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NUCLEAR REACTOR LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

**Formulation and Experimental Evaluation
of Closed-Form Control Laws for the
Rapid Maneuvering of Reactor Neutronic Power**

John A. Bernard, Ph.D.
MIT Nuclear Reactor Laboratory

Report No. MITNRL-030
September 1989

CAMBRIDGE, MASSACHUSETTS 02139

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on**

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Rapid Maneuvering of Reactor Neutronic Power**

Report No. MITNRL-030

by

**John A. Bernard, Ph.D.
MIT Nuclear Reactor Laboratory**

September 1989

**Prepared for
The Sandia National Laboratories**

Frank V. Thome and Francis J. Wyant

Exploratory Nuclear Energy Systems Directorate

MASTER

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When one is up to one's neck in alligators, it is sometimes difficult to remember that the original purpose was to drain the swamp.

- H. G. Rickover

Executive Summary

This report describes both the theoretical development and the experimental evaluation of a novel, robust methodology for the time-optimal adjustment of a reactor's neutronic power under conditions of closed-loop digital control. Central to the approach are the 'MIT-SNL Period-Generated Minimum Time Control Laws' which determine the rate at which reactivity should be changed in order to cause a reactor's neutronic power to conform to a specified trajectory. Using these laws, reactor power can be safely raised by five to seven orders of magnitude in a few seconds. The MIT-SNL laws were developed to facilitate rapid increases of neutronic power on spacecraft reactors operating in an SDI environment. However, these laws are generic and have other applications including the rapid recovery of research and test reactors subsequent to an unanticipated shutdown, power increases following the achievement of criticality on commercial reactors, power adjustments on commercial reactors so as to minimize thermal stress, and automated startups. The work reported here was performed by the Massachusetts Institute of Technology under contract to the Sandia National Laboratories. Support was also provided by the U.S. Department of Energy's Division of University and Industry Programs.

The achievement of time-optimal control implies that a system's output is changed both in minimum time and without violation of any operational constraints. Relative to nuclear reactors, techniques for the time-optimal adjustment of neutronic power were studied extensively in the 1960s. Examples include usage of Pontryagin's 'maximum principle' and Bellman's 'dynamic programming'. Unfortunately, when applied to realistic reactor problems, these methods are so computation-intensive that they often do not function in real time. That is, it takes longer to determine the optimal solution than is available for the calculation. As a result, the solution to an optimal control problem is usually not a closed-form law but rather a sequence of signals that are to be applied at each sampling interval. This is a form of open-loop control. Its deficiencies are that the sequence of signals will be valid only for the conditions that were present when the problem was first formulated and that no provisions exist for the use of feedback. Hence, no allowance is possible for modeling errors and unforeseen perturbations. A major accomplishment of the MIT-SNL program that is described in this report is that a method was found to obtain closed-form, time-optimal control laws for the adjustment of a reactor's neutronic power. As such, these laws are amenable to the use of feedback.

The work described in this report is significant in that a novel solution to the problem of time-optimal control of neutronic power was identified, in that a rigorous description of a reactor's dynamics was derived, in that the rate of change of reactivity was recognized as the proper control signal, and in that extensive experimental trials were conducted of these newly developed concepts on actual nuclear reactors.

The first unique aspect of the MIT-SNL research reported here was the method used to determine the reactor's time optimal response. Specifically, it was observed that if a system is subject to one or more constraints, then the time-optimal trajectory will move that system along the most limiting of those constraints. Hence, rather than solve the system's describing equations subject to both the constraint and a performance index such as a minimum time requirement, it was recognized that it would be more direct to define the physical conditions that corresponded to system movement along that limiting constraint. It was this insight that permitted the derivation of closed-form control laws because, using this approach, an actual equation can be written that describes the system's optimal behavior. Relative to the reactor-powered spacecraft now being considered for use by the United States, the limiting constraint is the reactor period which is a measure of the rate at which the reactor's power is being raised or lowered. The identification of a time-optimal trajectory therefore required that an expression be developed that gave the instantaneous reactor period as a function of the various parameters that described the core's dynamic behavior. Substantial progress had been made toward this objective as part of earlier research conducted at MIT under the auspices of the National Science Foundation. That effort was continued and it culminated in the elucidation of accurate mathematical models (the dynamic period equations) that describe reactor behavior during transients.

The second unique feature of the work described here was the derivation of the dynamic period equation which provides a rigorously valid description of the dynamics of reactors that are governed by space-independent kinetics. This includes research, test, spacecraft, multi-modular, and small to mid-size commercial reactors. The dynamic period equation, which may be written in either a standard or an alternate form, gives the instantaneous reactor period as a function of the rate of change of reactivity, the reactivity, and the rate of redistribution of the delayed neutron precursors. It is the parent relation for the well-known inhour equation which describes reactor behavior under the special case of asymptotic conditions. The dynamic period equation was the key to the practical realization of a closed-form law for a time-optimal response. In particular, it provided a description of each of the physical mechanisms that could affect the reactor period. The challenge then became one of selecting the appropriate mechanism for use as the control signal.

The third distinguishing feature of the work contained in this report was the choice of the rate of change of reactivity as the control signal. This was a significant advance because the traditional approach to the automated adjustment of reactor power is to use the reactivity itself as the control signal. The resulting control schemes are often difficult to implement because reactivity is not a directly controllable quantity. In particular, control mechanisms have finite speeds and reactivity, which is a function of the net change in each mechanism's position, can therefore not be altered on demand. In contrast, the rate of change of reactivity is directly controllable because its value is solely a function of the control

mechanism's speed. Adjustments in speed can be made essentially at once whereas a change in position requires a finite amount of time. Controllers designed in terms of the rate of change of reactivity are therefore very responsive. In particular, assuming no limitation on actuator speed, they can generate virtually any desired power profile.

Once the choice of a control signal was made, it became a simple matter to obtain the time-optimal laws. The terms in the dynamic period equation were rearranged so that the rate of change of reactivity was expressed as a function of the limiting parameter which is the instantaneous reactor period.

The MIT-SNL laws are mathematically tractable and hence readily implemented in real time. That is, using commercially-available computing equipment, these laws can be solved for the appropriate control signal in a few hundredths of a second. This is less than the needed sampling interval. The method of solution is as follows. Given a desired power trajectory, the instantaneous period associated with that trajectory is determined. This quantity is then substituted into one of the MIT-SNL laws. (Note: There are two mathematically-equivalent MIT-SNL laws. They are designated as standard and alternate. The alternate law is normally preferred because it is the easier to program.) Other independent variables such as the reactivity and the delayed neutron precursor distributions are then either measured or calculated. The law is then solved for the rate of change of reactivity needed to generate the period in question. The control devices are then moved at this rate thereby causing the specified power profile to be produced. Feedback is readily incorporated by reformulating the period at each sampling interval based on a comparison of the observed and specified power trajectories.

The MIT-SNL laws were first derived in October-November 1986. Experiments to verify the concept were begun in late December of that year on the 5 MWt MIT Research Reactor. These continued through the spring and summer of 1987. In parallel with this activity, preparations were made for conducting more extensive experiments on the Annular Core Research Reactor (ACRR) that is operated by the Sandia National Laboratories. A total of 89 control experiments were performed on the ACRR in October and December 1987. These culminated with a series of runs on 4 December 1987 in which the neutronic power of the ACRR was safely raised from 0.54 kW to 50 MW, a full five orders of magnitude, in 5.83 seconds. These runs unequivocally demonstrated that the MIT-SNL Period-Generated Minimum Time Control Laws did fulfill the primary requirement of the space nuclear effort. Namely, they were capable of raising a reactor's neutronic power by many orders of magnitude both safely and in minimum time.

Following introductory material, this report reviews the basic concepts of both control theory and reactor dynamics. Details of the derivation of the dynamic period equation, the formulation of the MIT-SNL laws, and the results of simulation studies are then presented. Next the experiment preparations are described with emphasis on safety considerations, the software-to-hardware interface, and the circuitry

needed to operate the variable speed motors that implement the control signals. Experimental results are then presented with data obtained from trials on both the Annular Core Research Reactor and the MIT Research Reactor. This material is followed by an assessment of the MIT-SNL laws. The report concludes with a discussion of future research needs. Emphasized is the importance of gaining experience with digital technology as applied to reactors by operating prototypes under conditions of closed-loop digital control.

The MIT-SNL research on reactor control is of importance to the space reactor community because the possibility of manipulating a reactor's neutronic power in a controlled manner on periods as short as 0.50 seconds was demonstrated. In particular, the validity of the MIT-SNL Period-Generated Minimum Time Control Laws was established and the concept of controlling neutronic power using the rate of change of reactivity, rather than the reactivity, was shown to be both practical and advantageous. Moreover, the generic nature of the MIT-SNL control laws was proven. Specifically, the same laws and essentially the same implementing software were used to control the neutronic power on two very different reactors. Impressive as these accomplishments are, it should also be recognized that the research reported here has a significance that extends beyond the control of spacecraft reactors. In particular, the present work, when combined with earlier research performed at MIT on the use of reactivity constraints for the supervisory control of reactor power, has established the basis of a theoretically valid, general theory for the control of neutronic power. Also, the results of the experimental program, which dates to 1983, are gradually forming the nucleus of a database from which the confidence may develop to operate both spacecraft and terrestrial reactors using digital control.

Preface

Barely fourteen months elapsed between the initial realization that time-optimal control of a reactor's neutronic power could be achieved by using the rate of change of reactivity as the control signal and the performance of closed-loop experiments on SNL's Annular Core Research Reactor in which that concept was used to raise reactor neutronic power by almost five orders of magnitude both in a few seconds and without overshoot. During those fourteen months the ultimate success of the program was never certain. The ideas being proposed were novel and their merit was not immediately recognized within the reactor community. Also, funding for space nuclear research was interrupted during the last quarter of 1987 when the crucial experiments were scheduled. Several members of the research team were officially 'on vacation' during the actual experiments because there was no other means available to fund the project. The success of this MIT-SNL program was therefore the direct result of the extraordinary determination of those involved.

Besides myself, the experiment team consisted of Mr. Frank V. Thome, Mr. Francis J. Wyant, Mr. Kwan S. Kwok, Mr. Paul T. Menadier, Mr. Michael S. Luker, and Mr. Pat Drozda. Frank Thome was the SNL Project Manager for the program. He is one of those rare individuals whose first instinct is to find a solution rather than to be awed by the obstacles. It was his zeal that kept the experiments on schedule. Francis Wyant, who is now the Project Manager for continuing MIT-SNL work on reactor control, assembled the hardware and built the simulator used to verify the software prior to its actual use on the ACRR. He also acted as the liaison between the experiment team and the Operations Staff of the ACRR. Kwan Kwok, who at the time was the Assistant Superintendent for the MIT Research Reactor, wrote the hardware-to-software interface and operated the computer during the actual experiments. Paul Menadier, Mike Luker, and Pat Drozda procured the required instrumentation, designed and built the special circuitry needed for the experiments, and assembled the apparatus needed for data acquisition. Paul had primary responsibility for the MITR design while Mike and Pat developed that for the ACRR. They are among the best, if not the best, in their field. Paul handled data acquisition for the MITR experiments while Mike did so for the ACRR ones. Both have that uncanny ability to produce whatever is needed on a moment's notice.

In addition to the experiment team, special thanks are reserved for the Operations Staffs of both SNL's Annular Core Research Reactor and the MIT Research Reactor. At the ACRR, those contributing to the success of the experiments were Dr. Theodore Schmidt who is the Manager for Reactor Development and Applications, Dr. Ted Luera who is the Reactor Applications Director, Mr. Max Morris who is the ACRR's Senior Supervisor, and other members of the Staff including Mr. Barry Estes, Mr. Charles Harmon, Mr. Mitch McCrory, and Mr. Ray Trowbridge. At the MITR-II, those contributing to the conduct of the experiments included Mr. William McDermott, Mr. Shing Hei (Eddy) Lau, and Mr. Mark Anderson. One apology is owed the ACRR Staff. For some of the runs

performed in December 1987 and in August 1988, it was thought that the ACRR was subcritical at the time of transfer to the digital controller and several figures were published with this as a tentative explanation of the controller's failure to achieve the specified period. Subsequent analysis has shown that the stepper motor frequency for the runs in question was inadequate.

Appreciation is also extended to the management of the Sandia National Laboratories' Exploratory Nuclear Energy Systems Directorate including Dr. David Ericson, Dr. Pat McDaniel, Dr. Eric Haskin, Dr. Wilhelm Gauster, and Dr. William Snyder. I would especially like to thank Dave Ericson for his spirited defense of the concepts described in this report during a particularly difficult review meeting.

There are many colleagues at MIT who deserve special mention for their contributions to this program. First is Professor David D. Lanning who was a co-Principal Investigator for the project. Prof. Lanning initiated the MIT program on reactor control and his emphasis on reactor safety has been responsible for much of the program's success. Professor Allan F. Henry was the first to derive the alternate form of the dynamic period equation. Also, he supervised students in the development of the numerical method for the real-time calculation of the precursor distributions. Professor John E. Meyer offered much useful advice, particularly in connection with feedback effects. Appreciation is also expressed to the members of the MIT Reactor Safeguards Committee, especially Professors Norman C. Rasmussen, Ronald C. Ballinger, Peter Griffith, and Mujid S. Kazimi. Both Professor Otto K. Harling and Mr. Lincoln Clark, Jr. warrant special mention. Otto, as Director of the Nuclear Reactor Laboratory, provided internal support for the project. Lincoln, who was then Director of Reactor Operations, reviewed the safety aspects of the experiments proposed for the MITR-II. Both also served on the Safeguards Committee.

Students contributing to the project included Brian Aviles, Myung Kim, Shing Hei (Eddy) Lau, and CDR Read Tuddenham. Brian did some of the simulation studies. Myung developed the numerical method for estimating the precursor concentrations. Read and Eddy worked on the feedback methodologies. (Note: This latter work occurred after the official end of the project in December 1987.)

Those who contributed to the preparation of this report included Mr. Kwan S. Kwok, Mr. Shing Hei (Eddy) Lau, and Mr. Ara Sanentz. Kwan wrote major portions of chapters eight and nine. Eddy did much of the data reduction and Ara, as always, edited the many drafts of this report. Mr. Leonard Andexler did the figures, Ms. Georgia Woodsworth and Mrs. Carolyn Hinds typed the report.

Finally, I would like to thank both Dr. Royal Rostenbach of the National Science Foundation and Dr. John Lewellen of the U.S. Department of Energy's Division of University and Industry Programs. Dr. Rostenbach provided support for the MIT research on reactor control during the program's formative years. His understanding of the importance of experiment to education and to scientific progress con-

tinues to inspire many of us. Dr. Lewellen is the Technical Manager for a related MIT research effort on the control of reactors characterized by spatial dynamics. His enthusiasm for this space-reactor project, particularly the experimental aspects, has always been most appreciated.

Material is presented in this report in the chronological order in which the work was performed. Those reading about the concepts for the first time may wish to omit Chapters Seven-Nine which describe experiment preparations. That material is quite detailed and by omitting it on a first reading, understanding of the relation between the practical advantages associated with the use of the MIT-SNL laws and the theoretical basis of those laws may be enhanced. (Note: The information on experiment preparations should of course be read by anyone wishing to replicate or extend the results.)

This report is complete in that both those aspects of the research that were successful and those that were less so are included. Among the latter were an ill-conceived feedback scheme and the decision not to use validated signals. Relative to the use of feedback, a colleague from Japan, Dr. Takashi Washio, subsequently performed a thorough stability analysis of the MIT-SNL minimum time laws. Building on that work, CDR Tuddenham and Mr. Lau then devised a properly formulated, robust feedback methodology. Some information on this is given in Chapter Twelve. Additional material will be available in subsequent reports.

There was a special excitement about this research and a sense that we were breaking new ground. Certainly the ROVER Project on nuclear rockets had achieved rapid increases of reactor power in the 1960s. However, we were the first to accomplish such increases using digital technology and the first to provide a rigorous theoretical foundation for our approach. It is our hope that this work will contribute to the exploration and eventual human settlement of space.

John A. Bernard, Ph.D
Massachusetts Institute of Technology
August 1989

Notice of Patent Protection

Certain of the control concepts described in this report are protected under United States and foreign patents. These include:

- (a) Apparatus and Method for the Closed-Loop Control of Reactor Power: Standard Dynamic Period Equation. (U.S. Patent No. 4,637,911 to Dr. John A. Bernard with rights assigned to the Massachusetts Institute of Technology; foreign patents pending.)
- (b) Apparatus and Method for the Closed-Loop Control of Reactor Power: Alternate Dynamic Period Equation. (U.S. Patent No. 4,710,341 to Dr. John A. Bernard, Dr. Allan F. Henry, Dr. David D. Lanning, and Mr. Kwan S. Kwok with rights assigned to the Massachusetts Institute of Technology; foreign patents pending.)
- (c) Apparatus and Method for the Closed-Loop Control of Reactor Power in Minimum Time: The MIT-SNL Period-Generated Minimum Time Control Laws. (U.S. Patent No. 4,781,881 to Dr. John A. Bernard with royalty-free rights to the U.S. Department of Energy, all other rights reserved; foreign patents pending.)

Those desiring further information should contact either Dr. Bernard or the MIT Technology Licensing Office, Building E32-300, the Massachusetts Institute of Technology, Cambridge, MA 02139.

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Formulation and Experimental Evaluation of Closed-Form Control Laws
for the Rapid Maneuvering of Reactor Neutronic Power

1. Scientific and Engineering Objectives

Both the exploration of space and the effective utilization of its resources for the benefit of mankind depend on the availability of reliable and abundant sources of energy. Most spacecraft, and especially those launched by Western nations, have to date relied on either solar cells or radioisotope generators to provide the necessary power. These sources have been extremely effective. However, as missions increase in complexity, there will be a concomitant rise in energy requirements. Whereas a few watts of electrical power were once sufficient, requirements on the order of 50 kW to perhaps even several hundred megawatts are now being forecast as necessary. Among the civilian space applications that would require kilowatt levels of electrical power are space-based radars for air traffic control, advanced meteorological systems, communication systems, and lunar-based stations. Permanent space colonies would of course need significantly more power as would most military defense applications. Detailed information on the energy requirements of both civilian and military space missions is available in a report, "Advanced Power Sources for Space Missions" that was recently issued by the National Research Council [1]. Both chemical and nuclear systems are among the alternatives being considered as possible solutions. If, in addition to the requirement to supply large amounts of power, the need for long duration is imposed, then nuclear energy is the only viable option. The work reported here was undertaken as part of the effort to make that option a reality and thereby perhaps open the frontiers of space to permanent human habitation and usage.

The implementation of nuclear energy in space will not be achieved by the mere transfer of the technology that has been developed for terrestrial applications. While the physical principles are of course the same, operation in space places emphasis on a different set of boundary conditions. These are for the most part related to difficulties associated with the rejection of heat in space, to the circulation of fluids in the absence of gravitational fields, and to the need to minimize system mass so that launching of the spacecraft will be economical. It is worth noting that the challenge to space-nuclear power is not to solve these problems separately but rather to fashion an integrated design that addresses them collectively.

Thus far, much research has been conducted in the areas of fuel development, energy conversion, and radiation-hardened electronics. Another area that warrants significant study is the design of instrumentation and control systems. The need here is for an unprecedented degree of reliability because reactor-powered spacecraft must be capable of operating autonomously. The capability for in-space repair by astronauts has of course been demonstrated. However, such servicing will be infrequent. Thus, the instrumentation should be designed so

that both abrupt failures and slow degradations can be identified and corrected. This will entail installing redundant sensors, developing models from which correct instrument performance can be inferred, and providing a methodology for reconfiguring valid sensors so as to eliminate failed ones while still fulfilling the mission. Similarly, the implementing equipment for the controller should be both fault-tolerant and designed with sufficient replication of function so as to permit continued system performance despite failure of an individual component. Finally, the control algorithms should be designed to accommodate changes in load. Base-loading, which is the mode in which most terrestrial reactors are now operated, may not be the case. This will be particularly true as mission complexity increases.

The research reported here is the result of a joint effort by the Massachusetts Institute of Technology (MIT) and the Sandia National Laboratories (SNL) to both formulate and demonstrate experimentally closed-form control laws for the rapid maneuvering of a reactor's neutronic power. The actual intent of the work was to provide a means for the autonomous control of spacecraft nuclear reactors. However, the resulting control techniques are generic and hence are applicable to terrestrial reactors as well. This research was funded by the United States Department of Energy as part of the Multi-Megawatt Reactor Program. Support was also received for certain aspects of the work from the U.S. Department of Energy's Office of Energy Research (Division of University and Industry Programs) under contract #DE-AC02-86-NE37962-A000. The period covered by this report is 1 July 1986 - 31 December 1987 although relevant material from other MIT research on reactor control is also presented. The control laws reported here are but one component of an on-going MIT program to develop and experimentally evaluate an integrated, fault-tolerant methodology for the closed-loop, digital control of power and temperature in nuclear reactors. Moreover, it is intended that the control laws developed as part of this joint MIT-SNL project be implemented within the context of such a system. In particular, the overall controller should provide signal validation, instrument fault detection, a supervisory algorithm that precludes challenges to the safety system, a number of selectable control laws, a means of reconfiguring those control laws depending on the mission, a method for verifying signal implementation, and some means for on-line performance assessment such as automated reasoning. In addition, there should of course be an independent safety system. Details of this approach, as applied to nuclear reactors, are given in an earlier report prepared by MIT for the National Science Foundation under grant #CPE-8317878 [2] and in section 12.4 of this report.

1.1 MIT Program on Reactor Control

The MIT program on advanced instrumentation and reactor control originated in the late 1970s under the direction of Professor David D. Lanning of the Nuclear Engineering Department. Studies at that time principally concerned the development of accurate, real-time models of various plant components such as pressurizers, steam generators, and condensers. The existence of accurate models is of course

fundamental to the construction of a controller. In 1980, Dr. John Deyst, Dr. John H. Hopps, and Dr. Asok Ray, who were all from the Charles Stark Draper Laboratory (CSDL), initiated an experimental program at the 5MWt MIT Research Reactor (MITR-II) to demonstrate signal validation and instrument fault detection. The 'parity space approach', which was subsequently used on both test and commercial reactors, was one result of this effort [3]. Once both validated signals and real-time models were available, it became possible to consider closed-loop, digital controllers. A supervisory algorithm that precluded challenges to the reactor's safety system was developed and demonstrated on the MITR-II by Dr. John A. Bernard in February 1983. Designated as the 'MIT-CSDL Non-Linear Digital Controller' or NLDC, this methodology uses reactivity constraints to determine if a change should be made to the present control signal in order to avoid a power overshoot at some future time [4-6]. The 'reactivity constraint approach' was licensed by the U. S. Nuclear Regulatory Commission for general use on the MIT Research Reactor in April 1985 and soon became the basis of an experimental protocol for the on-line testing of novel control strategies under conditions of closed-loop digital control [7]. Funding for the MIT control program was provided by the National Science Foundation (Dr. Royal Rostenbach, Division of Energy and Energetics) from 1984 to 1986. Other significant results achieved with NSF Support included the design and implementation of a rule-based controller, an on-line demonstration of control law reconfiguration, and the derivation by Professor Allan F. Henry of the alternate formulation of the dynamic period equation [8-11]. In July 1986, a project was initiated with the Sandia National Laboratories to develop control strategies for reactor-powered spacecraft. One result of that effort was the derivation by Dr. Bernard of the 'MIT-SNL Period-Generated Minimum Time Control Laws' which are closed-form expressions that permit a reactor's neutronic power to be raised by many orders of magnitude both without overshoot and within a few seconds. With the support of Mr. Kwan S. Kwok and Mr. Paul T. Menadier of MIT and Mr. Frank V. Thome and Mr. Francis J. Wyant of SNL, both these laws and the NLDC were subsequently demonstrated experimentally on SNL's Annular Core Research Reactor. The theoretical derivation of the MIT-SNL laws and their experimental evaluation are the subjects of this report. In September 1986, a project was begun under sponsorship of the U.S. Department of Energy to extend the reactivity constraint approach to large, spatially-dependent reactors. That work involves the development of 'faster-than-real-time' reactor models that use supernodal methods to describe the neutron flux behavior and fast-running thermal-hydraulic codes to characterize reactivity feedback effects. More recently, joint research has been initiated with the Oak Ridge National Laboratory concerning the closed-loop digital control of multi-modular reactors. Also, a joint program on automated reactor startups is in progress with the Sandia National Laboratories.

1.2 Rationale for Experimental Research

A distinguishing and perhaps unique characteristic of the MIT program on reactor control and also of the joint MIT-SNL research has

been the performance of on-line experiments. Every control concept has been evaluated experimentally on either the 5 MWt Research Reactor (MITR-II) or the Annular Core Research Reactor (ACRR) that is operated by the Sandia National Laboratories, or both. This was done in the firm conviction that true engineering progress can only be achieved through extensive experimentation and prototypic demonstration. Simulation studies are, of course, of importance. They serve to validate software, to provide initial evaluation of concepts, and to evaluate techniques beyond the allowed range of operation of actual systems. However, simulation is not a substitute for experiment. Simulations are successful only insofar as the process being studied can be modeled. If something is not known or not fully understood, it can not be simulated. Hence the need for well-designed and rigorously monitored experiments. Moreover, by performing on-line tests, confidence is built in the approach. It is unrealistic to expect either industry to adopt or regulators to accept advanced digital technology without there first being a history of reliable operation at the prototype level.

1.3 Overview of the MIT-SNL Control Research: 1986-1987

The principal objective of the 1986-1987 joint MIT-SNL research program was to develop a robust control technique that would permit a reactor's neutronic power to be raised by five to seven orders of magnitude both in a few seconds and without significant overshoot. The program was extremely successful with the principal result being the 'MIT-SNL Period-Generated Minimum Time Control Laws'. These laws function by altering the rate of change of reactivity so that the instantaneous reactor period is stepped from infinity to its minimum allowed value, held at that value until the desired power level is attained, and then stepped back to infinity. The resulting power transient is time-optimal because the reactor period is always at its limiting value. Following their extensive evaluation using computerized simulations, the MIT-SNL laws were demonstrated experimentally, first on the 5 MWt MIT Research Reactor or MITR-II and later on the Annular Core Research Reactor or ACRR that is operated by the Sandia National Laboratories. As of 4 December 1987, these laws had been used to raise the ACRR's neutronic power by almost five orders of magnitude on a constant period of 0.50 seconds without overshoot. The rationale, derivation, evaluation, and on-line trials of the MIT-SNL minimum time laws are discussed in the ensuing sections of this report. Listed here are the major accomplishments of the program:

- Development of a novel method for the identification of time-optimal solutions for systems bounded by a constraint.
- A rigorous derivation of the standard, dynamic period equation.
- Realization that the rate of change of reactivity rather than the reactivity itself should be used as the control signal for reactor power controllers.

- Derivation of the 'MIT-SNL Period-Generated Minimum Time Control Laws' which are closed-form expressions for the time-optimal adjustment of reactor neutronic power.
- Identification of an accurate, real-time method for estimating precursor concentrations.
- Feasibility studies of the MIT-SNL Period-Generated Minimum Time Control Laws using simulators.
- Characterization of the MIT-SNL Period-Generated Minimum Time Control Laws through parameter sensitivity studies.
- Demonstration of energy constraints for the closed-loop control of a nuclear reactor.
- Design, construction, and implementation of the hardware and hardware-to-software interface necessary for the conduct of rapid reactor transients under conditions of closed-loop, digital control.
- On-line, experimental trials of the MIT-SNL Period-Generated Minimum Time Control Laws on both the Annular Core Research Reactor and the MIT Research Reactor.
- Demonstration of the generic nature of both the 'MIT-SNL Control Laws' and the 'Reactivity Constraint Approach'.
- Incorporation of feedback in the MIT-SNL laws.

The principal application of the technology is, of course, space nuclear systems. However, as discussed in section 12.5 of the report, use of the MIT-SNL laws would also be of benefit to the control of terrestrial reactors including ones for both commercial use and research. Also, the methodology for identifying the time-optimal solution is applicable to non-nuclear systems.

1.3.1 Uniqueness of the MIT-SNL Control Laws

The MIT-SNL minimum time control laws are unique in two respects. First, they are closed-form expressions for the time-optimal control of power in reactors that are subject to a limitation on the allowed period. As such they represent a significant advance because previous efforts at identifying a time-optimal response have, for the most part, resulted in sequences of signals that must be applied in an open-loop manner and which are valid for only one set of boundary conditions [12-14]. In contrast, because the MIT-SNL expressions are actual control laws, they can be used with any initial conditions and they are amenable to feedback. This is unquestionably the most significant contribution of the MIT-SNL laws to reactor operation. If properly used, feedback ensures that a specified setpoint and/or trajectory will be maintained despite both errors in the model of the process in question and perturbations of unknown origin. Lacking

feedback, a controller may not drive a parameter to the specified value. Through use of the MIT-SNL laws, deviations in the period, reactivity, and precursor concentrations can be used in real-time to cause a reactor's neutronic power to adhere to a desired trajectory.

The second unique aspect of the MIT-SNL laws is that the control signal is the rate of change of reactivity. This too is a significant advance because the traditional approach to the automated adjustment of reactor power is to use the reactivity itself as the control signal. The resulting control schemes are often difficult to implement because reactivity is not a directly controllable quantity. Specifically, control mechanisms have finite speeds and reactivity, which is a function of the net change in each mechanism's position, can therefore not be altered on demand. In contrast, the rate of change of reactivity is directly controllable because its value is solely a function of the control mechanism's speed. Adjustments in speed can be made essentially at once whereas a change in position requires a finite amount of time. Controllers designed in terms of the rate of change of reactivity are therefore very responsive. In particular, assuming no limitation on actuator speed, they can provide the period necessary to achieve virtually any desired power profile.

1.4 Experimental Facilities

Experiments were conducted on both the 5 MWt MIT Research Reactor and on the Annular Core Research Reactor that is operated by the Sandia National Laboratories.

1.4.1 MIT Research Reactor (MITR-II)

The MITR-II is a 5 MWt, light-water cooled and moderated, heavy-water reflected, tank-type reactor that uses plate-type, uranium-aluminide fuel. The fuel is enriched to 93% U-235. Energy is continuously removed by forced circulation of the primary coolant. The maximum permitted operating temperature is 55°C. Figures 1.4.1-1 and 1.4.1-2 are isometric and vertical cross section views respectively of the facility.

The nuclear instrumentation used for the research described in this report consisted of three neutron flux sensors and a gamma-ray sensor that correlated neutron power with the radioactivity (N-16) of the primary coolant. All four sensors were directly proportional to the power over the range of interest. Measurements were also available of the coolant flow, coolant temperature, and control mechanism position. Four independent measurements of primary coolant flow were obtained from the pressure differences across orifices. Primary coolant temperatures were measured as follows: two sensors for the hot leg, two sensors for the cold leg, and one sensor for the temperature difference between the legs. In effect, three measurements were available for the temperature difference. All sensors were hard-wired to a portable LSI-11/23 minicomputer through appropriate isolators, signal conditioners, and A/D converters. None of the sensors that form the MITR-II's safety system were used for this research. Also,

Figure 1.4.1-1 Isometric View of MIT Research Reactor

Figure 1.4.1-2 Vertical Cross Section of MIT Research Reactor

it should be noted that the noise and statistical characteristics of the MITR-II's flow, temperature, and neutron flux instrumentation are similar to those in commercial reactors.

Coarse control of the power in the MITR-II is achieved by positioning a bank of six shim blades. Once critical, the neutron flux is normally maintained constant by adjusting the position of a fine-control regulating rod. Both the regulating rod and one of the shim blades were made available to the experimental program described in this report. Each is normally moved at a fixed speed of 4.25 inches per minute. However, for the research reported here, each was specially equipped with a variable speed stepping motor so that the rate of change of reactivity could be made to vary as specified by the control laws. The minimum allowed periods on the MITR-II are 50 seconds steady and 30 seconds dynamic. There is a negative coefficient of reactivity associated with the fuel, coolant, and reflector temperatures. However, its magnitude is rather small, averaging $-7.86 \cdot 10^{-5} \Delta K/K/^{\circ}C$. The MITR-II's effective delayed neutron fraction and prompt neutron lifetime are 0.00786 $\Delta K/K$ and 100 microseconds respectively.

1.4.2 Annular Core Research Reactor (ACRR)

The ACRR is a modified TRIGA that uses UO_2 -BeO fuel elements, enriched to 35% U-235. Its annular-shaped core is formed by 236 of these elements arranged in a hexagonal grid around the 23 cm diameter, dry, central irradiation cavity. The reactor operates in either a steady-state or a pulsed mode. For steady-state operation, the maximum allowed power level is 2 MWt. For pulsed operation, there is no restriction of the power. Rather, there is a limit of 500 MJ total energy per pulse and one of 1800 $^{\circ}C$ on the fuel temperature.

The ACRR is controlled by two fuel-followed safety rods, three poison transient rods, and six fuel-followed control rods. The transient rods, which are operated as a bank and which are driven by variable speed stepping motors, were used for the conduct of the experiments. There is an exceptionally large negative coefficient of reactivity associated with the fuel which is, in units of $\Delta K/K/^{\circ}C$, given by the expression $(-3.85 - 730/T) \cdot 10^{-5}$ where T is the temperature of the fuel in K. The ACRR's effective delayed neutron fraction and prompt neutron lifetime are 0.0073 $\Delta K/K$ and 24 microseconds respectively. Figure 1.4.2-1 is a vertical cutaway view showing the major components of the ACRR.

