circular cross section of diameter d and a

FIG. 21.—Factors G and H for single-layer rectangular coil, used in Eq. (40). radius of revolution $D/2$ inches, as shown in Fig. 22, is

$$L_0 = 0.01595n^2[D - (D^2 - d^2)^{1/2}] \text{ microhenrys} \quad (42)$$

For toroidal coils in which the turn diameter is much less than the toroid diameter (i.e., $d < 0.1D$), the inductance is approximately

$$L_0 = 0.007975 \frac{d^2 n^2}{D} \text{ microhenrys} \quad (43)$$

Toroidal Coil of Rectangular Cross Section.—When the axial depth of the winding is h and the inner and outer diameters d_1 and d_2 inches, respectively, as shown in Fig. 23, the low-frequency inductance is

$$L_0 = 0.01170 n^2 h \log_{10} \frac{d_2}{d_1} \text{ microhenrys} \quad (44)$$

Flat Spiral.—The low-frequency inductance of a

FIG. 22.—Toroidal coil of circular cross section showing notation of Eq. (42).

FIG. 23.—Toroidal coil of rectangular cross section illustrating notation of Eq. (44).

flat spiral of round wire of mean radius a and radial depth of winding c inches, as shown in Fig. 24, is conveniently expressed by

$$L_0 = a n^2 K \text{ microhenrys} \quad (45)$$

in which n is the number of turns and K is a quantity depending upon a/c as given in Fig. 24.

FIG. 24.—Factor K for flat spiral coil used in Eq. (45).

An approximate formula for this type of inductance is¹

$$L_0 = \frac{a^2 n^2}{8a + 11c} \text{ microhenrys} \quad (46)$$

This formula is accurate to within 5 per cent for $c > 0.2a$.

¹ See Wheeler, *loc. cit.*

The inductance of a flat spiral of *flat ribbon* is most readily found by using Eq. (51) for a short solenoidal multilayer coil. This gives results that are, in general, small by less than one per cent.

TABLE 14.—VALUES OF CORRECTION TERM A IN EQ. (48)

d/D	A	Difference	d/D	A	Difference
1.00	0.557	-0.051	0.20	-1.053	-0.051
0.95	0.506	-0.054	0.19	-1.104	-0.054
0.90	0.452	-0.057	0.18	-1.158	-0.057
0.85	0.394	-0.061	0.17	-1.215	-0.061
0.80	0.334	-0.065	0.16	-1.276	-0.064
0.75	0.269	-0.069	0.15	-1.340	-0.069
0.70	0.200	-0.074	0.14	-1.409	-0.074
0.65	0.126	-0.080	0.13	-1.483	-0.080
0.60	0.046	-0.087	0.12	-1.563	-0.087
0.55	-0.041	-0.095	0.11	-1.650	-0.096
0.50	-0.136	-0.041	0.10	-1.746	-0.105
0.48	-0.177	-0.043	0.09	-1.851	-0.118
0.46	-0.220	-0.044	0.08	-1.969	-0.133
0.44	-0.264	-0.047	0.07	-2.102	-0.154
0.42	-0.311	-0.048	0.06	-2.256	-0.173
0.40	-0.359	-0.052	0.05	-2.439	-0.223
0.38	-0.411	-0.054	0.04	-2.662	-0.288
0.36	-0.465	-0.057	0.03	-2.950	-0.405
0.34	-0.522	-0.061	0.02	-3.355	-0.693
0.32	-0.583	-0.064	0.01	-4.048	
0.30	-0.647	-0.069			
0.28	-0.716	-0.074			
0.26	-0.790	-0.080			
0.24	-0.870	-0.087			
0.22	-0.957	-0.096			

Flat Rectangular Coil.—The low-frequency inductance of a flat rectangular coil whose average dimensions are s_1 and s_2 with an average diagonal $g = \sqrt{s_1^2 + s_2^2}$ wound of n complete turns of wire of diameter d with a pitch of winding D , as shown in Fig. 25, is

$$\begin{aligned}
 L_0 = 0.02339n^2 & \left[(s_1 + s_2) \log_{10} \frac{2s_1s_2}{nD} - s_1 \log_{10} (s_1 + g) - s_2 \log_{10} (s_2 + g) \right] \\
 & + 0.01016n^2 \left(2g - \frac{s_1 + s_2}{2} + 0.447nD \right) \\
 & - 0.01016n(s_1 + s_2)(A + B) \text{ microhenrys}
 \end{aligned} \tag{47}$$

The dimensions are in inches. In the preceding, A and B are constants depending upon the wire spacing and number of turns, respectively. Values of A corresponding to various values of d/D are given in Table 14. Values of B corresponding to various values of n are given in Table 15.

Flat Square Coil.—If $s = s_1 = s_2$ are set in the previous section, the expression for inductance reduces to

$$\begin{aligned}
 L_0 = 0.02032n^2s & \left[2.303 \log_{10} \frac{s}{nD} + 0.2235 \frac{nD}{s} + 0.726 \right] \\
 & - 0.02032ns(A + B) \text{ microhenrys}
 \end{aligned} \tag{48}$$

TABLE 15.—VALUES OF CORRECTION TERM B IN Eq. (48)

Number of turns, n	B	Number of turns, n	B	Number of turns, n	B
1	0.000	15	0.286	80	0.326
2	0.114	20	0.297	90	0.327
3	0.166	25	0.304	100	0.328
4	0.197	30	0.308	150	0.331
5	0.218	35	0.312	200	0.333
6	0.233	40	0.315	300	0.334
7	0.244	45	0.317	400	0.335
8	0.253	50	0.319	500	0.336
9	0.260	60	0.322	700	0.336
10	0.266	70	0.324	1,000	0.336

where s is the average length of side, n is the number of turns, d is the diameter of the wire, D is the distance between turns, and A and B are constants that may be determined from Tables 14 and 15.

FIG. 25.—Flat rectangular coil.

FIG. 26.—Long multiple-layer coil of solenoidal form.

11. Inductance of Multiple-layer Coils. *Long Solenoidal Form.*—The low-frequency inductance of a long multiple-layer coil of solenoidal form, as shown in Fig. 26, where b and c are the axial length and radial thickness of the winding, both measured to the outside of the wires, and a is the mean radius, is given by

$$L_0 = Fn^2d - \frac{0.03193n^2ac}{b} (0.693 + B_s) \text{ microhenrys} \quad (49)$$

The dimensions are in inches. It will be recognized that the first term of this expression is the equation for the inductance of a single-layer solenoid given in Eq. (36), F being the factor given in Table 12. The second term is a correction for the thickness of the winding with the quantity B , the same quantity given in Table 13, except that b/c is substituted for l/c in the table. The foregoing expression is generally accurate to within 0.5 per cent.

If the insulation thickness is an appreciable fraction of the wire thickness, the following correction should be added:

$$\Delta L = 0.03193an \left(2.303 \log_{10} \frac{D}{d} + 0.155 \right) \text{ microhenrys} \quad (50)$$

where D is the distance between wire centers and d is the diameter of the bare wire in inches. This incremental correction increases as the insulation thickness increases.

FIG. 27.—Short multiple-layer coil of solenoidal form.

Short Solenoidal Form.—The inductance of the multiple layer coil of the form shown in Fig. 27, in which b and c are the axial length and radial thickness of the winding measured to the outside of the winding, d the diagonal of the winding cross section, and a the mean radius, may be expressed in several ways. If n is the number

FIG. 28.—Factor I for use in Eq. (51) applying to multiple-layer coil of solenoidal form.

of turns, l the total length of the wire, and D the distance between centers of adjacent wires, then the low-frequency inductance may be expressed by (dimensions in inches)

$$L_0 = \frac{l^2}{D^2} I \text{ microhenrys} \quad (51)$$

or

$$L_0 = an^2 J \text{ microhenrys} \quad (52)$$

where I and J are the factors given in Figs. 28 and 29, respectively, in terms of c/a and b/c . It will be observed that the maximum inductance is obtained from a given length of wire when the cross section of the winding is square and the side of the cross section is 0.662 times the mean coil radius.

Where greater accuracy is desired than may be obtained from the curves of Figs. 28 and 29, the following formulas may be used:

For $b > c$:

$$L_0 = 0.03193an^2 \left[2.303 \left(1 + \frac{b^2}{32a^2} + \frac{d^2}{96a^2} \right) \log_{10} \frac{8a}{d} - y_1 + \frac{b^2}{16a^2} y_2 \right] \quad (53)$$

where y_1 and y_2 are constants depending upon b/c and given in Table 16 below.

For $b < c$:

$$L_0 = 0.03193an^2 \left[2.303 \left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2} \right) \log_{10} \frac{8a}{d} - y_1 + \frac{c^2}{16a^2} y_2 \right] \text{ microhenrys } (54)$$

where y_1 and y_2 are constants depending upon b/c , as given in Table 16 below. The foregoing expressions are accurate to within 0.1 per cent. When the insulation

FIG. 29.—Factor J for use in Eq. (52) applying to multiple-layer coil of solenoidal form.

thickness of the wire is an appreciable fraction of the wire diameter, the following correction should be added:

$$\Delta L = +0.03193an \left(2.303 \log \frac{D}{d} + 0.155 \right) \text{ microhenrys } (55)$$

where D is the distance between adjacent wire centers and d is the diameter of the bare wire in inches.

A simple *approximate* formula for a short multiple-layer inductance of solenoidal form¹ is

$$L_0 = \frac{0.8a^2n^2}{6a + 9b + 10c} \text{ microhenrys } (56)$$

¹ See Wheeler, *loc. cit.*

where the symbols have the significance indicated in Fig. 27 and the dimensions are in inches. This formula is accurate to within one per cent when the terms of the denominator are of about the same size, i.e., for a coil of about the shape shown in Fig. 27.

Circular Coil of Circular Cross Section.—The low-frequency inductance of such a coil is

$$L_0 = 0.01595 D n^2 \left(2.303 \log \frac{8D}{d} - 1.75 \right) \text{ microhenrys} \quad (57)$$

TABLE 16.—VALUES OF y_1 , y_2 , AND y_3 FOR USE IN Eqs. (53) AND (54)

b/c or c/b	y_1	Difference	c/b	y_2	Difference	b/c	y_3	Difference
0	0.5000	0.0253	0	0.125	0.002	0	0.597	0.002
0.025	0.5253	0.0237						
0.05	0.5490	0.0434	0.05	0.127	0.005	0.05	0.599	0.003
0.10	0.5924	0.0386	0.10	0.132	0.010	0.10	0.602	0.006
0.15	0.6310	0.0342	0.15	0.142	0.013	0.15	0.608	0.007
0.20	0.6652	0.0301	0.20	0.155	0.016	0.20	0.615	0.009
0.25	0.6953	0.0266	0.25	0.171	0.020	0.25	0.624	0.009
0.30	0.7217	0.0230	0.30	0.192	0.023	0.30	0.633	0.010
0.35	0.7447	0.0198	0.35	0.215	0.027	0.35	0.643	0.011
0.40	0.7645	0.0171	0.40	0.242	0.031	0.40	0.654	0.011
0.45	0.7816	0.0144	0.45	0.273	0.034	0.45	0.665	0.012
0.50	0.7960	0.0121	0.50	0.307	0.037	0.50	0.667	0.013
0.55	0.8081	0.0101	0.55	0.344	0.040	0.55	0.690	0.012
0.60	0.8182	0.0083	0.60	0.384	0.043	0.60	0.702	0.013
0.65	0.8265	0.0066	0.65	0.427	0.047	0.65	0.715	0.014
0.70	0.8331	0.0052	0.70	0.474	0.049	0.70	0.729	0.013
0.75	0.8383	0.0039	0.75	0.523	0.053	0.75	0.742	0.014
0.80	0.8422	0.0029	0.80	0.576	0.056	0.80	0.756	0.015
0.85	0.8451	0.0019	0.85	0.632	0.059	0.85	0.771	0.015
0.90	0.8470	0.0010	0.90	0.690	0.062	0.90	0.786	0.015
0.95	0.8480	0.0003	0.95	0.752	0.064	0.95	0.801	0.015
1.00	0.8483		1.00	0.816		1.00	0.816	

where D is the mean diameter of the turns and d is the diameter of the circular winding cross section in inches. This will be recognized as being simply π^2 times the expression for a single turn of wire of diameter d . The expression neglects the space occupied by the insulation between wires.

Rectangular Coil of Rectangular Cross Section.—The low frequency inductance may be represented by

$$L_0 = p n^2 G \text{ microhenrys} \quad (58)$$

where p is to be defined as being the perimeter of the rectangle in inches, n is the number of turns, and G is a quantity depending upon the ratio s_1/s_2 of the length of the short side to the length of the long side, as given in Fig. 21. The quantity b/s_2 appearing in this figure is taken to be the circumference of the winding cross section divided by twice the length of the longer side (i.e., for b use $b = \text{circumference}/2s_2$).

Where greater accuracy is desired than may be obtained from the preceding formula, the following equation should be used. As before, let s_1 and s_2 be the short and long sides of the rectangle, respectively. Let t and w be the thickness and width of the cross section of the winding and g the diagonal of the coil in inches. Let n represent the number of turns. Then

$$L_0 = 0.02339(s_1 + s_2)n^2 \left[\log_{10} \frac{2s_1s_2}{t+w} - \frac{s_1}{s_1+s_2} \log_{10}(s_1+g) - \frac{s_2}{s_1+s_2} \log_{10}(s_2+g) \right] \\ + 0.01016(s_1 + s_2)n^2 \left[2 \left(\frac{g}{s_1+s_2} \right) - \frac{1}{2} + 0.447 \frac{(t+w)}{(s_1+s_2)} \right] \text{ microhenrys} \quad (59)$$

If the insulation thickness is an appreciable fraction of the wire diameter, the correction of Eq. (55) should be added.

Square Coil of Rectangular Cross Section.—The inductance of a square coil may be obtained by setting $s_1 = s_2 = s$ in Eqs. (58) and (59). With this substitution, Eq. (59) simplifies to

$$L_0 = 0.04678sn^2 \left[\log_{10} \frac{2s^2}{t+w} - \log_{10} 2.414s \right] \\ + 0.02032sn^2 \left[0.914 + 0.2235 \frac{t+w}{s} \right] \text{ microhenrys} \quad (60)$$

12. Mutual Inductance and Coefficient of Coupling.—When the relative position of two inductances is such that lines of flux from one inductance link with turns of the other, the two inductances are said to be inductively coupled, and mutual inductance exists between them. Mutual inductance may be defined in terms of the number of flux linkages in the second coil per unit current in the first coil, or vice versa. In practical units, the relation is

$$M = \frac{\left\{ \begin{array}{l} \text{flux linkages in second coil} \\ \text{produced by current in first coil} \end{array} \right\}}{\text{current in first coil}} 10^{-8} \quad (61a)$$

$$= \frac{\left\{ \begin{array}{l} \text{flux linkages in first coil} \\ \text{produced by current in second coil} \end{array} \right\}}{\text{current in second coil}} 10^{-8} \quad (61b)$$

where M is the mutual inductance in henrys.

Mutual inductance may also be defined as the voltage induced in the second circuit when the current in the first circuit is changing at a unit rate. If the current flowing in the first circuit is sinusoidal, then the voltage induced in the second circuit is

$$E_2 = -j\omega MI_1. \quad (62)$$

The negative of the ratio of the voltage induced in the second circuit to the current in the first circuit is called the *mutual impedance*. That is

$$Z_m = j\omega M = \frac{-E_2}{I_1} \quad (63)$$

where Z_m is the mutual impedance and the quantity $-j$ indicates that the induced voltage lags the current by 90° . The concept of mutual impedance can be generalized to give the mutual impedance between any two circuits coupled in any fashion no matter how complex. The generalized mutual impedance is the vector ratio $-E_2/I_1$, where E_2 is the voltage appearing across the opened terminals of the second circuit and I_1 is the current flowing in the first.

Coefficient of Coupling.—The maximum value of mutual inductance that can exist between two coils of inductance L_1 and L_2 is $\sqrt{L_1L_2}$, which occurs when all the flux of one coil links with all the turns of the other. The ratio of the mutual inductance actually present to the maximum possible value that can occur is called the *coefficient of coupling* and is written as

$$k = \frac{M}{\sqrt{L_1L_2}} \quad (64)$$

The coefficient of coupling k is a dimensionless quantity having a maximum value of 1.

Coaxial Concentric Solenoids—Outer Coil the Shorter.—When the coils have the configuration shown in Fig. 44, mutual inductance is again given by Eq. (86), provided g is defined as indicated in Fig. 44 and all dimensions are in inches.

Discussion of Formulas.—In the preceding formulas for the mutual inductance between solenoids, the expressions involve differences between quantities of nearly the same size. To obtain a high degree of accuracy it is hence necessary to carry out the calculations with a considerable number of significant figures. Other formulas are also available for calculating the mutual inductance of solenoids in which the calculations are not so exacting but involve a greater number of terms.¹ Where great accuracy is not demanded and many calculations are to be made, time can be saved by using a combined graphical and computational method. Formulas are also available for other cases of mutual inductance between solenoids than those given here.² These are invariably quite complex, involving elliptic integrals or series expansions with spherical harmonics.

18. Mutual Inductance between Multilayer Coils. Two Coaxial Circular Coils of Small Rectangular Cross

Coaxial Concentric Solenoids—Outer Coil the Shorter

FIG. 44.—Coaxial concentric solenoids, outer coil the shorter.

FIG. 45.—Two coaxial circular coils of small rectangular cross section.

Section.—For coils having the approximate configuration shown in Fig. 45, the mutual inductance is given by

$$M = n_1 n_2 M_0 \text{ microhenrys} \quad (88)$$

where n_1 and n_2 are the number of turns of the two coils and M_0 is the mutual inductance of the coaxial circles located at the centers of the cross sections of the two windings. Thus if a is the mean radius in inches of the small coil and A is the mean radius of the large coil and D the distance between their planes, the value of M_0 is that computed by Eq. (77) and Table 17 for the two coaxial circles thus defined. This formula is best suited for coils with square cross section and gives results in error by less than 0.5 per cent, even with relatively large cross-sectional dimensions, except when the coils are close together.

Two Coaxial Circular Coils of Rectangular Cross Section.—For coils whose cross-sectional dimensions are relatively large, compared to the coil diameters, as shown in Fig. 46, the mutual inductance is given by

$$M = \frac{n_1 n_2}{6} \left[(M_{1'2'} + M_{1'3'} + M_{1'4'} + M_{1'5'}) \right. \\ \left. + (M_{1'2'} + M_{1'3'} + M_{1'4'} + M_{1'5'}) - 2M_{1'1'} \right] \text{ microhenrys} \quad (89)$$

¹ H. B. Dwight and F. W. Grover, Some Series Formulas for Mutual Inductance of Solenoids, *Elec. Eng.*, Vol. 56, p. 347, March, 1937; F. W. Grover, Tables for the Calculation of the Mutual Inductance of Any Two Coaxial Single-layer Coils, *Proc. I.R.E.*, Vol. 21, p. 1039, July, 1933.

² F. W. Grover, Mutual Inductance of Coils not Coaxial, *Communications*, Vol. 18, p. 10, August, 1938. See also Chester Snow, Mutual Inductance of Concentric Solenoids, *Bur. Standards Jour. Research*, Vol. 1, p. 531, 1928.

where the various terms represent the mutual inductance between the coaxial circles corresponding to the subscripts as indicated in Fig. 46. The separate terms are calculated by Eq. (77) and Table 17.¹

FIG. 46.—Two coaxial circular coils of rectangular cross section.

FIG. 47.—Two identical coaxial coils.

Two Identical Coaxial Coils.—When the coils are identical and of rectangular cross section, as shown in Fig. 47, the mutual inductance is given by

$$M = \frac{n^2}{3} (M_{02} + M_{03} + M_{04} + M_{05} - M_{01}) \text{ microhenrys} \quad (90)$$

where n is the number of turns per coil and the separate terms are the mutual inductances between the coaxial circles corresponding to the subscripts, as indicated in the figure. As before, the separate terms are calculated by Eq. (77) and Table 17.

Two Coils of Equal Diameter.—When the coils are of equal diameter and radial depth but of different lengths, as shown in Fig. 48, the mutual inductance is given by

$$M = \frac{n_1 n_2}{6} (2M_{11'} + M_{11'} + M_{11'} + M_{11'} + M_{11'}) \text{ microhenrys} \quad (91)$$

where n_1 and n_2 are the number of turns of the two coils and the M terms are the mutual inductances between the coaxial circles corresponding to the subscripts as indicated in the figure as calculated by Eq. (77) and Table 17.

COILS WITH AIR CORES

17. Types of Windings Used in Air-cored Coils.—Most air-cored coils are single or multilayer arrangements wound on cylindrical forms. Other types occasionally used are toroids, flat spirals (or disks), and single or multilayer windings on square, hexagonal, etc., forms.

Single-layer coils are used to provide inductances for resonant circuits at broadcast and higher frequencies and for radio-frequency choke coils at very high frequencies. Multilayer coils are used when a large inductance is to be obtained in a small space. Such coils are employed in resonant circuits at broadcast and lower frequencies, as radio-frequency choke coils and as inductance standards for audio frequencies.

¹ Hak, op. cit., p. 61, 1938.

FIG. 48.—Two coaxial coils of equal diameter.

18. Losses in Air-cored Coils at Radio Frequencies. Method of Expressing Losses.—Coil losses can be expressed in terms of the equivalent series resistance R , in terms of power factor of the coil, or in terms of the ratio $\omega L/R = Q$. The coil power factor and Q are related by the equation

$$\text{Power factor} = \frac{1}{\sqrt{Q^2 + 1}} \quad (92)$$

When $Q > 5$, the power factor can be taken as $1/Q$.

Factors Contributing to Coil Losses.—The equivalent series resistance of a coil is greater than the d-c resistance at all but the very lowest frequencies, as a result of:¹ (1) skin effect in the conductor that modifies the current distribution over the cross section of the individual conductor; (2) proximity effect with respect to adjacent conductors in the coil, which produces an additional modification in the current distribution; (3) losses in dielectrics in the vicinity of the coil, particularly in the coil form; (4) eddy-current losses in neighboring metal objects. These effects tend to increase with frequency, with the result that the ratio $\omega L/R = Q$ varies only slowly with variations in frequency.

Q of Single-layer Coils.—The Q of a single-layer coil depends upon the frequency, size, and shape of the coil, and such constructional details as the number of turns, size, and character of the wire, etc. With a given coil, the Q , as observed experimentally, varies somewhat with frequency and goes through a broad maximum at a frequency that is usually either in or near the working frequency range of the coil. With a given inductance and coil diameter, the Q is maximum when the ratio (length)/(diameter) is of the order of $1/2:1$, although the relation is not at all critical. With everything fixed except the size of wire, it is usually found that maximum Q is obtained with wire somewhat smaller than the largest that could be used in the available space. When the size of a coil is increased while maintaining the inductance and shape constant, the Q will be roughly proportional to square root of the coil diameter, provided the wire diameter is always maintained at the optimum value.

Edgewise-wound strip and ribbon conductors are sometimes used in large single-layer and pancake coils, respectively. Such conductors can be expected to give lower Q than solid wire of the same surface area, but if the dimensions are large and the strip or ribbon of ample cross section, the losses will still be low.

Litz wire gives a higher Q than solid wire at the lower radio frequencies, while at very high frequencies solid wire is as good as or better than litz. The frequency at which litz loses its advantage depends upon circumstances, but is normally of the order of 1,000 to 2,000 kc. The proper size of litz wire to use depends upon the frequency and coil construction, and, in general, does not have the same cross section as does the corresponding solid wire of optimum proportions. The effect of broken or insulated strands in a coil wound with litz wire is far less than the loss in d-c conductivity.

The form, binder, etc., used with single-layer coils have relatively little effect upon the losses, except possibly at very high frequencies, provided the dielectrics involved do not absorb moisture. Thus bakelite, polystyrene, isolantite, etc., are quite satisfactory, whereas untreated paper or cardboard is not. Experiments indicate that there is no unusual advantage, even at very high frequencies, of using types of construction such as "basket-weave" coils, in which the amount of dielectric associated with the coil is reduced to the barest minimum.

¹ Considerable attention is sometimes paid to coil proportions that have the lowest possible d-c resistance. Such considerations are, however, of little significance at radio frequencies because of skin and proximity effect. Their only usefulness is in connection with inductances for low frequencies, such as standard inductances for audio-frequency bridge work.

FIG. 49.—Values of circuit Q for single-layer coils designed for different frequency ranges, as reported by various investigators. All of these data were measured on the assumption that the tuning condenser had zero losses (i.e., the condenser losses are charged against the coil).

Experimental data giving Q as a function of frequency for a variety of single-layer coils are given in Fig. 49. These data have been obtained from various sources, and are representative of what can be expected.^{1,2}

Q of Multilayer Coils.—The losses in a multilayer coil depend upon the frequency, size, and shape of the coil, the arrangement of the winding, the size and kind of wire, etc., just as in the single-layer coil. With a given coil, the Q varies only slowly with frequency and goes through a broad maximum at a frequency that is usually in or near the working range of the coil. The Q increases with the size of the coil providing the optimum wire size and arrangement is maintained, but the effect of size is not as

Actual Size of Coil	Litz Wire	Q
3/40 SS	74
	3/40 SS	79
7/41 E	87
	7/41 SS	87
7/41 SS	91
	5/38 SS	48
10/41 SS	87
	7/40 SS	74
10/40 SS	68
	5/40 SS	90
10/38 SS	64

FIG. 50.—Characteristics of typical multilayer coils such as used at intermediate frequencies.

great as might be expected, and is frequently overshadowed by other factors. The exact shape and arrangement of the winding space is not critical, and satisfactory multilayer coils can be built in such widely different arrangements as a thin winding having two or three layers of many turns, and a disk or pancake winding having many layers, each of only a few turns. Good efficiency will be obtained provided the proportions do not depart too greatly from the relation $3t + 2b = D$, where t is the radial thickness of the winding, b the axial length, and D the outer diameter.

¹ Values given by the curves involve in nearly every case the assumption that the condenser used to tune the coil to resonance when the measurements were being made had zero losses. As a result, the true coil losses are probably somewhat less than those presented in the figures, particularly toward the high-frequency end of the useful range of the coil.

² D. Pollack, The Design of Inductances for Frequencies between 4 and 25 Megacycles, *Elec. Eng.*, Vol. 56, p. 1160, September, 1937; D. Grimes and W. Barden, Coil Design for Short Wave Receivers, *Electronics*, Vol. 7, p. 174, June, 1934; F. E. Terman, "Radio Engineering," 2d ed., p. 38; A. J. Palermo and F. W. Grover, Study of the High-frequency Resistance of Single Layer Coils, *Proc. I.R.E.*, Vol. 18, p. 2041, December, 1930; E. L. Hall, Resistance of Conductors of Various Types and Sizes as Windings of Single-layer Coils at 150 to 6,000 Kilocycles, *Bur. Standards Tech. Paper* 330, October, 1926; August Hund and H. B. DeGroot, Radio-frequency Resistance and Inductance of Coil used in Radio Reception, *Bur. Standards Tech. Paper* 298, October, 1925.

The optimum size of wire ordinarily has a wire diameter that gives a cross section of copper considerably less than the cross section of the winding space, corresponding to a not too closely packed winding. Litz gives a higher Q than solid conductors at broadcast and lower frequencies, which are the frequencies where multilayer coils find their principal use. The size and number of the strands in litz for best results depends upon the frequency, physical size of the coil, etc., and ranges from three strands of No. 40 or five strands of No. 41 wire for very compact coils intended for broadcast and intermediate frequencies, to proportionately larger and more numerous strands as the physical size of the coil is increased.

Data on typical multilayer coils of the type used in radio receivers are shown in Fig. 50, and indicate the type of behavior that can be expected.¹

Q of Toroidal Coils.—The Q 's obtained with toroidal coils are always appreciably lower than for ordinary single-layer or multilayer coils of corresponding physical dimensions. In fact, an ordinary radio-frequency coil that is well designed and enclosed in a shield will have at least as high a Q and no more external magnetic field than a toroid of corresponding dimensions, and will also have zero external electrostatic field.

Maximum Q with given outer diameter is obtained in single-layer toroids of circular cross section wound with the best size of wire when the diameter of the winding section is approximately 0.35 times the over-all diameter. With single-layer toroids of rectangular cross section wound with the optimum size of wire, the optimum radial depth of winding is approximately 0.3 times the outside diameter, and the optimum height is approximately 0.4 times the outside diameter.²

The losses of toroidal coils can be reduced by using a single-layer winding and a relatively flat conductor so curved that the flat side follows exactly the surface of the toroid, as in Fig. 52.³ The maximum Q of such a toroid with circular cross section occurs at $r/R = 0.71$ for fixed outer diameter. For rectangular cross section, the optimum condition for given height h and outer diameter are $r_1/r_2 = 0.279$. The Q of such coils is theoretically proportional to the size and to the square root of the frequency. With large dimensions and very high frequencies, Q 's of the order of 1,000 to 10,000 are theoretically realizable. There are practical difficulties of construction, however.

19. Calculation of Copper Loss in Coils.—The most important loss in coils is the copper loss as influenced by skin and proximity effects. The problem of deriving formulas for calculating coil losses when skin and proximity effects are involved has been attacked by several investigators. The most extensive work of this character is that of Butterworth⁴, who considered a coil to be composed of a number of short cylindrical parallel sections of conductor, and then determined the losses in each of these cylinders as a result of the magnetic fields produced by the current in each cylinder and in the neighboring cylinders. The results of these calculations are summarized below.

Copper Losses in Single-layer Solenoids.—For not too closely wound single-layer solenoids employing solid wire, one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + u \left(\frac{d_0}{c} \right)^2 G \quad (93)$$

¹ D. Grimes and W. S. Barden, A Study of Litz Wire Coils, *Electronics*, Vol. 6, p. 303, November; p. 342, December, 1933.

² S. Butterworth, The High-frequency Resistance of Toroidal Coils, *Exp. Wireless and Wireless Eng.*, Vol. 6, p. 13, January, 1929.

³ F. E. Terman, Some Possibilities for Low Loss Coils, *Proc. I.R.E.*, Vol. 23, p. 1069, September, 1935.

⁴ S. Butterworth, Effective Resistance of Inductance Coils at Radio Frequencies, *Exp. Wireless and Wireless Eng.*, Vol. 3, p. 203, April, 1926; p. 302, May, 1926; p. 417, July, 1926; p. 483, August, 1926; On the Alternating Current Resistance of Solenoidal Coils, *Proc. Roy. Soc. (London)*, Vol. 107, p. 693, 1925. A summary of Butterworth's work is given by B. B. Austin, Effective Resistance of Inductance Coils at Radio Frequency, *Wireless Eng. and Exp. Wireless*, Vol. 11, p. 12, January, 1934.

TABLE 18.—VALUES OF THE FUNCTIONS H AND G
 d = diameter of wire, cm; ρ = resistivity, cgs units; f = frequency, cycles per sec;
 $x = \pi d \sqrt{2f/\rho}$. For copper of resistivity 1,700 cgs units $x = 0.1078 d \sqrt{f}$

x	H	G	x	H	G	x	H	G	x	H	G
0.0	1.000		2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641
0.1	1.000	$x^4/64$	2.6	1.201	0.3184	5.2	2.114	0.700	11.0	4.151	1.818
0.2	1.000		2.7	1.228	0.3412	5.4	2.184	0.826	12.0	4.504	1.995
0.3	1.000		2.8	1.256	0.3632	5.6	2.254	0.861	13.0	4.856	2.171
0.4	1.000		2.9	1.286	0.3844	5.8	2.324	0.896	14.0	5.209	2.348
0.5	1.000		3.0	1.318	0.4049	6.0	2.394	0.932	15.0	5.562	2.525
0.6	1.001	0.00202	3.1	1.351	0.4247	6.2	2.463	0.967	16.0	5.915	2.702
0.7	1.001	0.00373	3.2	1.385	0.4439	6.4	2.533	1.003	17.0	6.268	2.879
0.8	1.002	0.00632	3.3	1.420	0.4626	6.6	2.603	1.038	18.0	6.621	3.056
0.9	1.003	0.01006	3.4	1.456	0.4807	6.8	2.673	1.073	19.0	6.974	3.233
1.0	1.005	0.01519	3.5	1.492	0.4987	7.0	2.743	1.109	20.0	7.328	3.409
1.1	1.008	0.02196	3.6	1.529	0.5160	7.2	2.813	1.144	21.0	7.681	3.586
1.2	1.011	0.03059	3.7	1.566	0.5333	7.4	2.884	1.180	22.0	8.034	3.763
1.3	1.015	0.04127	3.8	1.603	0.5503	7.6	2.954	1.216	23.0	8.388	3.940
1.4	1.020	0.0541	3.9	1.640	0.5673	7.8	3.024	1.251	24.0	8.741	4.117
1.5	1.026	0.0691	4.0	1.678	0.5842	8.0	3.094	1.287	25.0	9.094	4.294
1.6	1.033	0.0863	4.1	1.715	0.601	8.2	3.165	1.322	30.0	10.86	5.177
1.7	1.042	0.1055	4.2	1.752	0.618	8.4	3.235	1.357	40.0	14.40	6.946
1.8	1.052	0.1265	4.3	1.789	0.635	8.6	3.306	1.393	50.0	17.93	8.713
1.9	1.064	0.1489	4.4	1.826	0.652	8.8	3.376	1.428	60.0	21.46	10.48
2.0	1.078	0.1724	4.5	1.863	0.669	9.0	3.446	1.464	70.0	25.00	12.25
2.1	1.094	0.1967	4.6	1.899	0.686	9.2	3.517	1.499	80.0	28.54	14.02
2.2	1.111	0.2214	4.7	1.935	0.703	9.4	3.587	1.534	90.0	32.07	15.78
2.3	1.131	0.2462	4.8	1.971	0.720	9.6	3.657	1.570	100.0	35.61	17.55
2.4	1.152	0.2708	4.9	2.007	0.738	9.8	3.728	1.605			
2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641	Large	$\frac{\sqrt{2x}+1}{4}$	$\frac{\sqrt{2x}-1}{8}$

TABLE 19.—VALUES OF u FOR SINGLE-LAYER SOLENOIDS OF MANY TURNS (SPACED)

D = diameter of coil

b = length of coil

u_1 = contribution of radial component of field

u_2 = contribution of axial component of field

b/D	u_1	u_2	$u = u_1 + u_2$
0.0	3.29	0.00	3.29
0.2	3.13	0.50	3.63
0.4	2.83	1.23	4.06
0.6	2.51	1.99	4.50
0.8	2.22	2.71	4.93
1.0	1.94	3.35	5.29
2	1.11	5.47	6.58
4	0.51	7.23	7.74
6	0.31	8.07	8.38
8	0.21	8.52	8.73
10	0.17	8.73	8.90
∞	0.00	9.87	9.87

where H = resistance ratio of wire when isolated, as given by Table 4 (or Table 18).

G = proximity effect factor, given by Table 18.

u = a constant given by Table 19 or 20.

$$d_0/c = \frac{\text{wire diameter}}{\text{spacing between centers of adjacent turns}}$$

This equation holds for single-layer solenoids having many turns provided u is obtained from Table 19 and $d_0/c \leq 0.6$ (i.e., turns not too closely spaced¹). It also applies to coils having a few spaced turns provided the length is small compared with the diameter and u is obtained from Table 20.

TABLE 20.—VALUES OF u FOR SINGLE-LAYER SOLENOIDS OF FEW TURNS (SPACED)

No. of turns.....	2	4	6	8	10	12	16	24	32	∞
u	1.00	1.80	2.16	2.37	2.51	2.61	2.74	2.91	3.00	3.29

TABLE 21.—VALUES OF α , β , γ IN EQ. (94)*

d_0/c	$x = 1$			$x = 2$			$x = 3$		
	α	β	γ	α	β	γ	α	β	γ
1.0	1.01	1.02	0.96	1.09	1.34	0.67	1.31	2.29	0.49
0.9	1.00	1.02	0.97	1.06	1.29	0.72	1.20	1.99	0.55
0.8		1.02	0.98	1.04	1.23	0.78	1.13	1.73	0.62
0.7		1.02	0.98	1.02	1.18	0.83	1.08	1.52	0.68
0.6		1.01	0.99	1.00	1.13	0.87	1.04	1.36	0.75
0.5		1.01	0.99		1.09	0.91	1.02	1.24	0.82
0.4		1.01	0.99		1.06	0.94	1.01	1.14	0.88
0.3		1.00	1.00		1.04	0.97	1.00	1.06	0.93
0.2					1.01	0.99		1.03	0.97
0.1					1.00	1.00		1.01	0.99

d_0/c	$x = 4$			$x = 5$			$x = \infty$		
	α	β	γ	α	β	γ	α	β	γ
1.0	1.43	3.61	0.43	1.50	4.91	0.41	1.71	inf.	0.35
0.9	1.30	2.75	0.49	1.37	3.39	0.46	1.55	12.45	0.30
0.8	1.21	2.12	0.55	1.25	2.48	0.53	1.41	4.83	0.44
0.7	1.12	1.71	0.62	1.15	1.91	0.60	1.27	2.87	0.52
0.6	1.07	1.51	0.70	1.09	1.60	0.68	1.16	2.03	0.60
0.5	1.03	1.32	0.78	1.04	1.37	0.76	1.08	1.59	0.69
0.4	1.02	1.19	0.85	1.02	1.22	0.84	1.03	1.33	0.78
0.3	1.00	1.10	0.91	1.00	1.11	0.90	1.01	1.17	0.87
0.2		1.04	0.96		1.05	0.96	1.00	1.07	0.94
0.1		1.01	0.99		1.01	0.99		1.02	0.98

* x has the same meaning as in Tables 4 and 18.

¹ At frequencies such that H does not differ greatly from unity, the formula holds even when the spacing is less.

With closely wound coils having very many turns of solid wire, the resistance ratio for any spacing between turns, even very close spacing, is

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = \alpha H + (\beta u_1 + \gamma u_2) G \left(\frac{d_0}{c} \right)^2 \quad (94)$$

where α , β , and γ depend on (d_0/c) and are given by Table 21, u_1 and u_2 depend upon the ratio of length to diameter of the coil, as in Table 19, and the remaining notation is as in Eq. (93).

At very high frequencies, Eqs. (93) and (94) reduce to the following:

For Eq. (93), assuming copper wire:

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = 38 \sqrt{f_m c d_0} \left[1 + \frac{1}{2} u \left(\frac{d_0}{c} \right)^2 \right] \quad (93a)$$

For Eq. (94), assuming copper wire:

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H \phi \quad (94a)$$

Here d_0 is the wire diameter in centimeters. The factor ϕ is given in Table 22, and depends upon the ratio of coil length to diameter and upon the closeness of the winding.

When litz wire is used, the resistance ratio of single-layer coils of many turns not too closely spaced is

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \left[k + u \left(\frac{d_0}{c} \right)^2 \right] \left(\frac{d_s}{d_0} \right)^2 n^2 G \quad (95)$$

where d_0 = diameter of cable.

c = spacing between centers of adjacent turns.

d_s = diameter of individual strand of cable.

n = number of strands in cable.

k = constant depending on n , given in Table 23.

G and H are the same as above, except that x in Table 18 is evaluated for the diameter of the individual strand.

TABLE 22.—VALUES OF FACTOR ϕ IN EQ. (94a)

$\frac{d_0}{c}$	$\frac{b}{D} = \frac{\text{coil length}}{\text{coil diameter}}$											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	∞											3.41
0.9	18.2	17.5	16.1	14.6	13.2	11.9	8.02	5.27	4.39	3.90	3.78	3.11
0.8	6.49	6.32	5.96	5.57	5.23	4.89	3.91	3.20	3.04	2.97	2.92	2.82
0.7	3.59	3.53	3.43	3.29	3.17	3.07	2.74	2.61	2.51	2.51	2.50	2.52
0.6	2.36	2.35	2.32	2.29	2.26	2.23	2.16	2.15	2.14	2.16	2.16	2.22
0.5	1.73	1.74	1.75	1.75	1.75	1.76	1.77	1.85	1.85	1.86	1.86	1.93
0.4	1.38	1.39	1.41	1.42	1.44	1.45	1.49	1.56	1.57	1.59	1.60	1.65
0.3	1.16	1.19	1.21	1.22	1.22	1.24	1.28	1.34	1.34	1.35	1.36	1.39
0.2	1.07	1.08	1.08	1.09	1.10	1.10	1.13	1.16	1.16	1.17	1.17	1.19
0.1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05

TABLE 23

No. of strands, n	3	9	27	∞
k	1.55	1.84	1.92	2

Calculation of Copper Losses in Multilayer Coils.—Butterworth has also derived a formula for the a-c/d-c resistance ratio of multilayer coils having many turns not too compactly wound. For solid wire one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \frac{1}{4} \left(\frac{Kbm}{D} \right)^2 \left(\frac{d_0}{c} \right)^2 G \quad (96)$$

FIG. 51.—Values of constant K in Eqs. (96) and (97) for multilayer coils.

where K = a constant depending upon the type of winding, obtainable from Fig. 51.

b = axial length of winding.

D = over-all diameter.

m = number of layers.

$$d_0/c = \frac{\text{diameter of wire}}{\text{spacing between centers of adjacent turns in same layer}}$$

H and G = constants given by Table 18.

For litz, one has

$$\frac{\text{A-c resistance}}{\text{D-c resistance}} = H + \left[k + \frac{1}{4} \left(\frac{Kbm}{D} \right)^2 \left(\frac{d_0}{c} \right)^2 \right] \left(\frac{d_i}{d_0} \right)^2 n^2 G \quad (97)$$

The notation in Eq. (97) is the same as in Eqs. (95) and (96).

Equations (96) and (97) apply to all shapes of coils, ranging from solenoids in one extreme ($t/D \rightarrow 0$) to disk or pancake coils in the other extreme ($D/b \rightarrow \infty$). The only restriction is that there must be many turns and these must be reasonably well spaced (i.e., not too compactly wound). In the particular case of a single-layer solenoid, the value of K is such that Eqs. (96) and (97) reduce to Eqs. (93) and (95).

Calculation of Copper Losses in Toroidal Coils.—The resistance ratio of toroidal coils is given by Eq. (96) when solid round wire is used, and by Eq. (97) when litz wire is employed, provided that the following modified definitions are used.¹

D = over-all diameter of toroid.

$b = \pi D$ = outer periphery of winding section.

c = spacing between centers of adjacent turns of same layer at outer periphery.

K = a constant given, depending upon the type of toroid and the proportions, as given in Table 24.

TABLE 24.—VALUES OF K FOR SINGLE-LAYER TOROIDS

Circular cross section		Rectangular cross section				
Diam. of winding Over-all diam.	K	Radial depth Over-all diam.	K			
			Axial length Over-all diam.			
			0.125	0.250	0.375	0.500
0.05	2.110					
0.10	2.243					
0.15	2.409	0.1	2.25	2.25	2.26	2.26
0.20	2.602	0.2	2.65	2.67	2.69	2.70
0.25	2.913	0.3	3.37	3.47	3.54	3.58
0.30	3.327	0.4	5.25	5.68	5.96	6.14
0.35	3.978					
0.40	5.180					
0.45	8.341					

In the case of single-layer toroids wound with conductor of the type illustrated in Fig. 52, the Q obtained with optimum proportions for Fig. 52a is²

$$Q \text{ for optimum } \frac{r_2}{r_1} = \frac{84.3 \sqrt{f} r_2}{\left(1 + 0.556 \frac{r_2}{h} \right)} \quad (98)$$

¹ S. Butterworth, *The High Frequency Resistance of Toroidal Coils*, *Exp. Wireless and Wireless Eng.*, Vol. 6, p. 13 January, 1929.

² Terman, *loc. cit.* See also G. Reber, *Optimum Design of Toroidal Inductances*, *Proc. I.R.E.*, Vol. 23, p. 1086, September, 1935.

For a toroid of circular cross section (Fig. 52b),

$$Q \text{ for optimum } \frac{r_2}{r_1} = 52.1b \sqrt{f} \quad (99)$$

The dimensions in these equations are in centimeters, and the frequency f is in megacycles.

Practical Usefulness of Formulas for Calculating Copper Losses.—Equations (93) to (99) take into account only the copper losses resulting from skin and proximity effects. At frequencies somewhat less than that giving maximum Q , these losses represent nearly the entire loss of energy in the coil. At higher frequencies, dielectric losses become important and lower the Q .¹

The chief value of the formulas for calculating losses is in indicating the relative effect that such factors as size, shape, wire size, and other proportions have upon

FIG. 52.—Toroidal coils wound with strip conductor that is curved to follow the surface of revolution formed by the toroid.

the coil Q . These formulas will satisfactorily compare the merit of different proposed designs, and indicate which are most promising, even under conditions where the absolute accuracy of the formulas is not particularly high.

The formulas are especially useful in indicating the size of wire for winding a given form. In the case of a single-layer solenoid in which Eq. (93a) applies, the wire diameter giving maximum Q when the length, diameter, and number of turns is fixed is

$$\frac{\text{Optimum diameter}}{\left\{ \begin{array}{l} \text{Spacing between centers} \\ \text{of adjacent turns} \end{array} \right\}} = \sqrt{\frac{2}{\mu}} \quad (100)$$

where μ is given by Table 19.

¹ A comparison of calculated and observed coil losses is given by W. Jackson, Measurements of the High-Frequency Resistance of Single-layer Solenoids, *Jour. I.E.E.*, Vol. 80, p. 844, 1937; see also Wireless Section, *I.E.E.*, Vol. 12, p. 133, June, 1937.

20. Distributed Capacity of Coils.—The voltage difference that exists between different parts of a coil produces an electrostatic field in the air and in the dielectric near the coil. The effect of the resulting storage of electrostatic energy upon the coil behavior is to a good approximation equivalent to the effect produced by a small capacity shunted across the terminals of the coil. Such a hypothetical shunting capacity is termed the *distributed capacity* of the coil. Under practical conditions, there are also capacities between the coil terminals, and between lead wires, which increase the total effective shunting capacity.¹

The distributed capacity of an isolated single-layer solenoid is approximately proportional to the diameter of the coil and decreases slowly with increasing length. The distributed capacity is largely independent of the number of turns if there are a considerable number of turns and these are not too closely spaced. Very close spacing increases the capacity somewhat, however, particularly if there are only a few turns.

FIG. 53.—Several types of multi-layer windings. The numbers in the drawing indicate the order of the turns.

Multilayer coils tend to have higher distributed capacity than do single-layer coils. In order to avoid excessive distributed capacity, the winding of a multilayer coil must be arranged so that the difference of potential between adjacent turns is only a small fraction of the total voltage applied to the coil. Thus a simple two-layer winding, as shown in Fig. 53a, will have very high distributed capacity, whereas with a winding in which the turns are wound as indicated in Fig. 53b (bank winding), the distributed capacity will be only moderate. Other means of minimizing distributed capacity consist of employing a narrow winding of many layers with few turns per layer, as shown in Fig. 53c, or the use of spaced layers, as in Fig. 53d. The common universal or honeycomb-type multilayer coil has a reasonably low distributed capacity, since it is equivalent to a coil of many layers with few turns per layer.

Dielectric in the field of a coil, such as the form upon which the coil is wound, the insulation of the wire, etc., increase the distributed capacity. Metal objects such as a shield, metal panel, etc., near a coil increase the distributed capacity, particularly if the distance from the coil is of the same order of magnitude as the coil dimensions or is less.

Effects of Distributed Capacity.—The presence of distributed capacity introduces losses, because some of the electrostatic energy storage involves solid dielectrics, which are never perfect. Dielectric losses have an effect equivalent to adding a resistance in series with the inductance coil according to the equation

$$\left. \begin{array}{l} \text{Equivalent series} \\ \text{resistance representing} \\ \text{dielectric losses} \end{array} \right\} = r\omega^2 L^2 C \quad (101)$$

where $\omega = 2\pi \times \text{frequency}$.

L = true coil inductance.

¹ The effective distributed capacity will also depend to some extent upon the current distribution in the coil, and will in general be larger when the coil is shunted with a large external tuning capacity than when the coil is resonated with its self-capacity.

C = distributed capacity.

γ = power factor of distributed capacity.

The resistance given by Eq. (101) is a part of the coil resistance R in Fig. 54a. Experiments indicate that the dielectric losses are not over 20 per cent of the total loss in the working range of the coil, unless the form or wire insulation has absorbed moisture.¹ When hygroscopic forms are used, and when the wire has cotton insulation, it is advisable to impregnate the coil with wax, shellac, or lacquer to prevent the absorption of moisture.

The presence of the distributed capacity causes a partial resonance that modifies the apparent resistance and reactance of the coil as viewed from the terminals. The apparent series resistance L_{eq} and R_{eq} that appear to be present when the coil is

FIG. 54.—Actual circuit of coil with distributed capacity, together with equivalent series circuit.

viewed from its terminals (see Fig. 54b) are related to the true resistance and inductance according to the equations

$$\left. \begin{array}{l} \text{Apparent inductance } L_{eq} \text{ of} \\ \text{coil with distributed capacity} \end{array} \right\} = \frac{L}{1 - \gamma^2} \quad (102)$$

$$\left. \begin{array}{l} \text{Apparent resistance } R_{eq} \text{ of} \\ \text{coil with distributed capacity} \end{array} \right\} = \frac{R}{(1 - \gamma^2)^2} \quad (103)$$

where L and R are the true coil inductance and resistance, respectively, and γ is the ratio of actual frequency to the frequency at which the coil inductance would be resonant with the distributed capacity.

21. Temperature Coefficient of Inductance.²—The inductance of a coil varies with temperature as a result of (1) changes in dimensions with temperature and (2) changes in current distribution in the wire as a result of the change of wire resistivity with temperature modifying the skin and proximity effects.

The relation between the dimensions of a coil and the temperature depend upon several factors. In the ideal case of a freely suspended coil expanding according to the coefficient of linear expansion of the metal, the temperature coefficient of inductance will be the same as the temperature coefficient of expansion of the metal. Actually, however, the mechanical strains existing in the conductor ordinarily cause added changes in dimensions, with the result that the temperature coefficient of such a coil is

¹ W. Jackson, Dielectric Losses in Single-layer Coils at Radio Frequencies, *Exp. Wireless and Wireless Eng.*, Vol. 5, p. 255, May, 1928.

² H. A. Thomas, The Stability of Inductance Coils for Radio Frequencies, *Jour. I.E.E.*, Vol. 77, p. 702, 1935; also *Wireless Section, Jour. I.E.E.*, Vol. 11, p. 44, March, 1936; D. A. Bell, Temperature Coefficient of Inductance, *Wireless Eng.*, Vol. 16, p. 240, May, 1939; H. A. Thomas, The Dependence on Frequency of the Temperature-coefficient of Inductance of Coils, *Jour. I.E.E.*, Vol. 84, p. 101, 1939; also, *Wireless Section, Jour. I.E.E.*, Vol. 14, p. 19, March, 1939; E. R. Moullin, The Temperature Coefficient of Inductances for Use in a Valve Generator, *Proc. I.R.E.*, Vol. 26, p. 1385, November, 1938; Janusz Groszkowski, The Temperature Coefficient of Inductance, *Proc. I.R.E.*, Vol. 25, p. 448, April, 1937; Janusz Groszkowski, Temperature Coefficient of Inductance, *Wireless Eng.*, Vol. 12, p. 650, December, 1935.

usually a number of times the coefficient of linear expansion. When the coil is wound on a form instead of being freely suspended, the changes of dimension tend to be controlled by the form, but even here it is found that the temperature coefficient of inductance will normally be considerably greater than the expansion coefficient of either the form or the wire.

The change in wire resistivity with temperature modifies the skin and proximity effect and so causes the inductance to vary. Inasmuch as skin and proximity effects depend upon the frequency, this causes the temperature coefficient of inductance to depend likewise upon the frequency. The contribution to the temperature coefficient resulting from resistance changes is low at low frequencies when the skin and proximity effect are small, and is also low at very high frequencies, where these effects are very large. The change in inductance is maximum at a frequency between the extremes, where the skin and proximity effects are only moderate and the current distribution is particularly sensitive to resistance changes.

It is frequently found that the variation of coil inductance with temperature is noncyclical; i.e., if the coil undergoes temperature cycles the inductance will not follow the temperature exactly, but, rather, will vary irregularly and will not return to its initial value when the temperature returns to normal. This is apparently the result of mechanical changes such as result from slippage of the wire over the form, and permanent changes in physical dimensions occasioned by the relieving of initial stress.

Practical Inductance Coils Having Low Temperature Coefficients.—Several methods have been suggested for designing coils to have a low and stable temperature coefficient. Probably the best arrangement is a single-layer solenoid with flat conductor wound directly on an insulating form having a low coefficient of expansion. The conductor can be a flat strip, but is preferably a conducting film deposited directly on the surface. If the radial depth of the flat conductor is very small and if the turns are well spaced axially, then the temperature coefficient of inductance will be of the same order of magnitude as that of the form, will be independent of frequency, and will be stable over repeated temperature cycles. Temperature coefficients of inductance of 5 to 15 parts per million per°C can be obtained in this way.

Another possible method of construction is to anneal the turns to relieve initial stress, and then mount them so that the coefficient of expansion in an axial direction is controlled by material having a relatively large temperature coefficient, while the coefficient of expansion in a radial direction is fixed by material having a low expansion coefficient. By suitable choice of the relative coefficients, it is then possible to obtain either zero or any desired temperature coefficient. Such coils must, however, be very carefully constructed or their behavior will not repeat on successive temperature cycles.¹

Small inductances having low temperature coefficient can be constructed by employing a single-layer coil in which the conductor is a copper- or silver-plated invar (or nilvar) wire wound on a ceramic form having a low temperature coefficient of expansion. It is possible in this way to obtain an inductance coefficient of about one part in a million per°C.²

22. Practical Air-cored Coils. Coils for Resonant Circuits in Radio Receivers.—Coils for resonant circuits of radio receivers must be compact, and at the same time have a moderate to high Q . At frequencies in excess of 1,500 kc, the coils are practically always single-layer solenoids using spaced turns of solid wire wound on insu-

¹ For further information on such coils, see W. H. F. Griffiths, Recent Improvements in Air-cored Inductances, *Wireless Eng.*, Vol. 19, p. 8, January, 1942.

² S. W. Seely and E. I. Anderson, UHF Oscillator Frequency-stability Considerations, *RCA Rev.*, Vol. 5, p. 77, July, 1940.

lating tubes. Coils for broadcast frequencies (550 to 1,500 kc) are usually also single-layer solenoids, although bank and universal windings are sometimes employed. At frequencies below 550 kc, the coils for resonant circuits are nearly always multilayer, usually of the universal or similar type.¹ Litz wire is generally preferred for these frequencies, although solid wire is often used because of lower cost.

Disk, toroidal, and other types of coils are practically never used for resonant circuits in receivers. At broadcast and lower frequencies, coils with finely subdivided iron cores are sometimes used (see Par. 27).

Coils for Resonant Circuits of Radio Transmitters.—Coils for transmitters are subjected to high voltage stresses and must handle large amounts of reactive circulating energy. This requires relatively large spacings in order to obtain the required voltage insulation and heat-dissipating ability. The size of coils for transmitters tends to be directly proportional to the power and inversely proportional to frequency.

In short-wave transmitters, it is customary to use coils wound with solid wire on ceramic forms when the power is low, and to use coils of copper tubing supported from the terminals for high power. In high-power broadcast transmitters, coils may be

FIG. 55.—Typical examples of radio-frequency choke coils.

either large single-layer solenoids wound with copper tubing or edgewise wound strip. Single-layer disk or pancake coils using ribbon or solid conductors are also sometimes employed. When large powers are to be handled at still lower frequencies, it is customary to use a very large solenoid, either single or multilayer, with conductor consisting of large copper tubing, or litz, of large cross section.

Radio-frequency Choke Coils.—A radio-frequency choke coil is an inductance designed to offer a high impedance to alternating currents over the frequency range for which coil is to be used. This result is obtained by making the inductance high, and the distributed capacity low. Radio-frequency choke coils are commonly used at frequencies in the neighborhood of the natural resonant frequency of the coil. Thus the coils will normally offer a capacitive impedance to an applied voltage over at least a part of their working range.

The usual radio-frequency choke consists of two or more universal wound coils mounted some distance apart on an insulating rod, as shown in Fig. 55a. Another method of construction is to use a series of "pies" wound in deep narrow slots (Fig. 55b). A long single-layer solenoid, as in Fig. 55c, is sometimes used at very high frequencies.

The important characteristic of a radio-frequency choke coil is the variation of impedance with frequency. This depends upon the inductance and capacity of the

¹ The design of such multilayer windings is discussed by A. A. Joyner and V. D. Landon, *Theory and Design of Progressive Universal Coils, Communications*, Vol. 18, p. 5, September, 1938; A. W. Simon, *Winding the Universal Coil, Electronics*, Vol. 9, p. 22, October, 1936; L. M. Hershey, *The Design of the Universal Winding, Proc. I.R.E.*, Vol. 29, p. 442, August, 1941.

sections, the mutual inductance and mutual capacities between sections, the capacity to ground, and the lead arrangements. The behavior in a simple idealized case is illustrated in Fig. 56, in which the choke is considered as consisting of two inductances each with distributed capacities, but with mutual inductance and mutual capacities:

FIG. 56.—Impedance characteristics of idealized radio-frequency choke coils.

ignored. If the resonant frequencies of the two sections are the same, i.e., if $L_1C_1 = L_2C_2$ in Fig. 56, then the choke acts as a single resonant circuit and has an impedance characteristic as shown by the solid line of Fig. 56. On the other hand, if the two sections have different resonant frequencies, the impedance will vary as a function of frequency in the manner shown by the dotted line. At a frequency some-

FIG. 57.—Brooks inductometer.

where between the two resonant frequencies, the choke has negligible impedance as a result of one section offering an inductive reactance of the correct magnitude to be in series resonance with the capacitive reactance of the second section. The actual situation existing in a practical radio-frequency choke coil is more complicated than

that of Fig. 56, because of mutual inductance and capacity effects and capacity to ground. The behavior is, however, of the same character, though more complex. By careful balancing of inductances, capacities, and couplings, it is possible to obtain performance approximating that corresponding to a single resonant frequency.¹

Practical radio-frequency choke coils are designed either to have a single resonant frequency or to be proportioned so that the "holes" in the impedance characteristic occur at frequencies other than those for which the coil is to be used. Where the choke coil is to be shunted across the resonant circuit of a receiver that is tuned over a frequency band, it is necessary that the choke be very carefully proportioned to avoid even minor irregularities in the impedance curve, as shown by the dashed line in Fig. 56.

Variable Inductances.—The most common type of variable inductance is the ordinary variometer or inductometer. This consists of two systems of coils that are connected in series and rotated with respect to each other in order to vary the mutual inductance. The total inductance is $L_1 + L_2 \pm 2M$, and if the coefficient of coupling is large, the ratio of maximum to minimum inductance obtainable will be of the order of 10:1.

Two methods of constructing variable inductances are shown in Figs. 57 and 58. In the variometer, the rotating coil is wound on a spherical section, and mounted inside a corresponding fixed coil. In the Brooks inductometer, the same result is achieved in a slightly different way. The Brooks inductometer has the advantage that the calibration is not appreciably affected by axial displacements of the rotating coil and, further, that by proportioning the coils as shown in Fig. 57, the inductance varies linearly with the angle of rotation.²

The inductance of single-layer coils wound with relatively large wire can be varied by providing a wheel or trolley assembly that rides on the wire and provides a sliding contact that is adjusted by rotating the coil to vary the position of contact.³

With proper mechanical design, such an arrangement provides a satisfactory and very fine control of the inductance.

Small variations in the inductance of coils used at radio frequencies can be obtained by rotating a copper disk or ring in the magnetic field of the coil. Such a copper object reduces the inductance in proportion to the magnetic flux lines that it intercepts, and so gives maximum inductance when the plane of the ring or disk is at right angles to the magnetic flux. The extra losses resulting from this method of controlling inductance are small, provided that the range of control is not excessive.

Fixed Inductances for Audio Frequencies.—Fixed inductances for such purposes as standards at audio frequencies are made of several types, as illustrated in Fig. 59. The conventional multilayer coil shown at Fig. 59a gives the largest inductance in proportion to the direct-current resistance, but it has the disadvantage of a large external field.⁴ Inductances of this type will have minimum loss when proportioned so that $b = c$, and $c/a = 0.66$. The toroid has no external field but requires a very

FIG. 58.—Variometer type of variable inductance.

¹ The design of radio-frequency choke coils for optimum performance is discussed by Harold A. Wheeler, The Design of Radio-frequency Choke Coils, *Proc. I.R.E.*, Vol. 24, p. 850, June, 1936; Herman P. Miller, Jr., Multi-band R-F Choke Coil Design, *Electronics*, Vol. 8, p. 254, August, 1935.

² For detailed design information on the Brooks inductometer see H. B. Brooks and F. C. Weaver, A Variable Self and Mutual Inductor, *Bur. Standards, Sci. Paper* 290.

³ Paul Ware, A New System of Inductive Tuning, *Proc. I.R.E.*, Vol. 26, p. 308, March, 1939; L. E. Herborn, A Precise High-frequency Inductometer, *Bell Lab. Rec.*, Vol. 17, p. 351, July, 1939.

⁴ The design of standard inductances of this type is thoroughly discussed by H. B. Brooks, Design of Standards of Inductance, and the Proposed Use of Model Reactors in the Design of Air-core and Iron-core Reactors, *Bur. Standards Jour. Research*, Vol. 7, p. 289, August, 1931.

large amount of wire in proportion to the inductance obtained, and so has high direct-current resistance. The double-*D* winding is intermediate in its characteristics between the toroid and the multilayer coil with respect to both external field and direct-current resistance. This type of coil has the advantage that the individual coils can be wound with a suitable number of turns, impregnated, taped, and mounted, all before final adjustment. The adjustment to give the exact value of inductance desired is then made by shifting the relative position of the two coils slightly with respect to each other before clamping them into final position.

Fixed inductances for use at radio frequencies should have low losses, good mechanical stability, and a minimum of dielectric loss. Litz wire is desirable for frequencies up to and including broadcast frequencies. The construction must be substantial, the form should be of good dielectric material, and the coil should be protected from moisture by suitable "dope." The distributed capacity should be low in order that the frequency at which the inductance is resonant with the distributed capacity will be as high as possible. In this way, the working range of frequencies over which the

FIG. 59.—Types of fixed inductances used in audio-frequency work.

apparent and the true inductance are approximately the same will be as great as possible.

The temperature coefficient of inductance standards should be low. However, it is impractical to achieve a particularly low coefficient in multilayer and toroidal coils, as there is no material for the coil form that has the requisite low temperature coefficient combined with other desirable characteristics, such as stability, machinability, etc. Wood impregnated with beeswax and certain grades of porcelain are to be preferred, while bakelite and hard rubber are not recommended because of their high coefficients of expansion.

COILS WITH MAGNETIC CORES

23. Fundamental Properties of Magnetic Substances.—The relationship between flux density and magnetizing force in the core of a coil when the core is of magnetic material that has been previously demagnetized is shown in Fig. 60a, and is termed the saturation or magnetization curve. When such a core is carried through a complete cycle of magnetization, as when the magnetizing force is supplied by an alternating current, the relationship between flux density and magnetizing force follows a loop, as illustrated in Fig. 60b. This is termed the hysteresis loop. The residual flux B_r remaining after a positive magnetizing force has been removed is termed the *retentivity* of the material. The negative magnetizing force H_c required to reduce the flux to zero is termed the *coercive force*.

Permeability and Incremental Permeability.—The permeability of a magnetic material is defined as the ratio B/H , where B is the flux density in lines per square centimeter (gauss) and H the magnetizing force in oersteds (1 ampere turn per centimeter = 1.256 oersteds). With these units the permeability of air is unity. The

FIG. 60.—Typical magnetization curve, permeability curve, and hysteresis loop of a magnetic material.

permeability μ depends upon the flux density, and is in general small at low flux densities, maximum at moderate values of B , and quite small when the core is saturated, as shown in Fig. 60a.

The permeability at very low magnetizing forces is termed the *initial permeability*, and is of considerable importance in communication equipment. With ordinary magnetic materials, the initial permeability is very small.

The permeability offered to an alternating magnetizing force superimposed upon a d-c magnetizing force is termed *incremental permeability*, apparent permeability, or a-c permeability. Under such conditions, the cycle of magnetization follows a small displaced hysteresis such as is illustrated in Fig. 61; and the permeability to the alternating magnetization is determined by the slope of the line joining the tips of this displaced hysteresis loop.¹ The incremental permeability increases with increasing alternating flux densities (up to the point at which the core is saturated) and decreases with increasing d-c magnetization. The behavior in a typical case is illustrated in Fig. 62.

When both alternating and direct magnetizing forces are acting simultaneously, the permeability offered to the d-c magnetization in the low or moderate flux-density range is increased by a small amount of superimposed a-c excitation, and decreased by a large amount.² With large d-c magnetizations, the permeability to d-c is reduced by a superimposed alternating magnetization.

FIG. 61.—Typical hysteresis loop showing displaced minor hysteresis loops 1, 2, and 3, obtained when a small alternating current is superimposed on a direct-current magnetization. The effective permeability to the superimposed alternating current is proportional to the slope of the line joining the tips of the displaced hysteresis loop.

¹ Thomas Spooner, *Permeability*, Trans. A.I.E.E., Vol. 43, p. 340, 1923.

² R. F. Edgar, *Silicon Steel with A-C and D-C Excitation*, Trans. A.I.E.E., Vol. 53, p. 318, February, 1934.

Core Loss.—When a core is subjected to an alternating magnetization, there is a dissipation of energy in the core as a result of hysteresis and eddy currents. The total power loss in core materials used in transformers is often expressed in curves,

FIG. 62.—Incremental permeability of a typical sample of silicon steel as a function of alternating-current magnetization, for several values of superimposed direct current.