1.5 Experimental Protocol

The testing of novel control strategies on the MIT Research Reactor is permitted if the following protocol is observed. First, the heat removal and the reactor safety systems are prepared for operation at full power, 5 MW. Second, the control strategy that is to be tested is permitted to raise or lower the power over some portion of the normal operating range, usually 1-4 MW. Third, the decisions of the novel controller are reviewed by the MIT-CSDL Non-Linear Digital Controller (NLDC) prior to their being implemented. The NLDC is based on

Annular Core Research Reactor Facility

Figure 1.4.2-1 Isometric View of Annular Core Research Reactor Facility

(Source: Sandia National Laboratories)

the 'reactivity constraint approach' [2]. The NLDC is programmed to intervene if a decision made by the novel controller could result in the power exceeding some fraction of the maximum allowed power, usually 4.5 MW. This arrangement guarantees that the novel controller will not challenge the safety system while permitting it to act as if it had full control. Hence, when examining the experimental results shown in this report, it should be realized that the fact that power overshoot the targeted value in some cases is significant to the experiment, but never to the reactor which was at all times operated conservatively.

It was necessary to modify this protocol for use on the Annular Core Research Reactor (ACRR). The limiting condition for the ACRR is not a specific power level but rather the total energy produced during the transient. Accordingly, it was necessary to develop an energy constraint which assured that the reactor would be at or below its allowed steady-state operating power prior to the limit on integrated power production being exceeded. Details are given in section 7.3.2.1 of this report.

2. Scope of this Report

This report covers both the scientific and the engineering progress achieved as a result of the joint MIT-SNL project "Formulation and Experimental Evaluation of Closed-Form Control Laws for the Rapid Maneuvering of Reactor Neutronic Power". The period covered is from 1 July 1986 to 31 December 1987. This report builds on an earlier one entitled, "Fault-Tolerant Systems Approach Toward Closed-Loop Digital Control of Nuclear Power Reactors" that was prepared by MIT for the National Science Foundation [2].

This report on the joint MIT-SNL research on reactor control begins with a review of control methodologies and reactor dynamics. The material on control is introductory in nature and is intended as an aid to readers who may not be experienced in that topic. Compared are the salient features of the major techniques now used in process control including proportional-integral-derivative action, transfer functions, state-space methods, and optimal control. The material on reactor operation stresses the non-linear, time-delayed nature of reactor dynamics. Emphasized is the importance of developing both supervisory and predictive methods for the control of such systems. The main body of the report then follows with four topics being extensively covered. These are:

- Theoretical foundations of the MIT-SNL laws.
- Preparations for experimental evaluations.
- The results of the on-line experimental trials of the MIT-SNL laws under conditions of closed-loop control.
- An assessment of the MIT-SNL laws.

Chapters Four-Six provide the theoretical foundations for the MIT-SNL Period-Generated Minimum Time Control Laws. Chapter Four includes the derivation of the standard and alternate dynamic period equations, an enumeration of the advantages associated with the use of these equations as a reactor model, and a discussion of the rationale for using the rate of change of reactivity as the control signal. With this material as background, the MIT-SNL laws are then derived. Chapter Five lists conditions important to the implementation of the MIT-SNL laws including the relation of these laws to the reactivity constraint approach, the calculational sequence for evaluating the various terms in the laws, and methods for the real-time estimation of precursor concentrations. Chapter Six then reports the results of simulation studies in which the performance of the MIT-SNL laws was assessed. Chapters Seven-Nine describe the preparations undertaken for the experimental evaluation of the MIT-SNL laws. Included is material on safety considerations and regulatory requirements, the hardware and the hardware-to-software interface, simulator studies, instrument calibrations, and the organization of the experiment team. Chapters Ten and Eleven present the results of the experimental trials in which the efficacy of the MIT-SNL laws was demonstrated on both the MITR-II

and the ACRR under conditions of closed-loop, digital control. Chapter Twelve provides an assessment of these laws. The report concludes with a discussion of further research needed relative to the utilization of digital controls for the operation of nuclear facilities.

3. Control Methodologies and Reactor Dynamics

Brief reviews of both the principal control methodologies now in use and of reactor dynamics are presented here as an aid to the reader. Relative to the former, a survey is given of the major control techniques with the intent of identifying trends and unresolved issues for the non-specialist. Relative to the latter, a discussion is given of reactors that can be described by space-independent kinetics. Reactors of this type, which have power levels ranging from a few watts to several hundred megawatts, are the ones presently being considered for use in space. This chapter concludes with a discussion of the contributions of both the current MIT-SNL program and previous MIT efforts to reactor control and to the realization of a general methodology for non-linear control.

3.1 Basic Control Concepts and Definitions

Central to the control of any entity or system is the concept of 'feedback'. On a formal basis, it is defined as the process whereby the difference between the output of a system and a reference input is used to maintain a prescribed relation between that output and the reference. Feedback is, of course, an everyday phenomenon. Relative to engineering, its use for the control of mechanical systems is generally considered to date from James Watt's invention of the ball-governor to control his newly developed steam engine.

Control systems can be broadly categorized according to the use that is made of feedback. Figure 3.1-1 shows a generally accepted scheme in which systems are classified as manual, closed-loop, or open-loop. Using manual control, a human being compares the output of the process to that desired and then adjusts the actuator so as to maintain that output as specified. The operator is using feedback in that he or she first monitors the instrumentation and then combines that information with knowledge of the plant dynamics to determine the appropriate signal for the actuator.

Closed-loop control is similar to manual operation except that the plant's output is used directly in the determination of the control signal. This is shown in the middle portion of the figure. The 'reference output' reflects the desired or specified manner in which the plant is to be operated. This is compared to the measured output which is the signal obtained from the instrumentation. The difference between these two quantities is the 'actuating error signal'. The control signal, which is sent to the actuator, is a function of this error. The error and control signals could be computed by either analog or digital computers. Or, as was the case with the ball-governor for the steam engine, they could be determined by mechanical means. Under closed-loop control, the signal to the actuator is generated as the process evolves. Hence allowance is automatically made for any perturbations that may affect the plant's dynamics. One final point that is worth noting about closed-loop control is that the control signal is a function of the measured rather than the actual output of the plant. Hence, if the instruments fail, so will the control sys-

Figure 3.1-1 Manual, Closed-Loop, and Open-Loop Control

tem. This may not matter during manual operation because human beings can sometimes recognize and compensate for failed sensors. However, under closed-loop conditions, the sensor's erroneous output will be used. The solution to this problem is sensor validation which is discussed in sections 10.2.1 and 10.4.1 of this report.

Open-loop control is shown in the lower portion of the figure. The distinguishing feature of this approach is that no use is made of feedback. Rather, a predetermined reference trajectory is specified. For example, an accurate model of the process in question could be constructed and simulations performed until the sequence of control signals needed to generate the desired output was identified. Open-loop control may or may not result in the desired plant behavior. Clearly if the model used to determine the trajectory is in error, then so will the control action. However, even with a perfect model, the results might be poor. There is, for example, no way to allow for unanticipated changes in the plant. Despite its drawbacks, open-loop control is extensively studied and, as discussed below, some sophisticated control strategies use it.

3.2 Control Methodologies

Enumerated here are the major control methodologies that are now in use. As general references, the texts 'State Functions and Linear Control Systems' by Schultz and Melsa (McGraw-Hill, 1967), 'Control System Design: An Introduction to State-Space Methods' by Friedland (McGraw-Hill, 1986), 'Modern Control Engineering' by Ogata (Prentice-Hall, 1970) and 'Process Dynamics and Control' by Seborg, Edgar, and Mellichamp (John Wiley & Sons, 1989) are recommended. The definitions given in section 3.2.3.1 of this report are from the first of these.

3.2.1 Proportional-Integral-Derivative Control

Proportional-Integral-Derivative or P-I-D control is the most commonly used method for the automated operation of process systems. Using this approach, the system itself is treated as a 'black box'. That is, the control signal is determined solely by comparison of the measured and reference outputs. Information on the dynamics of the process, even if available, is not used. Figure 3.2.1-1 is a block diagram of the approach. The error signal is taken as the difference between the reference and measured outputs of the process. If the controller output is only proportional to this error, then the control action is termed 'proportional'. If the controller output also contains a term that is a function of the accumulated error, then it is designated as 'proportional-integral'. Finally, if the controller output consists of terms representing both the integral and the rate of change of the error signal as well as the error signal itself, then it is referred to as 'proportional-integral-derivative' or P-I-D.

It is instructive to examine the mathematics of the P-I-D control approach. For simplicity assume that the actual and measured plant outputs are identical. That is, the instruments are functioning perfectly. Suppose that the controller is strictly proportional and that

Figure 3.2.1-1 Proportional-Integral-Derivative Control

the system and the controller can be represented respectively by the following relations:

$$n(t) = Hu(t) \quad (3.2.1-1)$$

$$u(t) = Ke(t) \quad (3.2.1-2)$$

where: $n(t)$ is the plant output,
H denotes the process dynamics,
 $u(t)$ is the control signal,
K is the gain of the controller, and
 $e(t)$ is the error signal.

Also note that the error signal is defined as:

$$e(t) = (\bar{n}(t) - n(t)) \quad (3.2.1-3)$$

where $\bar{n}(t)$ is the reference output. Combining the above relations yields:

$$\begin{aligned} n(t) &= Hu(t) \\ &= HKe(t) \\ &= HK(\bar{n}(t) - n(t)) \end{aligned} \quad (3.2.1-4)$$

Solving for $n(t)$ yields:

$$n(t) = \left[\frac{HK}{1 + HK} \right] \bar{n}(t) \quad (3.2.1-5)$$

This simple relation illustrates the enormous advantages to using feedback and closed-loop control. Specifically, note that by making the gain arbitrarily large, the quantity $(HK/(1+HK))$ will be driven to unity and the observed plant output, $n(t)$, will become equal to the reference or desired output, $\bar{n}(t)$. Of special significance is that this result is achieved without any knowledge of the process dynamics and that it is valid for any reference output. Moreover, even if the process dynamics change or are perturbed, it should be possible to maintain the output of the process at the desired value. Hence the appeal of closed-loop control.

Unfortunately, it is not possible to achieve all of the theoretical advantages associated with the proportional controller described above. For one thing, the controller gain can not be made arbitrarily large. So doing will make the system increasingly sensitive to small perturbations and may eventually cause instability. This creates an immediate problem because, for finite values of K, the quantity $(HK/(1+HK))$ will be less than unity and hence $n(t)$ will never quite equal $\bar{n}(t)$. This is a major disadvantage of proportional control. Such systems exhibit an offset between the desired and observed outputs. This problem can be rectified by adding integral action to the controller. That is, instead of a control signal that is directly

proportional to the error signal as in Equation 3.2.1-2, the control signal is of the form:

$$u(t) = Ke(t) + \frac{K}{T_i} \int_0^t e(t)dt \quad (3.2.1-6)$$

where the quantity T_i is the integral time. The inverse of the integral time, which is termed the 'reset rate' has a physical interpretation. It is the number of occasions per minute that the proportional part of the control action is duplicated. The advantage of adding an integral term to the control action is that the control signal can have a non-zero value even though the error is zero. Thus, the use of integral action resolves the offset problem. Any difference between the desired and observed outputs will eventually cause the total error to accumulate to the point where it drives system response to the specified value. The drawback to integral action is that it may induce oscillations as the system converges on the specified output.

Controller performance may be further improved by adding a derivative term to the control signal. Thus,

$$u(t) = Ke(t) + \frac{K}{T_i} \int_0^t e(t)dt + KT_d \frac{de(t)}{dt} \quad (3.2.1-7)$$

The quantity T_d is the derivative time. Physically, it is the interval by which the proportional part of the control action is advanced. Derivative action is anticipatory and thus its use may reduce oscillations. However, it may also amplify signal noise.

In summary, the advantages of the P-I-D approach are that the technique can be applied without regard to the process dynamics, that its basic concepts are readily understood, and that it can be implemented using either analog or digital equipment. Its disadvantages are that the selection of the control parameters (the gain and the integral and derivative times) is empirical and that these parameters will be valid for only one particular transient. Thus the use of the P-I-D approach for non-linear or time-delayed systems is impractical. Another drawback is that the technique can only handle one input and one output. Also, the theory of P-I-D control does not provide the basis for determining system stability.

3.2.2 Transfer Function Approach

The use of transfer functions in control system design represents a major advance over the P-I-D approach because the system is no longer treated as a black box. As a result, the stability of the controlled system can be analyzed and theoretically valid methods can be used to select the gain coefficient. However, whereas the P-I-D approach was in theory applicable to any process, use of the transfer function approach is limited to linear, time-invariant systems.

A transfer function is defined as the ratio of the Laplace trans-

form of the system output to the Laplace transform of the system input. For a given system, it is obtained by first identifying the plant's describing differential equation, then taking the Laplace transform, and finally rearranging terms so as to obtain the ratio of the output to the input. This process results in a transfer from the time domain (differential equation) to the frequency domain (transfer function). As an illustration of the utility of the approach, suppose that the system (actuator plus process) shown in Figure 3.2.1-1 was actually described by the following second-order, linear differential equation:

$$u(t) = \ddot{y}(t) + \dot{y}(t) \quad (3.2.2-1)$$

Taking the Laplace transform of this equation subject to the assumption that all initial conditions are zero yields:

$$U(s) = s^2Y(s) + sY(s) \quad (3.2.2-2)$$

Hence, the transfer function of the process is:

$$\frac{Y(s)}{U(s)} = \frac{1}{s(s+1)} \quad (3.2.2-3)$$

Further suppose that a control scheme is to be designed which uses the same feedback approach as did the P-I-D controller. That is, only the output of the system is fed back. The process is illustrated in Figure 3.2.2-1. The closed-loop behavior of the system is described by the relation:

$$[R(s) - Y(s)](K) \left[\frac{1}{s(s+1)} \right] = Y(s) \quad (3.2.2-4)$$

Hence, the closed-loop transfer function is:

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2 + s + K} \quad (3.2.2-5)$$

This can be written in factored form as:

$$\frac{Y(s)}{R(s)} = \frac{K}{(s-a)(s-b)} \quad (3.2.2-6)$$

where, using the quadratic formula, the quantities a and b are determined to be:

$$a = -0.5 + 0.5\sqrt{1-4K}; \quad b = -0.5 - 0.5\sqrt{1-4K} \quad (3.2.2-7)$$

Finally, by taking the inverse Laplace transfer of (3.2.2-6), the closed-loop behavior of the system can be deduced. So doing,

$$y(t) = \left(\frac{1}{a-b} \right) [ae^{at} - be^{bt}] \quad (3.2.2-8)$$

Much can be determined about system stability and response using this

Figure 3.2.2-1 Transfer Function Approach to Controller Design

analytic solution for the behavior of the closed-loop system. First note that the value of the gain can range from zero to infinity. If the gain lies between 0.00 and 0.25, then the quantities a and b , which are the roots of the closed-loop transfer function, are negative real numbers. If the value of the gain exceeds 0.25, then those roots are complex numbers with negative real parts. Hence, for any value of the gain, the exponential terms ($\text{EXP}(at)$ and $\text{EXP}(bt)$) will die out as time increases. The system is therefore unconditionally stable. (Note: For other systems, it is possible that for a certain range of gains, the roots would be positive implying unbounded exponential growth and instability.) Another important feature to note is that if the value of the gain exceeds 0.25, the system response will be underdamped meaning that the output will oscillate about its desired value before settling out. If the gain is equal to 0.25, the system should attain the desired response without overshoot. If the gain is less than 0.25, then there will still be no overshoot but the response time will be longer than is perhaps necessary.

The above example shows that by including a model of the system in the controller, much insight can be achieved. Using the P-I-D approach, the only guidance received was to make the gain as large as possible. But with the transfer function approach, it is possible to identify the range of gains for which the system will be stable and to select a gain within that range to achieve desired response characteristics. (Note: The system shown in Figure 3.2.2-1 was analyzed by obtaining the closed-loop response, Equation 3.2.2-8. In reality, this was not necessary because using methods such as 'root-locus' or Routh's stability criteria, the same information could have been obtained directly from the transfer function. This is a further strength of the technique.)

In summary, the transfer function approach can be applied to linear, time-invariant systems. It provides a rational means for analyzing system stability and for specifying certain system response characteristics. It can be implemented using either digital or analog equipment. Its disadvantages are that it is limited to linear systems, that it treats only single-input/single-output systems, and that it does not address certain fundamental issues such as what constitutes a proper choice of control signal. Also, the proper functioning of the controller is dependent on the accuracy of the model chosen as representing the system during the design stage. This raises a number of issues including model validation and maintenance.

3.2.3 State-Space Methods

State-space methods are arguably the most successful of the various control techniques now available. This methodology was the one used to design the control systems for the Apollo spacecraft and it is the one now used for many aerospace applications. The basic concept originated in the 1960s and, although nearly three decades have passed, the methodology is still referred to by the misnomer 'modern control theory'. As with the transfer function approach, the strength of state-space methods is the use of a system model. Only instead of

using a single differential equation to relate system output to system input, a set of first order differential equations is used. Thus, full advantage is taken of the knowledge given by the internal structure of the system. For example, in the preceeding section of this report, a system characterized by the following equation was analyzed:

$$u(t) = \ddot{y}(t) + \dot{y}(t) \quad (3.2.3-1)$$

This is a second order differential equation where y is the output and u is the input. Using the state-space approach, this is rewritten as two first order differential equations. The state variables, here denoted by the symbols x_1 and x_2 , are defined as:

$$x_1 = y \quad (3.2.3-2(a))$$

$$x_2 = \dot{y} \quad (3.2.3-2(b))$$

Thus,

$$\dot{x}_1 = x_2 \quad (3.2.3-3(a))$$

$$\dot{x}_2 = -x_2 + u \quad (3.2.3-3(b))$$

The second of these equations is obtained by solving (3.2.3-1) for the quantity $\ddot{y}$ and then substituting x_2 for $\dot{y}$. Equations 3.2.3-3 can be written in matrix form to facilitate solution on a digital machine:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad (3.2.3-4)$$

Note that by substituting (3.2.3-3(b)) into (3.2.3-3(a)) and then noting that $\dot{x}_1$ equals $\dot{y}$ and that $\dot{x}_2$ equals $\ddot{y}$, equation 3.2.3-1 is regenerated. Thus, both representations (3.2.3-1 and 3.2.3-3) provide descriptions of the system. The advantage associated with the state-space approach is that by using a set of n first order differential equations to describe an n th order system, the internal dynamics of that system become accessible. Figure 3.2.3-1 is a block diagram illustrating the value of this approach for controller design. Instead of feeding back merely the output of the process, each state variable is assigned a feedback coefficient. (Note: The symbol 'h' is used to denote the feedback coefficients in the figure.) Hence, for an n th order system, n degrees of freedom are introduced. This provides the control engineer with enormous flexibility. In particular, by the judicious choice of the feedback or gain coefficients associated with each of the state variables, the shape of the system's response as well as its stability can be specified. There are other advantages as well to the use of state-space methods. Using this approach, controllers can be designed for systems that have multiple inputs and out-

Figure 3.2.3-1 State-Space Approach to Controller Design

puts. Also, as is discussed below in section 3.2.3.1 of this report, the state-space methodology addresses fundamental issues such as the criteria by which it can be determined if a system is in fact controllable. The technique is however not without its drawbacks. For one thing, its use is limited to linear systems. For another, it may not be possible to give a physical meaning to each of the state variables and even if it is, there is no guarantee that the variable can be measured. For example, relative to a nuclear reactor, precursor concentrations are state variables that cannot be directly observed.

3.2.3.1 Controllability and Observability

The concepts of 'controllability' and 'observability' were first introduced by Kalman in the 1950s. The former concerns the capability of the control signal (i.e., the input to the controller) to affect each of the state variables. The latter is the capability of each state variable to affect the output. On a formal basis, these are defined as:

- (1) Controllability: A system is said to be 'controllable' if any initial state $x(0)$ can be transferred to any final state $x(t_f)$ in a finite time, $t_f \geq 0$, by some control, u .
- (2) Observability: A system is said to be 'observable' if every state $x(0)$ can be exactly determined from measurements of the output y over a finite interval of time, $0 \leq t \leq t_f$.

Most physical systems are both controllable and observable. Hence the need to verify these properties often appears to be of only academic interest. However, such should not be the case, especially when considering complex systems. Two issues are of concern. First, while it is true that most physical systems are controllable, their associated mathematical models may lack that property. This most often occurs when a linearized model is used to represent a non-linear system. The second issue is that some systems may be only partially controllable. This is a complex problem that has not yet been fully addressed on a theoretical basis. Specifically, the formal definition of controllability provides no information on the degree to which a system may or may not be controllable. This would be most desirable because many systems and their associated control mechanisms fulfill the definition of controllability in the sense that given enough time and with no restriction on the allowed trajectories, it is possible to cause the desired change using the specified control device. Yet, the resulting control action may be of poor quality. For example, suppose that a controller for a spacecraft nuclear reactor is to be constructed in which the reactor power is to be adjusted in response to a temperature sensor. Moreover, suppose that this sensor must be located some distance from the reactor so that radiation damage effects are minimized. Is the system controllable? Clearly, if the reactor power were to change, then so would the temperature of the reactor coolant. But the sensor is located on a boom and hence the temperature of the coolant in which it is immersed will not change until heat has been conducted along the entire length of the boom.

Thus, there will be a significant time delay associated with the use of such a sensor for control and the resulting control action may be oscillatory. The system is controllable according to the definition. Nevertheless, the resulting power trajectory may exhibit undesirable features.

Concerns related to controllability and in particular to the need to refine the concept so that the controller is capable of producing not only the desired end-state but also an acceptable trajectory have been a major aspect of the MIT program in the closed-loop digital control of reactor power. Much has been accomplished. In particular, the concept of 'feasibility of control' has been defined and the rate of change of reactivity (as opposed to the reactivity itself) has been recognized as the appropriate means of control. Both of these concepts are discussed in detail in Chapter Four of this report. For the present, it is sufficient to note that 'feasibility of control' is, like controllability, a property of the controller. Observance implies that the allowed states of a system are restricted such that it will always be possible to halt a transient on demand. Relative to nuclear reactors, completed work at MIT has provided a practical method for both defining and incorporating such a property in controllers for reactor neutronic power. The selection of the rate of change of reactivity as the control signal bears on this issue as well. Specifically, it is a directly controllable quantity in the sense that it can be altered on demand. In contrast, reactivity can only be changed over some finite interval of time.

3.2.3.2 Summary of State-Space Approach

In summary, the state-space approach is currently the most widely used technique for the design of controllers for complex systems. Its advantages are that it can be used both to assure system stability and to achieve a desired system trajectory. Moreover it is capable of handling multiple-input/multiple-output systems. Also, its underlying theory addresses fundamental issues such as what constitutes a controllable system. Its disadvantages are that its use is restricted to linear systems, that its implementation may require information on variables that cannot be measured, and that it does not address the degree to which a system may be controllable. Also, the solution of most state-space design problems requires the use of a digital computer.

3.2.4 Optimal Control

Control system performance is considered optimal in the sense that the resulting trajectory either minimizes or maximizes a designer-specified performance index. For example, it may be desired to complete a transition in minimum time or with as little fuel consumption as possible. The realization of optimal control is achieved by using a state-space representation. That is, the system (process plus actuator) is described using a set of first order differential equations. In the state-space approach, each state variable (i.e., the independent variables in the describing equations) is assigned a

feedback coefficient or gain. The user then selects the desired shape or trajectory of the system's response and solves for the gains so as to achieve that response. The burden of selecting the appropriate trajectory is therefore on the designer. With optimal control, the designer merely selects a performance criteria. The system's gain is chosen so as to minimize (or, if appropriate, maximize) this index. Thus, the designer is relieved of a major burden. Unfortunately, another one is imposed in that the mathematics of the approach are often intractable. For example, if the Pontryagin approach to optimal behavior is used, then it becomes necessary to solve a set of partial differential equations with split boundary conditions. That is, some conditions are known that apply to the initiation of the transient and some to its termination, but there is not a complete set at either the start or end of the transient. Hence, the problem must be solved by iteration. Figure 3.2.4-1 illustrates the problem. The objective is to identify a trajectory that optimizes some performance index. Assume that the optimal trajectory is the one indicated by the broken line. The curves denoted by the letters 'a' and 'b' represent successive attempts to identify the optimal solution. The separate pieces of each curve are generated by marching forward from the initial conditions and backwards from the final ones with the hope of the two pieces joining to form a single path. This process is iterative often with many successive attempts being required before the desired path is found. Thus, the solution to an optimal control problem is calculation-intensive. This means that, unless the problem is very simple, the calculations will take longer than the time that is available during each sampling interval. Hence, many optimal control problems are solved in their entirety prior to initiating the actual control action. The solution to an optimal control problem is therefore usually not a closed-form law but rather a sequence of signals that are to be applied at each sampling interval. The drawback to this approach is that this sequence will be valid only for the boundary conditions that were initially specified. No provisions exist for variations in those conditions and there is no continuous feedback of measured signals. In other words, this is a form of open-loop control. If the system model is perfect and if the boundary conditions have not changed, then the resulting control action will be as desired. Otherwise, an incorrect trajectory will be generated. Alternatives to the split boundary condition problem as formulated by Pontryagin do exist. For example, Bellman's dynamic programming avoids that problem. However, it too is calculation-intensive. This is the weakness of the optimal approach to system control.

A major accomplishment of the MIT-SNL program that is described in this report is that a method was found to obtain closed-loop, time-optimal control laws for the adjustment of a reactor's neutronic power. Details are given in Chapters Four and Twelve of this report.

In summary, the advantage of optimal control is that it allows the control engineer to obtain a desired response without having to specify the details of the trajectory. (Note: In the language of the state-space approach, "without having to select the locations of the closed-loop poles.") Another advantage is that the technique is

Figure 3.2.4-1 Successive Approximations to an Optimal Path

applicable to non-linear systems. Disadvantages are that the mathematics associated with the technique are often intractable and this in turn may cause resulting control action to be open-loop with no provisions for feedback.

3.3 Trends and Unresolved Issues in Control System Design

The brief review of control methodologies given in section 3.2 of this report considered the P-I-D approach, the use of transfer functions, state-space methods, and optimal control. There are of course many variations on these basic techniques as well as other distinct methodologies. However, the ones examined here are the dominant approaches. Several trends and unresolved issues are apparent from the review. These bear directly on the design of control systems for complex technologies such as nuclear reactors.

First, increasing reliance is being placed upon system models. The P-I-D approach avoids use of a model. Transfer functions are themselves a model with only the system input and output accessible. Both state-space and optimal control assume the existence of accurate models replete with details of the system's inner structure. As a general observation, it appears that the mathematics available for utilizing a model has outstripped the capability to delineate such models. That is, the limiting factor in taking advantage of the benefits of advanced control concepts is, in many respects, the difficulty associated with obtaining accurate representations of the plant. In particular, if these control techniques are to be applied to systems for which safety is of paramount importance, then issues such as model validation, calibration, and maintenance should be addressed on a formal basis.

A second issue is that of partial controllability. This was discussed in section 3.2.3.1 of this report. The issue is of particular importance in the control of systems that are either non-linear or time-delayed. In such systems the influence of one variable upon another may vary substantially as the result of small changes in the control signal.

A third issue is that of non-linear control. Most control methodologies are intended for use on linear systems. Moreover, even if a particular control approach is capable of treating the non-linear case, it is often applied to a linearized model in order to minimize mathematical complexity. This practice is certainly acceptable for static situations in which the objective of the control action is to maintain a specified condition. But it is not acceptable for transients. The now decade-old revolution in digital technology has made it possible to apply enormous computing power to individual control loops. Advantage should be taken of this new resource to design controllers in terms of the non-linear system.

The following section of this report provides a review of reactor dynamics with the emphasis on issues that affect reactor control. A section then follows in which the contributions to process control of

both the MIT-SNL work that is reported here and of previous MIT programs are enumerated.

3.4 Overview of Reactor Dynamics

A reactor's multiplication factor is defined as the ratio of the neutrons produced from fission to those lost by either leakage or absorption. The reactor is subcritical, critical, or supercritical depending on whether the multiplication factor is less than, equal to, or greater than unity. The quantity most fundamental to the control of power in a nuclear reactor is the reactivity. It is defined as the fractional departure of the multiplication factor from unity. A reactivity of zero therefore corresponds to the critical condition. Adjustments of power in reactors now being considered for use in space are accomplished by positioning neutron-absorbing control rods or rotating drums so as to temporarily alter the neutron multiplication rate. If a power increase is desired, a control rod would be withdrawn (or a drum rotated) so as to insert positive reactivity and thereby place the reactor on a period. (Note: Period is defined as the power level divided by the rate of change of power. Thus, a period of infinity corresponds to steady-state, while one equal to a small positive number indicates a rapid power increase.) Having established a period, the power is allowed to rise. Once the power level approaches the desired value, the control device is gradually returned to its original position in order to reduce the reactivity to zero and to level the power without overshoot. Reactor operation is characterized by several reactivity feedback mechanisms. One of the most important is that a rise in fuel temperature will result in the increased absorption of neutrons in reactions that do not lead to fission. Known as the 'Doppler' effect, this is an inherent safety feature that limits the potential for an accident. Specifically, an increase in the reactor power will cause the fuel temperature to rise which will in turn decrease the number of neutrons available to sustain the fission chain reaction. The net result is a negative reactivity insertion followed by either a lessening of the rate of rise of power or, for some reactors, an actual power decrease.

3.4.1 Manual Operation

Licensed MITR and ACRR operators rely on their understanding of reactor dynamics and their past experience to perform power changes. This requires considerable skill because reactor dynamics are governed by non-linear relations. In particular, the reactor period and hence the rate of change of power depend on both the rate of change of reactivity and on the net reactivity present. Also, the rate at which reactivity can be removed under normal control conditions is a function of the insertion speed of the control rods and therefore finite. This means that merely halting rod withdrawal, or in some situations even reversing the direction of rod motion, may not immediately stop a power increase. Some degree of preplanning is necessary in order to level the power at the desired value without overshoot. Additional complications are that reactivity is not directly measurable, that the differential reactivity worths of the control rods are, usually, non-

linear functions of position, that the reactivity is altered by thermal feedback effects resulting from power changes, and that the relation between power and period is exponential, not linear.

3.4.2 Closed-Loop Operation

A control algorithm should fulfill two major functions. First, it should specify the desired plant trajectory and, if the actual state of the plant differs from the specified one, generate a feedback signal that reduces the error. Second, it should both define the envelope of conditions under which it will be possible to halt the transient and preclude operation beyond that envelope. These two functions are referred to here as 'predictive' and 'supervisory' respectively. Supervisory control is generally only required if, as is the case with a nuclear reactor, a system exhibits either non-linear or time-delayed behavior. In order to appreciate the nature of this problem, it is useful to consider further the dynamics of a nuclear reactor. Figure 3.4.2-1 is a simplified schematic of the fission process. Relative to reactor control, the most significant feature is that there are three parallel but separate mechanisms for the production of neutrons. Prompt neutrons appear directly following the fission event and have lifetimes that are quite short. A typical value is 100 microseconds. (Note: Neutron lifetime is considered here to be the time required for the neutron to appear, thermalize, and be reabsorbed.) Delayed neutrons are produced following the decay by beta particle emission of certain fission products. Those fission products that undergo this process are referred to as 'precursors'. The delay in the appearance of a delayed neutron relative to the fission event is the result of the precursor half-life. There are six recognized groups of precursors with half-lives ranging from 0.23 to 55 seconds. The average value is 12.2 seconds. The third mechanism for neutron production is the interaction of fission product gamma rays with certain moderating materials, most notably heavy water and beryllium. The appearance of photoneutrons is delayed relative to the fission event because of the time required for the fission products to undergo radioactive decay and emit the needed gamma rays. Prompt, delayed, and photoneutrons are all produced at high energies. In order to sustain the fission reaction, these neutrons must be slowed down or thermalized. This is accomplished by designing the reactor so that the neutrons will collide with the nuclei of a moderating material, thereby giving up much of their kinetic energy. (Note: Photoneutrons may be represented mathematically in the same way as delayed neutrons. Hence, the term 'delayed' is subsequently used in this report to refer to both neutrons resulting from precursor decay and photoneutrons.)

The importance of delayed neutrons to reactor safety is well-recognized. They lengthen the average neutron lifetime so that changes in a reactor's dynamic behavior will occur over intervals that are long relative to human response times. The significance of delayed neutrons to reactor control is less well understood and is therefore discussed here at some length. A problem originates in that the respective appearances of prompt and delayed neutrons following

Figure 3.4.2-1 Fission Process Showing Prompt, Delayed, and Photoneutron Production

fission occur on very different time scales. As a result, at any given moment, the prompt neutron population will be proportional to the current power level while the delayed neutron population will be a function of the previous power levels. This difference has no significance during extended steady-state operation because the previous and current power levels are the same. However, such will not be the case during transient operation. Specifically, because of the interval that elapses between the creation of precursors and the appearance of delayed neutrons, the delayed neutron population will not be in direct proportion to the transient power. This non-equilibrium condition of the delayed neutrons has several implications. First, during power increases, the contribution of the delayed neutrons will always be less than it would be at equilibrium and the contribution of the prompt neutrons will therefore be correspondingly greater. Similarly, during power decreases, the delayed neutron population will be greater than its equilibrium value. Second, upon attaining the desired power level, the rate of increase of the delayed neutrons, unlike that of their prompt counterparts, can not be immediately halted. Rather, the delayed neutron population will continue to rise until it attains equilibrium with the precursor population that corresponds to the reactor's power level. Hence, if power overshoots are to be averted, it is essential to limit the delayed neutron contribution so that, upon attainment of the desired power, the insertion of the control mechanism will make the rate of change of the prompt neutrons sufficiently negative so as to offset the continued increase in the delayed neutrons. Assuming that these two rates can be balanced, the reactor period can be made infinite and the power kept constant while equilibrium of the neutron populations at steady-state is achieved. Physically, this condition of balance can be readily achieved because the prompt and delayed neutron populations are proportional to the rate of reactivity change and to the net reactivity respectively. Figure 3.4.2-2 illustrates the process. Shown is a transient obtained from the MIT Research Reactor. Initially, there is positive reactivity present, the reactor is on a positive period, and the power is rising. At approximately 95 seconds, the operator was told to halt the transient. The control mechanism is driven in causing the reactivity to decrease and the period to lengthen. There is a change in the slope of the power curve, but the power continues to rise. Delayed neutrons are appearing at a faster rate than the prompt ones are being cut back. The power does momentarily level off at 112 seconds. This occurs when the positive reactivity (i.e., the rising delayed neutron contribution) is balanced by the negative rate of change of reactivity (i.e., the decreasing prompt neutron contribution). The ability to identify and maintain this balance between reactivity and its rate of change is central to the control of reactor power. Controllers exhibiting this capability are designated as incorporating the property of 'feasibility of control' [2,10]. Subsequent to the 112 second mark, the reactivity decrease was continued. The period becomes negative and the power decreases, eventually being leveled at the desired value.