FIG. 63.—Core-loss curves of typical silicon steel sample.

such as those of Fig. 63, that give the watt loss per pound of material as a function of flux density for a given frequency. Such curves are of convenience in the design of power and audio transformers.

A coil having hysteresis and eddy-current losses can be represented by postulating series resistances R_h and R_e , respectively, in series with an inductance having no losses, together with a resistance R_c representing copper losses, as shown in Fig. 64b. In many cases, it is found convenient to represent the eddy-current losses as a resistance R_{eh} shunted across the inductance as in Fig. 64c, instead of as a series resistance. When the eddy-current losses are small, this shunting resistance is independent of frequency, whereas the series resistance R_e is proportional to the square of the frequency.

The eddy-current loss in a core is the result of currents induced in the core by the varying magnetic flux. The magnitude of the power loss per unit volume is proportional to the square of the frequency and to the square of the flux density, inversely proportional to the resistivity of the core material, and directly proportional to the square of the thickness (or diameter) of the individual core laminations (or particles). The eddy-current loss is not affected by d-c saturation of the core.

FIG. 64.—Equivalent circuits showing methods of representing hysteresis and eddy current losses in a coil having a magnetic core.

The equivalent series resistance R_e in Fig. 64b that can be considered as representing the eddy-current loss in a laminated core is¹

$$R_e = 0.00658 \frac{t^2}{\rho_1} \mu_m f X_L \quad (104)$$

where t = thickness of the laminations, cm.

ρ_1 = resistivity of core material, microhm-cm.

μ_m = permeability of core corresponding to the maximum flux density produced by the magnetization ($\mu_m = 1$ for air).

X_L = inductive reactance of the coil with its magnetic core.

The value of the equivalent series resistance R_e for the case of a powdered core, on the assumption that the individual magnetic particles are of spherical shape is

$$R_e = \frac{0.00197 t^2}{\rho_1 k^{3/2}} \mu_m f X_L \quad (105)$$

where μ_m , f , ρ_1 , and X_L have the same meaning as before, while t^2 is now defined as the mean square diameter of the spherical magnetic particles in cm and k is the fraction of the core volume that is occupied by the magnetic material. Equations (104) and (105) assume that the core is sufficiently subdivided so that magnetic skin effect is negligible.

Hysteresis is a form of magnetic friction, and for a given maximum flux density gives rise to a power loss per unit volume that is proportional to the frequency and to the area under the hysteresis loop. At flux densities sufficiently low that the permeability at the maximum flux density is not over 10 to 20 per cent greater than the initial permeability of the material, the hysteresis power loss per unit volume of most core materials is proportional to the fractional increase of the permeability over the initial value, and is approximately²

¹ V. E. Legg, *Magnetic Measurements at Low Flux Densities using the Alternating Current Bridge*, *Bell System Tech. Jour.*, Vol. 15, p. 39, January, 1936.

² This is a consequence of the Rayleigh hysteresis loop. See Legg, *loc. cit.*

$$R_h = 0.425 \left(1 - \frac{\mu_0}{\mu_m}\right) X_L \quad (106)$$

where μ_0 and μ_m are the initial permeability and the permeability corresponding to the maximum flux density, respectively, and X_L is the reactance of the coil with its iron core. With low magnetizing forces, the value of $\left(1 - \frac{\mu_0}{\mu_m}\right)$ is proportional to the magnetizing force, and hence to the peak flux density B_m . Under these conditions the equivalent resistance R_h representing hysteresis loss is proportional to B_m , and the power loss due to hysteresis¹ is proportional to B_m^3 . When the flux density is moderately high, the hysteresis loss increases less rapidly with flux density. Thus, for silicon steel at flux densities such as used in ordinary transformers, the hysteresis power loss per volume is approximately proportional to $B_m^{1.6}$, corresponding to an equivalent resistance R_h that decreases slowly with increase in B_m .

Hysteresis loss is affected by d-c saturation and also by the presence of currents of other frequencies. The relations are relatively complicated but can, in general, be summarized as follows: With alternating flux densities below a critical value (13,000 to 14,000 gauss for silicon steel), the hysteresis loss rises with increasing d-c magnetization, when constant alternating flux density is assumed. With alternating flux densities greater than this critical value the hysteresis loss with constant alternating flux density decreases with superimposed d-c excitation.² When high- and low-frequency magnetizations are superimposed, the hysteresis loss to the high frequency is greater than it would be in the absence of the low-frequency magnetization, and the hysteresis loss of the latter is reduced. This action, termed *flutter effect*,³ is of practical importance in loaded telephone lines carrying superimposed telegraph currents. The magnitude of the flutter effect is proportional to the area of the normal hysteresis loop and to the frequency of the high-frequency current.

*Harmonics and Cross-modulation Caused by Magnetic Cores.*⁴—The nonlinear relation existing between flux and magnetizing force in magnetic cores results in the production of harmonics when a sine-wave magnetizing force is applied to the coil, and also to cross modulation when two or more frequencies are applied.

With sine-wave excitation, the distortion components are in the form of odd harmonics, of which the third is the most important. With high maximum flux densities, the principal cause of such harmonic generation is the variation of permeability with flux density. With very low flux densities, hysteresis is the principal factor accounting for harmonic generation.

When the magnetizing force consists of a sine wave upon which a d-c current is superimposed, there results a second-harmonic distortion component having an amplitude depending upon the core material and upon the direct and alternating magnetizing forces involved.

When two alternating magnetizations are superimposed upon a core, the result is to produce combination frequencies (cross-modulation), the magnitude of which is dependent upon the amplitude of the magnetizing forces involved and upon the magnetic material. It has been determined experimentally that the third harmonic produced with a sinusoidal magnetizing force is an index of the tendency toward cross-modulation, which, accordingly, depends primarily upon hysteresis effects at low flux densities and upon the variation of permeability at high flux densities.

¹ There is also a component of hysteresis loss at low flux densities that is proportional to B_m^2 . This is termed a "residual loss" and is of importance only at very low frequencies. For further discussion, see Legg, *loc. cit.*

² See Edgar, *loc. cit.*

³ W. Fondiller and W. H. Martin, Hysteresis Effects with Varying Superimposed Magnetizing Forces, *Trans. A.I.E.E.*, Vol. 40, p. 553, 1921.

⁴ Eugene Peterson, Harmonic Production in Ferromagnetic Materials at Low Frequencies and Low Flux Densities, *Bell System Tech. Jour.*, Vol. 7, p. 762, October, 1928.

Permalloy is many times better than silicon steel in freedom from harmonic generation and cross-modulation, while permivar is many times better than permalloy.

24. Magnetic Materials Used in Communication.¹ *Silicon Steel.*²—The most widely used magnetic material in communication work is silicon steel. Silicon steel is available in various grades and types, according to the particular properties to be

FIG. 65.—Typical magnetization curves of various nickel-iron alloys.

emphasized. It is used for power transformers, filter chokes, and as a core material for most audio-frequency, interstage, and output transformers.

*Magnetic Alloys of Iron, Nickel, and Cobalt (Permalloys, Perminvar, and Permendur).*³—Alloys of iron, nickel, and cobalt have some remarkable properties that are very useful in specialized applications. The characteristics of these alloys depend

upon their composition and heat treatment, and a wide variety of properties are obtainable.

An important class of alloys of this type are those consisting of nickel and iron in the range 45 to 80 per cent nickel, with small amounts of chromium and molybdenum added in some cases. Such alloys go by such names as *Permalloy*, *Hipernik*, *Electric metal*, *Mu-metal*, etc., according to the exact composition and the manufacturer. When properly heat-treated, they are all characterized by high maximum permeability and high initial permeability.

FIG. 66.—Hysteresis loop of permivar at moderate flux density, showing the negligible coercive force and residual magnetization.

The flux density at which saturation occurs is, however, the same as or even slightly less than for silicon steel. Typical magnetization curves are shown in Fig. 65. The addition of molybdenum to permalloy increases the resistivity, and hence reduces the eddy-current losses, without serious sacrifice in the low hysteresis loss which permalloy has. Molybdenum per-

¹ V. E. Legg, Survey of Magnetic Materials and Applications in the Telephone Systems, *Bell System Tech. Jour.*, Vol. 18, p. 438, July, 1939.

² C. H. Crawford and E. J. Thomas, Silicon Steel in Communication Equipment, *Elec. Eng.*, Vol. 54 p. 1348, December, 1935.

³ G. W. Elmen, Magnetic Alloys of Iron, Nickel, and Cobalt, *Elec. Eng.*, Vol. 54, p. 1292 December, 1935.

malloy accordingly gives low losses at relatively high frequencies provided the flux densities are not too great.

Alloys of nickel, iron, and cobalt are termed *perminvars*, and when suitably heat-treated are characterized by substantially constant permeability and extremely low hysteresis loss at low flux densities. At medium flux densities, the hysteresis loop is severely constricted in the middle, as shown in Fig. 66, so that although the area of the hysteresis loop is not zero, the coercive force and retentivity are very small. At high flux densities, this constriction disappears, and the hysteresis loop has normal shape. Typical characteristics of *perminvar* are shown in Fig. 67.

Iron and cobalt in equal parts yield a useful magnetic alloy termed *permendur*. Its outstanding property is high permeability at high flux densities, corresponding to a saturation flux density higher than that of most materials. *Permendur* also has higher incremental permeability than most magnetic materials at relatively high d-c magnetizing forces. A magnetization curve of *permendur* is shown in Fig. 67.

FIG. 67.—Typical magnetization curves of *permendur* and *perminvar*, showing the high flux density at saturation of the former and the constant permeability of low flux densities of the latter.

Permanent Magnets.¹—The merit of a magnetic substance for use as a permanent magnet can be expressed in terms of the coercive force H_c , the retentivity B_r , and the maximum value of the energy product BH of the demagnetization curve. The value of the energy product BH gives the relative energy available to sustain flux in the external magnetic circuit per unit volume of magnetic material. For most efficient utilization of the magnetic material, it is accordingly necessary to make the cross section of the magnet such that the desired flux will be obtained in the external circuit with a flux density in the permanent magnet corresponding to the flux giving maximum energy product. The demagnetizing force corresponding to the flux density existing in the magnet represents the magnetizing force per unit length of magnet available to force this flux through the reluctance of the magnetic circuit external to the permanent magnet. As a result of these factors, the total cross section of the permanent magnet is determined by the total flux required in the external circuit, and the length of the magnet is determined by the reluctance of the circuit.

The most desirable materials for permanent magnets are listed in Table 25, and have characteristics such as shown in Fig. 68 in the demagnetization portion of their

¹ C. S. Williams, *Permanent Magnet Materials*, *Elec. Eng.*, Vol. 35, p. 19, January, 1936; K. L. Scott, *Magnet Steels and Permanent Magnets—Relationship among Their Magnetic Properties*, *Bell System Tech. Jour.*, Vol. 11, p. 383, July 1932.

hysteresis curves.¹ Alnico and the related Alnic and Alnico-cu, are the best compromises between high coercive force and high retentivity. These alloys are inexpensive, but have the disadvantage that they must be cast to shape and finished by grinding, since they are so brittle that drilling and machining are impossible. The oxide material has the highest coercive force. It is also the least expensive and has an

FIG. 68.—Demagnetization curves for permanent magnet materials.

unusually low specific gravity, but is even weaker mechanically than Alnico, since it consists of a sintered mass of oxides. Cobalt steel has a very high retentivity but low coercive force, is machinable, but is relatively expensive.

TABLE 25

Name	Component elements*	Coercive force, oersteds	Retentivity, gaussses
Alnic.....	Al-Ni-Fe	100-600	8 000-4,000
Alnico.....	Al-Ni-Co-Fe	450-650	7,000-6,000
Alnico-cu.....	Al-Ni-Co-Cu-Fe	550	7,000
Cobalt steel.....	Co-Fe	150-250	10,000
Cobalt-tungsten.....	Co-W-Fe	30-150	13,500-11,000
Chrome.....	Cr-Fe	60	10,000
Oxide.....	CoO, Fe ₂ O ₃ , Fe ₃ O ₄	300-1,100	6,000-1,500

* Ni = nickel; Al = aluminum; Fe = iron; Co = cobalt; W = tungsten

25. Calculation of Properties of Iron-cored Coils. *Inductance*.—The inductance of an iron-cored coil subjected to an alternating magnetization corresponding to a maximum a-c flux density B_{max} in the iron is as follows:

¹ Another permanent magnet material called Vicalloy has recently been announced, and promises to be of commercial importance. It is composed of vanadium, iron, and cobalt, and is described in *Bell Lab. Rec.*, Vol. 19, p. 38, September, 1940.

Dimensions in centimeters:

$$\text{Inductance in henrys} = L = \frac{1.256N^2}{\left(\frac{l_i}{\mu a_i} + \frac{l_a}{a_a}\right)} \times 10^{-8} \quad (107a)$$

Dimensions in inches:

$$\text{Inductance in henrys} = L = \frac{3.19N^2 \times 10^{-8}}{\left(\frac{l_i}{\mu a_i} + \frac{l_a}{a_a}\right)} \quad (107b)$$

where N = number of turns.

l_i = length of magnetic circuit in iron.

l_a = length of magnetic circuit in air.

μ = permeability of magnetic material evaluated at the appropriate maximum alternating flux density $B_{\max}$.

a_i = cross section of iron (not including the area of insulation between laminations).

a_a = effective cross section of air gap.

The effective cross section a_a of the air gap will in general be somewhat greater than the cross section a_i of the iron because of fringing flux, and because the insulation between laminations makes the effective cross section of iron less than the cross section calculated from the external dimensions of the core.

The relationship between the maximum alternating flux density in the core and the rms value of a sinusoidal voltage is

$$\text{Voltage (rms)} = 4.44fNAB_{\max} \times 10^{-8} \quad (108)$$

where f = frequency in cycles per sec.

N = number of turns.

A = net area of core.

$B_{\max}$ = maximum flux density in core in lines per unit area.

Incremental Inductance.—The incremental inductance, sometimes called also the apparent, or a-c, inductance, is also given by Eq. (107) for any given alternating flux density $B_{\max}$ provided the permeability μ appearing in the equation is interpreted to mean the incremental permeability evaluated at the appropriate a-c and d-c flux densities from data such as are presented in Fig. 62.

The value of d-c flux density B_0 existing in an iron core having a winding of N turns carrying a d-c current I_0 can be determined from the fact that it is necessary to satisfy the relation.

$$1.256NI_0 = H_i l_i + B_0 \left(\frac{a_i}{a_a}\right) l_a \quad (109)$$

The notation is the same as above, with the addition that H_i represents the magnetizing force in oersteds per centimeter required to support a d-c flux density of B_0 lines per square centimeter in the iron. The value of d-c flux density B_0 that satisfies Eq. (109) can be obtained by the cut-and-try process of calculating the value of the right-hand side of Eq. (109) for different assumed flux densities B_0 until a value is obtained that satisfies the equation. An alternative method is to employ the graphical solution shown in Fig. 69. Here B_0 is found as the intersection of the B/H magnetization curve of the magnetic material, with a straight line crossing the horizontal axis at

$$H = 1.256NI/l_i$$

and the vertical axis at $B = 1.256NI/[l_a(a_i/a_a)]$.

Leakage Inductance.—The leakage inductance existing between two coils wound on the same core depends upon the geometry of the windings. The leakage inductance

tance is independent of the character of the core material or of the d-c magnetization, since the paths of the leakage flux are largely in air, and the core contributes negligible reluctance to them.

FIG. 89.—Graphical method of determining the flux density B_0 existing in a magnetic core having a series air gap and subjected to a known magnetizing force.

FIG. 70.—Typical coil arrangements in audio transformers.

When the secondary is wound on the outside of the primary, as shown in Fig. 70a, the total leakage inductance of the windings, referred to the primary side, is¹

$$L'_p = \frac{2.9CN_p^2}{l} \left(h + \frac{a_1 + a_2}{3} \right) \times 10^{-8} \text{ henry} \quad (110)$$

where C is the length of the mean turn in inches, N_p the number of primary turns, and

¹ The total leakage inductance referred to the secondary side is obtained by replacing N_p in Eq. (110) with the number of secondary turns N_s . The total leakage inductance L_p' referred to the primary

the remaining symbols have the meaning indicated in Fig. 70a (all measured in inches). This equation applies to coils wound on both circular and rectangular forms.

If the primary and secondary windings are arranged side by side as in Fig. 70b, the leakage inductance referred to the primary side is

$$L'_p = \frac{3.2CN_p^2}{b} \left(h + \frac{\beta_1 + \beta_2}{3} \right) \times 10^{-8} \text{ henry} \quad (111)$$

where the notation is as in Fig. 70b, or is as above.

The leakage inductance can be reduced by dividing the primary and secondary coils into sections and interspersing these. Simple arrangements of this type are shown in Figs. 70c and 70d, and have leakage inductances referred to the primary that are

For Fig. 70c:

$$L'_p = \frac{0.72CN_p^2}{l} \left(h_1 + h_2 + \frac{\alpha_1 + \alpha_2 + \alpha_3}{3} \right) \times 10^{-8} \text{ henry} \quad (112)$$

For Fig. 70d:

$$L'_p = \frac{0.8CN_p^2}{b} \left(h_1 + h_2 + \frac{\beta_1 + \beta_2 + \beta_3}{3} \right) \times 10^{-8} \text{ henry} \quad (113)$$

Sectionalization of windings, although greatly reducing the leakage inductance, simultaneously increases the winding capacities. The reduction achieved in leakage inductance is accordingly not all net gain.

In push-pull output transformers, the primary is frequently sectionalized to maintain symmetry. When this is done, the leakage reactance between each half of the primary and the secondary can be kept at a minimum by dividing the secondary into two identical parts, as in Fig. 71a, and connecting these in parallel.

FIG. 71.—Transformers with divided windings.

Distributed and Other Capacities of Iron-cored Coils.—The distributed capacity of multilayer windings such as commonly wound on iron cores is made up primarily of the layer-to-layer capacities.¹ The turn-to-turn capacities can be neglected unless there are only a few layers. When the coil is wound in the normal manner so that the voltage between one end of the adjacent layers is twice the voltage per layer and the voltage between layers at the other end of the same layers is zero, the equivalent distributed capacity C , that can be considered as shunted across the terminals of the

side is $L_p'' + L_s''/n^2$, where n is the step-up ratio of the transformer and L_p'' and L_s'' are the leakage inductances of the individual primary and secondary windings, respectively.

In the textbooks where the leakage inductance for Fig. 70a is calculated for an idealized case, the numerical constant in the equation is found to be 3.2 instead of 2.0. This latter value is obtained by introducing an empirical modifying factor of 0.9 to give more accurate agreement with observed results, as recommended by J. G. Story, *Design of Audio-frequency Input and Intervalve Transformers*, *Wireless Eng.*, Vol. 15, p. 69, February, 1938.

¹ Glenn Koehler, *The Design of Transformers for Audio-frequency Amplifiers with Preassigned Characteristics*, *Proc. I.R.E.*, Vol. 16, p. 1742, December, 1928.

winding is

$$C_s = \frac{0.30PlK}{dT} \mu\mu f \quad (114)$$

where P = mean circumference of coil turns, in.

l = axial length of coil, in.

d = distance between copper of adjacent layers, in.

T = number of layers.

K = average dielectric constant of insulation between layers, in. For paper-insulated wax-impregnated wire, K is approximately 3.

It is to be noted that Eq. (114) gives the distributed capacity of the coil referred to its own terminals. The equivalent value of the same distributed capacity referred to

FIG. 72.—Example of incremental inductance as a function of air-gap length with various d-c magnetizing currents.

another winding on the core is obtained by converting in accordance with the inverse of the square of the turn ratio.

The coil capacity may be reduced by splitting the coil into two or more sections as in Fig. 71, each coil having the same number of layers as the single coil, but fewer turns per layer. In such an arrangement the capacity of each section can be determined by Eq. (114), and the capacities of the various sections then considered as acting in series. Thus subdivision into two equal sections quarters the distributed capacity. Subdivision will, however, increase the capacity of the winding to ground, and so may not improve the performance of the coil.

26. Design of Coils and Transformers with Magnetic Cores.—The design of coils with magnetic cores is normally carried out by a cut-and-try process. A typical procedure is to start by assuming a core cross section on the basis of previous experience. The space available for winding is then allotted to the various coils to be wound on the core, and a tentative selection is made of wire size and winding details in accordance with the insulation required, current to be carried, flux densities desired,

the space available, etc. The performance of the resulting reactor or transformer is then calculated, and the tentative design modified as required.

Winding Data.—The coils of chokes, transformers, etc., using magnetic cores are normally layer-wound by machine. The insulation employed depends upon the size of wire and the voltage between turns and between layers. In the case of small coils using relatively large wire, cotton-covered insulation is often sufficient. With very small wire, such as is used in audio-frequency transformers, the bulk of cotton or silk insulation is proportionately excessive, and the coils usually use enameled wire with paper insulation between layers in many cases.

TABLE 26a.—TURNS POSSIBLE PER SQUARE INCH OF WINDING SPACE, USING ONE LAYER OF PAPER OF PROPER THICKNESS AS SHOWN
Table Based on Average Commercial Practice

B & S gage	Plain enamel	Layer insulation	S.C.E.	D.C.C.	S.C.C.	S.S.E.	S.S.C.	D.S.C.
10	70	.010 in. Kraft	58	58	84	Sq. in. of winding space = $B \times F$
11	86	.010 in. Kraft	72	64	72			
12	105	.010 in. Kraft	90	90	90			
13	126	.010 in. Kraft	111	111	120			
14	162	.010 in. Kraft	143	125	143			
15	191	.010 in. Kraft	180	143	168			
16	238	.010 in. Kraft	208	180	208			
17	310	.007 in. Kraft	253	238	270			
18	396	.007 in. Kraft	304	288	340			
19	473	.007 in. Kraft	378	340	396			
20	621	.005 in. Kraft	490	418	504	1,596 1,974 2,392 2,064 3,591 4,340 5,025 6,068 7,280 8,910 10,272 12,051 13,800 14,112 15,182 16,184 18,270 19,140 21,855 25,092 15,800	1,800 2,200 2,640 3,355 4,020 4,894 5,737 6,864 8,245 10,085 12,051 13,800 15,182 16,184 18,270 22,504	1,800 1,800 2,150 2,538 3,074 3,648 4,216 4,801 5,688 6,794 7,728 8,722 10,379 11,845 14,022
21	750	.005 in. Kraft	572	504	621			
22	952	.005 in. Kraft	605	621	750			
23	1,140	.005 in. Kraft	832	725	980			
24	1,596	.0022 in. Glassine	1,120	928	1,258			
25	1,974	.0022 in. Glassine	1,330	1,054	1,517			
26	2,392	.0022 in. Glassine	1,596	1,258	1,800			
27	3,009	.0022 in. Glassine	1,886	1,440	2,150			
28	3,894	.0015 in. Glassine	2,397	1,720	2,046			
29	4,745	.0015 in. Glassine	2,750	1,978	3,074			
30	5,904	.0015 in. Glassine	3,300	2,254	3,648	10,272 12,051 13,800 14,112 15,182 16,184 18,270 19,140 21,855 25,092 15,800	7,728 8,722 10,379 11,845 14,022	7,728 8,722 10,379 11,845 14,022
31	7,189	.0015 in. Glassine	3,712	2,548	4,216			
32	8,888	.0013 in. Glassine	4,414	2,860	4,801			
33	11,172	.0013 in. Glassine	5,106	3,132	5,688			
34	14,336	.001 in. Glassine	6,150	3,658	6,715			
35	17,324	.001 in. Glassine	6,889	3,065	7,728			
36	21,330	.001 in. Glassine	7,840	4,352	8,722			
37	25,404	.001 in. Glassine						
38	31,878	.001 in. Glassine						
39	42,020	.0007 in. Glassine						
40	51,188	.0007 in. Glassine				Wire gage: 18-20 1/8 in. each end 30-37 3/32 in. each end 38-44 1/16 in. each end		
41	62,746	.0007 in. Glassine						
42	75,302	.0005 in. Glassine						
43	91,800	.0005 in. Glassine						
44	105,534	.0005 in. Glassine						

Data useful in the winding of coils are given in Tables 26a to 26d.¹

Some Factors Involved in the Design of Inductances Carrying Direct Current.—There is a particular air gap for which the incremental or a-c inductance is maximum when the core material and dimensions, the number of turns, and the d-c magnetizing current are fixed. As the d-c magnetizing current is increased, the optimum air gap becomes longer and the corresponding incremental (a-c) inductance less. The behavior in a typical case is shown in Fig. 72.²

This best length of air gap can be obtained by plotting curves of LI^2/V as a function of NI/l for various percentage air-gap lengths, as shown in Fig. 73, where L is the

¹ Data from *Inca Bull.* 3, Phelps Dodge Copper Products Corporation, Inca Mfg. Division.

² H. M. Turner, Inductance as Affected by the Initial Magnetic State, Air-gap, and Superimposed Currents, *Proc. I.R.E.*, Vol. 17, p. 1822, October, 1929.

TABLE 26b.—RANDOM WOUND COILS SUCH AS DYNAMIC SPEAKER FIELDS USING ENAMELED WIRE

Table Based on Average Commercial Practice

B & S gage	Turns possible per linear in. of winding space*	Turns possible per sq in. of cross section	Ohms possible per cu in. of winding space
10	9	85	0.00716
11	10	106	0.0112
12	11	131	0.01733
13	12	160	0.02673
14	14	209	0.0440
15	15	251	0.0666
16	17	319	0.1068
17	19	99	0.168
18	22	519	0.275
19	24	631	0.424
20	27	794	0.672
21	30	987	1.053
22	34	1,255	1.689
23	31	1,525	2.59
24	35	1,933	4.14
25	39	2,428	6.55
26	44	3,080	10.48
27	49	3,891	17.6
28	56	5,001	27.3
29	62	6,200	42.4
30	69	7,797	67.1
31	77	9,800	106.2
32	86	12,210	166.9
33	98	15,740	271.5
34	109	19,520	425
35	121	24,680	677
36	136	31,280	1,082
37	150	38,500	1,681
38	170	49,350	2,720
39	192	63,400	4,370
40	243	67,500	5,910
41	274	85,800	9,370
42	304	104,900	14,410
43	340	130,900	23,300
44	369	153,850	33,300

Linear in. of winding space = B

Sq in. of winding space = $B \times F$

Cu in. of winding space = $[(D^2 \times 0.7854) - (E + 2G)^2 \times 0.7854]B$

No insulation used between layers on random wound coils.
* Percentages give normal space factor.

incremental (a-c) inductance, I the direct current in the winding, V the volume of the core, N the number of turns, and l the length of the magnetic circuit.¹ The envelope of these curves, shown dotted in Fig. 73, gives the maximum incremental inductance per unit volume obtainable for a particular current I , with any given d-c magnetizing force per unit length NI/l , and corresponds to a percentage air gap given

¹ C. R. Hanna, Design of Reactances and Transformers which Carry Direct Current, *Trans. A.I.E.E.*, Vol. 46, p. 155, 1927. An alternative method of design is given by John Minton and Loury G. Maloff, Design Methods for Soft Magnetic Materials in Radio, *Proc. I.R.E.*, Vol. 17, p. 1021, June, 1929.

by the particular curve tangent to the envelope at the point in question. Information such as shown in Fig. 73 is particularly useful where a large number of cores are to be designed employing the same material, since then it is worth while to calculate or measure the special curves required for the particular material involved. Where only a single core is to be designed, it is simpler to determine the optimum air gap by assuming various values and calculating the behavior of each case.

TABLE 26c.—OHMS POSSIBLE PER CUBIC INCH IN LAYER INSULATED COILS*
Table Based on Average Commercial Practice

B & S gage	Plain enamel	S.C.E.	D.C.C.	S.C.C.	S.S.E.	S.S.C.	D.S.C.
10	0.00584	0.00466	0.00466	0.00534			
11	0.00904	0.00756	0.00756	0.00856			
12	0.0139	0.0119	0.0119	0.0139			
13	0.0210	0.01852	0.01852	0.02103			
14	0.0341	0.0301	0.0301	0.0341			
15	0.05075	0.04775	0.04775	0.05075			
16	0.0797	0.0696	0.0696	0.0797			
17	0.1305	0.1074	0.1074	0.1305			
18	0.21	0.1614	0.1614	0.21			
19	0.318	0.254	0.254	0.318			
20	0.526	0.3895	0.3895	0.526			
21	0.800	0.610	0.610	0.800			
22	1.275	0.936	0.936	1.275			
23	1.935	1.412	1.412	1.935			
24	3.41	2.395	2.395	3.41			
25	5.325	3.59	3.59	5.325	4.31	4.86	4.09
26	8.140	5.43	5.43	8.140	6.725	7.48	6.125
27	12.79	8.10	8.10	12.79	10.82	11.35	9.25
28	21.05	12.95	12.95	21.05	16.05	18.15	13.71
29	32.81	18.80	18.80	32.81	24.50	27.43	20.99
30	50.90	28.40	28.40	50.90	37.35	42.05	31.40
31	77.95	40.25	40.25	77.95	54.50	62.50	45.70
32	121.4	60.40	60.40	121.4	82.95	93.75	68.80
33	192.5	87.95	87.95	192.5	125.60	142.20	98.05
34	312.3	133.75	133.75	312.3	194.25	219.00	147.75
35	475	188.65	188.65	475	280.65	330.15	212.00
36	729	271.50	271.50	729	421.00	479.00	301.50
37	1,110			1,110	616	662	452
38	1,750			1,750	882	1,002	649
39	2,920			2,920	1,330	1,568	974
40	4,480			4,480	1,913	2,269	1,365
41	6,860			6,860			
42	10,390			10,390			
43	16,380			16,380			
44	22,850			22,850			

* This table based on using one layer of paper insulation of proper thickness.

In the design of filter reactors, the allowable d-c voltage drop in the winding is frequently the limiting factor rather than the heating of the winding. Under these conditions, a larger core and winding will be required to obtain a desired inductance than would otherwise be the case.

Factors Involved in the Design of Small Transformers.—In the design of transformers, the first step is to determine the volt-ampere ratings of the windings. In the case of

TABLE 26d.—TURNS POSSIBLE PER LINEAR INCH OF WINDING SPACE IN LAYER INSULATED COILS*

Table Based on Average Commercial Practice

B & S gage	Plain enamel†	S.C.E.†	D.C.C.†	S.C.C.†	S.S.E.†	S.S.C.†	D.S.C.†
10	9	8	8	8			
11	10	9	8	9			
12	11	10	10	10			
13	12	11	11	12			
14	14	13	12	13			
15	15	14	13	14			
16	17	16	15	16			
17	19	17	17	18			
18	22	19	18	20			
19	24	21	20	22			
20	27	23	22	24			
21	30	26	24	27			
22	34	29	27	30			
23	38	32	29	35			
24	42	35	32	37			
25	47	38	34	41	42	45	41
26	52	42	37	45	47	50	45
27	59	46	40	49	52	55	49
28	66	51	43	54	57	61	54
29	73	55	46	58	63	67	58
30	82	60	49	64	70	74	64
31	91	64	52	68	75	81	68
32	101	69	55	73	82	88	73
33	114	75	58	79	91	97	79
34	128	82	62	85	99	106	86
35	142	86	65	92	107	117	92
36	158	92	68	98	117	125	98
37	174				126	134	107
38	198				136	145	115
39	220				145	158	123
40	246				155	171	130
41	274						
42	304						
43	340						
44	369						

* These figures are for paper layer insulated coils.

† The percentages represent the space factor under normal conditions.

small transformers used for heating filaments, etc., this is done on the assumption of 80 to 90 per cent efficiency and 90 per cent power factor in the primary and unity power factor on the secondaries. In transformers used with rectifiers, it is commonly necessary to increase the volt-ampere ratings, particularly of the secondary, because the utilization factor is appreciably less than unity. The next step in the design is to select a suitable core area. Experience with small transformers shows that, with ordinary silicon steel, the net area of iron in the core, in square inches, should be at least¹ $0.16 \sqrt{\text{primary volt-amperes}}$. Where a conservative design is to be used, a somewhat larger core area than this minimum is recommended.

¹ The actual cross section of the core will be somewhat larger because of the insulation between laminations. The ratio of the area of iron to the gross area of the core, i.e., the stacking factor, is commonly of the order of 0.9.

The third step in the design is to select a suitable maximum flux density, and then to calculate the turns per volt from the equation

$$\text{Turns per volt} = \frac{10^8}{4.44fB_{\max}A} \quad (115)$$

The notation is the same as in Eq. (108). The maximum flux density $B_{\max}$ should normally not exceed 75,000 lines per square inch for 60-cycle operation and 90,000 lines per square inch for 25-cycle operation. Lower flux densities will reduce the core losses and also the exciting current, but require more copper and iron.

The next step is the determination of the wire size and winding arrangements. It is customary to allow from 500 to 1,000 circular mils per ampere for transformers under 50 watts, and up to 1,500 to 1,750 circular mils per ampere for larger trans-

FIG. 73.—Graphical method of determining best air-gap length.

formers. The insulation between turns and layers, and between the winding and core are arranged in accordance with the voltages involved.

A core is now selected that will have sufficient window space to accommodate the winding to be used. The core is customarily composed of laminations of standard *E-I* punchings. This gives a shell-type core that is assembled in interleaved fashion to eliminate butt joints and minimize the air gap. Standard laminations of this type are available in a wide variety of shapes and in sizes large enough for power transformers of moderate power transmitters. With 60-cycle operation, the laminations are normally 29 gage (0.014 inches) silicon steel.

This completes the formal design of the transformer. The copper losses of the various windings can now be calculated from the currents and resistances of the windings, and the core loss can be evaluated from the volume of the core, the frequency, and the flux density, with the use of curves such as those of Fig. 63. The efficiency and the power that must be dissipated in the transformer can then be readily calculated. Voltage regulation can be determined from the resistances of the windings, and the leakage inductance as calculated by Eqs. (110) to (113). After the design has been completed and the important features of the performance cal-

culated, it is always desirable to examine the design as a whole. This may indicate that modifications will give an improved performance or a satisfactory performance at a lower cost.

27. Magnetic Cores in Audio- and Radio-frequency Coils.—The permeability of iron (and other magnetic materials) drops off slowly with increasing frequency, but the reduction is not great at broadcast and even short-wave frequencies. Thus tests with iron and nickel have indicated that the permeability does not drop to unity until the frequency is in the range 10^9 to 10^{11} cycles.¹ Magnetic cores can accordingly be employed in high-frequency coils. It is merely necessary that the core be subdivided sufficiently that the ratio R_s/X_L in Eqs. (104) and (105) are not too great.

If the core is not sufficiently subdivided (*i.e.*, if tf in Eqs. (104) and (105) is large), not only will the eddy-current losses be excessive, but also the magnetic flux produced by the eddy currents will be sufficient to modify the flux distribution over the cross section of the magnetic material. The net result is then to reduce flux density at the center of the laminations or particles, producing a form of *magnetic skin effect* or shielding that reduces the effective cross section of the core and lowers the inductance.²

Hysteresis does not introduce any special problems at high frequencies, since the ratio of hysteresis loss to inductive reactance is independent of frequency, *i.e.*, the power loss is proportional to the number of cycles per second.

Use of Laminated Cores at Audio and Radio Frequencies.—The silicon steel laminations of No. 29 gage (0.014 inches thick) used in 60-cycle transformers, also have reasonably low losses at the lower audio frequencies. Such laminations are also used in filter reactors for smoothing rectified 60-cycle currents, for the cores of audio-frequency input, interstage, and output transformers, and in some cases in wave filters having pass bands below 1,000 to 1,500 cycles. At higher frequencies it is necessary to use laminations thinner than 0.014 inches if excessive eddy current losses are to be avoided. Thicknesses as small as 2 mils have been used, and in this way magnetic cores can be obtained that have fairly low loss even at moderately high radio frequencies.³ Thin laminations of this character were at one time used to a considerable extent in communication work, but have now been largely displaced by cores of compressed powdered magnetic material.

Coils Using Cores of Compressed Powdered Magnetic Material.⁴—The most effective way to employ magnetic material at the higher audio frequencies and at radio frequencies is to mix the magnetic material in the form of a very fine powder or dust

¹ For a discussion of the effect of frequency on permeability see G. W. O. Howe, Permeability at Very High Frequencies, *Wireless Eng.*, Vol. 16, p. 541, November, 1939.

² Analyses are available for calculating the extent of the magnetic shielding in the case of laminations of rectangular cross section. See K. L. Scott, Variation of Inductance of Coils Due to the Magnetic Shielding Effect of Eddy Currents in the Cores, *Proc. I.R.E.*, Vol. 18, p. 1750, October, 1930.

In practice there is a thin surface layer, normally about 1 mil thick, having lower permeability than the main body of the laminations. This surface film is apparently the result of the rolling operation used in producing the lamination, and is not affected by annealing. Its effect is important at high frequencies where the penetration of the flux is small, and at these frequencies the simple theory will accordingly fail to give correct results. Where the layer is important, it can be removed by etching.

For further discussion of magnetic skin effect, with particular reference to the oxide film layer, see E. Peterson and L. R. Wrathall, Eddy Currents in Composite Laminations, *Proc. I.R.E.*, Vol. 24, p. 275, February, 1936; M. Reed, An Experimental Investigation of the Theory of Eddy Currents in Laminated Cores of Rectangular Section, *Jour. I.E.E.*, Vol. 80, p. 567, 1937; also *Wireless Section, I.E.E.*, Vol. 12, p. 167, June, 1937; M. Reed, Losses in Ferromagnetic Laminas at Radio Frequencies, *Wireless Eng.*, Vol. 15, p. 263, May, 1938.

³ Information on losses of thin silicon steel laminations at radio frequencies is given by Thomas Spooner, High Frequency Iron Losses, *A.I.E.E. Jour.*, Vol. 39, p. 809, September, 1920; Leon T. Wilson, The Behavior of Iron in Alternating Fields of between 100,000 and 1,500,000 cycles, *Proc. I.R.E.*, Vol. 9, p. 56, February, 1921.

⁴ W. J. Polydoroff, Ferro-inductors and Permeability Tuning, *Proc. I.R.E.*, Vol. 21, p. 690, May, 1933; V. E. Legg and F. J. Given, Compressed Powdered Molybdenum Permalloy for High-quality Inductance Coils, *Trans. A.I.E.E.*, Vol. 59, p. 1940; W. J. Shackleton and I. G. Barber, Compressed Powdered Permalloy, *Trans. A.I.E.E.*, Vol. 47, p. 429, April, 1928; Buckner Speed and G. W. Elmen, Magnetic Properties of Compressed Powdered Iron, *Trans. A.I.E.E.*, Vol. 40, p. 596, 1921.

with an insulating binder, and then compress the combination into cores of the appropriate form. In such cores the insulating binder surrounds the magnetic particles, thereby giving a very complete subdivision of the core, and low eddy-current losses.

The size of the magnetic particles and the amount of magnetic material in proportion to insulating binder control the effective permeability of the resulting core and also the frequency at which the coil Q is highest. The smaller the particles the less the effective permeability, and also the higher the optimum frequency. The effective permeabilities used in practice range from 75 to 150 for coils designed to be optimum at

FIG. 74.—Variation of coil Q with frequency for several coils using toroidal dust cores.

audio frequencies, to values of 4 to 6 for coils intended for operation at frequencies of 500 to 1,500 kc.

The materials most frequently used in powdered cores are iron, permalloy, and molybdenum permalloy. Molybdenum permalloy has the lowest losses and hence gives the highest Q , while iron has the highest losses. The total core loss in any case will be quite small, however, provided that the core is used in its proper frequency range.

Coils employing powdered iron cores are often used in the intermediate-frequency and radio-frequency tuned circuits of radio receivers, up to and beyond the broadcast range.¹ For such radio-frequency application, the cores are commonly in the form of

¹ An excellent discussion of the properties of such cores and of the measurement of these properties is given by D. E. Foster and A. E. Newlon, *Measurement of Iron Cores at Radio Frequencies*, *Proc. I.R.E.*, Vol. 29, p. 266, May, 1941.

cylindrical slugs upon which the coil is wound. At intermediate (175 to 460 kc) and lower frequencies, the use of iron-dust cores makes it possible to realize a decidedly higher Q in a very small coil than is obtainable by the use of an air-cored coil of comparable size. In the case of intermediate-frequency transformers, magnetic cores also have the advantage that the resonant frequency can be controlled by adjusting the position of the core, thereby eliminating the trimmer condenser otherwise needed.

Toroidal coils using cores of compressed powdered molybdenum permalloy, permalloy, and iron-dust rings are widely used in telephone work for loading coils, in wave filters, etc. The losses of such coils are quite small, as shown by Fig. 74, with molybdenum permalloy the most desirable core material and permalloy next. Not only are the losses lower than with iron but also the inductance is less affected by the intensity of the magnetizing force, and the harmonics and cross-modulation are also lower than with iron. Another advantage is that a large d-c magnetization has less permanent effect upon the inductance.

CAPACITY AND CAPACITORS

28. Definitions of Capacity.—Any pair of electrical conductors that will store electrical charge when a difference in potential is applied to them constitutes a condenser. The defining relation for capacity may be written

$$C = \frac{Q}{E} \quad (116)$$

where C is the capacity in farads, Q is the charge in coulombs, and E is the potential in volts. Capacity may also be defined in terms of the energy stored in the electrostatic field about the condenser plates; thus

$$W = \frac{1}{2}CE^2 \quad (117)$$

where W is the energy stored in watt-seconds (joules), C is the capacity in farads, and E is the potential in volts. Since current is the time rate of change of charge, the current flowing into any condenser is proportional to the time rate of change of the potential across it. This relation is given by

$$I = C \frac{dE}{dt} \quad (118)$$

where I is the current in amperes, t is the time in seconds, and the other symbols have their previous significance.

Capacity has the dimensions of length when expressed in electrostatic units. As a result the capacities of similar condensers are proportional to the first power of their linear dimensions. Doubling all dimensions of a condenser doubles its capacity.

29. Condenser Losses and Their Representation.—A perfect condenser when discharged gives up all the electrical energy that was supplied to it in charging. Actual condensers never realize this ideal perfectly but, rather, dissipate some of the energy delivered to them. Most of the loss in ordinary condensers occurs in the dielectric. Other ways by which energy can be lost in a condenser are from the resistance of the leads and metal plates, from leakage resistance between plates, and as a result of corona.

The merit of a condenser from the point of view of freedom from losses is usually expressed in terms of the power factor or phase angle.¹ The power factor represents the fraction of the input volt-amperes that is dissipated in the condenser, while the phase angle is the angle by which the current flowing into the condenser fails to be 90°

¹ The merit of a condenser is occasionally expressed in terms of the condenser Q , which represents the ratio of the capacitive reactance of the condenser to the equivalent series resistance. For all practical purposes the condenser Q is the reciprocal for the power factor.

TITLE: LECTURE 3: PULSE VOLTAGE CIRCUITS

AUTHOR(S): Walter Lee Willis, AP-1

SUBMITTED TO: Power Conditioning Course

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Lecture 3
PULSE VOLTAGE CIRCUITS
by
W. Willis

PULSE VOLTAGE CIRCUITS

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PULSE VOLTAGE CIRCUITS

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Lecture 3
PULSE VOLTAGE CIRCUITS
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I. SCOPE

The problems associated with very high voltages may be avoided in part by a variety of pulse-multiplication techniques. These methods are quite varied but share two features, the use of modest dc voltages and the use of switching techniques, to produce in some manner a vectorial transient addition of voltages.

Many such methods exist, but only the Marx bank, spiral generator, inversion generator, and Blumlein will be addressed here in detail. Two other methods will be given a brief discussion as matters of interest. An important system, the pulse transformer, is the subject of a separate discussion. Also, some aspects of transmission lines will be taken for granted in this lecture and justified elsewhere.

II. MARX BANK

A. History

The history of this generator is interesting, particularly to those who are actually struggling to build a Marx bank, and also because the terminology describes circuits as an Erwin C_2 or Erwin R_3 , which recognizes Erwin Marx who described these generators in 1923. At about that time quantum mechanics also began to appear in publication.

Rather good performance was obtained even in those days. In 1926 Burov, of Russia, described a 100-kV pulse with a 10-ns duration. He did not describe the measurement technique. However, a 10-ns duration pulse, even if it is triangular in waveform, yields a 5-ns risetime. One hundred kilovolts in five nanoseconds is still respectable.

Burov used, incidently, a Hertzian shortwave peaking gap and is one of the earliest references for use of a peaking gap to sharpen the output of a Marx bank.

By 1938, Schering and Raske of Germany reported a 400-kV pulse with a 10-ns risetime. This is 40 kV/ns, which is also satisfactory. They described the five stage Marx bank as using CO₂-pressurized spark gaps, and there was no mention of sharpening gaps to obtain that performance. The first reference in American literature seems to be R. C. Fletcher, in 1949, who used a gaseous peaking gap and reached 20-kV output, which is not particularly high voltage but 0.4-ns risetime yields 50 kV/ns rate of rise. Again, there was no mention of how the output was measured. Other information reveals that he built his own special spectroscope for doing so, but it was never described in print.

Figures 1 and 2, from J. C. Martin, Nanosecond Techniques, show the state of the art in recent years. The note on Figure 1 indicates where Los Alamos Scientific Laboratory (LASL) was in 1975, about 8 years behind the state of the art.

There are two points that should be mentioned here. First, these systems are not commercially available but show that somebody worked extremely hard for a special application. Second, when you start looking at numbers, this can be misleading. Megavolt systems are required to get this kind of performance, not 100-kV or 50-kV systems, for reasons that will show up very quickly.

Fig. 1. Marx Bank State of the Art

Fig. 2. Marx Bank Erection Time

Figure 2 shows the erection time per megavolt for the best design and the accent is on best. LASL is not noted on this particular one. Erection time involves the manner in which Marx banks are fired, a critical parameter of Marx banks that has a lot to do with the mechanical layout and design. Erection time is not given by formulas very handily, but is a good measure of how well a bank will work. Again, in Fig. 2 it is not stated whether the erection time includes a sharpening gap or not. As an example, consider a megavolt bank in terms of 100 kJ. Using the relationship for energy storage, and a simple calculation:

$$U = \frac{1}{2} C V^2$$

so that

$$C = \frac{2 U}{V^2}$$

which, at 1 MV, leads to

$$C = \frac{2 \times 10^5}{10^{12}} = 0.2 \mu\text{F}$$

The Marx bank, which means quite a few capacitors in a series, then requires 100 nH in this system at 1970 performance levels shown in Fig. 1. That is impressive for a 10-stage Marx with 2- μF capacitors, only 10 nH per stage, including the inductance of the capacitor, the hook-up, the spark gap, etc. Now if this is operated as a 100-kV system, the same relationship only allows 1 nH per Marx bank stage. That is why at this point the performance curve does not describe physically practical systems. Very high voltage Marx banks are required to yield this kind of performance.

B. Basic Configurations

The fundamental concept of the Marx bank is quite simple--charge capacitors in parallel and discharge them in series. Figure 3 shows three basic arrangements. Figure 3a is perhaps the most economical of parts, since the output switch may be optional in some cases, where a modest level of dc on the output is acceptable. If the load happens to be a gas discharge, and will operate with some dc on it, the designer can save a switch. It is not a usual case, but it is possible. The circuit gives the same voltage polarity out as is put in if polarity is of interest. Normally, this need is avoided by reversing the power supply, but sometimes this is difficult for one reason or another. It has a drawback with respect to triggering, which will be discussed later. We are more interested in charging at this point. However, the first gap in the string goes high upon firing, producing a high-voltage transient phase in the trigger system.

Figure 3b inverts the supply voltage sign and, in this case, the output is at ground potential except during the charging transient. One advantage lies in the fact that one terminal of S1 is at ground potential. This may be an advantage in the triggering design because high-voltage isolation is not a problem. The Ion Physics MiniMarx line is of this design and employs a trigatron-type gap as the first switch. It generates a 100-kV pulse with a 2-ns risetime. The trigatron is a type of spark gap that is usually triggered with respect to ground. Figures 3a and 3b are drawn in the ladder system. Figure 3c is somewhat different. This is an even further step away from high-voltage dc supplies. It uses both a positive and a negative supply to charge and there are only half as many switches as there are in the single polarity case. Switching is not necessarily simpler. If the same voltage output is required as before, then the switch has to hold off twice as much voltage as in the other Marx banks, so it uses fewer switches but it is a more difficult switch to build. Do not be misled by apparently concealed advantages. A drawback on the output point is that the full erected

Fig. 3. Marx Bank Circuit Configurations

voltage appears across the output resistor. It is difficult to find a megohm resistor that will withstand a megavolt, even for a short time. One reason why these circuits are typically built ladder style is because when they erect, the voltage across any one resistor is the stage voltage rather than the bank voltage. The resistor to ground on each stage does have advantages for charging speed. Visualize the charging components as inductors rather than resistors. There is an obvious charging time problem for banks of 10 or 12 stages. Marx banks typically are not a high-repetition rate device. The old timers were after very high voltages for things like E-beam systems that tend to melt down, if operated at a high-repetition rate. If a fast Marx bank is desired, do not use resistors. Serious inefficiency problems appear, for one thing. Another is just getting the charge on the capacitors. Go to a set of inductors with another set of constraints. It is still possible to charge with a dc supply. In that case the first inductor normally becomes very large with respect to the rest of the system components, so that when the bank is fired, it does not dump back into the power supply. It makes the calculation of how long it takes to charge much simpler than for the RC case. Also, there is no real reason why, given inductors and capacitors, the Marx bank could not be designed as a high-voltage pulse-forming network (PFN).

C. Charging Methods

Figure 4a is an n-stage Marx bank with a step input U_t/n , where U_t is the desired output. From a charging analysis standpoint it does not really matter whether the resistors are given as $R_0/2$ or are replaced by R_0 with the lower elements eliminated. The output voltage available at any instant is, theoretically, the sum of the individual stage voltages. Thus, we have an RC line in which, except for the first stage, all forcing functions are time and position dependent. Two solutions are conveniently available. One relationship, according to Fitch, is

$$T_{CH} = R_0 C_0 n^2$$

Fig. 4. Marx Bank Charging Performance

Fig 4 A

$$V_S(t) = \sum_{i=1}^n V_i(t) \cong 1 - \frac{8}{\pi^2} \exp \left\{ - \left(\frac{\pi}{2} \right)^2 t \right\}$$

Fig 4 B

$$T_{CH} = R_o C_o n^2$$

where T_{CH} is the lag of the n th stage with respect to the input stage.

G. W. Swift presents a power series analysis that he takes to the limit as n goes to infinity. This is the relationship shown on Fig. 4b with a few finite cases shown. The $R_t C_t$ time constant is the total R and total C of the circuit shown. The fact that five or more stages are effectively infinite makes this useful for calculation of banks of many stages. Note that the time t is given in terms of the $R_t C_t$ time constant. For a simple case, take $1\text{ M}\Omega$ and $1\text{ }\mu\text{F}$ for R_o and C_o , which are typical numbers, and 10 stages. A 10-stage Marx of that type takes about 100 s after the first stage is charged before the last stage charges, almost 2 min. Now is that consistent? In Swift's analysis in the same case, R_o is 10^7 because there are ten $1\text{-M}\Omega$ resistors and 10 of C_o in parallel, for a total of 10^{-5} F . Thus, $t = R_t C_t$ and that still comes out to about 100 s on the normalized scale of Fig. 4b. Swift's analysis shows that the bank would be 93% charged at the end of that time constant. The agreement between the two methods is good--only a few per cent difference. In either analysis, something better than 100 s is required to get the bank charged. The expression given in Fig. 4b is useful. Express time in terms of the RC time constants and solve for the time required to charge to 98%, 94%, or whatever is desired. The analysis does not describe the final stage. It says what the output would be if all these capacitors are in series with switches as shown and fired at a given time. In that curve, if the Marx is fired at 100 s, 93% is the total output because the input capacitors are charged but the ones behind it, although nearly charged, are slightly lower. It really describes the output voltage of the Marx bank, not the final stage voltage. However, a designer probably would not use an RC bank at a very high repetition rate, but would use an inductive system.

In the case of an LC circuit, recourse may be taken to the PFN characteristics that will be covered in a subsequent lecture. Ideally, such a network has a characteristic time given by

$$\tau_{\text{CHG}} = \sqrt{L_{\text{MARX}} C_{\text{MARX}}}$$

and discharging into a matched impedance requires a time 2τ . In mismatched cases, oscillations occur that extend this time. More commonly, some command charge system is employed in which an external inductor, large with respect to the Marx inductors, is resonated against the total network capacitance. Charging is usually accomplished in a half cycle of the resonant frequency, leading to

$$\tau_{\text{CHG}} = \pi \sqrt{L_{\text{TOTAL}} C_{\text{MARX}}}$$

The point here, neglecting the switches, is that normally the pulse out is much, much faster, even at a kilohertz. There is perhaps 1-ms charge the bank. The output falls typically in a microsecond or in that order, so that there are orders of magnitude time differential between the discharge period and the charging period, which means the input inductance can be huge compared to the one in the Marx bank, and is used for isolation. So, effectively, the Marx inductances almost are not there. Simply look at the bank as a total capacitor and a charging inductance. Put a voltage step on it and it starts to ring in the classic resonating circuit fashion. It charges up in half a cycle at most--in a period that can be made fast compared to an RC time constant.

D. Discharging of the Marx

The inefficiencies of charging have a matching set of inefficiencies that have to do with the discharge. Figure 5 reveals that, in general,

Fig. 5. Marx Bank Discharge Relationships

$$\tau_D = \frac{R_o C_o}{2}$$

$$L = \frac{2 Z_o \tau}{n \left[\frac{i_{LC \text{ MAX}}}{I_L} \right]}$$

$$\frac{i_{LC}}{I_L}$$

a stage capacitor is paralleled by two charging impedances Z_0 in the resistive case, the self-time constant is just

$$\tau_{\text{DISCH}} = \frac{Z_0 C_0}{2}$$

and must be long compared to the output pulse for good efficiency. The two end cases are best, since each has only one resistor and therefore has twice the impedance. With many stages, that factor disappears in overall analysis. The point is the existence of a discharge time constant. The Marx discharges very quickly into the load with a much slower competing process because the impedances are large compared to the load impedance. Nonetheless the discharge is in there, drawing energy during the pulse and drawing current out of the capacitor back through the charging resistors, at the same time the bank is delivering current to the load. The competing process is an inefficiency. It has more than one repercussion and it can involve heating problems in the system.

When inductances have been used as charging impedances, the behavior is similar except for the appearance of resonance in place of the simpler RC case. The analysis in Glasoe and Lebacqz is straightforward and summarized here.

A given mesh will attempt to resonate with a current given by

$$i_{LC} = V_N \sqrt{\frac{C}{L}} \sin \frac{t}{LC}$$

where L and C are the mesh values of the Marx and V_n is the stage voltage. For good efficiency, the self-ring period T is chosen much greater than the discharge period τ . Thus, the sine of the angle may be replaced with the angle and

$$i_{LC} \approx V_N \frac{C}{L} \frac{t}{\sqrt{LC}} = \frac{V_N t}{L}$$

and t has a maximum value of τ , since the process terminates when the bank is discharged.

If the Marx discharges into a matched impedance (PFN-like behavior), the Marx current is

$$I_1 = \frac{n V_N}{2 Z_N}$$

and one can relate the efficiency by the ratio of i_{LC}/I_1 .

This case is more apt to be discharge inefficient than the resistive case. The resistive case normally has the loop currents swamped, almost nothing goes into the resistors.

E. Marx Triggering and Erection

Up to this point, the Marx circuit has seemed quite simple and straightforward. The subtleties appear in the relationships between triggering, mechanical assembly, and erection. Very little has been mentioned previously about how the necessary switch closures are accomplished. Although other switches have been used, spark gaps will be assumed for this discussion.

(Two papers, Marx and Marx-like High-Voltage Generators by R. A. Fitch, and Overvoltage and Breakdown Patterns of Fast Marx Generators by R. W. Morrison and A. M. Smith, have been drawn upon for the following material.)

This gets into the part of the Marx bank that is both nonquantitative and frustrating. Discussion will start with the backbone circuit, Fig. 6. It is the relationships among the mechanical assembly, triggering

Fig. 6. Backbone Circuit

$C_{g1} \triangleq$ gap electrode capacitance

$C_o \triangleq$ stage energy storage capacitance

BACKBONE CIRCUIT

FIGURE 6

and erection (closing of all of the switches) of the Marx bank to number of stages times the stage voltage, hopefully, of output voltage. Other switches have been used but spark gaps are assumed all the way through. Very few Marx banks are not made with spark gaps as a matter of fact, so there are not many exceptions to worry about. Assume now that somehow we trigger the first gap. Thus far there has been little discussion about the triggering. All the gaps here are two-element gaps, and the voltages can be increased until something collapses or the first one can have low pressure, or somehow be designed to be overvolted. It is possible to build a husky trigger system, pulse the first capacitor, overvolt that stage, and launch the bank that way. Argue that only the first gap is triggered on purpose.

When the first gap fires, all voltages around the loop must add up as before, to zero, by Kirchoff's Law. (Gaps are opposite in sense to capacitors.)

$$\begin{aligned} N V_o &= N V_{\text{gap}} && \text{initial} \\ N V_o &= (N - 1) V_{\text{gap}} && \text{one gap fired} \\ N V_o &= (N - 2) V_{\text{gap}} && \text{two gaps fired} \end{aligned}$$

or, alternatively, the voltage across an unfired gap should be

$$V_{\text{gap}} = \frac{N V_o}{N - n}$$

where n is the number of fired gaps.

This model is, fortunately, too simple. It predicts an accelerating rate of overvoltage firing, which is not always observed, does not predict order of firing, and is pessimistic for many-stage Marxes. The latter is due to the small overvoltage per stage produced by triggering one gap.

Figure 7 shows the physical layout and equivalent circuit of a simple stacked Marx. (In Fitch's notation, a C1 Marx, that is, capacitively coupled, single column.) Some simplification is still present. C_g is the gap capacitance, C_c the capacitance between stage capacitors, and C_s the stray capacitance to ground.

Because of voltage holdoff and assembly considerations, C_s is usually very small, only a few picofarads. C_c can be quite different. In order to reduce height and inductance, the capacitors C_o may be quite close. Two Maxwell pulse capacitors may exhibit up to 50 pF or so for C_c , and 1.25 μ F, 4-kV CSI metal cased units can have up to several hundred picofarads between the electrodes of adjacent units.

A trigger is shown in this particular case. That trigger is commonly brought in through a small capacitor for voltage isolation, and 250 pF is a typical number for the trigger. Assume a trigger is applied. In that case the lower electrode of CO_2 is driven with respect to ground to +V. Consequently, the upper electrode of CO_2 jumps to 2 V. It appears that this will overvolt gap C_{g2} because the charging string has the top of C_{g2} at dc ground before triggering. Unfortunately, the 2-V pulse sees divider C_c and C_{s3} to ground. The equation says that

$$V_{cg2} = V \left(1 + \frac{C_{s3}}{C_{s3} + C_c} \right)$$

If the stray capacitance is small and the coupling capacitor is large, what happens is obvious. At first V_{cg2} is small and the divider action takes it off. The step voltage will get there. The charging resistors are still present and eventually, since the charging resistor returns to ground, C_{s3} will discharge and the voltage will appear across C_c and C_{cg2} . Sooner or later the gap will go, but it goes with an RC delay and the capacitive divider slows it down. Fitch comments that if the bank erects at 60% of the self-break on the gap it is

Fig. 7. Layout and Circuit Relationships

FIG. 7

a healthy bank. It should run dependably at 70% of self-break and up. But if the divider is bad enough, it may not even erect. The transients may all filter off on RC time constants down the line to where the bank ends up on the backbone circuit with the overvoltage distributed equally across all the gaps and none of them fired. The solution for this is to go to what Fitch would call a C2 Erwin Marx, as shown in Fig. 8a. A zigzag pattern, mechanically, leaves the strays to ground from the capacitor the same as they always were. The stray coupling capacitor jumps stages, bridging two gaps in an alternating pattern. This can be done in a spiral of three and get a C3 Marx, etc. In practice, three is about as high as people normally go. We will come later to the use of resistors instead of capacitors and with physical components start calling these R1, R2 and R3 Marxes.

The behavior is a bit different in the C_2 case. Now when the first gap is triggered, the lower electrode of CO_2 jumps to 2 V, C_c is no longer a divider with respect to the next gap up. It does see C_s and has to charge it through the impedance of the stage. Basically, the voltage appears across C_{g2} , the capacitive divider has gone away and the next gap fires quite nicely. In a Russian paper describing an Arcadex/Marx, quite large strays are inserted on purpose to hold the upper electrode of C_{g2} at ac ground and virtually the entire voltage appears across the second gap to fire it. This looks very elegant; it is going to go right up the line, firing 1, 2, 3, 4, and 5. Again life is not that simple. To preview coming attractions, look what happens when lower C_{g2} has been driven to 2 V, fired C_{g2} , and now lower C_{g2} tries to jump to 3 V. Unfortunately, C_c has coupled the 2 V point around to the top of C_{g3} and partially unloaded the gap intended to fire next. The interactions start to chase themselves up the ladder.

This problem has been analyzed in detail by Morrison and Smith. The complexity of the problem can be seen by examining their analysis of a 10-stage Marx in which the first gap is triggered and the interactions of the capacitive dividers are carefully studied (Fig. 8b). They have

Fig. 8. Marx Triggering Sequence

IRWIN C2

FIG. 8A

Fig. 9. R3 Martin Marx

R3 MARTIN

FIG. 9

assumed ratios, by putting in a coupling capacitor large enough to give a ratio to the gap capacitor of 10 and a stray capacitance that is twice the gap capacitance. The stray is fairly easy to keep small, because normally the capacitors are well away from the ground plane. The coupling capacitor normally is not small because in fighting inductance the capacitors are close. These are given as $C_c/C_g = 10$ and $C_s/C_g = 2$. The analysis assumes firing the first gap and following the transients up a 10-stage Marx through all the capacitive dividers. After triggering gap 1, gap 2 is the most extremely overvolted of all gaps, so it will fire. When 2 breaks down, the voltages in the capacitive divider vary. At some points, for example, gap 4, they are inverted and at some they are not. But 3 is still normal and most overvolted. This, then, keeps on going to 3 then 4. When the fourth gap breaks down, the problem comes "What Happens?" Gap 5 is high and it goes. When 5 goes, 6 is high, 8 is high, 10 is high, and the question is "What goes next?" The outcome is that for different ratios in these dividers, the Marx bank may start erecting from the bottom, turn around and erect back down from the top and meet in the middle, or other kinds of unexpected patterns can occur. Now that is not a trivial consideration. Take any two capacitors with a little space left between them. A charging current flows and there is some inductance associated with that current. The bank has large capacitors with respect to the coupling values. This can result in a version of what we will hit later, in resonant charging. The behavior depends on whether the capacitor has a metal can or dielectric can. But the circuit can swing up to something like four times the stage voltage across that stray coupling capacitor. If it is closely spaced, arcs in the air or oil dielectric can occur. There will be arcs from stage to stage and firing patterns that are very hard to predict, in terms of voltages that are going to appear.

One consequence of this behavior is the use of mid-plane gaps in Marx banks. For triggering purposes, a nonsymmetric gap is desirable. However, this presupposes knowledge of the electrode voltages, a condition that is not, in general, met in banks of many stages. If the bank

uses dielectric case capacitors, and they often do, this may punch right through the dielectric case. The Maxwells, for example, have a preferred side up from the ground. But if the preferred side up is in the overvolted stray side, and four times rated voltage occurs, punch-through may destroy it. So, how is a bank made to operate dependably? This brings up the point of triggering.

The use of more than one triggered gap is common in many Marx banks in order to reduce jitter. This, combined with back coupling, yields a more predictable system. Figure 9 is an R3 Martin Marx (named for J. C. Martin, who pioneered the use of three-electrode gaps in Marx banks) in which the first three gaps are externally triggered. This circuit depends upon triggering by maintaining the trigger electrode potential while the electrodes change voltage.

Up until now these have been called Erwin Marx, because the Erwin Marx is a two-electrode system. The Martin Marx is characterized by spark gaps with trigger electrodes. In one of these a trigger system fires the first three stages so that at the top of C_4 the voltage rises to 4 V for overvolting on subsequent stages. The gaps have trigger electrodes and the circuit design reveals what is going on. The fourth gap trigger electrode is held down to the voltage of the bottom stage, which is going to be V, and the lower main electrode will jump to 4 V. What actually happens in this type of circuit is that the trigger voltage changes very little while the main electrode swings. In the first stages swing the trigger electrode to fire the bank, from there on up hold the triggers down to the potential of preceding stages with discrete components while the electrode itself swings away for triggering. These are quite dependable in jitter and triggering. There are a couple of penalties to pay. For example, shown is a very simple triggering with three input capacitors. When the plasma forms in the gap arc, the trigger electrodes are tied to stage voltages and are fed back down into the triggering system. It makes the triggering system a bit nasty. Also, voltages of 3 V, 2 V, and 1 V are summed, yielding miscellaneous

sloshing in the trigger system. Typically, there is some resistance in series with each of the trigger capacitors to limit the surge currents that flow in the coupling capacitor. Only $50\ \Omega$ or so do not affect the system (because they do not draw current until the arc is actually formed, at which point the triggering is over). Do not, however, put the trigger resistor back at the other end of the cable because then the cable capacitance and resistor produce an RC circuit that will slow the trigger pulse. Part of the precision of these systems is related to the risetime of the trigger generator. There is yet another approach for which no figure is included. A high-energy Marx runs to large capacitors and large strays are associated with them. It is not unusual to build another small Marx bank, which has the same basic characteristics and cross couple all the way up between stages. This small, very low energy Marx bank is very, very fast. It runs in parallel and you trigger the entire Marx bank with another Marx bank. That is an expensive and difficult way to go, but can be the ultimate in precision, because the risetime for erection is set by a very small fast Marx rather than a large sluggish Marx.

F. Output Considerations

G. A. Mesyats, in a translation of "Formation of Nanosecond Pulses of High Voltage," presents an interesting analysis of the effect of the spark gaps on the Marx output waveform. This circuit model (Fig. 10) replaces the bank with C_0 , an equivalent capacitor, which is the stage capacitance divided by the number of stages. An inductance associated with the Marx circuit is given. There are two resistors in the model. R_L is an equivalent fixed resistance. Sometimes a little resistance is added to a Marx bank for damping purposes to prevent ringing. There is always some fixed resistance in the connections. R_{sp} is a spark gap resistance and there is an output load resistance paralleled with a load capacitance, probably a stray. When the gap resistance is pertinent, he approaches the problem with

Fig. 10. Mesyats Circuit Model of Marx Bank

$$C_0 = C/n$$

$$U_0 = nU$$

R_L = Equivalent Fixed Resistance

FIG. 10

$$R_{sp}^2(t) = \frac{pd^2}{2a \int_0^t i^2 dt}$$

where $R_{sp}(t)$ is the gap resistance as a function of time,

p is gap pressure in atmospheres

d is gap spacing, in centimeters

i is current in amperes

a is a gas constant, ≈ 0.8 to $1 \frac{\text{atm cm}^2}{\text{V}^2 \text{ sec}}$

for air or nitrogen.

For a given breakdown voltage, as pressure goes up, R_{sp} goes down. (Paschen's Law)

If the spark resistance is considered, the output pulse amplitude becomes

$$U = U_0 \left[1 - \left(\frac{1}{\sqrt{1 + 2B}} \right)^2 \right]^{\frac{3}{2}} \sqrt{\frac{1 + 2B}{2B}}$$

where

$$B = \frac{R_{sp} C}{\Theta}$$

and

$$\theta = \frac{2 \rho d^2}{a U^2}$$

The pulse front width is defined as the ratio of pulse amplitude to maximum front curvature and is given by

$$t_f = \frac{128}{27} \frac{\left(1 - (2B)^{-\frac{1}{3}}\right)^{\frac{3}{2}}}{1 - \phi(A)} \quad \text{for } B \geq 5$$

where

$$A = \frac{L}{\theta R_H}$$

$$\phi(A) = 0.157A - 0.0108 A^2 + 0.00017 A^3$$

A small θ (called the spark time constant) yields a larger, faster rising pulse. Examples are:

$d \approx 1$ cm at atmospheric pressure

$t_f \geq 10$ ns, even with no circuit inductance

When the gap pressure is high enough, the output is determined by the remaining parameters. If C_H , the load capacitance, is neglected

$$t_f = 2.2 \frac{L}{R_H} \quad 10 \text{ to } 90\%$$

Further, if R_H is $\gg 2\sqrt{L/C_0}$, the half-width pulse height is $0.7 R_H C_0$. This defines L in the sense that a 1-ns risetime requires an L of $10^{-9} R_H$. For P it is the spark resistance that he has calculated. C is the total bank capacitance for parameter θ , and θ , in turn, is derived out of PP_2 .

Mesyats discusses the curvature of the front, where we would be more inclined to speak of the risetime. He calls θ the spark time constant, and it is the key parameter of this discussion. The smaller θ is, the faster the Marx bank is going to be. It does more or less agree with other relationships used to describe gap behavior. There is a relationship between geometry, pressure, and the voltage a gap will hold off. Going up in pressure increases θ , but the down-quadratically-with-voltage relationship agrees with other relationships that show a preference for small spacings and high pressures. Mesyats worked out a couple of examples in his paper. He took a case where d was a centimeter and p was an atmosphere. This is about a 30-kV spark gap so it is not an unreasonable design. He goes through all these equations and comes out with 10 ns using that kind of a gap for the Marx risetime. This is the minimum time, even if the inductance for his model is zero. If the pressure is set high enough, the time can be reduced to 1 ns. If you take this model, when the gap pressure is high enough, other parameters in the circuit determine risetime, which is the usual result in any of these spark gap switch systems. Increasing pressure and voltage with the spacing reduced generally yields a better system. If you are in a position to design your own spark gaps, you are better off to increase pressures and voltages, and decrease spacing. The key relationship defines L and says that a nanosecond risetime requires inductance of 10^{-9} times the load impedance. If the load impedance is very low, then a fast risetime is difficult. Another point -- can you neglect the stray capacitor C_H ? Current flow is given by $C(dv/dt)$. This current drops some voltage back in the Marx impedance and in the case of very,

very fast Marx banks, dv/dt can be a large number. Even small capacitors will start loading back in the system. There is an important comment related to that. One can very easily, in these pulse systems, come up with the situation where output voltage and output current wave forms do not remotely resemble one another. Sharpening gaps and so forth can produce a voltage pulse with virtually zero risetime. Then comes the discovery that the current pulse is performing differently because it is determined by the inductance of the system and the impedances scattered around. Power delivered to a load may be well out in time.

One way around this is to pulse charge a low impedance intermediate storage element with an output switch. If the bank is going to erect to 100 kV, try to set the output switch to break down at, say 95 kV. The Marx pulse is rising in some fashion for 100 kV. At 95 kV, the sharpening switch closes and what the outside world sees is something that looks extremely sharp. This is very common practice in E-beam systems where there is usually some kind of transmission line that the Marx charges in 2 or 3 μ s. On the output there is a spark gap that fires very quickly. When that breaks down, looking back into the system, you do not see the Marx bank, but the transition section, which has a very low impedance. This is capable of delivering a current pulse, similar to the voltage pulse, for many purposes. It is one way of getting around all of the inductance associated with the Marx bank and of stiffening up the risetime of the system.

III. INVERSION GENERATOR

Let us now consider the inversion generator shown in Fig. 11. It is related to the Marx bank and what Fitch calls a vector addition system. It has several advantages and disadvantages. It resembles a Marx bank but the switches are not in series. The charging circuit is not shown, but the capacitors are charged in alternating sequence, minus or

Fig. 11. Inversion Generator Circuit

plus, plus to minus, etc. One disadvantage is that there is no way to trigger one switch from the next and there is no capacitive coupling between the stages. They do not see one another from a triggering standpoint, which means on this sort of circuit, triggering every switch must be accomplished somehow. There is no convenient way of triggering only one switch. Let us look at the advantages first. Erection of the inversion generator is quite straightforward. Closing a switch starts a capacitor-inductor circuit ringing according to

$$I = V \sqrt{\frac{C}{L}} \sin \frac{t}{\sqrt{LC}}$$

This L is fairly large relative to the value in the discharge circuit, and it is usually put in deliberately for timing purposes. Each circuit should ring with the same period. After a half period, $T = \pi \sqrt{LC}$, the capacitor has reversed polarity, adding to the unswitched units. Since some losses are incurred in the switch and connections, the output is

$$V_o(t) = \frac{nV}{2} \left(1 - e^{-\alpha t} \cos \frac{\pi t}{T} \right) * \quad .9 \checkmark$$

where

$$\alpha = \frac{R}{2L}$$

One thing is immediately apparent. The output voltage begins to appear at once with a risetime set by the ringing portion of the circuit. This is usually fairly long with respect to the desired output time in order to minimize losses in the switches. As a result, an output switch, commonly an overvolted gap, is normally a part of the circuit.

Unlike the Marx bank, load currents do not have anything to do with the switches. So all the switch loss, switch inductance, resistance, etc., are out of the way. The output is from a string of capacitors in series. A low-inductance fast-current pulse can be produced by this generator. The generator has advantages and, buried away, a couple of big disadvantages. The triggering problem has been mentioned. The trigger must stand off the entire bank voltage on the upper stage. The really interesting fault occurs if one of the switches fails to go, for example, S_3 , then C_2 and C_4 invert and suddenly there is +5 V looking at a capacitance charged -1 V. If the switch tries to hold, the capacitor is going to be highly stressed. High-voltage capacitors just do not like to be reversed. Even if an excessive overvoltage did not occur, damage could result from excessive ringing. Another difficulty occurs if the bank is charged up and dumping becomes necessary. There is no really convenient way, in case of malfunctions, to save this circuit by dumping it in a hurry. Also, a prefire will ring one stage, probably without firing the other switches. The complexities of preventing these failures reduce the initial attractiveness of the LC inversion generator.

IV. BLUMLEIN

The Blumlein rates about one page in Glasoe and Lebacqz, as a special case of a Darlington discharge circuit. In this case, the description is in terms of a PFN. In very fast discharge networks, the Blumlein is more nearly a single-stage LC inversion generator. Since the PFN mode is the normal one, the discussion will begin there. Figure 12 is a somewhat redrawn version of Fig. 11.16 from Glasoe and Lebacqz.

Initially, all capacitors, $C1-C8$, are charged to the same sign and potential. It should be noted that $C5-C8$ must charge through the load. If the load is normally open, for example, a gas discharge, a parallel path must be provided. In many cases, the load is a pulse transformer primary for impedance conversion.

Fig. 12. PFN Blumlein

PFN BLUMLEIN

FIG. 12

Something similar to this is used with some laser loads. The pulse transformer is handy for klystrons, magnetrons, and similar devices to provide an impedance match. This type of a PFN is used in rather broad pulse applications because it is difficult to get very low inductances and very low inductance capacitors. One side is charged to ground and the other side through the load. If the load happens to be a gas discharge, a bypass must be provided for charging C5-C8. It can be an inductor or a resistor. When the switch is closed, the left stage, as a ringing stage, can ring until it has reversed the charge on C1. C1 is now plus on the bottom and minus on the top at which point C2 can ring and reverse. This ringing will progress down the line until eventually the charging capacitor C4 is charged inversely, and the whole line is effectively inverted. This reversal concept will be used several times in the rest of the discussion. The main point is that a short across the end of a transmission line causes the line to invert in polarity. At this point, the model of Fig. 13 shows a circuit in which effectively a battery that is inverse in sign to the original charge of transmission line impedance, the line impedances, and a battery of original polarity are across the load. It is now obvious that the plus to minus, plus to minus sequence yields 2 V across the load.

The lines commonly built have all of the capacitors of the same size for procurement reasons and the inductors are different because they are easier to tailor. There is a characteristic time given by $\sqrt{L_{\text{tot}} C_{\text{tot}}}$ that is the time it takes for a switching function to move one way down the line. It will turn out in most of these cases that the time of interest is down the line and back interval. With voltage doubling and 2Z in the source, if the load impedance is 2Z the output voltage equals the charge voltage. The circuit doubles the voltage and then divides it again, a necessarily characteristic operation on these. If it turns out that the load is not resistive but, in the case we are commonly interested, some kind of a gaseous discharge, then this has a feature of ringing up to 2 V to break down the gaseous discharge and then discharging current. If the overvoltage is needed for a sharpening gap or something of that sort the system doubles the charge voltage and in that sense acts like an inversion generator.

Fig. 13. Blumlein Transient Equivalent

TRANSIENT EQUIVALENT

FIG. 13

The switch requirements are never greater than V charge, an advantage in many applications, such as with thyratrons. On the other hand, the switch shorts the left PFN so that the current is nominally V/Z or twice the matched load current.

In faster circuits a PFN may not be practical. Then a distributed network, either cable or stripline, may be used. It is at this point that ambiguity begins. It was pointed out in the work by Mesyats that the switching time of a spark gap can affect the leading edge of a waveform in the several nanoseconds time scale. Also, the switch and connections will have an associated inductance. In the case of a lumped PFN, this may be included in the PFN calculation. Here it leads instead to an

$$e^{-\frac{R}{L} t}$$

type of behavior with R a function of time. The effect is to launch a wave down the Blumlein with a finite risetime. Since propagation velocities in lines tend to be 1.5 ns/ft, a cable of, say 10 ft, will depart from a transmission line behavior and begin to behave more like a simple LC circuit with a ringing waveform. The analogy to an LC inversion generator then becomes apparent.

In Blumlein systems, operation ranges all the way from real transmission-line behavior to real LC inversion circuit technique, depending on the trade-off in time constants of the system. The Tachisto laser is touted as a Blumlein system. In point of fact, the switch closure and gas breakdown occurs about half way through the inversion and it does not look like a transmission line at all. It looks like lumped L's and C's. When people talk about a Blumlein, look carefully at the closing time of the switch to see whether it is really a Blumlein or an LC inversion circuit.

A number of clever configurations that involve folding of the line have been developed. Figure 14 is a case in point. Initially the intermediate coaxial cylinder is charged as usual. (Upper half) When the switch is closed, the outer coaxial line undergoes the sign reversal from left to right previously described. At time τ the reversal reaches the right end so that the situation is as shown on the lower half, with both lines adding across the load. The two lines then discharge in series according to the usual procedure. One necessary condition in this is that the intermediate conductor is thick compared to the skin depth of the current that flows in the line. If that is not true, one side affects the other. One can also do this with stripline geometries and there are many, many geometries. Tom Burkes of Texas Tech has a number of these he uses for transformers and stepups.

V. SPIRAL GENERATORS

Spiral generators are a little hard to understand. This material is directly from a paper by Fitch. It does not follow his notation because it is difficult to carry his notation here. This is an approach to understanding two or three types of pulse generators other than Marx banks. In Fig. 15, if a set of striplines is charged in the fashion shown, alternating sign vectors zero to plus, as shown below $t = 0$ are established and the net output voltage is zero. The equations are for stripline. The inductance and the capacitance are in terms of spacing and width. From these arise the same impedance and transmission time relationships already mentioned. When the switches short the active lines in this system, at time τ reflections occur on the right, the vectors are reversed on each line, and a $2-nV$ output voltage results of $2\ nV$. The impedance is $2n\ Z$ since the lines appear to be in series electrically. The result is voltage multiplication from a, relatively speaking, high-impedance source. Also, there is a delay after closing the switch of one transmission time. The circuit will stay inverted for two transmission times, so it is fairly easy to decide how long the

Fig. 14. Folded Co-Ax Blumlein

Upper Half-as Charged
Lower Half- After Inversion

FOLDED CO-AX BLUMLEIN

FIGURE 14

Fig. 15. Stripline Inversion Circuit

$$L = 4\pi \cdot 10^{-7} \frac{a}{b} \text{ HY/M}$$

a = SPACING

b = WIDTH

$$C = \frac{\epsilon_R}{36\pi \cdot 10^9} \frac{a}{b} \text{ FD/M}$$

$$Z = \sqrt{\frac{L}{C}}$$

$$\tau = \frac{2 \times \sqrt{\epsilon_R} \times \text{LENGTH}}{C}$$

$$Z_0 = 2NZ = \frac{240\pi Na}{\sqrt{\epsilon_R} b}$$

$$V_0 = 2NV$$

FIG. 15

lines have to be for a desired pulse width. What has been done in Fig. 16 is to go one step further and wrap the lines around a central core. Initially, these lines are charged from ground to positive. When the switches close, counter-clockwise waves come around and invert in the same manner. We are approaching a spiral generator by wrapping the lines around a common core. The principle is the same. Note that there are multiple switches. That is a complexity we are going to eliminate.

In Fig. 17 the active lines, the ones to be switched, are on the left. The passive lines are on the right. In a sense, this is Fig. 15 unfolded. We want to see what occurs and why these are not ideal. The switch shorts points A and C, launching an inverted vector down the line. This occurs on the other narrow lines so that there are six vectors stacked up. Unfortunately, there is another set of lines A-B and these are shorted also. These vectors try to invert the passive vectors in the wrong sense. The impedance formula shows that the impedance depends upon the spacing. If the A-B spacing is large and the passive one small, A-B is a high-impedance line, the passive line is a low-impedance line, and when the vectors arrive, you have a voltage divider that only cancels a portion of the vector. The formulas are worked out in terms of geometry.

A single-switch arrangement yields a loss mechanism that relates a number of parameters and it comes out in a coerror function. Figure 18 shows this loss in the form of β . There is a different β for the spiral generator. This one is in terms of dielectric constant and length divided by spacing for copper. If the line is fairly long, the β will be quite decent. If the need is for a very fast line, then β will tend to be small. This expression will appear again in the spiral generator equations, but it is more complex. This is still the simple case of switching a set of parallel plate lines in parallel. In the spiral generator, the transmission line is wrapped on itself and another layer of dielectric added so it does not short against itself. The result is a pair of lines back to back, or one line for all intents and purposes.

Fig. 16. Wrapped Line Generator

FIG. 16: WRAPPED LINE GENERATOR

Fig. 17. Single Switch Inversion Circuit

Use of single switch

Fig 17

Fig. 18. Inversion Loss Mechanisms

$$V_0 = 2NV \operatorname{ERFC} \left[\frac{\text{LENGTH}}{480-A} \left(\frac{\rho \epsilon v_0}{t} \right)^{1/2} \right]$$

PLOTS OF $\frac{\sqrt{\epsilon} \text{ LENGTH}}{A}$

$$\rho = 1.7 \times 10^{-6} \text{ OHM-CM}$$

FIG. 18

One line is grounded and one charged as shown in Fig. 19. The case of a switch in the middle is a little easier to understand. Initially, there are alternating opposed vectors. The switch is closed and a vector is launched both ways from the switch. When these lines are shorted, the energy transfers from a voltage on a capacitor in an electrostatic field to a current in the inductor, a magnetic field. When the line reverses, the energy transfers back to the electrostatic field and progresses back down the line. In effect, the short launched by the switch runs around unwinding the coils in both directions at first and cancels the upward-directed vector, and as it goes around generates a ramp. Eventually, the waves reach the two ends of the line, reflect, and start back inward. As they start back down, they begin to invert the cancelled vectors and the voltage continues to climb. Finally, the transmission reflection arrives at the switch, reflects off the switches, starts in the other direction and the generator starts to wind down again in the same fashion, generating a saw-tooth output. These are not normally high-energy systems. They are used in trigger systems. The risetime is relatively slow, particularly if large multiplication is desired. Many turns with a lot of voltage multiplication generates a long ramp in time. A sharpening gap is common in the output. Normally, the switch is not in the center because that is physically inconvenient, but at one end of the line and you launch only one vector. The commercial variety of these uses a cryotron, the PT55. The cryotron is a great device that runs anywhere from one time to a couple of thousand and fails. The sharpening gap is electrically extremely noisy. The spiral generator does not produce a 2-nV output due to three loss mechanisms, switch impedance, resistive losses, and coupling effects.

Figure 20 treats the first case. If the switch closes exponentially with a response time τ , the voltage along the line at a given time t is described by $V_1(t)$ as shown. By manipulation, the voltage as a function of time at a position is then $V_t(l)$. Since the ideal switch charges the line to V , the area under the ideal voltage trace is just $V L$. Thus, the first loss is described by the area ratio defined as β_1 and plotted.

Fig. 19. Spiral Generator

$$0 < t < \tau, \quad V_0(t) = 2 \frac{t}{\tau_T} V$$

WHERE τ_T = ONE TURN TIME

τ_L = STRIPLINE ONE-WAY TIME

$$\tau = 2 \tau_L$$

$$\tau < t < 2\tau$$

$$V_0(t) = 2 \left(n - \frac{t - \tau}{\tau_T} \right) V$$

FIG. 19

Fig. 20. Spiral Generator Switch Loss

1. SWITCH EFFECTS

$$V_k(t) = V_k \left[1 - e^{-t/\tau_s} \right]$$

SO THAT

$$V_t(\ell) = V_t \left\{ 1 - e^{-\frac{(L-\ell)\sqrt{E}}{c \tau_s}} \right\}, \quad L = \text{TOTAL LENGTH}$$

$$e_1 = \frac{1}{VL} \int_0^L V_t(\ell) d\ell \quad \text{OR}$$

$$e_1 = 1 - \frac{\tau_s}{\tau} \left[1 - e^{-\tau/\tau_s} \right]$$

Resistive losses appear much as in the simple case treated before, except that the expression has been rearranged in Fig. 21. Again, area ratios with the ideal case lead to β_2 , plotted for the cases of copper and aluminum. The axis is times a thousand. If the line is 6000 cm (200 ft, ~200-ns risetime); this term alone is about 10%. This is worked out in terms of a spacing that is fairly realistic, 10 mils. Kaptan has an ultimate failure of about 6 to 7 kV per mil and the failure point is 60 kV in that case.

There is a third β term difficult to describe mathematically. For example, the system behaves as a pulse transformer whose natural frequency depends upon the leakage inductance between the first turn and its closest neighbor. Secondly, the active and passive lines are always coupled by the inductance of one turn. The passive lines can discharge through the switch by the inductance of the midturn, and, to an extent, the waves in the active lines enter the passive lines by inductance of inner and outer turn. This complex situation is not readily amenable to analysis. It should be possible to write an empirical formula. What Fitch concludes is that the overall data for the spiral generator are the product of the two for which there is an analysis plus a loss that is not readily analyzed. Numbers of around 1/2 for the product of the three terms are typical. People have done better than that. A β of 0.9 has been built in specific cases. Maxwell has a paper on high-voltage spiral generators. They built a couple of them and have a couple of basic relationships on output capacitance, etc. They discuss the problems they have had. The problems typically are related to the fact that the generator is wound with a stripline. That means there is a nasty tendency to go around the edge or, more dramatically, right through because there is a high field stress right at the edge of the foil. Much work was done grading edges with resistive paper or conducting paints, water, and copper sulfate solutions. There are a number of techniques to keep the field concentrations down. The stripline, particularly narrowly spaced striplines, are subject to edge effects because of the high-field stress at the edge of the metal. Flashover is a serious

Fig. 21. Spiral Generator Resistive Losses-Ideal Switch

$$V_T'(z) = V_{eRFC} \left[\frac{1}{120\pi} (\epsilon \epsilon^{1/2} \mu_0 c)^{1/2} \frac{z}{4a(L-z)^{1/2}} \right]$$

$$\beta_2 \triangleq \frac{1}{VL} \int_0^L V_T(z) dz$$

$$\text{FOR } V_0 = 2n \beta_2 V$$

$$a = 0.25 \text{ MM} \quad (10 \text{ MILS})$$

$$\epsilon = 2.2$$

problem. They quoted β 's of about 0.6 or 0.7 on some units. They have generators 1-1/2 ft in diameter and 3 ft overall length with two 10-mil conductors and 10-mil Mylar, vacuum impregnated and capable of repeated operation at about 700 kV. The 1-MW goal was obtained. However, edge failure prevented subsequent charge.

VI. COAXIAL GENERATORS

Figure 22 has sketches of two coaxial generators. The first is connected so that the center conductors are bridged at the right end and at the left end, a switch. This is not much different than the lines already discussed. Fitch reports an analysis that states that if $2N$ is more than 9, performance degrades. That means about four cables are all you can stack. The limit arises because of the stray capacitance between the lines. The impedance of a coaxial cable compared to stripline is so high that the generator impedance increases rapidly with n . If this is 50- Ω cable, a 200- Ω source impedance results, with only four of them. It does not take much stray capacitance at 200 to load the generator. These are typically relatively low-energy pulses. They might be used for trigger generators and applications of that sort. The second circuit is electrically connected differently. It is called a time-isolation transformer. Instead of tying all the grounds together as in the first system, a center conductor to ground, center-conductor-to-ground pattern is used all the way up. All of the centers and all of the braids tie together at the left. Instead of charging the cable in this case a fast pulse is launched down the cable. A 700-kV generator built this way had a very fast risetime. Again, it is a high-impedance source, but it goes along with Fitch's paper on the vector inversion concept. You are only limited by your ingenuity on how you can stack up elements and switch some of them to get the vectors all turned over and lined up for whatever period of time you need. The author of this paper quotes performance that is impressive. The paper consists almost entirely of the mathematical analysis on how to fit all the various stray

Fig. 22. Co-Ax Pulse Generators

Co-Ax Generators

$2n \times 9$ DUE TO Z , CAPACITANCE

F

Pulse Amplifier

A. Chodorow, Proc. IEEE, July 75

Figure 22

H

elements in order to optimize the system. Mathematically beautiful but there is not one word on how to physically implement the analysis.

Novel principle of transient high-voltage generation

R. A. Fitch, B.Sc., Associate Member, and V. T. S. Howell, B.Sc.

Synopsis

It is shown how voltage multiplication can be obtained by the transient reversal of the voltages in alternate units of series-connected systems. This leads to three groups of high-voltage generators whose performances are discussed theoretically. The results of preliminary experiments are reported briefly; these are in agreement with the theoretical predictions and show that voltages of about 1 MV are readily obtained by these methods.

List of symbols

- C = capacitance, F
- C_o = output capacitance, F
- c = velocity of light, cm/s
- D = mean diameter of spiral generator, cm
- E_{ix} = electric field through insulator, V/cm
- ϵ = permittivity
- L = inductance, H
- $l_{||}$ = length parallel to current flow, cm
- $l_{\perp}$ = length of current sheet perpendicular to current flow, cm
- l_s = length between current sheets, cm
- l_{xH} = length between adjacent lines, cm
- n = number of vector pairs or number of turns in spiral generator
- V = initial voltage per stage, V
- V_o = peak output voltage, V
- W = initial energy, J
- Z = transmission-line impedance, Ω
- Z_B = impedance of region between adjacent strip lines, Ω
- Z_o = output impedance, Ω
- $\beta = \beta_1\beta_2\beta_3$ = total loss factor
- μ_0 = permeability of free space
- ρ = resistivity, Ω cm
- τ = duration of stacked-line-generator pulse = $2\tau_i$; or rise time of spiral-generator pulse, sec
- τ_s = refolding time of current in switch, sec
- τ_T = mean propagation time per turn of spiral generator, sec
- τ_i = propagation time of stripline, sec

1 Introduction

Transient high voltages are conventionally generated either electromagnetically (e.g. induction coils¹ and pulse transformers^{2,3}) or electrostatically (e.g. Marx generators⁴ and stacked transmission-line transformers^{5,6}). Less common methods include the rapid conversion of dynamic energy either by the piezoelectric effect⁷ or by magnetic-field compression, and the abrupt release of charge buried in suitable

insulators by bombardment with high-energy electrons^{8,9}. This paper suggests a new way of looking at electrostatic generation which leads to a family of generators with interesting possibilities, particularly for pulses of about 10^5 – 10^7 V. Their performances are discussed on the basis of a simple theoretical analysis.

2 Principle of vector inversion

Pulse electrostatic generators can be regarded as devices in which a number of electric vectors are transiently rearranged to be in series addition. Thus, the Marx generator

Fig. 1
Disposition of E-vectors in Marx generator

- a Full circuit
- b Effective circuit initially
- c Effective circuit during pulse

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shown in Fig. 1a is, in its initial state, essentially as in Fig. 1b, and the vectors can be regarded as being in parallel (or as existing independently if the resistances are very high). During the pulse, the circuit is essentially as in Fig. 1c, and the vectors are clearly in series addition.

Consider a series of vectors initially arranged in series opposition (Fig. 2a), so that the net voltage is zero. If alternate

Fig. 2
Voltage generation by inversion of alternate E vectors

vectors are inverted (Fig. 2b), the resultant voltage is $2nV$, where V is the amplitude of one vector (i.e. the charging voltage) and n is the number of vector pairs. Such inversion is conveniently effected by an electromagnetic wave in a transmission line: for example, the closure of a switch at one end of a charged line leads to an inversion of the E vector at the open end which persists for twice the propagation time of the line. Alternatively the inversion can be achieved by the oscillation of lumped circuits.

Generators based on inversion principle

3.1 Stacked transmission-line generators

The simplest form of generator using this principle is illustrated in Fig. 3a. A stack of striplines is charged as shown; alternate lines are simultaneously switched so that their E vectors are reversed at the open ends and convert the initial series opposition to series concurrence.

In the ideal case the open-circuit voltage is readily seen to be a square pulse with the following properties:

Amplitude

$$V_o = 2nV \quad (1)$$

Output impedance

$$Z_o = 2nZ = \frac{240\pi n l_x}{\epsilon^{1/2} l_{\perp}} \quad (2)$$

and pulse duration

$$\tau = 2\tau_l = \frac{2l_{\parallel}\epsilon^{1/2}}{c} \quad (3)$$

In practice, the performance is limited mainly by the switch impedance and resistive losses in the lines. The switch impedance can generally be reduced to a negligible value by using parallel switches.¹⁰ In a single switch it is difficult to achieve a value of dI/dt much greater than about 10^{13} A/sec, and so parallel switches are necessary if rise times less than about $V/Z \times 10^{-13}$ sec are required.

The wave initiated at the switch is attenuated by resistive losses, and since the highest frequency components are attenuated the most, the rise time is degraded as the line length is increased. The output voltage is the sum of the

original static voltages and the incident and reflected voltages in the active lines (i.e. twice the incident voltage); thus

$$V_o = nV \text{ active} + nV \text{ passive} + 2nV \text{ active, attenuated}^{11} \quad (4)$$

But $V \text{ active} = -V \text{ passive}$.

Fig. 3
Stacked stripline generator
a Basic type: side view
b Closed type: end view

Therefore, within the interval $\tau_l < t < 3\tau_l$,

$$V_o = 2nV \operatorname{erfc} \left[\frac{l_{\parallel}}{480\pi l_x} \left(\frac{\rho \epsilon \mu_0}{t} \right)^{1/2} \right] \quad (5)$$

provided the strip thickness is much greater than the skin depth and the rise time due to the switch is negligible. The loss factor in eqn. 5 is plotted in Fig. 4; its effect is seen to be small for values of $\epsilon^{1/2} l_{\parallel}/l_x$ less than about 10^5 .

Fig. 4
Dependence of loss factor on line constants and time for transmission-line generators

$$\text{Loss factor} = \beta = \operatorname{erfc} \left[\frac{l_{\parallel}}{480\pi l_x} \left(\frac{\rho \epsilon \mu_0}{t} \right)^{1/2} \right], \beta = 1.7 \times 10^{-5} \text{ m}$$

A further limitation arises from the capacitive coupling between the edges of the lines. For example, a capacitance C_{AB} exists between the edges of lines A and B in Fig. 3a;

initially A and B are at the same potential, but when the switches close, a potential difference nV is propagated and C_{AB} becomes charged to this voltage. The energy can only come from the passive lines, so that some reduction in their voltage, and hence in the generator voltage, is produced. The potential difference between A and B rises to $2nV$ when the waves are reflected at the open end, resulting in a further energy drain and voltage reduction. The same argument applies to all the interline capacitances which are initially uncharged. A quantitative analysis of this process appears highly complex; qualitatively one can see that its effect is small where the capacitance of the lines is large compared with the edge capacitance, i.e. when $l_1 \gg nl_x$. However, while the output capacitance of the lines decreases as $1/n$, the edge capacitance decreases more slowly (cf. parallel wires where C varies inversely as the logarithm of the separation); so this loading sets a limit to the voltage multiplication for a given l_1 . We must therefore insert a factor γ in eqn. 5 which decreases from unity to zero as nl_x/l_1 increases from zero to infinity. This limitation is largely avoided if the edges of the lines close on themselves as shown in Fig. 3b; there then remains only the small capacitive loading at the ends of the lines.

The complication of multiple switching can be avoided with the circuit shown in Fig. 5a at the cost of some loss of efficiency. This arises because waves are simultaneously

Fig. 5
Stacked stripline generators with single-switch initiation
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initiated in the passive lines, as can be seen from the diagram: the switch closes across conductors A and D, completing the circuit formed by the lines AB, BC, CD in series. The line impedances are proportional to the conductor separations (provided $l_{xB} \ll l_1$), and so we must subtract a voltage $2Vl_x/(l_x + 2l_{xB}) \approx Vl_x/l_{xB}$ from each passive line. The attenuation of these waves is usually negligible because of the large value of l_{xB} ; the open-circuit voltage is therefore given approximately by the following expression:

$$V_o \approx 2n\gamma V \left\{ \operatorname{erfc} \left[\frac{l_1}{480\pi l_x} \left(\frac{\rho e \mu_0}{t} \right)^{1/2} \right] - \frac{l_x}{2l_{xB}} \right\} \quad (6)$$

If the passive lines are separated from the active lines as, for example, in Fig. 5b, the performance is much the same: the same fraction of the wave in AD will appear across BC as in Fig. 5a; they will, however, not be in phase unless the permittivities in l_x and l_{xB} are identical. A further variant is shown in Fig. 5c, the performance of which is the same except that the waves in the passive lines have amplitudes $2Vl_x/l_{xB}$; this means, as might be expected, that the total separation between adjacent pairs of active lines must be the same in both circuits for the same performance. In all three circuits the value of dl/dt in the switch is n times that of the basic circuit, so that multiple parallel switches are necessary for rise times less than about $nV/Z \times 10^{-13}$ sec. If it is necessary to use more than one switch, the advantages of the basic circuit probably warrant the slight extra complication of using series rather than parallel switching.

Coaxial lines can be used as shown in Fig. 6. The main difference arises from the fact that coaxial lines typically

Fig. 6
Stacked coaxial-line generator

have much higher impedances than striplines, while the capacitive loading of the coaxial-line stack is similar to that of the stripline stack; consequently the amplification is curtailed at relatively low values of n . A simple case is considered by Lewis and Wells³, who show that with conventional lines ($Z \sim 15$ ohms) it is unprofitable to make $2n > 9$. No advantage accrues from using isolated switches because the passive lines are completely decoupled from the active lines at the switch end.

The passive lines can be replaced by lumped capacitors as shown in Fig. 7; the performance is similar, except that in the stripline version the negative component in eqn. 6 becomes

Fig. 7
Stacked transmission-line-capacitor generator

$2nV(1 - \exp(-Z_0/C))$, and with both striplines and coaxial lines the output impedance becomes nZ in series with nC . Again no advantage accrues from using isolated switches.

All the above generators can be constructed with lines composed of lumped elements, but isolated switching is essential in all cases.

3.2 LC generators

The circuit shown in Fig. 8 illustrates how the vector-inversion principle can be applied to lumped circuits. The output voltage is

$$V_o(t) = nV(1 - e^{-\alpha t} \cos \omega t) \quad (7)$$

where $\omega^2 = 1/LC$, $\alpha = R/2L$, where R is the resistance in ohms.

The peak voltage is

$$V_o = nV(1 + e^{-\pi R/R_0}) \quad (8)$$

where $R_0 = 2(L/C)^{1/2}$; and, for times short compared with $(LC)^{1/2}$, the output impedance is

$$C_o = C/2n \quad (9)$$

Fig. 8
LC generator
 $\tau = \pi(LC)^{1/2}$

These equations are similar to those of a capacitor-transformer circuit³ with perfect coupling. The necessity for multiple switching is a disadvantage, but the higher efficiency, the simplicity of the coils compared with the transformer, and the fact that the capacitive energy storage does not have to be provided twice, make it an interesting alternative. In comparison with a Marx generator the output waveform is quite different, and the number of switches is halved; furthermore, if a fast high-voltage switch is used to connect the generator to the load, the output impedance can be made very low because it does not include the internal switches and their connections.

3.3 Spiral generator¹²

If we wind a stripline into a coil it is necessary to insulate between turns, and by making this insulation similar to that of the original line, we obtain twin spiral lines with common conductors. A switch at the midpoint (i.e. at $1/11$) initiates waves in one line only, as shown in Fig. 9. These waves convert the electrostatic energy to electromagnetic energy in the active line, leaving the electric vectors in the passive line unopposed, so that, ideally, the voltage between the inner and outer turns builds up to nV by the time the waves reach the ends of the line. To a first approximation the ends are unterminated, and so the reflected waves convert the electromagnetic energy back to electrostatic energy of opposite sign. The voltage is thus $2nV$ by the time the waves

have returned to the switch. At this instant the system is effectively $2n$ coaxial capacitors in series, with each capacitor charged to V . The waves reflected from the switch reverse the process until, at their second arrival at the switch, the system

Fig. 9
Cross-section of spiral generator showing waves propagating from the switch in alternate layers

is in its pristine state. The behaviour of this simplified model is described by the following equations:

$$0 < t < \tau, \quad V_o(t) = 2 \frac{t}{\tau} V \quad (10)$$

$$\tau < t < 2\tau, \quad V_o(t) = 2 \left(n - \frac{t - \tau}{\tau} \right) V \quad (11)$$

$$\tau = \pi n D \epsilon^{1/2} c^{-1} \quad (12)$$

$$\text{Thus } V_o = 2nV \quad (13)$$

$$\text{and } C_o = \left(\frac{1}{2n} \right)^2 C \quad (14)$$

In practice, the behaviour is complicated by (a) the switch impedance, (b) resistive losses and (c) the fact that each layer of the coaxial capacitor is not isolated but is connected to its neighbour by one turn of the conductor. The switch and the resistive losses affect the waves exactly as discussed in Section 3.1, but the effect on the output voltage is different because, in the spiral generator, this is the summation of the mean voltages in each $(1/2n)$ th section of the line. It is found experimentally that the output voltage is approximately triangular as predicted by eqns. 10 and 11 (see Fig. 14), and so we confine the theory to evaluating the peak voltage; this will be $2nV$ (eqn. 13) multiplied by the ratio of the area under the V/I curve at time $t = \tau$ to the ideal area V/I .

We assume that the switch imposes an exponential rise with an E folding time τ_s ; thus

$$V_o(t) = V_o[1 - \exp(-t/\tau_s)] \quad (15)$$

where t is measured from the instant the wave arrives at point I , and therefore

$$V_o(t) = V_o[1 - \exp[-(t_1 - \tau_s)^2/\tau_s]] \quad (16)$$

and the area ratio is

$$\beta_1 = \frac{1}{V_{11}} \int_0^{l_1} V_z(l) dl \quad (17)$$

therefore

$$\beta_1 = 1 - \frac{\tau}{T} [1 - \exp(-\tau/\tau_s)] \quad (18)$$

This expression is plotted in Fig. 10. For the switch to impose no more than a 10% loss it follows that the switch inductance, L_s , must be less than $\tau Z/20$. Substituting for Z and writing $I_s = V/E_{sx}$, we get the following condition for not greater than 10% loss:

$$\frac{L_s}{V} < \frac{6\pi\tau}{I_s \epsilon^{1/2} E_{sx}} \quad (19)$$

Assuming a practical limit for L_s/V to be $2 \times 10^{-13} \text{H/V}$ and $E_{sx} = 8 \times 10^5 \text{V/cm}$, we find that multiple switching is required for $\tau/l_1 < 10 \text{ns/cm}$ if the loss is to be not greater than 10%.

The resistive losses degrade the wave further; to simplify the analysis we calculate the reduction for a square wave (i.e. an ideal switch) and multiply the two loss factors together to obtain a slightly conservative approximation. Rearranging eqn. 5, we obtain the following voltage distribution along the line at time $t = \tau$:

$$V_z(l) = V \operatorname{erfc} \left[\frac{1}{120\pi} (\rho \epsilon^{1/2} \mu_0 c)^{1/2} \frac{l}{4I_s(l_1 - l)^{1/2}} \right] \quad (20)$$

and the resistive loss factor is

$$\beta_2 = \frac{1}{V_{11}} \int_0^{l_1} V_z(l) dl \quad (21)$$

This expression has been evaluated numerically for typical parameters, the results being given in Fig. 11.

The third loss factor stems from the fact that the generator is threaded with a pair of continuous conductors which interconnect regions of different potential, thus permitting currents to flow which tend to reduce these potential differences. This gives rise to various effects. Firstly, at time $t = \tau$ the system is a parallel LC circuit, where L is the inductance of the spiral and $C = C_0$ charged to V_0 . Such a system would degenerate in a damped cosine oscillation, and indeed a damped oscillation of approximately this frequency is observed after the initial pulse. However, this picture can only apply at the instant of peak voltage; at all other times the voltage does not appear uniformly across the series capacitor elements. For example, when the wave has travelled the first turn, the system behaves as a pulse transformer whose frequency depends on the leakage inductance between the first turn and its closest neighbours, and which acts such that the adjacent capacitor elements are charged in the same sense as the first element. Consequently the only energy lost (to the voltage generator) is that in the leakage magnetic field. Secondly, the active and passive lines are always coupled by the inductance of one turn, so that the two halves of the passive line can discharge through the switch via the inductance of the midturn, and at the extremities the waves from the active lines enter the passive lines via the inductances of the inner and outer turns. The currents flowing in these turns are in the same sense as the LC currents described above; they thus contribute by transformer action to the total generator voltage, and furthermore, the two sets of currents oppose each other through their mutual inductance. This complex situation is not readily amenable to analysis; with the accumulation of experimental data a simpler model may be

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evolved; at all events it should be possible to derive an empirical formula. An approximate analysis shows that β_3 should increase as D/l_s increases and n decreases, and this is borne out by preliminary experiments. Finally, therefore, we write

$$V_0 = 2n\beta_1\beta_2\beta_3V = 2n\beta V \quad (22)$$

Fig. 10
Effect of switch time constant on spiral-generator output
 $V_0 = 2n\beta V$

Fig. 11
Effect of resistive loss on spiral-generator output
 $V_0 = 2n\beta V$
 $L_s = 250 \mu\text{H}$
 $\epsilon = 2.2$

where β_1 is given by Fig. 10, β_2 by Fig. 11, and β_3 is found empirically (a high β_3 requires D/nl_s to be large). τ and C_0 are given by eqns. 12 and 14. The voltage multiplication is

$$\eta_V = 2n\beta \quad (23)$$

and the energy efficiency is

$$\eta_W = \beta^2 \quad (24)$$

At the instant of peak voltage the generator is effectively a short length of coaxial transmission line charged to V_0 . So, provided that the axial propagation time is much less than τ (i.e. $l_1 < l_0$), the output impedance is resistive if a fast high-voltage switch¹³ is used to connect the generator to its load at $t = \tau$. The design of this arrangement is discussed in the Appendix. It should be noted that the spiral generator switched at the midpoint, as in Fig. 9, is essentially two generators connected in series and fired from a single switch. It is sometimes more convenient to switch at either the outside or inside: this reduces β because D/nl_s per generator is reduced. Conversely, in a large generator, β can be increased by using several equispaced isolated switches.

4 Preliminary experimental results

A detailed account of the construction and performance of several of these generators will be published when experiments have been completed. Preliminary experiments carried out on most of the generators show that their behaviour is in approximate agreement with the theory, so that multiplications of at least ten times and voltages of the order of 1 MV are readily obtainable with the transmission-line group.¹³ Spiral generators have been operated within the ranges $10^4 \text{ V} < V_0 < 10^5 \text{ V}$, $10^{-8} \text{ s} < \tau < 10^{-6} \text{ s}$, $5 < \eta < 60$, $10^{-2} \text{ J} < W < 10^3 \text{ J}$. It is not difficult to achieve $\beta \sim 0.5$, and values in excess of 0.9 have been obtained with $D/nl_x \sim 70$. Insulating against the high output voltage is a particularly difficult problem because of the inherent compactness of the generator. Most of the work so far has been done with the smaller generators cast in epoxy resin and larger ones immersed in distilled water and pulse-charged. Fig. 12 shows

Fig. 12

Demonstration of 750 kV spark produced by 0.6 μF 20 kV spiral generator; 1 m rule shows scale

a cast spiral generator producing a 750 kV spark from an input of 20 kV ($C = 0.6 \mu\text{F}$, $C_0 = 52 \text{ pF}$, $n = 54$, $D/nl_x = 7.5$, $\beta = 0.4$); β cannot be inferred from V , V_0 and n , because of the capacitive loading of the spheres and a widespread corona loading which is clearly visible in the original photograph. It can be seen that the generator proper is compact in the context of 750 kV (the long horns are only necessary if the output is to be applied in air at atmospheric pressure, as in the demonstration). Fig. 13 is an oscillogram

of the output voltage from this generator on open circuit (it is taken at reduced voltage because it is difficult to get a clean picture at 750 kV). The waveform is only approximately triangular; this is due partly to the low value of D/nl_x and partly to the inductance of the switch connections, which has not been fully minimised. With good switch connections and a high value of D/nl_x , the waveform approximates closely to

Fig. 13

Waveform of output voltage from generator shown in Fig. 12; $D/nl_x = 7.5$

Time markers at 50 ns intervals

Fig. 14

Waveform of output voltage from spiral generator $D/nl_x = 60$

Time markers at 50 ns intervals

the theoretical triangle, as shown by Fig. 14. The lower limits of operation of the spiral generator are determined by the switch; the upper limits are largely a question of size. The scaling laws have not yet been established, but there seems to be no reason why the ranges studied so far should not be considerably extended.

5 Conclusions

The vector-inversion principle leads to a family of high-voltage generators with novel characteristics which may prove more convenient than conventional generators in many applications; for example, the stripline group is capable of producing voltages of several megavolts with rise times of about 10^{-9} sec; the spiral version can produce voltages up to at least 1 MV with characteristics similar to those of pulse transformers but with greater simplicity and compactness, while the LC arrangement offers a low-output-impedance alternative to the conventional Marx generator.

6 Acknowledgments

The authors acknowledge Mr. J. C. Martin's priority in the conception of many of the generators discussed in

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Section 3.1. Our independent development of this group followed from the conception of the spiral generator; the paper treats the groups achronologically in the interests of logical presentation. We are indebted to Mr. R. C. Smith for carrying out the preliminary experiments and calculations, and we should like particularly to thank Mr. I. Maddock for his interest and support.

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8 Appendix

If a spiral generator is to have a transmission-line output (via a fast, high-voltage switch) the design is determined by the following equations:

$$V_0 = 2n\beta E_{xx} l_x \quad (25)$$

$$\tau = 2l_x \epsilon^{1/2} c^{-1} \quad (26)$$

$$nl_x \leq D, \quad Z \approx 240nl_x(\epsilon^{1/2}D)^{-1} \quad (27)$$

Thus we have seven variables: three defining the performance (V_0 , τ , Z) and four the generator geometry (nl_x , l_x , D , β). The other quantities (E_{xx} , ϵ) are usually fixed by a compromise between performance, reliability and cost. Since β cannot be stated explicitly, the final answer must be obtained by successive approximation. We can choose any three of the six other variables, and the remaining three are then determined. The output energy is

$$W_0 = V_0^2 \tau / 4Z \quad (28)$$

and the initial energy is

$$W_i = W_0 \beta^2 \quad (29)$$

The empty space inside the former of such generators usually accounts for the bulk of the total volume. Some reduction in external dimensions can be achieved by using a ferromagnetic core, but this is costly and introduces additional insulation complications. An alternative is to use two lines, one inside the other. If they are connected in parallel, as

shown in Fig. 15, then individual impedances must obviously be doubled; the effect is to reduce D by roughly 30%. If they are connected in series, as shown in Fig. 16, the individual impedances remain the same; the main effect is to

Fig. 15
Twin spiral generators with transmission-line output, parallel connection

Fig. 16
Twin spiral generators with transmission-line output, series connection
a End view
b Side view

halve l_x , D is slightly increased and the total axial length is reduced by less than half because the total end-margin length is doubled. The arrangement of Fig. 15 can also be used for the parallel connection by taking the output from the left-hand end at the cost of losing the advantage of switching at the midpoint of each generator.

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Lecture 4

TRANSMISSION LINES AND PULSE FORMING NETWORKS

by

Bob Butcher

LECTURE 4

TRANSMISSION LINES AND PULSE FORMING NETWORKS

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LECTURE 4

TRANSMISSION LINES AND PULSE FORMING NETWORKS

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$$L = \frac{\mu_0 \mu_r \ln \frac{b}{a}}{2\pi}$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \frac{b}{a}}$$

TRANSMISSION LINES AND PULSE FORMING NETWORKS

$$2 \times 10^{-7} \ln \frac{b}{a}$$

Bob Butcher

$$Z = \sqrt{\frac{L}{C}} = \frac{1}{c}$$

The topic of this lecture is pulse forming networks. The first item of discussion will be transmission lines because they are so prevalent, even if only in the form of coaxial cable. From there the subject will proceed to pulse-forming networks: the practical problems encountered with them, their advantages, and disadvantages. Capacitors will be our final topic, as they are the limiting factor in lumped transmission elements.

I have referenced two masters' theses, and there is a bibliography available from Texas Tech University. These present a good description of the transmission-line theory and more detail than I can present in the limited space here.

Fig. 1 shows the most familiar type of transmission line — coaxial cable. By notation, there is an inner radius A and an outer radius B (which is actually the outer radius of the inner conductor and the inner radius of the outer conductor and the inner radius of the outer conductor). This is an important point in that surfaces nearest each other are used because high-frequency currents flow near the surface.

Formulas for capacitance and inductance of coaxial transmission lines are:

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln \frac{b}{a}} = \frac{5.56 \times 10^{11} \epsilon_r}{\ln \frac{b}{a}} \frac{F}{M} \quad (1)$$

FIG 1: COAXIAL TRANSMISSION LINE

$$L = \frac{\mu_o \mu_r \ln \frac{b}{a}}{2\pi} = 2 \times 10^{-7} \mu_r \ln \frac{b}{a} \frac{H}{m} \quad (2)$$

in units of Farads and Henries per meter of length. Since most high-voltage transmission lines do not use ferrite or other magnetic material as a filler, the value of μ_r is usually taken as 1, and it will be in the rest of this lecture. Given the inductance per unit length and the capacitance per unit length of a transmission line, the impedance of the transmission line can then be calculated.

$$Z_o = \sqrt{\frac{L}{C}} \quad (3a)$$

When this is reduced, it becomes:

$$Z_o = \frac{377}{2\pi\epsilon_r} \ln \frac{b}{a} \text{ ohms} \quad (3b)$$

where 377 ohms is recognized as the impedance of free space. The relative dielectric constant is sometimes referred to as K, although occasionally the total dielectric constant is simply lumped as ϵ . I generally tend to break this down as ϵ_o times ϵ_r , where ϵ_o is the permittivity of free space.

The one-way transit time of a transmission line is calculated by:

$$T_{1\text{-way}} = \sqrt{LC} = \sqrt{\mu_o \epsilon_o \epsilon_r} = \frac{\sqrt{\epsilon_r}}{C} \frac{s}{m} \quad (4)$$

This is the time it takes the pulse to travel one meter along the transmission line. Notice that this reduces to the square root of the relative dielectric constant divided by the speed of light (C) and has units of seconds per meter.

If the total inductance and the total capacitance of the transmission line are known, one can find the total one-way propagation time of that line, or it can be done on a per-meter basis. Usually the numbers in tables are given on a per-foot or per-meter basis, and the impedance is a dimensionless quantity. The impedance does not depend on the length of the line: the length is assumed to be infinite.

The second common type of transmission line is the strip transmission line, which is characterized by two parallel plates (Fig. 2). This can be thought of as a coaxial cable that has been unfolded and laid out flat. In general, the thickness (t) is assumed to be much less than the width (w) of the line. The formulas for inductance and capacitance of the strip transmission lines are:

$$L = \mu_0 \frac{t}{w} = 4\pi \times 10^{-7} \frac{t}{w} \frac{\text{H}}{\text{m}} \quad (5)$$

$$C = \epsilon_0 \epsilon_r \frac{w}{t} = 8.854 \times 10^{-12} \epsilon_r \frac{w}{t} \frac{\text{F}}{\text{m}} \quad (6)$$

The impedance of the transmission line is:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\epsilon_r}} \frac{t}{w} \text{ ohms} \quad (7)$$

one-way transmission time is again as it was in the coaxial line:

$$\tau_{1\text{-way}} = \sqrt{LC} = \mu_0 \epsilon_0 \epsilon_r \frac{t}{w} = \frac{\sqrt{\epsilon_r}}{c} \frac{t}{w} \quad (8)$$

A good approximation for impedance, if your thickness is not much less than the width is:

$$Z_0 \approx \frac{377}{\sqrt{\epsilon_r}} \frac{t}{w} \parallel \frac{377}{\sqrt{\epsilon_r}} \quad (9)$$

where $||$ denotes parallel impedance. It is obvious that this type of transmission line precludes impedances greater than 377 ohms. Even the 377-ohm level is difficult to attain if the dielectric material used is other than vacuum or air. Twin-lead line at 300 ohms is the highest value generally available.

Fig. 3 shows a battery switched onto the end of an uncharged transmission line. This transmission line is characterized by impedance Z_1 and a one-way transit time τ , and is terminated in an impedance Z_2 . When the battery is switched into the line, a current of magnitude I_1 flows to the right. The arrows on the transmission line indicate that under certain conditions a reflected voltage and current wave, which are denoted V_1' and I_1' occur. These travel in the opposite direction down the transmission line (to the left in Fig. 3). At $t = 0$, when the switch is closed, a current of magnitude:

$$I_1 = \frac{V_1}{Z_1} \quad (10)$$

begins to flow to the right in the line. When this wave reaches the end of the line (at time $t = \tau$), the voltage and current at the end of the transmission line must be continuous. The voltage and current are considered to be the algebraic sum of the wave traveling to the right and the wave traveling to the left. This can be written as:

$$V_2 = V_1 + V_1' \quad (11)$$

where V_2 is the voltage across Z_2 . Currents can be calculated in the same way, resulting in:

$$I_2 = I_1 + I_1' \quad (12)$$

Ohm's Law declares:

FIG 2: STRIP TRANSMISSION LINE **FIG 3: VOLTAGE & CURRENT WAVES IN TRANSMISSION LINE**

$$I_2 = \frac{V_2}{Z_2} \quad (13)$$

We can also show that the current traveling to the left is:

$$I_1' = -\frac{V_1'}{Z_1} \quad (14)$$

which is negative by convention, due to the direction of travel. If equations (10), (12), and (14) are combined:

$$V_2 = \frac{V_1}{Z_1} - \frac{V_1'}{Z_1} \quad (15)$$

Combining equations (11) and (15) and defining

$$K = \frac{Z_2}{Z_1} \quad (16)$$

allows derivation of the voltage reflection coefficient

$$\frac{V_1'}{V_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{1 - K}{1 + K} \quad (18)$$

The ratio of load current to line current, then, is

$$\frac{I_2}{I_1} = \frac{2Z_1}{Z_1 + Z_2} = \frac{2}{K + 1} \quad (19)$$

It is interesting to notice the effects of various configurations of transmission lines. In an open transmission line ($Z_2 = \infty$) that is initially uncharged (Fig. 4), when the switch is closed, current I_1

FIG 4: OPEN TRANSMISSION LINE

begins flowing into the line. Current travels down the line until when it reaches time $t = \tau$ (the one-way transmission time of the line) the current must be continuous at the end of the line. Since the line is open there can be no current flowing out of the line, and all the current must reflect and start back down the line in the opposite direction ($I'_1 = -I_1$). Given this situation and equations (10), (11), and (14), the output voltage that results is

$$V_2 = 2 V_1 \quad (19)$$

or twice the initial voltage. This can sometimes be put to practical use producing a voltage-doubling pulse. These pulses are reflected back down the line and current flows back into the battery. The net result is a voltage pulse of twice the initial amplitude that lasts for two transit times of the line. It is delayed by τ , and it lasts for 2τ . This can be either a help or a nuisance. It is disadvantageous when a fast wave is launched down a transmission line and hits the open end of the line, resulting in voltage doubling, which will track for tremendous distances on a coax cable in air. A resistor ($R \gg Z_0$) in series between the voltage source and transmission line prevents this.

Marx banks are complicated by this effect. In Marx banks, there are several biasing resistors associated with each spark gap. If all the resistors and all the capacitors are mounted separately and then connected with a coax cable, pulses traveling back down the coax cables hitting the few-megohm type resistors at the far end can give rise to monumental tracking problems.

Fig. 5 shows the case of a shorted transmission line. The switch is closed and the terminating impedance is zero, so the current builds up in a stairstep fashion. It goes to V_1/Z_1 and then, at time 2τ , it goes to three, then five, then seven times this value. This effect can be used to build a stairstep current generator.

FIG 5: SHORTED TRANSMISSION LINE

Fig. 6 shows a transmission line that is charged to some voltage V_1 and is shorted. The current initially flows in the negative direction. This is a perfect transmission line with no losses. The current in this transmission line will oscillate with a period of 4τ . If a terminating resistor R is inserted and the charged line is switched into this terminating resistor, we have Fig. 7. If the resistance is matched to the impedance in the transmission line, a square pulse of current will result of magnitude

$$I_1 = \frac{-V_1}{R + Z_1} \quad (20)$$

lasting for 2τ .

If the resistance is greater than the impedance of the transmission line, a staircase generator results where the initial current is found from equation (20). Then, since the energy was not dissipated in a single two-way transit time of the line, it'll staircase down.

If the terminating impedance is less than the characteristic impedance of the line, the initial current, from equation (20), will have somewhat greater magnitude than in the matched-impedance case, since R is smaller than Z_1 , and it will tend to reverse. Therefore, there is both a current reversal and a voltage reversal in the load, and the circuit will tend to oscillate.

These cases are mostly of academic interest, because in reality there cannot be a perfectly resistive termination. There will always be some inductance in the termination that will round off the leading and trailing edges to make the waveform more sinusoidal. In general, most of the energy eventually ends up in the resistor.

Also in reality some of the energy is dissipated in the line, although we've been assuming a lossless transmission line. Real transmission lines include a resistive term in series with the line (R_s) (Fig. 8), caused by the resistance in the skin of the line, which can become appreciable at high frequencies. This resistance tends to cause dissipation of the energy traveling down the line.

Another effect, which is usually small, is the parallel conductance or leakage resistance of the dielectric (R_L). Because dielectrics are not perfect they do permit some losses. Usually this is a very large resistance, while the series resistance is fairly small. We shall ignore these in the models: the analysis becomes very complicated if these terms are included. A recent measurement showed a loss of about 1% in a 10-foot length of cable at a frequency of 10 MHz, which is fairly typical of coax cables.

A charged cable loses charge to leakage resistance or corona. If the cable were charged to a high enough voltage to produce corona effects, or if there were voids in the dielectric, corona losses would occur. These may be voltage dependent, and there may be a fairly large resistance up to the point where corona begins, and then considerable losses occur. Such corona effects are detrimental to cable life.

Fig. 9 is a case where the transmission line becomes shorted when a battery is switched into an open transmission line. The current oscillates positive and negative with a period 2τ . This current will oscillate in the line, theoretically forever. But the resistive effects previously discussed eventually dissipate that energy, which will settle out at a zero current value and a V_1 voltage value. This current waveform found when a battery is switched into an open transmission line is the same basic current waveform found when shorting a charged transmission line. It appears inverted, but this is a result of the sign convention.

FIG 6: SHORTING OF CHARGED TRANSMISSION LINE

FIG 7: CHARGED TRANSMISSION LINE SWITCHED INTO A LOAD

FIG 8: LUMPED LOSS TERMS IN A NON-IDEAL TRANSMISSION LINE

FIG 9: CURRENT IN OPEN TRANSMISSION LINE

This leads to pulse-forming networks; in that if a circuit when shorted produces a current waveform of a particular shape, when the circuit is charged, it returns a current with the same shape when switched into a short circuit or a load resistor. This characteristic allows the design of a pulse-forming network in which the line is replaced with a lumped equivalent.

There are different types of pulse-forming networks. A current-fed line (Fig. 10) is of academic interest, although it is not of much practical use for high-voltage. This can be done with transistors or vacuum tubes to a few hundreds of volts — vacuum tubes permit kilovolt levels. A current source is switched into the shorted line, starting the current flowing into the line. At $t = 0$ the switch is opened. The line will then deliver a pulse into the load resistor. This is only of academic interest because although most switches are able to close under high-voltage conditions, they are difficult to open. It is difficult and impractical to build a 50-kV switch that can open quickly. Vacuum interrupters open in a few microseconds or tens of microseconds, but these times are not considered fast. Like most switches, they prefer closing to opening.

This leads to the voltage-fed line, which is charged with some voltage V_0 (Fig. 11). (Please notice the designation change.) The transmission line impedance is Z_0 , the one-way length is τ , and the voltage is V_0 . A closing switch switches into a resistive load at $t = 0$, producing a square pulse. The pulse will be similar to that of a matched load if the load resistance matches the impedance of the transmission line (i.e. the voltage pulse will have a magnitude $V_0/2$ and width of 2τ).

Fig. 12 shows how a lumped equivalent transmission line can be derived from series inductance and parallel capacitance. If the line is continued to the limit at which N approaches infinity and each of the inductances and the capacitances is very small, it looks like a true transmission line with distributed elements.

FIG 10: CURRENT FED LINE

FIG 11: VOLTAGE FED LINE

$$Z_0 = \sqrt{L/C}$$

$$\gamma_{1 \text{ WAY}} = \sqrt{LC}$$

FIG 12: N-SECTION LUMPED EQUIVALENT LINE

FIG 13: SERIES L-C CIRCUIT

Again, the impedance of the line is the square root of L/C , and this can be either the total series inductance divided by the total parallel capacitance or L_N/C_N . Similarly, the one-way transit time of the line will be the square root of the product of LC : the total series L times the total parallel C .

A Fourier series approximation for a square current waveform may be written as:

$$I(t) = I_m \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right] \quad (21)$$

This series has to extend to infinity to produce a true square pulse. The amplitudes of the higher harmonics diminish quite rapidly and the line is approximated by some finite series for pulse-forming networks, as it is awkward to have an infinite number of capacitors and inductors. If a battery is switched into a series inductance and capacitance (Fig. 13), the current will have the form:

$$I(t) = \frac{V_0}{Z_0} \sin \omega t \quad (22)$$

where Z_0 is:

$$Z_0 = \sqrt{\frac{L}{C}} \quad (23)$$

and W is:

$$W = \frac{1}{\sqrt{LC}} = \frac{1}{\tau} \quad (24)$$

FIG 14: 5-STAGE TYPE C GUILLEMIN NETWORK

In the case of a one-way transit time τ , ω appears as a $1/\tau$ term, which leads to Guillemin networks.

The Type-C Guillemin network (Fig. 14) is our first item of interest. The battery is switched into a five-stage network, numbered C1, C3, C5, C7, and C9. These stages are numbered to produce odd harmonics (e.g. the "third harmonic" is produced from the L3-C3 combination). For current oscillation as in a cable PFN, the proportionality is written:

$$I(t) \propto \left[\frac{1}{z_1} \sin_1 t + \frac{1}{z_3} \sin_3 t + \frac{1}{z_5} \sin_5 t + \frac{1}{z_7} \sin_7 t + \frac{1}{z_9} \sin_9 t \right] \quad (25)$$

You will notice that the $I_{\max}$ term has been temporarily deleted. Comparing terms with equations (21) and (25), it is evident that the conditions necessary for square-wave oscillations are:

$$z_1 = 1, \quad z_3 = 3, \quad z_5 = 5, \quad z_7 = 7, \quad z_9 = 9,$$

$$w_1 = w, \quad w_3 = 3w, \quad w_5 = 5w, \quad w_7 = 7w, \quad \text{and } w_9 = 9w.$$

By comparison to equations (23) and (24), we can generalize:

$$z_n = n \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{L_n}{C_n}} \quad (26)$$

and

$$w_n = \frac{1}{\sqrt{L_n C_n}} \quad (27)$$

We can now find the n th term by comparison to Equation (22) and by noting that all the impedances (Z_n) are in parallel:

$$Z_0 = Z_1 || Z_3 || Z_5 || Z_7 || Z_9 \quad (28)$$

One way of solving for the values in this circuit is to assume the capacitors are all charged to V_0 and the circuit is switched into a load resistor of value $R = Z_0$ (Fig. 15). By analogy to the transmission line model, the current could be assumed to be a square pulse of amplitude of $V_0/2Z_0$ and of a duration τ . τ now is used as the full width of the pulse instead of the half width. This is done to be consistent with the text. If there is matched impedance ($R = Z_0$), then all the energy stored in the PFN will be deposited in R during the pulse. This energy can be calculated from the definite integral of the power:

$$w = i^2 R = \left(\frac{V_0}{2Z_0} \right)^2 Z_0 = \frac{V_0^2}{4Z_0} \quad (29)$$

which is written

$$E = \int_0^{\tau} w \, dt = \frac{V_0^2 \tau}{4Z_0} \quad (30)$$

This energy may be equated to the energy stored on the total capacitance of the PFN

$$E = \frac{1}{2} C_T V_0^2 \quad (31)$$

and solve for the total capacitance

$$C_T = \frac{\tau}{2Z_0} \quad (32)$$

FIG 15: TYPE C GUILLEMIN NETWORK WITH MATCHED LOAD

FIG. 16. TYPE C WAVEFORM

where Z_0 is the load resistance in which the energy is deposited. In this type of network all the inductances are equal.

The difference in τ is visible in the capacitances of the coax lines. When τ is the one-way transit time, the capacitance of a coax or a regular transmission line is simply τ/Z_0 .

Because there is not an infinite number of elements in this Fourier series there will not be a perfectly square pulse, and the energy deposited will be slightly off because it is not a square pulse. In a five-stage pulse-forming network, the energy will be about 4% off. If there are fewer stages, the discrepancy will be somewhat greater.

The pulse width for this case can also be calculated from

$$\tau = \frac{\pi}{\omega_1} \quad (33)$$

this is from the fact that

$$\omega_1 = 2\pi f_1 \quad (34)$$

The period of the fundamental frequency is

$$T = \frac{1}{f_1} = \frac{2\pi}{\omega_1} \quad (35)$$

The pulse width (τ) is the half period of the fundamental frequency, and the pulse width from the PFN will be

$$\tau = \frac{T}{2} \quad (36)$$

Rearranging Equation (33) results in

$$\omega_1 = \frac{\pi}{\tau} \quad (37)$$

The inductance may be calculated from

$$L_n = L = \frac{1}{\omega_1^2 C_1} = \frac{1}{(\pi/\tau)^2 C_1} = \frac{\tau^2}{\pi^2 C_1} \quad (38)$$

From Equation (26) we can find

$$C_n = \frac{C_1}{n^2} \quad (39)$$

which puts the capacitors in a ratio of 1:1/9:1/25:1/49:1/81. Therefore:

$$C_T = C_1 + \frac{C_1}{9} + \frac{C_1}{25} + \frac{C_1}{49} + \frac{C_1}{81} = 1.184 C_1 \quad (40)$$

$$C_1 = \frac{C_T}{1.184} = \frac{\tau}{2.368 Z_0} \quad (41)$$

From Equations (32) and (40):

$$L = \frac{2.368 Z_0 \tau}{2} = 0.24 Z_0 \tau \quad (42)$$

The remaining capacitors are found from Equations (39) and (41). Fig. 16 shows a typical waveform for this type of circuit, done on the digital computer. It was calculated to produce a 1-microsecond pulse width with an amplitude of 10,000 A. The PFN is initially charged to 20,000 volts, and it has a 1-ohm impedance and is switched into a 1-ohm resistor, so it should have about a 10-kA current for a microsecond. There is quite a bit of overshoot from the leading edge and the trailing edge because

FIG 17: TRAPEZOIDAL WAVEFORM FLAT TOP WITH ~~PARABOLIC~~ RISE & FALL

FIG 18: PARABOLIC RISE AND FALL TIME

the eleventh harmonic is not there to reduce it. The ringing diminishes somewhat toward the middle of the pulse. As more stages are added to the pulse-forming network, there is less overshoot on corners and less ringing in the middle.