The above understanding of reactor dynamics explains the need for supervisory control in a nuclear reactor. Mere prediction of the

Figure 3.4.2-2 Reactor Transient Illustrating Need for Supervisory Control

anticipated power trajectory is not enough. The envelope of conditions under which a transient can be halted must also be continuously defined. This report focuses on the development of predictive control laws that permit adjustments in a reactor's neutronic power to be achieved in minimum time. That emphasis notwithstanding, it is important to recognize that predictive control is only one part of the solution. (Note: Two supervisory approaches, reactivity and energy constraints, are discussed in sections 5.1 and 7.3.2.1 of this report.)

3.5 MIT and MIT-SNL Contributions to the Theory of Reactor and Non-Linear Control

Thus far, a survey has been given of several of the major approaches now in use for the design of control systems. Also, certain physical characteristics of the dynamics of a nuclear reactor have been described. With this material as background, the contribution of both previous MIT programs and the current MIT-SNL effort to reactor control and to the theory of non-linear control can be viewed in the proper perspective.

The central premise of the MIT approach to reactor control is that reactor dynamics are non-linear and that reactor controllers should therefore be designed on a non-linear basis. This does not mean that linear techniques such as state-space methods are rejected. But it does mean that new techniques, perhaps to be implemented in conjunction with existing methods, are needed. Figures 3.5-1 and 3.5-2 illustrate the problem. Shown are profiles of the neutronic power and period obtained using a linearized model of reactor dynamics. Also given for comparison are the profiles obtained using a six-group point kinetics solution. The simulated transient was rather mild involving a ramp reactivity addition of only 100 mbeta. Yet the error resulting from the linearized model is substantial. Hence, the rationale for the non-linear approach.

Specific contributions of both the MIT and MIT-SNL programs are as follows:

- (a) Supervisory Control - Model-based controllers such as those designed using transfer functions and state-space techniques contain the means to predict a system's behavior. For second-order linear systems, prediction of a system's trajectory also implies that the capability exists to generate that trajectory. But, for non-linear and especially time-delayed systems, this may not be true. In particular, it may be necessary to define separately the envelope of conditions under which a transient can be halted. On a formal basis, this property has been designated as 'feasibility of control'. It is described in section 5.1.2 of this report and in [2]. The point is that for non-linear systems, separate predictive and supervisory algorithms are needed.
- (b) Constraint Approach - Maintenance of 'feasibility of control' requires that practical methods be developed for implementing

Figure 3.5-1 Power Prediction Using a Linearized Model

Figure 3.5-2 Period Prediction Using a Linearized Model

supervisory algorithms. The MIT solution is to utilize constraints that take the form of inequalities. The left side of the constraint is time required to establish conditions under which the driving force can be completely negated. The right side is the time remaining to attain the limiting condition. Hence, by observing the inequality, it is assured that conditions under which the transient can be halted will exist upon attaining the limit. Constraints have been developed for reactors subject to limitations on neutronic power, net energy, and temperature. Those on power and energy have been demonstrated experimentally both on the MIT Research Reactor and on Sandia's Annular Core Research Reactor. Details are given in section 5.1, section 7.3.2.1, and in Chapters Ten and Eleven of this report. A characteristic of the constraint approach is that only current information is needed. No use is made of predictive models. As a result, a real-time decision can be made as to the need to alter the present control signal in order to avoid a future challenge to a limiting condition.

- (c) Closed-Form Optimal Control Laws - A method was developed to derive closed-form control laws that yield a minimum time response. The approach is applicable to systems that are subject to a constraint during conduct of the transient. This technique is discussed at length in Chapter Four of this report.
- (d) Control Signal Controllability - The issue of controllability was discussed at length in section 3.2.3.1 of this report. Relative to reactor control, a major achievement of the MIT-SNL program was the selection of the rate of change of reactivity, as opposed to the reactivity itself, as the control signal. The former can be changed as desired. The latter can only be altered over some finite interval of time.
- (e) Rigorous Models of Reactor Dynamics - The standard and alternate dynamic period equations were derived. These equations, which are rigorous, give the instantaneous reactor period as a function of the rate of change of reactivity, the net reactivity, and the rate of redistribution of the delayed neutron precursors. These equations are of enormous utility because they provide the physical insight needed to design non-linear controllers for nuclear reactors. For example, it can be shown that the rate of change of reactivity corresponds to the effect of a changing prompt neutron population and that the reactivity itself is a measure of delayed neutron effects. Hence, it would be expected that the former would be directly controllable while such would not be the case with the latter. Additional information is given in sections 4.3 and 4.4 of this report.
- (f) Multi-Tiered Controller Design - A controller structure having both supervisory and predictive components as well as a means for automated reasoning has been proposed and many components of such a controller have been demonstrated. Such an approach allows plant-specific control laws to be used in conjunction with super-

visory algorithms that are based on general principles. This may in turn facilitate the standardized licensing of closed-loop digital controllers for nuclear reactors. Details are given in Chapter Twelve of this report.

- (g) Experimental Evaluations - Every control concept has been evaluated experimentally under closed-loop conditions on either the MIT Research Reactor or on Sandia's Annular Core Research Reactor or both. The resulting database is providing the confidence to implement these control techniques on safety-constrained systems.

The preceding are some of the major accomplishments of the MIT and MIT-SNL programs on reactor control. Much else has also been achieved including demonstration of the parity-space approach for signal validation, the on-line reconfiguration of control laws, the design and implementation of a rule-based controller, and the receipt of license approval from the U.S. Nuclear Regulatory Commission for general use of the 'reactivity constraint approach' on the MIT Research Reactor [2]. The following sections of this report provide a detailed discussion of many of these topics. The focus there is largely on the control of nuclear reactors. However, it should also be recognized that the MIT and MIT-SNL programs are contributing to the realization of general methodologies for non-linear control.

4. MIT-SNL Period-Generated Minimum Time Control Laws

Material is presented here as it relates to the rationale for and the formulation of the MIT-SNL Period-Generated Minimum Time Control Laws.

4.1 Statement of Problem

The principal objective of the MIT-SNL program was to develop and demonstrate experimentally techniques for the rapid adjustment of a reactor's neutronic power. It was, and is, intended that these methods be used for the transient control of reactor-powered spacecraft. Accordingly, the specifications imposed on these control techniques were quite rigid. First, it must be possible to raise the reactor power by many orders of magnitude both in a few seconds and without overshoot. Second, the control methodology must be sufficiently flexible to permit use under a variety of initial conditions. Third, it should be possible to implement the controller using actuators of low mass such as stepping motors. Fourth, the control approach should permit continuous operation, including power cycling. Fifth, the control technique should be sufficiently flexible to accommodate a wide range of feedback effects but, at the same time, not be dependent upon those effects. Sixth, these control laws should be compatible with any form of energy removal including continuous, aperiodic, and near adiabatic. Seventh, it must be possible to execute the control algorithm itself in real time.

Little guidance was provided as to the ultimate design of the reactor to which the control methodology would be applied. It was stipulated that the reactor would be describable in terms of space-independent kinetics. However, decisions as to the fuel and coolant type had yet to be made. Therefore, a final requirement imposed on the control techniques was that they be based on generic concepts and hence be applicable to many different types of reactors.

It was evident at the outset of the MIT-SNL program that conventional methods for changing a reactor's power would not be adequate. Specifically, adjustments of the neutronic power on land-based reactors are normally accomplished by manually inserting reactivity until a specified stable period has been attained. The power is then allowed to rise (or fall) exponentially on that period until the desired level is approached. The period is then gradually lengthened by removing the reactivity, usually in a stepwise fashion, so as to level the power smoothly. For remotely-operated reactors on board spacecraft, this simple control process is deficient in two respects. First, in lieu of manual operation, some method must be available by which the proper time for the initiation of reactivity removal can be reliably predicted. Second, the process does not achieve a minimum-time response because the rate of change of power will be less than the maximum allowed during the intervals of reactivity insertion and removal. The solution to the first of these problems is the 'Reactivity Constraint Approach' which is the subject of the aforementioned report prepared by MIT for the National Science Foundation [2]. The

solution to the second problem is the subject of this report. The basic idea was to develop a practical method for identifying a control signal that would result in a time-optimal response.

4.1.1 Definition of 'Reactor Power'

The expression 'reactor power' warrants careful definition because it could refer to either the neutronic power as measured by a fission or compensated ion chamber or to the thermal power as determined by a calorimetric balance. As a result of system heat capacity, neutronic and thermal power are generally only equal after prolonged operation at steady-state. For the work described in this report the term 'reactor power' refers to the instantaneous rate at which energy is being produced by the fission process. It is therefore the signal obtained from a fission or ion chamber.

4.2 Time-Optimal Control

The solution of reactor dynamics problems subject to the requirements of time optimization has been the subject of much research. Excellent discussions of the topic are given by Weaver [12], Mohler and Shen [13], and Ash [14]. However, as was the case with the conventional control methodology described in section 4.1 of this report, existing methods for time-optimal control were found to be inadequate for the MIT-SNL program.

In general, previous efforts to obtain a minimum-time response for reactor power have entailed assembling a set of equations that accurately models a reactor, specifying performance criteria, and applying the conditions for optimality such as those elucidated by the mathematician Pontryagin. So doing yields a set of partial differential equations with split boundary conditions that must be solved iteratively using complex numerical methods. Often it is necessary to linearize the model equations in order to make the mathematics tractable. The result is a sequence of control signals that are valid for a specific reactor model and one particular set of initial conditions. These signals are then applied in an open-loop fashion. Provisions do not exist for the continuous feedback of the power level or period and, should there be an appreciable change in the plant conditions, this predetermined sequence of signals will not produce the desired result. Moreover, both the computational equipment necessary to implement the technique and the time required for the performance of the calculations is often substantial.

4.2.1 Methodology for a Time-Optimal Response

The first of the seven design specifications listed in section 4.1 of this report mandated that the resulting methodology be some form of time-optimal control. As noted above, existing techniques for obtaining such a response were considered unsatisfactory because they resulted in open-loop sequences of signals and therefore could not comply with several of the other program requirements, particularly those concerning initial conditions, feedback, and computation time.

The MIT-SNL Period-Generated Minimum Time Control Laws were obtained using a very different and novel approach to the problem of time-optimal control. Specifically, it was observed that if a system is subject to one or more constraints, then the time-optimal trajectory will move the system along the most limiting of those constraints. This observation can be used to great advantage. Namely, rather than solve the system's describing equations subject to both the constraint and a performance index, it is more direct to define the physical conditions that correspond to system movement along that limiting constraint. Relative to control of a nuclear reactor, typical constraints are the allowed reactor period, the available rate of reactivity change, and permitted rates of change of temperature. The time-optimal control laws described here are for a reactor subject to a limitation on reactor period which is the constraining factor for the reactor-powered spacecraft now being considered by the United States. Hence, the immediate objective of the MIT-SNL program was to write an expression that gave the instantaneous reactor period as a function of the various parameters that described the core's dynamic behavior.

4.3 Description of Reactor Dynamics

The first distinguishing feature of the MIT-SNL control approach was the realization that a closed-form time-optimal expression for adjustment of a reactor's neutronic power could be obtained by describing system movement along its most limiting constraint, which is the instantaneous period. The second distinguishing feature was the derivation of the dynamic period equation which gives the instantaneous reactor period as a function of the rate of change of reactivity, the reactivity, and the rate of redistribution of the delayed neutron precursors. That equation may be written in either a standard or an alternate form. The details of both derivations are given here. The significance of the dynamic period equation to the MIT-SNL program was that, within the limitations of space-independent kinetics, it is a rigorous method for describing reactor dynamics. Without this equation, it would not have been possible to exploit the observation that a time-optimal response can be achieved by specifying movement along a limiting constraint.

4.3.1 Derivation of the Standard Dynamic Period Equation

Reactors which have close-coupled cores may be described by the space-independent kinetics equations. These are:

$$\frac{dT(t)}{dt} = \frac{(\rho(t) - \bar{\beta})}{\ell^*} T(t) + \sum_{i=1}^N \lambda_i C_i(t) \quad (4.3.1-1)$$

$$\frac{dC_i(t)}{dt} = \frac{\bar{\beta}_i}{\ell^*} T(t) - \lambda_i C_i(t) \text{ for } i = 1, N \quad (4.3.1-2)$$

where $T(t)$ is the amplitude function and is a weighted integral of all neutrons present in the reactor, and other symbols are defined as:

- $\rho(t)$ is the reactivity,
- $\bar{\beta}$ is the effective delayed neutron fraction,
- λ_i is the decay constant for the i th precursor group,
- $C_i(t)$ is the concentration of the i th precursor group normalized to the initial power,
- ℓ^* is the prompt neutron lifetime,
- $\bar{\beta}_i$ is the effective fractional yield of the i th group of delayed neutrons, and
- N is the number of groups of delayed neutrons, including photoneutrons.

The first step in the derivation is to define the instantaneous reactor period and to incorporate that definition in the neutron kinetics equation.

The instantaneous reactor period ($\tau = 1/\omega$) is defined as:

$$\dot{T}(t) \equiv \omega(t)T(t) \quad (4.3.1-3)$$

The point kinetics equations may therefore be written as:

$$\omega(t)T(t) = \frac{(\rho(t) - \bar{\beta})}{\ell^*} T(t) + \sum \lambda_i C_i(t) \quad (4.3.1-4)$$

$$\dot{\Sigma C}_i(t) = \frac{\bar{\beta} T(t)}{\ell^*} - \sum \lambda_i C_i(t) \text{ for } i = 1, N \quad (4.3.1-5)$$

The next step in the derivation is to define the standard, effective, multi-group decay parameter:

$$\lambda_e(t) \equiv \sum \lambda_i C_i(t) / \Sigma C_i(t) \quad (4.3.1-6)$$

This parameter is time-dependent because the relative concentrations of the delayed neutron precursor groups change as power is being raised and lowered. Applying this definition to the neutron kinetics equation yields:

$$\omega(t)T(t) = \frac{(\rho(t) - \bar{\beta})}{\ell^*} T(t) + \lambda_e(t) \Sigma C_i(t) \quad (4.3.1-7)$$

The next and crucial step in the derivation is to differentiate (4.3.1-7), the modified version of the neutron kinetics equation. So doing, and using the definition of the instantaneous period to eliminate the derivative of the amplitude function yields:

$$\begin{aligned} \dot{\omega}(t)T(t) + (\omega(t))^2T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}\omega(t)T(t) = \frac{\dot{\rho}(t)T(t)}{\ell^*} + \\ \dot{\lambda}_e(t)\Sigma C_i(t) + \lambda_e(t)\dot{\Sigma C_i}(t) \end{aligned} \quad (4.3.1-8)$$

A series of substitutions are now made. First, the rate of change of the precursor concentrations is eliminated by substitution of (4.3.1-5):

$$\begin{aligned} \dot{\omega}(t)T(t) + (\omega(t))^2T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}\omega(t)T(t) = \frac{\dot{\rho}(t)T(t)}{\ell^*} + \\ \dot{\lambda}_e(t)\Sigma C_i(t) + \frac{\lambda_e(t)\bar{\beta}T(t)}{\ell^*} - \lambda_e(t)\Sigma\lambda_i C_i(t) \end{aligned} \quad (4.3.1-9)$$

Next, the quantity $\Sigma C_i(t)$ is eliminated by use of (4.3.1-6), the definition of the effective, multi-group decay parameter. The quantity $\Sigma\lambda_i C_i(t)$ is then eliminated by substitution of (4.3.1-4), the modified version of the neutron kinetics equation. The result is:

$$\begin{aligned} \dot{\omega}(t)T(t) + (\omega(t))^2T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}\omega(t)T(t) = \frac{\dot{\rho}(t)}{\ell^*}T(t) + \\ \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}[\omega(t)T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}T(t)] + \frac{\lambda_e(t)\bar{\beta}T(t)}{\ell^*} - \\ \lambda_e(t)[\omega(t)T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}T(t)] \end{aligned} \quad (4.3.1-10)$$

Dividing by the amplitude function $T(t)$ and consolidating terms yields:

$$\begin{aligned} \omega(t)\left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*} + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\right] = \\ \left[\frac{\dot{\rho}(t)}{\ell^*} + \frac{\lambda_e(t)\rho(t)}{\ell^*} + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\left(\frac{\bar{\beta}-\rho(t)}{\ell^*}\right)\right] \end{aligned} \quad (4.3.1-11)$$

Solving for $\omega(t)$ yields:

$$\omega(t) = \frac{[\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta}-\rho(t))]}{[\bar{\beta}-\rho(t) + \beta^* \left(\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)} \right)]} \quad (4.3.1-12)$$

The expression for the instantaneous reactor period is therefore:

$$\tau(t) = \frac{\bar{\beta}-\rho(t) + \beta^* \left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)} \right]}{[\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta}-\rho(t))]} \quad (4.3.1-13)$$

This relation is the standard dynamic period equation. It is a rigorously derived exact relation. Nevertheless, it is not a valid equation because the dependent variable is the instantaneous reactor period and one of the independent variables is the inverse of that period. However, given that the prompt neutron lifetime is extremely small, terms containing β^* can be deleted and (4.3.1-13) readily reduces to a form that is a valid equation. Therefore, taking the prompt neutron lifetime to be zero (prompt-jump approximation):

$$\tau(t) \approx \frac{\bar{\beta}-\rho(t)}{[\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta}-\rho(t))]} \quad (4.3.1-14)$$

This result is a general relation that accurately predicts the instantaneous reactor period associated with any reactivity pattern provided that the prompt critical value is not approached.

4.3.1.1 Behavior of Component Terms in the Standard Dynamic Period Equation

It is useful to examine the behavior of the terms that comprise the denominator of the standard dynamic period equation for various reactivity insertions. Figure 4.3.1.1-1 shows the behavior of these terms for a tent-shaped reactivity insertion. Note that the initiating event for each segment of this transient was a step in the rate of change of reactivity and the the variation in the other terms was in response to those steps. The inference to be drawn from this observation is that the rate of change of reactivity, and not the reactivity itself, was the controlling factor. As is discussed at some length in section 4.4 of this report, this realization is central to the development of strategies for the control of neutronic power in nuclear reactors.

Figure 4.3.1.1-2 shows the behavior of the component terms of the

Figure 4.3.1.1-1 Behavior of Component Terms of the Dynamic Period Equation for a 'Tent-Shaped' Reactivity Pattern

Figure 4.3.1.1-2 Behavior of Component Terms of the Dynamic Period
Equation for a Reactivity Insertion Pattern Typical
of the MIT-CSDL Non-Linear Digital Controller

standard dynamic period for a transient typical of the MIT-CSDL Non-Linear Digital Controller, which is the implementing algorithm for the reactivity constraint approach. There is a rapid insertion, an extended hold, and finally a gradual removal of reactivity. Also shown are the corresponding changes in the standard, effective, multi-group decay parameter and two of the precursor groups. Three features stand out relative to the quantity $(\dot{\lambda}^e / \lambda^e)(\beta - \rho)$. First, it rises as reactivity is inserted. This behavior is in response to the shift of the effective, multi-group decay parameter toward the decay constants of the shorter lived precursor groups that dominate during power increases because they remain near equilibrium levels. Offsetting factors are the decreasing value of the term $(\beta - \rho)$ and the rising magnitude of the decay parameter. Second, this quantity tends to zero once the reactivity is held constant. This occurs because, in the absence of varying reactivity, the precursor groups approach an asymptotic state. Third, once reactivity removal is begun, the value of this quantity becomes negative. Physically, there is an immediate decrease in the prompt neutron population, which in turn causes first the rate of rise of the total neutron population and then that of the short-lived precursors to decline. The longer lived precursors are not immediately affected and therefore become more significant. The net result is that the effective, multi-group decay parameter decreases.

Relative to the design of the MIT-SNL minimum time laws, the significant feature is that, as was the case with Figure 4.3.1.1-1, the transient is being driven by changes in the prompt neutron population.

4.3.2 Derivation of the Alternate Dynamic Period Equation

Use of the standard dynamic period equation is made difficult by the presence of the term $(\dot{\lambda}^e / \lambda^e)(\beta - \rho)$. This quantity is noisy when evaluated on-line because it is a derivative. Professor Allan Henry (MIT Department of Nuclear Engineering) suggested an alternate formulation of the dynamic period equation that avoids this difficulty.

Both the standard and alternate formulations are derived from the point kinetics equations using the same overall approach. The two derivations differ in that (1) each uses a different definition of an effective, multi-group decay parameter, (2) the alternate formulation defers use of the multi-group decay parameter until after completion of the major step in the derivation which is differentiation of the neutron kinetics equation. It is the result of this last difference that the alternate formulation avoids the presence of a derivative-containing term in the resulting expression for the period.

The instantaneous reactor period ($\tau = 1/\omega$) is defined as:

$$\dot{T}(t) \equiv \omega(t)T(t) \quad (4.3.2-1)$$

where $T(t)$ denotes the amplitude function and is a weighted integral of all neutrons present in the reactor. The first and second point kinetics equations may therefore be written as:

$$\omega(t)T(t) = \frac{(\rho(t) - \bar{\rho})}{\beta^*} T(t) + \Sigma \lambda_i C_i(t) \quad (4.3.2-2)$$

$$\dot{C}_i(t) = \frac{\bar{\beta}_i}{\beta^*} T(t) - \lambda_i C_i(t) \text{ for } i = 1, N \quad (4.3.2-3)$$

The first step in the derivation is to differentiate the neutron kinetics equation. So doing, and using the definition of the instantaneous period to eliminate the derivative of the amplitude function yields:

$$\dot{\omega}(t)T(t) + (\omega(t))^2 T(t) + \frac{\bar{\beta} - \rho(t)}{\beta^*} \omega(t)T(t) = \frac{\dot{\rho}(t)}{\beta^*} T(t) + \Sigma \lambda_i \dot{C}_i(t) \quad (4.3.2-4)$$

The next step is to eliminate the rate of change of the precursor concentrations by substitution of the precursor kinetics equation. This yields:

$$\begin{aligned} \dot{\omega}(t)T(t) + (\omega(t))^2 T(t) + \frac{\bar{\beta} - \rho(t)}{\beta^*} \omega(t)T(t) &= \frac{\dot{\rho}(t)}{\beta^*} T(t) \\ &+ \frac{\Sigma \lambda_i \bar{\beta}_i}{\beta^*} T(t) - \Sigma \lambda_i^2 C_i(t) \end{aligned} \quad (4.3.2-5)$$

In order to eliminate the precursor concentrations, an effective, multi-group decay parameter is defined as:

$$\lambda'_e(t) \equiv \Sigma \lambda_i^2 C_i(t) / \Sigma \lambda_i C_i(t) \quad (4.3.2-6)$$

It is important to recognize that this effective, multi-group decay parameter is a time-dependent quantity. Its value will vary during transients because the relative concentrations of the various delayed neutron precursor groups change depending on the rate at which power is being raised or lowered. Combining this definition with the first point kinetics equation yields:

$$\Sigma \lambda_i^2 C_i(t) = \lambda'_e(t) [\omega(t)T(t) + \frac{\bar{\beta} - \rho(t)}{\beta^*} T(t)] \quad (4.3.2-7)$$

Substitution of (4.3.2-7) into (4.3.2-5) achieves the specified objective which is elimination of the precursor concentrations. The result is:

$$\begin{aligned} \dot{\omega}(t)T(t) + (\omega(t))^2T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*} \omega(t)T(t) &= \frac{\dot{\rho}(t)}{\ell^*}T(t) + \frac{\Sigma\lambda_i\bar{\beta}_i}{\ell^*}T(t) \\ &- \lambda'_e(t)[\omega(t)T(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*}T(t)] \end{aligned} \quad (4.3.2-8)$$

Dividing through by the amplitude function $T(t)$, noting that $\bar{\beta} = \Sigma\bar{\beta}_i$ and rearranging terms yields:

$$\omega(t)\left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \frac{\bar{\beta}-\rho(t)}{\ell^*} + \lambda'_e(t)\right] = \frac{\dot{\rho}(t)}{\ell^*} + \frac{\lambda'_e(t)\rho(t)}{\ell^*} + \frac{\Sigma\bar{\beta}_i(\lambda_i - \lambda'_e(t))}{\ell^*} \quad (4.3.2-9)$$

Solving for $\omega(t)$ yields:

$$\omega(t) = \frac{\dot{\rho}(t) + \lambda'_e(t)\rho(t) + \Sigma\bar{\beta}_i(\lambda_i - \lambda'_e(t))}{(\bar{\beta}-\rho(t)) + \ell^*\left(\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda'_e(t)\right)} \quad (4.3.2-10)$$

Noting that the quantity $\ell^*\left[\frac{\dot{\omega}}{\omega} + \omega + \lambda'_e\right]$ is small in comparison with the term $(\bar{\beta}-\rho)$ leads to the following expression for the instantaneous reactor period:

$$\tau(t) = \frac{\bar{\beta}-\rho(t)}{\dot{\rho}(t) + \lambda'_e(t)\rho(t) + \Sigma\bar{\beta}_i(\lambda_i - \lambda'_e(t))} \quad (4.3.2-11)$$

The inverse of equation 4.3.2-10 is the alternate formulation of the dynamic period equation and the quantity $\lambda'_e(t)$ is designated as the alternate, effective, multi-group decay parameter.

4.3.2.1 Behavior of Component Terms in the Alternate Dynamic Period Equation

Figure 4.3.2.1-1 shows the results of a simulation study in which the component terms of the alternate formulation of the dynamic period equation were determined. The reactivity pattern for which this evaluation was performed is representative of that obtained when raising power on the 5 MWt MIT Research Reactor using the MIT-CSDL Non-Linear Digital Controller. As shown in the lower portion of the figure, such transients are characterized by a rapid insertion, an extended hold, and finally a gradual removal of reactivity.

The simulated reactor was originally at steady-state under conditions of delayed neutron equilibrium. Hence, the period was infinite

Figure 4.3.2.1-1 Behavior of Component Terms - Alternate Dynamic Period Equation

and the three component terms in the denominator of the alternate formulation of the dynamic period equation (4.3.2-11) were each equal to zero. The equilibrium value of the effective, multi-group decay parameter was 0.407 inverse seconds. Reactivity was then added for twenty seconds at the rate of five millibeta per second (0.0039% $\Delta K/K$ per second), held constant at 100 millibeta for forty seconds, and then removed for forty seconds at the rate of 2.5 millibeta per second. The middle portion of the figure shows the behavior of the effective, multi-group decay parameter and the normalized concentrations of the shortest and longest-lived delayed neutron precursor groups. As expected, the short-lived group responds rapidly to changes in the reactivity insertion rate while the long-lived group's response is quite sluggish. The behavior of the decay parameter is dominated by that of the short-lived precursor groups and therefore conforms to the reactivity pattern. This is as expected because that parameter is defined in terms of the square of the precursor decay constants and is therefore quite sensitive to changes in the concentrations of the short-lived groups which have the largest decay constants. The upper portion of the figure depicts the behavior of the three terms that form the denominator of the alternate formulation of the dynamic period equation. The quantity $(\lambda' \rho)$ conforms to the pattern of the reactivity with some slight curvature resulting from the non-linearity of the decay parameter. The term $(\sum \beta_i (\lambda_i - \lambda'))$ is the more interesting. Its behavior follows that of (λ') except that it is of opposite sign. Thus, this term goes rapidly negative, remains almost constant, and then slowly goes to zero.

4.3.3 Background Information on the Dynamic Period Equations

The derivations of the dynamic period equations as given by (4.3.1-12) and (4.3.2-10) are believed to be one of the unique results of the MIT research program on the closed-loop control of reactor power. Simpler forms of these equations have appeared earlier in the literature. These are given here as a matter of reference.

If asymptotic behavior, one-group of delayed neutrons, and the prompt-jump approximation are assumed, then the following relation is obtained:

$$\tau \approx (\bar{\beta} - \rho) / \lambda_e \rho \quad (4.3.3-1)$$

This gives the asymptotic period that ultimately results from a step change in reactivity. It is a well-known, simple relation that is found in many nuclear engineering textbooks.

A less restrictive relation is given in many training manuals for nuclear operators. It is obtained by assuming both one-group of delayed neutrons and the prompt-jump approximation. Its derivation is given by Hetrick [15]. It is:

$$\tau(t) \approx \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e \rho(t)} \quad (4.3.3-2)$$

In 1982 the following relation was derived by Dr. Bernard assuming only the prompt-jump approximation:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t))} \quad (4.3.3-3)$$

This equation is the basis of the MIT-CSDL Non-Linear Digital Controller which was first demonstrated on the 5 MWt MIT Research Reactor in February 1983. Early in 1985, Professor Allan Henry derived the alternate dynamic period equation. This derivation was significant because it required no approximations. An alternate version of the MIT-CSDL Non-Linear Digital Controller was constructed and tested using this equation in the summer of 1985. In December 1986, Dr. Bernard derived the standard dynamic period equation without resort to any approximations. As reported here, both of these equations have now been used as the basis of control laws that permit reactor power to be changed in minimum time.

Most nuclear engineers are familiar with the inhour equation which gives the reactor period that is ultimately attained following a step insertion of reactivity. The inhour equation is in fact a very special case of the standard dynamic period equation [2,16]. Currently, only the inhour equation is routinely taught as part of nuclear engineering curricula. However, both equations merit inclusion in nuclear course work. The inhour relation, with its multiple roots represented explicitly, shows the effect of the various delayed neutron precursor groups. Also, it relates the stable reactor period to the reactivity. However, it does not describe the effect of a varying reactivity. The dynamic period equation shows that the period is a function of the rate of change of reactivity, the net reactivity, and the rate of redistribution of the delayed neutron precursor groups. Knowledge of this equation is particularly important for the proper design of reactor control systems. Additional information on the relation between the inhour and the dynamic period equations is given in section 6.2 and Appendix A of this report.

4.3.4 Limitations of the Dynamic Period Equation

The dynamic period equation is not without its limitations. Specifically, it is a form of space-independent kinetics and hence subject to the restrictions inherent in those relations. The space-independent kinetics equations are derived from the transport equation subject to two major approximations. The first is that all neutrons are assumed to have the same energy spectrum. In reality, delayed neutrons are produced at somewhat lower energies than their prompt counterparts. Hence, the delayed neutrons have a higher probability, perhaps 20%-30% for spacecraft reactors, of causing thermal fission. Allowance is made for this difference by the use of 'effective' delayed neutron fractions. That is, the yield of the delayed neutrons is taken as higher than that measured so as to reflect their greater probability for causing fission. The second and more serious approximation is that the spatial shape of the neutron flux is taken to be

constant. That is, the magnitude of the neutron flux varies during a transient but the shape of the flux profile is assumed to be unchanged. All sections of the core behave in unison. This assumption is valid for small homogeneous reactors. However, it breaks down if the reactor becomes either large or heterogeneous. The advantage associated with this approximation is that it allows the various parameters given in the space-independent kinetics equations to be treated solely as a function of time.

Is the assumption of constancy of the spatial flux shape reasonable for spacecraft reactors? Such reactors are small but the cores are not homogeneous. In particular, the control devices are distinct entities which, when moved, will cause perturbations in the local flux shape. Hence, pending the selection of an actual core design, all that can be stated is that the assumption of constancy of the flux shape will be reasonably valid for small reactivity perturbations in which the control rods are not moved significantly.

It should be noted that the validity of the space-independent kinetics model and hence that of the dynamic period equations can be enhanced by the judicious placement of the control devices. If the reactor is equipped with only one or two devices of significant integral reactivity worth, then the achievement of criticality will require that these be moved substantial distances. This will in turn cause a major perturbation in the local flux shape. A better approach would be to use many control devices, each of relatively low integral reactivity worth. Homogeneity could then be approximated by distributing these throughout the core. Moreover, if moved in unison, the impact on the flux shape would at least be uniform and perhaps small.

4.4 Reactor Control and the Rate of Change of Reactivity

Thus far, a method for the identification of a time-optimal response has been suggested and a rigorous means of describing a reactor's dynamic behavior has been developed. The next step in the MIT-SNL program was to consider the specific problem of designing a controller for the reactor power. In this regard, it is important to recognize that the dynamics of a nuclear reactor are both non-linear and time-delayed. Special approaches are therefore necessary if a robust controller capable of functioning over a wide operating range is to be constructed.

The normal approach to controller design is to represent the system as a set of first order differential equations. As was discussed in section 4.3 of this report, that route was not followed. Rather, the reactor's describing equations were combined through a process of differentiation and substitution so that the constrained parameter, in this case the instantaneous period, was expressed directly as a function of the system variables. Although cumbersome, this approach has the advantage that every process that affects the period is shown explicitly. This permits physical understanding to be utilized directly in the controller's design. Such an approach is today unorthodox.