This effect led Guillemin to write the equation for a trapezoid and assume that he had a linear risetime, a flat pulse, and a linear falltime. He made the risetime some percentage (a) of the total pulse length (Fig. 17). With " a " a percentage of the risetime, one can relax the requirements on a pulse-forming network. As the requirements are relaxed the " a " becomes larger, and the number of terms required in the pulse-forming network is reduced. A point is reached where the capacitor for the 9th harmonic becomes either zero or slightly negative, and the inductance becomes slightly negative or infinite and can be deleted. The contribution from those higher order terms are unnecessary to generate the trapezoidal waveforms.

Fig. 18 shows another case examined by Guillemin: a flat-topped waveform with parabolic rise- and falltime. The total risetime is used as some percentage of the total pulse width (a) and the only difference is the parabolic rise- and falltimes.

Fig. 19 shows some computer plots of the waveforms for the five-stage trapezoid and the five-stage parabolic generators terminated in the correct impedance. These waveforms are not ideal — in fact they look similar to Fig. 18. The main difference is that the overshoot is less. The few irregularities in the middle of the pulse are not exactly on the first, third, fifth, seventh, and ninth harmonics, but on some slight variation.

As one approaches the 10% trapezoidal waveform, the pulse becomes smoother and as he approaches the 10% parabolic, it becomes lumpier. These are still better than the rectangular pulse in Fig. 18. Although these are meant to be the same pulse width, this is difficult to compare because they are plotted on graphs of different scales. However, the

FIGURE 19: COMPUTER WAVEFORMS

risetime is obviously good. A 100-nH inductance put in series at the load effectively increases the last inductor by 100 nH, producing an L/R risetime of 100 ns, which is a 10% value for the parabolic load. It rounds off the leading edge of the waveform, flattens most of the ripples, and rounds off the trailing edge. For the most part, on all of these there is very little energy deposited in the resistance after the main pulse.

Guillemin found other circuits that produce the same general waveform (Fig. 20). Type A has parallel LC resonant circuits with one capacitor and one inductor in series.

All these circuits have contained capacitors of different values. Type B also has various inductances and capacitances, although it looks more like a classic transmission line. This configuration is requisite to generate the trapezoidal pulse with a nominal 8% risetime.

The Type F has all the capacitors in series in a transposition of the Type B network, replacing all the capacitors with inductors, etc. Unfortunately, these capacitors have some very unusual values.

The Type D network uses capacitors of equal value, but has negative values of inductance in series with several of the capacitors. Coupling between inductors can realize this physically and produce the Type E Guillemin network. In this network, there is a coefficient of coupling between the various inductors and all the capacitors are of equal value.

Total inductances desired to produce these mutual inductances can be found from:

$$L_T = \frac{\tau Z_0}{2} \quad (43)$$

and the capacitances are derived from

PFN's FIG 20: OTHER GUILLEMIN NETWORKS

$$C_T = \frac{\tau}{2Z_0} \quad (44)$$

This capacitance is divided equally among the number of stages. Then a coil is wound on a long cylindrical form having the total desired inductance, with 20%-30% additional inductance added to the coils on each end of this form. The ratio of length to diameter should be chosen to provide mutual inductance of about 15% of the self-inductance in each center section. Soldering the capacitors on results in the correct diameter-to-length ratio, winds the coil, and taps it at the appropriate number of turns.

There are certain minimum inductance limitations in the various types of networks. Some PFNs have inductors in series with capacitors; others, like the Type C network with its coil wound on a form, require a somewhat larger minimum inductance value. The importance of this minimum inductance number can be seen from rearranging Equation (43) to calculate a minimum pulse width-impedance:

$$\frac{\tau Z_0}{2} \geq L_{\min} \quad (45)$$

On a Type A network, if the minimum inductance is reduced to 100 nH, then the Z_0 product must be greater than or equal to 38-1/2 times 10^6 . Since the Type E network has mutual inductance, the absolute minimum inductance is about a microHenry, which produces a 1×10^6 τZ_0 product. Type C networks may have minimum inductances to the level found in the capacitors plus the connections.

Capacitors may be bought of almost any inductance, but vary greatly. A practical lower limit does exist for capacitors. Maxwell Series S

capacitors have about 20 nH in the capacitor and can usually be connected into a circuit in which the inductance of connections can be kept down to 20 nH. An inductance of 40 nH produces a τZ_0 of 0.16 microseconds - ohms.

This is plotted on a graph in Fig. 21. If the desired pulse width and impedance of load are known, this circuit will provide some idea of the practical minimums available in Guillemin-type networks. Notice that at impedances of one ohm, pulse width remains around a microsecond with a Type C Guillemin, but that generally low impedances dictate long pulse widths. High impedances allow short pulse widths. Ten ns is a reasonable minimum pulse width.

Also available on the graph is the useful range of distributed pulse-forming networks (coaxial cables). With enough coaxial cables in parallel, impedance can be reduced to 1 ohm. Because coax cables with a polyethylene dielectric have a one-way propagation time of about a nanosecond per foot, given a fast enough switch, a 1-ns pulse width can be attained on a cable or transmission line. Strip lines provide low impedances but are difficult to switch. Neither cable nor lumped PFNs will generally deliver the desired pulse unless the switch closes much faster than the desired pulse width. Switching, therefore, presents a problem.

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HIGH VOLTAGE/PULSE POWER TECHNOLOGY

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LECTURE 4 - APPENDIX
ENERGY STORAGE CAPACITORS

by

W. J. Sarjeant

Los Alamos Scientific Laboratory

PREFACE

With the recent increase in technological needs and the interest in the power conditioning arena, one of the problems facing workers in the field is the lack of texts or notes describing recent progress, particularly in the area of repetitive power conditioning. For this reason and because of expanding internal requirements, the University of New Mexico (UNM) and the Los Alamos Scientific Laboratory (LASL) have created a set of lecture notes based upon the graduate course taught recently at UNM. The objective of these notes is to create a record of many of the advances in the field since the last text in the field was published just after World War II. In this context, the lectures presented are oriented toward an introduction of the reader to each of the areas described and present sufficient background information to explain many of these advances. They are not intended to serve as design engineering notes, and thus the reader is referred to the references at the end of each lecture for detailed technical information in specific areas.

The preparation of these writings is a result of a considerable teamwork effort on the part of LASL and Sandia staff. In particular, Cathy Correll, in conjunction with Jo Ann Barnes and the rest of her efficient word processing staff, carried the major responsibility for preparation of the lectures while the lecturers did the proofreading and revisions. As course coordinator, it is a pleasure to acknowledge the strong support of Ray Gore, our E-Division Leader, and Shyam Gurbaxani who is the UNM Graduate Center Director, Los Alamos Campus.

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October 3, 1980

LECTURE 4 - APPENDIX

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I. INTRODUCTION

Energy storage capacitors (referred to as "capacitors" in the balance of this note) allow electrical energy to be stored over a slow charging time frame and then released as required over very short (nanoseconds to microseconds) periods of time under controlled conditions. In contrast to dc filter capacitors, pulse discharge or energy storage capacitors must provide large peak currents, often of an oscillatory nature, during this short release or discharge time. These large currents and their repetition rates can lead to significant internal power losses inside the capacitor, necessitating considerable care in design to achieve adequate heat transfer rates for long capacitor lifetime. This note will explore current capacitor technology and attempt to delineate some of the engineering considerations required to achieve user-specified lifetimes for the major classes of operation of the devices. A review of manufacturing techniques and their impact upon capacitor performance is presented and correlated with the known physical and chemical mechanisms responsible for observed capacitor electrical properties and lifetime.

Webster's Dictionary defines a "capacitor" as: "A device giving capacitance, usually consisting of conducting plates or foils separated by thin layers of dielectric with the plates on opposite sides of the dielectric layers oppositely charged by a source of voltage and the electrical energy of the charged system stored in the polarized dielectric."

An illustration of the elements essential to the realization of such a capacitor is sketched in the Fig. 1. We will accept as an experimental fact that closing S1 with S2 open will place equal and opposite charges of magnitude q upon the upper and lower plates. For the parallel plate geometry shown, the constant of proportionality between q and V_{dc} , namely C is:

Fig. 1. Parallel plate capacitor.

$$q = C \cdot V_{dc} \quad \text{where } C = \epsilon \cdot \ell \cdot \frac{W}{d} \quad \text{farads - MKS units}$$

$$= \epsilon_0 \cdot K \cdot \ell \cdot \frac{W}{d}$$

and ϵ_0 is the dielectric constant of free space

$$= 8.854 \times 10^{-12} \text{ C}^2/\text{N-m}^2$$

with K being named the relative dielectric constant or coefficient.

Table I is a brief listing of some representative materials giving the K and breakdown voltage or "Dielectric Strength" V_b , under an impressed dc voltage. The dielectric strength is expressed in volts per mil of dielectric thickness, at a voltage stress level where breakdown is virtually certain within 1 minute of application of voltage. Note that the dielectric strength for thicker pieces of material is considerably less as a result of bulk impurities, voids, and mechanical flaws.

As an example, consider a capacitor 1 x 1 m with a plate spacing of 1 cm and a K of 1. Then:

$$C = 8.854 \cdot 10^{-12} \cdot 1 \cdot 1/0.01 = 8.854 \cdot 10^{-10} \text{ farads.}$$

Since a farad is a large unit rarely encountered in high-voltage capacitors, smaller subdivisions have been created for convenience in calculations. These are the following:

$$\begin{aligned} 1 \text{ microfarad} &= 10^{-6} \text{ farad} = \mu\text{F} \\ 1 \text{ nanofarad} &= 10^{-9} \text{ farad} = \text{nF} \\ 1 \text{ picofarad} &= 10^{-12} \text{ farad} = \text{pF} \end{aligned}$$

TABLE I

DIELECTRIC CONSTANTS AND BREAKDOWN STRENGTHS OF INSULATORS

	<u>K</u>	<u>V_b</u> <u>V/mil</u>
Air	1.000585	75
Aluminum oxide	7.0	300
Bakelite (general purpose)	6.0	300
Castor oil	3.7	350
Ceramics	5.5-7.5	200-350
Ethylene glycol	39	500
High-voltage ceramic (barium titanate composite and filler)	500-1000	50
Kapton (Polyamide)	3.6	7000
Kraft paper (impregnated)	6	2000
Lucite	3.3	500
Mylar	2.5	5000
Paraffin	2.25	250
Polycarbonate	2.7	7000
Polyethylene	2.2	4500
Polypropylene	2.5	9600
Polystyrene	2.5	500
Polysulfone	3.1	8000
Pyrex glass	4-6	500
Quartz, fused	3.85	500
Reconstituted mica	7-8	1600
Silicone oil	2.8	350
Sulfur hexafluoride	1.0	200 (per atm)
Sulphur	4.0	--
Tantalum oxide	11.0	100
Teflon	2.0	1500
Titanium dioxide ceramics	15-500	--
Transformer oil	2.2	250-1000
Water	80	500 (pulse charged in 7 to 10 μ s)

Note in Table I the relatively high K of reconstituted mica, which combined with its high dielectric strength of 1.6 kV/mil makes it a rather interesting candidate for compact pulse capacitors.

Let us consider the work done in charging a capacitor of capacitance C to a potential difference of V_{dc} across the upper and lower metallic conductors. The incremental work done dW in adding an increment of charge dq to the capacitor when the plates are at a potential of V volts between them is:

$$dW = V \cdot dq$$

Then the total work done, in joules, in charging the capacitor to a potential difference of V_{dc} volts is the integral from 0 to q_{dc} , where q_{dc} is the charge on the capacitor plate (+ q on the top and - q on the bottom plate) at V_{dc} . That is,

$$W = \int_0^{q_{dc}} V \, dq = \int_0^{q_{dc}} \frac{q}{C} \cdot dq = \frac{1}{2} C \cdot (V_{dc})^2$$

where

$$q \equiv C \cdot V$$

This energy is stored in the dielectric between the capacitor plates and is released into the load when the discharging switch in Fig. 1 is closed. It is important to point out that the discharge of this stored energy gives rise to mechanical forces inside the capacitor, which can cause dielectric fracturing and, in repetitive operation, excite mechanical resonances in the capacitor structure, eventually resulting in destructive damage and electrical breakdown in the unit. For example, in the parallel plate capacitor the potential difference between the plates varies linearly with the distance from the bottom plate and the electric field $\underline{E}$ is constant:

$$\underline{E} = \frac{V_{dc}}{d} \cdot \hat{x}$$

and the energy in the volume $\ell \cdot w \cdot x$ is:

$$U = \frac{\epsilon}{2} \int E^2 dV$$

$$= \frac{\epsilon}{2} \cdot \frac{V_{dc}^2}{d^2} \cdot \ell \cdot w \cdot x$$

This is the energy stored in a volume $\ell \cdot w \cdot x$ of the capacitor dielectric where the base of this volume is situated on the upper electrode of the capacitor. Similarly, the energy W^1 stored in a volume contained between x_0 and x_1 , any two distances between the plates, can be calculated and found to be:

$$W^1(x = x_0 \text{ to } x = x_1) = \frac{C \cdot (V_{dc})^2}{2 \cdot d} \cdot (x_1 - x_0)$$

In discharging this energy there is a mechanical force that the dielectric exerts upon the mechanical conductors and container in which the capacitor is housed. For the nonconducting, that is, the very high dc resistance dielectrics we are considering here in this note, this force, F_x , is equal to minus the rate that the stored energy is changing in the x direction¹

$$F_x = -\nabla_x U$$

$$= -\frac{1}{2} \cdot C \cdot \frac{V_{dc}^2}{d} \cdot \hat{x}$$

Thus, discharging the capacitor causes the force in the dielectric to be opposite in sign to the direction that the current is flowing through the dielectric, meaning that the dielectric is being compressed and pulled away from the metallic conductors. In fact, these disruptive forces can be sufficiently large to cause rupture damage internally to the dielectric and destroy the intimate bond required between the dielectric and rigid metallic conductors. Consider the effects the dielectric would experience with small impurities:

1. Not ionized
2. Not ionized but polarized
3. Ionized
4. Ionized and polarized

We will discuss impurity effects as well as the effects that isolated packets of charge at the edges of capacitors have on lifetime for both dc and very short pulse-charged methods of bringing the capacitor potential up to V_{dc} . This potential, V_{dc} , will be used throughout this note to designate either type of charging conditions. Any differences these conditions may make, either to capacitor lifetime or restrictions on modes of operation in the discharge loop, will be made clear as we continue our discussions.

II. PROPERTIES OF CAPACITORS IN GENERAL

Whenever a capacitor is selected to perform in an electronic circuit, its characteristics have been optimized to provide the designer with a well-defined level of reliability of the component throughout the "design lifetime" of the circuit. Table II presents a listing of a number of the more important parameters that effect the capacitor designer's choice of geometry, impregnant and materials.

The selection of a capacitor design requires the matching of available capacitor characteristics and parameters to the application needs. In addition to the basic capacitance value and voltage rating, specifying all the characteristics presented in Table II by the user allows

TABLE II

CAPACITOR DESIGN PARAMETERS

the supplier to provide the most cost-effective capacitor for the given application. The fundamental parameters open to control by the designer, listed on the left of Table II, are controlled to a large degree by the environmental factors, such as temperature range, voltage, wave shape, and duty cycle. Essentially all these environmental factors impact upon the "life expectancy" of the capacitor and it is here that the user can effect considerable cost savings in providing the designer with all the data listed in Table II and an accurate assessment of the lifetime of the equipment into which the capacitor is to be placed.

It should be noted that very long lifetimes, in excess of 100 million shots, may often take the capacitor designer into regions where lifetime data are not available. Thus, it may be advantageous to allow for capacitor replacement at fixed time intervals as a function of allowable equipment downtime, utilization factor, and the specific cost-benefit ratios of the capacitor designs for longer lifetimes. The user who is faced with very long lifetime requirements will almost always find that he is restricted in utilizing capacitor designs that have been based upon the highly reliable insulation systems for power distribution systems. Low inductance and very fast discharge times at high repetition rates do not find an adequate data base in this wealth of dc and 60 Hz test information. With the advent of the pressing needs for high repetition rate capacitors, emphasis in this note is oriented in this direction. Those who require single-shot devices (i.e., 1 pulse/min or fewer) will find no difficulty in acquiring capacitors of high reliability for $> 100,000$ shots at modest reversal (less than 30%), peak currents of 50,000 A and voltages up to 100,000 V. Suffice to say the enormous data base available for designs utilizing this performance profile surface is very well documented, especially in the very thorough studies by G. Boicourt² and it would be presumptuous to attempt to summarize them here. A word of caution: It is worthwhile asking any supplier who provides a design that is smaller, cheaper, and has a longer

lifetime than all the competition, where similar devices have been made and used before, and then discuss with these users the performance they obtained.

The balance of this section of the note will address most of the individual headings in Table II in detail. At this point let us assume that specific recommendations concerning capacitor compositions, impregnants and lifetime will be incorporated under the umbrella of these headings. Thus, there may be some need to digress within each area while we discuss the physical, geometrical and chemical processes responsible for affecting each of these parameters.

As a preamble to this discussion, the basic characteristics of capacitors will be summarized and then related to the unique requirements of pulse capacitors.

A. Dielectrics

The capacitance relation presented earlier states that the capacitance C varies directly with the dielectric constant, the area of the plates and inversely with the distance d between the plates. In general, the plate area is determined by the surface dimensions the user can tolerate and the separation d is controlled almost totally by the lifetime the user specifies and the dielectric the manufacturer must use to provide him with the highest profit for this lifetime. It is here that several iterations with the potential suppliers and careful specification of the desired mean time between failure (MTBF) at a high confidence level, say 90%, can often save the user money. As a general rule, the smaller the capacitor is specified to be for a given lifetime, independent of repetition rate, the more expensive it will be. For high reliability, the user is generally better off to place the minimum of size constraints upon the capacitor manufacturer and to avoid unique sole-source designs at all costs unless there is no other alternative.

Now the dielectric constant of any material placed into an elementary parallel plate capacitor is a direct measure of the ability of the material to store electrical energy when charging of the plates to V_{dc}

is completed. This energy is stored in the dielectric's polarization, either permanent, as in water, or induced as in the case of aluminum oxide. Essentially, this polarization comes from either the alignment of permanent dipoles in the dielectric parallel to each other and in line with the internal electric fields (the field applied during charging, in Fig. 1, is parallel to the x axis and its unit vector originates from the top [positive] plate of the capacitor¹) or the generation of an induced polarization by the presence of the applied electric field.

B. Induced Polarization

In this case, the electric field is an external perturbation of a coulomb nature on the molecule or atom and tends to attract the electron cloud to the positive plate direction in the capacitor. On the other hand, the attractive forces between the electrons and the nucleus tend to resist this displacement of the electron cloud, giving rise to an equilibrium condition wherein the molecule has a finite dipole moment. This moment then is solely induced by the presence of our external (applied) electric field and is a result of an elastic displacement of the electronic charge distribution relative to the nucleus. To a first approximation, this polarization " $\underline{P}_{ind}$ " can be shown to be proportional to $\underline{E}$, with the constant of proportionality being χ_e , the susceptibility:

$$\underline{P}_{ind} \equiv \chi_{ind} \cdot \underline{E}$$

Note that both $\underline{P}_{ind}$ and $\underline{E}$ are vectors and for isotropic materials are parallel.

We will not discuss here the impact of anisotropic materials on any of the capacitor systems as such a discussion requires a tensor description for the susceptibility χ_{ind} and is beyond the scope of this note. Just for the sake of interest, such anisotropic materials tend to have rather large internal losses and low breakdown voltages. They may, however, be of considerable utility in the field of electromagnetic shock

lines. In the case of induced polarizability caused by electronic displacements within atoms and molecules of dielectric under the influence of an applied external electric field, it is generally true that the polarization is a true elastic process so that no bound internal energy states exist. Thus, the induced dipole moment susceptibility, χ_{ind} , is constant up to frequencies corresponding to the ultraviolet portion of the spectrum.

In the case of ions, χ_{ind} can be much larger as a result of atomic or ionic displacements within the molecules (i.e., changes in bond angles and interatomic distances caused by the application of $\underline{E}$). It is constant for frequencies up into the infrared portion of the spectrum and concerns us primarily in the case of ionic impurities in almost all capacitors. In this case, these ionic impurities experience a force proportional to the local electric field at their site and this force, constant during dc on-time and oscillatory during discharge, may be sufficient to cause mechanical rupture in the dielectric and electrical breakdown.^{2,3} Note that the ratio of the forces upon the interstitial ionic impurity and dielectric molecules is in direct proportion to their relative dipole moments, in the presence of the applied electric field. Thus, if all impurities have interstitial or lattice site polarizations less than those of the host dielectric, little chance of mechanical damage will exist. The question of ionization and breakdown from voids in the dielectric is rarely dominated by this class of polarizations and will thus be discussed later along with other corona processes.

C. Permanent Polarization

Dielectrics may also possess a permanent dipole moment $\underline{\mu}$, which is antiparallel to the applied electric field $\underline{E}$ for isotropic dielectrics. The external field will tend to exert a torque on each little dipole and try to orient it in the direction of the field.³ This orienting and ordering influence of the field is counteracted by the thermal motion of the particles and their local interaction of one dipole moment with

another. Analysis shows that, as one might expect, the application of the field $\underline{E}$ to such a dielectric results in a net orientation of some fraction of all these dipoles parallel to the field; thus:

$$\underline{P} = \chi_e \cdot \underline{E}$$

where $\underline{P}$ is the polarization per unit volume of dielectric resulting from the dipole orientation caused by $\underline{E}$; χ_e is the permanent dipole moment susceptibility generally referred to as K_p (relative dielectric constant) and $K_p = (1 + \chi_e)$, $\epsilon_p = K_p \epsilon_0$.

Since thermal equilibrium is the depolarizing perturbation on $\underline{P}$, increasing the temperature T of the dielectric can be shown to result in a linear decrease in χ_e . This accounts for the dramatic decrease in capacitance observed in many polar dielectrics. Note that operation at voltages above the level where all the dipoles are oriented parallel to the applied field is of little value in increasing the energy stored as the dielectric constant then falls to ϵ_0 . Let us state again that all the fields here are the external fields applied to the dielectric as a result of having a potential difference of V_{dc} volts across the capacitor electrodes. There exists no general qualitative theory of dipolar materials that can be applied to our problems in capacitors; first, because of difficulties in evaluating the internal fields; and second, the dipole rotatability possesses complex thermal and fabrication-induced anisotropies.³ The discussions thus become qualitative.

In the case of solids, the susceptibility decreases with increasing temperature, except for a large susceptibility increase at the melting point, where all the dipoles are free to rotate strongly in the field direction. If your capacitor is at this point it has likely melted and shorted, so further comment is irrelevant. On the other hand, ceramic materials such as barium titanate possess large susceptibilities as they are spontaneously polarized, giving rise to a very large permanent

dipole moment in the material. Hence, the dielectric constant is large and has a significant decrease with increasing temperature. Above a critical temperature known as the "Curie" temperature, this polarization all but disappears and the dielectric constant drops dramatically to close to ϵ_0 . We shall discuss the influence of E upon C of these types of capacitors later on and it is interesting to speculate how this property allows time-varying pulse forming networks to be synthesized.⁴ As a result of the binders employed in their manufacture, the power loss per pulse generally exceeds 5% at high frequencies (> 1 MHz) so that repetitive operation ought to be pursued with some care.

To sum up, we conclude that there are materials having large dielectric constants K_p , which in general one desires to maintain as constant as possible as a function of E in order to store the maximum energy in the capacitor. Almost all the major contribution to the dielectric constant for capacitor dielectrics can be shown to be made by the permanent dipole moment in the dielectric. This moment has a temperature, field, and frequency of discharge and repetition rate class of dependencies that are not yet subject to accurate physical modeling from fundamental principles. Thus, we are forced to adopt a parametric approach for each material of interest, which is a research problem that has been well addressed in the case of dc and single-shot dielectric tests and capacitor design. The field of high-repetition rate capacitors does not have an extensive data base and is very fertile field for further research.

III. CAPACITOR CHARACTERISTICS AND THEIR CHANGES WITH TEMPERATURE, CHARGING VOLTAGE, AND FREQUENCY

In circuit applications the capacitor is often subjected to electrical, mechanical, thermal, and other environmental stresses, some of all that are listed on the right-hand side in Table II. The capacitance value of the capacitor is generally measured on a low-voltage bridge at a frequency of 1000 Hz. Assuming the capacitor to be

mechanically well made, neither the area or plate separation will change in any significant degree during charge or discharge. Hence, all the variations in capacitance come about as a result of these environmental factors changing the dielectric constant of the material between the capacitor plates. The dielectric constant K is initially set by design in choosing the dielectric materials and impregnants optimum for the class of capacitor being constructed. The major environmental factors causing a change in K are:

- (1) Temperature range
- (2) Voltage
- (3) Humidity
- (4) Frequency
- (5) Storage temperature and time.

A. Temperature Range

Temperature variations in K , and thus the capacity value, are widely different for various dielectrics and depend also upon the liquid impregnant utilized to fill voids between the capacitor plates and dielectric as well as upon any small voids in permeable materials such as papers. In the latter case, permeation of the impregnant into the paper results in a very significant improvement of the dielectric strength, primarily as a result of suppressing the corona that would have caused breakdown in voids in the paper, by filling the voids with an impregnant of nearly the same K and thus preventing corona formation until bulk breakdown occurs at a much higher voltage. Figs. 2 to 4 illustrate average curves of K against temperature and the corresponding per cent change in capacitance from its value at 25°C . Note that special processing of the dielectrics in manufacture of the films, as well as utilization of proprietary impregnants, can be used to alter these curves very significantly and to reduce changes in K at the higher temperatures. The slowly varying plastic films such as Teflon, polystyrene, etc., (Fig. 2) find their K is changed very little upon impregnation with several of the available silicone dielectric fluids.⁵ In addition, these

fluids possess very small power losses that are essentially frequency independent. To maintain these low losses, a paper wick cannot be employed in such units. There are also significant manufacturing process innovations to be made in assembly packaging and impregnation of non-wicked capacitors, which will require considerable further development for high-reliability applications. We conclude then that temperature-induced variations in K for plastic materials can be made small enough to be insignificant for energy discharge capacitors, by the selection of the film and impregnant that gives the required lifetime/frequency loss characteristics. These techniques are to a large degree proprietary, and further on in this note we will try to discuss briefly the various selected combinations that manufacturers have chosen and tested.

At the moment, let us return to two particular classes of capacitors, paper impregnated with castor oil and high- K ceramics. The paper capacitor is vacuum impregnated with, for example in the case of Aerovox, a liquid named "Hyvol D" a specially processed and treated grade of castor oil specifically prepared for electrical applications. These capacitors exhibit temperature variations in K , Fig. 5, having a drastic decrease in the capacitance at low temperatures. This is also followed by very large increases in the internal losses, which are a result of increasing intermolecular losses as the temperature decreases. Castor oil capacitors used below 0°C ought to be provided with appropriate heaters if constancy of capacitance is a problem. If not, then the increase in internal dissipation will, for high-repetition rate operation, generally heat the dielectric sufficiently to increase the temperature, reducing the capacitance variation and as well the power loss.

Let us now turn our attention to the classes of ceramic capacitors having published characteristics. The capacitance variation with temperature for the old Sprague 715 series illustrated in Fig. 6, is large indeed. Through gradual developments over the last few years, the manufacturer is now selling a very stable unit whose designation is the 720/722 series (C-57 formulation). The improvement is clearly evident

Fig. 5. Typical characteristics of Kraft paper/castor oil impregnated capacitors.

(a)

TYPICAL DISSIPATION FACTOR AND CAPACITANCE CHANGE
AS A FUNCTION OF TEMPERATURE

(b)

TYPICAL DISSIPATION FACTOR AND CAPACITANCE CHANGE
AS A FUNCTION OF D-C OPERATING VOLTAGE

Fig. 6. Characteristics of Sprague type 715 ceramic capacitors.

in Fig. 7 where the variation from 25 to 70°C is less than 3% compared to 30% for the 715 series as illustrated in Fig. 6. We have tested a special version of the 720 series having double silvering, half-inch studs soldered to the silvering after the silver has been hot tin dipped. The edges are varnish coated and there is a 1-mm nonsilvered margin for operation in Dow Corning 200 silicone dielectric fluid, 20-50 centistokes viscosity. Under these conditions, operation to 50 kV dc for single shot applications is feasible provided fingers, smoke, and solder flux are kept out of the oil and most especially any silicone rubber. The exception is the special silicone RTV made by Dow Corning for use on copper surfaces as no corrosive materials are given off during cure (types 3140 and 3145). This material requires the use of a rosin/ketone surface adhesion preparation available from the same company known as type 1201 primer. Note that cure times are 1 to 2 days for the RTV, which can be reduced somewhat through gentle heating. Pulse charged (less than 100 ns) operation avoids corona limitations on lifetime. We found then that lifetime scales by about the 12th power of the ratio of the voltage stresses. As a datum point, 20 units, nominally 1800 pF, will have 10 failures at a pulse charge voltage of 190 kV and 30% reversal, with a discharge period of 50 ns FWHM, after a total of 50 discharges. This correlated to a lifetime of two to three shots at 210 kV charging voltage at about a 50% reversal, obtained from a sample of three units which were not preconditioned at lower voltages. In contrast, the 715 series was not usable above 130 kV and had a lifetime of about 100 shots at 110 kV and 20% reversal. There is virtually no application where the 715 cannot be replaced by the 720/722 series for pulse duty.

An example of the temperature properties of a high-frequency, low-loss capacitor using plastic film impregnated with (as usual) the "proprietary" oil, is shown in Fig. 8. Polysulfone wins top marks and these units from High Energy Corporation are rated to pass up to 47 kVar of reactive power at a rms current of 25 A. From this figure it is also

(a)

**MINIMUM INSULATION RESISTANCE
(MEASURED AT 180 VOLTS D-C AND +25°C)**

(b)

**TYPICAL CAPACITANCE CHANGE AS A
FUNCTION OF TEMPERATURE**

(c)

**TYPICAL CAPACITANCE CHANGE AS A
FUNCTION OF APPLIED D-C VOLTAGE**

Fig. 7. Characteristics of Sprague type 720/722 ceramic capacitors.

Fig. 8. Characteristics of type "CK" high-frequency High Energy Corporation capacitors.

quite evident why Mylar capacitors are not suitable for high prf operation, both from the standpoint of capacitance stability and increasing internal losses with temperature. These are impressive specifications for the polysulfone, but check with the users as to how they have performed in practice, particularly at the higher voltages, in the 25-kV range. This should not be taken as a criticism of the company but rather as a suggestion based upon general past experience with all suppliers.

B. Voltage

The voltage rating of the capacitor, along with the other environmental factors will determine the lifetime the user can expect from the unit. For most dielectrics, except ceramics, there is an insignificant effect of the applied voltage upon the capacitance value of the capacitor. To illustrate the difficulties with ceramic capacitors, refer to Figs. 6, 7, and 9, wherein the per cent change in capacitance as a per cent of rated dc voltage is shown for Sprague type 715 and 720/722 capacitors, and Murata type DHS capacitors. Suffice to say they are all about as poor so that the nameplate capacitance, measured at 1000 Hz and 1 V ac applied must be reduced according to the curves, as the dc voltage is increased. For those applications where pulse charging is used, Fig. 10 shows the decrease in capacitance for the 715 and 720/722 series up to 120 kV charge voltage. Note that the capacitance decreased only by 30% at the nameplate voltage of 40 kV compared to the 45% reduction shown on the dc curves. There appears to be no straightforward explanation for the smaller decreases in capacitance under pulse charging. These measured decreases we obtained by calculating energy deposited into a low-inductance resistive load for each voltage, operating single shot, at a peak current of about 10,000 A at 120 kV. The 720 series has a capacitance that drops to about 1200 pF, for the nameplate 1800 pF unit, at about 150 kV. We have consistently found that the capacitance dropped to its nameplate value at 40 kV dc so the curves for these units are conservative. In contrast, the 715 series was consistently somewhat worse in its capacitance drop at 40 kV dc, but this matters little since

Fig. 9. Characteristics of Murata type "DHS" pulse discharge duty ceramic capacitors.

Fig. 10. Variation of capacitance with voltage of Sprague series 715 and 720/722 ceramic capacitors.

the advent of the Murata DHS and Sprague 720/722 series units allows the 715 units to be used where they ought to be in dc filter capacitor service for which they were originally designed.

C. Capacitor Model

Only for ceramic capacitors is the amount of energy lost per pulse inside the capacitor a function of the applied voltage, as is shown in Figs. 6, 7, and 9 for both the Sprague and Murata units. The maximum loss per pulse is 2% and varies little with frequency up to 1 MHz as shown in Fig. 9 for the DHS Murata capacitor. In order to discuss the dissipation factor (DF) in more detail, we must refer to the physical model of the capacitor illustrated in Fig. 11. A practical capacitor possesses, in addition to capacitance, an inductance, and a resistance. In this equivalent circuit:

- R_s : The series resistance due to leads, contact terminations, and the electrodes on the capacitor dielectric. R_s is generally below 0.1Ω and has a frequency and peak current dependence.
- R_p : The leakage resistance of the capacitor due to the resistivity of the dielectric and case material as well as to dielectric losses. Generally the dc leakage resistance is several hundred megohms per microfarad of capacitance and is much larger than the frequency dependent dielectric losses.
- L : The measured inductance of the capacitor, generally having a negligible frequency or voltage dependence below the self-resonant frequency, as it is primarily a geometrically determined quantity.
- C : The capacitance of the capacitor, usually measured by the manufacturer on a bridge at a 1 V ac level and a frequency of 1000 Hz. Due note must be taken of the voltage coefficient of capacitance in ceramic units.

ALL FREQUENCIES HIGH FREQUENCIES

note: ESR is frequency dependent

Fig. 11. Capacitor equivalent circuit.

1. Equivalent Series Resistance (ESR). The ESR is the ac resistance of the capacitor, essentially R_s and R_p in parallel, at a given frequency so that the loss in the capacitor can be expressed as the loss in a single (generally frequency dependent) resistance R in the equivalent circuit, as shown in Fig. 11. Unless soldered or other eutectic bonds are made, this R will have a significant current dependence when peak currents rise into the multikiloamp region. In these cases, compression contacts have small oxide layers, which give rise to superheating where ohmic contact is made, and hence increases the power lost per pulse over soldered connection configurations. Whether or not this is important depends upon the peak and rms currents, as well as upon the repetition rate. The persistent usage of inserted tab designs at high-repetition rates in liquid impregnated capacitors causes severe heating that can seriously degrade the lifetime. In addition, the gases released in this process can cause dangerous pressures to build up inside the capacitor case, often causing explosions with the force of a small munitions shell. Flexible cases are the rule here or solid cases with pressure relief vents. Experience to date with the latter has been inconclusive as to the protection level achieved. Only in the case of reconstituted mica capacitors (Fig. 12), where the tabs are under very high pressures as a result of the fabrication process, has the inserted tab design reliably sustained high rms current loadings for far in excess of 10^{10} shots. The number of tabs per end connection determines the rms and peak-current carrying capability of the capacitor connection, exact numbers depending upon the tab width and compression pressure.⁶

2. Capacitive Reactance. The reactance of the capacitor, X_C , at frequency f is given as:

$$X_C = -j/2\pi fC \text{ ohms} \quad \text{where} \quad j = \sqrt{-1}$$

$$= -\frac{j}{\omega C} \quad \text{and} \quad \omega \equiv 2\pi f$$

(a)

INSULATION RESISTANCE

(Ref. Mil-STD-202, Method 302)

Electrification 500 VDC for a time period not greater than 2 minutes.

(b)

DISSIPATION FACTOR

Frequency and equipment same as that for capacitance.

← 1 % max for all types

(c)

Fig. 12. Characteristics of Axel mica pulse duty capacitors.

3. Impedance. At high frequencies, the inductive reactance component must be taken into account. From the equivalent circuit of this real capacitor (Fig. 11) the impedance, Z , at frequency f is:

$$Z = R + j(\omega L - 1/\omega C) \text{ ohms}$$

and the magnitude of this impedance is

$$(ZZ^*)^{1/2} = [R^2 + (\omega L - 1/\omega C)^2]^{1/2} \text{ ohms}$$

Clearly, at very high frequencies the inductive term dominates. As the frequency decreases, a point is reached where the inductive and capacitive reactance are equal so that at this point

$$Z = R$$

and the capacitor acts as a pure resistor at that frequency, generally referred to as the "self-resonant frequency" (SRF) of the capacitor.

It is best to stay far below the SRF, as the very large rms currents that can flow in a low-inductance discharge loop at the SRF, means large power losses inside the capacitor. Note that any capacitor may have more than one SRF, depending upon the construction geometry and the transmission line properties of each series section inside the capacitor. Detailed discussions on this matter are very complex and beyond the scope of this note. We only remark that a section inside a capacitor is generally a rolled up parallel plate capacitor of some length ℓ , which forms a transmission line and can only discharge in a time equal to the two-way transit time, t , along the line where:

$$t = \frac{2 \cdot \ell}{c} \cdot \sqrt{K}$$

and c is the velocity of light in free space and K the relative dielectric constant.

This time is independent of the lumped inductance of the capacitor, and clearly shows why a supposedly "10 nH capacitor" of this type, and 0.1 μ F capacitance may take, depending upon ℓ , much longer than the calculated 100 ns to totally ring up and down in current.⁷ The only way to make a "fast" capacitor is to manufacture it with short-length sections whose discharge times are significantly shorter than the desired "lumped element" discharge time of the capacitor under construction. Each capacitor manufacturer has his own bag of proprietary tricks, but just for fun, ask him how he would make the above capacitor for, say a 100,000 A peak current discharge application with the discharge time of 20 ns (zero to zero of current). Then ask him how long each of his series packs are, and calculate t for yourself. There are ways of fabricating very low inductance capacitors utilizing many very fast discharge, small capacitors, in parallel. However, a few moments of reflection on the required reliability of each of these capacitors in the assembly in order to achieve a specified system reliability, should give one pause for thought. In contrast to this requirement, there are a large number of very complex, single-shot systems in existence that work very well and have few reliability problems with their capacitors. It is only fair to point out that this is after significant amounts of money were spent to achieve high reliability capacitor designs and many research groups actively participated in and, in many cases, were instrumental in solving the major hurdles in these capacitor development programs.² This has not been the case for low-inductance, high-repetition rate capacitor development.

The SRF of several ceramic capacitors has been measured in a recent experiment here.⁸ For both the Murata type DH and the Sprague 720 of capacitance values 2890 and 2380 pf respectively, the SRF was 20 ± 0.5 MHz and their series inductance 18 and 20 nH, respectively. Thus, many fast discharge circuits (~ 50 ns pulse width) employ these capacitors near their SRF, resulting in increased circuit losses and capacitor heating.

4. Dissipation Factor. The dissipation factor (DF) is defined as the ratio of the effective series resistance R to the capacitive reactance X_c at a frequency f . It is normally expressed in per cent:

$$\begin{aligned} DF &= (R/X_c) \times 100\% \\ &= (R\omega C) \times 100\% \end{aligned}$$

The DF increases with frequency and is always a measure of the power loss inside the capacitor when multiplied by X_c times the (rms current)² through the capacitor. Normally, this calculation of power loss is straightforward, since the discharge loop defines the discharge frequency for a half-sine pulse:

$$(I_{rms})^2 = (I_{peak})^2 \cdot \tau \cdot \frac{f^2}{2}$$

where I_{peak} is the peak current and τ its duration, assuming no current reversal. The repetition rate is f hertz.

In complicated discharge systems where shaped pulses are required, a similar calculation can be effectively carried out by integrating the square of the discharge current over the discharge time and multiplying by the repetition rate. Typically, ceramic capacitors have dissipation factors of 2 to 3% in the 0.1 to 1 MHz frequency region, castor oil paper around 5%, polysulfone-silicone and other proprietary impregnants about 0.5%, and reconstituted mica from 0.4 to 1.0%, depending also somewhat upon the geometry and compactness the user demands. The only capacitors to have compiled a long lifetime record at high-repetition rates are the two low loss type of units described above. All in all, there are very sound arguments in favor of the reconstituted mica type up to, say 100 kV where high peak currents and nonflammable impregnants are demanded. High currents, up to 200 A rms can be handled⁶ by carefully fabricated mica units in low-inductance geometries, but they are

somewhat more expensive than modules of liquid impregnated polysulfone capacitors; they also do not blow up, they just short and melt when overloaded.

Since the dissipation factor must be measured at the frequency of discharge of the capacitor it is important that the supplier provide the user either with this number or the power loss at the specified repetition rate from which DF and hence R may be calculated. This R then determines the ultimate Q of your circuit

$$Q = \frac{1}{DF} = \frac{1}{\omega RC}$$

for DF expressed as a pure number and not in per cent.

Some manufacturers provide the power factor (PF) of the capacitor at the discharge frequency. The PF is defined as the ratio of R to Z at this frequency. When the PF is less than 10% the DF and PF are essentially the same.

In closing this part of the note, let us recall that, except for reconstituted mica capacitors,⁶ the internal resistance of a tabbed capacitor is current dependent. Thus, at short discharge times, high peak currents and high-repetition rates, Q's are found to be lower than shown by calculations based on low-level measurements. Once again, we have a strong argument for extended foil, soldered, and swaged geometries in all types of capacitors.

In addition to the frequency dependence of the dissipation factor, there is a temperature effect, which simply stated says: the higher the temperature, the higher the level of power you lose inside the capacitor for the same stored energy and current waveform.⁹ In the case of ceramic capacitors, this is illustrated in Figs. 6, 7, and 9, for castor oil in Fig. 5, and for some low-loss plastic films in Fig. 8. For mica the curve is slowly varying (Fig. 12) from 0.5% at 25°C up to 1.0% at 125°C, at which temperature the capacitor is still happily operated without any derating. The variation in DF with temperature is illustrated in Fig. 13 for both dry dielectrics and liquid impregnated kraft

(a)

(b)

Electrocube Corp.

Fig. 13. Dissipation factors for film and impregnated papers as a function of temperature.

paper. The manufacturer can and does use the DF as a quality control tool. Variations in DF above normal values for a particular lot of the same capacitors generally indicates a possible loss of fabrication or material control.

To sum up, the user operating a capacitor in ac or high-repetition rate pulsed discharge applications is intimately concerned with the dissipation factor since the series resistance factor in DF is the heat producing element in these applications.

5. Humidity. For hermetically sealed capacitors, this is not a problem except in so far as high humidity does cause some decrease in the surface breakdown voltage of the terminal insulators. Moisture in the capacitor dielectric will cause a large reduction in leakage resistance, and increases in the DF and internal gas pressures at high-repetition rates. A long-term dc leakage test or corona level test before and after operation will pick out such defective units.

6. Frequency. The variation of C and DF with frequency has been discussed in Secs. 3 and 4 above. When utilizing capacitors in pulse-forming networks, bear in mind that, particularly in tapered PFN designs wherein the impedance is constant but the capacitor values are staggered to achieve fast risetimes, each capacitor sees a different Fourier frequency distribution in the currents that pass through. This must be taken into account in power-loss calculations and is best done by computer analysis.⁴

7. Storage Temperature and Time. All capacitors have a range of storage temperatures beyond which either impurity migration, internal gasification from voids in the dielectric, or chemical changes in the impregnant will cause reductions in lifetime. Ceramic capacitors also suffer from depolarization with time, decreasing their capacitance. These variations are summarized in Table III.

8. Lifetime. Here, we finally come to the main problem area in capacitor technology. If it is assumed that the manufacturer utilized proven devices from which his specials are derived, then the lifetime for capacitors in general is limited by gasification from arcing at non-soldered connections, resultant chemical reduction in dielectric strength, and corona-induced treeing in and on the dielectric,

TABLE IV

<u>Capacitor Type</u>	<u>Storage Temperature Range - °C</u>	<u>Capacitance Decrease Rate with Time in Per Cent per Year</u>
1) Castor oil/Kraft paper	+10 to +40	nil
2) Plastic film/pro- prietary impregnant	-20 to +60	nil
3) Ceramic type DHS from Murata	-40 to +125	-3% and equilibrium at 2,000 hours
4) Ceramic type 715 from Sprague	-55 to +125	-20% in first year, decreasing logarithm- ically. Can be pre- stabilized by mfr.
5) Ceramic type 720	-55 to +125	As above
6) Reconstituted mica	-65 to +125 storage and operating - no derating needed.	nil

particularly at the edges of the capacitor sections. These points are illustrated schematically in Fig. 14. Arcing at tabs or between stacked, pressure-contacted sections can be eliminated with soldering, metalizing or cold flow eutectic bonding. These are listed in order of increasing cost but will eliminate gas formation and arc-drop heating, particularly at high-repetition rates. There is absolutely no excuse for the continued utilization of inserted tab constructions in very high peak current or/and high-repetition rate capacitors where high reliability is desired. Void ionization and impurity migration under the influence of the external electric field can be minimized through the use of high quality dielectrics and the elimination of the unnecessary Kraft paper wick. This wick is retained in many film capacitor designs because of its very effective oil wicking properties, which make impregnation of the capacitor faster and thus less expensive. Implementation of "impregnation while winding" would eliminate this problem, but requires substantial modification to existing winding machinery. There are several new classes of silicone dielectric fluids of low loss which may be a very good dielectric constant match at the higher frequencies. These, in conjunction with several of the new "proprietary" fluids,¹⁰ could be combined with the above winding technique to eliminate almost completely the possibility of corona damage from impregnant voids between dielectric sheets or dielectric edges and the electrode. Note that, normally, several thin (0.001 in.) layers of dielectric are used rather than one thick sheet as the latter results in much higher breakdown voltages because thicker sheets have a much higher percentage of imperfections, inclusions, and small voids. This is a technological limitation that could likely be overcome at some considerable cost.

Now we are left with only the problem of surface tracking at the edge of the dielectric as sketched in Fig. 14. Charlie Martin at Aldermaston, has a simple way of grading edges; namely, you cover them with blotting paper impregnated with a dilute solution of copper sulphate in

Fig. 14. Physical factors affecting capacitor lifetime.

water. Having tried this on numerous occasions it can be confirmed that it is very effective indeed. What the paper does is reduce the concentration of the electric field at the interface of the electrode edge and dielectric, essentially by providing a resistor-like layer from the electrode out to the edge of the dielectric. The results are dramatic and stresses of much higher values than in commercial capacitors (which generally operate at less than 3,400 V/mil) are achievable. The difficulty with this technique is that such resistively graded systems must be pulse charged, owing to the dc resistance of the copper sulphate solution shunting the capacitor. Thus, we see that some form of resistive grading will be necessary to minimize this effect and this must be tempered by the increase in the loss factor that may follow, placing more power loss inside the capacitor.

When the user experiences failures in his pilot run of capacitors, the above notes may provide some help in identifying the problem areas and allow working with the supplier to rectify the problem. It is a general observation that larger size and very long lifetime are currently directly related.⁹ In general, the internal material cost is relatively a smaller portion of the price compared to the labor to make a unit very compact, often with special high-stress headers. Unless space constraints dominate, it is in the interests of reliability to allow the manufacturer to recommend a capacitor volume for that specific application. Often, the volume ratios of different supplier's capacitors provide a clue to the design margins each is using. If all suppliers utilize paper wicking, then the cost per unit capacitor element volume is a good selection factor to be included in the assessment of each potential supplier. Combine this with the knowledge of the stress level in each, the method of electrical connection, and the mechanical package pressure control upon the capacitor packs, and you have most of the factors needed to make the most cost-effective choice.

a. Long-Life Service. In dc and ac service, capacitor lifetimes of 30 to 40 design years have been achieved in the power industry and dc

filter applications. These are obtained in most cases by using highly purified mineral oils with stabilizing additives and the capacitor is vacuum baked, dried, and then oil impregnated at an exceptionally low moisture level.¹¹ All these units use Kraft paper for historical reasons which need not concern us here. The oil impregnated capacitor is hermetically sealed and may include a bellows or other thermal expansion device. In Fig. 15 is shown a curve of the internal impulse strength against years for this type of long-life design compared to a typical short-life design representative of what one might expect from a dc or ac filter or coupling capacitor for commercial service.¹¹ The differences are in the quality of manufacture and in the electrical stress levels in the dielectric. The long-life design may well operate below 300 V/mil (and let us note this makes for a large capacitor) and the short life in the neighborhood of 900 V/mil. The BIL is the unity probability of failure impulse strength level of the capacitor and all units must meet this standard after 30 to 40 yr of service (at the "end of life"). In the case of film capacitors in the same duty, lifetimes of 10 to 15 yr are likely, based upon accelerated life tests. A real advantage of film units is their very much lower power losses at higher frequencies and higher insulation resistances, both factors serving to reduce internal heating and corona currents. They are smaller, but considerably more expensive.

We have now reached the point in our notes where the question of lifetime of energy discharge capacitors must be addressed. Up to now, the discussion has centered about capacitor parameters in general, with some notes on points of interest for pulse capacitors interjected where they related to the parameters under discussion. The rest of the note looks at the characteristics of discharge circuits having ideal switches and various types of loads. It will be assumed that the discharge loop can be described by some lumped inductance and that the switch instantly closes. Although this is most clearly a gross over simplification of

INDICATED IMPULSE STRENGTH vs TIME

SEALED, OIL-PAPER INSULATION SYSTEM.
EFFECT OF AGING ON INTERNAL IMPULSE STRENGTH.

Fig. 15. Impulse strength as a function of time for very long lifetime capacitors for 60 Hz ac duty.

the true state of affairs, it will serve to illustrate some of the salient features of the circuitry and the currents and voltages the capacitor will likely see.

IV. APPLICATIONS OF ENERGY STORAGE CAPACITORS

The energy storage capacitor can be used singly, or with many units, in parallel, to provide short pulses of electrical energy up to many megajoules of energy. The electrical energy stored in a capacitor of capacitance C is $1/2 CV^2$, where V is the final voltage across the capacitor at the end of the charging cycle. Applications for high-peak-current capability capacitors include particle accelerators, metal forming, laser drivers, and x-ray generators. In these situations, discharges are often highly oscillatory and voltage reversals up to 90% are not uncommon. Flash tubes, some lasers, and numerous welding applications have capacitors discharging into resistors or other critically damped loads, with relatively longer discharge times and small reversals compared to the previous case.

A. Discharge Circuit

A simplified discharge circuit is sketched in Fig. 16; this illustration shows a resistively charged capacitor being discharged into a load, which has a resistance R and an inductance L . The question of what happens when the load is a laser where a peak voltage must be reached before current flows is very complex.¹² In the case in Fig. 16, closing the switch discharges the capacitor into the load in a nonoscillatory, oscillatory, or overdamped manner depending upon the values of R , L , and C as illustrated in Fig. 17.¹³ The terminology is defined just below the heading and differs slightly because E is substituted for V_c . Using these relations, the user can calculate the ringing frequency, peak current, voltage reversal, and number of post-discharge rings in the oscillatory case.

CIRCUIT: TYPICAL DISCHARGE WAVESHAPES FOR THE UNDERDAMPED OR OSCILLATORY CASE:

Fig. 16. Simplified circuit diagram for charging and discharging an energy storage capacitor.

EQUATIONS Definitions of Symbols

$i(t)$ - current at any time, t
 I_p - peak current (in amp)
 C - capacitance (in farads)
 L - total circuit inductance (in henries)
 E - charge voltage (in volts)
 f - ringing frequency of the circuit
 R - total circuit resistance (in ohms)
 Q - merit factor
 ESR - equivalent series resistance of capacitor
 R_c - Capacitor ESR

1. Stored energy (in joules) = $1/2 CE^2$

2. Voltage reversal (in percent) =

$$-\frac{R}{4Lf}, \text{ where } f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

3. Voltage reversal (in percent) = $\left(1 - \frac{\pi}{2Q}\right) \times 100$
(for high % reversals)

4. Q of the capacitor (at resonance) = $\frac{1}{ESR} \sqrt{\frac{L}{C}}$

5. $Q = \frac{X_c}{ESR}$

NONOSCILLATORY	OVER-DAMPED	OSCILLATORY
Critically damped, $R = R_c = 2\sqrt{\frac{L}{C}}$ $i(t) = \frac{E}{L} t e^{-\frac{Rt}{2L}}$	$R > 2\sqrt{\frac{L}{C}}$ $i(t) = \frac{E}{2j\omega L} \left[e^{-\frac{Rt}{2L} + j\omega t} - e^{-\frac{Rt}{2L} - j\omega t} \right]$	Under-damped, $R < R_c$ $i(t) = \frac{E}{\omega L} e^{-\frac{R}{2L}t} \sin \omega t$ WHERE $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
$I_p = 0.736 \frac{E}{R_c} = 0.368 E \sqrt{\frac{C}{L}}$ Time to reach $I_p = \frac{2L}{R_c}$	$I_p = 0.736 \frac{E}{R} = 0.368 \sqrt{\frac{C}{L}} \left(\frac{E}{R} \right)$ Time to reach $I_p = \frac{2L}{R}$	$I_p \approx \frac{E}{\omega L} e^{-\frac{R}{4\omega L} \pi}$ WHEN $R \ll R_c$ $I_p = E \sqrt{\frac{C}{L}}$ IDEAL CASE WHEN $R = 0$ $I_p = \left[1.00 + \frac{\% \text{ REVERSAL}}{100} \right] \times \frac{\omega CE}{2}$
		for high values of voltage reversal. Time to reach $I_p \approx \frac{\pi}{2} \sqrt{LC}$ WHEN $R \ll R_c$

Fig. 17. Capacitor discharge circuit relationships for a load having a lumped resistance and inductance of fixed values.

B. Ringing Frequency

For oscillatory discharges and small load resistances the ringing frequency is:

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad \omega^2 LC = 1$$

It is very difficult to make this larger than 100 MHz in lumped capacitor systems for high-power applications, at the multikilovolt level. The reasons for this have been discussed before and are the lumped inductance limitations in high-voltage geometries inside the capacitor and the transmission line discharge time of each pack in the capacitor. There are some kilojoule or so, 100-kV systems capable of 25 to 50 ns discharge times operating at low-repetition rates (less than 10 Hz). These were developed a number of years ago for laser drivers and they worked very well, but are rather expensive.¹⁴ High-repetition rate versions would require pulse charging and grading development, particularly if the 85% reversal duty cycle was retained.

C. Peak Current

Resistance in the discharge loop represents a power loss term, which one generally wishes to minimize for maximum energy transfer. The limit here is the resistance of the load, be it a laser, diode, or pure inductor. If the resistance is made small in order to increase the peak current and reduce the pulse width (ref. Fig. 17) then the peak current I_p is approximately:

$$I_p = V \cdot \sqrt{\frac{C}{L}}$$

Increasing the value of R, where:

$$Q \cdot R = \sqrt{\frac{L}{C}}$$

gradually reduces the peak current, until, at critical damping, I_p is reduced by a factor of 0.4 from the oscillatory case. Note that the current waveform may be clamped when a unidirectional switch such as a thyatron is used. In this case the switch opens when the voltage across it turns negative and the device turns off. The rms current is thus significantly reduced and the voltage reversal experienced by the capacitor is controlled primarily by the inverse voltage-limiting circuits placed across the switch. Each case here must be individually analyzed and this subject, for modulator and laser drivers, finds computer simulation a very useful tool.

D. Voltage Reversal

The voltage reversal has a first negative peak given by:

$$\frac{V_{\text{reversal}}}{V_{\text{charge}}} = \exp \left(-\frac{\pi \cdot R}{2} \cdot \sqrt{\frac{C}{L}} \right)$$

Normally, this is expressed as a percentage of the charge voltage. For most high-repetition rate capacitor geometries, 85% reversal is the cost-effective limit and should cost you, the user, a small premium over the standard 20% reversal units. Even if these large reversals are experienced on a small percentage of the pulses, reliability can be significantly enhanced by requiring the unit to live with a 1-cycle reversal of 85% for all pulses. Reversal primarily causes corona damage at dielectric edges in a mechanically well-designed capacitor. This damage is currently minimized by designing such capacitors for ac duty stresses, making them rather large but at a slight increase in cost.

E. Lifetime

Lifetime for most liquid impregnated capacitors follows the relation:^{2,5}

$$\text{Lifetime} = \left(\frac{V_{\text{charge}}}{V_{\text{breakdown}}} \right)^{-8} \cdot Q^{-2.2} \text{ (shots)} \quad \underline{\text{DC charging}}$$

For a given capacitor geometry, the manufacturer has accurate data on the unit probability breakdown voltage, generally after 1 minute at that dc charge voltage.² The user should note that this lifetime dependence upon stress results from corona and dielectric surface tracking, with occasional bulk breakdown and the exponent may be significantly increased through pulse charging. The same law holds true for all corona limited systems; this applies to almost all highly stressed discharge capacitors-paper, mica, ceramic and true liquid dielectrics. We have ascertained that ceramic capacitors follow the same law but with an exponent of 12, for pulse charging times less than a few hundred nanoseconds, which is shorter than the corona inception time:

$$\text{Lifetime} = \left(\frac{V_{\text{charging}}}{V_{\text{breakdown}}} \right)^{-12} \cdot Q^{-2.2} \text{ (shots)} \quad \underline{\text{Pulse Charging}}$$

One is tempted to hazard a guess that this relation may hold for oil impregnated and mica capacitors, but this remains to be shown. Q is the circuit quality factor and is related to the per cent reversal by:

$$\% \text{ Reversal} = \left(1 - \frac{\pi}{2Q} \right) \cdot 100$$

V. CLOSING NOTE

As a closing note, low-inductance kilohertz repetition rate mica capacitors are now being produced for operation at several hundred amps peak current, with measured lifetimes of 10^{10} shots at very low failure levels. Extension to multikilohertz operation appears feasible, but will require great care in keeping internal power losses sufficiently low to conserve this long life. Further developmental work is required to extend this class of capacitor into the tens of kiloamps of peak current at voltages in the region 100 to 150 kV.

Differential Measurement of Fast Energy Discharge Capacitor, Inductance, and Resistance

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Abstract—A differential method to measure, simply and accurately, the equivalent series inductance (ESL) and the equivalent series resistance (ESR) of energy discharge capacitors is described and compared with other available methods. The differential method measures the ESL over a wide range of inductance, including extremely low inductance values, without elaborate equipment or intricate data interpretation. The differential method also determines the ESR of fast energy capacitors at frequencies approaching the capacitor self-resonant frequency.

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THE CAPACITANCE C , the effective series resistance (ESR), and the effective series inductance (ESL) of fast energy discharge capacitors are important parameters in pulsed power system design. The capacitance is easily measured, but the ESR and the ESL are sometimes difficult to obtain. The ESR is highly dependent upon the frequency, especially in the high energy density capacitors, but is expressed in terms of the loss factor at a frequency of only 1 kHz by most companies. Pulsed power systems that rely on low inductance, high energy density capacitors usually operate in the hundred kilohertz to megahertz range.

The ESL is not measured but estimated by most capacitor companies and is of major concern. There are

Fig. 1. Standing wave method.

several methods for determining the capacitor ESL. The standing wave method [1], shown in Fig. 1, imposes a standing wave upon a transmission line terminated by the test capacitor. The frequency of voltage null at the capacitor terminals determines the self-resonant frequency and thus the ESL. This method is valid until the capacitor plates begin to act as transmission lines ($ESL \leq 40$ nH) resulting in internal reflections which lead to erroneous results [1].

The variable short method [1] is used for extremely low inductance. The test capacitor is fitted with an adjustable external short circuit and spark gap, as shown in Fig. 2. The capacitor is charged to a voltage V_0 and discharged through the spark gap while dI/dt is monitored with a calibrated probe. The external circuit inductance L_e is varied in exact increments and the maximum $dI/dt \approx V_0/L$ is plotted versus the external circuit inductance, as in Fig. 3, to determine the capacitor series inductance, L_s , or ESL.

The two RF methods sometimes require elaborate and powerful equipment as well as accurate measurement and may be invalid in the very low inductance range. The variable short method requires a well-designed and machined, adjustable short, as well as careful measurement of the time rate of change of the current. The exact inductance of the external circuit and spark gap must be known to determine the ESL accurately.

The differential method explained below is simple and accurate, but does not require exact machining or elaborate equipment. This method requires a low inductance external circuit with spark gap as shown in Fig. 4. The only requirement on the external circuit is that the inductance be on the order of the capacitor inductance

Fig. 2. Variable short method.

Fig. 3. Variable short data interpretation.

to be measured. Two low inductance capacitors, C_1 and C_2 , are connected in series in a low inductance arrangement as shown in Fig. 4(a). The circuit is charged and discharged while monitoring the capacitor voltage or the current to determine the oscillatory waveform. Next the capacitor to be measured is removed and replaced by a direct short that duplicates the capacitor terminals as shown in Fig. 4(b). The circuit is again charged and discharged while monitoring the voltage or current. The ESL of the removed capacitor is the difference in the inductance of the two circuit arrangements. Assuming that the series resistance is negligible, the inductance L_2 of the circuit, shown in Fig. 4(a), with two capacitors is

$$L_2 = \frac{C_1 + C_2}{\omega_2^2 C_1 C_2}$$

where ω_2 is the resultant resonant frequency. When the test capacitor C_1 is removed, the circuit inductance is

Fig. 4. Differential capacitor measurement.

$$L_1 = \frac{1}{\omega_1^2 C_2}$$

and the ESL is defined by

$$ESL = L_2 - L_1$$

or

$$ESL = 1/C_2 \left[\frac{C_1 + C_2}{\omega_2^2 C_1} - \frac{1}{\omega_1^2} \right]$$

Using the period of oscillation $T = 2\pi/\omega$, the ESL can be determined by

$$ESL = 1/(4\pi^2 C_2) \left[\frac{(C_1 + C_2)}{C_1} T_2^2 - T_1^2 \right]$$

For identical capacitors, $C_1 = C_2 = C$,

$$ESL = 1/(4\pi^2 C) (2T_2^2 - T_1^2)$$

The magnitude of the circuit inductance external to the test capacitor is constrained to be on the same order as the test capacitor. This is necessary to ensure that the observed change in inductance is greater than the errors in measurement. The external circuit inductance should, therefore, be small enough to satisfy the following relationship:

$$\frac{L_2 - L_1}{L_2} = \frac{ESL}{L_2} > 0.1$$

The ESR of the removed capacitor can be determined from the difference in the series resistance of the two circuits. The series resistance R_T of a series RLC circuit is given by

$$R_T = \frac{2L_T \ln \frac{Q(t_1)}{Q(t_2)}}{t_1 - t_2}$$

where L_T is the total series inductance and $Q(t_1)$ and $Q(t_2)$ are the values of the measured quantity, voltage, or current, at times t_1 and t_2 , respectively, as shown in Fig. 4(c). The ESR of the removed capacitor C_1 is given by

$$ESR = R_2 - R_1$$

where R_2 is the circuit series resistance with two capacitors and R_1 is the series circuit resistance with one capacitor. The resistance of the external circuit must be on the same order or smaller than that of the test capacitor. Again, this is necessary to ensure that the observed change in resistance is greater than the error of measurement. A reasonable constraint is

$$\frac{R_2 - R_1}{R_2} = \frac{ESR}{R_2} > 0.1$$

This constraint is easily obtainable in most circuits. The external circuit resistance is due mainly to the spark gap (10–20 mΩ) and the external circuit capacitor. The capacitor resistance increases with frequency and will usually dominate the spark gap resistance at the frequencies involved. Thus the test capacitor resistance (ESR) is approximately equal to the external circuit resistance.

The series resistance of the circuit affects the resonant frequency. The percent of error in a measurement of inductance ($ESL = L_2 - L_1$) due to the series resistances R_1 and R_2 , is given by

% error in ESL =

$$\frac{R_1^2 C_1}{4L_1} \left[\frac{1 - \left(\frac{R_2 \omega_1}{R_1 \omega_2} \right)^2 \frac{L_1}{L_2}}{1 - \left(\frac{\omega_1}{\omega_2} \right)^2 \frac{C_1 + C_2}{C_1}} \right] \times 100$$

There is a slight change in the external circuit resistance due to exponentially decreasing current through the spark gap. A more accurate method for determining the circuit resistance requires plotting the absolute amplitudes of successive maxima and minima of the voltage or current versus time on log paper. The slope of this line of points is the circuit resistance. It is possible to increase the accuracy of this method by taking into account the change in inductance and circuit resistance due to the change in the skin depth at different frequencies, caused by removing the test capacitor, and measuring the capacitance of C_1 and C_2 at the resulting ringing frequency on an LC bridge.

The differential method determines the ESL of a low inductance capacitor exactly and the ESR at frequencies approaching the capacitor self-resonant frequency without using complicated external shorts, low inductance spark gaps, or marginal RF methods.

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Report of Work Performed by M. P. Maley (Q-10)
for AP-1 on Electromagnetic Shock Lines

Introduction

This constitutes an informal report of a study performed on electromagnetic shock lines during the period 13 Nov.-22 Dec. 1978. The purpose of this study is to provide information necessary to assess the feasibility of using electromagnetic shock lines for generating high-power, high-repetition-rate, fast rise-time pulses. The first part of this effort was a general literature survey to determine the present state-of-the-art in this field. This was followed by a study of the material requirements of such a line, including a survey of US manufacturers and suppliers. The final stage was an attempt to develop some general design considerations based on Kataev's theoretical treatment and available materials.