However, given that there is no general theory of non-linear control, it is the only viable approach.

4.4.1 Physical Interpretation of the Dynamic Period Equation

The space-independent kinetics equations were given in section 4.3.1 of this report. Approximating the amplitude function as the total neutron population, the first of these equations becomes:

$$\frac{dn(t)}{dt} = \frac{(\rho(t) - \beta)}{\Lambda^*} n(t) + \sum_{i=1}^N \lambda_i C_i(t) \quad (4.4.1-1)$$

Equation 4.4.1-1 is merely a statement that the rate of change of the total neutron population is the sum of the rates of change of the prompt and delayed neutron populations. This is shown by the following, admittedly qualitative, argument. Consider the quantity $[n(t)(\rho(t) - \beta)/\Lambda^*]$. The reactivity can be thought of as the fractional change in the total neutron population per generation. The symbol β denotes the effective fraction of delayed neutrons. Hence, $(\rho(t) - \beta)$ is the fractional change in the prompt population per generation. Given that there are $(1/\Lambda^*)$ neutron generations per second and that the total neutron population is $n(t)$, the quantity $[n(t)(\rho(t) - \beta)/\Lambda^*]$ is the rate of change of the prompt neutron population. A similar argument can be used to show that $\sum \lambda_i C_i(t)$ is the rate of change of the delayed neutron population. Specifically, that quantity is the rate at which precursors decay and given that there is one delayed neutron produced per decay, it must also be the rate at which delayed neutrons appear.

The physical meaning of the terms in the denominator of the standard dynamic period equation can now be determined. This is most easily accomplished by tracing each term to its point of origin in the derivation. So doing, it is apparent that the rate of change of reactivity ($\dot{\rho}$) corresponds to the effect of a changing prompt neutron population. Similarly, the quantities $\lambda_e \rho$ and $(\dot{\lambda}_e / \lambda_e)(\beta - \rho)$ correspond respectively to the effect of a changing delayed neutron precursor population and to a changing distribution of delayed neutron precursors within the defined groups. These observations have certain practical consequences. In particular, the precursor populations are a function of the power history and therefore can not be altered on demand. Hence, the terms $\lambda_e \rho$ and $(\dot{\lambda}_e / \lambda_e)(\beta - \rho)$ can not be manipulated at will and are not subject to direct control. In contrast, the prompt neutron population is essentially a function of only the current power level and it is therefore immediately controllable. Hence, it is apparent that if an immediate change is required in the period, then an adjustment should be made in the rate of change of reactivity rather than in the reactivity itself. This observation is central to the development of algorithms for the control of the neutronic power in a nuclear reactor.

4.4.2 Selection of the Rate of Change of Reactivity as the Control Signal

It was argued in section 4.4.1 of this report that the rate of change of reactivity ($\dot{\rho}$) is directly controllable. That conclusion followed logically from the realization that the quantity $\dot{\rho}$ is a measure of prompt neutron effects and that those effects are proportional to the reactor's current power level. In contrast, terms containing the reactivity were shown to be indicative of delayed neutron contributions and were therefore a function of the reactor's power history. This same conclusion can be attained using a less esoteric argument in which the consequences of moving a reactor's control mechanism are examined. Actuation of the mechanism will result in movement of the associated control rod and hence immediately alter the rate of change of reactivity. In contrast, for there to be a change in the reactivity itself, movement of the control device must be sustained for some finite interval of time. That is, the control device must be moved a certain distance for there to be any effect on the reactivity.

In addition to being directly controllable, the rate of change of reactivity ($\dot{\rho}$) has another desirable attribute. Specifically, a variation in that parameter will result in an immediate change in the instantaneous reactor period which is the quantity that must be controlled if a time-optimal response is to be achieved. While this is evident from the dynamic period equation, it is nevertheless useful to visualize the effect. Figure 4.4.2-1 depicts a computer simulation in which the reactor was modeled using the six-group point kinetics equations with a time step of 0.0001 seconds. A rod of linear differential worth was withdrawn for 20 seconds so as to add reactivity at the rate of 5 mbeta/second (0.004% $\Delta K/K$ per second), held constant for 40 seconds, inserted for 40 seconds at half-speed thereby removing reactivity at the rate of 2.5 mbeta/second, and then held constant. Feedback effects were omitted in order not to obscure the basic relationship between the rod motion and the reactor power. The figure shows the reactivity, startup rate (SUR), and power as functions of time. (Note: Startup rate, which equals (26.06/period) and is measured in decades-per-minute (DPM), is a measure of the inverse of the instantaneous reactor period. It, like the concept of period, applies not only to reactor startups but to power transients and steady-state operation as well. It is shown because its steady-state value is zero, not infinity, and therefore easier to display on a graph.) Note the following:

- (1) The startup rate initially increases almost as a step and then rises smoothly. The reactor power rises smoothly at an ever-increasing rate.
- (2) When the reactivity insertion is halted at 20 seconds, the startup rate remains positive but drops, almost discontinuously, and then remains nearly constant. The reactor power continues to increase but there is a sharp decrease in slope corresponding to the decrease in the startup rate. Power then rises almost exponentially. The startup rate (or

Figure 4.4.2-1 Reactor Power Increase - Computer Simulation

period) present when the reactivity was being changed is referred to as 'dynamic' while that which exists when the reactivity is constant is called 'steady'.

- (3) When the reactivity decrease is started at 60 seconds, the startup rate drops sharply and then continues to decrease, eventually becoming negative. The reactor power continues to increase but at an ever-lessening rate until the startup rate goes negative indicating a decreasing power. Note that this occurs when there is still positive reactivity in the core. This demonstrates the decoupling between the observed startup rate (or period) and the actual reactivity due to the delayed neutron effect.
- (4) Reactivity removal stops at 100 seconds when the reactivity is zero. The startup rate increases rapidly and then approaches zero. The reactor power curve again exhibits a discontinuous change of slope and the power levels off.

Figure 4.4.2-2 shows another simulated run. This time the rod was driven in for 20 seconds thereby inserting negative reactivity at the rate of -5 millibeta per second, held constant for 40 seconds, and then withdrawn for 40 seconds at half-speed. Feedback effects were again omitted. Similar relations exist between the reactivity, startup rate, and power as were found for the initial case shown in Figure 4.4.2-1. Note that although the reactivity change was the mirror image of the one in Figure 4.4.2-1, the reactor's response was not a mirror image. This difference occurs because, in the absence of a negative rate of reactivity insertion, reactor power can not be decreased at a rate faster than the decay of the longest-lived delayed neutron precursor group. That group has a half-life of 55 seconds which corresponds to a period of -80 seconds.

It is apparent from the proceeding arguments that the rate of change of reactivity fulfills the conditions necessary for it to be used as the control signal. Specifically, it is itself alterable on demand and it will, upon being varied, cause an immediate change in the instantaneous reactor period which is the parameter being controlled. This choice is in sharp contrast to traditional designs for reactor power controllers which use the reactivity as the control signal. As argued above, such a choice is inappropriate because reactivity is not a directly controllable quantity.

There are a number of specific advantages that accrue as a result of utilizing the rate of change of reactivity as the control signal. For example, control laws based on this concept will provide information on both the speed and the magnitude of desired control rod movements. In contrast, were laws that use reactivity as the control signal to be employed, no information would be generated as to the appropriate speed of withdrawal (or insertion) of the control devices. This and other advantages are further discussed in Chapter Twelve of this report which provides an assessment of the MIT-SNL minimum time laws.

Figure 4.4.2-2 Reactor Power Decrease - Computer Simulation

4.5 Formulation of the MIT-SNL Minimum Time Control Laws

Having obtained the dynamic period equations and realizing which term in those relations is directly controllable, it becomes a simple matter to obtain the time-optimal control laws. The terms in the dynamic period equation are rearranged so that the control signal, which is the rate of change of reactivity, is expressed as a function of the controlled parameter, which is the instantaneous reactor period. Thus, upon transformation of the standard dynamic period equation (Equation 4.3.1-13), the following is obtained:

$$\begin{aligned} \dot{\rho}(t) = & (\bar{\beta} - \rho(t))\omega(t) - \lambda_e(t)\rho(t) - (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t)) + \\ & \mathcal{L}^* \dot{\omega}(t) + \mathcal{L}^* [(\omega(t))^2 + \lambda_e(t)\omega(t) - (\dot{\lambda}_e(t)/\lambda_e(t))\omega(t)] \end{aligned} \quad (4.5-1)$$

Written in this manner, the law is difficult to implement because the quantity $\dot{\rho}(t)$ is the sum of the rates of change of reactivity that result from both movement of the control devices and feedback effects. The former is directly controllable, the latter is not. Accordingly, it is desirable to represent these two effects as separate entities. So doing, (4.5-1) becomes:

$$\begin{aligned} \dot{\rho}_c(t) = & (\bar{\beta} - \rho(t))\omega(t) - \lambda_e(t)\rho(t) - (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t)) - \dot{\rho}_f(t) + \\ & \mathcal{L}^* \dot{\omega}(t) + \mathcal{L}^* [(\omega(t))^2 + \lambda_e(t)\omega(t) - (\dot{\lambda}_e(t)/\lambda_e(t))\omega(t)] \end{aligned} \quad (4.5-2)$$

This is the standard MIT-SNL Period-Generated Minimum Time Control Law. A mathematically-equivalent but nonetheless different form of this law can be obtained from the alternate dynamic period equation. That relation, which is designated as the Alternate MIT-SNL Period-Generated Minimum Time Control Law is as follows:

$$\begin{aligned} \dot{\rho}_c(t) = & (\bar{\beta} - \rho(t))\omega(t) - \lambda'_e(t)\rho(t) - \Sigma \bar{\beta}_i (\lambda_i - \lambda'_e(t)) - \dot{\rho}_f(t) + \\ & \mathcal{L}^* \dot{\omega}(t) + \mathcal{L}^* ((\omega(t))^2 + \lambda'_e(t)\omega(t)) \end{aligned} \quad (4.5-3)$$

The quantities $\lambda_e(t)$ and $\lambda'_e(t)$ are the standard and alternate effective, multi-group decay parameters. As was discussed in section 4.3 of this report, they are defined respectively as:

$$\lambda_e(t) \equiv \Sigma \lambda_i C_i(t) / \Sigma C_i(t) \quad (4.5-4)$$

and

$$\lambda'_e(t) \equiv \Sigma \lambda_i^2 C_i(t) / \Sigma \lambda_i C_i(t) \quad (4.5-5)$$

These quantities are time-dependent because the relative distribution of the delayed neutron precursor groups changes as power is either raised or lowered. Other symbols are defined as:

- $\bar{\beta}$ is the effective delayed neutron fraction,
- $\rho(t)$ is the net reactivity,
- ℓ^* is the prompt neutron lifetime,
- $\omega(t)$ is the inverse of the specified reactor period,
- $\dot{\omega}(t)$ is the rate of change of the inverse of the specified reactor period,
- $\dot{\rho}_c(t)$ is the rate of change of reactivity associated with the control mechanisms necessary to either achieve or maintain the specified period,
- $\dot{\rho}_f(t)$ is the rate of change of reactivity associated with feedback effects,
- $\dot{\lambda}_e(t)$ is the rate of change of the standard effective, multi-group decay parameter,
- λ_i is the decay constant of the i th precursor group,
- $C_i(t)$ is the concentration of the i th precursor group, and
- $\bar{\beta}_i$ is the effective yield of the i th precursor group.

4.5.1 Illustrative Example of the MIT-SNL Minimum Time Laws

The MIT-SNL Period-Generated Minimum Time Control Laws provide the rate of change of reactivity necessary to either achieve or maintain the specified reactor period. Figures 4.5.1-1 and 4.5.1-2 are from a simulation study in which the capabilities of these laws were investigated. Shown is a power increase from 1 kW to 1 GW, six orders of magnitude, accomplished in 4.15 seconds using the alternate MIT-SNL minimum time law. The specified period was 0.30 seconds. The reactor parameters used for this simulation study were those of the MIT Research Reactor. That is, the value of the prompt neutron lifetime (ℓ^*) was 100 microseconds and that of the effective delayed neutron fraction (β) was 0.00786 $\Delta K/K$. Thermal feedback effects were excluded from the model in order to observe the basic physics.

Figure 4.5.1-1 shows the resulting power and reactivity profiles. The power increase was completed without overshoot in the allotted time of 4.15 seconds. Note that the power was leveled in the presence of substantial positive reactivity by making the rate of change of reactivity negative. Physically, the prompt neutron population was intentionally decreased in order to balance exactly the continuing rise in the delayed neutrons. Figure 4.5.1-2 shows the behavior of the control signal and that of the delayed terms. Recall that the quantity $\dot{\rho}$ corresponds to the effect of the changing prompt neutron population while the terms $\lambda'_e \rho$ and $\Sigma \beta_i (\lambda_i - \lambda'_e)$ represent the combined

Figure 4.5.1-1 Power Increase Using Minimum Time Control Laws

Figure 4.5.1-2 Behavior of Control Signal and Delayed Terms

effects of a change in the precursor population and a changing distribution of precursors. It is apparent from this figure that the transient was initiated using a very high rate of change of reactivity. As a result, the prompt neutron population readjusted rapidly and caused power to rise at the specified rate. Delayed effects then began to build in and it became possible to gradually reduce the required rate of change of reactivity. A second feature worth noting in this figure is that the reactivity approached the prompt critical value. Had a smaller value of the delayed neutron fraction or a larger prompt neutron lifetime been used, the prompt critical value would have been surpassed. However, this would not have caused any difficulty because neither the derivation of the dynamic period equations nor that of the MIT-SNL minimum time laws required use of the 'prompt-jump' or any other approximation. Hence, the controller would have continued to function properly even if the reactivity were to have exceeded the prompt critical value. (Note: This topic is further explored in section 6.4 of this report.) A final feature worth noting from this figure is that the rate of change of reactivity, which is the control signal, was changed discontinuously at both the outset and termination of the transient. This behavior is typical of the "bang-bang" response associated with time-optimal control. The conduct of the transient was in fact time-optimal because the instantaneous period was stepped from infinity to its minimum allowed value, held at that value until the desired power level was attained, and then stepped back to infinity thereby causing the power adjustment to be continuously made at the maximum permitted rate.

Details of the implementation, characterization, experimental evaluation, and assessment of the MIT-SNL minimum time laws are given in subsequent chapters of this report. Nevertheless, several aspects of these control laws warrant mention here. First, it may seem strange to write the control signal as a function of the controlled parameter. However, because a time-optimal response has been specified, the desired behavior of the controlled parameter is known. The objective of the control law is therefore to determine the value of the control signal necessary to generate that behavior. Second, the rate of change of reactivity, rather than the reactivity itself, is being used as the control signal. This is logical because, as was discussed in section 4.4 of this report, a changing reactivity corresponds to the effect of the prompt neutrons on the transient and the prompt effect is being used to drive the transient. Third, algorithms based on the alternate control law are easier to implement than those obtained from the standard version because the latter has a term containing the derivative of its effective, multi-group decay parameter and that term is noisy when evaluated in real time. Fourth, the treatment of the term $\lambda^* \dot{\omega}(t)$ requires special consideration. Ideally, the change in $\omega(t)$ should occur as a step. However, the presence of the quantity $\lambda^* \dot{\omega}(t)$ shows that this is not physically attainable. Precise numerical studies of both control laws have shown that the immediate effect of a step change in $\dot{\rho}(t)$ is a corresponding step change in $\dot{\omega}(t)$. As a result, the quantity $\omega(t)$ readjusts on a rapid exponential to its specified value. Relative to the control laws, the quantity $\dot{\omega}(t)$ is determined from the expression:

$$\dot{\omega}(t) = [\omega(t) - 1/\tau]/[n(\Delta t)] \quad (4.5.1-1)$$

where τ is the measured reactor period, Δt is the sample interval, and n is the number of sample intervals permitted for $\omega(t)$ to attain its specified value. Given that the quantity $n(\Delta t)$ can be made small, the readjustment of $\omega(t)$ will occur so rapidly (a hundredth of a second or less) as to be essentially a step. (Note: This issue is discussed at length in section 6.1.3 of this report.) Fifth, it should be recognized that the MIT-SNL laws are model-based. Accurate knowledge of the reactivity, decay parameters, and differential rod worth is essential to the proper functioning of these laws.

5. Considerations Regarding Implementation of the MIT-SNL Minimum Time Laws

Material is presented here as it relates to the implementation of the MIT-SNL Period-Generated Minimum Time Control Laws. Included is a discussion of the relation of the MIT-SNL minimum time laws to the reactivity constraint approach, a description of the implementing procedure for those laws, a methodology for the real-time calculation of the multi-group decay parameters, and results of the initial experimental trial of these laws on the MIT Research Reactor.

5.1 Relation of the MIT-SNL Laws to the Reactivity Constraint Approach

The MIT-SNL Period-Generated Minimum Time Control Laws can be used to generate virtually any desired power profile. They accomplish this by determining the rate of change of reactivity necessary to produce the instantaneous reactor period that corresponds to the specified power trajectory. As such, these laws are predictive in nature. That is their strength. Their weakness is that they do not provide any mechanism for verifying in advance that the requested rate of change of reactivity will in fact be available. For transient initiation, this is an operational but not a safety concern. Failure to achieve the desired rate of reactivity change merely means that the rate of power increase will be less than that specified. However, for transient termination, inability to attain the desired rate of reactivity removal would be both an operational and a safety issue. Specifically, failure to remove reactivity at the requisite rate would result in a power overshoot making use of the reactor's independent safety system potentially necessary. In order to preclude the latter scenario, the MIT-SNL laws are normally implemented in conjunction with the 'reactivity constraint approach' which restricts the net reactivity to that which can be offset by the dynamic effect of rod insertion [2]. As an aid to readers who may not be familiar with that methodology, a brief review of the concept is given here.

5.1.1 Overview of the Reactivity Constraint Approach

The objective of the reactivity constraint approach is to provide a means for the closed-loop digital control of a reactor's neutronic power during transients so that there will not be a challenge to the reactor's safety system. Currently, the technique has been implemented for situations in which the prompt-jump approximation is valid. Under this condition, the standard dynamic period equation becomes:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta} - \rho(t))} \quad (5.1.1-1)$$

The avoidance of safety system challenges can be achieved if it is possible to make the instantaneous reactor period infinite upon attainment of the desired power level. This goal can be realized if the net reactivity is constrained so that:

$$[\lambda_e(t)\rho(t) + (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta}-\rho(t)) + \dot{\rho}_f] \leq |\dot{\rho}_c| \quad (5.1.1-2)$$

where the term $\rho(t)$ is the net reactivity, both that added deliberately by the control mechanisms and that present indirectly from feedback effects. The quantity $\dot{\rho}_f$ denotes the rate of change of reactivity due to thermal-hydraulic feedback effects, and the symbol $|\dot{\rho}_c|$ denotes the maximum available rate of change of reactivity that could be obtained were a control mechanism to be moved. As such, $|\dot{\rho}_c|$ is always a non-zero finite number regardless of whether or not the mechanism is actually being moved. If the above inequality is observed, then upon the insertion of the control devices, the quantity $\dot{\rho}_c$ will be negative and the sum of the terms in the denominator of the standard dynamic period equation will be zero. As a result, the reactor period will be driven to infinity. Equation 5.1.1-2 is basically a statement that delayed neutron effects, which are not subject to direct control, must be restricted to that which can be offset by an induced change in the prompt population.

For reasons that have been previously discussed [2], it is preferable from a control viewpoint to rewrite equation 5.1.1-2 as:

$$[\lambda_e(t)\rho(t)] \leq |\dot{\rho}_c| \quad (5.1.1-3)$$

Equation 5.1.1-3 is an 'absolute reactivity constraint'. If observed, it will be possible to level the reactor power at any time during a transient by reversing the direction of movement of the control devices. The quantity $(|\dot{\rho}_c|/\lambda_e(t))$ is therefore said to be the amount of reactivity for which control is 'continuously feasible'. A less stringent constraint can be written that specifies that there should be sufficient time available to eliminate whatever reactivity is present beyond the amount that can be immediately negated by reversal of the direction of motion of the designated control mechanism before the desired power level is attained. This requirement, a 'sufficient reactivity constraint', can be written for power increases as:

$$[\rho(t) - |\dot{\rho}_c|/\lambda_e(t)]/|\dot{\rho}_c| \leq \tau(t)\ln(P_F/P(t)) \quad (5.1.1-4)$$

where P_F and $P(t)$ are the desired and current power levels, respectively, and $\tau(t)$ is either the observed (instantaneous) reactor period or the asymptotic period that corresponds to the net reactivity, whichever results in a more conservative decision. The quantity on the left of equation 5.1.1-4 is the time that will be required to reduce the net reactivity to the amount that can be offset by continuous insertion of the control mechanism. It is designated as the 'required time'. The quantity on the right is a measure of the time remaining to attain the specified power. It is designated as the 'available time'. If the inequality represented by the constraint is

satisfied, then control mechanism movement as specified by an associated control law is permitted. If the constraint is not satisfied, then the control device is inserted. A major advantage of the reactivity constraint approach is that it determines if a change should be made to the present control signal in order to avoid an overshoot at some future time. Moreover, it does this without the use of predictive models.

Reactivity constraints may also be derived from the alternate dynamic period equation. These are of the form:

$$[\lambda'_e(t)\rho(t)] \leq |\dot{\rho}_c| \quad (5.1.1-5)$$

$$[\rho(t) - |\dot{\rho}_c|/\lambda'_e(t)]/|\dot{\rho}_c| \leq \tau(t)\ln(P_F/P(t)) \quad (5.1.1-6)$$

where all symbols are as previously defined except that $\lambda'_e(t)$ is the alternate, effective multi-group decay parameter. Unlike the MIT-SNL minimum time laws, the standard and alternate forms of the MIT-CSDL Non-Linear Digital Controller are mathematically different. For example, using the alternate approach, the amount of reactivity for which control is 'continuously feasible' is given by $(|\dot{\rho}_c|/\lambda'_e(t))$. Owing to the different definitions of the multi-group decay parameters, this quantity is not the same as when calculated using the standard version. In fact, it differs by roughly a factor of five. (Note: Refer to equations 4.3.1-6 and 4.3.2-6 for the definitions of the multi-group decay parameters.) This point is often a source of confusion. Why should the amount of reactivity for which control is feasible depend on whether it is calculated using the standard or alternate form of the dynamic period equation? The point is that both the quantities $(|\dot{\rho}_c|/\lambda_e(t))$ and $(|\dot{\rho}_c|/\lambda'_e(t))$ are approximations and that the approximation obtained from the alternate equation is the more conservative. The selection of a standard or an alternate constraint is, as discussed in [2,11], a matter of the user's preference.

5.1.2 Feasibility of Control

Controllers for a reactor's neutronic power that utilize the 'reactivity constraint approach' are said to incorporate the property of 'feasibility of control'. Specifically, a reactor together with a specified control mechanism is defined here as constituting a system that is 'feasible to control' if the system can be transferred from a given power level and rate of change of power (i.e., period) to a desired steady-state power level without overshoot (or conversely, undershoot) beyond specified tolerance bands, if any. This concept has two important attributes. First, it applies to a reactor and to the specific control mechanism designated for use in accomplishing a given transient. Second, not all states are allowable intermediates through which the system may pass while transiting from some initial to some final power. Excluded are both those states that represent actual overshoots and those from which overshoots could not be averted by manipulation of the specified control mechanism. It is important

to realize why certain intermediate states must be avoided. As noted in section 3.4.2 of this report, the rate of rise of reactor power is a function of the rates of change of both the prompt and delayed neutrons. The former are directly controllable. The latter are not owing to the delay in their production. Power changes can only be halted if the rate of rise of the delayed neutrons can be offset by making the rate of change of the prompt neutrons negative. Physically, this means that it must be possible to counter positive reactivity by a negative reactivity insertion rate. Hence, the need to restrict operation to only certain combinations of reactivity and available reactivity insertion rates. (Note: It should be recognized that the concept of 'feasibility of control' is distinct from the more general property of 'controllability'. That term has a specialized meaning in that a system is said to be controllable if "any initial state can be transferred to any final state in a finite time by some control sequence" [17]. This definition does not place any restrictions on intermediate states.)

5.1.3 Demonstration of the Reactivity Constraint Approach

The implementing algorithm for the reactivity constraint approach is the MIT-CSDL Non-Linear Digital Controller (NLDC). Figure 5.1.3-1 shows a power increase of 1-4 MW that was accomplished on the MIT Research Reactor using the NLDC. The control device was a shim blade which had integral and differential reactivity worths of 1.56% $\Delta K/K$ and $0.75 \cdot 10^{-4}$ $\Delta K/K$ per second. The controller consisted of the standard sufficient constraint and a directive to withdraw the blade continuously subject to a thirty second limitation on the observed (instantaneous) period. Shown in the upper portion of the figure are the power and reactivity profiles. Shown in the lower portion are the required and available times. As is evident, power rose rapidly and leveled off without overshoot. The period approached 30 seconds, which is the minimum allowed dynamic period. Note that the dynamic effect of blade insertion permitted the power to be held constant despite the presence of 0.078% $\Delta K/K$ of positive reactivity. (Note: 1 beta equals 0.78% $\Delta K/K$.) Also shown are the required and available times which illustrate the role of the sufficient constraint. Note that when these two times became equal, the direction of blade motion was reversed and reactivity removal commenced. There was a corresponding change in the rate of rise of the reactor power. The effect of the limit on dynamic period is apparent in the intermediate stages of this transient. It caused a gradual reduction in the rate of increase of the reactivity.

Relative to the actual conduct of the transient, it is worth noting that control is continuously feasible only if the reactivity is within the bounds of the absolute constraint. (Note: These bounds will not be symmetric about the origin because the magnitude of the effective, multi-group decay parameter depends on whether power is being raised or lowered.) Once the reactivity exceeds these bounds, control is only guaranteed to be feasible at the desired power level. At the outset of the transient, the reactivity is zero. Given that there is some rate of change of reactivity available were the designated control device to be moved, the term $\left| \dot{\rho}_c \right|$ is non-zero. Hence,

Figure 5.1.3-1 Power Increase via Closed-Loop Digital Control
Using the MIT-CSDL Non-Linear Digital Controller

the required time is negative and equal to $(-1/\lambda_e)$ or -12.2 seconds. This indicates that control is already continuously feasible. The available time is initially zero because the desired power equals the actual power. Once the change is made in the demanded power, the available time becomes infinite because the period in the previously steady-state reactor was infinite. Control mechanism motion then commences and the reactivity becomes positive. The required time becomes less negative, passes through zero, and then becomes positive indicating that some finite interval must now elapse before the transient can be halted. The transition from the absolute to the sufficient constraint occurs when the required time is zero. The available time tends towards zero because the period is becoming shorter and the power is rising. Once the time required to restore continuous feasibility of control equals the time remaining to attain full power, continued control mechanism withdrawal is prohibited and insertion is begun. The required time is continuously bounded by the available time indicating that the control mechanism is being more or less constantly inserted. The required and available times eventually both become zero. When this occurs, the reactor power can be leveled because control is again within the range of the absolute constraint. There is still positive reactivity present in the core at this time and the dynamic effect of control mechanism insertion is required to counter this positive reactivity. Hence, the control mechanism must be driven in continuously at this time. As the reactor power settles out, the available time remains zero. The required time becomes negative, eventually resuming its original value of $(-1/\lambda_e)$.

The preceeding example illustrates the complementary roles of the supervisory and predictive components of the control law. The former's contribution is that it reviews the decision of the predictive component in terms of current reactor conditions and intervenes as necessary. Its deficiencies are that it lacks both the ability to project values of power and reactivity forward in time and the capability to cause the shape of the transient power curve to conform to a particular set of specifications. The predictive portion of the controller can provide those functions in which the supervisory program is deficient. However, it can not guarantee the safety of its own actions unless the reactor is operating within the envelope of conditions for which it was designed. Both components are necessary.

5.2 Calculational Sequence for the MIT-SNL Minimum Time Laws

Implementation of both the standard and the alternate MIT-SNL Period-Generated Minimum Time Control Laws is readily accomplished using the following method to determine the sequence of the digital control signals:

- (1) The quantity $\omega(t)$ is taken to be the desired reactor period. Usually, this is the minimum allowed value of the instantaneous reactor period. Attainment and maintenance of this specified period is the objective of the control law.

- (2) The time allowed to attain this desired period is specified. This is usually taken as one sample interval. However, as discussed in section 6.1.3 of this report, larger values are sometimes preferred.
- (3) Knowing the current period, the specified period, and the allowed time to attain that period, the quantity $\dot{\omega}(t)$ is calculated using equation 4.5.1-1.
- (4) The reactivity and the appropriate effective, multi-group decay parameter are either measured or calculated.
- (5) Each term in the appropriate control law is evaluated.
- (6) Either the standard or the alternate MIT-SNL minimum time law (either equation 4.5-2 or 4.5-3 depending on the user's preference) is solved to obtain the rate of change of reactivity necessary to establish the specified period.
- (7) Using previously obtained knowledge of the differential reactivity worths of the associated control mechanisms, the required velocity of the control mechanism is determined. The control device is then moved at this rate. As a result, the period changes from infinity to its specified value in the number of sample intervals stipulated from step 2.
- (8) The reactivity, decay parameter, and period are now either measured or calculated in order to update each term in the control law.
- (9) The selected control law (either standard or alternate) is solved to obtain the rate of change of reactivity necessary to maintain the specified period. Again using premeasured curves of the differential reactivity worths of the control mechanism, the needed velocity of those mechanisms is determined. The mechanisms are then moved at this rate.
- (10) Steps 8 and 9 are repeated until the reactor power attains the desired level.
- (11) The specified value of the reactor period is then set to infinity so as to achieve a zero rate of change of power.
- (12) Steps 2 through 7 are then repeated to obtain the rate of change of reactivity necessary to drive the period to infinity.
- (13) The control mechanism is moved so as to provide the needed rate of change of reactivity. As a result, the period moves back to infinity in the specified number of sampling intervals. Again, this occurs essentially as a step.
- (14) Steps 8 and 9 are repeated for as long as it is necessary to

maintain the period infinite and thereby hold the power constant.

The above methodology was the one employed for both the MIT and the ACRR experiments. (Note: A modification to this methodology is given in section 10.5.4.1 of this report.)

Equations 4.5-2 and 4.5-3 are suitable for use with any reactor period including ones that are as short as a few tenths of a second. If relatively long periods that correspond to slow rates of power increase are desired, then equations 4.5-2 and 4.5-3 may be simplified by omitting the terms that contain the prompt neutron lifetime. The standard and alternate MIT-SNL minimum time control laws then become respectively:

$$\dot{\rho}(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda_e(t)\rho(t) - (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t)) \quad (5.2-1)$$

and

$$\dot{\rho}(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda'_e(t)\rho(t) - \Sigma \bar{\beta}_i(\lambda_i - \lambda'_i(t)) \quad (5.2-2)$$

The procedure used to implement equations 5.2-1 and 5.2-2 is the same as that given above except that the determination of the rate of change of the inverse of the specified period ($\dot{\omega}(t)$) is omitted.

5.2.1 Incorporation of Feedback

Assuming that the reactor is accurately modeled, the implementation sequence described above will automatically compensate for unanticipated perturbations. For example, if the coolant temperature were to rise, the measured value of the reactivity would decrease thereby causing the rate of change of reactivity necessary to maintain the specified power to be increased. Thus, one approach is for the specified period to be altered only to reflect a change in the specified power trajectory. Unfortunately, modeling imperfections are a reality and it is usually necessary to include some means for on-line correction of the control signal. The approach selected for the experiments conducted in October and December 1987 entailed recalculating the specified reactor period at every sampling interval. This was done using the relation:

$$\omega_s(t) = \ln[n_s(t)/n(t)]/\Delta t \quad (5.2.1-1)$$

where: $\omega_s(t)$ is the inverse of the specified reactor period,

$n_s(t)$ is the specified value of the reactor's neutronic power,

$n(t)$ is the observed value of the reactor's neutronic power, and

Δt is the duration of the sample interval.

Subsequent to the completion of the December 1987 experiments, it was realized that equation 5.2.1-1 was in fact causing an oscillatory response. Following an extensive stability analysis, an improved method for incorporating on-line corrections in the calculation of the specified period was devised. This is described in section 12.1.3 of this report. Basically, it entails making any needed correction over a certain number of sampling intervals. Thus,

$$\omega_s(t) = \ln[n_s(t+\delta)/n(t)]/\Delta t \quad (5.2.1-2)$$

where δ is the time interval by which the calculation of the specified power should be advanced. It should be at least two sampling intervals. Five sampling intervals appears to yield the best results. Further information is given in sections 10.5.4.1 and 12.1.3 of this report.

5.2.2 Specification of Sample Interval

One of the objectives of the MIT-SNL research program was, as mentioned, to develop control laws capable of increasing a reactor's neutronic power by many orders of magnitude both in a few seconds and without overshoot. Periods as short as 0.30 seconds were among those to be considered. When dealing with such short periods it is important to recognize that both the design of the control law and the method of its implementation are essential to the prevention of power overshoots. As regards the design of the MIT-SNL minimum time laws, there is no inherent problem. The laws themselves are rigorous, closed-form expressions that provide the rate of change of reactivity necessary to generate any desired period. Hence, if it were possible to implement these laws in a continuous manner and assuming that the requested rate of reactivity change is in fact available, there should not be a power overshoot. Unfortunately it is of course impossible to apply these laws in a continuous manner. A digital system is necessary and hence the actions specified by the MIT-SNL laws are both calculated and applied in a discrete manner. One consequence of this discrete implementation is that a power overshoot may occur even if the control law is functioning properly. For example, suppose that the observed power is slightly below the specified level at the start of a sample interval of duration Δt_s . The power will continue to rise on the specified period throughout that entire sampling interval. It will only be at the start of the next sampling interval that the control law will be capable of adjusting the rate of reactivity change so as to drive the period to infinity. Hence, when implemented in discrete fashion, even a properly formulated control law may not be able to avoid some degree of overshoot. Once this fact is recognized, it becomes possible to determine the necessary sample (or cycling) time for the digital computer system. Suppose that overshoots of 5% are acceptable and that the minimum specified period will be 0.30 seconds. Then, the largest acceptable sampling interval will be:

$$\Delta t_s = \tau_s \ln[n_m/n_d] \quad (5.2.2-3)$$

where: Δt_s is the sampling interval,

τ_s is the minimum specified reactor period,

n_m is the maximum allowed power, and

n_d is the desired power level.

Taking the ratio of (n_m/n_d) to be 1.05 and τ_s to be 0.30 seconds, the largest acceptable sample interval is 15 milliseconds. As is discussed in section 8.2 of this report, this requirement bears directly on the selection of the computer system and the design of the hardware to-software interfaces.

5.3 Estimation of the Effective, Multi-Group Decay Parameters

A precondition for the implementation of control methodologies deduced from either the standard or the alternate dynamic period equation is that it be possible to calculate accurately the effective, multi-group decay parameters in real time during transients. As reported previously [2], two methods for estimating the decay parameters were developed as part of the MIT program that resulted in the reactivity constraint approach. These were:

- (a) Direct Solution of the Kinetics Equations: Both the neutron and the precursor kinetics equations are solved and the parameters are then calculated from their definitions. This approach requires extremely small time steps (0.0001 seconds or less) because the neutron kinetics equation is quite stiff. The advantage of this approach is that the behavior of the neutron population over each sampling interval is calculated accurately. (Note: Sampling intervals are typically 0.05 seconds.) The disadvantage is that significant computational resources are necessary.
- (b) Correlation Methods: If asymptotic behavior is assumed, then correlations can be developed between the decay parameters and the reactivity. For example, regarding the standard decay parameter, this can be accomplished by noting that the inhour equation relates the asymptotic reactor period to the reactivity. Rewriting the inhour equation subject to the prompt-jump approximation and rearranging terms yields:

$$\lambda_e^* = (\bar{\beta} - \rho) / \tau_p \quad (5.3-1)$$

where terms are as previously defined except that:

λ_e^* is the asymptotic value of the standard, effective, multi-group decay parameter, and

τ is the asymptotic period.

Given the period, the inhour equation can be used to obtain

the reactivity. Using the period and reactivity, equation 5.3-1 can be solved for the asymptotic value of the standard, effective, multi-group decay parameter. Thus, a correlation can be developed between the decay parameter and the reactivity. A similar approach can be used for the alternate decay parameter. The resulting correlations are shown in Figures 5.3-1 and 5.3-2. The advantage to this approach is that the computation is trivial. The disadvantage is that it is not accurate except under near-asymptotic conditions. In particular, it can not be used for rapid adjustments of reactor power.

Neither of these methods was suitable for the conduct of rapid transients such as would be performed using the MIT-SNL laws. Accordingly, various numerical methods for solving the precursor kinetics equations both accurately and rapidly were investigated. This work was performed by Myung-Hyun Kim under the direction of Professor Allan F. Henry of the MIT Department of Nuclear Engineering. (Note: This MIT work was an independent development. However, the contributions of others, such as Chao [18], in this area should be recognized.)

5.3.1 Standard Difference Technique

The first method investigated was to write both the neutron and precursor kinetics equations as difference equations using the same value of the weighting parameter, θ , for both relations. So doing, the following expression is obtained for the precursor concentrations:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t_n} = \frac{\bar{\beta}_i}{\ell^*} [\theta T^{n+1} + (1-\theta)T^n] - \lambda_i [\theta C_i^{n+1} + (1-\theta)C_i^n] \quad (5.3.1-1)$$

Rearranging terms, this becomes:

$$C_i^{n+1} = \frac{1}{(1/\Delta t_n + \theta \lambda_i)} \left[\frac{\bar{\beta}_i}{\ell^*} (\theta T^{n+1} + (1-\theta)T^n) + \left(\frac{1}{\Delta t_n} - (1-\theta)\lambda_i \right) C_i^n \right] \quad (5.3.1-2)$$

If the sampling interval is a constant, then (5.3.1-2) can be further simplified by defining the following constants:

$$\begin{aligned} A_{1i} &\equiv [1/\Delta t_n - (1-\theta)\lambda_i] / [1/\Delta t_n + \theta \lambda_i] \\ A_{2i} &\equiv \bar{\beta}_i / [\ell^* (1/\Delta t_n + \theta \lambda_i)] \\ A_{3i} &\equiv (1-\theta)\bar{\beta}_i / [\ell^* (1/\Delta t_n + \theta \lambda_i)] \end{aligned} \quad (5.3.1-3)$$

where the parameter i varies from 1 to N , the number of precursor and photoneutron delayed neutron groups. Hence, (5.3.1-2) reduces to the following, easily-programmed, expression:

Figure 5.3-1 Standard, Effective, Multi-Group Decay Parameter

Figure 5.3-2 Alternate, Effective, Multi-Group Decay Parameter

$$C_i^{n+1} = A_{1i}C_i^n + A_{2i}T^{n+1} + A_{3i}T^n \quad (5.3.1-4)$$

5.3.2 Time-Integration Method with Linear Power Profile

The central idea is to decouple the neutron and precursor kinetics equations by assuming a particular shape for the neutronic response during each sampling interval. The precursor kinetics equations, which are not stiff, can then be solved using time steps that are the same size as the sampling intervals, 0.01 seconds. (Note: The technique is quite robust and time steps as long as 1.0 second are possible.)

The precursor kinetics equation is:

$$\dot{C}_i(t) = \frac{\bar{\beta}_i}{\ell^*} T(t) - \lambda_i C_i(t) \text{ for } i = 1, N \quad (5.3.2-1)$$

where:

$\dot{C}_i(t)$ is the rate of change of the concentration of the ith precursor group,

$\bar{\beta}_i$ is the effective yield of the ith precursor group,

ℓ^* is the prompt neutron lifetime,

$T(t)$ is the amplitude function which is a weighted integral of all neutrons present in the core,

λ_i is the decay constant of the ith precursor group,

$C_i(t)$ is the concentration of the ith precursor group, and

N is the number of precursor and photoneutron delayed neutron groups.

Equation (5.3.2-1) is now treated using analytical methods. This yields:

$$C_i^{n+1} = C_i^n e^{-\lambda_i \Delta t_n} + \int_{t_n}^{t_{n+1}} \frac{\bar{\beta}_i}{\ell^*} T(t) e^{-\lambda_i(t_{n+1}-t)} dt \quad (5.3.2-2)$$

and

$$\Delta t_n = t_{n+1} - t_n \quad (5.3.2-3)$$

Note that (5.3.2-2) can not be solved unless the quantity $T(t)$ is known. If $T(t)$ were to be obtained from the neutron kinetics equation, then the solution would be quite stiff and would require small time steps. Therefore, in order to avoid that problem, a shape is assumed for $T(t)$. The shape selected should approximate as much as

possible the anticipated behavior of the power. Thus, a high order polynomial might be appropriate. However, as the order increases, so does the number of data points needed to describe the polynomial. This in turn causes the calculation, which is of course done in discrete form, to lag present time. For this reason, the use of a linear interpolation for the shape of the power profile was chosen. Advantages are that it is readily implemented and gives reasonable results. Thus, $T(t)$ is approximated linearly as:

$$T(t) \approx \left(\frac{t-t_n}{\Delta t_n}\right)T^{n+1} + \left(\frac{t_{n+1}-t}{\Delta t_n}\right)T^n \quad (5.3.2-4)$$

Substituting (5.3.2-4) into (5.3.2-2) yields the following for the second term of (5.3.2-2):

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right)\left(\frac{T^{n+1}}{\Delta t_n}\right)\left\{\int_{t_n}^{t_{n+1}} t e^{-\lambda_i(t_{n+1}-t)} dt - \int_{t_n}^{t_{n+1}} t_n e^{-\lambda_i(t_{n+1}-t)} dt\right\} + \quad (5.3.2-5)$$

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right)\left(\frac{T^n}{\Delta t_n}\right)\left\{\int_{t_n}^{t_{n+1}} t_{n+1} e^{-\lambda_i(t_{n+1}-t)} dt - \int_{t_n}^{t_{n+1}} t e^{-\lambda_i(t_{n+1}-t)} dt\right\}$$

Consolidating terms, this becomes:

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right)\left(\frac{T^{n+1}}{\Delta t_n}\right)\left\{\frac{\Delta t_n}{\lambda_i} - \frac{1}{\lambda_i^2}(1-e^{-\lambda_i \Delta t_n})\right\} + \quad (5.3.2-6)$$

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right)\left(\frac{T^n}{\Delta t_n}\right)\left\{\frac{1}{\lambda_i^2}(1-e^{-\lambda_i \Delta t_n}) - \frac{\Delta t_n}{\lambda_i} e^{-\lambda_i \Delta t_n}\right\}$$

Thus (5.3.1-2) becomes:

$$C_i^{n+1} = C_i^n e^{-\lambda_i \Delta t_n} + \left(\frac{\bar{\beta}_i}{\ell^*}\right)T^{n+1}\left\{\frac{1}{\lambda_i} - \frac{1}{\lambda_i^2 \Delta t_n}(1-e^{-\lambda_i \Delta t_n})\right\} + \quad (5.3.2-7)$$

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right)T^n\left\{\frac{1}{\lambda_i^2 \Delta t_n}(1-e^{-\lambda_i \Delta t_n}) - \frac{1}{\lambda_i} e^{-\lambda_i \Delta t_n}\right\}$$

Note that this expression is a simple two-point formula. If the sampling interval is a constant, (5.3.2-7) can be simplified by defining the following constants:

$$\begin{aligned}
 K_{1i} &\equiv e^{-\lambda_i \Delta t_n} \\
 K_{2i} &\equiv \frac{1}{\lambda_i^2 \Delta t_n} (1 - e^{-\lambda_i \Delta t_n}) \\
 K_{3i} &\equiv \frac{\bar{\beta}_i}{\rho^*} \left[\frac{1}{\lambda_i} - K_{2i} \right] \\
 K_{4i} &\equiv \frac{\bar{\beta}_i}{\rho^*} \left[K_{2i} - \left(\frac{1}{\lambda_i} \right) K_{1i} \right]
 \end{aligned} \tag{5.3.2-8}$$

where the parameter i varies from 1 to N , the number of precursor and photoneutron delayed neutron groups.

Thus, (5.3.2-8) reduces to:

$$C_i^{n+1} = K_{1i} C_i^n + K_{3i} T^{n+1} + K_{4i} T^n \tag{5.3.2-9}$$

This relation is easily programmed.

The values of the standard and alternate, effective, multi-group decay parameters are readily found from their definitions once the precursor concentrations are obtained. Thus, the major steps in the calculation are to (1) measure the power level, (2) assume a linear relation between the current and previous measured values of the power, (3) calculate the precursor concentrations, and (4) evaluate the decay parameters.

5.3.3 Time Integration Method with Exponential Power Profile

This method is similar to that developed in section 5.3.2 of this report except that the shape of the neutronic response during each sampling interval is assumed to be exponential, not linear. The rationale for choosing an exponential shape is that this is the profile associated with use of the MIT-SNL minimum time laws. Hence, while it was expected that this method would function quite well with the MIT-SNL laws, it was also recognized that this method might not be appropriate for the general case.

The precursor kinetics equation is again solved analytically to obtain:

$$C_i^{n+1} = C_i^n e^{-\lambda_i \Delta t_n} + \int_{t_n}^{t_{n+1}} \frac{\bar{\beta}_i}{\rho^*} T(t) e^{-\lambda_i (t_{n+1}-t)} dt \tag{5.3.3-1}$$

where all symbols are as previously defined. The amplitude function is now approximated exponentially as:

$$\ln T(t) \approx \left(\frac{t-t_n}{\Delta t_n}\right) \ln T^{n+1} + \left(\frac{t_{n+1}-t}{\Delta t_n}\right) \ln T^n \quad (5.3.3-2)$$

That is, a linear interpolation of the logarithm of the amplitude function is being used. Solving for $T(t)$, this becomes:

$$T(t) \approx \text{EXP}[\alpha^{n+1}(t-t_n) + \alpha^n(t_{n+1}-t)] \quad (5.3.3-3)$$

where the quantities α^n and α^{n+1} are defined respectively as:

$$\begin{aligned} \alpha^n &\equiv (\ln T^n) / \Delta t_n \\ \alpha^{n+1} &\equiv (\ln T^{n+1}) / \Delta t_n \end{aligned} \quad (5.3.3-4)$$

Substituting (5.3.3-3) into (5.3.3-1) yields the following for the second term of (5.3.3-1):

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right) \left[\text{EXP}[\alpha^n t_{n+1} - \alpha^{n+1} t_n - \lambda_i t_{n+1}] \right] \left[\int_{t_n}^{t_{n+1}} [\text{EXP}(\alpha^{n+1} - \alpha^n + \lambda_i)t] dt \right] \quad (5.3.3-5)$$

Integrating, this becomes:

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right) \left[\frac{1}{\alpha^{n+1} - \alpha^n + \lambda_i} \right] \left[e^{(\alpha^{n+1} \Delta t_n)} - e^{(\alpha^n \Delta t_n - \lambda_i \Delta t_n)} \right] \quad (5.3.3-6)$$

Substituting the definitions given by (5.3.3-4), (5.3.3-6) can be rewritten as:

$$\left(\frac{\bar{\beta}_i}{\ell^*}\right) \left[\frac{1}{(1/\Delta t_n) \ln(T^{n+1}/T^n) + \lambda_i} \right] \left[T^{n+1} - T^n e^{-\lambda_i \Delta t_n} \right] \quad (5.3.3-7)$$

Thus, the expression for the precursor concentrations becomes:

$$C_i^{n+1} = C_i^n e^{-\lambda_i \Delta t_n} + \left(\frac{\bar{\beta}_i}{\ell^*}\right) \left[\frac{\Delta t_n}{\lambda_i \Delta t_n + \ln(T^{n+1}/T^n)} \right] \left[T^{n+1} - T^n e^{-\lambda_i \Delta t_n} \right] \quad (5.3.3-8)$$

Again, if a constant sampling interval is assumed, the precursor expression can be simplified. So doing, (5.3.3-8) becomes:

$$C_i^{n+1} = P_{1i} C_i^n + \left[\frac{P_{2i}}{P_{3i} + \ln(T^{n+1}/T^n)} \right] [T^{n+1} - P_{1i} T^n] \quad (5.3.3-9)$$

where the following constants are defined as:

$$P_{1i} \equiv e^{-\lambda_i \Delta t_n}$$

$$P_{2i} \equiv \left(\frac{\beta_i}{\phi^*} \right) \Delta t_n$$

$$P_{3i} \equiv \lambda_i \Delta t_n \quad (5.3.3-10)$$

Note that, as was the case with the linear version, (5.3.3-9) is easily programmed.

5.3.4 Comparative Evaluation of Numerical Methods for Decay Parameter Estimation

Preliminary numerical studies had shown that both a direct solution of the kinetics equations and use of any of the three methods described above would yield the same value for the effective, multi-group decay parameters provided that the time step was quite small. Accordingly, a step size of 0.0001 seconds was taken for the reference case. Studies were then performed using time steps of 0.001, 0.01, 0.1, 0.5, and 1.0 seconds. Three power trajectories were considered. These were (1) an increase on a 0.30 second period from 30 W to 30 MW followed by an extended hold at 30 MW; (2) an increase on a 0.50 second period from 10 W to 1 kW, a hold at 1 kW for 1.0 second, a decrease to 10 W on a -0.50 second period, and an extended hold at 10 W; and (3) a 'zig-zag' profile in which power was repeatedly raised and lowered from 30 W to 1 MW on periods of 0.30 or -0.30 seconds. For all three trajectories, the value of the weighting parameter (θ) in the standard difference technique was taken as 0.50. The findings of these studies were as follows:

- (1) For a time step of 0.001 seconds, there was no discernable difference apparent between any of the three methods under investigation and the reference case.
- (2) For time steps of 0.01 seconds and greater, differences between the three methods under investigation and the reference case were apparent. In general, the standard difference technique was the least accurate and the integration method with the exponential power profile was the most accurate.

Table 5.3.4-1 provides representative results obtained from the third of the three power trajectories studied. The time step shown is 1.0 second. Figures 5.3.4-1 through 5.3.4-8 provide additional results

Table 5.3.4-1

Comparison of Methods for the Estimation of the Effective, Multi-Group Decay Parameters

Time (s)	Power (W)	Period (s)	λ_e				λ'_e			
			Standard	Linear	Exponential	Reference	Standard	Linear	Exponential	Reference
0.00	0.30000E+02	0.3000E+00	0.76798E-01	0.76798E-01	0.76798E-01	0.76798E-01	0.40529E+00	0.40529E+00	0.40520E+00	0.40529E+00
1.00	0.84095E+03	0.3000E+00	0.18512E+00	0.19352E+00	0.16545E+00	0.16545E+00	0.91664E+00	0.95455E+00	0.94245E+00	0.94245E+00
2.00	0.23573E+05	0.3000E+00	0.29410E+00	0.30590E+00	0.31728E+00	0.31728E+00	0.10412E+01	0.10745E+01	0.11424E+01	0.11424E+01
3.00	0.66079E+06	0.3000E+00	0.30130E+00	0.31322E+00	0.33072E+00	0.33072E+00	0.10457E+01	0.10789E+01	0.11504E+01	0.11504E+01
3.12	0.10000E+07	0.3000E+00	0.30344E+00	0.31078E+00	0.33090E+00		0.10549E+01	0.10714E+01	0.11505E+01	
4.00	0.53987E+05	-0.3000E+00	0.25421E+00	0.23906E+00	0.23564E+00	0.23564E+00	0.83878E+00	0.74259E+00	0.71492E+00	0.71492E+00
5.00	0.19259E+04	-0.3000E+00	0.16804E+00	0.17466E+00	0.17330E+00	0.17330E+00	0.31067E+00	0.42483E+00	0.41365E+00	0.41365E+00
6.00	0.68705E+02	-0.3000E+00	0.14379E+00	0.14369E+00	0.14322E+00	0.14322E+00	0.31692E+00	0.30547E+00	0.30242E+00	0.30242E+00
6.25	0.30000E+02	-0.3000E+00	0.13806E+00	0.13823E+00	0.13786E+00		0.29140E+00	0.28884E+00	0.28659E+00	
7.00	0.36720E+03	0.3000E+00	0.12425E+00	0.12462E+00	0.12442E+00	0.12442E+00	0.24769E+00	0.25414E+00	0.25316E+00	0.25316E+00
8.00	0.10293E+05	0.3000E+00	0.11245E+00	0.11281E+00	0.11266E+00	0.11266E+00	0.24741E+00	0.25337E+00	0.25402E+00	0.25402E+00
9.00	0.28853E+06	0.3000E+00	0.15305E+00	0.15765E+00	0.15793E+00	0.15793E+00	0.64789E+00	0.68150E+00	0.71004E+00	0.71004E+00
9.37	0.10000E+07	0.3000E+00	0.20249E+00	0.20635E+00	0.22512E+00		0.87642E+00	0.89495E+00	0.97050E+00	
10.00	0.12364E+06	-0.3000E+00	0.20748E+00	0.20035E+00	0.20726E+00	0.20726E+00	0.80267E+00	0.74377E+00	0.75274E+00	0.75274E+00
11.00	0.44107E+04	-0.3000E+00	0.14431E+00	0.14871E+00	0.15072E+00	0.15072E+00	0.33807E+00	0.42344E+00	0.41284E+00	0.41284E+00
12.00	0.15735E+03	-0.3000E+00	0.12051E+00	0.12110E+00	0.12326E+00	0.12326E+00	0.28810E+00	0.28929E+00	0.28905E+00	0.28905E+00
12.50	0.30000E+02	-0.3000E+00	0.11157E+00	0.11210E+00	0.11418E+00		0.25163E+00	0.25715E+00	0.25836E+00	
13.00	0.16034E+03	0.3000E+00	0.10433E+00	0.10465E+00	0.10663E+00	0.10663E+00	0.23053E+00	0.23497E+00	0.23674E+00	0.23674E+00
14.00	0.44945E+04	0.3000E+00	0.93236E-01	0.93528E-01	0.95184E-01	0.95184E-01	0.21170E+00	0.21590E+00	0.21724E+00	0.21724E+00
15.00	0.12599E+06	0.3000E+00	0.10398E+00	0.10587E+00	0.10512E+00	0.10512E+00	0.42233E+00	0.44512E+00	0.43790E+00	0.43790E+00

Notes: (1) Data shown are for a time step of 1.0 seconds.

(2) The trajectory examined is a 'zigzag' in which the power was repeatedly raised and lowered from 30 W to 1 MW on periods of 0.30 or -0.30 seconds respectively.

Figure 5.3.4-1 Reactor Power Trajectory for Case One

**Figure 5.3.4-2 Calculation of Decay Parameters Using a 0.50
Second Time Step - Power Trajectory One**

**Figure 5.3.4-3 Calculation of Decay Parameters Using a 1.0
Second Time Step - Power Trajectory One**

Figure 5.3.4-4 Reactor Power Trajectory for Case Two

**Figure 5.3.4-5 Calculation of Decay Parameters Using a 1.0
Second Time Step - Power Trajectory Two**

Figure 5.3.4-6 Reactor Power Trajectory for Case Three

Figure 5.3.4-7 Calculation of Decay Parameters Using a 0.50
Second Time Step - Power Trajectory Three

**Figure 5.3.4-8 Calculation of Decay Parameters Using a 1.0
Second Time Step - Power Trajectory Three**

for each of the power trajectories studied. It was concluded that the time integration method using an assumed linear power profile was the most appropriate approach for the planned experiments. The reasons for this selection were as follows. First, the time steps being considered were on the order of 40 ms. Hence, the accuracy of the method was not an issue. Second, the approach is both easily programmed and capable of rapid execution by a digital computer. Third, the additional accuracy offered by the use of an exponential profile was judged to be the result of using power trajectories characterized by constant periods. For the general case, it was suspected that the exponential approach would offer no inherent advantage.

5.4 Initial Experimental Trial of the MIT-SNL Laws

Experimental trials for the MIT-SNL Period-Generated Minimum Time Control Laws began in December 1986 on the 5 MWt MIT Research Reactor (MITR-II) and continued through October 1987 when the first of several intensive sets of experiments were conducted on Sandia's Annular Core Research Reactor (ACRR). Figures 5.4-1 and 5.4-2 give the results of one of the original MITR runs. Shown is an actual power increase of 100 kW that was performed under conditions of closed-loop, digital control using a variable speed motor to drive the reactor's regulating rod. The alternate MIT-SNL law was used. The protocol described in section 1.5 of this report was observed. The data shown are from a trial in which the specified period was 600 seconds. The period was stepped to this value by initially requiring a rate of reactivity insertion of about 1.7 millibeta per second. Then, as the delayed terms grew in magnitude, the rate of change of reactivity was reduced. On reaching full power, the rate of change of reactivity was made negative, the period stepped back to infinity, and the power increase stopped. Note that the initial and final power levels were 992 kW and 1100 kW. Hence, from the definition of reactor period, the minimum possible transit time was $\ln(P_F/P_0)$ or 62 seconds, where P_F and P_0 are the final and initial power levels. This is in excellent agreement with that obtained from the experiment. Moreover, given that the reactor period was maintained constant at 600 seconds for the entire transient, the power change was accomplished in the minimum time possible for a reactor limited to a 600 second period. This transient ran under the MIT-CSDL Non-Linear Digital Controller (NLDC). That is, the NLDC was monitoring the transient. The NLDC did not intervene indicating that control remained feasible at all times because the available rate of change of reactivity was always adequate for halting the transient. Hence, the MIT-SNL minimum time law was compatible with the 'reactivity constraint approach'. Figure 5.4-1 shows the power and reactivity profiles. Note that there was no power overshoot. Figure 5.4-2 shows the behavior of the terms that comprise the control law and which were dominant during this experimentation. The transient shown in these figures was quite mild. Its significance is that it was one of the very first successful demonstrations of the concept.

Figure 5.4-1 Initial Demonstration of the MIT-SNL Period-Generated Minimum Time Control Laws

Figure 5.4-2 Behavior of Control Signal and Delayed Terms
for Experiment Shown in Figure 5.4-1

6. Characterization of the MIT-SNL Minimum Time Laws

The MIT-SNL Period-Generated Minimum Time Control Laws were first derived in October-December 1986 with the initial experimental trials being conducted at MIT in December of that year. Following those successful tests, an intensive program was undertaken to characterize these laws using computer simulation. The transient selected for the reference case was a power increase of six orders of magnitude on a specified period of 0.30 seconds. If controlled correctly, the response time for this transient should be 4.15 seconds. The sampling time, which is defined as the interval over which measured data would be updated on a real reactor, was 0.01 seconds. Details of these simulation studies are described here. (Note: A summary of this material has been previously provided [19].)

6.1 Quantitative Description of the MIT-SNL Laws

The standard and alternate MIT-SNL Period-Generated Minimum Time Control Laws are:

$$\dot{\rho}_c(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda_e(t)\rho(t) - (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t)) - \dot{\rho}_f(t) + \quad (6.1-1)$$

$$\ell^* \dot{\omega}(t) + \ell^* [(\omega(t))^2 + \lambda_e(t)\omega(t) - (\dot{\lambda}_e(t)/\lambda_e(t))\omega(t)]$$

and

$$\dot{\rho}_c(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda'_e(t)\rho(t) - \Sigma \bar{\beta}_i (\lambda_i - \lambda'_i(t)) - \dot{\rho}_f(t) + \quad (6.1-2)$$

$$\ell^* \dot{\omega}(t) + \ell^* ((\omega(t))^2 + \lambda'_e(t)\omega(t))$$

The quantities $\lambda_e(t)$ and $\lambda'_e(t)$ are the standard and alternate effective, multi-group decay parameters and are defined respectively as:

$$\lambda_e(t) \equiv \Sigma \lambda_i C_i(t) / \Sigma C_i(t) \quad (6.1-3)$$

and

$$\lambda'_e(t) \equiv \Sigma \lambda_i^2 C_i(t) / \Sigma \lambda_i C_i(t) \quad (6.1-4)$$

These quantities are time-dependent because the relative distribution of the delayed neutron precursor groups changes as power is either raised or lowered. Other symbols are defined as:

$\bar{\beta}$ is the effective delayed neutron fraction,

$\rho(t)$ is the net reactivity,

ℓ^* is the prompt neutron lifetime,

$\omega(t)$ is the inverse of the specified reactor period,

$\dot{\omega}(t)$ is the rate of change of the inverse of the specified reactor period,

$\dot{\rho}_c(t)$ is the rate of change of reactivity associated with the control mechanisms necessary to either achieve or maintain the specified period,

$\dot{\rho}_f(t)$ is the rate of change of reactivity associated with feedback effects,

$\dot{\lambda}_e(t)$ is the rate of change of the standard effective, multi-group decay parameter,

λ_i is the decay constant of the ith precursor group,

$C_i(t)$ is the concentration of the ith precursor group, and

$\bar{\beta}_i$ is the effective yield of the ith precursor group.

Given current values of the net reactivity, the delayed neutron precursor distribution, and the desired (or specified) reactor period, these laws generate the rate of change of reactivity necessary to either achieve or maintain that specified period. Ideally, the change in $\omega(t)$ should occur as a step. However, the presence of the quantity $\dot{\omega}^*(t)$ shows that this is not physically attainable. Rather, the immediate effect of a step change in $\dot{\rho}(t)$ is a corresponding step change in $\dot{\omega}(t)$. As a result, the quantity $\omega(t)$ readjusts on a rapid exponential to its specified value. Relative to the control laws, the quantity $\dot{\omega}(t)$ is determined from the expression:

$$\dot{\omega}(t) = [\omega(t) - 1/\tau]/[n(\Delta t)] \quad (6.1-5)$$

where τ is the measured reactor period, Δt is the sample interval, and n is the number of sample intervals permitted for $\omega(t)$ to attain its specified value.

A qualitative description of the performance of the MIT-SNL laws relative to the reference trajectory (1 kW - 1 GW on a 0.30 second period) was given in section 4.5.1 of this report. That material is further discussed here with emphasis on more quantitative aspects of the behavior of the laws. As an aid to the reader, the figures given in section 4.5.1 are reproduced here.

Figure 6.1-1 shows the power and reactivity profiles achieved using the alternate MIT-SNL law for the reference transient assuming a prompt neutron lifetime of 100 microseconds and an effective delayed neutron fraction of 0.00786 $\Delta K/K$. Figure 6.1-2 depicts the control signal ($\dot{\rho}$) and the two delayed terms which are $\lambda'_i \rho$ and $\sum \bar{\beta}_i (\lambda'_i - \lambda'_e)$. The simulated reactor was initially at steady-state. Apparent in these figures are several of the distinctive features that are characteristic of the MIT-SNL laws. These are:

- There are abrupt changes in slope of the power profile upon

Figure 6.1-1 Reference Trajectory for Simulation Studies

Figure 6.1-2 Behavior of Control Signal and Delayed Terms
for Reference Trajectory

both transient initiation and termination. These occur because the period is first stepped from infinity to the specified value and then from that value back to infinity. A step change in the period results in a discontinuity in the slope of the power profile.

- The shape of the power profile during the transient is exponential if recorded on a linear plot or a straight line if recorded on a logarithmic scale. This results because the control law maintains a constant value for the period during the entire transient.
- At the outset of the transient, there is a rapid insertion of reactivity reflecting the need to use a high rate of change of reactivity to generate the specified reactor period.
- During the intermediate stages of the transient, there is a gradual cessation of reactivity addition indicating that the reactivity is approaching its asymptotic value for the specified period.
- Upon transient termination, there is a rapid decrease in reactivity indicating that the dynamic effect associated with control device insertion is being used to halt the transient.
- There are discontinuities in the rate of change of reactivity upon both transient initiation and termination.
- Prompt neutron effects ($\dot{\rho}$) dominate during transient initiation and termination. Otherwise, delayed effects ($\lambda'_e \rho$) and $\Sigma \beta_i (\lambda_i - \lambda'_e)$ are preeminent.

6.1.1 Behavior of Control Signal and Delayed Terms

Relative to the simulated transients shown in Figures 6.1-1 and 6.1-2, the reactor was initially at steady-state with a net reactivity of zero and an equilibrium distribution of the delayed neutron precursors. Thus, at the very outset of the transient, the only non-zero terms in the control law were those dependent on ω , the inverse of the specified period. Hence, the alternate law reduced to:

$$\dot{\rho}_c = \bar{\beta}\omega + \ell^* \dot{\omega} + \ell^* (\omega^2 + \lambda'_e \omega) \quad (6.1.1-1)$$

The value of ω was 3.33 because a period of 0.30 seconds was specified. Also, it was required that this period be attained within one sample interval or 0.01 seconds. Hence, from equation 6.1-5 the initial value of the $\ell^* \dot{\omega}$ term was 4237 millibeta per second (3.33% $\Delta K/K/\text{second}$). Those of the $\bar{\beta}\omega$ and $\ell^* (\omega^2 + \lambda'_e \omega)$ terms were 3330 and 158 millibeta per second (2.62 and 0.12% $\Delta K/K/\text{second}$) respectively. Thus, upon initiation of the transient, the alternate law caused the simulated control device to be moved so as to add reactivity at the

rate of 7725 millibeta per second (6.07% $\Delta K/K/\text{second.}$) This step change in $\dot{\rho}$ caused a corresponding step change in ω which in turn forced the period to its desired value of 0.30 seconds within 0.01 seconds. Once this occurred, the $\lambda^*\omega$ term became zero. Also, the delayed terms ($\lambda'\rho$ and $\Sigma\beta_i(\lambda - \lambda')$) began to increase in magnitude. Hence, the rate of change of reactivity necessary to maintain the 0.30 second period decreased significantly within the first few hundredths of a second. As the transient continued, the delayed terms continued to grow in magnitude and eventually became dominant. Physically, a changing prompt neutron population was used to initiate the transient whereas a changing delayed one was used to sustain it. Once the power attained the desired level of $1 \cdot 10^6$ kW, the prompt population was made to change so that its rate of decrease exactly offset the continuing rise in the delayed population. The required rate of change of reactivity was -4587 millibeta per second (-3.61% $\Delta K/K/\text{second.}$) Of that, -4237 millibeta (-3.33% $\Delta K/K$) was for the $\lambda^*\omega$ term. As was the case with transient initiation, the magnitude of the required rate of change of reactivity rapidly decreased as the delayed terms responded.

6.1.2 Behavior of Terms Dependent on the Specified Period

Both the standard and the alternate minimum time control laws contain terms that are dependent on the inverse of the specified period. Figures 6.1.2-1 and 6.1.2-2 show the behavior of these terms for the reference transient using the alternate law for two different values of the prompt neutron lifetime. These are 1000 and 100 microseconds. The former resulted in the simulated reactor's being prompt critical. Shown are the quantities $(\beta - \rho)\omega$ and $\lambda^*(\omega^2 + \lambda'\omega)$. Several features are worth noting. First, given that the reactivity is initially zero, the quantity $(\beta - \rho)\omega$ has the same initial value for both cases. Moreover, this term decreases rapidly in magnitude to a value set by the limiting reactivity. If that reactivity exceeds the prompt critical value, then the asymptotic value of $(\beta - \rho)\omega$ will be negative. Second, the quantity $\lambda^*(\omega^2 + \lambda'\omega)$ is nearly zero for short neutron lifetimes. Its magnitude was appreciable in the prompt critical case only because λ^* was taken to be 1000 microseconds. This quantity's slight slope, apparent in the prompt critical case, occurs because of the variation of the effective multi-group decay parameter (λ') with the power increase. Finally, both quantities go to zero once the desired period is set to infinity.