Aside from some interest in the early 60's (Landauer and Hatfield), there does not appear to have been any development of the principle of electromagnetic shock waves for shaping pulses in the US. Following the comprehensive treatment by Kataev in "Electromagnetic Shock Waves," workers in the USSR have carried on a modest but steady development effort (see 27 references below). From a perusal of the Russian literature, it appears that the use of electromagnetic shock lines for producing fast rise times on pulses is now a well established and much used technique. One of these papers (see 12) describes what appears to be a "commercial" pulser based upon this principle. The most active work in this area has been performed by I. G. Kataev and his disciples, I. I. Rozhkov, and A. N. Meshkov. Although, as the reference list indicates, a number of other groups are involved in this field. Recently, references to work underway in Czechoslovakia by O. Benda and J. Oravec have appeared. Unfortunately none of these references have been translated from the Slovak. Toward the end of my investigation, I discovered, by accident, the existence of work being carried out at LLL by Nick J. Chakakis. A telephone conversation with Chakakis, described below, confirmed that he has successfully developed fast-rise-time pulsers based on e.m. shock lines during the past two years. He is also of the opinion that an effort along these lines is underway at CERN. Unfortunately none of the work at LLL or CERN has been published. Nevertheless, I am of the opinion that the Soviets are significantly ahead of anyone else in this field.

Literature Search

I used the Physics Abstracts and the Electrical and Electronics Abstracts to search for references containing the words electromagnetic shock waves and for articles authored by Kataev. Later key words used were pulse generator or shaper in conjunction with ferrite, shock or non-linear (also with high power or high rep-rate). Key authors searched for were Kataev, Meshkov, Rozhkov, Benda, Veghova, Hatfield, Landauer, Mesyats and Meierovich. After a thorough search of the abstract indices, I gave this same list of key words and people to Lara Baker at TD-7 to run through their search program. Some of these same names were run through the NTIS system by the Main Library.

A particularly rich source of references on the subject was found in the Russian journal Priory i Teknika Eksperimenta (PTE). A complete subject index listing of publications by this journal through 1976 proved very useful. The journal Izvestia VUZ Radioelektronika contains somewhat more comprehensive articles. However, this journal was not translated from 1968-1978 and translation has only recently been resumed. Two of the articles listed below are not available from this period and have been ordered through library request forms. Similarly, none of the Czech references are available at LASL and they have also been ordered. In addition, I came across references to Russian books or monographs on the subject of pulse compression and magnetic thyristor pulsers (see Shirman and Meierovich below) which are not available here. These are also on the library search list.

Summary of Russian Work

1. The use of the electromagnetic shock wave principle for producing fast rise times (~ 1 ns) on pulses is a well established and rather conventional technique in the Soviet Union.
2. The work by I. G. Kataev "Electromagnetic Shock Waves" is still the "Bible" in this field. Subsequent experimental work has largely confirmed the theoretical treatment contained in this book.
3. The usual arrangement for high-power pulse generators is a conventional generator incorporating a thyratron or thyristor switch, followed by a magnetic pulse compression network using ferrite cores, followed by a non-linear (shock wave) transmission line using rectangular loop ferrites. The "shock line" itself does not provide any pulse compression but merely compresses the leading edge of the pulse. Pulse amplitude gain is all achieved in the lumped element compression network.

4. Rise times on pulses ~ 1 ns are routinely achieved in shock lines. The highest pulse power reported in these references is 300 kW in a 2-30 ns length corresponding to 800 A into a low-ohm load or 10 kV across a capacitive load and at a rep-rate of 100 Hz. Rep-rates up to 25 kHz have been achieved for lower power pulsers. Typical applications for these pulsers have been for pumping injection lasers and for the electron gun of a linac.
5. Although VT-6 (magnesium-manganese) ferrite appears to be the favorite material for shock line development in the USSR, NN (nickel-zinc) and NM (manganese-zinc) ferrites have also been used successfully. In Ref. 2 below, the statement is made that "a magnetic material that is less inertial than these 3 ferrites evidently does not exist in nature."
6. The Russian ferrites appear to be nearly identical in characteristics to the rectangular-loop ferrites developed in the US, Netherlands, and Japan for computer memory cores.

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E. M. Shock Line Work at LLL

Mr. Nick J. Chakakis has been developing a pulser based on the e.m. shock wave principle for application to an electron-gun for the past two years. His place of work has been building 194 (the electron-positron accelerator). He has recently been transferred to a new division (Tel. #415/422-8381). He informed me that he had developed a pulser that delivers a 40 A, 2000 V pulse into a 50- Ω load. He has achieved a 1.5 ns rise and fall time and a rep-rate of 1400 Hz. The shock line is coaxial, filled with small (3 mm o.d.) MgMn ferrite rings obtained from Transtek. He was enthusiastic about the performance of the shock line and saw no reason why it could not be scaled up for much higher power pulses. None of this work has been published, but he has written progress reports. He agreed to send me some written material but has not done so yet.

Ferrite Materials

The requirements for a suitable ferrite material for use in an e.m. shock line are as follows:

1. A rectangular hysteresis loop with a low coercive force (1.0 Oe or less) and a high remanence (80% of saturation).
2. High electrical resistivity ($\rho_0 \geq 10^8 \Omega\text{-cm}$) and high dielectric strength ($\geq 10^5 \text{ V/cm}$).
3. A moderately high dielectric constant (6-12).
4. A saturation magnetization $M_0 \sim 100\text{-}300 \text{ Oe}$.
5. Fast switching properties (switching constant $S_w \leq .1 \mu\text{s-Oe}$).
6. Low anisotropy and magnetostrictive coefficients.

These requirements are generally satisfied by a number of ferrites that were developed for computer-memory cores. Magnesium-manganese ferrites are one class of these materials that exhibit very high electrical resistivity and a very rectangular hysteresis loop. Because of the economic and technological importance of these materials, they have been exhaustively studied. In addition to the wide variations in elemental composition and stoichiometry available, processing techniques are also important in determining properties. This is an extremely mature field. The wealth of options available in preparing these materials makes it possible to select characteristics such as saturation magnetization, resistivity and dielectric constant almost independently. Obtaining a rectangular hysteresis loop appears to be still somewhat of an art, but can be accomplished with great

reliability by a number of commercial firms. In addition to the spinel class of which MgMn-ferrite is a member, the garnets are also promising candidates. In general, garnets have higher resistivities than spinels and may have higher breakdown strengths.

In view of the above considerations, I would recommend a close collaboration with a commercial firm with expertise in the field of developing and manufacturing ferrites for microwave and computer applications. A partial list of such firms follows:

1. Trans-Tech, Inc.
12 Meem Avenue
Gaithersburg, MD 20760
Tel: (301)948-3800
*Russ West has worked with Chakakis at LLL and seems very knowledgeable about e.m. shock lines - has read Kataev's book).
2. Airtron/Dv Litton
200 E Hanover
Morris Plains, NJ 07950
(201)539-5500
3. Krystinel Corp.
Fox Island Road
Port Chester, NY 10573
(914)937-5253
4. Countis Ind
P. O. Box 1444
San Luis Obispo, CA 93406
(805)544-1770
5. Indiana General Corp.
General Dynamics Div.
Keasbey, NJ 08832
(201)826-5100

Design Considerations

In order to discuss factors affecting the design of an e.m. shock line, it will be necessary to state the basic relations derived in Kataev's book.

Basic Equations

$$1. \quad v_p = \frac{v_0}{\sqrt{\mu_{sh}}}$$

where v_p is the velocity of propagation of the shock front, v_0 is the e.m. propagation velocity in the saturated ferrite and μ_{sh} is the dynamic permeability in the shock front (defined in 2). $\mu_{sh} \gg 1$ and therefore $v_p \ll v_0$.

$$2. \quad \mu_{sh} = 1 + \frac{8KM_0}{H_{sh}}$$

where M_0 is the saturation magnetization of the ferrite, H_{sh} is the magnitude of the "jump" in magnetic field across the shock front and $0 \leq K \leq \pi$ is a parameter describing the fraction of the area between the two conductors which is occupied by ferrite polarized in the opposite direction to the field of the incoming pulse. μ_{sh} is the most important parameter in describing the characteristics of a shock line.

$$3. \quad V_{sh} = \rho_0 \sqrt{\mu_{sh}} I_{sh}$$

where V_{sh} is the magnitude of the voltage step and I_{sh} that of the current step across the shock-wave front. ρ_0 is the value of the characteristic impedance of the line when the ferrite is saturated. This leads to:

$$4. \quad Z = \rho_0 \sqrt{\mu_{sh}} = \sqrt{\frac{L_0}{C_0}} \frac{\sqrt{\mu_{sh}}}{\sqrt{\epsilon_{eff}}}$$

where Z is the characteristic impedance seen at the shock front, L_0 and C_0 are respectively the inductance and capacitance of the unfilled line and ϵ_{eff} is the effective dielectric constant of the ferrite and any other filler in the line.

$$5. \quad \tau_{sh} = \frac{6.7 M_0}{\lambda H_{sh}}$$

where τ_{sh} is the rise time of the current and voltage in the shock front, H_{sh} is the magnitude of the jump in magnetic field and M_0 is the saturation magnetization in Oe. λ is a material dependent parameter. For the rectangular loop M_{gn} ferrites of interest $\lambda \sim 1 \times 10^{10} \text{ s}^{-1}$, $M_0 \sim 200 \text{ Oe}$ and $\tau_{sh} \sim 1 \times 10^{-7} \text{ s}$.

$$6. \quad l_c = \frac{V_c P}{4 K M_o} \int_0^{\infty} I_{sp}^i dt$$

where l_c is the length of line required to envelop the entire leading edge of the pulse in a shock front, $p = \frac{H}{I}$, and $I_{sp}^i(t)$ is the expression for the leading edge of the input current pulse. For an input current with an exponential rise τ_i , this becomes:

$$6a. \quad l_c = \frac{V_c P}{4 K M_o} I_o \tau_i, \quad \text{where } I_o \text{ is}$$

the input current amplitude.

$$7. \quad I_{sh} = \frac{2 I_{sp}^i}{1 + \sqrt{\mu_{sh}}}$$

This equation gives the relation between the input current amplitude I_{sp}^i and the amplitude I_{sh} of the resulting shock front. The difference is caused by reflections caused when, during the shock development, the overtaking simple wave traveling through an impedance ρ_o hits the shock front with impedance Z .

$$8. \quad W_L = \frac{1}{4} L_o I_{sh}^2 (\mu_{sh} - 1)$$

where W_L is the energy loss per unit length of line which occurs during the switching of the ferrite. This loss is a fraction of the energy associated with the shock front given by:

$$8a. \quad f = \frac{W_L}{W} = \frac{1}{4} \frac{\mu_{sh} - 1}{\mu_{sh}}$$

These 8 equations contain most of the information needed to design an e.m. shock line. Before considering a detailed design, I would like to list some general observations concerning these equations.

1. From Eq. 4, it is clear that a small value for μ_{sh} and a large value for E_{eff} will be important for low impedance applications.
2. Small μ_{sh} will also be essential for reducing losses (Eq. 8) and for keeping reflections from the shock front to a minimum (Eq. 7).
3. The value of μ_{sh} (Eq. 2) is determined by M_o/H_{sh} and by K . Thus low values for M_o and configurations in which H_{sh} are high will be helpful. It should be pointed out, however, that large values of H_{sh}/I_{sh} necessarily imply a large value for L_o , the saturated inductance, thus raising Z . The value of K may be varied from 0- π by filling part of the space between the conductors with a nonmagnetic

dielectric or by biasing part of the ferrite in the direction of the incoming wave field.

4. Low values for μ_{sh} will lower impedance, losses, and reflections but will increase the required length of the line (Eq. 6). This length can be reduced by keeping the input-wave rise time τ_i at a low value.

In order to better understand the interrelationships of the parameters in determining line characteristics, it will be useful to consider a specific design. For this purpose I have chosen a 3-conductor strip line as shown in Fig. 1.

In this figure, W is the width of the center conductor, D is the overall thickness, a is the thickness of the ferrite, c is the thickness of ferrite polarized in the direction of the incoming wave, and b is the thickness of the dielectric. The structure is symmetric with the same arrangement of ferrite and dielectric (not shown) above the center conductor.

If we neglect fringing fields the following expressions can be easily derived using Eqs. 1-8 above:

$$9. L_0 = \frac{\mu_c D}{4W}$$

where L_0 is the saturated inductance per unit length.

$$10. C = 4 \epsilon \frac{W}{D}$$

where C is the capacitance per unit length.

$$11. \rho_0 = \sqrt{\frac{L_0}{C}} = \sqrt{\frac{\mu_c}{\epsilon}} \frac{D}{4W}$$

$$12. \epsilon_{eff} = \frac{D}{2} \frac{\epsilon_f \epsilon_D}{a \epsilon_D + b \epsilon_f}$$

$$13. Z = 94 \frac{D}{W} \sqrt{\frac{\mu_{sh}}{\epsilon_{eff}}}$$

$$= 94 \frac{D}{W} \sqrt{1 + \frac{16 \pi M_s (a-c)}{H_{sh} D}} \sqrt{\frac{2(a \epsilon_D + b \epsilon_f)}{D \epsilon_f \epsilon_D}}$$

$$14. V_{\max} = \frac{\mu \epsilon_D + 6 \epsilon_F}{\epsilon_D} E_S$$

where $V_{\max}$ is the maximum voltage and E_S is the dielectric strength of the ferrite.

Equation 13 illustrates the importance of maintaining a large ϵ_{eff} and a small μ_{sh} in order to achieve values of $Z < 1.0 \Omega$. Although the breakdown strength for a given D will be improved by choosing a dielectric with a low value of E_D , this will have the effect of lowering ϵ_{eff} and thus raising Z.

As an illustration let us consider a strip line with the following parameters:

$$D = .02 \text{ m}$$

$$a = .002 \text{ m}$$

$$b = .008 \text{ m}$$

$$M_O = 200 \text{ Oe}$$

$$W = 1.0 \text{ m}$$

$$E_D = 9$$

$$E_F = 12$$

$$\tau_i = 20 \text{ ns}$$

$$I_{\text{sh}} = 10^5 \text{ A}$$

For these parameters

$$\epsilon_{\text{eff}} = 9.47$$

$$\mu_{\text{sh}} = 2.6$$

$$Z = .98 \Omega$$

$$V_{\text{sh}} = 98 \text{ kV}$$

$$I_{\text{sp}} = 130 \text{ kA}$$

$$l_c = 10.0 \text{ m}$$

$$\tau_{\text{sh}} = 1.7 \times 10^{-10} \text{ s}$$

$$E_{\max} = 77.4 \text{ kV/cm}$$

This example illustrates some of the trade-offs involved in designing such a shock line. The length l_c of 10. m is somewhat too large, but lowering it considerably by increasing K and μ_{sh} can only be done by increasing Z.

Summary and Recommendations

1. The use of e.m. shock lines for shaping pulses with rise times $\sim 1.0 \text{ ns}$ is a proven technology for peak power $\leq 300 \text{ kW}$, and scaling up to substantially higher power does not seem to offer any insurmountable problems.

2. A rectangular-loop ferrite with $E_S > 10^5$ V/cm and $E_F \sim 12$ would be required to achieve simultaneously $Z < 1.0$, $l_c < 10$. m and acceptably low losses and reflection coefficient at the current and voltage levels of interest.
3. Any developmental effort should be carried out in close collaboration with one of the commercial firms that specializes in ferrite material development.
4. A development effort should include a complete characterization of the ferrite materials. Measurements of ϵ_p , $M(H)$, E_S , and electrical resistivity should be made as functions of frequency.
5. Limitations on average power transmitted in the shock line will likely be determined by losses in the ferrite material.

Addendum

Since preparing this report, I received a copy of the paper "The Transmission of Pulses Through a Network Containing a Line Incorporating a Rectangular Hysteresis loop Ferrite," I. G. Kataev, A. N. Meshkov, and I. I. Rozhkov Izv. VUZ Radioelektronika 11, 6, 1968 pp. 570-576. This paper develops the conditions for proper impedance matching of the line at the input and output ends. The abstract reads:

"The circuit considered consists of a linear transmission line followed by a line containing a rectangular hysteresis loop ferrite, then another linear line. It is shown that when the characteristic impedances of the lines ρ_1 and ρ_3 and the nonlinear line ρ_0 are all equal, ignoring the magnetization of the ferrite and certain other conditions, the wave is transmitted at full amplitude from the generator to the load but with a sharpened wavefront. The range of values of ρ_1/ρ_0 over which power compression takes place is evaluated. Methods are suggested for measuring the value of ρ_0 ."

It is evident that the development in the main body of this report (based on Kataev's book, Ref. 26) is much too pessimistic as regards designing a line which matches a load impedance of a few tenths of ohms. The impedance of the saturated line ρ_0 and not Z must be matched to the load impedance. This can clearly be accomplished without paying the penalty of making the line impractically long and thus increasing the losses.

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DATA TRANSMISSION INFORMATION

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INTRODUCTION

Digital signals transmitted any distance, only a few inches in some cases or up to several miles, must enter the analog world of transmission lines before arriving at their digital destinations. Therefore, special attention should be given to the interface between the digital and analog areas—often one of the least understood considerations in a system design. Interface problems occur every time a digital signal is sent from a printed circuit board and, indeed, transmission line theory must often be considered when connecting devices on the same board when using very high-speed logic such as ECL or Schottky TTL. This section is intended to shed some light on this important design procedure—interfacing of systems (or subsystems) for the transmission of information from one location to another via line drivers, transmission lines and line receivers.

To design a data transmission interface, the systems designer must be familiar with several subjects. He needs a good understanding of the effects of pulse excitation on transmission lines, knowledge about the various fundamental forms and modes of data-transmission-line circuit operation, and familiarity with the problems encountered when operating with long transmission lines. He must also have a good working knowledge of line drivers and line receivers, their electrical characteristics, and how and where these devices should be used.

This section is not intended to be a textbook in the formal sense, nor is it a 'cookbook' of applications since each system must be designed based on its unique requirements. It is presented as a refresher on the basics of pulse transmission line theory combined with methods for determining signal quality and feasibility of the particular data transmission system. Only those areas that directly affect the transmission of digital signals are covered. This section contains the essential elements to aid the system designer in making an informed decision regarding the quality of data received at the end of a transmission line.

Section two discusses the general characteristics of transmission lines and their derivations. Here, using a transmission line model, the important parameters of characteristic impedance and propagation delay are developed in terms of their physical and electrical parameters.

The third section explores another important characteristic—reflection coefficient. This concept is combined with the material in section two to present graphical and analytical methods for determining the voltages and currents at any point on a line with respect to distance and time after signal application. The effects of various source resistances and line termination methods on the transmitted signal are discussed.

In section four, the logical progression from the ideal transmission line to the real world of the long transmission line with its attendant losses and problems is made; specifically, the methods to determine the practicality of a certain length of line at a given data rate is discussed. Transmission line effects on various data formats are examined as well as the effects of several types of sources (drivers) on signal quality. A practical means is given to measure signal quality for a given transmission line using readily available test equipment. This, in turn, leads to a chart that provides the designer a way to predict the feasibility of a proposed data-transmission circuit when twisted-pair cable is used. A method to estimate signal quality on coaxial-cable interconnections is also discussed.

Section five describes the various combinations of forms and modes of operation in data-transmission systems. Also described are the circuits that conform to industry standards—EIA RS232C, MIL STD 188C and IBM 360/370 I/O interfaces. A selection guide for available line drivers and receivers integrated circuits and some helpful suggestions for incorporating them into a system complete the section.

In section six, new EIA standards of RS-422 and RS-423 specifications are discussed.

Section 2

DATA TRANSMISSION LINES AND THEIR CHARACTERISTICS

INTRODUCTION

A data transmission line is composed of two or more conductors transmitting electrical signals from one location to another. A parallel transmission line is shown in Figure 2-1. To show how the signals (voltages and currents) on the line relate to as yet undefined parameters, a transmission line model is needed.

TRANSMISSION LINE MODEL

Because the wires A and B could not be ideal conductors, they therefore must have some finite resistance. This resistance/conductivity is determined by length and cross-sectional area. Any line model, then, should possess some series resistance representing the finite conductivity of the wires. It is convenient to establish this resistance as a per-unit-length parameter.

Similarly, the insulating medium separating the two conductors could not be a perfect insulator because some small leakage current is always present. These currents and dielectric losses can be represented as a shunt conductance per unit length of line. To facilitate development of later equations, conductance is the chosen term instead of resistance.

If the voltage between conductors A and B is *not variable* with time, any voltage present indicates a static electric field between the conductors. From electrostatic theory it is known that the voltage V produced by a static electric field E is given by

$$V = \int E \cdot dl \quad (2.1)$$

This static electric field between the wires can only exist if there are free charges of equal and opposite polarity on both

wires as described by Coulomb's law.

$$E = \frac{q}{4\pi\epsilon r^2} \quad (2.2)$$

where E is the electric field in volts per meter, q is the charge in Coulombs, ϵ is the dielectric constant, and r is the distance in meters. These free charges, accompanied by a voltage, represent a capacitance ($C = q/V$); so the line model must include a shunt capacitive component. Since total capacitance is dependent upon line length, it should be expressed in a capacitance per-unit-length value.

It is known that a current flow in the conductors induces a magnetic field or flux. This is determined by either Ampere's law

$$\oint H \cdot dl = I \quad (2.3)$$

or the Biot-Savart law

$$dB = \frac{\mu Idl \times r}{4\pi r^3} \quad (2.4)$$

where r = radius vector (meters)
 l = length vector (meters)
 I = current (amps)
 B = magnetic flux density (Webers per meter)
 H = magnetic field (amps per meter)
 μ = permeability

Fig. 2-1. Infinite Length Parallel Wire Transmission Line

If the magnetic flux (ϕ) linking the two wires is variable with time, then according to Faraday's law

$$V = \frac{d\phi}{dt} \quad (2.5)$$

A small line section can exhibit a voltage drop — in addition to a resistive drop—due to the changing magnetic flux (ϕ) within the section loop. This voltage drop is the result of an inductance given as

$$V = L \frac{di}{dt} \quad (2.6)$$

Therefore, the line model should include a series inductance per-unit-length term. In summary, it is determined that the model of a transmission line section can be represented by two series terms of resistance and inductance and two shunt terms of capacitance and conductance.

From a circuit analysis point of view, the terms can be considered in any order, since an equivalent circuit is being generated. Figure 2-2 shows three possible arrangements of circuit elements.

For consistency, the circuit shown in Figure 2-2c will be used throughout the remainder of this chapter. Figure 2-3 shows how a transmission line model is constructed by series connecting the short sections into a ladder network.

Before examining the pertinent properties of the model, some comments are necessary on applicability and limitations. A real transmission line does not consist of an infinite number of small lumped sections — rather, it is a distributed network. For the lumped model to accurately represent the transmission line, (see Figure 2-3) the section length must be quite small in comparison with the shortest wavelengths (highest frequencies) to be used in analysis of the model. Within these limits, as differentials are taken, the section length will approach zero and the model should exhibit the same (or at least very similar) characteristics as the actual distributed parameter transmission line. The model in Figure 2-3 does not include second order terms such as the increase in resistance due to skin effect or loss terms resulting from non-linear dielectrics. These terms and effects are discussed in the references rather than in this chapter, since they tend to obscure the basic principles under consideration. For the present, assume that the signals applied to the line have their minimum wavelengths a great deal longer than the section length of the model and ignore the second order terms.

INPUT IMPEDANCE OF A TRANSMISSION LINE

The purpose of this section is to determine the input impedance of a transmission line; i.e., what amount of input current i_{in} is needed to produce a given voltage v_{in} across the line as a function of the LRCG parameters in the transmission line, (see Figure 2-4).

Combining the series terms IR and IL together simplifies calculation of the series impedance (Z_s) as follows

$$Z_s = l(R + j\omega L) \quad (2.7)$$

Likewise, combining IC and IG produces a parallel impedance Z_p represented by

$$Z_p = \frac{1}{Y_p} = \frac{1}{l(G + j\omega C)} \quad (2.8)$$

Fig. 2-5. Cascaded Network to Model Transmission Line

Since it is assumed that the line model in Figure 2-5a is infinite in length, the impedance looking into any cross section should be equal, that is, $Z_1 = Z_2 = Z_3$, etc. So Figure 2-5a can be simplified to the network in Figure 2-5b where Z_0 is the characteristic impedance of the line and Z_{in} must equal this impedance ($Z_{in} = Z_0$). From Figure 2-5b,

$$Z_{in} = Z_s + \frac{Z_0 Z_p}{Z_0 + Z_p} = Z_0 \quad (2.9)$$

Multiplying through both sides by $(Z_0 + Z_p)$ and collecting terms yields

$$Z_0^2 - Z_s Z_0 - Z_s Z_p = 0 \quad (2.10)$$

which may be solved by using the quadratic formula to give

$$Z_0 = \frac{Z_s \pm \sqrt{Z_s^2 + 4Z_s Z_p}}{2} \quad (2.11)$$

Substituting in the definition of Z_s and Z_p from Equations 2.7 and 2.8, Equation 2.11 now appears as

$$Z_0 = \frac{l(R + j\omega L)}{2} \pm \frac{1}{2} \sqrt{l^2(R + j\omega L)^2 + 4 \frac{R + j\omega L}{G + j\omega C}} \quad (2.12)$$

Now, as the section length is reduced, all the parameters (R , L , G , and C) decrease in the same proportion. This is because the per-unit-length line parameters R , L , G , and C are constants for a given line. By sufficiently reducing l , the terms in Equation 2.12 which contain l as multipliers will become negligible when compared to the last term

$$\frac{R + j\omega L}{G + j\omega C}$$

which remains constant during the reduction process. Thus Equation 2.12 can be rewritten as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{Z_s Z_p} \quad (2.13)$$

particularly when the section length l is taken to be very small. Similarly, if a high enough frequency is assumed,

$$\frac{\omega}{2\pi} > 100 \text{ kHz}$$

such that the ωL and ωC terms are much larger respectively than the R and G terms, $Z_s = j\omega L$ and $Z_p = 1/j\omega C$ can be used to arrive at a lossless line value of

$$Z_0 = \sqrt{\frac{L}{C}} \quad (2.14)$$

In the lower frequency range,

$$\frac{\omega}{2\pi} \approx 1 \text{ kHz}$$

the R and G terms dominate the impedance giving

$$Z_0 = \sqrt{\frac{R}{G}} \quad (2.15)$$

A typical twisted pair would show an impedance versus applied frequency curve similar to that shown in Figure 2-6. The Z_0 becomes constant above 100 kHz, since this is the region where the ωL and ωC terms dominate and Equation 2.13 reduces to Equation 2.14. This region above 100 kHz is of primary interest, since the frequency spectrum of the fast rise/fall time pulses sent over the transmission line have a fundamental frequency in the 1-to-50 MHz area with harmonics extending upward in frequency. The expressions for

Fig. 2-6. Characteristic Impedance versus Frequency

Fig. 2-7. Input Current Into a 96Ω Transmission Line for a 2 V Input Step for Various Line Lengths

Z_0 in Equations 2.13, 2.14 and 2.15 do not contain any reference to line length, so using Equation 2.14 as the normal characteristic impedance expression, allows the line to be replaced with a resistor of $R_0 = Z_0 \Omega$ neglecting any small reactance. This is true when calculating the initial voltage step produced on the line in response to an input current step, or an initial current step in response to an input voltage step.

Figure 2-7 shows a 2 V input step into a 96Ω transmission line (top trace) and the input current required for line lengths of 150, 300, 450, 1050, 2100, and 3750 feet, respectively (second set of traces). The lower traces show the output voltage waveform for the various line lengths. As can be seen, maximum input current is the same for all the different line lengths, and depends only upon the input voltage and the characteristic resistance of the line. Since $R_0 = 96\Omega$ and $v_{in} = 2 \text{ V}$, then $i_{in} = v_{in}/R_0 \approx 20 \text{ mA}$ as shown by Figure 2-7.

A popular method for estimating the input current into a line in response to an input voltage is the formula

$$C(dv/dt) = i$$

where C is the total capacitance of the line ($C = C$ per foot $\times$ length of line) and dv/dt is the slew rate of the input signal. If the 3750-foot line, with a characteristic capacitance per unit length of 16 pF/ft is used the formula $C_{total} = (C \times l)$ would yield a total lumped capacitance of .06 μF . Using this $C(dv/dt) = i$ formula with $(dv/dt = 2 \text{ V}/10 \text{ ns})$ as in the scope

photo would yield

$$I = \frac{2 \text{ V}}{10 \text{ ns}} \times .06 \mu\text{F} = 12 \text{ A}$$

This is clearly not the case! Actually, since the line impedance is approximately 100Ω, 20 mA are required to produce 2 V across the line. If a signal with a rise time long enough to encompass the time delay of the line is used, ($t_r \gg \tau$) then the $C(dv/dt) = i$ formula will yield a reasonable estimate of the peak input current required. In the example, if the dv/dt is 2 V/20 μs , ($t_r = 20 \mu\text{s} > \tau = 6 \mu\text{s}$) then $i = 2 \text{ V}/20 \mu\text{s} \times 0.06 \mu\text{F} = 6 \text{ mA}$, which is verified by Figure 2-8.

Figure 2-8 shows that $C(dv/dt) = i$ only when the rise time encompasses the capacitance to be lumped. The maximum input current requirement will be with a fast rise time step, but the line is essentially resistive, so $v_{in}/i_{in} = R_0 = Z_0$ will give the actual drive current needed. These effects will be discussed later in Chapter 3.

PHASE SHIFT AND PROPAGATION VELOCITY FOR THE TRANSMISSION LINE

There will probably be some phase shift and loss of signal v_2 with respect to v_1 because of the reactive and resistive parts of Z_s and Z_p in the model (Figure 2-5b). Each small section of the line (l) will contribute to the total phase shift and amplitude reduction if a number of sections are cascaded as in Figure 2-5a. So, it is important to determine the phase shift and signal amplitude loss contributed by each section.

Fig. 2-8. Input Current Into Line with Controlled Rise Time $t_r > 2\tau$

Using Figure 2-5b, v_2 can be expressed as

$$v_2 = v_1 \frac{Z_p Z_0}{Z_p + Z_0} \frac{1}{Z_s + Z_p Z_0 / (Z_p + Z_0)} \quad (2.16)$$

or

$$\frac{v_1}{v_2} = \frac{Z_s(Z_p + Z_0) + Z_p Z_0}{Z_p Z_0} \quad (2.17)$$

and further simplification yields

$$\frac{v_1}{v_2} = 1 + Z_s \left[\frac{1}{Z_0} + \frac{1}{Z_p} \right] \quad (2.18)$$

Remember that a per-unit-length constant, normally called γ is needed. This shows the reduction in amplitude and the change in the phase per unit length of the sections.

$$\gamma_l = \alpha_l + j\beta_l \quad (2.19)$$

Since

$$v_2 = v_1 \exp(-\gamma_l) = v_1 \exp(-\alpha_l) + v_1 \exp(-j\beta_l) \quad (2.20)$$

where $v_1 \exp(-\alpha_l)$ is the signal attenuation and $v_1 \exp(-j\beta_l)$ is the change in phase from v_1 to v_2 .

$$\ln \left[\frac{v_1}{v_2} \right] = \ln \exp(\alpha_l + j\beta_l) = \alpha_l + j\beta_l = \gamma_l \quad (2.21)$$

Thus, taking the natural log of both sides of Equation 2.18

$$\ln \left[\frac{v_1}{v_2} \right] = \ln \left[1 + Z_s \left(\frac{1}{Z_0} + \frac{1}{Z_p} \right) \right] \quad (2.22)$$

Substituting Equation 2.13 for Z_0 and Y_p for $1/Z_p$

$$\gamma_l = \ln \left[1 + Z_s \left(\sqrt{\frac{Y_p}{Z_s}} + Y_p \right) \right] \quad (2.23)$$

Now when allowing the section length l to become small,

$$Y_p = l(G + j\omega C)$$

will be very small compared to the constant $\sqrt{Y_p/Z_s} = 1/Z_0$, since the expression for Z_0 does not contain a reference to the section length l . So Equation 2.23 can be rewritten as

$$\gamma_l = \ln \left(1 + Z_s \sqrt{\frac{Y_p}{Z_s}} \right) = \ln \left(1 + \sqrt{Y_p Z_s} \right) \quad (2.24)$$

By using the series expansion for the natural log:

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \text{etc.} \quad (2.25)$$

$\approx x$ for small x

and keeping in mind the $\sqrt{Z_s Y_p}$ value will be much less than one because the section length is allowed to become very small, the higher order expansion terms can be neglected, thereby reducing Equation 2.24 to

$$\gamma_l = \sqrt{Z_s Y_p} = l \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.26)$$

If Equation 2.26 is divided by the section length,

$$\gamma = \frac{\gamma_l}{l} = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.27)$$

the propagation constant per unit length is obtained. If the resistive components R and G are neglected by assuming the line is reasonably short, Equation 2.26 can be reduced to read

$$\gamma_l = j\beta_l = j\omega l \sqrt{LC} \quad (2.28)$$

Equation 2.28 shows that the lossless transmission line has one very important property: signals introduced on the line have a constant phase shift per unit length with no change in amplitude. This progressive phase shift along the line actually represents a wave traveling down the line with a velocity equal to the inverse of the phase shift per section. This velocity is

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad (2.29)$$

for lossless lines. Because the LRCG parameters of the line are independent of frequency except for those upper frequency constraints previously discussed, the signal velocity given by Equation 2.29 is also independent of signal frequency. In the practical world with long lines, there is in fact a frequency dependence of the signal velocity. This causes sharp edged pulses to become rounded and distorted. More on these long line effects will be discussed in Chapter 3.

SUMMARY — Characteristic Impedance and Propagation Delay

Every transmission line has a characteristic impedance Z_0 , and both voltage and current at any point on the line are related by the formula

$$Z_0 = \frac{v}{i}$$

In terms of the per-unit-length parameters LRCG,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Since $R \ll j\omega L$ and $G \ll j\omega C$ for most lines at frequencies above 100 kHz, the characteristic impedance is best approximated by the lossless line expression

$$Z_0 \approx \sqrt{\frac{L}{C}}$$

The propagation constant, γ , shows that signals exhibit an amplitude loss and phase shift with the latter actually a velocity of propagation of the signal down the line. For lossless lines, where the attenuation is zero, the phase shift per unit length is

$$\beta = \frac{\beta_l}{l} = \omega \sqrt{LC}$$

This really represents a signal traveling down the line with a velocity

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

This velocity is independent of the applied frequency.

The larger the LC product of the line, the slower the signal will propagate down the line. A time delay per unit length can also be defined as the inverse of v

$$\delta = \frac{1}{v} = \sqrt{LC} \quad (2.30)$$

and a total propagation delay for a line of length l as

$$\tau = l\delta = l\sqrt{LC} \quad (2.31)$$

For a more detailed discussion of characteristic impedances and propagation constants, the reader is referred to the references below.

Hamsher, D.H. (editor); Communications System Engineering Handbook; Chapter 11, McGraw-Hill, New York, 1967.

Reference Data for Radio Engineers, fifth edition; Chapter 22; Howard T. Sams Co., New York, 1970.

Matick, R.F.; Transmission Lines for Digital and Communications Networks; McGraw-Hill, New York, 1969.

Metzger, G. and Vabre, J.P.; Transmission Lines with Pulse Excitation; Academic Press, New York, 1969.

Section 3

REFLECTIONS: COMPUTATIONS AND WAVEFORMS

INTRODUCTION

In Section 2 it was determined that transmission lines have two important properties: one, a characteristic impedance relating instantaneous voltages and currents of waves traveling along the line and, two, a wave propagation velocity or time delay per unit length. In this chapter, both Z_0 and δ are used to compute the line voltages and currents at any point along the line and at any time after the line signal is applied. Also, concepts of reflections and reflection coefficients are explored along with calculating methods for voltages and currents.

THE INITIAL WAVE

Section 2 also showed that for most practical purposes, where fast rise and fall time signals are concerned, the characteristic impedance of the line actually behaves as a pure resistance ($R_0 = \sqrt{L/C}$).

Figure 3-1a shows a generator comprised of a voltage source (magnitude V), a source resistance of R_S ohms, and a switch closing at time $t = 0$ connected to a lossless, infinite length transmission line having a characteristic resistance, R_0 . Because the relationship of v_{in} to i_{in} is known as $v_{in} = R_0 i_{in}$, the lossless transmission line can be replaced with a resistor as shown in Figure 3-1b. The loop equation is

$$i_{in} (R_S + R_0) = V \quad (3.1)$$

Substituting v_{in}/R_0 for i_{in} and collecting terms shows

$$v_{in} = V \left(\frac{R_0}{R_0 + R_S} \right) \quad (3.2)$$

This shows that both source and characteristic resistances act as voltage dividers for the source voltage V . Figure 3-2 shows voltage and current steps for the various source resistances. Source resistances of less than R_0 produce initial voltage steps on the line which are greater than half the compliance of the source voltage, V . A matched source ($R_S = R_0$) produces voltage steps exactly half of V and source resistances greater than R_0 produce an initial voltage step less than one half V in magnitude. Generators can be classified into three categories:

- Voltage source types where $R_S < R_0$
- Matched source types $R_S = R_0$
- Current source types $R_S > R_0$

Waveforms of these types will be discussed more fully in section 4 on long line effects. Suffice to say that initial voltage wave amplitude depends greatly on source resistance. Voltage source type drivers produce higher amplitude initial voltage waves in the line than either matched source or current source type drivers.

CUT LINES AND A MATCHED LOAD

In examining an infinite, lossless line (Figure 3-3), it is already known that the ratio of line voltage to current is equal to the characteristic resistance of that line. The line is lossless, and the same voltages and currents should appear at point x down the line after a time delay of $x\delta$. If the line at point x is cut, and a resistor of value R_0 is inserted, there would not be a difference between the cut, terminated finite line and the infinite line. The v_x and i_x waves see the same impedance (R_0) they were launched into at time $t = 0$, and indeed, the waves are absorbed into $R_L (= R_0)$ after experiencing a time delay of $\tau = x\delta$. So, from an external viewpoint, an infinite-length lossless line behaves as a finite-length lossless line terminated in its characteristic resistance.

KIRCHOFF'S LAWS AND LINE-LOAD BOUNDARY CONDITIONS

The principle of energy conservation, widely known and accepted in the sciences, applies as well to transmission line theory; therefore, energy (as power) must be conserved at boundaries between line and load. This is expressed in an English language equation as follows.

$$\left[\begin{array}{c} \text{Power available at} \\ \text{the line end} \end{array} \right] = \left[\begin{array}{c} \text{Power absorbed} \\ \text{by the load} \end{array} \right] + \left[\begin{array}{c} \text{Power not absorbed} \\ \text{by the load} \end{array} \right]$$

Figure 3-4 shows power available at the line end is derived by the following formula. (This is assuming in-phase current and voltage).

$$P_x = i_x \cdot v_x = \frac{v_x^2}{R_0} \quad (3.3)$$

The power absorbed by the load will be

$$P_L = v_L \cdot i_L = \frac{v_L^2}{R_L} \quad (3.4)$$

while power not absorbed by the load is represented by

$$P_r = v_r \cdot i_r = \frac{v_r^2}{R_0} \quad (3.5)$$

Here, the r subscript stands for reflected (not absorbed) power, voltage or current, respectively.

Applying Kirchoff's laws to point x in Figure 3-4, the current to the load is

$$i_L = i_x - i_r \quad (3.6)$$

and voltage across the load is

$$v_L = i_L R_L = v_x + v_r \quad (3.7)$$

To find the ratio of v_r to v_x so that it can be ascertained how much power is absorbed by the load, and how much is not absorbed (therefore, reflected), substitute v_x/R_0 for i_x and v_r/R_0 for i_r into Equation 3.6.

$$i_L = \frac{v_x}{R_0} - \frac{v_r}{R_0} \quad (3.8)$$

Rearranging Equation 3.7 and substituting for i_L in Equation 3.8 yields

$$\frac{v_x + v_r}{R_L} = \frac{v_x}{R_0} - \frac{v_r}{R_0} \quad (3.9)$$

The minus sign associated with v_r/R_0 means, in this case, that the reflected voltage wave v_r travels in the $-x$ direction toward the generator.

Collecting like terms of Equation 3.9 yields

$$v_x \left(\frac{1}{R_0} - \frac{1}{R_L} \right) = v_r \left(\frac{1}{R_0} - \frac{1}{R_L} \right) \quad (3.10)$$

So,

$$v_r = v_x \frac{\left(\frac{R_L - R_0}{R_0 R_L} \right)}{\left(\frac{R_L + R_0}{R_0 R_L} \right)} = v_x \left(\frac{R_L - R_0}{R_0 + R_L} \right) \quad (3.11)$$

and the desired relation for v_r/v_x is

$$\frac{v_r}{v_x} = \frac{R_L - R_0}{R_0 + R_L} \quad (3.12)$$

This ratio is defined as the voltage reflection coefficient of the load ρ_{VL}

$$\rho_{VL} \equiv \frac{v_r}{v_x} = \frac{R_L - R_0}{R_0 + R_L} \quad (3.13)$$

Fig. 3-3. Voltages and Current on an Infinite Length Line

Fig. 3-4. Boundary Conditions at the Line/Load Interface

A similar derivation for currents shows

$$\rho_{IL} = -\frac{R_L - R_0}{R_L + R_0} = -\rho_{VL} \quad (3.14)$$

For the remainder of the chapters, the v or i subscript on the reflection coefficient is dropped, and ρ_L is assumed to be the voltage reflection coefficient of the load. Similarly, applying Kirchhoff's laws to the source-line interface, the voltage reflection coefficient of the source is

$$\rho_S = \frac{R_S - R_0}{R_S + R_0} \quad (3.15)$$

The current reflection coefficient of the source has the same magnitude as ρ_S , but is opposite in algebraic sign.

When a traveling wave v_x , i_x meets a boundary such as the line load interface, a reflected wave is instantaneously generated so that Kirchhoff's laws are satisfied at the boundary conditions. This is the direct result of the conservation of energy principle. Referring again to Figure 3-4, the effects of three different termination resistance R_L values are shown.

Case 1, $R_L = R_0$

In this case, R_L is equal to the characteristic resistance of the line. Using Equation 3.13, the voltage reflection coefficient of the load ρ_L is

$$\rho_L = \frac{R_0 - R_0}{R_0 + R_0} = \frac{0}{2R_0} = 0 \quad (3.16)$$

Since $v_r/v_x = \rho_L$, then $v_r = \rho_L v_x = 0$ and no reflection is generated. This agrees with the discussion of cut lines and matched load where a line terminated in its characteristic impedance behaves the same as an infinite line. All power delivered by the line is absorbed into the load. The waveforms appear as shown in Figure 3-5. The wave starting at the source at time $t = 0$ is reproduced at point x down the line after a time delay of $t = x\delta = \tau$.

Case 2, $R_L > R_0$

To simplify this case, assume that $R_S = R_0$. This means that

the initial voltage is

$$V \frac{R_0}{R_0 + R_0} = \frac{V}{2} \quad (3.17)$$

Also assume $R_L = 3R_0$, then the load voltage reflection coefficient is

$$\rho_L = \frac{3R_0 - R_0}{3R_0 + R_0} = +\frac{1}{2} \quad (3.18)$$

The voltage wave arriving at point x at time $t = x\delta$ generates a reflected voltage wave of magnitude

$$v_r = \rho_L v_x = \left(+\frac{1}{2}\right)\left(\frac{V}{2}\right) = \frac{V}{4} \quad (3.19)$$

and the load voltage is

$$v_L = v_x + v_r = \frac{V}{2} + \frac{V}{4} = \frac{3V}{4} \quad (3.20)$$

The reflected voltage wave v_r generated at $t = x\delta = \tau$ travels back down the line toward the source arriving at the source at time $t = 2x\delta = 2\tau$. This wave will be absorbed without generating another reflection because R_S was picked to equal R_0 , making ρ_S equal to zero. The source voltage is now

$$v_S + v_r = \frac{V}{2} + \frac{V}{4} = \frac{3V}{4} \quad (3.21)$$

and equilibrium is achieved.

If the circuit in Figure 3-4 is analyzed using simple circuit theory and neglecting the transmission line effects, it is easily seen that

$$v_S = v_L = V \frac{R_L}{R_0 + R_L} = \frac{3V}{4} \quad (3.22)$$

This agrees exactly with Equation 3.21 and will always be the case. After all reflections cease and the circuit reaches equilibrium, the steady state voltages and currents on the line are the same as those produced using simple dc circuit analysis. Waveforms for $R_L > R_0$ (specifically $R_L = 3R_0$) appear in Figure 3-6.

In general, the case where $R_L > R_0$ is viewed in the following manner. Because the line is capable of delivering more power

Fig. 3-5. Waveforms for $R_L = R_0$

Fig. 3-6. $R_S = R_0$, $R_L = 3R_0$

er than can be instantaneously absorbed by the load, the excess power is returned to the source and absorbed in the source resistor (assuming $R_S = R_0$).

An upper limit on the voltage reflection coefficient is found by allowing R_L to go to infinity. In this case, Equation 3.13 goes to +1.

Case 3, $R_L < R_0$

In this case, again set $R_S = R_0$ and allow R_L to equal $R_0/3$. The initial wave, as before, is

$$v_s = V \frac{R_0}{R_0 + R_S} = \frac{V}{2} \quad (3.23)$$

and the load voltage reflection coefficient is

$$\rho_L = \frac{R_L - R_0}{R_L + R_0} = \frac{\frac{R_0}{3} - R_0}{\frac{R_0}{3} + R_0} = -\frac{1}{2} \quad (3.24)$$

Therefore, the reflected voltage wave v_r is

$$v_r = \rho_L \frac{V}{2} = -\frac{V}{4} \quad (3.25)$$

which starts propagating back toward the source at time $t = \tau$. The load voltage at time $t = \tau$ is

$$v_x + v_r = \frac{V}{2} + -\frac{V}{4} = +\frac{V}{4} \quad (3.26)$$

The $(-V/4)$ reflected wave arrives back at the source at time $t = 2\tau$. Because R_S is set equal to R_0 , ρ_S is, then, equal to zero and no reflected wave will be generated. The voltage at the source is now

$$v_s + v_r + \rho_S v_r = \frac{V}{2} + -\frac{V}{4} + 0 = \frac{V}{4} \quad (3.27)$$

From a dc circuit analysis, the steady state voltage is

Fig. 3-7. $R_L = \frac{R_0}{3}$

$$V_{ss} = V \frac{R_L}{R_L + R_0} = \frac{V}{4} \quad (3.28)$$

This agrees with the result of Equation 3.27. The waveforms for Case 3 ($R_L < R_S$) appear in Figure 3-7.

An interpretation of the actions occurring when load resistance is less than the characteristic line resistance is as follows: when power available at the line end is less than the power the load can absorb, a signal is sent back to the source saying, in essence, "send more power".

It has been shown that a ratio of line and load resistance (ρ) can be used to calculate the voltages and currents in terms of a wave arriving at the boundary, possibly generating a reflected, reverse-traveling wave to satisfy the conservation of energy principle at the line-to-load boundary. This ratio is

$$\rho_B = \frac{R_B - R_0}{R_B + R_0} \quad (3.29)$$

where R_B represents the resistance into the boundary, R_B is R_S when considering the source-to-line interface and R_B would be R_L when considering the line-to-load interface. It is obvious that if discussing impedances, then Z_S would be substituted for R_S in Equation 3.29, and there may be some phase angle between the voltage and current waves.

The forward traveling wave, v_x , plus the reflected wave, v_r , is equal to the load voltage (V_L). Since v_r is $\rho_L v_x$, this can be expressed as

$$v_x(1 + \rho_L) = v_L \quad (3.30)$$

This quantity $(1 + \rho)$ can be defined as the voltage transmission coefficient of the load and it is known that

$$\frac{v_L}{v_x} = (1 + \rho_L) \quad (3.31)$$

The cases with various load resistances can be summarized.

Condition	Circuit at time $t = \tau$ (one line delay time)
1. $R_L = R_0$ $\rho_L = 0$	No reflection is produced—circuit reaches steady state immediately.
2. $R_L > R_0$ $\rho_L > 0$	Positive voltage reflection—wave is sent back toward source. Voltage at load is higher than steady state voltage (overshoot).
3. $R_L < R_0$ $\rho_L < 0$	Negative voltage reflection—wave is sent back toward source. Voltage at load is lower than steady state voltage (undershoot).

FUNDAMENTAL PRINCIPLES

Before examining the algorithm for keeping track of reflections, there are two principles to keep in mind.

- Energy (as power) is conserved at boundary conditions (as explored previously)
- The principle of linear superposition applies. This means any arbitrary excitation function can be broken down into step functions, or ramps. The reaction of the circuit to each part can be analyzed, and the results can be added together when finished. This means that a positive pulse of duration t is examined, by superimposing two step functions,

Summarizing this lattice diagram method, any time $t = m\tau$ and $m > 1$, the following relationships exist:

If m is odd, the $v_i(m)$ wave is arriving at the load and generates a reflected wave

$$v_r(m) = \rho_L v_i(m)$$

This becomes $v_i(m+1)$ as it starts toward the source. The voltage at the load at time $t = m\tau$ will be

$$v_L(m) = v_L(m-2) + v_i(m) (1 + \rho_L)$$

This is the sum of the voltage that was there before the wave arrived, i.e., $v_L(m-2)$, plus the wave arriving $v_i(m)$ and the reflected wave $v_r(m)$ departing.

If m is even, the $v_i(m)$ wave is arriving at the source and generates a reflected wave

$$v_r(m) = \rho_S v_i(m)$$

This becomes $v_i(m+1)$ as it starts toward the load. The voltage at the source is now

$$v_S(m) = v_S(m-2) + v_i(m) (1 + \rho_S)$$

This is the sum of the voltage that was present $v_S(m-2)$ plus the incident wave arriving $v_i(m)$ plus the reflected wave departing $v_r(m)$.

The voltage and current at the source end of the line for a lossless line can be expressed as a summation.

$$v_S(t) = \frac{R_0}{R_S + R_0} \bullet \quad (3.32)$$

$$\left[e(t)u(t) + \left(1 + \frac{1}{\rho_S}\right) \sum_{n=1}^{\infty} \rho_S^n \rho_L^n e(t-2n\tau)u(t-2n\tau) \right]$$

$$i_S(t) = \frac{I}{R_S + R_0} \bullet \quad (3.33)$$

$$\left[e(t)u(t) + \left(1 - \frac{1}{\rho_S}\right) \sum_{n=1}^{\infty} \rho_S^n \rho_L^n e(t-2n\tau)u(t-2n\tau) \right]$$

where $e(t)$ is the generator voltage as a function of time, and $u(t)$ is the unit step function.

Likewise, the load voltage and load current for the lossless line can be expressed as a summation.

$$v_L(t) = \frac{R_0}{R_S + R_0} \bullet \quad (3.34)$$

$$(1 + \rho_L) \left[\sum_{n=0}^{\infty} \rho_S^n \rho_L^n e(t-(2n+1)\tau)u(t-(2n+1)\tau) \right]$$

$$i_L(t) = \frac{I}{R_S + R_0} \bullet \quad (3.35)$$

$$(1 - \rho_L) \left[\sum_{n=0}^{\infty} \rho_S^n \rho_L^n e(t-(2n+1)\tau)u(t-(2n+1)\tau) \right]$$

A similar expression of summation can be developed for the voltage (or current) at any point along the line at any time.

Because the lattice diagram is tabular in method, a computer program can be written relieving the designer of bookkeeping and repetitive calculations. A BASIC computer program for lattice diagrams appears in Figure 3-13.

LIMITATIONS OF THE LATTICE DIAGRAM METHOD

Before using the lattice diagram to explore reflection effects with various source and load characteristics, it is necessary to pause at this point and examine the models used by the lattice diagram.

First, both the line driver and receiver are simulated either by a constant input or output resistance. The source has two voltage sources and a switch representing the internal source voltage at a time less than zero and equal to (or greater than) zero. The receiver is represented by a single resistor shunting the line end opposite the driver site. The line itself is represented by its characteristic resistance R_0 and its total one-way time delay (τ). This is equal to length times propagation delay per unit length. This model is shown in Figure 3-11.

Because most data communication circuits are voltage types, that is, the receiver senses the line voltage to decide if a logic One or logic Zero is present, the primary interest is in voltages at the source and load as a function of time. Major exceptions include the current loops used in teletypewriters, telegraphs, and burglar alarm systems. The majority of data communication circuits used in computers, peripherals, and general controllers are voltage types.

The lattice diagram method cannot easily use source or receiver current/voltage relationships that are non-linear, i.e., not purely resistive. For non-linear current/voltage character-

Fig. 3-11. Model used for Lattice Diagram Method

istics such as found in diodes, a graphic method can be used called the reflection diagram or the Bergeron* method.

Signals exchanged using lattice diagrams are of the unit step variety. When ramps or more complex waves are exchanged, the complexity of the bookkeeping increases dramatically. Additionally, the lines are presumed to be lossless, although a constant line attenuation factor could be accommodated without excessive bookkeeping. These limitations should be kept in mind when examining various source and load resistance combinations and their reflection characteristics.

There are three classes of source resistance, $R_S < R_0$, $R_S = R_0$ and $R_S > R_0$. There are also three classes of load resistance, $R_L < R_0$, $R_L = R_0$ and $R_L > R_0$. This gives nine types of single driver, single receiver line circuits. Each circuit will be examined in turn to determine reflection effects for these combinations with evaluations of each combination for voltage type communications.

REFLECTION EFFECTS FOR VOLTAGE SOURCE DRIVERS

Initial waves launched by a voltage source type driver ($R_S < R_0$) are greater than one-half the magnitude of the internal voltage source. Referring to Figure 3-11, the initial voltage wave is derived as follows.

$$v_i(1) = (V_{0+} - V_{0-}) \cdot \frac{R_0}{R_0 + R_S} \quad (3.36)$$

while the voltage at the source at $t = 0+$ is

$$v_S(0+) = v_S(0-) + v_i(1) = V_{0-} + \frac{R_L}{R_L + R_S} \cdot v_i(1) \quad (3.37)$$

If the receiver switching point is at the mean of the driver voltage swing, the initial wave always has sufficient magnitude to indicate the correct logic state as it passes the receiver site. This maximizes the noise margins of the receiver.

Since $R_S < R_0$, the source voltage reflection coefficient ρ_S is less than zero. Any voltage waves, then, arriving back at the source are changed in sign, reduced in amplitude (assuming $R_S > 0\Omega$), and sent back toward the load. If the load resistance equals the characteristic line resistance ($R_L = R_0$), the voltage reflection coefficient of the load is

$$\rho_L = \frac{R_L - R_0}{R_L + R_0} = \frac{0}{2R_0} = 0$$

No reflections, therefore, are generated at the load. The voltage wave produced at the source is reproduced at the load after a time delay of $\tau = \ell\delta$, and the line assumes a steady state condition. Figure 3-12b illustrates the source and load voltage waveforms for this case.

If R_L is greater than R_0 , ρ_L is positive. Waves arriving at the load generate the same polarity reflections as the arriving waves. ρ_S and ρ_L are of opposite signs, so a dampened oscillatory behavior of the load voltage is expected. The oscillation period or ringing is 4τ . The overshoot of v_L from $t = \tau$

to 3τ may cause breakdown of the input circuitry of a receiver, depending on the receiver voltage rating. The undershoot at $t = 3\tau$ to 5τ can reduce the noise immunity of a receiver or even cause a logic level misinterpretation — an error in the data. These waveforms are shown in Figure 3-12a.

If R_L is less than R_0 , then ρ_L is negative and a wave arriving at the load generates a reflection opposite in polarity to the incident wave. This causes the voltage at the source to overshoot steady state voltage at $t = 0$. Each reflection returning from the load causes the source voltage to continually step down toward the steady state voltage V_{SS} . These steps last for 2τ , or one round trip delay. Load voltage starts an increasing step-up waveform towards V_{SS} at time $t = \tau$, with steps again taking one round trip delay, 2τ . A line receiver placed in the middle of the line sees an entirely different waveform — dampened oscillations much like the load voltage in Figure 3-12a. This is caused by the negative signs of both source and load voltage reflection coefficients. Each time an incident wave arrives at either source or load, the reflected wave generated at that time has a sign opposite to the sign of the incident voltage wave. The voltage at a distance half way down the line is composed of these forward and reverse traveling waves arriving at that point commencing at time $t = 0.5\tau$, and with each new wave passing that point after one line delay (τ). These waveforms are shown in Figure 3-12c.

The optimum load resistance for voltage signal communications on transmission lines driven by a low impedance source ($R_S < R_0$) is equal to the characteristic line resistance. Large signal line voltages are produced and there are no reflection effects complicating the waveforms (Figure 3-12b).

However, a matched load ($R_L = R_0$) is a dc load on the driver, thus it increases system power dissipation. But, it does preserve signal fidelity and amplitude allowing use of multiple bridging receivers ($R_{in} \gg R_0$) along the line.

The unterminated case ($R_L > R_0$) reduces dc driver loading and also reduces system power dissipation over the matched load case. The unterminated case does, however, allow the load signal to exhibit pronounced overshoot and undershoot around the steady state voltage. If the load signal undershoot places the receiver in its threshold uncertainty region, data errors result. There is a way to "civilize" the voltage waveform of the unterminated line load by trading off signal rise time versus line time delay. This is discussed later.

The final case of $R_S < R_0$ and $R_L < R_0$ is not generally useful in terms of voltage signals produced (Figure 3-12c.) Systems using this case consume more power than the previous two cases and have no particular advantage for voltage mode communications.

REFLECTION EFFECTS FOR MATCHED-SOURCE DRIVERS

In all three cases under discussion here, the initial voltage produced by the driver onto the line is

$$v_i(1) = (V_{0+} - V_{0-}) \cdot \frac{R_0}{R_0 + R_S} = \frac{1}{2}(V_{0+} + V_{0-}) \quad (3.38)$$

*A French hydraulic engineer, L.J.B. Bergeron developed the method to study the propagation of water hammer effects in hydraulics. See references, Chapter 2.

Fig. 3-12. Source and Load Voltage Waveforms for Various R_S and R_L

since $R_S = R_0$. The voltage at the source at time $t = 0+$ is

$$v_S(0+) = v_S(0-) + v_i(1) = V_{0-} \cdot \frac{R_L}{R_L + R_S} + v_i(1) \quad (3.39)$$

Assume, for clarity, that initial voltage (V_{0-}) is zero, thus Equation 3.39 simplifies to

$$v_S(0+) = \frac{V_{0+}}{2} \quad (3.40)$$

Since $R_S = R_0$, ρ_S is equal to zero. This means that load-generated reflections due to load mismatch are absorbed at the source when, at time $t = 2\tau$, the reflected wave arrives back at the source. The line then assumes a steady state throughout. This back match or series termination effect of a matched source allows a wide latitude in choice of load resistance without sacrificing the signal fidelity of the load voltage waveform.

If the load resistance equals the characteristic line resistance $R_L = R_0$, then ρ_L equals zero and no load site reflections are generated. The initial voltage wave arrives at the load at time $t = \tau$ (one line delay) and voltages (and currents) on the line immediately assume steady state conditions (see Figure 3-12e). The optimum receiver threshold here is one-half the steady state voltage or $V_{0+}/4$. The main advantage over the voltage source type driver with matched load case ($R_S < R_0$, $R_L = R_0$) is that R_S and R_L resistance tolerances may be relaxed without incurring much signal *ringing*. This effect is due primarily to the termination provided by both line ends, rather than just one line end. Any reflected voltage wave on either system is attenuated by the product of ρ_S and ρ_L for each round trip line delay time. Since the $\rho_S \rho_L$ product for the fully matched case is smaller than the $\rho_S \rho_L$ product for the single matched case, the reflections are attenuated and die out in fewer round trips. For example, if 20% tolerance resistors are used in both cases, ρ_S and ρ_L values for the fully matched case become 0.0 ± 0.0909 , which is a $\rho_S \rho_L$ product of ± 0.0033 . This means that after one round trip (2τ), the reflection amplitude starting back toward the load would be less than 0.33% of the initial wave.

Using $R_S = 10 \Omega$, $R_L = 100 \Omega$, and $R_0 = 100 \Omega$ as for Figure 3-12a, shows the same 20% tolerances applied to the single matched case

$$-0.8519 < \rho_S < -0.7857$$

and

$$-0.0909 < \rho_L < +0.0909$$

$$|\rho_S \rho_L| < 0.0774$$

The voltage reflection amplitude after one round trip is a maximum of 7.7% of the initial wave.

The choice between using the single and fully matched system should be carefully considered because the fully matched system does sacrifice signal voltage magnitude to get a decreased dependence on absolute resistor values.

If the load resistance for a matched driver circuit is made much greater than the line resistance, the initial wave arriving at the load at time $t = \tau$ will be almost doubled since ρ_L

will be close to +1.0. Because source resistance is set equal to line resistance, ρ_S becomes zero, the reflected voltage wave from the load is absorbed by the source at time $t = 2\tau$, and steady state conditions prevail. Waveforms for this case are shown in Figure 3-12d. This is called *back matching* or *series termination*.

The main advantage of series termination is a great reduction in steady state power consumption when compared with the parallel terminated case ($R_S < R_0$, $R_L = R_0$). At the same time, series termination provides the same signal fidelity to a receiver placed at the line end. Compare the load voltage waveforms for the two cases in Figure 3-12b and 3-12d. The main disadvantage to series termination is that receivers placed along the line see a waveform similar to that shown for the source in Figure 3-12d. That is, receivers along the line see the $V_{0+}/2$ initial wave as it passes that point on the line, and do not see a full signal swing until the load end reflection passes that point. Consequently, receivers along the line do not see a signal sufficient to produce the valid logic state output until the load reflection returns. Depending on actual line length and receiver characteristics, the receiver may even oscillate, having been placed in its linear operation region. With the benefit, then, of reducing system power, the series termination method has a constraint of allowing only one line receiver located at the line load end. The parallel termination method should be used if other receivers along the line are required.

The final case of matched source drivers is with the use of a load resistance less than the characteristic line resistance. The waveforms for this case are shown in Figure 3-12f. A line receiver with a threshold of $V_{0+}/4$ placed at the source responds like a positive, edge triggered one-shot and produce a pulse in response to a $+V/2$ initial wave of 2τ duration. Aside from its use as a one-shot, this circuit doesn't seem to offer any advantages for voltage mode communications.

REFLECTION EFFECTS FOR CURRENT-SOURCE DRIVERS

The name *current source drivers* is somewhat of a misnomer, and might be more properly called *current-limited voltage source drivers*. True *current source* drivers such as the 75110 are normally used in conjunction with parallel termination resistors to create a matched source.

The *current source* drivers ($R_S > R_0$) discussed resemble true current sources in the respect that their output resistance is usually much greater than the characteristic line resistance. The initial voltage step produced on the line is thus usually small $v_i(1) = (i_c(1)R_0)$. This is due to the voltage divider action of the driver source resistance and the characteristic line resistance.

Voltage waveforms for a current source type driver either step up to V_{SS} , reach steady state after 2τ , or execute a damped oscillation around V_{SS} , depending on whether the load resistance R_L is greater, equal, or less than R_0 , respectively. The second case $R_L = R_0$ provides signals much the same as the other two cases where $R_L = R_0$, that is, the source voltage steps immediately to V_{SS} , with the load voltage following after one line time delay. Here the amplitude of

the signal is much smaller than previous matched load cases. Since the current source type drivers (75109/110) have high off-state impedances, they allow multiple drivers on the line to produce a data bus or parity line. This aspect will be discussed in more detail in Chapter 5. Waveforms for the matched load case are shown in Figure 3-12h.

The case $R_L < R_0$ really provides no useful advantage for voltage mode communications. The negative sign for ρ_L and the positive sign for ρ_S lead to dampened oscillatory behavior, or ringing. The maximum perturbation takes place at the source end of the line. Waveforms for this case are similar to those shown in Figure 3-12a, and are shown to scale in Figure 3-12i. With the given values used to produce the figure, the maximum amplitude ringing appears at the source line end.

The $R_L > R_0$ case is of interest because it is representative of DTL driving a transmission line with the output going from LOW to HIGH. DTL has a high value R_S (2 k Ω or 6 k Ω) in the HIGH logic state. Since both R_S and R_L are greater than R_0 , both ρ_S and ρ_L are positive. A small voltage step starts from the source at $t = 0+$; its magnitude is

$$v_i(t) = V \frac{R_0}{R_0 + R_S}$$

Upon arrival at the load at time $t = \tau$, this initial wave generates a positive voltage reflection since $\rho_L > 0$. The voltage reflection arrives back at the source site at time $t = 2\tau$. Since ρ_S is also positive, another positive voltage reflection is launched back toward the load. The process repeats, and the source and load voltages both execute a step-up approach toward steady state voltage V_{SS} . These waveforms are shown in Figure 3-12g.

In examining voltage at the line midpoint ($x = \ell/2$), a step-up type waveform is seen which is the sum of all the incident voltage waves passing the line midpoint up to the time of examination. The midpoint voltage is expressed as follows.

$$v_m(t) = V_{SS} (1 - \exp[-(t + 0.5\tau)/T]) \quad (3.41)^{**}$$

for $t = n + 0.5\tau$ with $n = 0, 1, 2, 3, \text{etc.}$ V_{SS} in Equation 3.41 is the steady state line voltage

$$V_{ss} = V_0 + \frac{R_L}{R_S + R_L}$$

and T is a time constant given by

$$T = \frac{2\tau}{\ln(\rho_S \rho_L)} \quad (3.42)^{**}$$

with τ being one line delay ($\tau = \ell \delta$).

Equation 3.41 provides an exact solution for odd multiples of n ($n = 1, 3, 5, \dots$, so $t = 1.5\tau, 3.5\tau, 5.5\tau, \dots$), while it approximates $v_m(t)$ for even multiples of n ($n = 0, 2, 4, \dots$, so $t = 0.5\tau, 2.5\tau, 4.5\tau, \dots$). The closer the $\rho_S \rho_L$ product is to 1, the better Equation 3.41 predicts $v_m(t)$, particularly for even multiples of n . To illustrate the fitting, the two tables in Figure 3-13 are generated by the BASIC language computer program (Table C) and their data is plotted in Figure 3-14.