Figure 6.1.2-3 gives the same information for a simulation study conducted on Sandia's Annular Core Research Reactor or ACRR. The principal difference between this figure and the earlier ones is that here the prompt neutron lifetime is quite small, 24 microseconds.

It is important to recognize the distinction between the specified and the instantaneous (observed) periods. The former is an imposed requirement and it therefore can be changed as a step. The latter is a physical quantity which can not change discontinuously. Specifically, the instantaneous (observed) period, denoted here by the symbol τ , readjusts to a new value after times greater than $5(\lambda^*/\beta)$.

Figure 6.1.2-1 Behavior of Terms Dependent on the Specified
Period: Prompt Neutron Lifetime at 1000 μs
 (Alternate Minimum Time Law with Specified
 Period of 0.30 Seconds)

Figure 6.1.2-2 Behavior of Terms Dependent on the Specified
Period: Prompt Neutron Lifetime at 100 μs
 (Alternate Minimum Time Law with Specified
 Period of 0.30 Seconds)

Figure 6.1.2-3 Behavior of Terms Dependent on the Specified
Period: ACRR Design Parameters
 (Alternate Minimum Time Law with Specified
 Period of 0.30 Seconds)

This is because if the reactivity is initially zero, then even if $\dot{\rho}(0)$ is to take on a very large value instantaneously at $t=0$, the point kinetics equations show that $\tau(0)$ is zero. An exact analytical calculation for the early stages of the transient shows that $\tau(0)$ is indeed initially equal to zero and that after a term $\text{EXP}(-\beta/\lambda^*)$ dies off, $\tau(t)$ 'jumps' to a much larger value.

6.1.3 Treatment of the λ^* Term

As noted earlier, the presence of the $\dot{\omega}$ term in the control laws means that the period cannot be altered as a step. However, through use of equation (6.1-5), the period can be forced to its new value within a prespecified number of sample intervals. Increasing the number of sample intervals over which the adjustment is permitted to occur has the obvious advantage of decreasing the magnitude of the $\lambda^*\dot{\omega}$ term and thereby lessening the required rate of change of reactivity. Unfortunately, so doing will entail two unwanted consequences. First, the specified reactor period will not be attained at once and, as a result, system response will be slightly less than optimal. Second, the rate of reactivity change available for halting the power increase will be less than the rate that is physically required. This will result in an overshoot. Simulation studies were conducted to assess these problems. A comparison was made of the effect on transient initiation of permitting the readjustment of the period to occur over one sample interval (0.01 seconds) and over ten sample intervals (0.1 second total). The results are given in Table 6.1.3-1. There was essentially no difference in the resulting power levels between the two cases. This was true both during the transient and upon termination of the transient at 4.15 seconds. (Note: The existence of a factor of two difference in the power levels after 4.15 seconds is immaterial to the design specifications given that the reactor is on a 0.30 second period.) So, nothing was gained by forcing the readjustment to occur extremely rapidly. Moreover, two disadvantages were apparent. First, the required rate of change of reactivity was 4465 mbeta per second (3.51% $\Delta K/K/\text{second}$) for the one-interval adjustment and 3480 mbeta per second (2.74% $\Delta K/K/\text{second}$) for the ten-interval adjustment. So, the former would necessitate a more stringent design. Second, the one-interval insertion was so rapid that some oscillation resulted. This happened because the 4465 mbeta per second insertion rate was held for a full hundredth of a second. As a result, the period was driven beyond the desired value of 0.30 seconds and the rate of change of reactivity had to be cut back drastically. Such oscillations would not only cause damage to the actuators but also be counterproductive in that the resulting rate of rise of power may actually be slowed too much.

A similar comparison was made for the termination of the transient. The results of that study are given in Table 6.1.3-2. Here, there was a major difference in the power profiles. There was an overshoot of 10% for the ten-interval case as compared with 3% for the one-interval case. (Note: Given that the period is 0.30 seconds and the sampling time is 0.01 seconds, the power may exceed the targeted level by $\text{EXP}(0.01/0.30)$ or by a factor of 1.034 even if the control

Table 6.1.3-1

Effect of Treatment of the Term (λ^*) on Transient Initiation

<u>Time (sec)</u>	<u>Power (kW)</u>		<u>Rate of Change of Reactivity (mbeta/sec)</u>	
	<u>One Interval</u>	<u>Ten Intervals</u>	<u>One Interval</u>	<u>Ten Intervals</u>
0.01	1.00	1.00	4465	3480
0.02	1.03	1.02	2874	3239
0.03	1.07	1.06	3161	3117
0.04	1.10	1.10	2933	3001
0.05	1.14	1.13	2856	2889
0.06	1.18	1.17	2741	2782
0.07	1.22	1.21	2643	2679
0.08	1.26	1.26	2544	2579
0.09	1.31	1.30	2451	2484
0.10	1.35	1.34	2350	2392
.				
.				
.				
0.20	1.89	1.88	1625	1643
.				
.				
.				
1.00	27.92	27.69	89.0	88.0
.				
.				
.				
4.15	$0.48 \cdot 10^6$	$0.99 \cdot 10^6$	0	0

Table 6.1.3-2

Effect of Treatment of the Term (λ^*) on Transient Termination

<u>Time (sec)</u>	<u>Power (kW)</u>		<u>Rate of Change of Reactivity (mbeta/sec)</u>	
	<u>One Interval</u>	<u>Ten Intervals</u>	<u>One Interval</u>	<u>Ten Intervals</u>
4.15	$0.98 \cdot 10^6$	$0.99 \cdot 10^6$	-1407	-422
4.16	$1.02 \cdot 10^6$	$1.03 \cdot 10^6$	-158	-380
4.17	$1.03 \cdot 10^6$	$1.06 \cdot 10^6$	-310	-351
4.18	$1.03 \cdot 10^6$	$1.08 \cdot 10^6$	-287	-330
4.19	$1.03 \cdot 10^6$	$1.09 \cdot 10^6$	-287	-315
4.20	$1.03 \cdot 10^6$	$1.10 \cdot 10^6$	-283	-305
4.21	$1.03 \cdot 10^6$	$1.11 \cdot 10^6$	-280	-297
4.22	$1.03 \cdot 10^6$	$1.11 \cdot 10^6$	-277	-290
4.23	$1.03 \cdot 10^6$	$1.11 \cdot 10^6$	-274	-285
4.24	$1.03 \cdot 10^6$	$1.11 \cdot 10^6$	-271	-281
4.25	$1.03 \cdot 10^6$	$1.11 \cdot 10^6$	-269	-273

law is perfect.) The solution to this problem is quite simple provided that the net reactivity is below the prompt critical value. Under that condition and as shown in section 6.3 of this report, the rate of change of reactivity necessary to step the period back to infinity will be significantly less than the rate that was required to initiate the transient. Hence the readjustment of the period should be treated differently depending on whether the transient is being initiated or terminated. A further simulation study was conducted to demonstrate this concept. The period readjustment was forced to occur over ten sample intervals when going from infinity to 0.30 seconds and over one interval when going from 0.30 seconds to infinity. As expected, the results were excellent.

6.2 Demonstration of Asymptotic Behavior

As noted in section 4.3.3 of this report, the inhour equation is a special case of the dynamic period equation. The latter gives the relation between the reactivity and the reactor period subject to the conditions that the reactivity is constant and that sufficient time has elapsed since the reactivity addition so that asymptotic conditions have been obtained. Accordingly, several simulation runs were performed to demonstrate that, in the limiting case of asymptotic behavior, the reactivity values ultimately attained using the MIT-SNL minimum time control laws were the same as would be predicted from the inhour equation. The reference transient was run for prompt neutron lifetimes of 1000, 100, and 10 microseconds, each with an effective delayed neutron fraction of 0.00786 $\Delta K/K$. In each instance, asymptotic conditions were approached and limiting reactivities of 1333, 952, and 913 millibeta were obtained. (Note: These figures are 1.05, 0.75, and 0.72% $\Delta K/K$ respectively.) This behavior is in accordance with that predicted by the inhour equation as shown by Glasstone [20]. Namely, achievement of a specified reactor period requires a greater reactivity addition as the prompt neutron lifetime becomes larger. Additional simulations were performed in which the effect of varying the effective delayed neutron fraction was observed while holding the prompt neutron lifetime constant. As expected, for a given period, the reactivity addition must be increased as the delayed neutron fraction decreases. Figure 6.2-1, which is from Glasstone [20] shows the relationship between the asymptotic reactor period and the reactivity for various neutron lifetimes. Tables 6.2-1 and 6.2-2 summarize the results of the simulation runs performed to demonstrate that use of the MIT-SNL law does, under asymptotic conditions, yield the same result as does the inhour equation. (Note: As an aid to the reader, the argument showing that the inhour equation is a special case of the dynamic period equation is given in Appendix A to this report.)

6.3 Effect of Prompt Neutron Lifetime on Required Rate of Change of Reactivity

Studies were conducted to assess the effect of the prompt neutron lifetime on the rate of change of reactivity needed to both initiate and terminate transients using the MIT-SNL laws. The results are summarized in Table 6.3-1. Shown are the magnitudes of the terms that

Figure 6.2-1 Relationship Between Reactor Period and Reactivity
for Various Prompt Neutron Lifetimes

(Source: Glasstone and Sesonske, Fig. 5.4, p. 244,
1981 edition.)

Table 6.2-1

Effect of Generation Time on Limiting Reactivity
to Achieve a Specified Period

<u>Period</u>	<u>Generation Time</u>	<u>Limiting Reactivity</u>
0.30 seconds	$1 \cdot 10^{-3}$ seconds	1333 millibeta
0.30 seconds	$1 \cdot 10^{-4}$ seconds	952 millibeta
0.30 seconds	$1 \cdot 10^{-5}$ seconds	913 millibeta

(Note: Effective delayed neutron fraction ($\bar{\beta}$)
constant at 0.00786 $\Delta K/K$.)

Table 6.2-2

Effect of Delayed Neutron Fraction on Limiting Reactivity
to Achieve a Specified Period

<u>Period</u>	<u>Delayed Neutron Fraction</u>	<u>Limiting Reactivity</u>
0.30 seconds	0.00650 $\Delta K/K$	961 millibeta
0.30 seconds	0.00786 $\Delta K/K$	952 millibeta

(Note: Generation time constant at 10^{-4} seconds.)

Table 6.3-1

Effect of Prompt Neutron Lifetime on Required Rate of Change of Reactivity

Period Stepped from Infinity to 0.30 Seconds (Transient Initiation)

<u>Run</u>	<u>Neutron Lifetime</u>	<u>$\dot{\rho}$</u>	<u>$(\bar{\beta}-\rho)\omega$</u>	<u>$\lambda_e'\rho$</u>	<u>$\bar{\Sigma}\beta_i(\lambda_i-\lambda_e')$</u>	<u>$l^*\dot{\omega}$</u>	<u>$l^*(\omega^2+\lambda_e'\omega)$</u>
N/A	$1 \cdot 10^{-8}$	3769	3330	0	0	423	15.8
ACRR	$2.4 \cdot 10^{-8}$	4465	3330	0	0	1094	40.8
MITR	$1 \cdot 10^{-4}$	7724	3330	0	0	4236	158
Prompt	$1 \cdot 10^{-3}$	47,278	3330	0	0	42,366	1582

Period Stepped from 0.30 Seconds to Infinity (Transient Termination)

<u>Run</u>	<u>Neutron Lifetime</u>	<u>$\dot{\rho}$</u>	<u>$(\bar{\beta}-\rho)\omega$</u>	<u>$\lambda_e'\rho$</u>	<u>$\bar{\Sigma}\beta_i(\lambda_i-\lambda_e')$</u>	<u>$l^*\dot{\omega}$</u>	<u>$l^*(\omega^2+\lambda_e'\omega)$</u>
N/A	$1 \cdot 10^{-8}$	-729	0	1051	-745	-423	0
ACRR	$2.4 \cdot 10^{-8}$	-1407	0	1050	-745	-1094	0
MITR	$1 \cdot 10^{-4}$	-4586	0	1094	-745	-4236	0
Prompt	$1 \cdot 10^{-3}$	-43,154	0	1533	-745	-42,366	0

Note: Neutron lifetime in seconds. All other units in millibeta per second.

comprise the alternate constant period control law for a range of neutron generation times. Data is given for the instant immediately after the period was stepped from infinity to 0.30 seconds and from 0.30 seconds to infinity. The following is apparent.

- (1) As the prompt neutron lifetime rises, the required rate of change of reactivity also rises.
- (2) For a power increase, the quantity $(\bar{\beta}-\rho)\omega$ establishes the minimum value for the required rate of change of reactivity. The quantities $\beta^*\omega$ and $\beta^*(\omega^2 + \lambda'\omega)$ then add to this minimum. The use of a small prompt^s neutron lifetime naturally lessens their effect.
- (3) The rate of change of reactivity necessary to step the period back to infinity is always less than the rate that was necessary to establish the original positive period. This is an important observation with ramifications for the safety of any automatic control action. Specifically, assuming only negative reactivity feedback effects, then the time of maximum demand on the control system's actuators should occur at the outset of the transient and not at the time that power must be leveled.

The information shown in Table 6.3-1 and that from the preceeding section of this report suggest that a small value for the prompt neutron lifetime is desirable. In particular, a smaller neutron lifetime will result in both a smaller overall reactivity insertion and a lesser required rate of reactivity change. This in turn means that the control mechanisms and their actuators can be less massive and therefore presumably less expensive. However, it is important to recognize that there is a balancing viewpoint to this argument. Specifically, a short neutron lifetime will also make the reactor sensitive to perturbations. Thus, there may be a tradeoff between ease of controller design and reactor stability.

6.4 Performance of the MIT-SNL Laws While Prompt Critical

The MIT-SNL minimum time control laws were derived in a completely rigorous manner. No simplifying assumptions such as the 'prompt-jump' or 'one delayed neutron group' approximations were made. Hence, these laws should function properly regardless of the magnitude of the reactivity insertion. This was demonstrated by simulating the reference transient with a prompt neutron lifetime of 1000 microseconds and an effective delayed neutron fraction of 0.0786 $\Delta K/K$. This study showed that the control laws were, as expected, capable of maintaining the period constant at 0.30 seconds even if the reactivity exceeded the prompt critical value. The required rates of change of reactivity to initiate and to halt the transient were 47,278 and -42,366 milli-beta per second respectively. (Note: These figures are 37.2% and -33.3% $\Delta K/K/\text{second}$.) These rates of reactivity change are so enormous that the reactivity itself behaves as if it were being changed in a discontinuous manner. Therefore, leaving aside the safety aspects of

operating while prompt critical, it is apparent that such operation is not practical even though the control laws remain functional. That is, the size and complexity of the equipment that would be required to operate a reactor in the prompt critical region would be prohibitive.

Figure 6.4-1 shows the power and reactivity profiles that resulted from a simulation in which the neutronic power was increased by six orders of magnitude on a reactor with a delayed neutron fraction of 0.00786 $\Delta K/K$ and a prompt neutron lifetime of $1 \cdot 10^{-3}$ seconds. A reactivity of 1.33 Beta was added. Figure 6.4-2 shows the behavior of the control signal and delayed terms for this case. As noted above, it is apparent that the MIT-SNL laws are capable of functioning even under prompt critical conditions. However, it is also obvious that such operation could not, as a practical matter, be achieved. (Note: Figures 6.4-1 and 6.4-2 should be contrasted with Figures 6.1-1 and 6.1-2 which give the same transient except for a prompt neutron lifetime of 100 microseconds. As a result, prompt conditions are not attained.)

6.5 Power Decreases

Figure 6.5-1 shows the power and reactivity profiles for a power decrease from 7.0 to 1.0 kW on a -0.30 second period. The control law functioned perfectly. Power decreased in the allowed time (0.58 seconds) and leveled off within the allowed band. However, a negative reactivity insertion of more than 5 Beta (3.93% $\Delta K/K$) was required. Also, the required rate of change of reactivity grew more negative as the transient progressed, attaining a value of -18.7 Beta per second (-14.7% $\Delta K/K/\text{second}$). The reason was, of course, that the longest-lived delayed neutron groups were resisting the power decrease. It is apparent from this simulation that, as was the case with operation while prompt critical, the control law does generate the proper signals for the rapid power decrease. However, it is doubtful that a control system capable of implementing those signals could be constructed.

6.6 Reactor Design Considerations

The simulation studies reported here suggest certain desirable design features for reactors intended for spacecraft propulsion. First, the core and its control devices should be designed so that the differential reactivity worth of the control mechanisms is maximized over as wide a range of travel as possible. Second, the actuators used to move the control devices should be variable speed motors capable of both achieving the specified rate of travel and of reversing direction within a single sampling interval. That time may be as short as a hundredth of a second. Third, the reactor should be configured so that the prompt neutron lifetime is small. There are several reasons for this. Foremost is that the asymptotic reactivity associated with the design period specification of ≈ 0.30 seconds will be less than the prompt critical value. This has important safety implications. Also, by keeping the reactivity less than the prompt critical value, the rate of change of reactivity necessary to step the

Figure 6.4-1 Power Increase in Minimum Time: Reactivity
in Excess of Prompt Critical

Figure 6.4-2 Behavior of Control Signal and Component Terms:
Reactivity in Excess of Prompt Critical

Figure 6.5-1 Power Decrease Using Alternate Minimum Time
Control Law on a Specified Period of -0.30 Seconds

period back to infinity will be significantly less than the rate that was necessary to establish the original positive period. This is an important consideration with ramifications for the feasibility of any automatic control action. Specifically, it means that if there are only negative reactivity feedback effects, then the time of maximum demand on the control system's actuators should occur at the outset of the transient and not at the time that power must be leveled. Expressed more simply, if it is possible to initiate the transient, then it should be possible to halt it. A further reason for desiring a short prompt neutron lifetime is that, in addition to a smaller overall reactivity insertion, the required rates of change of reactivity will be more manageable. Specifically, for a power increase, the quantity $(\beta - \rho)\omega$ establishes the minimum value for the required rate of change of reactivity. The quantities $\beta^*\omega$ and $\beta^*(\omega^2 + \lambda'\omega)$ then add to this minimum. The use of a small generation time naturally lessens their effect. (Note: It should be realized that there is a major opposing argument to the use of a short prompt neutron lifetime. Namely, the shorter the lifetime, the more sensitive the reactor will be to perturbations. Thus, there may be a trade-off between ease of controller design and reactor stability.) Additional factors that bear on reactor design are discussed in section 12.3 of this report.

7. Safety Considerations in the Implementation of the MIT-SNL Period-Generated Minimum Time Control Laws

Safety considerations as they related to the conduct of the experimental evaluations of the MIT-SNL Period-Generated Minimum Time Laws are described here. As was discussed in section 5.1 of this report, the MIT approach to the design of controllers for reactor power is to utilize both supervisory constraints and predictive control laws. The former restrict operation to the envelope of conditions from which no challenge to the safety system will result. The latter provide the actual control signal, subject to override by any constraint. That distinction is maintained here in that safety considerations related to the implementation of the constraints are discussed separately from the larger issue of use of the MIT-SNL minimum time laws. Following a brief discussion of the concept of an 'unreviewed safety question' material is presented as it relates first to the MIT Research Reactor and then to Sandia's Annular Core Research Reactor.

7.1 Regulatory Perspective: Definition of an Unreviewed Safety Question

In the United States, research and test reactors may be operated under either the Nuclear Regulatory Commission (NRC) or the Department of Energy (DOE). The MIT Research Reactor (MITR-II) is licensed through the NRC while Sandia's Annular Core Research Reactor (ACRR) is operated subject to DOE regulations. Both the U.S. Nuclear Regulatory Commission and the U.S. Department of Energy use similar approaches for determining whether or not a particular experiment can be performed on a given reactor. Specifically, both organizations have in place a methodology that ensures the proper operation of the facility and hence the safety of the general public while at the same time providing as much flexibility as possible to the facility's users. The basic idea is for each facility to predefine the envelope of conditions within which experiments can be conducted. Following a detailed safety analysis that is performed by the facility in question and thoroughly reviewed by either the Nuclear Regulatory Commission or the Department of Energy as appropriate, the restrictions associated with the proposed envelope are issued as technical specifications. These specifications are part of a reactor's operating license and have the force of law. As specific experiments are then proposed, each is evaluated for compliance with the technical specifications. If the proposal falls within the established bounds, it can be performed subject to an internal review. If not, prior approval must be obtained from the relevant regulatory agency (NRC or DOE). In both cases, a safety analysis must be prepared and documented. The process used to decide whether or not a given proposal falls within the scope of existing, preestablished approvals is formally designated as an "unreviewed safety question determination". Specifically, an unreviewed safety question is considered to exist if the probability of occurrence or the consequences of an accident or malfunction of equipment important to safety previously evaluated in the facility's safety analysis report is increased, or the possibility for an accident or malfunction of a different type than any evaluated previously is

created, or if the margin of safety as defined in the basis of any technical specification is reduced. The need to observe this criteria was central to the design of both the hardware and the hardware-to-software interfaces used to conduct the experimental trials of the MIT-SNL minimum time laws on both the MITR-II and the ACRR.

7.2 MIT Research Reactor

The MIT Research Reactor (MITR-II) had, as part of its original license, approval to use automatic controllers subject to a limitation on the reactivity worth of the associated absorber. The absence of challenges to the safety system was assured by restrictions on the actuator rather than by requirements on the design of the controller. Starting in February 1983, closed-loop digital control experiments were conducted on the MITR-II using the 'reactivity constraint approach' and its implementing algorithm, the MIT-CSDL Non-Linear Digital Controller. (Refer to section 5.1 of this report.) Initially, use of this digital controller was limited to actuators that met the restriction of reactivity worth on the associated absorber. Once evidence of the reliability of the technique accumulated, an application was made to the U.S. Nuclear Regulatory Commission for general use of the reactivity constraint approach on the MITR-II. That is, the absence of challenges to the safety system would be assured by incorporating the concept of 'feasibility of control' in the controller's design rather than by limiting the reactivity worth of the associated actuator. This approval was received in April 1985. Details of the safety analysis, which have been previously provided [2,7], are summarized here. Detailed information is then provided on the changes necessary to allow on-line testing of the MIT-SNL minimum time laws.

7.2.1 Description of MITR Control System

The control system of the MIT Research Reactor (MITR-II) provides the means of inserting and withdrawing six shim blades and one regulating rod. The former are used for startup, shutdown, xenon compensation, and the accomplishment of large power changes. The latter is used for steady-state operation and the performance of small power adjustments. The technical specifications that pertain to the control system stipulate that only one shim blade be withdrawn at a time, that the six shim blades be maintained as a bank with no individual blade's position deviating from the bank average by more than an allowed amount, and that the maximum positive reactivity insertion rate be less than a specified value. The reactor may be operated in either a manual or an automatic mode. Prior to April 1985, the technical specifications limited use of the automatic control system to the regulating rod which typically had integral and maximum differential reactivity worths of $0.17\% \Delta K/K$ and $0.167 \cdot 10^{-4} \Delta K/K$ per second respectively. The reason for this restriction was that, given the regulating rod's low worth, even its continuous withdrawal would only create a relatively slow transient that would be significantly mitigated by the reactor's negative temperature coefficient. The situation with respect to blades was quite different because each

typically had integral and maximum differential reactivity worths of 1.56% $\Delta K/K$ and $0.75 \cdot 10^{-4}$ $\Delta K/K$ per second. Blades, which were only used under manual control, were chosen for movement by means of a manually-operated 'blade-selection' switch. Each blade was driven at fixed-speed by a 120 volt AC motor. The blades were, and are, coupled to the drives by electromagnets. Should the reactor's safety system be tripped, the magnets deenergize thereby dropping the blades and shutting down the reactor. Thus far, for all MITR digital controllers, the manually-operated 'blade-selector' switch has been retained so as to preclude the possibility of a relay failure that results in more than one blade being simultaneously connected to the automatic controller.

7.2.2 Safety Considerations: Supervisory Control of Reactor Power

Relative to the control system, general use of the MIT-CSDL Non-Linear Digital Controller entailed installing the capability to move a shim blade using a fixed-speed motor under conditions of closed-loop, digital control. This proposed change was reviewed in terms of the definition of an unreviewed safety question and the following two issues were identified as requiring resolution:

- (1) The possibility that continuous blade withdrawals would be more probable.
- (2) The implications, if any, of substantially increasing the worth of the absorber attached to the closed-loop control system.

Although both of these concerns were resolved almost three years before the experimental trials of the MIT-SNL minimum time laws were initiated, it is useful to recapitulate the actions that were taken. Specifically, the breadth of the issues that must be addressed in order to implement a digital controller is worth noting.

7.2.2.1 Non-Existence of Continuous Blade Withdrawals

A continuous blade withdrawal is one form of a 'loss of regulation' accident. That term encompasses malfunctions in which the software is properly written but nevertheless the wrong action is taken. Loss of regulation can occur because of improper sequencing of the software, hardware failures, or malfunctions in the interface between the digital system and the blade drives. One of the most important considerations in preventing 'loss of regulation' accidents is the installation and use of a properly designed and calibrated system for signal validation and instrument fault detection. Included in such a system should be rationality checks to ensure that the signals are reasonable for the given reactor condition. In addition, signals should be compared both to each other and to the output of a real-time reactor model. The existence of an inconsistency over several sampling intervals would be indicative of an instrument failure. As an example of the necessity of signal validation methodologies, suppose that a neutron sensor were to saturate midway during a transient.

Without some means of identifying that detector as having failed, an automated controller would continuously withdraw the blade in a vain effort to cause the reactor's neutronic power, as seen by the failed sensor to increase. As a result, the true power would soon exceed that which was specified making use of the independent safety system necessary. Signal validation is essential to any form of digital control.

Signal validation concerns the correctness of sensor readings being sent to the digital computer. Equally important to the prevention of controller malfunctions is the verification that directives received from the computer are properly implemented. Signal verification is necessary to preclude failures in the interface between the fast-acting, electronic computer and the slow-acting, mechanical, blade drive systems. It is most readily accomplished by comparing the current response of the reactor to the control action specified in the previous time step. If the response is not as desired, or at least not in the general direction of that specified, then it should be assumed that a malfunction has occurred.

A third area useful to the prevention of 'loss of regulation' accidents is the incorporation of various protective features in both the hardware and software. Some of the more important ones are listed here:

- (1) Selection of a Default State: On failure of the digital system, should control revert to manual? Or, should the control devices be continuously inserted? For the MIT Reactor the former was chosen because the anticipated transients are mild and there is sufficient reaction time available to the operator.
- (2) Transparency of Automatic Control: The adjective 'transparent' means that the operation and function of the facility's manual controls should appear unchanged to those using them. Specifically, should an operator desire to establish manual control, he (or she) should only have to do those actions with which he (or she) is already familiar.
- (3) Computer-Interlock Circuits: Special circuits can be built to provide protection against hardware malfunctions of both the digital-to-analog converter and the computer itself.
- (4) Protection Against Software Indecision: Signal validation systems do not protect against the simultaneous, identical failure of several sensors. Protection can be obtained by substituting the maximum (i.e., most conservative) possible value for a parameter for the actual signal, should such a situation occur.
- (5) Automatic Control Permit Circuits: A special circuit can be installed to enable the use of a closed-loop digital system. It would consist of a series of relays, each representing a prerequisite for the closed-loop mode of operation.

- (6) Test and Calibration Schedules: Digital controllers are similar to any other operating system in that periodic maintenance is required. A combination of operability tests and scheduled calibrations are necessary to assure system reliability.
- (7) Auxiliary Trips: If appropriate, hard-wired circuits can be installed to cause a trip to the default state prior to attainment of conditions that would necessitate safety system action. For example, the use of such circuits on the reactor period might be desired.

If the closed-loop, digital control system is to be used solely for experimental purposes, then the features described above (signal validation, signal verification, and the various protective features) are probably sufficient. However, if the system is to be used for routine operation then, in addition to the above, the digital controller should probably be implemented in a manner that is both fault-tolerant and reconfigurable [9]. That is, it should be possible for the system to sense internal faults and to reconfigure its available components to permit continued operation.

7.2.2.2 Implications of Increased Absorber Worth

Restrictions on the reactivity worth of a reactor's absorbers should not be a function of the mode of control. Rather, the principal concern should be that the safety system is capable of performing its intended function. Assuming that use of a digital, closed-loop controller in lieu of manual operation neither alters the potential rate of reactivity insertion nor changes the requirements on the safety system's response times, it should be realized that the nature of the initiating event is immaterial. That is, the challenge posed to the safety system does not depend on whether the root cause is operator error or a malfunction of an automatic controller. Therefore, the only action necessary to ensure the integrity of the safety system is to maintain it "separate" from the control system with the word "separate" defined as meaning that "the output of an instrument used in the safety system will not be influenced by interaction with the control system". The fact that the safety system should not be compromised by a failure in the control system means that it should always be capable of meeting any challenge posed by a malfunction of the controller.

7.2.2.3 Summary of Safety Arguments Concerning Supervisory Controller

The basic argument contained in the safety evaluation supporting general use of the MIT-CSDL Non-Linear Digital Controller (NLDC) on the MIT Research Reactor was that (1) the safety and control systems would remain separate and the capability of the former would therefore be unchanged, (2) the functions of the control system would be unchanged and hence there was no possibility of a new type of accident

nor would the consequences of an existing (potential) accident be increased, and (3) the probability of occurrence of any analyzed (potential) accident would not be increased. Evidence for this last portion of the argument was that the NLDC had been successfully tested via simulation, ex-core mockups, and extensive on-line trials under the original technical specification. The new technical specification which was issued in April 1985 and which governs both general use of the NLDC and the MIT-SNL Period-Generated Minimum Time Control Laws contains five stipulations. These are (1) that the controller incorporate the principle of 'feasibility of control', (2) that each proposed control law be the subject of an internal safety analysis, (3) that the safety system be separate from the control system, (4) that should a limiting condition (in this case too short a period) be approached, control revert to manual, and (5) that the operability of the period trip be tested at approved intervals. There is no limitation on the reactivity worth of the associated absorbers. This digital controller has now been used for several hundred power increases and decreases of 20% or more. No challenge to the safety system, not even an incipient one, has ever occurred.

Appendix B to this report contains additional material, excerpted from [2], concerning the receipt of license approval for general use of the MIT-CSDL Non-Linear Digital Controller on the MIT Research Reactor.

7.2.3 Safety Considerations: MIT-SNL Period-Generated Minimum Time Control Laws

As regards the control system, the principal difference between the MIT-CSDL Non-Linear Digital Controller and the MIT-SNL Period-Generated Minimum Time Control Laws is that the latter require use of a variable speed stepper motor. Use of this type of motor introduced the possibility of a new type of malfunction in that positive reactivity might be inserted in excess of the maximum allowed rate should the motor overspeed. With a fixed speed motor, as had been previously used, such a condition could not occur because such motors are in effect 'locked' to the 60 Hz frequency of the electric power grid.

Analysis of the proposed MITR experiments showed that the desired testing of the MIT-SNL minimum time laws could be achieved if it were possible to use a specified period of 30 seconds. For shim blades positioned near the peak of the differential reactivity worth curves, achievement of a 30 second period as a 'step' change meant that the selected blade would have to be driven at a speed of 12.75 inches per minute. This was a factor of three greater than the normal speed of 4.25 inches per minute. Accordingly, it was decided to monitor operation of the variable speed motor using a hard-wired 'over-speed' detection circuit. This circuit was designed so that if the stepper motor's speed were to exceed the normal speed by more than a certain factor, control would revert to manual (the default state) and automatic blade motion would stop. Reactivity addition rates in excess of that allowed should therefore not be possible. The over-speed trip was designed to be functional for both the manual and closed-loop

modes of operation. If tripped while in manual, it would block out-motion of the blade. In-motion, which was the desired conservative action, would not be impeded.

7.2.3.1 Over-Speed Trip Circuit

Figure 7.2.3-1 is a block diagram showing the over-speed trip circuit for the stepper motor. This circuit was designed, built, and tested by Mr. Paul T. Menadier. Figure 7.2.3-2 shows the relation between it and the overall control circuit for variable speed control of one of the MITR-II's shim blades. As shown in Figure 7.2.3-1, the overspeed trip circuit monitors the pulse output of the stepper motor translator and causes a trip from the digital automatic mode of operation to manual control (the default state) if the pulse rate exceeds a preset limit. The circuit functions in the following manner. A sample of the translator drive frequency is supplied to the monitor circuit via an optical isolator and counted for 100 ms out of each 200 ms clock period. If the input pulse rate exceeds the preset limit for two successive counting periods, the over-speed trip is activated. This circuit, which must be manually reset, will cause the computer auto-control-permit to open, thus switching from auto to manual control and removing the computer-generated drive pulses from the translator. An audible signal will also occur if this trip activates. The over-speed circuit also monitors the manual control single speed pulses which are generated in the translator itself. Included as part of this circuit is a self-testing feature. The circuit checks the frequency every 200 ms. For the first 100 ms of each interval, it checks for an actual over-speed condition. For the other 100 ms it does a self-test in which over-speed is specified. Detection of this condition continuously resets a trip. Hence, should the circuit fail, a trip will result.

Prior to its implementation on the MIT Research Reactor, the over-speed trip circuit was tested off-line over an interval of several weeks using an ex-core mockup of a shim blade motor and drive. This approach allowed its operation to be verified under conditions of realistic signal noise. Once installed, a requirement was established for periodic testing of this trip.

7.2.3.2 Conduct of Experiments

In addition to the over-speed trip circuit on the variable speed stepper motor, several other requirements were imposed on the conduct of experiments including demonstrations of the MIT-SNL Period-Generated Minimum Time Control Laws. Foremost among them was the requirement to observe the experimental protocol described in section 1.5 of this report. Specifically, the minimum time laws are used only in conjunction with the MIT-CSDL Non-Linear Digital Controller. This approach assures that the overall controller (minimum time law and supervisory reactivity constraint) encompasses the concept of 'feasibility of control' and hence satisfies the conditions of the MITR-II's technical specifications. Relative to the actual conduct of the experiments, the standard sufficient reactivity constraint (Equation

Figure 7.2.3-1 Stepper Motor Over-Speed Circuit

Figure 7.2.3-2 Variable Speed Shim Blade Computer Control
Circuit

5.1.1-4) is usually used as the supervisory algorithm. It is programmed to intervene if the reactor's neutronic power could approach 4.5 MW. The MIT-SNL control law, either the standard or the alternate form, is programmed to maneuver power between 1 and 4 MW. Hence, the control law can be realistically tested (i.e., it can overshoot) and the NLDC will prevent power from exceeding 4.5 MW. Reactor heat removal is set for full power operation (5 MW) and all components of the reactor's safety system are, as required, kept separate from the controller.

Other requirements imposed on the testing of the MIT-SNL minimum time laws were that both positive and negative period trips be operable. The former was to prevent an excessive rate of rise of reactor power. The latter's purpose was more subtle. It was installed to alert the operator to a sudden, anomalous insertion of negative reactivity and to prevent the automated controller from compensating for that reactivity.

7.3 SNL Annular Core Research Reactor

As was the case with the MIT Research Reactor, Sandia's Annular Core Research Reactor had, as part of its original design, a method for automatic control. The ACRR's device was an eight-step programmable controller that could be used to move that reactor's three transient rods. If a particular power trajectory was desired, the user could approximate it as eight distinct segments. For each segment, the speed, direction, and duration of movement of the transient rods would be specified. Unlike the MIT Research Reactor, no distinction was made between the amount of reactivity that could be associated with this eight-step controller and that which could be inserted when on manual control. Both modes of operation, manual and automatic, were subject to the same limiting conditions.

It was soon recognized that testing of the MIT-SNL Period-Generated Minimum Time Control Laws could be performed within the existing envelope of conditions. The major difference was that, instead of an experimenter specifying eight individual pre-programmed segments, an algorithm running on a digital computer would provide both the direction and speed of the control device at every sampling interval. Relative to control, performance would be significantly improved because on-line feedback would be possible. Relative to safety, nothing would be changed because the output of the digital system would merely be substituted for the pre-programmed instructions. The existing control system's hardware, interlocks, and administrative restrictions would all be retained.

7.3.1 Description of ACRR Control System

As noted in section 1.4.2 of this report, Sandia's Annular Core Research Reactor is a modified TRIGA that uses UO_2 -BeO fuel elements. The ACRR is ideal for the testing of control strategies that involve the adjustment of neutronic power on periods of less than a second because it has an exceptionally large negative coefficient of reactivity

associated with fuel temperature. Hence, if despite all the precautions, a control system malfunction were to occur during one of these tests, the reactor would shut itself down. That is, as the power started to rise the fuel would heat up thereby inserting negative reactivity and terminating the transient before any fuel damage could occur.

The ACRR can be operated in either a steady-state or pulsed mode. Relative to the former, power is limited to 2 MW. Relative to the latter, there is a limit of 500 MJ total energy per pulse and one of 1800°C on the fuel temperature. The majority of the experiments performed to evaluate the MIT-SNL minimum time laws were conducted subject to the limitation of the pulsed mode of operation because power was in excess of 2 MW.

The ACRR is equipped with two safety rods, six control rods, and three transient rods. The safety and control rods are made of B₄C followed by fuel. The transient rods also are made of B₄C but followed by air (a hollow aluminum tube sealed on both ends). The transient rods are the only ones that can be manipulated using automatic control. They may either be ejected from the core using a pneumatic system or they may be withdrawn at specified rates using variable-speed stepper motors. The latter method of operation, which is the one used for the MIT-SNL experiments, is called the "TRW" or 'transient rod withdrawal' mode. The standard approach to the procedure for performing experiments on the ACRR is as follows:

- a. Pull out both safety rods.
- b. Position the transient rods such that the full withdrawal of the transient rods will provide the necessary reactivity to either obtain the desired pulse of energy or to produce the specified power trajectory.
- c. Take the control rods to the estimated critical position and obtain a "delayed" critical. That is, the reactor is at this time critical on both prompt and delayed neutrons.
- d. Start the timer and count down from 120 seconds.
- e. The transient rods will be withdrawn in the specified manner at the end of the countdown.

The transient rods are normally operated as a bank (gang) and, as mentioned, can be moved either pneumatically or via stepper motors. Relative to the first method, nitrogen gas is used to pressurize each rod's piston chamber and thereby eject the three rods in times as short as 55 milliseconds. When operated in this manner, the maximum allowed reactivity insertion is 3 Beta. This limit is assured by inserting pedestals in the piston chamber assemblies. These pedestals act as zero-stops and limit the worth of the 'full-in' position of the transient rods so that they are capable of holding down no more than 3 Beta of reactivity in the critical reactor.

The alternate means of moving the transient rods is the 'Transient Rod Withdrawal' or TRW method. Using this approach, the transient rods are raised by fork-shaped devices which are each connected to variable speed stepper motors via an electromagnet and a rack and pinion gear. The maximum speed of withdrawing the transient rods without any chance of magnet separation is 750 pulses per second. Higher speeds are possible provided that the contact between the fork-shaped device (attached to the motor) and the lifting collar (connected to the absorber) has been made.

If moved from the 'full-in' to 'full-out' position, the total reactivity associated with the transient rods is 4.25 Beta. Transient rod location on the ACRR is given in "position units" with 'full-in' being 2031 units and 'full-out' being 7464 units. Every pulse delivered to the stepper motors moves the rods 2.661 units. The motors revolve one complete turn for every 200 pulses.

The basic components of the safety system for the ACRR are an NVT calculator and a scram circuit. The NVT calculator determines the energy release associated with operation in excess of 2 MW. (Note: The acronym 'NVT' stands for the product of neutron density, neutron velocity, and time. This product gives the integrated neutron fluence which is directly proportional to the total energy release that occurs during the transient.) The scram circuit will cause an automatic shutdown to prevent either the energy release from exceeding 500 MJ or the fuel temperature from rising above 1800°C.

7.3.2 Safety Considerations: Supervisory Control of Energy Production

As was noted in section 7.2 of this report, the major tenets of the MIT philosophy regarding the application of digital controllers to the operation of nuclear reactors are that the automatic controller should be structured so that there will not be a challenge to the safety system as the result of any automatic control action and that the safety systems should be independent of the control system so that the safety system's capability to perform its intended function will not be impaired. Achievement of the first of these objectives involves three equally important subgoals. These are (1) the creation of a methodology for sensor validation, signal verification, and hardware reliability; (2) the development of a supervisory algorithm that will provide a means for determining if the present control signal should be changed so as not to cause a safety system challenge at some future time, and (3) the incorporation of special design features, such as over-speed trips on stepper motors, to ensure the proper functioning of the control law. This approach to safety was also adopted for the testing of the MIT-SNL minimum time laws on Sandia's Annular Core Research Reactor. However, because of the design differences between the MITR-II and the ACRR, certain changes had to be made in the safety envelope. This was particularly true of the supervisory control algorithm where energy as well as reactivity constraints were necessary.

A major concern in the operation of any reactor is that the maxi-

imum allowable fuel temperature not be exceeded. For the MIT Research Reactor, which is cooled by forced convection, this concern translates to a limitation on the allowed power level. For Sandia's Annular Core Research Reactor, which has specially designed high temperature fuel and which operates essentially adiabatically, this concern entails a restriction on the net energy production. Accordingly, it was necessary to develop an 'energy constraint approach' that paralleled the 'reactivity constraint' methodology.

7.3.2.1 Energy Constraint Approach

The purpose of the supervisory routine is to restrict reactor operation to the envelope of conditions for which control will always be feasible. That is, it must be possible to halt a transient prior to exceeding any limiting parameter. For reactors subject to a restriction on power, the supervisory algorithm previously developed at MIT requires that the time necessary to reduce the net reactivity to the level such that the transient can be immediately halted always be less than the time remaining to attain the specified power level. These two times have been designated as 'required' and 'available' respectively. The overall methodology is known as the 'reactivity constraint approach' and the implementing algorithm is the MIT-CSDL Non-Linear Digital Controller. Relative to reactors subject to a limitation on energy, the same approach can be applied. Only now, the supervisory algorithm would specify that the time required to make the reactor subcritical always be less than the time remaining to generate the maximum allowed amount of energy.

Time to generate maximum allowed amount of energy: The behavior of a reactor's neutronic power level as a function of time is given by the following relation:

$$n(t) = n_0 e^{A(t)} \quad (7.3.2.1-1)$$

$$A(t) = \int_0^t d\sigma/\tau(\sigma) \quad (7.3.2.1-2)$$

where $n(t)$ is the instantaneous reactor power,

n_0 is the initial reactor power, and

$\tau(t)$ is the instantaneous reactor period.

Assuming that reactor conditions will be unchanged, the energy that will be produced can be found by integration of (7.3.2.1-1) subject to the restriction that the reactor period is a constant. Thus,

$$E(t) = n_0 \tau [e^{t/\tau} - 1] \quad (7.3.2.1-3)$$

The quantity needed for the formulation of the energy constraint is the time remaining to generate some maximum allowed value of the energy. Denoting the limiting value of the energy as $E_{\max}$ and the energy produced prior to the current time by the symbol $E_p(t)$, then the time

(ΔT_A) remaining to generate the limiting amount of energy is given by the expression:

$$\Delta T_A = \tau(t) \ln \left[\frac{(E_{\max} - E_p(t))}{n(t)\tau(t)} + 1 \right] \quad (7.3.2.1-4)$$

where $n(t)$ and $\tau(t)$ are the current values of the instantaneous neutronic power and reactor period respectively. The quantity ΔT_A is designated as the 'available time'.

Time to make the reactor subcritical: Also needed for formulation of an energy constraint is an estimate of the time that will be required to make the reactor subcritical and hence terminate the further production of energy.

Assuming that it is desired to make a reactor core subcritical by an amount ρ_{sd} , the expression for the required time is:

$$\Delta T_R = [\rho(t) + \rho_{sd} - |\dot{\rho}_c|/\lambda_e(t)]/|\dot{\rho}_c| \quad (7.3.2.1-5)$$

where $\rho(t)$ is the net reactivity,

ρ_{sd} is the reactivity necessary to assure a subcritical reactor,

$|\dot{\rho}_c|$ is the maximum available rate of change of reactivity that could be obtained were the control devices to be moved, and

$\lambda_e(t)$ is the standard, effective, multi-group decay parameter.

Energy constraints: The expression for supervisory control of a reactor that is subject to a limitation on net energy production is given by the relation:

$$[\rho(t) + \rho_{sd} - |\dot{\rho}_c|/\lambda_e(t)]/|\dot{\rho}_c| \leq \tau(t) \ln \left[\frac{E_{\max} - E_p(t)}{n(t)\tau(t)} + 1 \right] \quad (7.3.2.1-6)$$

It is intended that this energy constraint be used in conjunction with a predictive control law. The decisions of that control law are implemented provided that the inequality represented by the constraint is satisfied. Otherwise, the control rods are to be inserted. For the constraint to be effective, it should be evaluated at every sampling interval. The update frequencies used at the MIT Research Reactor and at Sandia's Annular Core Research Reactor were 1.0 second and 40 milliseconds respectively.

Equation (7.3.2.1-6) is a standard, sufficient constraint. It could also be written in absolute form or with use of the alternate multi-group decay parameter. As is the case with the various possible reactivity constraints, each of these forms of the energy constraint would result in a different response. For example, alternate con-

straints always yield a slower response than do standard ones [11]. The point is that each form of constraint ensures the absence of a challenge to the safety system. The choice of a particular form depends on the needs of the experiment.

7.3.3 Safety Considerations: MIT-SNL Period-Generated Minimum Time Control Laws

Relative to possible accident scenarios involving the control law itself, the one identified as being of concern was a situation in which an incorrect frequency was supplied to the stepper motors and a continuous rod withdrawal resulted that inserted the full reactivity worth of the transient rod bank. Accordingly, the safety aspects of using the MIT-SNL Period-Generated Minimum Time Laws to raise the power of Sandia's Annular Core Research Reactor were investigated by first modeling the ACRR and then simulating a series of continuous rod withdrawals at various stepper motor frequencies. Also, to the maximum extent possible, the results of these simulations were compared to data obtained from actual transients. This latter step served to confirm the validity of the model. Each simulation assumed that the full worth of the transient rod bank (4.25 Beta) was available for insertion. The results of these simulations for certain frequencies are shown in Table 7.3.3-1. Note that the fuel temperature level of 1800°C (2073 K) was never exceeded. Another feature worth noting is that even though 4.25 Beta of reactivity was available for insertion, the reactivities present upon attaining peak power never approached this amount. The reason was of course the strong negative reactivity feedback effect associated with the rising fuel temperature. Table 7.3.3-2 provides a comparison between experimental data and certain of the simulations. The results indicate that the model used in the simulations was realistic. The large mismatches in the 'time to peak power' between experiment and simulation were the unavoidable result of differences in the mode of reactivity insertion, pulsed vs. linear ramp, for the two cases. (Note: These studies were performed by Mr. Francis J. Wyant of SNL.)

These studies showed that even if a sustained, excessive frequency were to be supplied to the stepper motors from the digital controller, the resulting transients would not create a challenge to the ACRR's safety system. Relative to the actual conduct of experiments involving the MIT-SNL minimum time laws, an upper limit on the allowable frequency of 1100 Hz was selected. This figure was chosen because it is the value listed as acceptable for the TRW mode of operation in the ACRR's Safety Analysis Report and because the range of frequencies needed for the testing of the MIT-SNL minimum time laws was approximately 400-600 Hz. Accordingly, an over-speed frequency trip circuit, similar to the one designed for the MIT Research Reactor, was built for the Annular Core Research Reactor[21].

7.3.3.1 Conduct of Experiments

Testing of the MIT-SNL Period-Generated Minimum Time Control Laws on Sandia's Annular Core Research Reactor was approved subject to the following:

Table 7.3.3-1

(Source: Sandia National Laboratories: 09/28/87)

Effect of Stepper Motor Frequency on Continuous Rod Withdrawal

Accident

(Simulation Study)

<u>Parameter</u>	<u>Stepper Motor Frequency</u>			
	<u>100 Hz</u>	<u>1000 Hz</u>	<u>2500 Hz</u>	<u>10,000 Hz</u>
Time to Peak Power (s)	14	1.6	0.7	0.21
Peak Power (MW)	80	1900	6300	31,000
Reactivity Inserted (Beta)	1.10	1.40	1.70	2.70
Fuel Temperature (Average)(K)	950	900	970	830
Time to Attain Fuel Temperature (s)	30	4	4	0.4

Table 7.3.3-2

(Source: Sandia National Laboratories: 09/28/87)

Comparison of Experimental and Simulated Results for ACRR Pulses of

Various Reactivity Insertions

(Simulated / Experimental (%Δ))

<u>Parameter</u>	<u>Pulse Reactivity</u>			
	<u>\$1.50</u>	<u>\$2.00</u>	<u>\$3.00</u>	<u>\$3.50</u>
Time to Peak Power (s)	0.143/ 0.127 (+13)	0.078/ 0.047 (+66)	0.012/ 0.004 (+200)	-0.006/ ? (NA)
Peak Power (MW)	1860/ 1500 (+24)	7500/ 6400 (+17)	31000/ 33000 (-6.1)	48000/ 52000 (-7.7)
Full Width at Half Maximum (ms)	24.5/ 25.5 (-3.9)	12.5/ 12.8 (-2.3)	6.5/ 6.4 (+1.6)	5.2/ 5.2 (0.0)
Energy (MW-s)	52/90 (-42)	110/154 (-29)	230/310 (-26)	290/380 (-24)

- (1) The instrumentation that formed the ACRR's safety system would not also be used for the controller. The safety system was to be maintained as a separate, independent entity.
- (2) Signals from the digital computer would replace those from the existing eight-step programmable controller. The hardware and circuitry previously used for control of the ACRR would be retained.
- (3) An independent, hard-wired over-speed trip circuit would be installed to preclude withdrawal of the transient rod bank at frequencies in excess of 1100 Hz.
- (4) A 'Loss of Computer Enable' circuit which would result in an insertion signal to the transient bank should there be a malfunction of the computer or the associated hardware would be installed.
- (5) The following restrictions would be incorporated in the software:
 - Absolute limit on the net reactivity, maximum neutronic power level, minimum observed reactor period, and fuel temperature.
 - Use of the standard, sufficient reactivity constraint in a supervisory role.
 - Use of the absolute, sufficient energy constraint in a supervisory role.
- (6) Specification of a transient duration limit following which control would revert to manual.
- (7) A requirement to test both the over-speed trip and the 'loss of enable' circuit prior to each set of runs and periodically during the runs.
- (8) The pneumatic mode of transient rod bank withdrawal would be disabled and officially tagged out so as to preclude its inadvertent use.
- (9) Control would never be transferred from manual to automatic with the reactor already on a positive period. The purpose of this restriction was to limit the excess reactivity to that inserted by the transient rod bank.
- (10) Reactor operation would at all times be monitored by licensed personnel.

As described in Chapters Ten and Eleven of this report, the experimental evaluation of the MIT-SNL minimum time laws was accomplished both efficiently and with excellent results using the above arrangement.

8. Experimental Apparatus and Hardware-to-Software Interface⁽¹⁾

As noted at the outset of this report, a major objective of the joint MIT-SNL experimental program was to develop and demonstrate the capability to adjust a reactor's neutronic power by many orders of magnitude both without overshoot and on periods as short as a few tenths of a second. Of crucial importance to the demonstration of this capability was the design, assembly, and installation of the hardware and hardware-to-software interface necessary to the conduct of the experiments. Described here are the salient features of the system developed for Sandia's Annular Core Research Reactor.

8.1 MIT-SNL Philosophy Concerning Hardware and Software Design

Hardware and software development are conventionally viewed as distinct tasks and are usually performed by separate engineering groups. The end result is two separate products each of which is designed to fulfill a set of general criteria such that compatibility will presumably be assured. While this approach does produce a product that is functional and while it does represent an efficient division of labor, it may not result in an integral unit optimized to perform a particular task. Given the demanding requirements of the proposed MIT-SNL experiments, it was determined at the outset of the program that optimization of both the hardware and software would be an absolute necessity. Accordingly, the MIT-SNL team adopted an integrated approach whereby both hardware and software development was performed using a single set of design criteria. Thus, while Sandia had overall responsibility for hardware and MIT for software, there was a continuous exchange of ideas with members of both organizations contributing freely to all aspects of the project. The end result was an optimized package that fulfilled the stringent requirements of the experiments by utilizing the inherent advantages in the hardware and software to the fullest extent.

8.2 Hardware Configuration

As was discussed in section 7.3 of this report, the Annular Core Research Reactor was originally equipped with an eight-step programmable controller that could be used to specify the withdrawal sequence of the transient rod bank. When ready to operate in this manner, a 'TTL-Permissive' signal is sent from that eight-step controller to the ACRR control console. (Note: 'TTL' is an acronym for 'Transistor-to-Transistor Logic'.) Receipt of this signal, together with operator concurrence, initiates a timer that is usually set for 120 seconds. Upon completion of the 120 second countdown, a TTL trigger 'GO' signal is sent from the console to the programmable controller. This causes actual control of the reactor to be transferred to the eight-step controller which in turn provides a pulsating DC signal to the translator. The output of that unit is the specific sequence of pulses that will cause the stepper motors to move the transient rods in the desired manner. Performance of the MIT-SNL experiments entailed sub-

(1) This chapter was written by Mr. Kwan S. Kwok.

stituting a digital system for the eight-step controller. The translator and stepping motors as well as the use of the 'permissive' and 'go' signals were retained as specified in the safety analysis.

Figure 8.2-1 is a schematic diagram showing the major hardware components employed during the course of the MIT-SNL experiments. The reactor's neutronic power, fuel temperature, and transient rod bank position were transmitted to the computer via analog-to-digital (A/D) conversion devices. These signals were processed by both the data acquisition and controller logic software to determine the desired movement of the transient rod bank. This information, encoded as a 16 bit word, was sent to the interface circuit via a digital input/output port. The principal function of this specially designed interface was to interpret the frequency and direction information contained in the 16 bit word so that the proper signal would be provided to the translator unit. In addition, this interface circuit also fulfilled two safety functions. These were detection of an over-frequency condition and loss of a 'computer enable' signal.

Each of the major hardware components is described below.

8.2.1 Instrumentation

Relative to the MIT-SNL experiments, information on the ACRR's neutronic power level was obtained from three fission chambers which were located in three different chamber channels within the core region. The signals from these neutron detectors were transmitted to preamplifiers and then to log signal conditioning amplifiers which provided both visual readouts and 0 to 10 volt conditioned signals. The latter were transmitted to the analog-to-digital input ports of the computer. The three fission chambers were positioned such that at least one detector was on-scale in the source, intermediate, and power ranges. The computer monitored all three chambers continuously. A range selection algorithm was used to select the proper detector or combination of detectors depending on the power level.

Fuel temperature was monitored by thermocouples which were connected to signal conditioning amplifiers that were located in the ACRR's control room. These amplified voltages were used to drive recorders. The temperature range of these thermocouples was 0°C to 1000°C. Two temperature channels were available. One was fast acting, the other somewhat slower. The former was used exclusively for the ACRR's safety system. The latter was made available for the MIT-SNL experimental program. Its output was transmitted to one of the computer's analog-to-digital converters.

Transient rod bank position indication was obtained by means of an electro-mechanical analog system that was capable of resolving each rod's position to 0.01 inches. A direct current (DC) signal conditioned to ± 10 volt was made available to the computer as an analog input signal. This signal was read by the analog-to-digital converter which had a 12 bit resolution. It is important to realize that rod position information was obtained from the stepper motor drives and not from the rods themselves.

Figure 8.2-1 Schematic Diagram of the Hardware Components for
MIT-SNL Experiments

8.2.2 Digital Computer and Real-Time Operating System

A DEC LSI-11/73 digital computer together with an RT-11 real-time operating system was selected for use in the MIT-SNL experiments. The principal reason for choosing this system was its real-time operating capability. The LSI-11/73 system achieved an average cycle time of 40 milliseconds when used during the MIT-SNL experiments. This was found to be acceptable. However, a cycle time of 15 milliseconds or less would have been preferred because, as discussed in section 5.2.2 of this report, a cycle time of 15 milliseconds corresponds to a minimum expected overshoot of 5%. At 40 milliseconds, the overshoot that could occur during a single sample interval is 8%. (Note: In the event that shorter cycle times are eventually required, they could probably be achieved through the use of a 'multiple-computer-single-task' approach. That is a more powerful computer, but one that does not operate in real time, could be used to perform some of the computational overhead thereby relieving the LSI-11/73 of this burden and improving overall throughput.)

The computer system used for the MIT-SNL experiments was equipped with an LSI-11/73 processor, a floating-point math coprocessor, four megabytes of random access memory, four serial RS-232 communication ports, two RL-02 disk drives, a programmable real-time clock, a 16 channel digital-to-digital input/output board, a 16 channel analog-to-digital input board, a four channel digital-to-analog output board, and a 22 bit Q-bus back plane. The system's components are tabulated in Table 8.2.2-1. A functional diagram of the system is given in Figure 8.2.2-1.

Relative to Figure 8.2.2-1, the computer system and hardware interface function in the following manner. The analog-to-digital (A to D) board continuously provides instrument signals to the computer. Once the computer is properly configured, a signal is sent in integer form via the digital input/output output board (DIO1-OUT) to the ACRR's control console which would be in the transient rod withdrawal or TRW mode. This is the 'TTL Permissive' signal. As was the case with the programmable controller, this action initiates a 120 second countdown. Upon completion of the countdown, a signal, also in integer form, is transmitted from the ACRR's console via the digital input/output input board (DIO1-IN) to the computer. This is the TTL 'Trigger Go' signal. The reactor's signals are then processed and the direction and speed of movement of the transient rod bank is determined in software. This information is then sent out as a 16 bit word via the digital input/output output board that is labeled 'motor' in the diagram. This latter process is then repeated at every sampling interval.

8.2.2.1 Data Acquisition System

The data acquisition system provided communication between the LSI-11/73 processor and the analog equipment external to the computer. Information was exchanged within the data acquisition system via the input-output lines on the Q-bus. The more bits available on the

Table 8.2.2-1

Component Parts for MIT-SNL Computer System

<u>DEC* Item #</u>	<u>Description</u>	<u>Size</u>
KDJII-AB	LSI-11/73 Processor	Dual Width
FPJII-AA	Floating Point Math Coprocessor	On CPU Board
MRVII-D	Multifunction Controller Board	Dual Width
MXVII-B2	Bootstrap Chip	On Controller Board
CMV4000	4 Megabytes of Memory	Quad Width
DLVJI-M	4 Serial RS-232 Ports	Dual Width
AAVII-C	Digital-to-Analog Output Board	Dual Width
EPO88	Cable for AAVII-C	
ADVII-C	Analog-to-Digital Input Board	Dual Width
EPO98	Cable for ADVII-C	
DRVII-C	Digital-to-Digital Input/Output Board	Dual Width
KWVII-C	Programmable Real-Time Clock	Dual Width
EPO67	Cable for KWVII-C	
BA23-A	Back Plane (Bus)	
BA23A-AR	Rack Mount Hardware for BA23-A	
RL02-BA	10 Megabyte Permanent Disk Drive	
RQLX2-BA	RL02 Disk Controller	
LA-210	Serial Line Printer	
VT-100	CRT Terminal	

* DEC stands for Digital Equipment Corporation

Figure 8.2.2-1 Functional Diagram of the Computer System
(Source: Sandia National Laboratories)

Q-bus, the higher is the total addressable memory and thus the data transfer rate. A 22 bit Q-bus was therefore used because it allowed a much faster data transfer rate as compared to the older 18 bit Q-bus. One of the distinct advantages of this 22 bit Q-bus was the larger addressable memory, in this case 4 megabytes, which allowed implementation of virtual memory disk storage. This was used in the MIT-SNL experiments in order to reduce the cycle time. Normally when writing the collected data to a permanent storage device such as a disk drive, the access time required for each "read" or "write" statement is at least 20 milliseconds. It may be on the order of hundreds of milliseconds for a tape drive. The virtual memory disk concept involves partitioning a portion of the random access memory and using it as a disk drive for the executing program's "read" and "write" commands. The access times for this type of RAM "read" and "write" are only a few milliseconds each. The 4 megabytes of physical memory in the LSI-11/73 allowed 500 blocks of data to be collected in RAM for each transient. This is equivalent to a file equal in size to quarter of a megabyte. Each file was copied to one of the permanent disk drives upon the completion of each transient.

The serial RS-232 communication ports were used to support peripheral devices such as the LA-120 printer and the VT-100 terminal. The printer was configured at 1200 baud which is equivalent to a print rate of about 120 characters per second. It was used primarily as a tool for debugging any hardware or software errors. Experimental data were normally transferred to transportable disk media such as floppy disks for later analysis. The terminal was configured at the highest rate possible which was 9600 baud. The terminal also displayed any diagnostic messages to the experimenter who might then intervene and/or terminate the transient depending upon the severity of the message. (Note: Most of the transients required only a few seconds for completion. Hence, operator intervention during the actual transient was not practical. However, the runs (of which the transients were part) often lasted several minutes making operator intervention practical. Run time includes both transient completion time and operation at steady-state both before and after a transient.)

The analog-to-digital (A/D) converter board was configured to scan five input signals at the highest possible rate. The five signals were the transient rod position, fuel temperature, and three neutron power detectors. These A/D converters had a 12 bit resolution. The system was also equipped with a strip chart recorder. Inputs to the recorder were driven by voltages obtained from the analog side of the A/D converters.

The programmable real-time clock was used to control and maintain the data acquisition system in real time. The actions of this real-time clock regulated the sample collection process and assigned interrupt priorities to the interrupt-driven functions in the software such that all completion routines and the main program were executing in real time.

8.2.3 Interface Controller Board

The interface controller board was designed and constructed by Mr. Michael S. Luker and Mr. Pat Drozda of the Sandia National Laboratories. Its principal function was, as the name implies, to serve as an interface between the newly installed digital computer and the existing translator unit. The specific functions of the interface board were:

1. Generation of a frequency source (oscillator) for driving the translators of the stepper motors for the transient rods.
2. Provisions to implement the control law decision by interpreting the digital word received from the digital output port of the computer and translating it into pulses per second.
3. Provision of direct pulses to the appropriate stepper motor translators which determine the direction of travel for the transient rods.
4. Verification of pulses sent to the translators as a frequency check.
5. Provisions for an electrical enable path to satisfy the existing ACRR control console and the TRW programmer circuit continuity interlocks.

The first three of the above functions were for control purposes. The interface circuit took frequency and direction information from the computer, which solved the MIT-SNL minimum time laws for the necessary rate of change of reactivity, and provided a pulsed signal to the stepper motor translator input. The fourth function was safety related. The transient rod bank would be continuously inserted if an over-frequency condition were detected. This feature protected against withdrawal of the control rods at a rate greater than that analyzed in the ACRR's Safety Analysis Report. The last of the five functions performed by the interface controller board was also safety related. The transient rod bank was to be driven in upon failure of the computer system. That is, the default state for rod motion while on digital automatic control was insertion.

The design specifications imposed on the interface controller board were that it be possible to move the stepper motors over the frequency range of -1000 Hz to +1000 Hz, that the deviation of the output frequency from the required frequency not exceed 5%, and that the over-frequency trip be set at 1100 Hz. (Note: There was a dead-band over the range of ± 15.26 Hz. This did not impact on any of the experiments.) In addition, it was stipulated that the interface be optically isolated from the ACRR console. This was done because of the high level of electrical noise present in the ACRR control room.

Diagrams of the interface controller board are given in Figures 8.2.3-1 and 8.2.3-2. The board consists of four functional units. These are the frequency generator, the pulse steering logic, the over-frequency trip, and the loss of 'enable' trip. The operation of the frequency generator and pulse steering logic is as follows. The MIT-SNL minimum time laws, which are represented in software in the computer, are used to determine the required transient rod speed. This information, expressed as a 16 bit word, is sent to the interface board's frequency generator which contains an oscillator that produces a constant high frequency. A programmable divider is used to reduce this signal to the value that corresponds to the desired rotation speed of the stepper motor. The resulting reduced frequency together with the desired direction of rod movement is sent to the pulse steering logic which in turn transmits the information to the correct stepper motor translator line input. Operating in parallel with the frequency generator and steering logic are the over-frequency and loss of 'enable' trip circuits. (Note: The signal sent from the computer is a 16 bit word. However, as shown in Figure 8.2.3-1, only 14 bits of binary data are available to the frequency generator. The problem is that the computer has only 16 available digital input/output lines and two of these are needed for transient rod bank directional information. The two least significant bits of the signal are used for this purpose.)

The interface controller board performed as specified. However, despite its having been designed to accommodate demanded frequencies of as much as 1000 Hz, the MIT-SNL experiments were restricted to 400 Hz. The problem was that step changes of frequency in excess of 400 Hz would sometimes cause the stepper motors either to slip or to stall. Modifications to the interface circuit that will allow for smooth acceleration to frequencies as high as 1000 Hz are being considered as part of the continuing MIT-SNL joint experimental program. Additional information on the design and performance of the interface controller board is given in [21].

8.3 Software Configuration

The software utilized for the conduct of the MIT-SNL experiments consisted of a main program, its attendant subroutines, and the algorithms to solve the MIT-SNL minimum time laws. Information on the organization and functioning of the main program, as installed on Sandia's Annular Core Research Reactor, is given here.

The software was written in either the FORTRAN 77 or the MACRO-11 ASSEMBLY language. In addition, the standard and math FORTRAN libraries, as well as a commercially available library of machine language routines for real-time operation, were used. (Note: The latter was a product of Data Translation, Inc.). Figure 8.3-1 is a block diagram showing the relation of the main program ('MITSNL') to the other subroutines. The implementing algorithms for the MIT-SNL laws are called from the routine 'CONPER' which in turn calls a number of other subroutines that are not shown in the diagram. The functions of the main program are as follows:

Figure 8.2.3-1 Block Diagram of the Interface Controller Board

(Source: Sandia National Laboratories, Ref. 21)

LSI 11/73
COMPUTER

Figure 8.2.3-2 Functional Diagram of the Interface Controller
Board

Figure 8.3-1 Block Diagram of Software Structure

- Provision of a hardware-to-software interface.
- Data acquisition
- Data scaling and conditioning
- Error detection
- Prioritization of interrupt levels
- Motor speed control
- Provision of information to the experimenter (man-machine interface).

8.3.1 Main Program 'MITSNL'

A functional description of the main program, 'MITSNL', is given here. One of the major considerations in the software design was the size of the sample interval. The code contains provisions for use of either a constant or a variable time step. It was shown during the actual experiments that the variable mode was superior. Accordingly, that mode is described in the write-up that follows. Section 8.3.1.1 of this report provides a discussion of the relative merits of the two approaches and the procedure needed to implement a constant time step.