*Since the input diode is not represented, the representation of DTL input as a single resistor to ground is not strictly correct. For purposes of approximation, this simple representation is used. Treatment of non-linear current/voltage sources and loads is covered by Metzger & Vabre (op cit.)

**This equation is presented without derivation, but a procedure similar to that used by Matuck (Ref 2, Chapter 2) can be used

Table (a). ($R_S = 2000 \text{ ohms}$, $R_0 = 100 \text{ ohms}$, $R_L = 4000 \text{ ohms}$)

TIME	V(I)	VAPPX	%DIFF
0.5	0.04762	0.04762	+1.220%
1.5	0.09292	0.09292	+0.000%
2.5	0.13590	0.13440	+0.375%
3.5	0.17844	0.17288	+0.000%
4.5	0.20610	0.20566	+0.207%
5.5	0.24170	0.24170	+0.000%
6.5	0.27200	0.27243	+0.136%
7.5	0.30093	0.30093	+0.000%
8.5	0.32700	0.32737	+0.097%
9.5	0.35190	0.35190	+0.000%
10.5	0.37539	0.37546	+0.013%
11.5	0.39577	0.39577	+0.000%
12.5	0.41512	0.41536	+0.057%
13.5	0.43353	0.43353	+0.000%
14.5	0.45018	0.45038	+0.045%
15.5	0.46602	0.46602	+0.000%
16.5	0.48035	0.48035	+0.000%
17.5	0.49399	0.49399	+0.000%
18.5	0.50632	0.50647	+0.030%
19.5	0.51800	0.51800	+0.000%
20.5	0.52867	0.52860	+0.024%
21.5	0.53877	0.53877	+0.000%

Table (b). ($R_S = 500 \text{ ohms}$, $R_0 = 75 \text{ ohms}$, $R_L = 10 \text{ kilohms}$)

TIME	V(I)	VAPPX	%DIFF
0.5	0.13043	0.13071	+7.112%
1.5	0.25893	0.25893	+0.000%
2.5	0.35380	0.36066	+1.809%
3.5	0.44146	0.44146	+0.000%
4.5	0.51661	0.52153	+0.952%
5.5	0.58473	0.58473	+0.000%
6.5	0.63509	0.63867	+0.564%
7.5	0.68469	0.68469	+0.000%
8.5	0.72335	0.72396	+0.361%
9.5	0.75147	0.75147	+0.000%
10.5	0.78416	0.78606	+0.242%
11.5	0.81046	0.81046	+0.000%
12.5	0.82990	0.83128	+0.167%
13.5	0.84904	0.84904	+0.000%
14.5	0.86320	0.86420	+0.117%
15.5	0.87714	0.87714	+0.000%
16.5	0.88744	0.88818	+0.083%
17.5	0.89759	0.89759	+0.000%
18.5	0.90510	0.90563	+0.059%
19.5	0.91249	0.91249	+0.000%
20.5	0.91955	0.91934	+0.042%
21.5	0.92434	0.92434	+0.000%

Table (c). BASIC Program Listing

```

100 PRINT "ENTER RS,R0,RL"
110 INPUT R1,R0,R2
120 P1=(R1-R0)/(R1+R0)
130 P2=(R2-R0)/(R2+R0)
140 V1=V0/(R1+R0)
150 K1=2/(1+P1*P2)
160 V2=V1/(1+P1)
170 PRINT "RS=";R1;"R0=";R0;"RL=";R2;"TAU=";K1
180 PRINT "V1=";V1;"V2=";V2;"VSS=";V1+V2
190 V=V1
200 PRINT "TIME" V(I) VAPPX %DIFF
210 FOR I=0 TO 20 STEP 2
220 V2=V2*(1-EXP(-(I+.5)/K1))
230 V=V1+(V2-V1)/V
240 PRINT USING "20.0,20.0,20.0,20.0" V,I,V2,P
250 IF I=20 THEN GOTO 260
260 V=V2
270 NEXT I
280 END
290 PRINT "SOURCE END"
300 V2=V2*(1-EXP(-(I+.5)/K1))
310 PRINT USING "20.0,20.0,20.0,20.0" V,I,V2,P
320 V=V1+V2
330 NEXT I
340 END
350 PRINT
360 END
370 GOTO 100
380 END

```

Fig. 3-13. Comparison of v_m Formula to Computed Midline Voltage

Designers familiar with DTL circuits should quickly recognize that the waveforms shown in Figure 3-14 are very similar to the rising edge waveforms found when a DTL gate output goes from the LOW to HIGH state. This characteristic waveform has usually been attributed to the series RC circuit (a gate output resistance driving a lumped transmission line capacitance.) The time constant for this approach, based on the $C(dv/dt) = i$ rule from simple circuit theory, provides only an approximation. The actual cause of the waveform shape, however, is due to reflection effects. Unfortunately, the only way to speed up the rising edge is to reduce source resistance, (providing an initial step greater than the receiving threshold) and terminate the line to eliminate the load reflections.

DTL inability to drive transmission lines at high repetition rates is the direct result of the signal rise time limitation caused by positive reflection coefficients for both the source and load. A transmitted positive pulse may be missed if its duration is less than the time required for the load signal to reach the receiver threshold.

The $R_S > R_0$ and $R_L > R_0$ case provides no definite advantages as voltage mode communication is concerned. This case, in fact, poses a definite hazard to high speed data communications because the reflections cause, in effect, a slow, exponential signal transition. Because line delay is a factor, longer lines will only increase the effect.

Fig. 3-14. Approximation of Midline Voltage with $R_S > R_0$ and $R_L > R_0$

SUMMARY — Which are the Advantageous Combinations?

In examining the basic combinations of source, line and load resistances, and typical waveforms characteristic of each case, advantageous combinations can be determined. The primary results are tabulated in Figure 3-15. Those combinations generally used in voltage mode communications circuits are as follows.

1. Underterminated case ($R_S \ll R_0$, $R_L \gg R_0$). This situation provides low steady state power dissipation and large signal levels, but also shows pronounced "ringing" effects. The "ringing" can be reduced by controlling signal rise/fall time versus τ , or by clamping diodes to limit load signal excursions. This case is representative of TTL circuits and is thus widely employed.
2. The parallel terminated case ($R_S \ll R_0$, $R_L = R_0$) provides large signal levels, and excellent signal fidelity. However, it is power consuming with most of that power dissipated in the load resistor. This case is useful for cleaning up the

reflection effects of Case 1 but, at the same time, does require a driver circuit to have its internal current limits set at greater values than those required to produce the desired signal level into the minimum line resistance used. Thus, this case requires power buffer gates such as the 9N40 or 9S140 or specific line driver devices such as the 9612, 9614 9621 or 8T13/23. Ordinary TTL, except for the above mentioned circuits, has too low a current limit point to adequately drive 50 Ω lines.

3. The series terminated or backmatched driver case $R_S = R_0$, $R_L \gg R_0$ provides a low steady state power dissipation system for use with one receiver located at the load end of the line. The positive reflection coefficient of the load is used to approximately double the initial wave arriving at the load. Setting $R_S = R_0$ terminates the reflected wave when it arrives back at the source site after two line delays, and the line then assumes steady state conditions. The use of other receivers located along the line is not recommended, because they will not see the full driver signal swing until the reflection from the load passes their particular

Configuration Name (if any)	(Driver) Source Resistance	(Receiver) Load Resistance	Signal Characteristics	Optimum Receiver Threshold	Line Receivers Allowed at Other Than Load End of Line?	Comments
Underterminated	$\ll R_0$	$\gg R_0$	Ringing pronounced	0.5 V_{SS}	Yes	Undershoot may cause data errors
Parallel terminated	$\ll R_0$	$= R_0$	Excellent fidelity	0.5 V_{SS}	Yes	Load resistor consumes power $P_L = \frac{(V_{SS})^2}{R_L}$
	$\ll R_0$	$\ll R_0$	Awful — different signals at each point on the line	NA	No	Not generally useful
Series terminated or backmatched driver	$= R_0$	$\gg R_0$	Load signal excellent	0.5 V_{SS}	No	Reduced power consumption over parallel termination
Fully matched	$= R_0$	$= R_0$	Excellent fidelity	0.25 V_{SS}	Yes	Greater tolerances on resistors allowed for same fidelity as parallel termination
	$= R_0$	$< R_0$	Load signal like a oneshot	NA	NA	Not generally useful for data, is useful as pulse generator
	$\gg R_0$	$\gg R_0$	Exponential like signal waveforms	0.5 V_{SS}	Yes	Low power consumption. Increased delay due to signal 'rise' times
	$\gg R_0$	$= R_0$	Small signal amplitude. Excellent fidelity	0.5 V_{SS}	Yes	Produces only small signal voltages compared with other methods. Uses current sinking drivers such as the 75110A, 75112
	$\gg R_0$	$< R_0$	Very small signal amplitudes, also ringing	NA	NA	Not generally useful

Fig. 3-15. Summary of Effects

bridging points. Such receivers could malfunction, as they would see a voltage very close to their threshold, and perhaps even place the line receiver in its linear operating region. This could make the line receiver sensitive to oscillatory, parasitic feedback. If these constraints are acceptable, the series termination method can be used to good advantage in providing the same signal fidelity and signal amplitude as with the parallel termination method, while at the same time, contributing a significant savings in steady state power consumption.

4. The fully matched case $R_S = R_0$, $R_L = R_0$ not only provides excellent signal fidelity all along the line, but also has reduced signal amplitude over that of the parallel terminated case. Additionally, the power consumption is somewhat less than the parallel termination case and the power is divided equally by the source and load. The primary advantage of the fully matched system is that termination resistor tolerances can be relaxed somewhat without incurring large amounts of ringing. This is because both the source and load act as line terminations.

EFFECT OF SOURCE RISE TIME ON WAVEFORMS

Previously, it was assumed that the source-produced signal rise time was always much less than the line time delay (τ). Because the waveforms for the source and load voltage were the superposition of incident and reflected waves occurring at their proper times, and because the shape of each wave was a square edged step function, the resultant source and load waveforms were thus also square edged, or *ideal* in nature. In many practical cases, particularly when line length is short, the source excitation possesses a finite, and non-negligible, rise time. Therefore, depending on the ratio of rise time to line delay, it is possible to have a new wave start arriving at the point of interest *before* the previous wave can reach its final value. The net waveform for voltage or current at that point, then, would consist of the superposition of two or more waves during their time of overlap. To study the superposition effect on signal waveforms, the source excitation is represented as a simple linear ramp rise to its final value of V_{0+} , so

$$\begin{aligned} e(t) &= 0 \text{ for } t < 0 \\ e(t) &= V_{0+} \cdot t/t_r \text{ for } 0 < t < t_r \\ \text{and} \quad e(t) &= V_{0+} \text{ for } t > t_r \end{aligned}$$

where t_r represents the 0-to-100% source rise time. The circuit model and its lattice diagram are shown in Figure 3-16. The values of R_S , R_0 and R_L were chosen to equal those of an actual circuit on hand, allowing the theoretical waveforms, obtained by graphical superposition, to be compared with the measured response of an actual circuit.

Figure 3-17 shows the load voltage v_L , source voltage v_S and source current i_S waveforms versus time for a circuit with a source rise time very much less than τ . The actual waveforms for v_L , v_S and i_S are composed of the superposition of both incident and reflected waves in their proper time sequence. In the figures, these waves are shown as dotted lines. Each wave represents the sum of the incident wave plus its reflection. The resultant v_L , v_S and i_S waveforms

(shown as solid lines) are the superposition of the waves represented by the dotted lines. With the exception of a slight rounding of the edges, the actual waveforms for the circuit, shown in the oscilloscope photograph in Figure 3-17, closely approximate the waveforms predicted by theory.

If the source excitation is adjusted so that its 0-to-100% rise time t_r is equal to 2τ , each of the $v_i + v_r$ and $i_i + i_r$ waveforms must be modified to include this rise time. The waves will have the same final value as predicted by the lattice diagram, but they now require two line time delays to reach this final value. The v_L , v_S and i_S waveforms consist of the superposition of these linear ramps. Because each wave reaches its final value just as a new wave arrives, their superposition converts the square edged v_L , v_S and i_S waveforms into triangular waveforms. This is shown in Figure 3-18. The accompanying oscilloscope photograph shows the close correspondence between the actual and theoretical waveforms whereas an additional oscilloscope photograph in Figure 3-18 shows the actual waveforms for the case where $t_r = \tau$. Not surprisingly, the $t_r = \tau$ case changes the v_L , v_S and i_S waveforms of the $t_r \ll \tau$ case into trapezoidal forms because each arriving wave reaches its final value well before a new wave arrives.

If the source excitation is adjusted such that its rise time equals three line delays $t_r = 3\tau$, the $v_i + v_r$ and $i_i + i_r$ waves overlap for a period of time equal to τ . That is, each wave reaches only 2/3 of its final value when a new wave starts arriving. Considering the waveform, the load voltage from time τ to 3τ is

$$v_i(1) (1 + \rho_L) e^{-(t - \tau)}$$

Starting at $t = 3\tau$, the wave

$$v_i(3) = v_i(1)\rho_S\rho_L e^{-(t - 3\tau)}.$$

begins arriving from the source, and the load voltage then is the superposition of these two waves. Because $v_i(3)$ is a negative wave ($\rho_S < 0$), the algebraic sum of the last third of the first wave and the first third of the second wave $v_i(3)$ arriving at the load causes the load voltage to reduce in amplitude from the ($t_r \ll \tau$) case. Likewise, the source voltage and source current show reduced amplitudes over the ideal case, due to the overlap period of the waves arriving at the source.

Theoretical and actual waveforms for the $t_r = 3\tau$ case are shown in Figure 3-19. Notice that load voltage perturbations and source current i_S requirements are reduced from those of the $t_r \ll \tau$ case. Similarly, the ratio of t_r to τ , can be successively increased. This results in reduced ringing on the load voltage and reduced source current due to the overlapping of more and more $v_i + v_r$ (or $i_i + i_r$) waves. Actual and theoretical waveforms for t_r equal to 4τ , 6τ and 8τ are shown in Figures 3-20, 3-21 and 3-22, respectively. In each case, as the t_r to τ ratio is increased, the instantaneous source and load voltages become more equal. The source current is also reduced so that the circuit exhibits fewer reflection effects and the transmission line itself can be considered as a simple interconnection from dc circuit theory.

Using the t_r : τ ratio to reduce reflection effects has many practical advantages in digital design. The low source and high input resistance of TTL or ECL circuits allows one gate to drive many receiving gates. The reflection effects of this unterminated combination, however, can cause data errors or at least lead to reduced noise immunity due to the pronounced load voltage undershoot. Since the rise and fall times of these devices are easily measured, a maximum line length can be set such that the resulting t_r to τ ratio provides the desired reduction in ringing. This is the primary basis for the wiring rules of each logic family and, usually, the t_r to τ ratio is chosen somewhere between 3:1 and 4:1. As an example, the rise and fall time for normal TTL (9N/74N series) is $t_{10\%-90\%} = 6$ ns. When this is converted to an equivalent linear 0 to 100% time, $t_r = 8$ ns. A common propagation delay of 1.7 ns/ft, in combination with the requirement that $t_r = 3\tau$, gives the maximum line length of approximately 18 inches. This corresponds with the published recommendation of the various manufacturers for the 9N/74N series TTL circuits. A similar computation of the rise and fall times for other logic families yields their respective line length recommendations. The faster families require shorter line lengths for the same t_r to τ ratio, and slower logic families allow relatively longer line length. This ratio can also be used to make stubs or taps on lines "disappear". In other

words, if the stub's time delay is made very short when compared to the t_r of the signal at the stub line location, the stub reflections will have a minimal effect on the line signals. A stub length to generate a t_r : τ ratio of greater than 8:1 is usually considered adequate to negate the stub reflections.

The third primary application of the t_r to τ ratio for controlling reflection effects is that used in some standard data communications interfaces such as EIA-RS232-C. Here, driver slew rate is explicitly controlled. This, along with the implied maximum interconnect cable length serves to produce a t_r : τ ratio of 3:1 or greater. This, in turn, reduces the reflection effects inherent in a voltage source driver, unterminated line system. The main disadvantage of using the t_r : τ ratio to control reflection effects is in the overall time for the signal representing the data to rise above the receiver threshold level. With the parallel terminated method, the minimum time delay was τ or one line delay. When the t_r : τ ratio is used, an additional delay time of approximately $0.5 t_r$ is added to the line delay yielding, therefore, a greater effective signal propagation delay. This increased delay may or may not be acceptable in the desired system so the trade-off between ease of usage of the unterminated case must be weighed against the increased effective signal delay over that delay obtainable with the terminated case.

Source					Load				
t in (τ)	$V_i + V_r$ (V)	$i_i + i_r$ (mA)	V_S (V)	I_S (mA)	t in (τ)	$V_i + V_r$ (V)	$i_i + i_r$ (mA)	V_L (V)	I_L (mA)
0	0.9400	12.53	0.9400	12.53	1	1.8500	0.40	1.8500	0.40
2	0.1224	-22.64	1.0624	-10.10	3	-1.5500	-0.34	0.3000	0.06
4	-0.1026	18.97	0.9599	8.87	5	1.2986	0.28	1.5986	0.35
6	0.0859	-15.90	1.0458	-7.03	7	-1.0881	-0.24	0.5106	0.11
8	-0.0720	13.32	0.9738	6.29	9	0.9116	0.20	1.4222	0.31
10	0.0603	-11.17	1.0341	-4.87	11	-0.7638	-0.17	0.6584	0.14
12	-0.0505	9.36	0.9836	4.48	13	0.6399	0.14	1.2983	0.28
14	0.0424	-7.84	1.0259	-3.36	15	-0.5362	-0.12	0.7622	0.16
16	0.0355	6.57	0.9904	3.21	17	0.4492	0.10	1.2114	0.26

Fig. 3-16. Transmission Line Model and Its Lattice Diagram

DOTTED LINES ARE $v_i(n)(1 + \rho_g)$ WAVES ARRIVING AT LOAD—THEIR SUM IS THE LOAD VOLTAGE $v_L(t)$ (SOLID LINE)

SOME WAVES OMITTED FOR CLARITY

DOTTED LINES ARE $i_i(n)(1 + \rho_g)$ WAVES AT SOURCE—THEIR SUM IS THE SOURCE CURRENT INTO THE LINE $i_s(t)$ (SOLID LINE)

Fig. 3-17. Waveforms for $t_r = 2 \ll \tau$

SOME WAVES OMITTED FOR CLARITY

Fig. 3-18. Waveforms for $t_r = 2\tau$

Fig. 3-19. Waveforms for $t_r = 3\tau$

Fig. 3-20. Waveforms for $t_r = 4\tau$

Fig. 3.21. Waveforms for $t_r = 6\tau$

Fig. 3.22. Waveforms for $t_r = 8\tau$

cases, so the primary contribution is made by the skin effect. Thus the resistance per unit length becomes

$$R = R_{dc} + K_s^m \quad (4.2)$$

where $0 < m < 1$.

For 2-wire lines (twisted pair, parallel wire), the resistance per unit length is increased by the skin effect. For closely spaced wires, however, the proximity effect also contributes significantly to a resistance increase. Radiation loss should also be included, but is very difficult to calculate because it depends on the surroundings of the line.

Inductance Per Unit Length, L. It can be shown^{5,6} that, as the frequency is increased, the skin effect, proximity effect, and radiation loss effect cause a reduction in the effective per-unit-length self-inductance of the line.

Capacitance Per Unit Length, C. This depends primarily on the dielectric constant of the insulating medium and conductor geometry. This term is constant over a wide range of frequencies for most dielectrics (Teflon®, Polyethylene). For Polyvinylchloride (PVC) insulation, the relative dielectric constant shows a decrease as frequency increases ($\epsilon_r \approx 4.7$ @ 1 kHz, $\epsilon_r \approx 2.9$ @ 100 MHz). The capacitance per unit length, therefore, will show a decrease corresponding with increasing frequency for PVC insulation and little change for most other dielectrics.

Conductance Per Unit Length, G. Because resistance per unit length usually has a much greater magnitude, this value is negligible. When this term cannot be neglected, it is represented as

$$G = \omega C \tan \phi \quad (4.3)$$

where C is capacitance per unit length, ϕ is the angular frequency ($= 2\pi f$) and $\tan \phi$ is a dielectric material coefficient. The angle ϕ is called the dielectric loss angle. This angle is usually quite small ($< .005$ radians) for the majority of dielectrics up to several hundred megahertz.

VARIATIONS IN Z_0 , $\alpha(\omega)$, AND PROPAGATION VELOCITY

The variations in the primary line parameters as a function of frequency shown by Figure 4-2 have a profound influence on the three secondary line parameters of characteristic impedance, attenuation, and velocity of propagation.

In the expression for the characteristic impedance of a line,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

at low frequencies, $j\omega L$ is small compared to R, and G is small compared to $j\omega C$. So the characteristic impedance is $\sqrt{R/j\omega C}$. At high frequencies, the increase in R is overshadowed by $j\omega L$ even though L is being reduced. With G still much smaller than $j\omega C$, the characteristic impedance is almost a pure resistance $R_0 = \sqrt{L/C}$. The behavior of the characteristic impedance as a function of frequency ($Z_0 = R_0 - jX_0$) is shown in Figure 4-3.

*Teflon is a registered trademark of E.I. du Pont de Nemours Company

Fig. 4-2. Variations in Primary Parameters as a Function of Frequency (22 AWG Polyethylene Insulated Twisted Pair)

Fig. 4-3. Typical Variation in Z_0 as a Function of Frequencies

Typical behavior of the line attenuation as a function of frequency is shown in Figure 4-4. This line attenuation is the real part of the equation

$$\gamma(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)}.$$

The change in resistance is the primary contributor to the attenuation increase as a function of frequency. For coaxial cables, this resistance increase is due primarily to the skin effect ($R_{SK} = Kf^m$). The slope of the attenuation curve on a log-log graph (log dB vs log frequency), therefore, is essentially linear and, at the same time, equal to m . For twisted pair and parallel wire lines, proximity effects and radiation losses make the curves less linear, but for high frequencies (over 100 kHz), the attenuation expressed in nepers per unit length is approximated by

$$\alpha \approx \frac{R}{2} \sqrt{\frac{C}{L}} \quad (4.4)$$

The R term is, of course, the sum of the dc resistance, plus the incremental resistance due to skin, proximity and radiation loss effects. This R term usually varies as follows.

$$R_{SK} = Kf^m$$

where $0.6 \leq m < 1.0$

The signal velocity propagation ($v = \omega/\beta$) is given by the imaginary part of the propagation constant γ . As shown in Section 3, v is a constant given by $v = \sqrt{LC}$ for lossless lines. For real lines, this value is approached at high frequencies. At low frequencies, however, (when ω is small compared to R/L or G/C), then $v_{LF} \approx (C/2)\sqrt{R/G}$ and the velocity is reduced. The propagation velocity as a function of frequency is shown in Figure 4-5. This variation in signal velocity as a function of signal frequency is *dispersion* which was previously discussed.

The signal at a point down the line represents the sum of that original signal's Fourier spectrum. Because both the attenuation and propagation velocity of these Fourier components increase with frequency, the resultant signal shape at that point down the line depends greatly on the winners of the

race to get to that point. The high frequency components, with their faster propagation velocities, arrive first, but the increased attenuation minimizes their effect. The low frequency signals arrive later, but the reduced attenuation allows them a greater influence on the resultant signal. In general, the output signal from the line should show a relatively fast rise up to some signal value (20 to 50% of the final value). This is due to arrival of the high frequency components, followed by a more leisurely rise to the final value as the slower, low frequency components arrive.

SIGNAL QUALITY — TERMS

Before the concepts presented in the previous sections can be used to answer the "how far—how fast" question, some familiarity with the terms describing data and signal quality is necessary.

The primary objective of data transmission is the transfer of information from one location to another. The information here is *digital* in nature; i.e., a finite number of separate states or choices. This is in contrast to *analog* which has an infinite number of separate states or a continuous range of choices. The digital information is *binary* or two-valued; thus two different, recognizable electrical states/levels are used to symbolize the digital information. A binary symbol is commonly called a binary-digit or *bit*. A single binary symbol or bit, by itself, can represent only one of two possible things. To represent alphabetic or numeric characters, a group of bits is arranged to provide the necessary number of unique combinations. This arrangement of bits which is then considered an information unit is called a *byte*. In the same manner that a group of bits can be called a byte, a collection of bytes, considered as a unit, is called a *word*. Selective arrangement of seven bits will provide 2^7 (or 128) distinct character combinations (unique bytes). The American Standard Code for Information Interchange (ASCII) is an excellent example of just such an arrangement—upper and lower case alphabetic, zero to nine numeric, punctuation marks, and miscellaneous information-code control functions.

Now with the means for representing information as bits or bytes, and the means for transmission of the bits (symbols) from one location to another (transmission line), the remain-

Fig. 4-4. Attenuation vs Frequency

Fig. 4-5. Propagation Velocity vs Frequency

ing task is to ensure that a particular bit arriving at its destination is interpreted in the proper context. To achieve this, both the sender and receiver of the data must accomplish the five following requirements.

1. Agree upon the nominal rate of transmission; or how many bits are to be emitted per second by the sender.
2. Agree upon a specified information code providing a one-to-one mapping ratio of information-to-bit pattern and vice versa.
3. Establish a particular scheme whereby each bit can be properly positioned within a byte by the receiver of the data (assuming that bit-serial transmission is used).
4. Define the protocol (handshaking) sequences necessary to ensure an orderly flow of information.
5. Agree to the electrical states representing the logic values of each bit and the particular pulse code to be used.

These are by no means all of the points that must be agreed upon by sender and receiver—but these are probably the most important. Items 2, 3 and 4 are more or less "software" type decisions, because the actual signal flow along the transmission line is usually independent of these decisions. Because items 1 and 5 are much more dependent on the characteristics of line drivers, line receivers, and transmission lines, they are the primary concern here.

Figure 4-6 represents the components of a typical data transmission system. The *information source* can be a computer terminal or a digitized transducer output, or any device emitting a stream of bits at the rate of one bit every t_b seconds. This establishes the *information rate* of the system at $1/t_b$ bits per second. The information source in the figure feeds a *source encoder* which performs logic operations not only on the data, but also on the associated clock and, perhaps, the past data bits. Thus, the source encoder produces a binary data stream controlling the *line driver*. The line driver interfaces the source internal logic levels (TTL, MOS, etc.) with transmission line current/voltage requirements. The transmission line conveys signals produced by the line driver to the line receiver. The line receiver makes a decision on the signal logic state by comparing the received signal to a decision threshold level, and the *sink decoder* performs logic opera-

tions on the binary bit stream recovered by the line receiver. For example, the *sink decoder* may extract the clock rate from the data or perhaps detect and correct errors in the data. From the optional sink decoder, the recovered binary data passes to the *information sink*—the destination for the information source data.

Assume for the moment that the source encoder and sink decoder are "transparent"; that is, they will not modify the binary data presented to them in any way. Line driver signals, then, have the same timing as the original bit stream. The data source emits a new bit every t_b seconds. The *pulse code* produced by the source encoder and line driver is called *Non-Return to Zero (NRZ)*, a very common signal in TTL logic systems. A sample bit pattern with its NRZ representation is shown in Figure 4-7a. The arrows at the top represent the *ideal instants*, or the times the signal can change state. The term *unit interval* is used to express the time duration of the shortest signaling element. The shortest signaling element for NRZ data is one bit time t_b , so the unit interval for NRZ data is also t_b . The rate at which the signal changes is the *modulation rate* (or signaling speed), and *baud* is the unit of modulation rate. A modulation rate of one baud corresponds to the transmission of one unit interval per second. Thus the modulation rate, in baud, is just the reciprocal of the time for one unit interval. A unit interval of 20 ms, therefore, means the signaling speed is 50 baud. The reason for differentiating between the *information rate* in bits per second (bps) and the *modulation rate* in baud will be clarified after examining some of the other pulse codes later in this chapter.

NRZ data should always be accompanied by a clock signal, Figure 4-7b, which tells the receiver when to sample the data signal and thus determine the current logic state. For the example in Figure 4-7b, the falling edge of the clock corresponds to the middle of the data bits, so it could be used to transfer the line receiver data output into a binary latch. The falling edge of the clock is thus the *sampling instant* for the data. The line receiver does have a *decision threshold* or *slicing point* so that voltages above that threshold level produce one logic state output, while voltages below the threshold produce the other logic state at the receiver output. The receiver may incorporate positive feedback to produce *hysteresis* in its transfer function. This reduces the possibility of oscillation in response to slow rise or fall time signals applied to the receiver inputs.

Fig. 4-6. Data Transmission System

Previously in this chapter, it was stated that the fast rise and fall times of signals, corresponding to the transitions between data bits, are rounded out and slowed down by a real transmission line. Each transition of the signal applied to the line by the line driver is transformed to a rounded out transition by the dispersion and attenuation of the transmission line. The resultant signal at the load end of the line consists of the superposition of these transformed transitions. The waves arriving at the load end of the line are shown in Figure 4-7c and their superposition is shown in Figure 4-7d. It is assumed that the line is terminated in its characteristic resistance so that reflections are not present. The receiver threshold level is shown here, superimposed on the resultant load signal, and the re-converted data output of the line receiver is shown in Figure 4-7e along with the ideal instants for the data transitions (tick marks).

Comparing the original data (Figure 4-7a) to the recovered data (Figure 4-7e) shows that the actual recovered data transitions may be displaced from their ideal instants (tic marks on Figure 4-7e). This time displacement of the transitions is due to a new wave arriving at the receiver site before the previous wave has reached its final value. Since the wave representing a previous data bit is *interfering* with the wave representing the present data bit, this phenomenon is called *intersymbol interference* (in telegraphy it is called *characteristic distortion*). The intersymbol interference can be reduced to zero by making the unit interval of the data signal quite long in comparison to the rise/fall time of the signal at the receiver site. This can be accomplished by either reducing the modulation rate for a given line length, or by reducing the line length for a given modulation rate.

Signal quality is concerned with the variance between the ideal instants of the original data signal and the actual transition times for the recovered data signal.

For synchronous signaling, such as NRZ data, the *isochronous distortion* of the recovered data is the ratio of the unit interval to the maximum measured difference irrespective of sign between the actual and theoretical significant instants. The isochronous distortion is, then, the peak-to-peak time jitter of the data signal expressed as a percentage of the unit inter-

val. A 25% isochronous distortion means that the peak-to-peak time jitter of the transition is .25 unit interval (max).

Another type of received-signal time distortion can occur if the decision threshold point is misplaced from its optimum value. If the receiver threshold is shifted up toward the *One signal level*, then the time duration of the One bits shortens with respect to the duration of the Zero bits, and vice versa. This is called *bias distortion* in telegraphy and can be due to receiver threshold offset (bias) and/or asymmetrical output levels of the driver. This effects is shown in Figure 4-8.

Bias distortion and characteristic distortion (intersymbol interference) together are called *systemic distortion*, because their magnitudes are determined by characteristics within the data transmission system. Another variety of time distortion is called *fortuitous distortion* and is due to factors outside the data transmission system such as noise and crosstalk, which may occur randomly with respect to the signal timing.

SIGNAL QUALITY MEASUREMENT—THE EYE PATTERN

To examine the relative effects of intersymbol interference on random NRZ data and a "dotting" pattern, see Figure 4-9. The top two waveforms represent the NRZ data and dotting pattern as outputs into two identical long transmission lines. The middle two traces illustrate the resultant signals at the line outputs and the bottom two traces show the data output of the line receivers. The respective thresholds are shown as dotted lines on the middle two traces. The arrows indicate the ideal instants for both data and dotting signals.

Notice that the dotting signal (D) is symmetrical, i.e., every One is preceded by a Zero and vice versa, while the NRZ data is random. The resultant dotting signal out of the line is also symmetrical. Because, in this case, the dotting half-cycle time is less than the rise/fall time of the line, the resultant signal out of the line (E) is a *partial response*—it never reach-

*The term dotting pattern is from telegraphy and means an alternating sequence of 1-bits and 0-bits (the "dot dot dot" etc.). Note that an NRZ dotting pattern generates a signal which has a 50% duty cycle and a frequency of $1/2 t_b$ (Hz)

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Fig. 4-7. NRZ Signaling

Fig. 4-8. Bias Distortion

Fig. 4-9. Comparison of NRZ Random Data and "Dotting" Signals

es its final level before changing. The dotting signal, due to its symmetry, does not show intersymbol interference in the same way that a random NRZ signal does. The intersymbol interference in the dotting signal shows up as a uniform displacement of the transitions as shown in Figure 4-9f. The NRZ data shows intersymbol interference, in its worst light, due to its unpredictable bit sequence. Thus, whenever feasibility of a data transmission system is to be tested, a random data sequence should be used. This is because a symmetri-

cal dotting pattern or clock signal cannot always show the effects of possible intersymbol interference.

A very effective method of measuring time distortion through a data transmission system is based on the eye pattern. The eye pattern, displayed on an oscilloscope, is simply the superposition, over one unit interval, of all the Zero-to-One and One-to-Zero transitions, each preceded and followed by various combinations of One and Zero, and also constant One

and Zero levels. The name *eye pattern* comes from the resemblance of the open pattern center to an eye. The diagrammatic construction of an eye pattern is shown in Figure 4-10. The data sequence can be generated by a pseudo-random sequence generator (PRSG), which is a digital shift register with feedback connected to produce a maximum length sequence. The PRSG, requiring only two devices (Figure 4-11), generates a sequence that repeats after $2^{20}-1$ bits. Feedback

connections for PRSG shift registers from 4 to 20 bits in length are shown in Figure 4-12.

Several features of the eye pattern make it a useful tool for measuring data signal quality. Figure 4-13 shows a typical binary eye pattern for NRZ data. The spread of traces crossing the receiver threshold level (dotted line) is a direct measure of the peak-to-peak transition jitter—isochnous distor-

Fig. 4-10. Formation of an Eye Pattern by Superposition

Fig. 4-11. Bench Set-up to Measure Data Signal Quality

tion in a synchronous system—of the data signal. The rise and fall time of the signal can be conveniently measured by using the built-in 0% and 100% references produced by long strings of Zeros and Ones. The height of the trace above or below the receiver threshold level at the sampling instant is the noise margin of the system. If no clear transition-free space in the eye pattern exists, the eye is closed. This indicates that error-free data transmission is not possible at that data rate and line length with that particular transmission line without resorting to equalizing techniques. In some extreme cases, error-free data recovery may not be possible even when using equalizing techniques.

The eye pattern can also be used to find the characteristic resistance of a transmission line. The 250 Ω printed circuit-type potentiometer termination resistor (Figure 4-11) can be adjusted to yield the minimum overshoot and undershoot of the data signal. Figure 4-14 shows the NRZ data eye patterns for $R_T > R_0$, $R_T = R_0$ and $R_T < R_0$. The 100% and 0% reference levels are again provided by long strings of Ones and

Zeros, and any overshoot or undershoot is easily discernible. The termination resistor is adjusted so that the eye pattern transitions exhibit the minimum perturbations (Figure 4-14b). The resistor is then removed from the transmission line, and its measured value is the characteristic resistance of the line.

By using the eye pattern to measure signal quality at the load end of a given line, a graph can be constructed showing the tradeoffs in signal quality—peak-to-peak jitter—as a function of line length and modulation rate for a specific pulse code. An example graph for NRZ data is shown in Figure 4-15. The graph was constructed using eye pattern measurements on a 24 AWG twisted pair line (PVC insulation) driven by a differential voltage source driver (9614) with the line parallel-terminated in its characteristic resistance (96 Ω). The oscilloscope photographs in Figure 4-16 show the typical eye patterns for NRZ data with various amounts of isochronous distortion. The straight lines represent a "best fit" to the actual measurement points. Since the twisted pair line used was not specifically constructed for pulse service, the graph

Fig. 4-12. Feedback Connection for Various Maximal Sequence Length PRSG

Fig. 4-13. NRZ Data Eye Pattern

Fig. 4-14. Using Eye Pattern to Determine Characteristic Resistance of Line

Fig. 4-15. Signal Quality as a Function of Line Length and Modulation Rate for Terminated 24 AWG Twisted Pair (PVC Insulation)

Fig. 4-16. Eye Patterns for NRZ Data Corresponding to Various Peak-to-Peak Transition Jitter

probably represents a reasonably good worst-case condition insofar as signal quality vs line length is concerned. Twisted pair lines with polyethylene or Teflon® insulation have shown better performance at a given length than the polyvinyl chloride insulation. Likewise, larger conductors (20 AWG, 22 AWG) also provide better performance at a given length. Thus, the graph in Figure 4-15 can be used to estimate feasibility of a data transmission system when the actual cable to be used is unavailable for measurement purposes. The arbitrary cutoff of 4000 feet on the graph was due to an observed signal amplitude loss of 6 dBV (1/2 voltage) of the 24 AWG line at that distance. The cutoff of 10 Mbaud is

based on the propagation delays of the typical TTL line drivers and receivers. Field experience has shown that twisted pair transmission systems using TTL drivers and receivers have operated essentially error-free when the line length and modulation rate are kept to within the shaded recommended operating region shown in Figure 4-15. This has not precluded operation outside this region for some systems, but these systems must be carefully designed with particular attention paid to defining the required characteristics of the line, the driver, and the receiver devices. The use of coaxial cable instead of twisted pair lines almost always yields better performance, i.e., greater modulation rate at a given line length

and signal quality. This is because most coaxial cable has a wider bandwidth and reduced attenuation at a given length than twisted pair line (one notable exception is RG 174/U cable). In the next section, a method is discussed that allows a rough estimation of the maximum "safe" modulation rate for a given length of a particular coaxial cable when actual eye pattern measurements cannot be made on that cable.

It should be remembered that, in some ways, the eye pattern gives the *minimum* peak-to-peak transition jitter for a given line length, type, pulse code, and modulation rate. This is because the eye pattern transition spread is the result of intersymbol interference and reflection effects (if present) and this minimum jitter is only obtainable if the following conditions are met.

- The One and Zero signal levels produced by the line driver are symmetrical, and the line receiver's decision threshold (for NRZ signaling) is set to coincide with the mean of those two levels.
- The line is perfectly terminated in its characteristic resistance to prevent reflections from altering the signal threshold crossings.
- The time delays through driver and receiver devices for both logic states is symmetrical and there is no relative skew in the delays (difference between $L \rightarrow H$ and $H \rightarrow L$ propagation delays = 0). This is especially important when the device propagation delays become significant fractions of the unit interval for the applicable modulation rate.

If any one of these conditions is not satisfied, the signal quality is reduced (more distortion). The effects of receiver bias or threshold ambiguity and driver offset can be determined by location of the decision threshold(s) on the oscillograph of the eye pattern for that driver/cable modulation rate combination. For eye patterns displaying more than 20% isochronous distortion, the slope of the signal in the transition region is relatively small. Therefore, a small amount of bias results in a large increase in net isochronous distortion. See Figure 4-17 for a graphic illustration of this effect. In the interest of conservative design practices, systems should always be designed with less than 5% transition spread in the eye pattern. This allows the detrimental effects due to bias to be minimized, thus simplifying construction of line drivers and receivers.

ESTIMATION OF SIGNAL QUALITY IN LIEU OF EYE PATTERN MEASUREMENTS

In many design situations it is not always convenient to measure signal quality using the eye pattern. This can be caused by lack of equipment, time, or unavailability of the transmission line to perform the necessary measurements. In such cases, the following rule of thumb applies: the unit interval should be greater than twice the 10 to 90% rise or fall time of the line, i.e.,

$$t_{ui} \geq 2t_{10-90\%}$$

Eye patterns that meet this criterion show that adherence to this rule will keep the peak-to-peak jitter to less than 5% of the unit interval. If the 10% to 90% rise time cannot be easily measured—as is the case with coaxial cables whose dominant loss mechanism is simple skin effect—then the 10 to 80% rise time may be used instead of the 10 to 90% rise time. The corresponding peak-to-peak jitter, due to intersymbol interference with the $t_{ui} \geq 2t_{10-80\%}$ rule, is usually less than 10%. In any event, if the unit interval is less than the 0 to 50% rise time of the signal at the line end ($t_{ui} < t_{0-50\%}$), then the error-free recovery of NRZ data is not possible without using equalizing techniques. Sometimes, changing to a pulse code which allows partial response recovery will enable data transmission at an otherwise unusable line length-speed combination. When actual eye pattern measurements cannot be made, an estimate of feasibility can still be obtained by calculating the signal transition time.

M.S. Nahman^{2,4} presented a simple method to predict the transient response of a coaxial cable from the cable attenuation. The procedure was based on a graphical analysis technique allowing prediction of the step response of a network from a graph of the imaginary part of the transfer function, in this case the imaginary part of γ (s). The normalized step responses (Figure 4-18) are plotted with the abscissa in normalized time (x)

$$x = t/a \quad (4.5)$$

where

$$a = \frac{1}{2\pi f_0} \left[\frac{\alpha(f_0)}{\cos(m\pi/2)} \right] \frac{1}{m} \quad (4.6)$$

and

t = real time.

Fig. 4-17. Receiver Bias Effect on Total Isochronous Distortion

Fig. 4-18. Normalized Step Responses for m Law Coaxial Cables (After Nahman²)

$\alpha(f_0)$ = attenuation of the coaxial cable in nepers at frequency (f_0) for length l . If $\alpha(f_0)$ is known in dB, then division by 8.686 will convert the dB/length into nepers/length (1 neper = 8.686 dB).

m = slope of the attenuation vs frequency curve for the particular coaxial cable when plotted on log-log graph paper (log dB vs log frequency).

$$m = \frac{\log \alpha(f_2) - \log \alpha(f_1)}{\log(f_2) - \log(f_1)}$$

where $f_2 > f_1$ and $\alpha(f_2)$ is the attenuation at frequency (f_2) , and log is the common (base 10) logarithm.

Slope m will have a range of $0 < m < 1$.

Nahman pointed out that this method applies to cables where either the attenuation or the phase characteristic can specify the transfer function. This implies that knowledge of either characteristic alone provides sufficient information for the other characteristic; in particular, the Bode condition must be met.

$$\lim_{s \rightarrow \infty} \frac{\gamma(s)}{s} = 0$$

For details, see Nahman's paper². In general, a reasonably accurate modified transfer function may be specified satisfying Bode's condition for coaxial cables. Twisted pair and parallel wire lines cannot usually be fully characterized by either attenuation or phase characteristics alone. This is because each has a significant contribution to the overall transfer function. Thus Bode's condition cannot usually be met, so Nahman's method will not yield accurate results.

In a more recent paper⁴, he asserted that the primary contributors to the shape of the step response waveform are those frequency components transmitted with less than 20 dB of attenuation. Thus, the predominant slope of the attenuation curve up to 20 dB at that cable length should be used. For short cables (1000 ft) the 20 dB attenuation may occur in the 10 to 100 MHz region. Equation 4.6 may be modified by

factoring out the cable length to produce

$$a = \frac{1}{2\pi f_0} \left[\frac{\alpha_1(f_0)}{\cos(m\pi/2)} \right]^{\frac{1}{m}} \cdot \left[l \right]^{\frac{1}{m}} \quad (4.7)$$

where $\alpha_1(f_0)$ is now the attenuation (in nepers per unit length and l is the line length. The time for the line signal to rise to 50% is obtained by combining Equations 4.5 and 4.7 (Equations 5 and 7 in Reference 4).

$$t_{50\%} = \frac{x_{50\%}}{2\pi f_0} \left[\frac{\alpha_1(f_0)}{\cos(m\pi/2)} \right]^{\frac{1}{m}} \cdot \left[l \right]^{\frac{1}{m}} \quad (4.8)$$

where $x_{50\%}$ is obtained from Figure 4-18 or the table in Figure 4-19. Likewise, combining the expressions for $x_{10\%}$ and $x_{80\%}$ and subtracting gives

$$t_{10-80\%} = \frac{x_{80\%} - x_{10\%}}{2\pi f_0} \left[\frac{\alpha_1(f_0)}{\cos(m\pi/2)} \right]^{\frac{1}{m}} \cdot \left[l \right]^{\frac{1}{m}} \quad (4.9)$$

The 10 to 90% rise time can be found similarly by substituting $x_{90\%}$ for $x_{80\%}$ in Equation 4.9.

As an example of the use of these equations to predict step response of a coaxial cable, consider RG59A/U cable. The listed attenuation for RG59A/U is¹³

f_0	$\alpha(f_0)/100$ ft (dB)
10 MHz	1.07
100 MHz	3.4
400 MHz	7.0

The slope of the attenuation curve is

$$m = \frac{\log 7.0 - \log 1.07}{\log(400 \times 10^6) - \log(10 \times 10^6)} = \frac{.82}{1.60} = 0.51$$

The attenuation per unit length (in nepers) at 400 MHz is

$$\alpha_1(400 \text{ MHz}) = \frac{7.0}{8.686} \cdot \frac{1}{100 \text{ ft}} = 0.008059 \text{ nepers/ft}$$

Thus, the normalization constant, a , is

m	$\times 50\%$	$\times 10-90\%$	$\times 10-80\%$
0.4	1.30	*	14.5
0.5	1.15	*	7.1
0.6	1.00	*	3.5
0.7	0.92	6.6	2.4
0.8	0.89	3.0	1.4
0.9	0.89	1.0	0.7

*90% points not given on graph for these values (Figure 4²)

Fig. 4-19. Normalized Time to Reach Various Percentages of Signal Final Value for Various Values of m

$$a = \frac{1}{2\pi(400 \times 10^6)} \left[\frac{.008059}{\cos[(.51)\pi/2]} \right]^{\frac{1}{.51}} = 6.355 \times 10^{-14}$$

A 140-foot length of line, therefore, should have a $t_{50\%}$ time

$$t_{50\%} = (1.15) (6.355 \times 10^{-14}) (140)^{\frac{1}{.51}} = 1.18 \text{ ns}$$

and the 10% to 80% rise time is

$$t_{10-80\%} = (7.1) (6.355 \times 10^{-14}) (140)^{\frac{1}{.51}} = 7.29 \text{ ns}$$

For 1000 ft of RG59A/U, the previous results for 140 ft can be scaled.

$$t_{50\%} = 1.18 \cdot \left[\frac{1000}{140} \right]^{\frac{1}{.51}} = 55.7 \text{ ns}$$

and

$$t_{10-80\%} = 7.29 \cdot \left[\frac{1000}{140} \right]^{\frac{1}{.51}} = 344.2 \text{ ns}$$

If the desired modulation rate is 1 Mbaud ($t_{ui} = 1.0 \mu s$) and the line length is 1000 feet, then the system using RG59/U cable would probably operate satisfactorily. Be aware, however, that if the single slope (m) does not accurately approximate the actual attenuation characteristic of the coaxial cable, then Nahman's method will show considerable error in predicting the signal transition time. RG174/U is a coaxial cable with an attenuation characteristic that has a slope in the 0.3 to 0.65 range depending on the frequency. Thus the method cannot be accurately applied to RG174/U cable.

Also, Nahman's method cannot be applied to commonly available twisted pair or parallel wire lines. This is because knowledge of either the line attenuation or its phase characteristics is not normally sufficient to completely specify the transfer function. Thus, the previously mentioned Bode condition is not satisfied, and the analysis previously used cannot be applied satisfactorily.

Experimental tests on various twisted pair cables from 10 to 4000 feet in length have revealed that these cables show a law of $m \approx 0.6$ for attenuation, but follow an almost linear increase in rise time as cable length is increased. The 10 to 90% rise time usually falls in the following range.

$$\tau \leq t_{10-90\%} \leq 2\tau$$

where τ is the time delay of the cable per unit length times the cable length. Combining this with the suggestion that the unit interval should be at least twice the 10-90% rise time of the line, then the minimum unit interval should be

$$t_{ui} \geq 4\tau$$

This corresponds with a boundary for recommended operating regions shown in Figure 4-15. Experience has shown that adherence to the above relationship with twisted pair cables results in less than 10% peak-to-peak jitter due to intersymbol interference.

OTHER PULSE CODES AND SIGNAL QUALITY

In the preceding sections, the discussion of signal quality has been centered around the use of NRZ signaling, because it represents the simplest and most commonly used pulse code. Other pulse codes have been developed which provide one or more of the following desirable features:

- Compress the overall bandwidth normally required to adequately transmit the signal yet still ensure recovery of the binary data.
- Eliminate the need for a dc response in the transmission medium so that transformer coupling can be used for phantom power distribution on repeated lines. (The elimination of a dc characteristic of the pulse code also allows ac coupling of amplifier circuits).
- Provide a clocking scheme within the signal so that no separate clock channel is required for synchronization.
- Provide built-in error detection.

The following discussion is restricted to the binary class of baseband signals. This simply means that each decision by the line receiver yields one bit of information. The M -ary schemes ($M \geq 3$) can encode more than one bit of information per receiver decision*, but these schemes are seldom applied to baseband signaling due to the complexities of the driver and receiver circuits (especially for $M > 3$). M -ary schemes, however, are applied to high speed non-baseband data transmission systems using modems. The price to be paid for the increased bit-packing with multi-level signaling is decreased immunity to noise relative to a binary system. This is because a smaller relative threshold displacement (or amount of noise) is required to produce a signal representing another logic state in the M -ary schemes.

In general, the binary class of pulse codes can be grouped into four categories;

- Non-Return to Zero (NRZ)
- Return to Zero (RZ)

*It can be shown that, for M levels, the information per receiver decision will be $S = \log_2 M$ bits/decision. Thus, three levels theoretically yield 1.58 bits, four levels yield 2 bits of information, eight levels yield 3 bits, etc.

- Phase Encoded (PE) (sometimes called Split Phase)
- Multi-Level Binary (MLB). (The MLB scheme uses three levels to convey the binary data, but each decision by the line receiver yields only one bit of information.)

A secondary differentiation among the pulse codes is concerned with the algebraic signs of the signal levels. If the signal levels have the same algebraic sign for their voltages (or currents) and differ only in their magnitudes, the signaling is called *unipolar*. A very common example of unipolar signaling is TTL or ECL logic. TTL uses two positive voltages to represent its logic states, while ECL uses two negative voltages for its logic states. The complement of unipolar signaling is *polar* signaling. Here, one logic state is represented by a signal voltage or current having a positive sign and the other logic state is represented by a signal with a negative sign. For binary signals, the magnitude of both signals should be equal, ideally. Their only difference should be in the algebraic signs. This allows the receiver to use ground as its decision threshold reference.

Non-Return to Zero (NRZ) Pulse Codes

There are three NRZ pulse codes: NRZ - Level (NRZ-L), NRZ - Mark (NRZ-M), and NRZ - Space (NRZ-S). NRZ-L is the same pulse code as previously discussed. In NRZ-L signaling, data is represented by a constant signal level during the bit time interval, with one signal level corresponding to one logic

state, and the other signal level corresponding to the opposite logic state. In NRZ-M or NRZ-S signaling, however, a change in signal level at the start of a bit interval corresponds to one logic state and no change in signal level at the start of a bit interval corresponds to the opposite logic state. For NRZ-M pulse codes, a change in signal level at the start of the bit interval indicates a logic One (Mark), while no change in signal level indicates a logic Zero (Space). NRZ-S is a logical complement to NRZ-M. A change in signal level means a logic Zero and no change means logic One. With NRZ-M and NRZ-S pulse codes, therefore, there is no direct correspondence between signal levels and logic states as there is with NRZ-L signaling. Any of the NRZ pulse codes may, of course, be used in unipolar or polar form. The NRZ codes are shown in Figure 4-20, along with their generation algorithms*, signal levels vs time, and their general power density spectrum.

The degradation in signal quality caused by intersymbol interference for NRZ-L signaling was discussed earlier. Since the minimum signaling element (unit interval) for all three NRZ pulse codes is equal to t_b , the previous signal quality discussion for NRZ-L also applies equally to NRZ-M and NRZ-S pulse codes. The following is a capsule summary of the previous discussion on NRZ signal quality.

*The generation algorithm showing the sequence of signal levels on the line, represented by the set $\{b_n\}$ is determined by the sequence of input logic states, represented by the set $\{a_n\}$. See Bennett¹⁴ for detailed usage of this notation.

- When t_b is less than the 0-50% rise or fall time of the signal at the line end, the open space in the eye pattern closes, thereby indicating error-free data transmission is unlikely.
- When t_b is less than the 10-90% rise or fall time of the line end signal, some intersymbol interference is present and thus, some time jitter in the transitions of the recovered data will be present.

NRZ codes are simple to generate and decode because no precoding or special treatment is required. This simplicity makes them probably the most widely used pulse codes, with NRZ-L the leader by far. NRZ-M has been widely used in digital magnetic recording where it is usually called NRZI for Non-Return to Zero, Invert-on-Ones. In terms of the four desirable features for a pulse code listed at the start of this section, however, none of the NRZ codes are all that great—NRZ codes do possess a strong dc component, and have neither intrinsic clocking, nor error detection features. Even so, their power frequency spectra are used as references for comparison with other pulse codes.

Return to Zero (RZ) Pulse Codes

The RZ group of pulse codes are usually simple combinations of NRZ-L data and its associated single or double frequency clock. By combining the clock with data, all RZ codes possess some intrinsic synchronization feature. Three representative RZ pulse codes are shown in Figure 4-21. Unipolar RZ is formed by performing a logic AND between the NRZ-L data and its clock. Thus a logic Zero is represented by the absence of a pulse during the bit time interval, and a logic One is represented by a pulse as shown. Pulse Position Modulation

(PPM) uses a pulse of $t_b/4$ duration beginning at the start of the bit interval to indicate a logic Zero, and a $t_b/4$ pulse beginning at the middle of the bit interval to indicate a logic One. Pulse Duration Modulation (PDM) uses a $t_b/3$ duration pulse for a logic Zero and a $(2/3)t_b$ pulse for a logic One, with the rising edge of both pulses coinciding with the start of the bit interval. PDM with $t_b/4$ pulse widths is also used but better results are usually obtained with the $t_b/3$, $2t_b/3$ scheme.

The reason for differentiating between information rate and modulation rate can now be further clarified. Each of the RZ pulse codes in Figure 4-21 has the same information rate; i.e., $1/t_b$ bits per second. Their respective minimum signaling elements (unit intervals) however, are all less than t_b so the modulation rate for the RZ pulse code is greater than the information rate. Remember that with NRZ signaling, the unit interval and the bit time interval are equal in duration, so the information rate in bps is equal to the modulation rate in bauds. For isochronous NRZ signaling, the measures bps and baud are both synonymous and interchangeable.

Inspection of unipolar RZ signaling reveals that the unit interval is $1/2$ bit interval ($t_{ui} = t_b/2$). When this unit interval is less than the 0-50% rise or fall time of the line, the data is likely to be unrecoverable. With a fixed modulation rate, the price paid to include clocking information into unipolar RZ is reduced information rate over that for NRZ signaling. Likewise, for PPM with its unit interval of $t_b/4$, the information rate reduces to $1/4$ that of NRZ data under the same conditions. This is because the maximum modulation rate is determined by the 50% rise time of the line which is constant

for a given length and type of line. PDM has a unit interval of $t_b/3$ so, for a given maximum modulation rate, the resulting information rate is $1/3$ that of NRZ data.

The preceding argument should not be taken as strictly correct—since the actual intersymbol interference patterns for the three RZ codes discussed differ somewhat from the pattern with NRZ codes. A random sequence of NRZ data can easily consist of a long sequence of Zeros followed by a single One and then a long sequence of Zeros, so the $t_{50\%}$ limit can be accurately applied. Unipolar RZ, in response to the same long data sequence, produces a $t_b/2$ pulse, so the $t_{50\%}$ argument can be applied here too. With PPM and PDM, the maximum time that the line signal can be in one state is quite reduced from the NRZ case. For PPM, this time is $1.25 t_b$ (O10 data sequence) while for PDM, it is $.67 t_b$ (see Figure 4-21). With PPM and PDM, then, the line signal may never reach the final signal levels that it does with NRZ data. So, the PPM and PDM signals have a head start, so to speak, in reaching the threshold crossing of the receiver. Because of the reduced time that PDM and PPM signal levels are allowed to remain at one signal level, their signaling may still operate at a modulation rate slightly above that where the NRZ data shows 100% transition jitter. Even with this slight correction to the previous discussion, the RZ group of pulse codes still sacrifice information rate in return for synchronization. The PPM scheme appears to be a poor trade in this respect, since PDM allows a greater information rate while retaining the self-clocking feature. Unipolar RZ, because it provides no clocking for a logic Zero signal, is not generally as useful as PDM for baseband data transmission. However, unipolar RZ is used in older digital magnetic tape recorders.

Examination of RZ codes shows only one more desirable feature than NRZ codes: clocking. RZ codes still have a dc component in their power density spectrum (Figure 4-21) and their bandwidth is extended (first null at $2/t_b$) over that of NRZ (first null $1/t_b$). RZ codes do not have any intrinsic error detection features.

Phase Encoded (PE) Pulse Codes

The PE group of pulse codes uses signal level transitions to carry both binary data and synchronization information. Each of the codes provides at least one signal level transition per bit interval aiding synchronous recovery of the binary data. Simply stated, Biphase-Level (Bi ϕ -L) code is binary phase shift keying (PSK) and is the result of an Exclusive-OR logic function performed on the NRZ-L data and its clock; it is further required that the resultant signal be phase coherent (i.e., no glitches). Biphase-Mark (Bi ϕ -M) and Biphase-Space (Bi ϕ -S) codes are essentially phase coherent, binary frequency shift keying (FSK). In Bi ϕ -M, a logic One is represented by a constant level during the bit interval (one-half cycle of the lower frequency $1/(2 t_b)$), while a logic Zero is represented by one-half cycle of the higher frequency $1/t_b$. In Bi ϕ -S, the logic states are reversed from those in Bi ϕ -M. Another way of thinking of Bi ϕ -M or Bi ϕ -S is as follows.

- Change signal level at the end of each bit interval regardless of the logic state of the data.
- Change signal level at the middle of each bit interval to mean a particular logic state.

In Bi ϕ -M (sometimes called diphas), a mid-bit interval change in signal level indicates a logic One (Mark), while no

change indicates a logic Zero. For Bi ϕ -S, no signal level change in the middle of the bit interval means a logic One, while a change means a logic Zero.

In Bi ϕ -L (also called Manchester Code), a positive-going transition at the middle of the bit interval means a logic Zero, while a negative-going transition there indicates a logic One.

The fourth member of the PE family is Delay Modulation (DM)^{15,16} sometimes referred to as Miller code. Here logic One is represented by a mid-bit interval signal level change, and a logic Zero is represented by a signal level change at the end of the bit interval if the logic Zero is followed by another logic Zero. If the logic Zero is immediately followed by a logic One, no signal level transition at the end of the first bit interval is used. The waveforms encoding algorithms, and general power density spectra for the PE pulse code family are shown in Figure 4-22.

A brief inspection of the signal waveforms for the three Bi-phase pulse codes reveals that their minimum signaling element has a duration of one-half bit interval ($t_{ui} = t_b/2$); the longest duration of either signal level is one bit interval. Similarly, DM is seen to have a minimum signaling element of one bit interval ($t_{ui} = t_b$) and the maximum duration of either signal level is two bit intervals (produced by a 101 pattern). Biphase codes should exhibit eye closure (they would not be recoverable without equalization) when $t_{ui} \leq 10-50\%$. So, a 50% jitter on NRZ signaling approximately corresponds to the Biphase codes non-operation point. Biphase codes, therefore, provide one-half the information rate of NRZ signals at a given maximum modulation rate. This is in exchange for synchronization information and a dc-free spectrum when used in polar form.

DM should have essentially the same intersymbol interference characteristics as NRZ, since the unit interval is the same for both codes. DM may perform slightly better than NRZ, because the maximum duration of either signal level is two bit intervals. Overall, DM is better coding scheme than the Bi ϕ . It does not require as much bandwidth as Bi ϕ and still possesses the desirable dc response and synchronization qualities.

Both Bi ϕ and DM are good choices for digital magnetic recording¹⁶; Bi ϕ is widely used in disc memory equipment, and DM is rapidly gaining acceptance where high bit packing densities are desired. Overall scoring, in terms of the four desirable characteristics, shows the PE pulse codes with three primary features: bandwidth compression, no dc, and intrinsic synchronization.

The Bi ϕ family does not possess any intrinsic error detection scheme. DM does possess the capability of detecting some—but not all—single bit errors. This detection process is accomplished by checking to see if a single level persists longer than two bit intervals, in which case, an error is indicated. DM detection requires two samples per bit interval.

Multi-Level Binary (MLB) Pulse Codes

The pulse codes in the MLB group discussed have a common characteristic of using three signal levels (expressed in shorthand notation as +, 0, -) to represent the binary information, but each receiver decision yields only one bit of information. These are sometimes called *pseudoternary* codes to distinguish them from true ternary codes wherein each receiver decision can yield 1.58 information bits.

¹⁶Delay Modulation^{15,16} has a maximum of $2 t_b$ without a signal level transition.

MPST coding was developed primarily to speed up the framing process, i.e., selecting which two successive pulses constitute a valid pair, when the probability for a Zero and a One are not equal.

Duobinary^{23, 24} is an example of a correlative level coding technique, wherein a correlation exists between successive signal levels. Duobinary uses three signal levels with the middle level corresponding to a logic Zero, and the other two levels corresponding to a logic One. The pseudoternary signal is generated by precoding the input data, which results in constraining the line signal to change only to the neighbor-

ing level, i.e., the (+) to (-) and (-) to (+) level changes are not allowed. This precoding process uses controlled intersymbol interference as part of the coding scheme. The benefit is an effective doubling of the bit rate for a given bandwidth and concentration of the power spectrum toward dc (Figure 4-23). Duobinary has the capability to detect single errors which violate the encoding rules. In terms of bandwidth utilization, Duobinary ranks first among all the binary and MLB codes²⁰, but its strong dc component prohibits the use of ac-coupled transmission media. Synchronization properties are similar to NRZ, thus external clocking must be used to recover the data.

Name	Data to be Sent (a_n)	Line Signal	t_{ui}	Line Signal Sequence (b_n) Generation																		
Polar Return to Zero (PRZ)	$a_n = 1$ $a_n = 0$		$tB/2$	$b_n = (+) \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$ $b_n = (-) \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$																		
Bipolar (BP)	$a_n = 1$ $a_n = 0$		$tB/2$	The polarity of b_n depends on the polarity of b_{n-k} corresponding to the last $a_{n-k} = (1)$. If the last b_{n-k} was a positive pulse (1), then b_n will be a negative pulse (2). If the last b_{n-k} was a negative pulse (2), then b_n will be a positive pulse (1). 1) $b_n = (+) \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$ 2) $b_n = (-) \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$																		
Dicode DI	NA	NA	tB	$b_n = 0 \quad 0 \leq t < tB$ If $a_n = a_{n-1}$, $b_n = 0$ or else $b_n = -b_{n-k}$ with b_{n-k} corresponding to last $a_{n-k} \neq a_{n-k-1}$																		
Pair Selected Ternary (PST-L)	NA	NA	tB	<table><tr><th>Pair $a_n a_{n+1}$</th><th colspan="2">Polarities for $C_n C_{n+1}$</th></tr><tr><th></th><th>+ MODE</th><th>- MODE</th></tr><tr><td>1 1</td><td>+</td><td>-</td></tr><tr><td>1 0</td><td>+</td><td>0</td></tr><tr><td>0 1</td><td>0</td><td>+</td></tr><tr><td>0 0</td><td>-</td><td>+</td></tr></table> $b_n = C_n \begin{cases} 0 \leq t < tB \\ \text{[Change mode after transmission of a 10 or 01 } a_n a_{n+1} \text{ pair.]} \end{cases}$	Pair $a_n a_{n+1}$	Polarities for $C_n C_{n+1}$			+ MODE	- MODE	1 1	+	-	1 0	+	0	0 1	0	+	0 0	-	+
Pair $a_n a_{n+1}$	Polarities for $C_n C_{n+1}$																					
	+ MODE	- MODE																				
1 1	+	-																				
1 0	+	0																				
0 1	0	+																				
0 0	-	+																				
PST-RZ	NA	NA	$tB/2$	Same pairing and $C_n C_{n+1}$ as above for PST-L. $b_n = C_n \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$																		
Modified PST-L (MPST-L)	NA	NA	tB	<table><tr><th>Pair $a_n a_{n+1}$</th><th colspan="2">Polarities for $C_n C_{n+1}$</th></tr><tr><th></th><th>+ MODE</th><th>- MODE</th></tr><tr><td>1 1</td><td>+</td><td>0</td></tr><tr><td>1 0</td><td>+</td><td>-</td></tr><tr><td>0 1</td><td>-</td><td>+</td></tr><tr><td>0 0</td><td>0</td><td>+</td></tr></table> $b_n = C_n \begin{cases} 0 \leq t < tB \\ \text{[Change mode after transmission of 00 or 11 } a_n a_{n+1} \text{ pair.]} \end{cases}$	Pair $a_n a_{n+1}$	Polarities for $C_n C_{n+1}$			+ MODE	- MODE	1 1	+	0	1 0	+	-	0 1	-	+	0 0	0	+
Pair $a_n a_{n+1}$	Polarities for $C_n C_{n+1}$																					
	+ MODE	- MODE																				
1 1	+	0																				
1 0	+	-																				
0 1	-	+																				
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Modified PST-RZ (MPST-RZ)	NA	NA	$tB/2$	Same pairing and $C_n C_{n+1}$ table as above for MPST-L. $b_n = C_n \begin{cases} 0 \leq t < tB/2 \\ b_n = (0) \end{cases} \begin{cases} tB/2 \leq t < tB \end{cases}$																		
Duobinary	$a_n = 0$ $a_n = 1$		tB	If $a_n = 0$ then $b_n = 0$. If $a_n = 1$, then the polarity of b_n depends on the polarity of b_{n-k} for last $a_{n-k} = 1$. If a_n and a_{n-k} are separated by an even number of zeros, then $b_n = b_{n-k}$, if not, then $b_n = (-)b_{n-k}$.																		

Fig. 4-24. Multilevel Binary Pulse Coding Table

Fig. 4-24. Multilevel Binary Pulse Coding Table

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ELECTRON AND ION BEAM ACCELERATORS

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CHAPTER 13. ELECTRON AND ION BEAM ACCELERATORS

13.1 INTRODUCTION

A major portion of the advances in technology that allowed significant increases in the peak power of pulsed systems during the 1960s and 1970s was a direct result of the development of a new class of electron and ion beam accelerators.¹⁻⁶ These accelerators were developed in the 1960s to satisfy the requirement to simulate the radiation output of nuclear weapons in the laboratory.^{7,8,9} In the 1970s inertial confinement fusion research, with electrons and light ion beams as the energy source for driving the reaction, led to the invention of terawatt electron and ion beam accelerators.^{1,4,5,10-13} Requirements for improved radiography sources, studies of ion acceleration techniques, and the feasibility of directed energy weapons led to the development of different classes of high-current, linear induction electron accelerators.¹⁴⁻²¹ Potential commercial markets for the technology in the late 1980s and 1990s are the generation of electron and gamma-ray beams for sterilization of food, containers, hospital equipment, and sewage. Electron and ion beams are also used to cure plastics and modify metallic surfaces. Each new application requires a different set of beam parameters and leads to the development of unique new portions of the pulsed power technology. The new portions are always extensions of previous research.

In these accelerators, energy is initially stored over a relatively long time (seconds to minutes) and then delivered to a load in a short pulse duration (tens to hundreds of nanoseconds) at very high powers. In an efficient system, the power will increase inversely proportional to the change in pulse duration. The power amplification takes place in one or several stages that are connected in either series or parallel. In each stage the voltage, current, or impedance is changed to achieve the necessary power amplification. The energy storage section of the systems that are described in this chapter is a capacitor bank, either configured as a Marx generator or connected to the primary of a high-voltage, air-core transformer. Intermediate stages are usually high-voltage capacitors that can be configured as either a pulse-forming network (PFN) or a pulse-forming transmission line (PFL). These stages supply electromagnetic waves to electron or ion beam diodes, or, in the case of linear induction accelerators, to accelerating gaps that incrementally add energy to drifting beams.

The remainder of this chapter includes system and load descriptions with specific accelerator examples. In Section 13.2 a description of pulse power issues that are generic to all high-power accelerators is given. In Section 13.3 high voltage (~ 10 MV) accelerator designs are described. In Section 13.4 the design aspects of moderate-voltage (~ 1 MV), high-current accelerators are reviewed. Section 13.5 contains information on electron beam generators with microsecond pulse durations. Section 13.6 contains the physics of

high-power electron and ion beam diodes. The operating principles of high current, linear induction accelerators is covered in Section 13.7. Issues that are unique to repetitively operated accelerators are covered briefly in Section 13.8. The chapter concludes with a brief summary.

13.2 SYSTEM CONSIDERATIONS

Electron and ion beam accelerators consist of generic subsystems that are used to store energy, form it into a short pulse, and deliver it to a load. Figure 13-1 is a block diagram of an electron or ion beam accelerator indicating the principal purpose of the major subsystems. In these systems the energy is stored in 10^{-2} to 10^2 seconds and delivered to the load in 10^{-5} to 10^{-8} seconds depending on the particular application. Thus three to ten orders of magnitude, time compression, and power amplification are achieved. Energy storage for these systems varies from 1.5 kJ to 14 MJ. Output powers span an impressive range from a few gigawatts to 100 terawatts.

The energy storage system in virtually all of the existing pulse power systems is a capacitor bank. This bank can be configured as a Marx generator (see Chapter 2) or a parallel bank used in conjunction with a pulse transformer (see Chapter 11). Voltage multiplication is achieved with these energy storage systems. Energy storage capacitors are usually charged to voltages in the range of 10 to 100 kV. The output voltage of these systems range from 0.3 to 12 MV for Marx generators and from 0.1 to 3.0 MV for transformers.

These capacitor banks (see Chapters 2 and 10) are usually charged with direct-current power supplies (see Chapter 3). For some repetitively operated systems additional control or energy storage elements are needed to achieve the appropriate pulse rates. In the future it may be necessary to use rotating machines, MHD generators, magnetic flux compression devices, and other prime power systems to meet the high average power requirements for new applications.

After the energy has been stored and the voltage increased to the desired level, a pulse is generated with the desired pulse duration. Either PFLs (see Chapter 5) or PFNs (see Chapters 4 and 5) are used to generate these pulses.

For pulse durations greater than one microsecond, PFNs are used. Two options exist for PFN systems. In the first system the voltage multiplication takes place by erecting a Marx generator or charging one or more high voltage capacitors. The PFN is composed of these capacitors and other elements in the high voltage circuit. In the second case the pulse is formed with a low-voltage PFN that discharges its energy into the primary of a high-voltage pulse transformer with the secondary connected directly to the load. The second case is used for relatively low energy systems. The impedance of the PFN in the primary circuit of these low energy systems is $1/n^2$ times the load impedance where n is the primary to secondary voltage step-up ratio. As the system energy is increased, the PFN impedance rapidly becomes very small, and efficient operation becomes difficult to achieve, because of the finite inductance of the energy storage capacitors and

transformers. Therefore, higher energy systems are either composed of many parallel modules, or more frequently become the first case systems with a Marx generator for the major element of the PFN; and either other Marx generators or LC networks to form the remaining components for pulse shaping purposes.

If pulse widths less than 0.1 microseconds are desired, PFLs are included in the power train. The voltage and output impedance establish the appropriate dielectric for these PFLs. Figure 13-2 shows the system voltage versus PFL impedance, with the areas that are appropriate for transformer oil, water, and polyester film indicated.²² Because high-voltage (10 MV) Marx generators are insulated with transformer oil, the system design is simplified if transformer oil is also used for insulating the PFLs. To minimize electric field enhancement at the edges of the electrodes, these PFLs usually have coaxial, cylindrical electrodes. Equations 13-1 and 13-2 give the impedance, Z , and peak electric field, $E_{\max}$ in the dielectric for coaxial transmission lines with a charge voltage V_c ,

$$Z = \sqrt{\frac{\mu}{\epsilon}} \ln(b/a) , \quad (13-1)$$

$$E_{\max} = \frac{V_c}{a \ln(b/a)} . \quad (13-2)$$

In these equations ϵ and μ are the permittivity and permeability of the medium; b and a are the radii of the outer and inner cylinders respectively. For a fixed diameter outer cylinder, $E_{\max}$ at radius a

has a minimum when $\ln(b/a) = 1$ (i.e., $b/a = e$). For transformer oil, this condition corresponds to $Z = 41 \Omega$, an impedance that is in the range of interest for high-voltage systems. For moderate voltage, high-current systems, either water with its high dielectric constant or polyester film with its high-dielectric strength are usually used to achieve the required low impedance PFLs.

13.3 HIGH-VOLTAGE ACCELERATORS

As stated earlier, high-voltage accelerators were developed as gamma-ray simulators.²³⁻³¹ Hermes II²³ is an example of a high-voltage accelerator that has been reliably operating at Sandia National Laboratories since 1969. It produces a 12 MV, 100 kA, 87 ns electron beam that is converted into bremsstrahlung radiation producing 5000 rads at one meter from the tantalum converter. Figure 13-3(a) is an artist's sketch and 13-3(b) is a cross-section drawing of this accelerator--note the two men in Fig. 13-3(a) that indicate the large size of this accelerator. The major components of the system are the Marx generator, the Blumlein PFL, and the electron beam diode.

13.3.1 High-Voltage Marx Generator

The energy is transferred to the Marx generator capacitors on a very slow time scale (~1 minute) compared to the remainder of the accelerator energy transfer times ($< 5 \mu\text{s}$). In addition the energy is stored in these capacitors at a low voltage ($< 100 \text{ kV}$). The Marx generator increases the voltage to several megavolts and then

efficiently transfers that energy to the next stage in a minimum amount of time. The voltage multiplication is achieved by connecting the capacitors in series with spark gaps. This process is referred to as the Marx generator erection. The Marx generators are configured to erect quickly (typically about 200 ns) to prevent excessive energy losses in the charge resistors. In high-voltage Marx generators with a large number of stages, the erection is initiated by externally triggering only a few spark gaps to minimize energy losses in the resistor coupling in the trigger signals.

If we assume the triggered spark gaps connected m capacitors in series and these capacitors are charged to V_0 , then a voltage mV_0 will exist at a single point in the Marx generator. If the Marx generator is properly configured, the mV_0 voltage can be used to self-breakdown or self-trigger the remaining spark gaps that need to close to fully erect the Marx generator. (See Chapter 2 and References 13-23, 13-28, 13-29, 13-30, 13-32 and 13-42 for discussions of Marx generator configurations.)

Each spark gap has a statistical spread in its breakdown voltage. In some circumstances a sufficient number of gaps can break down during charge to cause the Marx to erect. These circumstances are called prefires. Because of the large number of spark gaps in a Marx generator, the Marx generator self-breakdown distribution is considerably broader than the breakdown distribution of a single-spark gap.

For reliable operation without prefires, the Marx generator must erect at voltages substantially below the mean self-breakdown voltage of each spark gap. The remaining Marx generator concerns are that the capacitors, resistors, and spark gaps have sufficient mean time to failure and that arcing does not occur within the Marx generator.

During the design of Marx generators, attention must be given to the electric fields, the type of insulation, and the electrical circuit throughout the accelerators, since arcing can occur between components during charging, between components or stages (the common grouping of components) during erection, between high voltage stages and ground after erection. In addition, transient voltage pulses generated during the discharge of the PFL can cause arcing between stages on the high voltage end of the generator. Although this transient voltage is applied for a much shorter period of time, it does not divide linearly across the Marx generator due to the capacitance between stages and from the stages to ground.

In high-voltage accelerators, the PFL insulation is transformer oil. The dielectric strength of transformer oil under microsecond pulse conditions varies as $t^{-1/3}$, where t is the application time of the high voltage (see Section 13.3.2). To minimize t , the Marx generator inductance must be minimized. The dominant factor affecting the inductance is the spacing between components and from the components to ground. Therefore, for fast energy transfers, one minimizes these dimensions thereby increasing the electric fields in the Marx generator.

The designer must satisfy this conflicting set of requirements to reduce the PFL charge time and to prevent arcing to optimize the performance of the Marx generator in an accelerator. For additional information on components see Chapters 6 and 17. For design techniques see Chapter 2, Refs. 13-22 to 13-31, and the description of the accelerators referenced in Section 13.4.

The Hermes-II Marx generator³⁰ contained 372, 0.5 μF capacitors arranged in 93 positively charged stages and 93 negatively charged stages. Each stage had two capacitors in parallel. The generator was operated at 70 kV charge, producing a 13 MV open circuit voltage. The equivalent series capacitance of the fully erected Marx generator was 5.3 nF, and its inductance was 80 μH . This generator had 93 spark gaps. The generator was arranged in an $n = 3$ configuration^{23,30} and operated satisfactorily at 43% of the average self-breakdown voltage of the spark gaps.

The Hermes-II Marx generator charges the Blumlein PFLs in approximately 1 μs . Figure 13-4 is a simplified equivalent circuit of the erected Marx generator and the PFLs. A Blumlein PFL consists of two transmission lines that are charged in parallel and discharged in series. In Figure 13-4, C_2 is the Blumlein PFL capacitance; C_m is the Marx generator capacitance; and L_m is the Marx inductance. V_{cm} is the Marx generator open-circuit voltage. Equation 13-3 describes the charging voltage waveform of the Blumlein PFLs with the circuit shown,

$$V_{C_2} = V_{cm} \frac{C_m}{C_m + C_2} (1 - \cos \omega t) . \quad (13-3)$$

In this equation $\omega = 1/\sqrt{L_m C_s}$ where C_s is the equivalent series capacitance $C_m C_2 / (C_m + C_2)$. The peak voltage occurs when $\omega t = \pi$ and $1 - \cos \omega t = 2$. The voltage gain of this circuit is usually referred to as the ringing gain, RG. RG is given in Eq. 13-4 and is obtained by finding the ratio of V_{C_2} to V_{Cm} at $\omega t = \pi$,

$$RG = \frac{2C_m}{C_m + C_2} . \quad (13-4)$$

In Hermes II the PFL capacity is 5.6 nF, giving $RG = 0.97$. The capacitance between the high voltage end of the Marx generator and ground is also charged to the full Blumlein voltage. This adds 0.5 nF to C_2 and reduces RG to 0.93. When the resistive losses of the Marx generator are included, the peak voltage is reduced to 11.2 MV. The Blumlein switch is usually closed at 90% of the peak charge voltage, giving a 10 MV PFL charge voltage.

13.3.2 Pulse-Forming Transmission Line

The peak voltage, charge time, and dielectric strength of the insulating material determine the dimensions of the PFL. The dielectric strength of transformer oil and water are dependent on the area of the electrodes and the time the voltage is applied across these electrodes. J. C. Martin^{22,32} has found an empirical

relationship (Eq. 13-5) that describes this dependency for areas from 10^{-4} m^2 to 500 m^2 and for effective times from 0.05 to 1 μs ,

$$E_B t_f^{1/3} A^{1/10} = k . \quad (13-5)$$

In this equation E_B is the peak macroscopic electric field in the dielectric; A is electrode area that is stressed above 90% of E_B ; and t_f is the effective time the voltage is applied. For a $1-\cos \omega t$ waveform the effective time is defined as the time the voltage is greater than 63% of its peak value. The k on the right side of Eq. 13-5 is a constant for a given liquid and voltage polarity as given in Table 13-1. The values of k in Table 13-1 apply for all quantities in Eq. 13-5 in mks units. The more general usage for this formula is with mixed units: E_B in MV/cm, t in μs , and $A \text{ cm}^2$, in which case k is 0.3 for the positive polarity of water, 0.6 for the negative polarity of water, and 0.5 for transformer oil.

TABLE 13-1. Constants for Equation 13-5.

<u>Dielectric</u>	<u>Polarity</u>	<u>$\frac{k}{\epsilon}$</u>
water	+	1.2×10^5
water	-	2.4×10^5
transformer oil	+, -	2×10^5

Other authors have proposed an area exponent of 0.058 for water with 0.4×10^4 positive polarity constant.³³ There is little difference between these two forms of this equation with areas in the range of 0.1 to 1 m². Many of the water-dielectric machines discussed in Section 13.4 have PFL electrode areas in this range. The smaller exponent for the area term gives higher breakdown fields for larger areas than one predicts from Eq. 13-5. At least one accelerator has been operated at electric fields that are predicted as the breakdown level with this less conservative formulation.³⁴ Equation 13-5 is intended to predict the electric field at which dielectric breakdown will occur 50% of the time. The operating electric field of the PFLs is reduced below this value to lower the probability of breakdown to a suitable fraction of the pulses. This safety factor with Eqs. 13-2 and 13-5 will determine the dimensions of the PFLs. For Hermes II $0.8 E_B$ was used, resulting in the three coaxial cylinders with 2.2 m, 3.6 m, and 4.9 m diameters. The impedance of these two transmission lines are 21 Ω and 11 Ω respectively.

The length of the PFL is determined by the pulse duration, T , and the electromagnetic wave transient time (reciprocal of the wave velocity), $s = \sqrt{LC}$, the inductance and capacitance per unit length in the dielectric medium of that PFL (see Chapter 5). Because the wave travels the length of the PFL twice during the output pulse, the PFL length is $T/(2s)$. The relative dielectric constant of transformer oil is 2.1 resulting in a 4.95 ns/m wave transient time. The Hermes-II PFLs are 8.8 m long to produce an 87 ns pulse. The major portion of

existing high-voltage electron beam generators have 50 to 120 ns pulse durations.

13.3.3 PFL Switches

The rise time of the output pulse is usually an appreciable fraction of the pulse duration. It is limited by the high-voltage switch in the PFL and the load resistance, inductance, and capacitance. In the high-voltage generators, the PFL switch is either a self-breaking, single-channel, oil-dielectric spark gap or a triggered, multichannel oil-dielectric spark gap. Equation 13-5 is used to determine the spacing, d , of the electrodes in a self-breaking switch. This spacing is usually set for closure to occur at 85% to 90% of the peak charging voltage to minimize the time the voltage is across the PFL electrodes without strongly effecting the energy transfer efficiency. The breakdown voltage in these spark gaps with nearly uniform electric fields has approximately a 5% standard deviation. For a 1 μ s charge time, as Hermes II has, this voltage breakdown variation results in a ± 50 ns jitter in output pulse arrival at the load.²³

The Aurora electron beam accelerator^{24,35} produces four 12 MV, 400 kA, 125 ns electron beams. This accelerator has four, 20 Ω , 125 ns PFL Blumleins that are 7 m in diameter and configured as shown in Fig. 13-5. The pulses from these PFLs have to be synchronized within 20 ns to produce the desired radiation pulse. This

synchronization is achieved with three-electrode, triggered, oil-dielectric spark gaps that produce 4 to 6 arcs on each closure. The trigger electrode is a sharp-edged disk that is located about 17% of the main electrode distance from the ground electrode, as shown in Fig. 13-6. It is capacitively biased such that there is only a small enhancement of the mean electric field at the edges of the disk during charging of the PFLs. The gas switch is a SF_6 -insulated spark gap that can be triggered by a 200 kV pulse. When the gas switch closes, the voltage polarity on the trigger disk reverses in about 200 ns. This reversal time is determined by the disk capacitance and the circuit inductor. This rapid change in voltage on the trigger disk and the large field enhancement on the disc edge results in closure of 4 to 6 channels with a 10 ns rms jitter. With the multichannels this switch can carry the 1 MA current and produce the desired pulse rise time.^{24,35}

When an arc forms in a spark gap, the current is initially limited by both the arc conductivity and by the arc and electrode inductance. The time required to increase the arc conductivity to a value where the resistive voltage drop across the switch is small with respect to the PFL voltage is usually referred to as the resistive phase time,³² τ_R . Equation 13-6 is an empirical formula expressing the resistive phase time in terms of the mean electric field, E , in the dielectric medium between the electrodes at peak voltage and the impedance, Z , of the circuit supplying energy to the arc,

$$\tau_R = \frac{232}{E_B^{4/3} Z^{1/3}} \quad (13-6)$$

All quantities are in mks units. For a multichannel spark gap with n channels, Z is replaced by nZ because each spark is supplied energy by a portion of the PFL with nZ impedance, i.e., (n PFLs with impedance nZ in parallel have an impedance Z). The number of channels that will close is determined by the spread in breakdown voltage of a single spark and the rate of fall of voltage across the spark.

Experimentally it has been shown that if an arc closes after the voltage across the arc has dropped to 90% of its peak value, this arc carries a small fraction of the total switch current.³⁶ Equation 13-7 is an expression,

$$2\sigma_V V/(dv/dt)^{-1} < 0.1 (\tau_R + \tau_L) + 0.8 T_S, \quad (13-7)$$

from which the number of channels that will close can be estimated.

In this expression σ_V is the standard deviation of the breakdown voltage (V), τ_L is the inductive time constant for the switch circuit, L/nZ , and T_S is the average electromagnetic wave transient time between arcs if multiple arcs form. The 0.8 accounts for the nonuniformity of spacing between the multiple arcs. For highly field enhanced electrodes, σ_V is 1% to 2% of the charging voltage V ; τ_R , τ_L , and T are functions of n . Although the inductive rise time frequently dominates for single-channel spark gaps, the resistive phase quickly becomes the dominant factor for multiple-channel gaps because of the

$n^{-1/3}$ dependence as compared to the $1/n$ for the inductive time constant. The 10 MV single-channel spark gap in the 22 Ω PFL in Hermes II has a 5.1 ns resistive time and a 19 ns inductive time constant.²³

13.3.4 High-Voltage Diode and Vacuum-Insulator Flashover

The other major contributing factor to the rise time of the load current is the inductance of the electron or ion beam diode. Because the diode inductance is proportional to the cross-sectional area between the high voltage and ground electrodes and inversely proportional to the mean radius of these electrodes, this area/radius ratio must be minimized to achieve fast-rising pulses. Because electron or ion beam diodes require a vacuum to function, this region includes a solid interface between the liquid dielectric of the PFL and the vacuum. The electric field at which flashover of this interface occurs is relatively low and usually controls the dimensions of the above area.

Anderson and Brainard³⁷ have described a model of this flashover process. In this model, electrons are field-emitted at the triple junction and strike the solid-dielectric interface. A greater number of secondary electrons leave the surface and charge it positively. This positive charge enhances the probability of subsequent electrons to strike the surface. These electrons desorb gas from the insulator and ionize this gas to form a conducting plasma. The angle of the initial electric field with respect to the dielectric surface strongly

affects this process. Experimentally it has been shown that flashover strength varies with this angle as shown in Fig. 13-7.³⁸⁻⁴⁰ The shape of this curve is similar for many different dielectrics, but the magnitude of the flashover strength varies considerably; and the exact angle at which minimums and maximums occur varies slightly. J. C. Martin⁴¹ has also shown that the flashover strength for insulators with a positive 45-degree angle, as defined in Fig. 13-7, varies with the area of the dielectric and the duration of the voltage pulse, as expressed in Eq. 13-8,

$$E_B t_f^{1/6} A^{1/10} = 7 \times 10^5 . \quad (13-8)$$

(The original version of Eq. 13.8 used a mixed but convenient set of units with E in kV/cm, t in μ s, and A in cm^2 . In this case the constant is 175 rather than 700,000, as given here for mks units.) In Eq. 13-8 the effective time is the duration that the voltage is greater than 89% of the flashover value. This is a higher percentage of the peak value than used in Eq. 13-6. The different values result from finding an equivalent square wave representation for the different root dependence.¹³⁻³² Equation 13-8 gives the expected value of E for 50% probability of flashover, and insulators must be designed with operating stresses lower than this number. The data used to infer this relationship was for polymethyl methacrylate and cross-linked polystyrene with areas covering the range from 10^{-4} m^2 to

10^2 m^2 ($1 \text{ cm}^2 - 10^6 \text{ cm}^2$) and times from 3 ns to 5 μs and with the positive 45-degree angle between the equipotentials and the surface. These materials have relatively high flashover strength and good mechanical strength. They are the most commonly used dielectrics in high-voltage accelerator vacuum interfaces.

The operating electric field for insulators with their surfaces at a 45-degree angle to the electric field and with the large areas that are required for high-voltage accelerators is usually in the 5 to 8 MV/m (50 to 80 kV/cm) range. This field results in 1.25 to 2.0 m long insulator in a 10 MV diode. With a single insulator and this long length, it would be very difficult to maintain the 45-degree angle between the electric field and the insulator and achieve a uniform electric field along the complete length of the insulator. Therefore a stack of short insulators (usually in lengths between 1.3 to 10.8 cm) separated by thin metal grading rings (usually 0.25- to 1.3-cm thick) is used for these interfaces, with a sufficient number of insulators to obtain the desired length. Grading rings force the desired angle between the field and the insulator. The high voltage and ground return electrodes are shaped to achieve nearly the same mean electric field on each insulator.

The Hermes-II diode insulator stack is 2 m long and 1.8 m in diameter (see Fig. 13-3). The cathode shank is a 15-cm diameter tube that extends through the insulator stack and a 1.2 m diameter, 1.2 m long cylinder that is used to attach the vacuum pump. As shown in Fig. 13-3, the return current path outside of the insulator stack is a

4.8 m diameter hemisphere. The approximate inductance for this configuration is $1.7 \mu\text{H}$. The diode is usually operated with 100Ω impedance giving a 29 ns, 10 to 90 rise time. With an input voltage waveform to the diode that has a 47 ns, 10 to 90 rise time, the current rise time is about 55 ns.

13.3.5 Prepulse in High-Voltage Generators

In an ideal pulsed power system the load voltage would be zero until the main pulse arrived. It would rise to a peak value very rapidly, have a flat top, and then decay rapidly. Because electron and ion beam diodes operate in a transient mode with conditions continually changing (see Section 13.6), any voltage applied between the anode and cathode prior to the main pulse can degrade the beam parameters. An analysis of the equivalent electrical circuit of an accelerator reveals that a prepulse voltage is always applied to the load during the charging of the PFLs. In a Blumlein PFL this voltage is due to an imbalance in the charging circuits of the two PFLs. Figure 13-8 is a more detailed charging circuit for a Blumlein PFL than that given in Fig. 13-4. The inductor, L_3 , in series with capacitor, C_3 , isolates that PFL high voltage electrode from ground when the high-voltage switch is closed. Since this inductor is in parallel with the load, the voltage across it during charging of the PFLs also appears across the electron beam diode. From Fig. 13-8, it is apparent that minimizing the ratio of L_3/L_M minimizes the prepulse. The inductance of L_3 must be large enough that the time constant,

L_3/Z_{B2} , is much greater than the pulse duration where Z_{B2} is the impedance of the PFL shown as capacitor C_2 in Fig. 13-8, or the amplitude and shape of the main pulse will be seriously degraded.

Because the circuit in Fig. 13-8 is a bridge circuit, the voltage across the load could be reduced to zero, by balancing the bridge. However, point A in Fig. 13-8 is the tank wall that contains the accelerator and is the nominal ground for the circuit. To balance the bridge one must connect L_3 and another inductor in series with the low-voltage end of the Marx generator. If this inductance is adjusted to completely balance the bridge circuit, the prepulse will be minimized but never becomes zero because of the effects of stray capacitance from the Marx generator to the tank wall. Figure 13-9 is a circuit diagram of Hermes II, including stray capacitance from the Marx generator to the wall. The Marx generator is represented by four sets of capacitors, inductors, and resistors in series. The stray capacitances to ground are shown with voltages V_1 , V_2 , V_3 , and V_4 across them. The current used to charge these capacitances flows through the inductor in series with the Marx generator but not through the inductor in the other leg of the bridge, effectively unbalancing the bridge. The amplitude of the prepulse is strongly affected by the amount of charge that is supplied to the stray capacitance near the output end of the Marx generator.²⁹

In Hermes II the Marx generator was triggered at the stage where two-thirds of the peak voltage appeared when the generator was erected. With this scheme the voltage across the stray capacitance at

the output end of the generator cannot exceed one-third of the open-circuit voltage during erection. The isolating inductor, L_3 , was minimized but attempts were not made to balance the bridge. The prepulse amplitude is 5% of the output voltage. As shown in Fig. 13-3, L_3 is a pipe located above the tank floor. During charge of the PFLs this pipe is a 6.5 μH inductor. During the discharge of the PFL, it appears as a 100 Ω transmission line that is in parallel with the 12 Ω PFL, giving an 11% loss in voltage across this PFL for the full duration of the pulse. The net result is a 5.5% loss in output voltage due to this charging inductor.

13.4 MODERATE-VOLTAGE, HIGH-CURRENT ACCELERATORS

13.4.1 Coaxial Pulse-Forming Lines for Moderate Voltage Generators

In this class of beam generators^{34,42-58} the primary emphasis is on achieving high currents (0.1 to 15 MA) at moderate voltages (~ 1 MV) leading to relatively low impedance PFLs. From Eq. 13-1, it is apparent that either ϵ_R must be large or $\ln b/a$ must be small for low impedance coaxial PFLs. As b/a decreases, E increases (see Eq. 13-2), and it is often necessary to increase the inner cylinder radius, a , to keep those fields below the breakdown limit. As a increases, the area term in Eq. 13-5 increases and the allowable breakdown field decreases, resulting in a further increase in a . The dimensions can be determined through an iterative process. The diameters of the coaxial cylinders are quite large for low impedance ($\sim 1 \Omega$) accelerators.

The high dielectric constant liquid that is predominantly used in these low impedance PFLs is deionized water. The dielectric constant ϵ_R of water is 81 at room temperature, and it is nearly constant for frequencies from 1 to 100 MHz. The resistivity, ρ , of water can be kept in the $10^4 - 5 \times 10^4 \Omega - m$ range with simple deionizing systems. Resistivities in this range are acceptable for most applications, where the charging times are $\sim 1 \mu s$. The equivalent circuit for a water-dielectric PFL during the charge cycle is a capacitor in parallel with a resistor, as shown in Fig. 13-10. For a coaxial transmission line the resistance and capacitance are given by Eqs. 13-9 and 13-10, where L is the length of the transmission line,

$$R = \frac{\rho \ln b/a}{2\pi L}, \quad (13-9)$$

$$C = \frac{\epsilon 2\pi L}{\ln b/a}. \quad (13-10)$$

Because the geometrical factors are set by the desired impedance, voltage and pulse width, ρ must be increased until the resistive losses are at an acceptable level. It is readily seen from Eqs. 13-9 and 13-10 that the RC time constant for the PFL is $\rho\epsilon$.

The intrinsic resistivity of water at room temperature is $1.8 \times 10^5 \Omega - m$ yielding a $128 \mu s$ time constant. If the water is chilled to near its freezing point, ρ increases to $8.4 \times 10^5 \Omega - m$ and ϵ_R increases slightly to 90, resulting in a $670 \mu s$ time constant.⁵⁹ The dielectric strength decreases at a much slower rate than indicated

by Eq. 13-5 for pulse durations greater than a few microseconds.^{59,60} Experiments have shown that PFLs with areas up to 10 m^2 and a chilled water dielectric can be operated reliably with 5 MV/m electric fields with 100 μs charge times.⁶¹ Other high dielectric constant liquids that have been used, but much less frequently, are ethylene glycol ($\epsilon_R = 38$) and glycerine ($\epsilon_R = 40$).^{22,32,60} Ethylene glycol has essentially the same dielectric strength as water. The constants in Eq. 13-5 for water are applicable for ethylene glycol. The resistivity of ethylene glycol increases with decreasing temperature, giving time constants in the millisecond range with subzero degree centigrade temperatures. Experiments show that ethylene glycol PFLs can be operated reliably with 5 MV/m electric field strengths in this time regime.⁶² These properties of chilled dielectrics may result in considerable reduction in complexity of charging systems in future applications.

A high dielectric constant increases the electromagnetic transient time in the liquid (45 ns/m for water). The PFLs with the high dielectric liquid are substantially shorter than with oil-dielectric. Sketches of the HYDRA electron beam accelerator are shown in Fig. 13-11.⁴² This accelerator has two PFLs that are charged by one Marx generator. Each PFL was designed to provide energy for a 1 MV, 0.5 MA, 80 ns pulse of electrons. HYDRA became operational in 1972 and has been used for a large variety of experiments. The 4 Ω , 80 ns PFLs are only 1.35 m long compared to the 8.1 m for the Hermes-II PFLs.

The PFLs are charged to 3 MV, producing a 1.5 MV output pulse that is subsequently altered to a 1 MV, 0.5 MA pulse with a transmission line transformer.^{63,64} This transformer is a coaxial transmission line with the outer conductor tapered such that the impedance, given by Eq. 13-1, is 4 Ω at the switch end and 2 Ω at the output end (see Fig. 13-11b). This type of impedance transformer can cause a droop on the top of the output voltage pulse.⁶⁵ The percent droop, D, is a function of the voltage transformation ratio, V_x/V_o , the length of the transformer, x, the pulse duration, T, and the electromagnetic transient time, s, as given by Eq. 13-11. Thus the allowable droop and transformation ratio establishes the dimensions of this type of transformer.

$$D = 50 \left(\ln \frac{V_x}{V_o} \right)^2 \frac{T}{xs} \quad (13-11)$$

13.4.2 Switches for Coaxial PFLs

Spark gap switches with gas, liquid, and solid dielectric insulation are used in this class of accelerator. The 3 MV switch in HYDRA is an SF₆-insulated spark gap that has been operated either with two electrodes or in a trigatron configuration.^{42,66} In the trigatron mode the switch required a 200 kV trigger signal and produces a 2 ns rms jitter.⁶⁶ The inductance of this switch is 160 nH, producing a 20 ns $L/2Z_o$ rise time. The $2Z_o$ is used because the current flows from a PFL with a Z_o impedance through the switch into a transmission line

with Z_0 input impedance. The resistive phase³⁶ for gas switches is given in Eq. 13-12, where ρ_1 is the density of the gas dielectric, ρ_0 is air density at standard atmospheric conditions, and E_B , Z , and τ_R are in mks units. The impedance, Z , is the series combination of charged PFL and load impedance, $2Z_0$ for a PFL with impedance Z_0 discharging into a matched load.

$$\tau_R = \frac{8.8 (\rho_1/\rho_0)^{1/2}}{E_B^{4/3} Z^{1/3}} \quad (13-12)$$

For PFLs with single-channel gas-dielectric spark gaps, the inductive term usually dominates. The 10 to 90 rise time of the PFL voltage pulse in HYDRA is 44 ns. Four trigatron switches were used in the 1 MV, 0.5 MA, 50 ns Gamble-I accelerator⁵⁶ to reduce the rise time to about 12 ns. Five to ten gas spark gaps are triggered simultaneously in the water-dielectric Blumlein in the 1 MV, 1 MA, 50 ns Angara-V module⁵³ at the Kurchatov Institute.

Single-channel water switching was used on the 2 MV, 1 MA, 80 ns Gamble-II accelerator⁴³ at the Naval Research Laboratories. The 1 MV, 0.5 MA, 120 ns OWL accelerator⁴⁴ at the Physics International Company has a triggered water-dielectric switch very similar to the one described for Aurora. Higher power accelerators of this type, such as Python⁴⁵ at Physics International and the Blackjack machines^{34,46} at Maxwell Laboratories, use several stages of pulse compression with self-breaking water switches at the output of each stage. The number

of channels increases rapidly with the decrease in charge time for each subsequent stage.

Another switching technique that has been used on PFLs with charge voltages up to 0.5 MV is a stabbed-solid-dielectric switch. This type of switch was developed for solid-dielectric, strip transmission lines and has both multichannels and high electric fields to give very fast rise times.⁶⁷ It was adopted for the PFL switch in the 0.4 MV, 80 kA, 10 to 60 ns Nereus accelerators.⁵⁰ In the Nereus switch, a 0.32-cm thick slab of polyethylene is stabbed to a prescribed depth that determines the switching voltage. This polyethylene sheet is inserted between the main switch electrodes. About ten channels close when the PFL voltage reaches 0.50 MV in 550 ns. With ten channels and 160 MV/m electric fields in the polyethylene, the 10 nH inductance of the electrodes controls the rise time. If the two-ohm Nereus PFL was discharged into another two-ohm transmission line, the 10 to 90 rise time would be 5.5 ns. The output rise time for all of the above switches can be estimated quite accurately using the techniques outlined with the description of the Aurora switch in Section 13.3.3 of this chapter.

13.4.3 Strip-Transmission Line PFLs

When $a \gg b$, only a small portion of the total volume inside the outer cylinder of the coaxial PFL is used to store energy, and other PFL geometries may be more appropriate. If the PFL impedance is less than 30% of the minimum electric field value, other geometries such as

strip transmission lines should be considered (see Chapter 5). The impedance of a strip transmission line is given in Eq. 13-13, where d is the separation between electrodes, and w is the width of the electrodes,

$$Z = \sqrt{\frac{\mu}{\epsilon}} \frac{d}{d + w} . \quad (13-13)$$

The $d + w$ term is a result of the fringing fields associated with the edges of these transmission lines and of course becomes w for transmission lines with $w \gg d$.

These PFLs are either operated in a triplate geometry, as shown in Fig. 13-12, with two transmission lines in parallel, or with five electrodes configured as two Blumleins in parallel. These configurations have the high-voltage electrodes between two ground electrodes, avoiding energy losses into stray capacity to ground. For liquid dielectric PFLs, d is determined using Eq. 13-5. The electric fields are greater on the edges of the electrodes. The electric fields can be calculated with a numerical code, such as the one described in Ref. 13-68, to determine the shape of the electrode edge that will give acceptable electric fields. Solid dielectric PFLs are usually insulated with either polyethylene⁶⁹ or polyester film (Mylar),^{54,55} and the bulk breakdown strength of these solids, in conjunction with edge-grading techniques, determines d .

13.4.3.1 Liquid-Dielectric Strip Pulse-Forming Lines.

Proto I,^{70,71} Proto II,^{72,73} PBFA I,⁷⁴⁻⁷⁶ and Eagle^{77,78} are examples of accelerators with liquid-dielectric strip PFLs. The first three of these accelerators were developed in the Inertial Confinement Fusion (ICF) Program, which has the dual requirements of several terawatts of output power and a short pulse duration. These ICF accelerators are modular systems with the output power of each PFL less than 1 TW. Proto I is a 2 MV, 0.54 MA, 24 ns accelerator with transformer oil-insulated PFLs configured as two Blumleins in parallel (Fig. 13-13). It has six sets of these Blumlein PFLs feeding two diodes. The PFL switches are triggered, multichannel, oil-dielectric rail gaps.⁷⁹ Ten simultaneous channels are generated across the four 1.2 m width of each of the twelve switches producing 8 ns, 10 to 90 rise time pulses at the load. This rise time includes effects from the diode inductance. Each of these switches has approximately 1 ns jitter with a trigger pulse amplitude about half of the peak Blumlein PFL charge voltage.

Proto II is a 1.5 MV, 5.5 MA electron beam generator with either a 20 or 50 ns output pulse. Figure 13-14 is a sketch of this accelerator. The eight Marx generators transfer their energy to 16 intermediate storage capacitors (ISCs) in 750 ns. These ISCs transfer their energy through trigatron switches to the 50 ns PFLs in 220 ns. Three channels close in each of the 16 water-dielectric, self-closing transfer switches. The rms jitter of the trigatrons and these water switches is about 2 ns. In the 20 ns mode the 50 ns PFLs are used as

a second set of ISCs and transfer their energy to another set of PFLs in 60 ns. Each switch on these PFLs closes with about 20 channels giving a 6 ns, 10 to 90 rise time for the 340 kA pulse.^{80,81} The 30 TW PBFA-I accelerator shown in Fig. 13-15 is an extension of Proto-II technology, but each of the 36 modules has two 1.3 MV triplate PFLs connected in parallel. The output power of these PFLs is about 0.85 TW with a 40 ns pulse duration. The vacuum insulator of each module is 7 m from the center of the accelerator. The energy from each module is transmitted to the central diode region through 7-m long magnetically insulated, vacuum transmission lines (see Chapter 15). In this accelerator all 36 modules can be operated separately or tied together in parallel at the diode.

Eagle and Double Eagle are modular accelerator systems developed by the Physics International Company for the Defense Nuclear Agency. The accelerators are used for radiation effects studies. The output parameters of the single-module Eagle are 3 MV, 1.6 MA, 75 ns, and 4.6 TW. Eagle is designed to allow up to twenty of these high-power modules to be arranged to feed a single diode in future machines. Double Eagle has two of these modules with output parameters of 7 TW, 1.7 MV, 5.6 MA, 90 ns, and 510 kJ. The Eagle module consists of a 650 kJ Marx generator that charges a triplate water dielectric transfer capacitor in 1.3 μ s. The energy is then transferred through a high voltage gas switch to the first of two triplate, water dielectric PFLs. These PFLs have water-dielectric, self-closing spark gaps. After the pulse is formed it is transported to the diode with

0.3- Ω triplate transmission line transformer. Since the radius of the vacuum insulator stack and the width of the transmission are both large to accommodate the high-module current, it is difficult to obtain good azimuthal symmetry to the current flow into the diode. A transit time equalizer has been added to the center portion of the transmission line transformer near the point where it attaches to the diode to alleviate this feed symmetry problem. The transit time equalizer is formed by bending this center portion of the horizontal transmission lines in the vertical plane. This bend increases the transit time to the load until it is nearly equal to the transit time of the outer portions of the transmission line.⁷⁸

13.4.3.2 Solid-Dielectric Strip Transmission Lines. Two types of accelerators have been built with solid dielectric insulated PFLs. In the first, 200 kV Blumlein PFLs were stacked in series to deliver 1 MV to 4 MV pulses to a diode. In the second type, these PFLs were used to produce very low impedance systems with 0.3 to 0.6 MV output pulses. The first type was developed in the early 1960s by J. C. Martin's group at the Atomic Weapons Research Establishment (AWRE) in Great Britain. Examples of this type of accelerator are the 2.0 MV, 0.2 MA, 20 ns SMOG device⁶⁷ and the 4 MV, 0.15 MA, 30 ns SPASTIC accelerator jointly designed and built by the AWRE and Sandia National Laboratories. Figure 13-16 is a sketch of the PFL arrangement in this type of accelerator. Prior to switching the PFLs, the stray capacities indicated by dotted line connections in the Fig. 13-16(b)

are uncharged. When the PFLs are connected in series, these stray capacities must be charged. The energy that is used to charge these capacities is not delivered to the load and is a strong factor in determining the overall efficiency of a stacked transmission line accelerator. If the energy stored in the Nth-stage stray capacitance is small with respect to the energy stored in N PFLs, its value is given by Eq. 13-14, where V_o is the stage voltage,

$$E_n = \frac{1}{2} C_s (N V_o)^2 . \quad (13-14)$$

If one assumes that the stray capacitances are nearly constant, the energy stored in the stray capacitance increases as N^2 while the available energy from this PFL only increases linearly with N, as stages are added. Therefore this energy loss must limit the amount of voltage multiplication that is possible.

It is difficult to calculate the value of these stray capacitances. One way to obtain a reasonably good estimate for their value is to consider the difference of the impedance of a transmission line with Nd spacing, as compared with NZ_o , where Z_o is the impedance of a single line. Since these PFLs are usually 0.5 to 2 m wide, w, and the insulation thickness, d, is 0.5 to 2 cm. The $w + d$ term in Eq. 13-13 is essentially w for a single PFL. When 10 to 20 Blumleins are stacked in series, the total spacing is 10 to 80 cm, an appreciable fraction of w. The impedance calculated using these dimensions is the parallel combination of the loss impedance,⁶⁷ Z_B

(sometimes referred to as back impedance), and the output impedance NZ_o . From these two values one can determine Z_B and the expected voltage gain. The open-circuit voltage gain of this circuit becomes,

$$G = \frac{2N}{1 + NZ_o/Z_B}, \quad (13-15)$$

where G is the stacked-line output voltage divided by a single-line output voltage.

The 0.55 MV, 1 MA, 50 ns SNARK accelerator⁵⁵ developed at the Physics International Company and the 0.3 MV, 0.7 MA, 50 ns SLIM accelerator⁵⁴ developed at Sandia National Laboratories are examples of systems with low impedance, solid dielectric PFLs. Several layers of 0.025-cm thick polyester film are used to insulate these PFLs. The electric fields at peak charge voltage in the polyester film were about 1.6 MV/cm and the charge time was 1 to 2 μ s. The electrodes are thin copper sheets, as shown in Fig. 13-17. With the large field enhancements at the edges of the electrodes and the high mean electric fields, tracking around the edge of the insulation can be a limiting factor in this PFL design. The electric fields at the edges of the copper sheets can be reduced with field grading techniques to a level where the breakdown strength of the surrounding fluid is not exceeded and thereby preventing tracking.

One technique that has been used to reduce the fields in these PFLs is to immerse the PFLs in a resistive copper-sulfate solution. The area above (or below) each electrode in Fig. 13-17 becomes a

resistor and the voltage divides between a resistor above (or below) a high-voltage electrode and a ground electrode. By adjusting the length of the insulation margin (m in Fig. 13-17), the electric field in the copper-sulfate can be reduced below the breakdown level. The concentration of the copper-sulfate solution is adjusted to give a resistivity that is sufficiently high for the losses in these resistors to be acceptable during the charging of the PFL. This sets the lower limit on the resistivity of the solutions.

When the PFLs discharge, these resistors and the polyester film form an RC transmission line, as indicated in Fig. 13-17, with R_x and C_x the resistance and capacitance per unit length. The voltage as a function of time along this transmission line is determined by the solution to the diffusion equation, as given in Eq. 13-16,

$$\frac{dV}{dt} = - \frac{1}{R_x C_x} \frac{d^2 V}{dx^2} . \quad (13-16)$$

The solution to this equation gives a maximum electric field of $V_o/2 \cdot (R_x C_x / \tau)^{1/2}$, where V_o is the applied voltage and τ is the rise or fall time of the voltage pulse. Thus, the diffusion time constant must be comparable to the PFL rise time, or the electric fields at the electrode edges will exceed the breakdown strength during discharge. This time constant sets the upper limit on the copper-sulfate resistivity.

The switches in the PFLs are either three-electrode, solid-dielectric units or multichannel, gas-insulated, rail gaps. The

solid-dielectric switches are similar to those discussed for the Nereus accelerator in Section 13.4.2, except there are three electrodes and two pieces of polyethylene so that large numbers of them can be triggered simultaneously with about 1 ns rms jitter. These switches are located in as many places across the width of the PFLs as are necessary to achieve the desired rise time. The switch dielectrics must be replaced after each pulse. A multichannel rail gap switch⁵⁵ was developed by the Physics International Company to avoid this replacement process. The rail gap has 30-cm long cylindrical electrodes and a sharp blade trigger electrode that is located between these cylinders. With a trigger pulse amplitude approximately one-half the charge voltage, sufficiently low jitter was obtained so that two switches could be used on the 1.5 m wide PFLs. The inductance of one switch was 2 nH and produced about a 20 ns, 10 to 90 rise time pulse on the 0.4 to 0.55 MV, 0.4 to 0.5 Ω PFLs in SNARK and SLIM.

13.4.4 Prepulse Voltage in Moderate Current Accelerators

The prepulse amplitude for moderate voltage accelerators with Blumlein PFLs can be calculated as outlined in Section 13.3.5. The Marx generators for the accelerators with solid-dielectric Blumlein PFLs are frequently insulated with freon or SF_6 gas and are not housed in a common metal tank with the PFLs. In those cases it is possible to balance the charging bridge circuit and prepulse amplitudes can be substantially less than 1% of the charge voltage.⁵⁴

When single-coaxial PFLs or triplate-strip PFLs are used, the prepulse voltage results from the voltage division between capacitance of the output switch, C_g , and the capacitance of the transmission line between this switch and the diode, C_t , and of the diode, C_d , as shown in Fig. 13-18 and Eq. 13-17.

$$V_{pp} = \frac{C_g V_c}{C_g + C_t + C_d} \quad (13-17)$$

In this equation V_c is the charge voltage, and one should keep in mind that it is twice the amplitude of the output voltage into a matched load. All capacitance in parallel with the switch electrodes, both in the switching dielectric and the PFL dielectric, are included in C_g . From Fig. 13-11, it is apparent that this capacity can be quite large in a water dielectric PFL. In the case of HYDRA, the transmission line is long and the prepulse is relatively small. If the electrical length of this transmission line is comparable to the charge time of the PFL, then C_t and C_d in Fig. 13-18 should be replaced with a resistor, Z_t , whose value is the impedance of the transmission line. In this case, if $C_g Z_t$ is an appreciable fraction of the PFL charge time, the charging current through C_g and Z_t will be relatively large, resulting in a substantial prepulse. If the load is located near the switch, Eq. 13-16 applies and C_t is small. Usually $C_d < C_g$ and a significant percentage of the charge voltage can appear across the load. In any of the above cases, the prepulse amplitude can be substantially reduced by placing a resistor, R_p , in parallel with the

diode and selecting the value of R_p such that $R_p (C_t + C_d)$ is much shorter than the charge time. Frequently, a resistor whose value is two orders of magnitude greater than the diode impedance during the PFL discharge will give the desired small time constant and the energy lost during the main pulse is only 1%. Another design feature that has frequently been used with gas-dielectric spark gaps is to exclude all of the water from around the switch by extending the plastic switch housing to the outer cylinder of the PFL.⁸² By this technique, in water dielectric PFLs, C_g can be reduced by the ratio of the dielectric constants of water and the plastic.

In water-dielectric triplate PFLs, C_g is usually relatively large and the charge times short, resulting in prepulse amplitudes of about $0.1 V_c$. A prepulse shield for these point plane switches that reduces the prepulse amplitude to less than 1% of V_c is shown in Fig. 13-19.⁸¹ With the shield in place, the major portion of the capacitance from the switch end of the PFL and the switch shanks is to ground. The energy stored in this capacitance remains available as part of the main pulse. The electromagnetic waves from the PFLs are transmitted through the holes around the switch electrodes. This arrangement produces a PFL voltage pulse, with a 20 ns rise time, on Proto II and PBFA I.

13.4.5 Vacuum Interface

The principal element limiting power flow for a moderate-voltage, high power accelerator is the allowable electric field at the vacuum

interface. The flashover electric field, E_B in Eq. 13-8, is considerably lower than the dielectric strength of the insulating medium in the transmission lines that feed the diode. If a wave is propagating through a medium, Poynting's vector, $\bar{S}$, is the power per unit area normal to the direction of propagation of the wave. The general expression for $\bar{S}$ is given in Eq. 13-18,

$$\bar{S} = \bar{E} \times \bar{H} , \quad (13-18)$$

where $\bar{E}$ and $\bar{H}$ are the electric and magnetic field intensities. In a nondispersive, lossless medium, the peak value of S is given by Eq. 13-19,

$$S_p = \sqrt{\frac{\epsilon}{\mu}} E^2 , \quad (13-19)$$

where E and S_p are the magnitude of $\bar{E}$, $\bar{S}$. Solid dielectrics used in transmission lines have a similar value of ϵ to the interface material, but E is 2.5 to 15 times larger in the PFL than E_B . Therefore the peak value of Poynting's vector in the transmission line, S_p , is 6 to 225 times greater than the value of this vector at the interface, S_B . Transformer oil has a similar ϵ to the interface, but E can be up to five times greater than E_B , giving 25 times greater value of S_p than S_B . Water and ethylene glycol have much larger ϵ 's and E 's, and S_p in these dielectrics can be as much as two orders of magnitude greater than S_B . The low values of S_B are the result of the

low flashover strength (E_B) and imply that a substantially larger cross-sectional area is necessary for the vacuum interface than for the transmission lines.

This change in area results in an impedance discontinuity and causes energy to be reflected back into the PFL. Since the time it takes the wave to transit this impedance discontinuity is short with respect to the pulse duration, it is usually treated as a series inductor. This inductance limits the output pulse rise time or impedance of the system if a short pulse is necessary. Usually this vacuum interface region is designed to minimize this inductance. If an insulator stack similar to those described in Section 13.3.4 is used, the surrounding metallic structure should be shaped to give the same mean electric field for each insulator and thereby minimize the stack height. The inductance is directly proportional to this height. As stated in Section 13.3.4, the ratio of the area of the insulator stack region to the mean radius of this area is directly proportional to the inductance. Within certain mechanical constraints this ratio can be varied to decrease the inductance.

Because the interface voltages are much lower in these accelerators, single-radial insulators have been used effectively for low inductance designs. Successful examples of this design technique are the Gamble machines^{43,56} and HYDRA.⁴² The HYDRA diode and vacuum interface are shown in Fig. 13-20(b). With this design the electrodes on both the water and vacuum sides of the insulator are shaped to produce the 45-degree angle between the equipotential lines and the

insulator surface. It is difficult to maintain both a uniform electric field and the desired 45-degree angle; thus operating electric fields have been lower than for well-graded insulator stacks. J. Shipman⁸³ and D. McDaniel⁸⁴ found that variations in this angle over a very small length of the insulator-vacuum interface near either electrode could significantly decrease the flashover voltage of the insulator.

13.5 MODERATE-VOLTAGE, MICROSECOND PULSE DURATION SYSTEMS

In the preceding portions of this chapter the major emphasis has been on systems that required high peak powers and 150 ns or shorter pulse durations. Many applications of high power electron beams require pulse durations in the microsecond range.⁸⁵⁻¹⁰⁶ One of the primary applications of long pulse electron beams is for pumping gas lasers. Energetic electron beams can provide efficient pumping of high power lasers. These systems require excitation times from 0.3 to 4.0 μ s. This need has led to a class of electron accelerators with pulse durations in this regime and with electron energies from 0.15 to 1.0 MV. In the USSR, efforts are under way to extend this technology into the 10 to 100 μ s range.¹⁰⁷⁻¹¹⁰

The three generic types of systems that are used to satisfy the above requirements are: (1) Marx generator-PFL-diode, (2) Marx generators as PFN-diode, and (3) PFN-pulse transformer-diode. Each system is useful in a particular part of the impedance-voltage parameter space.

13.5.1 Marx Generator-PFL-Diode Systems

The Marx generator-PFL-diode system is the same as the water-dielectric PFL accelerators discussed in the preceding section. These systems can be used for high power lasers that are driven by low impedance electron beam accelerators with pulse durations up to about 0.8 μ s.¹⁰⁰⁻¹⁰⁴ Each meter of length of the PFL will add 59 ns to the pulse duration. Therefore the PFLs become quite long for the larger pulse durations. If a flat top pulse is desired at the diode, the charge time must be several times the pulse duration (i.e., several microseconds). It is necessary to keep the water resistivity considerably higher in these PFLs than in the short-pulse duration PFLs to avoid excessive resistive losses.

Because short rise time pulses are generally not required in these systems, single-channel, gas-dielectric spark gaps are used to switch the PFLs. Similarly, the flashover strength for the vacuum interface is low for these long pulses, and the additional inductance caused by the increased length is usually not a major problem. The major advantage of this type of system is that a single high-energy Marx generator or transformer system can charge several PFLs, resulting in a small number of components in the complete accelerator.

13.5.2 PFN High-Voltage Systems

The second type of system consists of PFNs that are directly connected to the diode.^{85,86,88-94,97-99} PFNs are discrete inductor-capacitor circuits configured to produce the necessary components of

the Fourier series required to produce a particular wave shape. For these moderately high-voltage applications, at least one stage of these networks is a Marx generator.

Figure 13-21 shows two configurations of pulse forming networks.¹¹¹ The component values are given in the figure with the appropriate procedure for converting to values that are required for particular pulse widths and impedances for a five-stage generator with a pulse rise time equal to 20% of the pulse duration given in the figure caption. The complete design formulation for these networks is given in Chapter 4 of this text and Ref. 13-111.

Type A and Type C networks are used extensively in laser applications. In the Type A network, only the primary stage is a Marx generator. The Type C network has Marx generators for all stages. The inductance requirements for the third and subsequent stages of Type A networks become sufficiently small that it is difficult to achieve for generators with impedances less than 10 Ω and voltages in the range considered in this section. This consideration has led to the development of Type C systems for a large portion of the high energy gas lasers that are excited by electron beams.

If a cold cathode electron beam diode is used for these long pulse systems, the impedance decreases with time (see Section 13.6). The standard PFN designs will produce a voltage pulse that decreases in amplitude as time increases with this type of load. For maximum efficiency and laser beam uniformity, it is important to optimize the voltage for uniform electron energy deposition in the gas and to keep

it constant during the pulse. The constant voltage can be achieved with a decreasing impedance if the voltage pulse is trapezoidal in shape. Because PFN circuits can be synthesized to produce any smooth pulse shape, PFNs can be designed to produce this flat top pulse. Similar effects can be accomplished with PFLs by varying the charge along the length of the PFL. This is usually accomplished by charging rapidly with respect to the discharge time.

If the pulse duration is in the 1 to 3 μs regime, PFN systems are always preferable to Marx generator-PFL systems. Between 0.3 μs and 1.0 μs , the voltage and impedance of the system become strong driving factors in selecting the type of system.^{102,103} Equations 13-20 and 13-21 give the impedance and pulse duration of the primary leg of a Type C PFN. The inductance, L_1 , to be used in these formulas, depends almost linearly on the voltage, because the inductance of almost all Marx generators is between 0.5 and 2.0 $\mu\text{H/MV}$. If L_1 is known, C_1 is fixed by the pulse duration. The energy stored in the primary stage, U_1 , is 85% of the total stored in that PFN and is given by Eq. 13-22,

$$Z = \sqrt{\frac{L_1}{C_1}}, \quad (13-20)$$

$$T = \pi \sqrt{C_1 L_1}, \quad (13-21)$$

$$U_1 = \frac{1}{2} C_1 V_0^2. \quad (13-22)$$

If U_1 is greater than the total system energy, U_0 , then the appropriate C_1 is chosen and an external inductor added to the Marx generator to give the desired pulse duration. If $U_1 < U_0$, n PFNs must be operated in parallel where n is $0.85 U_0/U_1$. Each Marx generator has V_0/V_c capacitors per PFN stage where V_c is the charge voltage. Collecting these facts and expressing them in a single equation, we get the total number of capacitors in a system as a function of T , U_0 , V_c , and L as given in Eq. 13-23,

$$N_c = N_m n = \frac{1.7 \pi^2 U_0 L}{T^2 V_c}, \quad (13-23)$$

where $L_1 = LV_0 = L N_m V_c$ and $N_m = V_0/V_c$.

Figure 13-22 is a plot of the number of capacitors versus the pulse duration for a megajoule PFN system that was investigated as a scoping study for a 100 kJ excimer laser for inertial confinement fusion.¹⁰² The number of components for a single Marx-PFL system is also shown. Since the cost of capacitors is not a strong function of their energy storage capability and the total cost of a PFN system is strongly related to the number of capacitors, one would choose the Marx generator-PFL system for pulse durations less than 800 ns for this megajoule system. For pulse durations in the 300 to 500 ns regime, reliability of individual components could be considerably less with the PFL modules to achieve a particular overall system reliability. The final decision on which configuration to use would

involve these factors as well as the overall size of the system and facility.

13.5.3 Low-Voltage PFN Systems

When the output voltage is at the low end of the range considered in this section and the diode impedance is relatively high, the third option, PFN-transformer-diode, is frequently employed.⁸⁷ The majority of electron beam generators with this design have pulse duration in the 1 to 3 μ s range. The same type of PFN circuits are used. The stages have individual capacitors that are charged between 10 and 100 kV. Step-up pulse transformers are used to increase the voltage to the desired level for the diode. The impedance of the PFN is Z_d/n^2 where Z_d is the diode impedance and n is the transformer step-up ratio. Usually the factor that determines if it is appropriate to use this system is the degree of difficulty required to achieve the impedance of this PFN.

The inductance of the transformer primary limits the pulse rise time. As in the preceding case, the inductance of each stage determines the impedance of each module, and it may be necessary to operate several modules in parallel to obtain the desired system output impedance. Thyratrons are frequently used in these circuits as the PFN switch. Other switching alternatives are ignitrons or spark gaps. Technology for these systems builds on the extensive work for radar modulators, which have essentially identical systems with lower voltages and longer pulse durations. Design details for these

circuits and the components used in them are given in Chapters 2, 4, 7, 8, 11 and 17 of this text.

13.6 ELECTRON AND ION BEAM DIODES

In this section the conversion of the electromagnetic pulses into electrons or ion beams is discussed. Intense electron beams are used for the major portion of the applications discussed in the introduction. Very high power ion beams are presently used for inertial confinement fusion research. Electron beams are generated by applying the high-voltage pulse between an anode and cathode and obtaining space-charge-limited flow. If the anode is a source of ions, the ratio of the electron current to the ion current is inversely proportional to the square root of their mass ratios. The ratio of the proton current to the electron current in such a diode is: $(1837)^{-1/2}$ (about 2.3%). To obtain intense ion beams, it is necessary to have a magnetic field transverse to the anode-cathode electric field. This magnetic field must have sufficient intensity to produce a Larmor radius of the electrons that is less than the spacing between the anode and cathode. The heavier ions have a much larger Larmor radius and become the major current carrier with up to 95% of the current in ions. The transverse magnetic field can be created from the normal beam currents within the diode or from field coils with an external energy source.

13.6.1 Space-Charge-Limited Electron Diodes

The high current densities (10^6 to 10^{10} A/m²) required for most of the electron beam applications have almost eliminated the use of thermionic emitters and has led to extensive development of cold cathodes. The widely accepted model for beam production from these devices involves the following steps:¹¹²⁻¹²²

1. Electrons are field emitted from metallic whiskers that have highly enhanced electric fields.
2. The current flowing through these whiskers quickly resistively heats and vaporizes them.
3. The vapor is ionized to form a plasma and becomes the electron source for the diode.
4. A space-charged-limited electron beam flows from this plasma surface to the anode.
5. This plasma drifts across the anode-cathode gap, causing an increase in current with time for a diode with a constant voltage.
6. Usually the beam is fully or partially stopped by an anode, and after sufficient energy deposition in the anode to desorb gases, an anode plasma also forms, decreasing the anode-cathode gap at a faster rate and allowing ion current to flow.

7. When the beam current becomes sufficiently large that its self-magnetic field causes the Larmor radius of the electrons to be smaller than the anode-cathode gap (cathode plasma surface to anode), the beam pinches.

At this stage the current amplitude is controlled by the electron drift across the electric field (parapotential flow) and not strictly by space-charge limits. The remainder of this section is a discussion of the effects of each of these steps on the diode current and impedance variations during the pulse.

Field emission of cold electrons occurs when the conduction electrons tunnel through the potential barrier at the surface of the metal. Appreciable current densities occur when the electric field at the surface approaches 10^{10} V/m. In cold-cathode diodes significant current is measured with mean electric fields above 10^7 V/m.^{114,116,118} Initial electron flow has been observed to flow from metallic whiskers that are a few microns in diameter.¹²² If the height to diameter ratio of these whiskers is greater than ten, field enhancements of several hundred are possible, and significant electron flow will occur.

The small diameter of these whiskers causes them to have high electrical and thermal resistance. A few amps flowing from a whisker causes it to vaporize in a few nanoseconds. This explosive vaporization occurs at many locations over the complete cathode surface and creates a plasma that drifts towards the anode at a velocity of $\sim 2 \times 10^4$ m/s. An alternate explanation for the formation

of this plasma is that dielectrics are either coated on the cathode surface or are imbedded in it, and the electric field causes these surfaces either to breakdown or to flashover, creating the cathode plasma.¹²³

In either case space-charge-limited electron current¹²⁴ flows from this plasma. The turn-on time of the diode is defined as the time required to reach the space-charge-limited current density calculated with the physical boundaries of the cathode and anode. The turn-on time is a strong function of the voltage rise time, the cathode macroscopic field enhancement, and the cathode material. In diodes with a current density of a few amps per square centimeter, the electric field strength required to maintain space-charge limited flow is marginal for turning on the cold cathode. The turn-on time can become an appreciable part of the pulse, and the current density can be very non-uniform. In these cases cathodes with substantial macroscopic field enhancement or preformed plasmas are used.

With the plasma surface supplying the electrons, the current is limited by the change in potential in the anode-cathode gap, caused by the electrons in the anode-cathode gap. The cloud of electrons near the cathode surface builds up a potential barrier (a reversal in the electric field) proportional to the thermal energy of the plasma electrons. In high-voltage diodes this potential is a very small percentage of the applied voltage, V_0 . Space-charge-limited flow is calculated assuming that the electric field is zero at the cathode. Using Poisson's equation (13-24) with distance measured from the

cathode and expressions for the conservation of charge (13-25) and energy (13-26), the space-charge-limited current for a one-dimensional parallel plane diode that produces a non-relativistic electron beam can be calculated.

$$\frac{dV}{dx} = \rho_c / \epsilon_0 , \quad (13-24)$$

$$J = \rho_c v , \quad (13-25)$$

$$\frac{1}{2} m_e v^2 = eV , \quad (13-26)$$

where V is potential as a function of x , ρ_c is the charge density, v is the electron velocity, e is the electron charge, m_e is the electron mass, and J is the current density. Equations 13-24, 13-25, and 13-26 can be combined to give Eq. 13-27,

$$\frac{d^2V}{dx^2} = \frac{J}{\epsilon_0 \sqrt{2eV/m_e}} = \frac{k}{V^{1/2}} , \quad (13-27)$$

where k is $J/(\epsilon_0 (2e/m_e)^{1/2})$. Equation 13-27 can be integrated to give Eq. 13-28,

$$\left(\frac{dV}{dx} \right)^2 = 4kV^{1/2} + C_1 . \quad (13-28)$$

Because we have assumed that $dV/dx = 0$ at $x = 0$, $C_1 = 0$, and Eq. 13-28 can be rewritten as follows,

$$\frac{dV}{dx} = 2 \sqrt{k} V^{1/4} . \quad (13-29)$$

This can be integrated to give Eq. 13-30,

$$\frac{4}{3} V^{3/4} = 2 \sqrt{k} x + C_2 . \quad (13-30)$$

Because $V = 0$ at $x = 0$, $C_2 = 0$. Substituting for k , and inserting the boundary conditions $V = V_0$ when $x = d$, and then solving for J , gives the expression for the space-charge-limited electron current density,¹²⁴

$$J = \frac{4\epsilon_0}{9} \left(\frac{2e}{m_e} \right)^{1/2} \frac{V_0^{3/2}}{d^2} . \quad (13-31)$$

This is the nonrelativistic, one-dimensional, space-charge-limited equation. For voltages above a few megavolts, the relativistic form of the equation should be used.^{113,125}

Transforming Eq. 13-31 into an expression for the diode impedance makes its use more convenient when designing loads for PFLs. This expression is given in Eq. 13-32, where Z_d is the diode impedance (V/I) and A is the area of the beam at the anode,

$$Z_d = \frac{4.29 \times 10^5}{v^{1/2}} \frac{d^2}{A} . \quad (13-32)$$

The movement of the cathode and anode plasmas causes d to decrease with time. To account for this change in spacing in Eqs. 13-31 and 13-32, d is replaced by $d_0 - ut$, where d_0 is the initial anode-cathode gap and u is the average plasma closure velocity.¹¹⁴⁻¹¹⁸ It is apparent from this substitution that Z_d varies throughout the pulse and can be exactly matched to the PFL impedance at only one instant of time. Because the energy delivered from the PFL to the diode is not a strong function of the mismatch ratio (Z_d/Z_0), accelerators can be designed with efficient energy transfer to the diode.

Equation 13-32 with d replaced by $d_0 - ut$ is usually a good expression for making initial estimates of Z_d , d , and cathode dimensions. Since this expression is based on a one-dimensional analysis, the current at the edges of the cathode is underestimated, and Z_d from Eq. 13-32 is usually larger than the diode impedance measured in experiments. Space-charge expressions for cylindrical and spherical diodes are given in Ref. 13-124 and can be used in conjunction with Eq. 13-32 to improve the estimate of total current, I . This technique is time consuming and only slightly improves the estimate of I . A better approach for predicting I is to use one of the excellent 2-D diode codes that have been developed to calculate electron trajectories, J and I . Both quasi-static and fully electromagnetic codes have been developed.^{5,126-132}

13.6.2 Pinched Electron Beam Diodes

The self-magnetic field of the electron beam affects the trajectory of the electrons as they travel from the cathode to the anode, as sketched in Fig. 13-23. The force from the magnetic field is always perpendicular to the electron velocity and is related to the centripetal acceleration in Eq. 13-33,

$$|\vec{F}| = e |(\vec{v} \times \vec{B})| = \frac{m_e v^2}{r} . \quad (13-33)$$

If we substitute the relativistic values for the mass and velocity of the electrons and $\mu_0 I / 2\pi R$ for B and solve for r , we obtain the Larmor radius, r_L , of the electrons at the outer edge of a circular cathode of radius R_c ,

$$r = \frac{\gamma m_e \beta c (2\pi R_c)}{e \mu_0 I} , \quad (13-34)$$

$$I_c = 8500 \beta_o \gamma_o \frac{R_c}{d} . \quad (13-35)$$

The critical current (I_c) for electron beam pinching occurs when $r = d$ and is given in Eq. 13-35, where β_o and γ_o are the values of relativistic parameters when the electrons have crossed the gap and gained energy eV_o electron volts.^{113,120,122,126,133-135} With currents larger than I_c , electrons leaving the outermost areas of the cathode do not strike the adjacent anode area but drift radially

toward the cathode axis (see Fig. 13-23). If the diode has a constant current density, B decreases linearly with R , and the electrons will cross the gap at some smaller radius. As the electrons drift inward, the potential change from the additional space charge in the anode-cathode gap decreases the current density from areas of the cathode that the electrons are passing. When the radially drifting electrons cross the gap, B increases, and subsequent electrons travel further towards the axis. In the limit of this process, all of the diode current originates from the outer rim of the cathode and crosses the gap near the axis. Very high current densities (1 to 10 MA/cm²) have been achieved in these pinched electron beam diodes.^{10,11,21,51,54,113,120,121,136-143}

The actual electron trajectories and current amplitudes are very complicated, but detailed predictions of these values are routinely made with the same quasi-static particle-in-cell or fully electromagnetic diode codes mentioned in the preceding section. An analytic treatment of the drift of the electrons transverse to the electric and magnetic fields yields a parapotential impedance expression (Eq. 13-36) that agrees quite well with experiments.¹³⁵

$$I = 8500 \frac{R}{d} \gamma_0 \ln [\gamma_0 + (\gamma_0^2 - 1)^{1/2}] \quad (13-36)$$

13.6.3 Electron Diodes with Axial Magnetic Fields

In gas laser applications, large area uniform electron beams are required. The total current of these diodes frequently exceeds the

critical value and pinching will occur unless something is done to prevent the pinching.^{93,94,100,101,103,105,144} If an axial magnetic field (B_z) is applied, a force is exerted on the electrons any time their velocity is not in the axial direction. If B_z is comparable in amplitude to B_s , the self-field, pinching can be prevented. Under these conditions the beam will rotate, and if a rectangular beam is used with supporting structure for the extraction foil, substantial beam losses to this structure can occur. If B_z is increased to several times the amplitude of B_s , then this loss can be minimized. However, the energy required to set up this axial field can be large.^{100,101}

In high current induction linacs and some intense microwave generation techniques, an electron beam is formed in an axial magnetic field and propagated in a vacuum. In this case foilless diodes can be used.^{113,145,146} Diode geometries with the accelerating electric field (E) either perpendicular to B_z or parallel to B_z have been used in experiments and extensively analyzed. Figure 13-24 shows a parallel B_z geometry that allows adjustment of the diode current by varying the anode-cathode spacing, L . The current is limited by the space-charge potential depression both near the cathode and in the vacuum drift tube. For the particular geometry shown in Fig. 13-24, an approximation for the current for various limiting conditions is given by Eqs. 13-37, 13-38, and 13-39.¹³⁻¹⁴⁶

$$L \ll \delta^*, \delta/a > 1.65 \quad I = \frac{17000 (\gamma_o^{2/3} - 1)^{3/2} \frac{r_2}{\delta}}{\ln 8\delta/a} \quad (13-37)$$

$$L \ll \delta^*, \delta/a < 1.65 \quad I = \frac{17000 (\gamma_o^{2/3} - 1)^{3/2}}{1 - \frac{2r_1^2}{r_2^2 - r_1^2} \ln \frac{r_2}{r_1} + 2 \ln \frac{R}{r_2}} \quad (13-38)$$

$$L \ll \delta^*, \delta/a < 1.65, \begin{matrix} a \ll r_1 \\ \delta \ll r_2 \end{matrix} \quad I = 17000 (\gamma_o^{2/3} - 1)^{3/2} \frac{R}{a + 2\delta} \quad (13-39)$$

For $L > \delta^*$, replace δ with δ^* in the above equation. The beam thickness, a , is essentially controlled by the cathode geometry and the single particle Larmor orbits accounting for both E_r and E_z in the anode-cathode gap. The above equations show that to generate large currents (> 20 kA) and propagate them through drift tubes, a and δ must be small with respect to R , or γ_o must be large. As with other diodes, one should use 2-D numerical simulations to get a more accurate description of the beam current and beam trajectories.¹⁴⁷

13.6.4 Ion Beam Diodes

Intense ion beams can be produced from diodes very similar in configuration to the intense electron beam diodes if an anode plasma is provided to supply the ions and a transverse magnetic field exists in the gap to inhibit electrons from crossing the gap.^{5,113} Ion beam diodes can be distinguished by the method in which the transverse

magnetic field is produced. Three of these methods that have been extensively studied are the following:

1. Pinched Beam Diode--from self-pinched electron beam flow
2. Applied-B Diode--from coils built into the diode structure and powered by capacitor banks
3. AMPFION--from the diode current flowing in shaped grids (electrodes)

13.6.4.1 Pinched Beam Diodes. The expressions for ion beam currents can be obtained by continuing the discussion on pinched electron beam diodes (13.6.2). Prior to the time when the electron current reaches I_0 , electrons are deposited in the anode, heating it until gas is desorbed and forms an anode plasma.^{114,119,120,136-138} Space-charge-limited ion flow is initiated when this plasma is formed. This positive charge density neutralizes some of the electron charge density and allows more electron current to flow. If the ions are included in the same formulation as used in Eqs. 13-24 to 13-32, the electron current density, J_e , and ion current density, J_i , for bipolar flow are

$$J_e = 1.86 J , \quad (13-40)$$

$$J_i = 1.86 J \left(\frac{m_e}{M_i} \right)^{1/2} , \quad (13-41)$$

where J is the current density without ion flow, as given in

Eq. 13-31. Fully developed bipolar flow implies that there are equal numbers of electrons and ions in the gap since the net charge density in the diode is zero. Because there are equal numbers of electrons and ions in the gap at any time, the ratio of I_i/I_e is inversely proportional to the time spent in the gap. When the pinched flow is completely formed, the electrons originate at the cathode edge and travel a distance R_c (see Fig. 13-23) to cross the gap (assuming $d \ll R_c$) while the ions travel a distance d . An expression for the ion-electron current ratio under these circumstances is given in Eq. 13-42, where A_i is the ion charge state and M_i is its mass.¹⁴⁸ It is apparent that for large R_c/d ratios and high voltages, the ion current can be considerably larger than the electron current.^{148,149}

$$\frac{I_i}{I_e} = \frac{R_c}{d} \left(\frac{2A_i V}{M_i c^2} \right)^{1/2} \quad (13-42)$$

For experiments in the 2 MV and 1 TW regime, this ratio has been close to one. If the anode foil is thin and the electrons reflex through it due to their own space charge, the ion production is further improved by decreasing the time to form the anode plasma and increasing the electron space charge near the anode.^{5,113,150-153}

13.6.4.2 Applied-B Diode. The ratio I_i/I_e can be further enhanced and made less dependent on voltage by applying a transverse magnetic field.^{5,113,153-158} Figure 13-25 is a sketch of a cross section of

one of these diodes with the anode a cylinder. When this diode is arranged to focus the ion beam at the center of the diode, the anode is a section of a sphere with modifications to account for beam deflection by the magnetic field.¹⁵⁷ The coils are spaced periodically around the circumference of the cathode, producing a uniform B field. The required field to produce magnetic insulation is approximately

$$B_{cr} = \frac{m_e c}{e d} (\gamma_o^2 - 1)^{1/2} . \quad (13-43)$$

Ratios of I_i/I_e as high as 19 have been observed experimentally with these diodes (i.e., 95% of the total current is ion current).

13.6.4.3 AMPFION Diode. The automagnetic, plasma-filled ion diode, AMPFION, has the field coils in series with the ion beam, such that the diode current creates the necessary transverse magnetic field without excessive loss of energy to electron flow.^{5,153,159,160}

Figure 13-26 is a sketch of the spherically focusing AMPFION diode for PBFA I. The surface of the field coil is parallel to the anode, and the pitch of the windings are adjusted to give a constant amplitude magnetic field over the complete surface of the anode. The constant amplitude magnetic field provides uniform plasma motion and uniform ion emission over large anode areas. The fields produced by the coil and by the ion beam are considered to select the coil pitch. With the coils immediately adjacent to the complete cathode surface, the volume

occupied by the magnetic field is relatively small, and the magnetic field energy requirements are minimized.

The AMPFION diode is designed to operate either with a high-density (10^{17} to 10^{18} ions/cm³) plasma at the anode surface or a low-density (10^{14} to 10^{15} ions/cm³) plasma completely filling the gap between the anode and the first cathode. In the high-density case, the operation is similar to the other diodes discussed in this section. In the plasma-filled case, the diode initially appears as a short circuit, and the impedance increases with time due to an erosion of the plasma front near the first cathode. This increasing impedance creates an accelerating voltage that increases with time. An increasing accelerating voltage can produce ion beam bunching during the drift of the ions to the center of the diode structure, thereby amplifying the ion beam current.

13.7 LINEAR INDUCTION ACCELERATOR

Linear induction accelerators (LIAs) are used to generate high energy beams. As the name implies, the accelerating potential is created from the time rate of change of the magnetic fields. These accelerators have been developed for radiography sources,¹⁸ electron ring ion acceleration experiments,^{14,15} electron beam propagation experiments,^{16,17,20,21,161,162,163} and heavy ion beam inertial confinement fusion.^{164,165} Similar to other linear accelerators, these devices can be used to generate electron or ion beams with arbitrarily high energies, while the pulsed power voltages are less

than a few megavolts. Compared to other linear accelerators, the currents generated with LIAs can be large: 1 to 80 kA electron beams^{15-21,161-163} and up to 2 kA ion beams.¹⁶⁵⁻¹⁷¹ With these beam currents and accelerating voltages for each stage of 0.25 to 3.0 MV, the power input to each stage is sufficiently high that the same type of pulse-forming circuits as discussed in the preceding portion of this chapter can be used. Two types of LIAs have been developed-- ferrite core accelerators^{14-18,163-171} and radial pulse line accelerators,^{19-21,161,162,172-174} sometimes referred to as air core LIAs.

13.7.1 Ferrite Core Accelerators

A sketch of a ferrite core accelerating cavity is shown in Fig. 13-27. Energy is supplied from 1 to 4 coaxial cables to the outer edge of the center disc. Two different ways of explaining the operation of this device exist. In the first explanation, the toroid, abcd, that encloses the ferrite is viewed as a single-turn primary of a transformer, and the toroid, bcdef, is the single-turn secondary of this transformer. The accelerating voltage is induced across the output terminals of the secondary, bf. Figure 13-28 is an equivalent circuit for the primary when the secondary terminals are open. The battery and resistor represent the input cable characteristics, and the inductor is the primary circuit inductance. An expression for the current in this circuit is given in Eq. 13-44, where τ is L/Z_0 ,

$$I = \frac{V_0}{Z_0} (1 - e^{-t/\tau}) . \quad (13-44)$$

Because the ferrite is operated on the unsaturated portion of the B-H curve, μ_0 is very high, and consequently L is very large. The pulse duration for this type of accelerator is always much less than τ . Therefore we can approximate $e^{-t/\tau}$ as $1 - t/\tau$. I and dI/dt become

$$I = \frac{V_0}{L} t , \quad (13-45)$$

$$\dot{I} = V_0/L . \quad (13-46)$$

Because the secondary completely encloses the primary magnetic flux and the leakage inductance of this unusual transformer is negligible, the secondary terminal voltage is V_0 . When a beam is accelerated and secondary current flows, the source voltage and impedance are reflected directly into the secondary circuit, and the accelerating potential becomes $V_0 - I_b Z_0$. This accelerating voltage will exist until either the input pulse is over or until the ferrite saturates. If the ferrite saturates, L becomes very small. In that case τ becomes short with respect to the pulse duration, and I and the accelerating potential quickly become zero (i.e., $e^{-t/\tau}$ goes to zero).

The alternate explanation is much more aligned with the rest of this chapter. In that explanation we treat the cavity as two transmission lines. One of these transmission lines, ab-ef, is short

with respect to the pulse duration multiplied by the speed of light and directly connects the input cables to the accelerating terminals, bf. When the beam current flows across the accelerating terminals, the same amplitude current is induced in this transmission line and flows through the input cables. The input cable also supplies energy to a second transmission line (ab-dc) that is shorted at its output end (bc). This transmission line is filled with a high permeability material, so it has both a high impedance and a very long wave transit time. The full accelerating voltage is supported across this line; but with low current, it draws very little energy from the input cables for either the pulse duration or until the ferrite saturates. The condition for the ferrite to saturate is given in Eq. 13-47, where A is the cross-section area of the ferrite (i.e., that area in abcd of Fig. 13-27 that is occupied with ferrite), and B_s is the allowed flux swing before saturation.

$$\int V dt = B_s \cdot A \quad (13-47)$$

Thus the area (or amount) of ferrite required is directly related to the accelerating voltage and pulse duration, and inversely proportional to the magnetic flux swing between the two saturation levels. Amorphous glass is a magnetic material with a large flux swing (see Section 13.8.3) and appears attractive for use in future systems. It has been tested in systems with up to four cavities at the 1 MV level.^{175,176}

Only a limited number of ferrite core LIAs have been developed. The 50-MV, 10-kA, ATA accelerator^{17,163} that has been built for the Defense Advanced Research Projects Agency at Lawrence Livermore Laboratory is the largest LIA that has been constructed at this time. This accelerator has 190, 250 kV accelerating stages. Ten 250 kV stages are connected in series to supply 2.5 MV to form the beam in the injector. In contrast to most of the other accelerators discussed in this chapter, ATA operates at a repetition rate of 0.5 Hz.

13.7.2 Radial Pulse Transmission Line Cavities

The second type of linear induction accelerator is the radial pulse transmission line accelerator.^{19-21,161,162,172-174} In this concept two or more transmission lines form the cavity and are arranged to symmetrically feed energy into the accelerating gap. The impedances of these transmission lines are nearly equal. Electromagnetic pulses can be injected into these cavities,¹⁷⁷ as in the ferrite core induction linacs, or the PFL and cavity can be combined into a single entity. In either case, to extract energy efficiently from the cavities, the accelerating pulse must be bipolar and the beam accelerated on the second half-cycle of the pulse.¹⁷⁸⁻¹⁸⁰ Accelerators that have been built using this concept use the combined PFL-cavity.

13.7.2.1 Induced Voltages.

The relationships between the induced voltage of these cavities and the PFL parameters are given in

Eqs. 13-48 to 13-52 and Fig. 13-29,

$$V = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A} . \quad (13-48)$$

For a strip transmission line with a constant spacing, a , and width, w , Eq. 13-48 becomes Eq. 13-49,

$$V = \frac{\partial}{\partial t} \left(\frac{\mu I}{w} a l \right) = \frac{\partial}{\partial t} (LI) . \quad (13-49)$$

If I is constant, Eq. 13-49 can be written as Eqs. 13-50 or 13-52.

$$V = \mu I \frac{a}{w} \frac{dl}{dt} \quad (13-50)$$

$$\frac{dl}{dt} = c = \frac{1}{\sqrt{\mu\epsilon}} \quad (13-51)$$

$$V = \sqrt{\frac{\mu}{\epsilon}} \frac{a}{w} I = Z_0 I \quad (13-52)$$

13.7.2.2 Cavity Physics. The completely enclosed cavities can be designed with either radial or coaxial transmission lines, as shown in Fig. 13-30. Design considerations for coaxial PFLs have been covered in other sections of this chapter. The impedance of the radial cavity PFL is given by the equation for a bicone transmission line, Eq. 13-53,

$$z_0 = \sqrt{\frac{u}{\epsilon}} \frac{1}{2\pi} \ln \frac{1}{(\tan \theta_1)^{1/2}}, \quad (13-53)$$

where θ_1 is the half angle of the conical electrode.¹⁸¹ An excellent approximation for this formula when the cones are mildly diverging and θ_1 is nearly equal to π is

$$z_0 = \sqrt{\frac{u}{\epsilon}} \frac{d}{2\pi r}, \quad (13-54)$$

where d/r is the slope of the cone. In this configuration the electric field is highest at the inner edge of the PFL. The pulse length establishes the outer radius and the width of the outer portions of the transmission lines. Therefore the accelerating gradient, defined as the ratio of the total beam energy to the length of the accelerator, is inversely proportional to the pulse duration for a given dielectric material in the PFL.

The accelerating voltage is determined by considering the PFLs, as one would in any other pulsed power circuit. The circuit shown in Figs. 13-30a1 and 13-30a2 was introduced by Pavlovskii and his coworkers^{19,172-174} and was the basis for the LIU-10¹⁷⁴ and the RADLAC-I accelerators.²⁰ The center disc is charged to a voltage V_0 with a Marx generator, transformer, or intermediate storage capacitor. While the PFL is being charged, both sides of the accelerating gap are at the same potential and the accelerating voltage, V_a , is zero. After the PFLs are charged, the PFL switches are closed, shorting one

of the PFLs, and V_o appears across the open-circuit accelerating gap. The accelerating voltage remains this polarity until the electromagnetic wave originating from the shorted PFL travels the length of the PFL and arrives at the accelerating gap. At this time the voltage reverses polarity and has an amplitude $-V_o$, if the gap remains an open circuit. Subsequently the wave propagates back to the short circuit, reflects, and returns to the accelerating gap. As shown in Fig. 13-31a, the duration of the first pulse is $2T$ and the second and subsequent pulse durations are $4T$ where T is the one-way wave transit time for a single PFL. If an electron beam is present in the accelerating gap, the load for the PFL is $Z_b = V_a/I_B$ where I_B is the beam current. If $Z_b = Z_o$, and the beam arrives at time $2T$, then $V_a = V_o/2$, as shown in Fig. 13-31b. Because of the pulse duration difference, the acceleration must occur on the second pulse to extract nearly all of the stored energy from the cavity.

The voltage across the vacuum-insulator stack surrounding the accelerating gap must withstand a voltage V_o for a time $2T$, and a reverse polarity voltage $V_o/2$ for $4T$ without flashover. For unipolar pulses the flashover strength is lowered by 60% if the polarity is reversed for the same pulse duration and a 45-degree insulator stack.³⁸⁻⁴¹ RADLAC I was designed to operate with the consideration of this 60% reduction, but flashover occurred during the $4T$ pulse. Experiments with this accelerator indicated that the flashover strength with the bipolar pulses varied as $t^{0.55}$ rather than $t^{1/6}$, as given for unipolar pulses in Eq. 13-8.¹⁸² Further experiments for

pulse durations from 0.01 to 1 μ s indicate that the flashover voltage for bipolar pulses is given by Eq. 13-55 for 2-cm thick insulators and a pulse waveform with a 25 ns pulse duration for each polarity portion of the bipolar waveform.¹⁸³ Thinner insulators (4 to 5 mm thick) improve the flashover strength by 50%.¹⁸⁴ The breakdown strength with these thinner insulators is given in Eq. 13-56. One possible explanation for this improvement is that ions formed above the insulator are accelerated to an electrode rather than trapped between the electrodes by the reversing waveform. Removal of the ions reduces their effect on increasing the ionization and forming a plasma.

$$E_B t_f^{1/2} A^{1/10} = 1400 \quad (13-55)$$

$$E_B t_f^{1/2} A^{1/10} = 2100 \quad (13-56)$$

13.7.2.3 Alternate Cavity Configurations. Other circuits/cavity configurations shown in Fig. 13-30b - 13-30d produce a bipolar waveform that has an accelerating potential into a matched load that is greater than the potential on the first half-cycle and greater than or equal to the charging voltage.¹⁷⁸⁻¹⁸⁰ By properly matching the impedance and reflection coefficients, 100% of the stored energy can be extracted from ET1, ET2, and S3. Accelerating potentials for maximum energy transfer from ET1, ET2, and S3 with V_0 charge voltage are V_0 , $1.5 V_0$, and V_0 respectively. These higher accelerating voltage to charge voltage ratios allow higher accelerating gradients

but requires the PFL switches to conduct currents several times larger than the beam current.

Figure 13-30e is an isolated Blumlein circuit^{20,182} that produces a unipolar pulse, if maximizing the voltage across the insulator stack is the most important consideration. This circuit functions similarly to a standard Blumlein, as discussed in the beginning of this chapter, except only half of the PFL supplies energy to the beam. The other half is used to transit time isolate the wave formed in the PFL that is switched from the unswitched PFL. The distance from the switch to the load is T through the normally shorted PFL in the Blumlein and $3T$ through the other PFL. The maximum efficiency of this cavity is 50%, but it has a full $2V_0$ open circuit voltage. This PFL configuration produces pulses with essentially zero prepulse and has been used to achieve that goal in IBEX, a 4 MV, 100 kA single anode-cathode accelerator.¹⁸⁵

The major LIAs that have been built using this technology had type P1 cavities (see Fig. 13-30a). LIU-10 has water-insulated cavities that are charged to 0.5 MV and then switched with gas-insulated spark gaps. There are 52 of these cavities in the accelerator which produce a 13 MeV, 50 kA electron beam.¹⁷⁴ RADLAC I, shown schematically in Fig. 13-32, had four oil-insulated cavities that each provide 1.8 MV acceleration to a 28 kA beam. The beam was formed with a 1.7 MV injector that had a coaxial PFL. This accelerator produced a 28 kA, 9.8 MeV electron beam.^{20,21,161} The isolated Blumlein cavity (see 13.7.2.3) was also tested during the

development of this accelerator. The low impedances that are readily achieved with these radial transmission lines make them suitable for accelerating high current electron beams (i.e., 50 to 100 kA beams).

13.7.3 Electron Beam Transport Issues

The high current beams in both the ATA and RADLAC accelerators are susceptible to instabilities that can cause disruption of the beam. Preventing these instabilities is an active area of research. Two of the most important instabilities are the beam breakup¹⁸⁶⁻¹⁹² and the image displacement.^{187,193-196} The beam breakup instability is the result of the cavity transverse magnetic mode that is excited by the arrival of the beam at the accelerating gap and then traverses back and forth from reflection points in the cavity. The fields associated with this wave can cause a transverse displacement of the beam. Because all cavities are identical, this transverse motion can continually grow until the beam is destroyed. In the ferrite cavities, the major reflection point is the cavity wall, because transmission lines from the feed points to the beam are much shorter than the pulse duration. The Q of these cavities at the frequency of the excited modes is made as low as is practical to maximize the attenuation of these high-frequency waves during their propagation through the cavities and thus minimize the beam perturbation. In the RADLAC accelerator, it takes the full pulse duration for a wave to propagate to the outer wall of the cavity and back, thereby eliminating this reflection as a cause for beam breakup.

Discontinuities exist at the vacuum interface that can cause reflections resulting in beam breakup. If the cavity is insulated with water, these waves are almost completely absorbed at the interface rather than reflected.¹⁹⁶ The image displacement instability results from the beam axis not being aligned with the beam tube axis, and the image forces increasing this displacement.^{187,193-195}

The high current beams that are transported through the RADLAC accelerator must be hollow to prevent space-charge forces from stopping the beam in the drift tubes at the low energy end of the accelerator. A foilless diode (see 13.6.3) is used to generate these hollow beams, and a strong axial magnetic field is used to maintain them close to the wall. Radial oscillations of the beam envelope are induced at the accelerating gaps because the radial force balance that existed in the drift tube is disrupted. By configuring the gap electrodes to minimize the magnitude and duration of the force unbalance, the designer can make the amplitude of these radial oscillations negligible.^{20,21,147,161,196-199}

13.7.4 Ion Beam Accelerators

Induction linacs are also used for accelerating ion beams.^{5,164-171} Because of the lower velocities of these beams, space-charge limits the ion currents to much smaller values than for relativistic electron beams. Current amplification occurs as the ion velocity increases. The accelerating voltage can be shaped to

slightly bunch the beam to further increase the current. In this type of accelerator, the required pulse duration decreases throughout the length of the accelerator. If the ion space charge is neutralized with electrons in the drift regions, large ion beam currents can be generated and transported through accelerators similar to those described for electrons. This concept has been explored in the Pulselac Program at Sandia National Laboratories.^{5,165-171} A plasma is formed on the walls of the drift tubes, and the radial electric field from the space charge of the ion beam draws electrons from this plasma into the body of the ion beam until it is neutralized. Similar to ion beam diodes (Section 13.6.4), a transverse magnetic field is applied at the accelerating gap that prevents the low energy electrons from crossing the gap but allows the heavier ions to be accelerated. A new set of electrons is introduced into the beam from the wall plasma on the far side of the gap. In the Pulselac C accelerator, 2 kA proton beams have been accelerated through four 250 kV gaps. A sketch of this accelerator is shown in Fig. 13-33. The accelerating cavities are similar to the ATA cavities, and the accelerating pulses are generated with coaxial, water-dielectric PFLs that are charged with Marx generators.

13.8 REPETITIVELY OPERATED ELECTRON BEAM SYSTEMS

13.8.1 Systems Considerations

The major portion of the requirements for high peak-power electron and ion beam accelerators has been satisfied with accelerators that produce a single pulse and are ready to produce a second pulse minutes to hours later. Success in present-day research for essentially all applications leads to the requirement for repetitively operated beam generators. In addition to the fusion²⁰⁰ and directed-energy weapons²⁰¹ applications, there is a potential for using the electron, ion, or gamma-ray radiation for preservation of food,²⁰² sterilization of hospital equipment,²⁰³ sterilization of sewage,²⁰⁴ cross-linking or curing plastics,²⁰⁵ and modifying metallic surfaces.²⁰⁶ These facts lead to the conclusion that there will be vastly greater emphasis on developing the technology for repetitively operating, high peak-power accelerators in the next few years than there has been in the past.

Although support for developing repetitive technology has been limited, some notable accomplishments have been reported. The electron-beam, ferrite core LIAs discussed in the preceding section of this chapter operate with repetition rates between 0.5 and 100 Hz with average beam powers between 0.08 and 16 kW. The ASTRON accelerator¹⁴ ran with 150 J/pulse (4 MV, 150 A, 250 ns) and 60 Hz for 9 kW average power; ATA produces 24 kJ (47 MV, 10 kA, 50 ns) per pulse and operates at 0.5 Hz or 12 kW average power.^{17,163} The technology for these

accelerators is based on the technology that has been developed for RF accelerators and radar modulators (see Chapter 4) and on the high peak-power technology that is discussed in this chapter. In general, each module supplies 100 to 300 J to the beam and, in the case of ATA, 200 of these modules operate in the desired sequence. Extensive development and testing of the ATA modules provided highly reliable units that operate for millions of pulses without failure. Handling this relatively small amount of energy per module increases the complexity and probably the overall cost of the system. Magnetically switched modules (Section 13.8.3) have the potential for higher voltage per module, higher energy per pulse, and higher repetition rates which would produce much higher average powers for this type of LIA.²⁰⁷⁻²⁰⁸

In the single anode-cathode beam generators, both long (1 μ s) and short (~50 ns) pulse duration, repetitively operated accelerators have been developed. In the short pulse regime, Marx-PFL-diode systems²⁰⁹⁻²¹¹ exist with output voltages up to 1.0 MV (5 Hz, 2.5 kW), and 10 kW (1.0 MV, 1 Hz) average power. Pulse transformer-PFL-diode systems²¹¹⁻²¹⁸ have also been developed with 1.5 MV, 100 Hz, 45 kW the highest operating parameters^{213,214}. A 1.5 MV, 5 kJ pulsed power system without an elution beam diode has been operated at 20 Hz for a 100 kW average power delivered to a resistor.²¹⁶ For ~1 μ s pulse duration, Marx-PFN systems have been reported with 350 kV the maximum output voltages and 30 kW the highest average power.²¹⁹⁻²²¹ PFN-transformer systems have operated with up to 300 kV, 2 μ s, 40 A,

100 Hz for 4 kW average powers.^{86,91,106} A high voltage (250 kV), dc power supply and two PFNs connected as a Blumlein have been used to produce a 200 kV, 8 kA, 5 μ s electron beam at 50 Hz.²²² A notable feature about all of these systems is the rather low energy per pulse (10^4 J/pulse maximum) and low average powers (100 kW maximum) compared to forecast needs of megajoules/pulse and 10 to 100s of megawatts average power.^{102,103,200-203,223-225} Nevertheless, significant advancements have been made in understanding the difficulties in operating various components and systems repetitively.

Operating levels for single-pulse accelerators are established such that a small fraction of the pulses are lost due to accelerator component failures. The total number of pulses for the life of many of these accelerators is fewer than 10^5 pulses. Repetitively operated accelerators that run between 1 to 100 Hz generate 3×10^7 to 3×10^9 pulses/year. Thus the lifetime of components for these systems must be increased by 2 to 5 orders of magnitude. In addition, any failure modes, whether high or low probability, will rapidly occur. If the system doesn't fully recover between pulses, the same failure will recur until the components involved in the fault mode, and perhaps others, are destroyed.

The major portion of the applications for high average power accelerators require high efficiency for economic and/or size constraints. The accelerator designer is concerned about efficiency independent of these issues. Energy that is not delivered to the load is deposited in various components resulting in a temperature rise in

those components. Because the dielectric strength of essentially all materials varies inversely with temperature, these temperature increases can cause dielectric breakdown. For the above reasons, one must design for highly efficient systems and have extensive knowledge of the breakdown probability distribution as a function of electric field, current, pulse durations, and pulse rate to design reliable systems.

In particle beam accelerators composed of a Marx generator, a PFL, and a diode, the efficiency can be improved by closing the high-voltage switch at peak voltage and decreasing the Marx generator energy losses. The latter can be accomplished by increasing the resistance of interstage resistors until the losses during Marx generator charging and discharging are almost equal. Further improvements can be made by replacing the resistors with inductors.²¹¹ Because the Marx components are usually immersed in transformer oil, cooling of most components occurs naturally. Internal heat flow can be a problem for capacitors at high repetition rates. In addition, spark gaps may require both gas and liquid cooling. Since these gaps are distributed throughout the Marx generator, it is often difficult to provide adequate cooling to many of the stages.

An alternate highly efficient PFL charging system is the high-voltage, air core transformer described in Chapter 11. With dual resonance charging, 95% of the capacitor bank energy has been transferred to the PFL.²¹⁶ The spark gaps in transformer systems are at the dc charge voltage and are more accessible for cooling. These

systems operate reliably at the 100 kW level (1.5 MV, 5 kJ, 20 Hz) during development experiments.^{216,226} Many applications will require an order of magnitude greater energy transfer per pulse, and additional development work will to be required.

The components in accelerators can be divided into two categories--active (switches such as spark gaps or thyratrons and electron or ion beam diodes) and passive (capacitors, PFLs, resistors, and inductors). In the active components a discharge or beam must be formed for each pulse. The discharge, beam formation, and beam deposition usually result in electrode erosion that can determine the lifetime of these components. In addition, these changes in dielectrics and electrodes during the pulse result in the physical conditions varying slightly from pulse to pulse. Therefore there is a statistical spread in the operating parameters of these devices. Passive components operate with functions resulting from changes in electric or magnetic fields within the component.

13.8.2 Repetitive Switching--Spark Gaps

The high-voltage switches on essentially all high peak-power systems are either liquid or gas-dielectric spark gaps. Energy is always deposited in the arcs of spark gaps during the breakdown process, creating shock waves. With liquid-dielectric spark gaps, the shock wave is strongly coupled to the surrounding structures. These shock waves make it difficult to achieve long life for these

structural components. Thus liquid-dielectric spark gaps are not considered to be viable switches for repetitively operated systems.

Although shock waves also occur in gas-dielectric spark gaps, the intensity of the shock is lower, and it does not couple to support structures. Because the inductance is relatively high for gas-dielectric spark gaps, large numbers of parallel channels must be used to achieve the desired rise times for many applications.²²⁷ The number of channels can be estimated from Eqs. 13-6 and 13-7 just as in the single-pulse case. Technical issues that arise in repetitive operation that are not usually a concern for single-pulse systems are dielectric strength recovery, dielectric and electrode heating, electrode erosion, and low-probability prefires.

When energy is discharged through a gas-dielectric spark gap, the small channel of gas in the arc is heated to roughly 10^4 K. When current stops flowing, this gas expands and cools. Since the dielectric strength of the gas is inversely proportional to its density, this gas must return to near its nominal temperature before its dielectric strength recovers. If significant forced flow of the gas is not employed, the gas will take about 20 ms to recover about 90% of its strength.²²⁸⁻²³² This data implies that one can operate systems below 50 Hz with nominal gas flow to carry away the heat deposited in the spark gap. Above 50 Hz the spark gap will not function without forced mixing or removal of the gas dielectric. Even at the lower rates, the breakdown probability distribution can change substantially if the electrode temperatures are not controlled. If

one removes all of the gas, rates as high as 1 KHz have been demonstrated for 250 kV gaps with a burst of five pulses²³³ and 50 kV gaps continuously.²³⁴

Each time an arc forms in a spark gap, a small amount of electrode material is vaporized. The amount varies from 50 to 1000 $\mu\text{g}/\text{coul}$ depending on the operating conditions.^{211,215,223,234-241} The higher rates are usually associated with shorter pulses and a relatively low number of coulombs passed. Even though the erosion rate is smaller (50 $\mu\text{g}/\text{coul}$) for longer pulses, more material is frequently removed because the total number of coulombs is usually higher. Electrode erosion is a concern, because a very large number of pulses can infer a rather large amount of electrode material removal. One can counteract effects of this removal on the electrode spacing by using large-area electrodes and varying pressure to maintain the operating voltage. Deposition of the eroded conducting material on insulating surfaces can be a shorter term problem. This problem can be alleviated by using controlled gas flow patterns to keep the insulating surfaces clean.^{211,212,215,216,226}

One additional problem that electrode erosion can cause is a change in the spark gap triggering characteristics. Triggering with nanosecond jitter relies on strongly exceeding the breakdown electric field when the fast rise time trigger pulse is applied. To keep the trigger pulse amplitude reasonable, high-voltage spark gaps are usually operated as follows:

1. A sharp-edged trigger electrode is biased on an equipotential line such that the electric field enhancement is minimized.
2. The trigger pulse changes the potential a small fraction of the total voltage, but a large enhancement of the electric field occurs at the sharp edge of the trigger electrode.
3. The breakdown initiates near this electrode and rapidly proceeds across the gap.

If the erosion process reduces the sharpness of the trigger electrode, the field enhancement can be substantially reduced, and the spark gap jitter increases. One way to avoid changes in field enhancement is to make the trigger electrode out of a very thin cylinder or disc. As the trigger electrode erodes away, the arcs change location on the main electrodes, but the gap maintains its low-jitter operation. This technique is used in a 250 kV cylindrical spark gap on ATA, and low-jitter operation exists for $> 5 \times 10^6$ pulses.²³³ Heat flow and repetition rate can limit the dimensions of the third electrode and thus the total number of pulses. This triggering problem can be avoided in low-voltage (< 100 kV) spark gaps by using uniform field gaps and a relatively large trigger pulse (> 100 kV). Rohwein has developed such a spark gap system for switching the energy into the primary of high-voltage transformers that are used to charge PFLs.²¹⁶ Three of these gaps have operated for more than 10^6 pulses with jitter maintained at less than 10 ns. Twenty-five such gaps have operated effectively in parallel on a repetitively operated 50 kJ capacitor bank.²⁴²

An active area of research on gas spark gaps is an attempt to determine causes for prefires occasionally occurring at less than 50% of the mean breakdown voltage.²⁴³⁻²⁴⁵ Although the breakdown distribution changes during repetitive operation of a spark gap, essentially all of the breakdowns occur within 10% to 15% of the mean breakdown voltage level. Data for spark gaps that are charged in many milliseconds shows prefires occur as low as 50% of the mean breakdown level about once in every 1,000 pulses. There is two probability distributions for these low-voltage breakdown events, one that depends on the gap flow rate and another that depends on the amplitude of the discharge current. The causes of these low-voltage breakdowns are still being explored, but the disruptions caused from their occurrence will be a greater problem for repetitive systems than single-pulse systems.

13.8.3 Repetitive Switching--Magnetic

A passive alternative to spark gaps for switching PFLs is magnetic switches, sometimes referred to as saturable reactors.^{207,208,246-249} With magnetic switching, a section of the PFL dielectric is replaced with magnetic material as shown in Fig. 13-34. The large change in permeability, when the magnetic material saturates, allows this section on line to be a switch. A typical B-H curve and plot of μ versus H are shown in Fig. 13-35. If the material is biased at $-B_p$, the permeability is very high and the

inductance, L_s , of this section of the line is large. For a coaxial transmission line, L_s is given by Eq. 13-57,

$$L_s = L_1 + \mu_0 \mu_r \ell_2 \ell_n a_1/b_2 . \quad (13-57)$$

L_1 is the inductance of Sections I and II of the PFL (see Fig. 13-34) where the dimensions were changed to accommodate the magnetic material; a_1 , b_2 , and ℓ_2 are given in Fig. 13-34. Since L_s is very large, only a small current flows through the switch to charge the transmission line beyond the PFL. The voltage across the switch changes the core flux, ϕ , as given by Eq. 13-58,

$$V = - \frac{d\phi}{dt} = -A \frac{dB}{dt} , \quad (13-58)$$

where A is the cross-sectional area of the magnetic material. It follows that

$$\int V \cdot dt = -A \Delta B , \quad (13-59)$$

where ΔB is the change in magnetic induction. With a knowledge of the B-H curve and by choosing A , one can arrange for the material to saturate when the PFL is fully charged. At saturation μ approaches μ_0 (see Fig. 13-35), and L_s decreases to a relatively small value. The PFL then discharges rapidly through this low inductance into the transmission line. The time compression per stage of magnetic

switching is proportional to the ratio of the unsaturated core inductance to the saturated core inductance. Calculations indicate that about a factor of 5 time compression can be expected for reasonable switch operation.^{247,248} Because accelerators usually require time compressions of about 50 from a few microseconds to a few tens of nanoseconds, two to four stages of switching are required, as shown schematically in Fig. 13-36.

Although saturable reactor switching has been in use since the 1940s,^{111,246} it has only recently been considered for high-voltage PFL switches. The development of Metglas 2605SC, an amorphous ferromagnetic metallic glass material,²⁵⁰ has helped spark this interest. This material has a relatively large flux swing, low loss, and the potential for very low cost. These factors make it attractive for PFL switches. It is especially attractive for repetitive operation because the passive nature of the switching allows high-frequency operation and long life.

At the present time these switches have only been used in development systems or accelerator trigger generators. Two examples of development of these switches are: 1) the high-voltage, high-energy experiments at Sandia National Laboratories^{251,252} where a two-stage switching system passed 135 kJ after switching at 5 MV and compressed the energy delivery time from 300 ns to 35 ns; and 2) the 1 KHz operation of a 800 J system producing 450 kV, 80 ns pulses at Lawrence Livermore National Laboratories.^{207,208}

13.8.4 Repetitively Operated Diodes

In high peak-power diodes the electron or ion beam is drawn from a plasma. Because the formation of this plasma causes portions of the cathode (for electron beams) or the anode (for ion beams) to erode away, the reproducibility of beam turn-on and current density can be a problem in repetitive systems. Diffusion of this plasma and the associated neutral gas that is generated from both electrodes on each pulse can be the limiting factor on operating frequency and can force requirements for vacuum systems with very high pump rates. Those portions of the beam that are deposited in the diode heat surfaces where the deposition occurs, and cooling must be provided to remove the heat. This beam deposition may be due to beam loss to anode support structure during normal operation or to beam interception in extraction foils. The accelerator designer must have knowledge of any changes in vacuum breakdown or insulator flashover due to repetitive operation. Because essentially all of the data for repetitively operated diodes involve electron beam systems, these diodes will be discussed in the remainder of this section.

Significant changes in the current density uniformity or impedance characteristics occur within 10^4 to 10^5 pulses for most of the short-pulse duration (30 to 70 ns) electron beam diodes that have been developed. In 1976, the Physics International Company developed a 500 kV, 40 kA, 50 ns, 1 Hz accelerator²¹⁰ from which a 200 A/cm^2 electron beam was extracted for pumping a KrF laser. The beam uniformity requirements were $\pm 20\%$. With three saw blades as the

cathode, this uniformity was maintained for $2-3 \times 10^4$ pulses with continuous operation. The reduction in uniformity was accompanied by significant erosion of these blades, resulting in a reduction in the enhancement of the electric field.

Subsequent investigations²⁵³ with a 300 kV, 30 kA, 30 ns diode operating at 30 Hz and 1.5 kA/cm^2 showed that the impedance of the diode at peak voltage continuously increased with the number of pulses as shown in Fig. 13-37. The data were taken with grooved brass cathodes and with a cathode consisting of a matrix of brass roll-pins. This data indicate that, as the number of pulses increases, it becomes increasingly more difficult to form the cathode plasma and achieve space-charge-limited flow over the complete cathode. If the operation is suspended after a large number of pulses and the cathode is coated with diffusion pump oil, the impedance returns to its nominal value and the aging process proceeds just as it did with a new cathode (Fig. 13-37). This data imply that plasma formation involves breakdown of the dielectric coating on the cathode surface. The erosion process includes removing this material from the surface as well as metallic material that decreases the field enhancement of the cathode surface.

An experiment was conducted in which an oil coating was continuously replenished and the diode impedance at peak voltage remained constant for 3×10^5 pulses, as shown in Fig. 13-37.²⁵³ In this experiment the cathode was made of bearing stock consisting of a porous steel material impregnated with oil. Significant amounts of

decomposition products coated the diode at the end of these experiments. These products may eliminate the use of this particular cathode material for many applications, but the experiments indicate that coating the cathode with a dielectric in a controlled fashion could result in cathodes with a long lifetime.

The cold cathodes used extensively in ETA and ATA²⁵⁴ are flashboard cathodes that have a lifetime of more than 10^5 pulses before significant changes in the total current occurs. A 10 kA beam is drawn from a 500 cm^2 cathode when an 80 kV pulse is applied to the grid of this triode and 2.5 MV to the anode. These cathodes consist of a matrix of metallic dots surrounded by a thin insulator surface in a metallic background. Each dot is connected to a resistor through which a fast-rising 40 kV voltage pulse is applied about 100 ns before the grid is pulsed. Dielectric flashover of the insulator surface occurs for more than half of these spots. The plasma formed from this flashover becomes the electron source. The electrons are accelerated to the grid and then compressed to 30 cm^2 during the acceleration to the anode. Difficulties in achieving beams with uniform current densities and high emittance with this cathode and beam optics arrangement have led to changes in both. A smaller diameter carbon fiber⁹⁷ or felt cloth cathodes coupled with elimination of the grid has produced a better quality beam.²⁵⁵

In diodes requiring $1 \text{ } \mu\text{s}$ pulses and $10\text{--}20 \text{ A/cm}^2$ at about 300 kV, it is difficult to obtain uniform current densities. The production of uniform current-density beams for repetitive operation is an active

area of research. Carbon felt cathodes are the most extensively used for single-pulse application and produce the most uniform beams for repetitive operation. As with the short pulse cathodes, the beam uniformity can degrade significantly in less than 10^4 pulses.

In addition to the uniformity problem, rapid collapse of the diode impedance can be prevalent in these diodes. One source for the shorting can be ionization of the residual anode and cathode plasma and neutrals from the preceding pulse if the pumping speed isn't substantially greater than used for similar single-pulse operation. Evidently more cathode and anode material is removed, and more effective ionization of the gas occurs during the long pulse operation. Pressure measurements indicate that several molecules of gas are evolved for each electron that is accelerated.¹⁰⁶ Impedance collapse or shorting can also occur from random spots on the cathode holder and support structure. The plasma closure velocity from these isolated plumes is frequently much larger than the average closure velocity in the anode-cathode gap.

Many of the above problems disappear with a well-designed thermionic cathode and diode structure. Large area diodes that produce 150 kV, $< 1 \text{ A/cm}^2$ beams have been operated for $> 10^4$ pulses without extensive problems.¹⁰⁶ Operation at current densities up to 30 A/cm^2 , 250 kV, 1 μs , and 25 Hz has been achieved with barium-oxide dispenser cathodes.²⁵⁶ With the higher current densities, gas evolves from grids or anodes. This gas can poison the thermionic emitter and limit its effective lifetime. This problem caused the switch from hot

cathodes to cold cathodes on ETA, and its effects are under investigation for long-pulse operation.

Most of the vacuum-insulator interfaces in operating repetitive systems have electrical fields less than 50% of the flashover electric fields predicted from Eq. 13-8. With these conservative designs, insulator flashover is not usually a problem. Experiments²⁵⁷ on small area samples and with repetition rates up to 50 Hz indicate the flashover strength remains constant as a function of frequency (see Fig. 13-38), and Eq. 13-8 can be used for designing insulator stacks for repetitive systems. Larger safety factors should be used for repetitive systems than for single-pulse diodes to ensure that the probability of breakdown is sufficiently low.

13.9 CONCLUDING REMARKS

In this chapter, the design issues for high peak-power electron and ion beam accelerators for a variety of parameter ranges have been addressed. Detailed information on most of the components of these systems is given in other chapters of this handbook and in the extensive reference list at the end of the chapter. The technology is sufficiently developed at this time that PBFA II, a 100 TW accelerator with a 13 MJ energy storage, is presently being tested to bring it to full operational power levels during 1986. A large variety of lower power needs are being fully satisfied for single-pulse applications. Only a very limited number of repetitive systems based on this technology are operating now. One of the major systems in this

category is the 50 MV ATA. Extensive development to more fully extend the high peak-power accelerator technology into a repetitively operated regime will occur during the next decade.

Fig. 13-1. Block diagram showing the basic subsystems and their functions for electron or ion beam accelerators.

Fig. 13#2. Voltage and impedance graph for PFLs with the areas appropriate for various dielectrics identified.

(a)

Fig. 13-3. Hermes II, a 10 MV, 100 kA, 88 ns electron beam accelerator. This accelerator consists of a 188-stage Marx generator that charges a Blumlein PFL to 10 MV. The Blumlein PFL has a self-closing oil-dielectric spark gap that initiates the energy transfer to the electron beam diode.

(a) Artist's drawing

(b)

Fig. 13-3. Hermes II, a 10 MV, 100 kA, 88 ns electron beam accelerator. This accelerator consists of a 188-stage Marx generator that charges a Blumlein PFL to 10 MV. The Blumlein PFL has a self-closing oil-dielectric spark gap that initiates the energy transfer to the electron beam diode.

(b) Cross-sectional sketch

Fig. 13-4. Simplified circuit of Marx generator-PFL during charging of the PFL. C_m is the erected Marx generator capacitance, L_m is the Marx generator inductance, C_2 is the PFL capacitance.

Fig. 13-5. Aurora, a 12 MV, 1.6 MA, 125 ns electron beam accelerator.

This machine has four Marx generators, four Blumlein PFLs and four electron beam diodes.

Fig. 13-6. The 12 MV, multichannel, oil-dielectric spark gap on the Aurora Blumleins. The switch consists of two main electrodes labeled positive and negative and one sharp-edged trigger electrode. The gas spark gap and inductor used to trigger the Aurora spark gap is also shown.

Fig. 13-7. Plot of the insulation flashover field versus angle θ for 7740 glass and cross-linked polystyrene. The angle θ is defined in the sketch. The pulse rise time is about 5 μ s. The data is compared to results for 7740 glass with pulse durations of 25 to 100 ns.¹³⁻³⁹

MARX GENERATOR

Fig. 13-8. Simplified Hermes-II equivalent circuit. Same notation as Fig. 13-4 except that the Blumlein PFL is shown as two capacitors, C_2 and C_3 .

Fig. 13-9. A more detailed equivalent circuit of Hermes II with the Marx generator represented by four blocks consisting of a capacitor (0.021 μF), an inductor (20 μH), and a resistor (4 Ω). Stray capacitance to ground from each of these sections of the Marx generator are included and shown with V₁, V₂, V₃, and V₄ across them. C₂ and C₃ from Fig. 13-8 are shown as 3600 pF and 2000 pF capacitors respectively. L₃ is 5.5 μH.

Fig. 13-10. Equivalent circuit of Marx generator and water dielectric PFL during charging of the PFL. R_p represents the resistance of the water dielectric.

Fig. 13-11. HYDRA, a 1 MV, 1 MA, 80 ns electron beam accelerator.

Each PFL-transmission line transformer-diode supplies a 500 kA beam.

(a) Artist's drawing

Fig. 13-11. HYDRA, a 1 MV, 1 MA, 80 ns electron beam accelerator.

Each PFL-transmission line transformer-diode supplies a 500 kA beam.

(b) Cross-section of PFL, gas switch, transmission line transformer, and diode

Fig. 13-12. Triplate strip transmission line configuration with one high voltage electrode and two ground electrodes forming two PFLs that are operated in parallel.

Fig. 13-13. Proto I, a 2 MV, 540 kA, 24 ns electron beam accelerator. The accelerator is shown with a Marx generator, 3 intermediate storage capacitors (ISC), 12 oil-dielectric strip Blumlein PFLs, and 2 electron beam diodes (or 1 ion beam diode). The 12 Blumlein PFLs are arranged in 6 stacks containing 5 electrodes that form 2 Blumleins in parallel with the output in the center of the stack. The high voltage distribution network for triggering the Blumlein rail gaps is also shown. Normal operation was without the ISC.

Fig. 13-14. Proto II, a 1.5 MV, 5.5 MA, modular electron beam accelerator. Each of the eight modules contains a Marx generator, two intermediate storage capacitors, two 50 ns PFLs, two 20 ns PFLs and two triplate transmission line transformers. The accelerator has either two electron or one ion beam diodes that are connected to the 16 transmission line transformers.

Fig. 13-15. PBFA I, a 36 module, 30 TW, 2 MV, 15 MA, 40 ns accelerator. Each module consists of a 3.2 MV, 100 kV Marx generator located in the oil tank. They also have an ISC, a 2.4 MV spark gap, 2-triplate PFLs and a transmission line transformer all located in the water tank. The output of each module is transported to the ion beam diode with a 7-m long magnetically insulated, vacuum transmission line.

$$V_{OUT} = 2NV_0 \frac{Z_L}{Z_L + 2NZ_0}$$

Fig. 13-16. Stacked Blumlein transmission line voltage adder.

- a) Three Blumlein PFLs showing potential vectors after the PFLs are charged but before their switches are closed. Load voltage is zero at this stage.
- b) Potential vectors after PFL switches have closed and the waves have inverted at the load end of the PFL. All Blumleins are connected in series, and the output voltage is given by the equation. The equation neglects effects of the stray capacitance shown as dotted lines in the figure.
- c) Equivalent circuit for b with stray capacities represented by impedance Z_B . The stacked Blumleins have an impedance $2NZ_0$ and an open circuit voltage $2NV_0$, where N is the number of stages.

Fig. 13-18. Equivalent circuit of a Marx generator, PFL, high-voltage switch, transmission line and diode. This circuit is the appropriate representation during charging of the PFL. The parameters are C_m -Marx capacitance, L_m Marx inductance, C_2 -PFL capacitance, C_g spark gap and surrounding structure capacitance, C_t -transmission line capacitance and C_d -diode capacitance.

Fig. 13-19. Prepulse shield for self-breaking, water-dielectric spark gaps in triplate strip PFL.

- a) Configuration without prepulse shield
- b) Configuration with prepulse shield

Fig. 13-20. HYDRA diode with radial vacuum insulator. The transmission line electrode diameters are labeled a and b. Vacuum exists in the volume formed by the cathode lucite sheet and anode. The size of this volume controls the magnitude of the inductance of the diode. The lucite sheet is the radial insulator which vacuum flashover is a concern.

Fig. 13-21. Pulse-forming network configurations for Type A and Type C networks. Multiply capacitors by T/Z and inductors by ZT to obtain circuit values for a network with impedance Z that will produce a pulse of duration T . The primary frequency legs of these circuits referred to as C_1 , L_1 in the text have $C_1 = 0.456$ and $L_1 = 0.0781$ in the Type A, and $C_1 = 0.4003$ and $L_1 = 0.253$ in the Type C.

Fig. 13-22. Plot of number of capacitors versus pulse duration for a one-megajoule Type C PFN system (Eq. 13-23). Two curves are plotted showing spreads in charging voltage and normalized inductance. Number of capacitors in a Marx generator-PFL-system is also shown.

Fig. 13-23. Typical electron trajectories for high current electron beam diode. The trajectories were calculated with a particle-in-cell code and include effects of anode and cathode plasmas.

Fig. 13-24. Foilless diode configuration that allows adjustment of impedance by varying anode-cathode spacing, L . High voltage is applied to the cathode, and the hollow, cylindrical, electron beam that is formed propagates in the tube with radius R .

Fig. 13-25. Cross-section of cylindrical Applied-B ion diode. The magnetic insulation field coils prevent electrons from crossing the anode-cathode gap. The ions are created at the anode, accelerated to the virtual cathode, pass through the gas cell window, and drift radially to the center of the diode. Electrons are emitted from the cathode near the gas cell window, trapped on the magnetic field lines, and drift between the gas cell window and the anode forming the virtual cathode.

Fig. 13-26. AMPFION ion beam diode. The diode current passes through the field coil to form the insulating magnetic field. The pitch of the coil is adjusted to give a uniform magnetic field at the anode.

Fig. 13-27. Cross-section of a ferrite-core accelerator cavity for an induction linac. The voltage pulse is fed into the cavity at two points on its circumference. The accelerating gap is the break in the beam line.

Fig. 13-28. Equivalent circuit for ferrite-core cavity. Components to represent losses in the ferrite are not shown since these losses are small.

Fig. 13-29. Strip transmission line sketch.

Fig. 13-30. Cavity configurations for transmission line linear induction accelerators--a1 and a2 are referred to as P1, b is ET-1, c is S3, d is ET-2, and e is the isolated Blumlein. The configurations are distinguished by the number of transmission lines and the impedance of these lines as shown on the figures.

a) OPEN CIRCUIT

b) MATCHED LOAD

Fig. 13-31. Accelerating waveform for Pavlovskii-type accelerating cavity.

(a) Open Circuit

(b) Matched Load

Fig. 13-32. RADLAC I, a 9.5 MV, 28 kA, four-cavity, radial pulse transmission line, linear induction accelerator.

Fig. 13-33. Pulselac C, a 1 MV, 2 kA ion beam linear induction accelerator. The accelerator has an injector and four accelerating cavities. A hollow-core beam is accelerated to allow room for the magnetic field coils required for the radial magnetic field.

Fig. 13-34. Sketch of a coaxial PFL with magnetic switch. The magnetic material is located in the section with axial dimension l_2 . The impedance of this section is very high compared to the PFL impedance if the magnetic material is not saturated. It is slightly higher than the PFL impedance if the material is saturated.

Fig. 13-35. Typical B versus H curve and μ versus H curve for magnetic switch material.

Fig. 13-36. Circuit diagram and calculated voltage waveforms for three stages of time compression with magnetic switching.

Fig. 13-37. Normalized impedance at peak voltage versus number of pulses for repetitively operated cathodes. The curves show the impedance increases rapidly for bare metal cold cathodes but remains constant with cathodes made out of oilite-bearing stock.

ACRYLIC FLASHOVER

Fig. 13-38. Vacuum-insulator flashover voltage as a function of repetition rate for acrylic rings.

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FIGURES CAPTIONS

Fig. 13-1. Block diagram showing the basic subsystems and their functions for electron or ion beam accelerators.

Fig. 13-2. Voltage and impedance graph for PFLs with the areas appropriate for various dielectrics identified.

Fig. 13-3. Hermes II, a 10 MV, 100 kA, 88 ns electron beam accelerator. This accelerator consists of a 188-stage Marx generator that charges a Blumlein PFL to 10 MV. The Blumlein PFL has a self-closing oil-dielectric spark gap that initiates the energy transfer to the electron beam diode.

(a) Artist's drawing

(b) Cross-sectional sketch

Fig. 13-4. Simplified circuit of Marx generator-PFL during charging of the PFL. C_m is the erected Marx generator capacitance, L_m is the Marx generator inductance, C_2 is the PFL capacitance.

Fig. 13-5. Aurora, a 12 MV, 1.6 MA, 125 ns electron beam accelerator. This machine has four Marx generators, four Blumlein PFLs and four electron beam diodes.

Fig. 13-6. The 12 MV, multichannel, oil-dielectric spark gap on the Aurora Blumleins. The switch consists of two main electrodes labeled positive and negative and one sharp-edged trigger electrode. The gas spark gap and inductor used to trigger the Aurora spark gap is also shown.

Fig. 13-7. Plot of the insulation flashover field versus angle θ for 7740 glass and cross-linked polystyrene. The angle θ is defined in the sketch. The pulse rise time is about 5 μ s. The data is compared to results for 7740 glass with pulse durations of 25 to 100 ns.¹³⁻³⁹

Fig. 13-8. Simplified Hermes-II equivalent circuit. Same notation as Fig. 13-4 except that the Blumlein PFL is shown as two capacitors, C_2 and C_3 .

Fig. 13-9. A more detailed equivalent circuit of Hermes II with the Marx generator represented by four blocks consisting of a capacitor (0.021 μ F), an inductor (20 μ H), and a resistor (4 Ω). Stray capacitance to ground from each of these sections of the Marx generator are included and shown with V_1 , V_2 , V_3 , and V_4 across them. C_2 and C_3 from Fig. 13-8 are shown as 3600 pf and 2000 pf capacitors respectively. L_3 is 5.5 μ H.

Fig. 13-10. Equivalent circuit of Marx generator and water dielectric PFL during charging of the PFL. R_p represents the resistance of the water dielectric.

Fig. 13-11. HYDRA, a 1 MV, 1 MA, 80 ns electron beam accelerator.
Each PFL-transmission line transformer-diode supplies a 500 kA beam.

(a) Artist's drawing

(b) Cross-section of PFL, gas switch, transmission line
transformer, and diode

Fig. 13-12. Triplate strip transmission line configuration with one
high voltage electrode and two ground electrodes forming two PFLs that
are operated in parallel.

Fig. 13-13. Proto I, a 2 MV, 540 kA, 24 ns electron beam accelerator.
The accelerator is shown with a Marx generator, 3 intermediate storage
capacitors (ISC), 12 oil-dielectric strip Blumlein PFLs, and 2
electron beam diodes (or 1 ion beam diode). The 12 Blumlein PFLs are
arranged in 6 stacks containing 5 electrodes that form 2 Blumleins in
parallel with the output in the center of the stack. The high voltage
distribution network for triggering the Blumlein rail gaps is also
shown. Normal operation was without the ISC.

Fig. 13-14. Proto II, a 1.5 MV, 5.5 MA, modular electron beam accelerator. Each of the eight modules contains a Marx generator, two intermediate storage capacitors, two 50 ns PFLs, two 20 ns PFLs and two triplate transmission line transformers. The accelerator has either two electron or one ion beam diodes that are connected to the 16 transmission line transformers.

Fig. 13-15. PBFA I, a 36 module, 30 TW, 2 MV, 15 MA, 40 ns accelerator. Each module consists of a 3.2 MV, 100 kV Marx generator located in the oil tank. They also have an ISC, a 2.4 MV spark gap, 2-triplate PFLs and a transmission line transformer all located in the water tank. The output of each module is transported to the ion beam diode with a 7-m long magnetically insulated, vacuum transmission line.

Fig. 13-16. Stacked Blumlein transmission line voltage adder.

- a) Three Blumlein PFLs showing potential vectors after the PFLs are charged but before their switches are closed. Load voltage is zero at this stage.
- b) Potential vectors after PFL switches have closed and the waves have inverted at the load end of the PFL. All Blumleins are connected in series, and the output voltage is given by the equation. The equation neglects effects of the stray capacitance shown as dotted lines in the figure.
- c) Equivalent circuit for b with stray capacities represented by impedance Z_B . The stacked Blumleins have an impedance $2NZ_0$ and an open circuit voltage $2NV_0$, where N is the number of stages.

Fig. 13-17. Cross-section of two, low-impedance, solid-dielectric, strip, Blumlein PFLs. Each of the four transmission lines consist of two, thin, copper electrodes and several layers of polyester sheet insulation. Only the upper portion of a vertical set of PFLs is shown. The assembly is submerged in a copper sulfate solution. The high voltage charge leads are attached to g. The Blumleins are operated in parallel and electrode n is the output high voltage electrode, R_x and C_x are the capacitance and resistance per unit length in the margin of the PFLs.

Fig. 13-18. Equivalent circuit of a Marx generator, PFL, high-voltage switch, transmission line and diode. This circuit is the appropriate representation during charging of the PFL. The parameters are C_m -Marx capacitance, L_m Marx inductance, C_2 -PFL capacitance, C_g spark gap and surrounding structure capacitance, C_t -transmission line capacitance and C_d -diode capacitance.

Fig. 13-19. Prepulse shield for self-breaking, water-dielectric spark gaps in triplate strip PFL.

- a) Configuration without prepulse shield
- b) Configuration with prepulse shield

Fig. 13-20. HYDRA diode with radial vacuum insulator. The transmission line electrode diameters are labeled a and b. Vacuum exists in the volume formed by the cathode lucite sheet and anode. The size of this volume controls the magnitude of the inductance of the diode. The lucite sheet is the radial insulator which vacuum flashover is a concern.

Fig. 13-21. Pulse-forming network configurations for Type A and Type C networks. Multiply capacitors by T/Z and inductors by ZT to obtain circuit values for a network with impedance Z that will produce a pulse of duration T. The primary frequency legs of these circuits referred to as C_1 , L_1 in the text have $C_1 = 0.456$ and $L_1 = 0.0781$ in the Type A, and $C_1 = 0.4003$ and $L_1 = 0.253$ in the Type C.

Fig. 13-22. Plot of number of capacitors versus pulse duration for a one-megajoule Type C PFN system (Eq. 13-23). Two curves are plotted showing spreads in charging voltage and normalized inductance. Number of capacitors in a Marx generator-PFL-system is also shown.

Fig. 13-23. Typical electron trajectories for high current electron beam diode. The trajectories were calculated with a particle-in-cell code and include effects of anode and cathode plasmas.

Fig. 13-24. Foilless diode configuration that allows adjustment of impedance by varying anode-cathode spacing, L . High voltage is applied to the cathode, and the hollow, cylindrical, electron beam that is formed propagates in the tube with radius R .

Fig. 13-25. Cross-section of cylindrical Applied-B ion diode. The magnetic insulation field coils prevent electrons from crossing the anode-cathode gap. The ions are created at the anode, accelerated to the virtual cathode, pass through the gas cell window, and drift radially to the center of the diode. Electrons are emitted from the cathode near the gas cell window, trapped on the magnetic field lines, and drift between the gas cell window and the anode forming the virtual cathode.

Fig. 13-26. AMPFION ion beam diode. The diode current passes through the field coil to form the insulating magnetic field. The pitch of the coil is adjusted to give a uniform magnetic field at the anode.

Fig. 13-27. Cross-section of a ferrite-core accelerator cavity for an induction linac. The voltage pulse is fed into the cavity at two points on its circumference. The accelerating gap is the break in the beam line.

Fig. 13-28. Equivalent circuit for ferrite-core cavity. Components to represent losses in the ferrite are not shown since these losses are small.

Fig. 13-29. Strip transmission line sketch.

Fig. 13-30. Cavity configurations for transmission line linear induction accelerators--a1 and a2 are referred to as P1, b is ET-1, c is S3, d is ET-2, and e is the isolated Blumlein. The configurations are distinguished by the number of transmission lines and the impedance of these lines as shown on the figures.

Fig. 13-31. Accelerating waveform for Pavlovskii-type accelerating cavity.

(a) Open Circuit

(b) Matched Load

Fig. 13-32. RADLAC I, a 9.5 MV, 28 kA, four-cavity, radial pulse transmission line, linear induction accelerator.

Fig. 13-33. Pulselac C, a 1 MV, 2 kA ion beam linear induction accelerator. The accelerator has an injector and four accelerating cavities. A hollow-core beam is accelerated to allow room for the magnetic field coils required for the radial magnetic field.

Fig. 13-34. Sketch of a coaxial PFL with magnetic switch. The magnetic material is located in the section with axial dimension l_2 . The impedance of this section is very high compared to the PFL impedance if the magnetic material is not saturated. It is slightly higher than the PFL impedance if the material is saturated.

Fig. 13-35. Typical B versus H curve and μ versus H curve for magnetic switch material.

Fig. 13-36. Circuit diagram and calculated voltage waveforms for three stages of time compression with magnetic switching.

Fig. 13-37. Normalized impedance at peak voltage versus number of pulses for repetitively operated cathodes. The curves show the impedance increases rapidly for bare metal cold cathodes but remains constant with cathodes made out of oilite-bearing stock.

Fig. 13-38. Vacuum-insulator flashover voltage as a function of repetition rate for acrylic rings.