Execution of the main program begins with the necessary variable and memory declarations. An input file which specifies the various control options, such as use of either the standard or alternate law, is then defined. These options, which are also referred to as external control parameters, will direct the execution of the control law software. The next action of the main program is to open an output file in virtual memory with an initial size of 500 blocks. This file's purpose is to provide temporary storage for all data collected during the transient. (Note: Once the transient is completed, all data recorded in this temporary file are transferred to a permanent disk storage device.) The program next clears all bits on the digital-to-digital input/output board and calls the subroutine INPUT1. This subroutine is programmed to request information concerning conduct of the transient from the experimenter. The experimenter's responses are entered interactively via the keyboard. Information so gathered includes the neutron detectors that are in use, the desired sample time if a value other than the default is to be used, and the target power level. The main program then defines and calculates the necessary variables for operation of the real-time clock based on the specified sample time. Parameters such as clock frequency, clock overflow control, preset counter, and control flags are calculated and checked for consistency before program initialization is completed. The system clock and calendar are then accessed to display the date and time on the video screen together with values for each of the control options that were specified in the input file. The experimenter should at this point cross-check the displayed parameters against the experimental data sheet to verify the accuracy of the software setup. (Note: Experimental data sheets are manually prepared summaries showing the setpoints of every control option.) If any mistakes are found, the experimenter now has the option of either terminating the

program or else correcting the errors. The program at this point pauses until the experimenter keys in a preselected symbol. An octal word which corresponds to a 'high' TTL signal is then sent to the ACRR control console to satisfy the enable-permissive circuit so that the trigger counter can begin the 120 second countdown. This 'high' signal is sent out via bit number one on the digital-to-digital output board. The program at this point enters a dynamic loop that continuously monitors bit number zero of the digital-to-digital input board. The objective is to identify the return of the TTL 'high' signal from the ACRR trigger timer. Once bit number zero on the digital-to-digital input board is set 'high' by the trigger timer, the software breaks out of the dynamic loop and prints a message on the screen to notify the experimenter that the transient has begun. The system clock and calendar are again accessed and the exact time and date are recorded in the virtual memory data file for documentation purposes. The program at this point enters another loop within which the five sensors (transient rod bank position, temperature, and power) are scanned and their values stored in a data buffer array. Information in the data buffer is passed to other subroutines via common blocks. A 'Get Time' system subroutine is used to obtain an absolute time reading. This absolute time is compared to the previous reading and the difference is used as the size of the time step. If the calculated time step is less than 10 milliseconds, the program resets the time step size to 10 milliseconds. This is done to preclude possible numerical instabilities in other parts of the software. Time step sizes determined using this method will vary from sample cycle to sample cycle because they are functions of the time that was actually required for program execution and execution time will vary depending on the use of write statements and the number of iterations required to achieve convergence. This method of determining the sampling interval was therefore designated as the 'Variable Time Step' approach. Continuing with the functional description of the main program, the completion routine ACRR1 is now called to initiate the utilization of the control law portion of the software. This is done by calling the subroutine CONPER. It is important to realize that the 'Get Time' functions must be called and the size of the time step determined before calling the completion routine ACRR1. This is necessary because the size of the time step is an important parameter in the control law calculations and were it not known, erroneous results would be obtained. The program stays in this loop until the transient is either completed or else terminated by either the experimenter or by the program timer which is in subroutine ACRR1.

8.3.1.1 Constant Versus Variable Time Step

One of the major implementation issues that affected the design of the software for the MIT-SNL experiments was the selection of the method for determining the sampling interval or cycle time. The code was originally written to use a constant time step because this mode facilitated execution of the control law algorithms. Use of a constant time step meant that a predetermined time interval was assigned for both data acquisition and software execution. If the performance of those tasks took less than the assigned time, then the computer

would simply pause until the time was up. Conversely, if more time was needed, an error message to that effect would be printed. The code was optimized to permit use of as small a sampling interval as possible. This was done by configuring the software to run in parallel with the data acquisition process. (Note: Obviously, complete parallel operation is impossible because the control law algorithms can't be executed until the sensor readings have been obtained.) The system functioned as follows. The data acquisition software was setup as an 'interrupt-driven' process. At the outset of each sampling interval, the computer would begin parallel execution of both the data acquisition routines and the other software. The real-time clock would monitor the interrupt-driven data acquisition process and initiate an 'interrupt-hold' instruction when the data collection process was completed. Parallel processing would then end and the computer's full resources would be devoted to executing the remaining software. Once that process was completed, the computer would halt until any remaining time in the specified sampling interval had elapsed. The interrupt-hold instruction would then be terminated and the overall process repeated.

The 'constant time step' approach functioned properly as long as the time required to execute the software remained constant. However, the concepts involved in the MIT-SNL minimum time laws were new and, as with any experimental program, many diagnostic statements were included in the software package. These would only be activated if the unanticipated occurred. A problem arose in that if several diagnostics were activated during the same sampling interval, the time needed to complete the execution of the software would exceed the allowed sampling interval. Whenever this occurred the code would still run to completion but it would no longer be operating in real time. That is, the actual elapsed time would be perhaps 50 milliseconds while the sampling interval, which is the amount of time that the code 'thinks' has elapsed, would be 30 milliseconds. This mismatch in times could result in oscillatory behavior of the controlled parameter because the required control action is computed some time after the moment at which it should have been implemented. One solution would be to increase the sampling interval to allow for the longest possible execution time. However, that approach would unnecessarily slow down the entire run. Another was to employ a 'variable time step' in which the sample interval for the present time step was assumed to be that which had been observed for the previous interval. That is, upon completion of both data acquisition and code execution, the program would compute the elapsed time and use that quantity as the next time step. The 'constant time step' approach was used for all experiments conducted on the MIT Research Reactor and for the first set of runs performed on Sandia's Annular Core Research Reactor. The 'variable time step' method was used for the second and subsequent sets of runs performed at Sandia. Its advantage was that its use resulted in a shorter average time step while providing flexibility for the periodic printing of error messages. Also, on average, the assumption that the current time step was equal to the previous one was valid.

The completion subroutine ACRRT1 was used to control the mode of time step determination. It is an interrupt-driven routine external to both the main program and the other subroutines. ACRRT1 is capable of being executed either in the foreground sequentially with the other subroutines or in the background simultaneously as the remaining parts of the program are being executed. The foreground or serial execution mode is implemented whenever the variable time step mode is used. The background or parallel execution mode is used whenever the constant time step mode is in effect. As noted above, the background execution method in conjunction with the constant time step was found to be more demanding on the system overhead and thus sometimes not able to maintain real-time execution. The variable time step method was found to be a better time-keeping method whenever cycle times on the order of a few milliseconds were required.

8.3.2 Subroutine ACRRT1

In addition to regulating the use of either a constant or variable time step, the subroutine ACRRT1 was used to process all sensor readings. The main program, 'MITSNL' scans all sensors and stores the acquired data in a storage buffer array. This information is then made available to other parts of the program via common blocks. The subroutine ACRRT1 is written in FORTRAN. It begins with a 'DO' loop which converts all data in the data collection buffer to real numbers and stores them in a separate array. There were, as mentioned, three measured quantities. These were the transient rod bank position, the neutronic power, and the fuel temperature. Signals from the first of these quantities were used as obtained and are not discussed further. Signals from the latter two required processing as described in sections 8.3.2.1 and 8.3.2.2 of this report.

Once sensor readings have been properly processed, ACRRT1 calls the subroutine CONPER which initiates solution of the algorithms that describe the MIT-SNL minimum time laws. No arguments are needed when calling CONPER because all pertinent parameters are shared with the appropriate subroutines through the use of common blocks. Return from CONPER is via the subroutine CONTRL which, as discussed in section 8.3.5 of this report, generates the appropriate signal for the stepper motors. From CONTRL, program execution is returned to CONPER and thence to ACRRT1. A comparison is then made of the elapsed time since the start of the transient against the time that was allotted for the entire run. (Note: This latter quantity was entered as one of the control options specified prior to initiating the run.) If the allotted time has been exceeded, the subroutine HALT is called and the run is terminated. Otherwise, program execution is returned to MITSNL, the sensors are again scanned and the entire sequence is repeated. (Note: ACRRT1 may also call CONTRL directly. It would do so if it were desired to drive the transient rod bank in continuously.)

8.3.2.1 Estimation of Neutronic Power

The reactor's neutronic power was determined using the following relationship:

$$P(t) = \frac{10^{S_1} \cdot W_1 + 10^{S_2} \cdot W_2 + 10^{S_3} \cdot W_3}{W_1 + W_2 + W_3} \quad (8.3.2.1-1)$$

where $P(t)$ is the instantaneous reactor power, and other symbols are defined as:

- S_1 is the signal from logarithmic amplifier number 1,
- W_1 is the weighting factor assigned to detector number 1,
- S_2 is the signal from logarithmic amplifier number 2,
- W_2 is the weighting factor assigned to detector number 2,
- S_3 is the signal from logarithmic amplifier number 3, and
- W_3 is the weighting factor assigned to detector number 3.

The weighting factors were a function of the operable range of each sensor and were selected so as to ensure a bumpless transition between sensors. The arrangement used is shown in Figure 8.3.2.1-1 where the quantities CS1 - CS4 are defined as follows:

- CS1 is the lowest power level at which sensor #2 is always accurate,
- CS2 is the highest power level at which sensor #1 is always accurate,
- CS3 is the lowest power level at which sensor #3 is always accurate, and
- CS4 is the highest power level at which sensor #2 is always accurate.

The power levels represented by the CS quantities should not be the extremes at which the sensors could be used. They should be (and were) conservatively selected so that each detector's proper operability was certain within each of their assigned ranges. Moreover, CS2 and CS4 were selected so that it would not be possible to raise power from CS1 to CS2 or from CS3 to CS4 in one time step. That is,

$$CS2 > CS1 \text{ EXP}(\Delta t/\tau) \quad \text{and} \quad (8.3.2.1-2)$$

$$CS4 > CS3 \text{ EXP}(\Delta t/\tau) \quad (8.3.2.1-3)$$

where τ is the shortest anticipated period, and

Δt is the longest possible sample interval.

It is desirable that equations 8.3.2.1-2 and 8.3.2.1-3 be satisfied by at least a factor of two or three. This will assure a smooth transi-

Figure 8.3.2.1-1 Weighting Factors for Multiple Power Sensors

tion from one power sensor to the next. It was also assumed that sensor #1 and sensor #3 did not overlap. In the event that they did, CS3 would have been selected so that no overlap existed as far as the power estimation algorithm was concerned. Figure 8.3.2.1-2 shows the ranges of the source, intermediate, and power range detectors used for the closed-loop control experiments conducted on the ACRR in December 1987.

8.3.2.2 Estimation of Fuel Temperature

As was discussed in section 8.2.1 of this report, the fuel temperature was measured using thermocouples. The channel available to the MIT-SNL experiments was characterized by time lags of several seconds and hence of little value during fast transients in which the power level was increased by many orders of magnitude. Accordingly, it was necessary to incorporate an analytic model of the fuel temperature. The model chosen was taken from the ACRR simulator developed by Mr. Francis J. Wyant of the Sandia National Laboratories.

The fuel temperature was calculated using the following lumped parameter model:

$$T_{\text{fuel}}^i = T_{\text{fuel}}^{i-1} + T_{\text{gain}} \cdot t_{\text{sam}} \cdot [P(t) - h \cdot A_{\text{fuel}} \cdot T_{\text{water}}] / (M_{\text{fuel}} \cdot c_p) \quad (8.3.2.2-1)$$

where:

- T_{fuel}^i is the current estimate of the fuel temperature (K),
- T_{fuel}^{i-1} is the previous estimate of the fuel temperature (K),
- T_{gain} is an adjustable gain (initialized at unity) which can be varied from run to run to allow sensitivity studies of the fuel temperature model,
- t_{sam} is the elapsed time (s),
- $P(t)$ is the instantaneous reactor power (W),
- h is the surface heat transfer coefficient of the fuel rods to the pool water (W/m²-K),
- A_{fuel} is the heat transfer area of the fuel (m²),
- T_{water} is the bulk water temperature (K),
- M_{fuel} is the mass of fuel (kg), and
- c_p is the specific heat of the fuel material (J/kg-K)

This calculated fuel temperature was then compared within the code to

Figure 8.3.2.1-2 Calibration of ACRR Neutron Power Sensors:

December 1987

(Source: Sandia National Laboratories)

the measured value and the larger of the two was used in the control law calculations.

8.3.3 Subroutine INPUT1

As noted in the functional description of the main program, the subroutine INPUT1 is used to gather information concerning the transient from the experimenter interactively through the keyboard. Specifically, INPUT1 is a FORTRAN subroutine intended to be executed once prior to the beginning of each run. It reads the program control options which are stored in a file in virtual memory and accepts keyboard entry of parameters necessary for the initialization of the transient. Next, the scale factors necessary for the rod speed determination are calculated and stored in the proper common variables. (Note: Rod speed is calculated as a fraction with unity corresponding to the maximum desired stepper motor frequency. That frequency is one of the control options. Hence the need to adjust the scale for rod speed at the outset of each run.) Queries are then made to the CRT screen for the experimenter to choose one of the three neutron detectors to be used at the outset of the transient and to specify the size of the sample interval if operating in the constant time step mode. All sensors are scanned once and the values of their associated parameters determined based on the information that was read and entered from the keyboard. The calculated neutron power is printed on the CRT screen and the experimenter is asked to enter the target power in units of kW. The entered value is then checked against predetermined target power limits. The low limit was 50 watts and the high limit was 100 MW. (Note: 300 MW was used for some of the later experiments.) The code execution halts unless the target power value falls within these limits. INPUT1 at this point returns control to the main program.

8.3.4 Subroutine DISPLY

This FORTRAN subroutine was written for controlling the screen of the VT100 terminal. Special MACRO-11 ASSEMBLY language routines were written to reset some of the keyboard and screen controls of the terminal so that a customized screen display was obtained. This subroutine requires one argument. If that argument is equal to 1, the screen is cleared and scroll boundaries are defined such that the upper half of the screen lists the important control options and the bottom half of the screen flashes out in real time any messages that are generated in the software as the transient progresses. If the argument is equal to 2, the cursor is brought to the lower half of the screen. The purpose of this feature was to provide the experimenters with a means of writing customized messages on the screen for information or debugging purposes. INPUT1 returns control to the main program after either of these two options has been exercised.

The subroutine CHARAC, also written in MACRO-11 ASSEMBLY language, may be called by DISPLY. It allows resetting of the keyboard and screen control functions on the VT-100 terminal. There are four FORTRAN callable routines in CHARAC. These are TTINIT, TTRSET,

LLCHAR, and NOCHAR. These are direct FORTRAN calls which perform the necessary reset functions on the terminal.

8.3.5 Subroutine CONTRL

This subroutine is written in FORTRAN. Its purpose is to generate the information that is used to control the speed of the transient rod bank motors. CONTRL first determines if 'full speed in', 'full speed out', or 'full stop' signals are required. Any one of these requests would result in direct output of the specified motor action without any calculation. This feature was considered highly desirable should fast action be required in order to halt a power excursion. If none of these actions are appropriate, then the digital word that corresponds to the desired motor speed is calculated using the following formula:

$$W = \frac{250000}{F_{\max} \cdot |Y|}$$

$$\text{and } |Y| \geq \frac{15.26}{F_{\max}} \quad (8.3.5-1)$$

where W is the digital word which is transmitted to the controller interface board,

$F_{\max}$ is the maximum frequency to be used for the transient (<1000 Hz with the trip set at 1100 Hz), and

Y is the desired rod speed calculated by the control law portion of the software and expressed as a fraction ranging from -1.0 to 1.0.

The minimum speed at which the motor could be driven was ± 15.26 Hz. This was the reason for the constraint on Y in equation 8.3.5-1. The quantity 32768 was subtracted from the calculated digital word (W) if the desired motor motion was out of the core. The quantity 16384 was added to the calculated digital word if the desired motor motion was into the core. (Note: This may seem counterintuitive but is correct. The 74193 binary counters are used as bit-wise dividers in the circuit in order to read the digital word sent from the digital-to-digital output port. The functional relationship is inverted because of the division operation.) CONTRL returns program execution to the calling subroutine at this point.

One built-in feature of this subroutine is the use of a flag to check the validity of the control signal. The flag is set to unity in the algorithms that are used to solve the MIT-SNL minimum time laws. This is done once the desired rod speed Y has been assigned. The CONTRL subroutine checks this flag at the very beginning of the routine and prints out an error message on the CRT screen to warn the experimenter if it was not equal to unity. If it is equal to unity, CONTRL resets it to zero and proceeds with the calculations. Hence,

if the value of the flag is not continually shifted from zero to unity and back, the control law algorithm is probably not functioning properly. The cycling of this flag is a safety check that provides protection against any unforeseen hardware or software failures which might occur during the transient.

8.3.6 Subroutines BLOCK DATA, XWAIT, HALT, and CHARAC

The subroutine BLOCK DATA defines reactor physics parameters that are needed by virtually every other subroutine in the code. Included are the prompt neutron lifetime, the effective delayed neutron fraction, precursor yields, and precursor decay constants.

The subroutine XWAIT was written in FORTRAN. Its purpose is to generate a time delay that permits the completion of all interrupt-driven routines to be accomplished before termination of the program at the end of the transient. Without this delay, any hardware/software inconsistencies that arose at the end of the transient might cause the processor to enter an infinite loop with a resulting loss of data from the experiment run. (Note: The length of the time delay is equal to XWAIT's argument in seconds.)

The subroutine HALT was written in MACRO-11 ASSEMBLY language. It allows termination of the program in the event of a hardware error by using a single FORTRAN call. The subroutine CHARAC was also written in MACRO-11 ASSEMBLY language. It allows resetting of the keyboard and screen control functions on the VT-100 terminal. There are four FORTRAN callable routines in CHARAC. These are TTINIT, TTRSET, LLCHAR, and NOCHAR. These are direct FORTRAN calls which perform the necessary reset functions on the terminal.

9. Experiment Preparations Including Preliminary MITR Trials⁽¹⁾

Described here are the specific preparations undertaken prior to conducting experimental trials of the MIT-SNL Period-Generated Minimum Time Control Laws under conditions of closed-loop, digital control using variable speed stepper motors. For the MIT Research Reactor, this meant on-line tests using a computer system that was already installed. These tests were performed, albeit crudely, using fixed speed motors as permitted by the experiment protocols that were then in effect. For Sandia's Annular Core Research Reactor, this meant the construction of a simulator for testing both the software and the hardware, transfer of the hardware to the reactor site, and calibration of both the requisite instrumentation and the data acquisition system.

9.1 MIT-SNL Philosophy Concerning Conduct of Experiments

The MIT-SNL philosophy regarding the conduct of reactor control experiments is to use a graded approach in which each concept is evaluated under realistic conditions and yet not used in an essential role until it has been determined to be reliable. Thus, each of the MIT-SNL laws was first tested extensively off-line via simulation and then on-line at the 5-MWt MIT Research Reactor using that facility's regulating rod which was of low reactivity worth. Only after they had been shown to function properly and routinely under those circumstances were the MIT-SNL laws used in conjunction with an actuator of high reactivity worth such as one of the MITR-II's shim blades or the ACRR's transient rod bank. This graded approach was advantageous because, at each level of application, it resulted in the accumulation of a database that provided confidence for the next undertaking. In this regard, recognize that the use of digital technology for the direct control of reactor power is relatively untried in the United States. Thus, the experiments which are the subject of this report are significant not only as a demonstration of the MIT-SNL minimum time laws but also as an application of digital control to reactor operation.

9.2 MITR Preparatory Program

As was described in sections 1.5 and 7.2.2 of this report, the basic regulatory approval necessary to conduct closed-loop digital control experiments already existed at the time that the MIT-SNL minimum time laws were first enumerated. Experiments could be performed using either the reactor's regulating rod or one of its six shim blades. The former, being of low reactivity worth, could be driven by either a fixed or a variable speed motor. Operation of the latter was limited to fixed speed motors pending construction and implementation of the overspeed trip circuit that was described in section 7.2.3.1 of this report. Once that circuit was functional, approval was requested from the MIT Reactor Safeguards Committee to conduct closed-loop evaluations of the MIT-SNL laws using a shim blade driven by a variable speed motor. Unlike the original approval to operate a shim

(1) Section 9.3 of this chapter was written by Mr. Kwan S. Kwok.

blade using the MIT-CSDL Non-Linear Digital Controller, this request did not involve a change to one of the MIT Research Reactor's technical specifications nor did it entail an unreviewed safety question. Accordingly, prior approval of the U.S. Nuclear Regulatory Commission (NRC) was not a prerequisite. Rather, it was acceptable to report the use of a variable speed motor to drive a shim blade to the NRC under the provisions of 10 CFR 50.59. Thus, preparations at the MIT Research Reactor to conduct on-line trials of the MIT-SNL Period-Generated Minimum Time Control Laws were relatively straightforward consisting of (1) the characterization studies described in Chapter Six of this report, (2) the installation of the overspeed trip circuit, and (3) receipt of approval from the MIT Reactor Safeguards Committee with subsequent reporting of that action to the NRC. Unfortunately, difficulties encountered in the course of constructing the overspeed trip, and in particular its sensitivity to noise, delayed the overall approval process. In the interim, it was decided to continue testing using the MIT Research Reactor's variable speed regulating rod and to conduct tests using a shim blade driven by a fixed speed motor. The purpose of these trials was to obtain at least some information on the performance of the control methodology on an actual reactor. Relative to the use of a fixed speed motor, the MIT-SNL laws were implemented using the approach given in section 5.2 of this report except that if the control signal were positive the shim blade would be withdrawn at full speed and if it were negative, the blade would be inserted at full speed. This resulted in significant oscillatory movement of the blade in question. However, because the time step (one second) was small relative to the duration of the transient, the resulting reactivity profiles were essentially as desired and it was possible to obtain useful information. The results of one of these runs are shown in the following section of this report. Also shown are two runs, a power increase and a decrease, in which the MIT Research Reactor's regulating rod was used as the actuator.

9.2.1 Preliminary Trials of the MIT-SNL Laws on the MIT Research Reactor

As was noted in section 5.4 of this report, the initial on-line trials of the MIT-SNL Period-Generated Minimum Time Control Laws were conducted in December 1986 using the MIT Research Reactor's regulating rod which was driven by a variable speed stepper motor. The transient shown in Figure 9.2.1-1 is an extension of those initial trials. Shown is an actual power increase of 500 kW achieved on the MIT Research Reactor using the alternate MIT-SNL minimum time law. The data are from a trial in which the specified period was 300 seconds. The period was stepped to this value by initially requiring a rate of reactivity insertion of about 3.3 millibeta per second. Then, as the delayed terms grew in magnitude, the rate of change of reactivity was reduced. On reaching full power, the period was stepped back to infinity by making the rate of change of reactivity negative. The power increase then stopped. Given that the reactor period was maintained constant at 300 seconds for the entire transient, the power change was accomplished in the minimum time possible for a reactor limited to a 300 second period. As proof, note that the initial and final power

Figure 9.2.1-1 Power Increase of 500 kW Using Alternate Minimum Time Control Law with a Specified Period of 300 Seconds

levels were 1030 and 1500 kW. Hence, from the definition of reactor period, the minimum possible transit time was $\tau \ln(P_F/P_0)$ or 113 seconds. This is in excellent agreement with the 114⁰ second figure obtained from the experiment. Shown are the power and reactivity profiles. Note that there was no power overshoot. The reactivity is that of the control mechanism. The rate of change of reactivity is quite rapid at both the outset and termination of the transient. This occurs because the prompt effect (the ρ term) is being used to drive the period from infinity to 300 seconds and from 300 seconds to infinity respectively. The rate of change of reactivity decreased during the interval 10-114 seconds because the delayed neutron contributions to the period were increasing. However, it did not go to zero because the temperature of the primary coolant was rising and the controller automatically added reactivity in order to compensate for this effect.

Figure 9.2.1-2 shows the results of a power decrease performed using the alternate MIT-SNL minimum time control law. There was some difficulty at transient initiation, but overall the law's performance was adequate. As was the case with the previous figure, the most significant feature shown is the rapid change in reactivity that occurred at both the outset and termination of the transient.

Figure 9.2.1-3 is from an experimental study in which one of the MIT Research Reactor's shim blades was used as the actuator. The reactivity worth of the shim blades is roughly an order of magnitude greater than that of the regulating rod. The controller was programmed so that the alternate time-optimal law would repeatedly raise and lower the power by 1 MW while on a period of ± 100 seconds as appropriate. As shown, the performance of the control law was excellent.

9.3 ACRR Preparatory Program

The preparatory program at the Annular Core Research Reactor consisted of two phases. The first involved the construction of a simulator for testing both hardware and software under as realistic conditions as possible. The second was to install the computer system at the ACRR and then to calibrate the various sensors needed for the experiments. It should be noted that, unlike the MIT Research Reactor, equipment used in experiments at the Annular Core Research Reactor is not allowed to remain at the site for extended periods. Rather, each experimenter is allocated a fixed amount of time during which he or she must install the requisite apparatus, conduct the experiment, and vacate the premises. Schedules are very tight. Hence, the emphasis on testing not only the software but also the hardware via simulation before attempting the actual experiments.

9.3.1 Simulation of the Experiment

The MIT program on reactor control has a long history of evaluating control schemes via computer simulation. Prior to the MIT-SNL joint program, every simulation had used a single computer with one piece of software running on it. The software typically consists of a main program and two principal subroutines. One subroutine is a model

Figure 9.2.1-2 Power Decrease of 500 kW Using Alternate
Minimum Time Control Law with a Specified
Period of -300 Seconds

Figure 9.2.1-3 Power Cycling on ± 100 Second Period Using
Alternate Minimum Time Control Law

of the reactor while the other contains the controller. The main program is used to first call one subroutine and then the other. In effect, it serves as a clock by first directing the output of the model to the controller and then returning the resulting control signal to the model. Such simulators need not run in real time and they do not provide a test of the hardware that will be used for the actual experiments.

As noted above, it was important to conduct as realistic a test as possible of the hardware prior to installing it on the Annular Core Research Reactor. Accordingly, Mr. Francis J. Wyant of the Sandia National Laboratories constructed a simulator that used two computers. These were an LSI-11/23 and an LSI-11/73. The former was used to model the ACRR. The latter was the actual computer on which the controller software would be running during the actual experiments. Given below is a brief description of the simulator, typical results, and its uses.

9.3.1.1 Simulator Design

The simulator consists of an LSI-11/23 and an LSI-11/73 computer. The former, which is used to represent the reactor, is a complete system in itself having its own 16 channel digital-to-digital input/output board, a four channel digital-to-analog output board, and a real-time clock. The digital-to-digital input/output board performs the function of mimicking the ACRR control console. Channel zero of the input port detects the 'TTL Permissive' signal which indicates that the computer on which the control laws are programmed is ready. This action initiates a countdown (120 seconds in the actual reactor). The equivalent of the TTL 'Trigger Go' signal is then transmitted via the digital output port. The digital-to-analog output board provides analog output signals to simulate the neutron power, fuel temperature, and transient rod positions of the ACRR. The real-time clock maintains the code execution in real time. A point kinetics model was used for the neutronics calculations and a convective heat transfer model for the fuel temperature calculations. The rod position and the power level are initially set by the experimenter. The fuel temperature and the other necessary parameters for a critical reactor are then calculated. The reactor power, fuel temperature, and rod position are sent out via the digital-to-analog converters.

The LSI-11/73 computer system, which includes the data acquisition system and the interface controller board that is used in the actual experiments, is connected directly to the LSI-11/23. This physical arrangement, in which two separate computer systems are used, allows far more realistic testing of the control methodologies than if the simulation code were running on the LSI-11/23 together with the control laws. Signals sent out by the LSI-11/23's digital-to-analog converter are read by the LSI-11/73 computer's analog-to-digital converters as if they were the real signals coming from the ACRR. The control laws programmed on the LSI-11/73 then determine the desired transient rod speed which is sent out as a 16-bit digital word via the digital-to-digital output. This 16-bit word is thus transmitted to

the interface controller board and thence to the digital-to-digital input port of the LSI-11/23. The interface controller board converts this 16-bit digital word to the desired pulsating DC pulse rate. This pulse rate is monitored by a high speed frequency counter and displayed for diagnostic purposes. The LSI-11/23 reads the digital word using the digital-to-digital input port and decodes the desired transient rod speed using equation 8.3.5-1. The rods are then moved in the ACRR model at this speed. The reactivity change due to both the movement of rods and the change of fuel temperature is then calculated and the reactor power level updated. This completes one cycle of the calculations.

9.3.1.2 Simulator Studies

Prior to performing the initial set of experiments on the Annular Core Research Reactor, the software package brought to the Sandia National Laboratories from MIT was tested extensively using the simulator. A total of twelve man-days were spent changing, debugging, and refining the software. This series of simulator tests was very important because complete testing of the software package at MIT had not been possible. The principal reasons for this were limitations on the types of transients that could be conducted on the MIT Research Reactor and differences in the hardware set-up between the computer system installed at the MIT Research Reactor and that scheduled for use on the ACRR. Also, a portion of this test period was spent changing software to reflect modifications made to the interface controller board as a result of some component changes during the final stages of that board's construction.

An important feature of the simulator was the use of one of its analog output ports to drive a strip chart recorder. This allowed plotting of the power and rod position signals in real-time. The performance of the control law was therefore immediately apparent to the experimenter. Forty-three simulator runs were performed prior to conducting the first experiment on the ACRR. Several of these are shown in Figure 9.3.1.2-1 which is a composite of the recorder strip charts from four successful runs. Shown are the power profiles resulting from simulated transients conducted using both the standard and alternate MIT-SNL Period-Generated Minimum Time Control Laws and the standard and alternate versions of the MIT-CSDL Non-Linear Digital Controller. The characteristic features of each of the four control laws are evident in these strip chart recordings:

1. The recordings of the standard and alternate MIT-SNL Minimum Time Control Laws show that the transients were completed without overshoot. Also, the straight slope of the power trace (log signal) indicates that the transients were in fact conducted on constant periods. Finally, note the sharp cornering at the initiation and termination points of each transient. (Note: The standard and alternate MIT-SNL minimum time laws are mathematically equivalent, although the latter is easier to implement. Hence, the recordings for these two laws appear almost identical.)

Alternate MIT-CSDL NLDC Algorithm

Standard MIT-CSDL NLDC Algorithm

Alternate MIT-SNL Minimum Time Control Law

Standard MIT-SNL Minimum Time Control Law

Figure 9.3.1.2-1 Examples of Simulation Transients

2. The recordings of the standard and alternate MIT-CSDL Non-Linear Digital Controller (NLDC) algorithms also showed that the transients were completed without overshoot. These traces were obtained by using sufficient reactivity constraints in conjunction with a control law that withdrew the rods continuously. (Note: As always with tests of the NLDC, a poorly conceived control law is utilized so as to be certain of testing the constraint.) Note that the slopes of the power signals during the transients were not constant indicating that, as would be expected, the period was not a constant. Again, the cornering was quite good. Finally note that the alternate NLDC began leveling the reactor power sooner than did the standard version. As noted in section 5.1.1 of this report, this occurred because the alternate effective decay parameter is significantly larger than the standard one. Hence, the alternate sufficient constraint allows less reactivity to be present during the final stages of a transient. Additional information is given in [10,11]. (Note: Unlike the MIT-SNL minimum time laws, the standard and alternate versions of the NLDC are mathematically different.)

9.3.1.3 Simulator Utilization

Prior to the initiation of the actual experimental trials of the MIT-SNL minimum time laws on the Annular Core Research Reactor, the simulator was used to provide as broad a range of trials as was possible of both the software and the hardware. Specific tests are enumerated below. Note that in addition to the standard safety studies, many feasibility tests were performed. This was made possible by the special design of the ACRR simulator in which two computers, one representing the reactor and the other the controller, were coupled. The capability for the off-line performance of feasibility tests, such as the determination of time step size, resulted in a substantial savings of on-line time. As a result, the actual experiment sessions were more productive.

The ACRR simulator was used for the following types of studies:

1. Sensitivity study to determine proper size of sample interval:

It was desired to use as small a sampling interval as possible and yet not cause the control system, which was to be operated on the LSI-11/73, to not operate in real time. Intervals of about forty milliseconds were found to be optimal.

2. Throughput study of the controller's computer:

This was a study of the overall system which included hardware, software, and the necessary interfaces. An internal clock was set up in the software to monitor the time re-

quired for the execution of each cycle. Although this time changed with each version of the software, it provided a means of ranking each software optimization method. The one that performed the best was used in the final version of the controller code.

3. Functional test of hardware interlocks and safety features:

Each hardware interlock was actuated individually by either a simulated signal or by a loss of signal. The overall response of the controller was then observed and compared to that intended. Also, each safety feature was triggered by the use of either hardware or software so that each safety response was observed. For example, the software was changed to output an 1100 Hz signal to activate the comparator circuit on the interface controller board. It was observed that a full speed insertion signal was sent to the transient rod stepper motors.

4. Validation of hardware/software interface:

Each analog and digital channel was programmed to receive a special signal sent out by the simulator. The response of each channel was then monitored and displayed on the video screen, and compared to the signal that was originally sent. For example, if the power level in the simulator reactor (LSI-11/23) were 10 MW, was a 10 MW signal being sent via the digital output port and the digital-to-analog converter? Also, was a 10 MW signal being received by the controller's (LSI-11/73) analog-to-digital converter and its digital input port? In this way, each of the physical hardware connections, its proper software address, the assigned data buffer, and the corresponding variables were all tested.

5. Functional test of software interlocks and built-in limits:

All software interlocks were individually actuated through software. This was done by deliberately changing the software to force testing of the control law's response. For example, the limit on net energy production was intentionally made quite low so as to necessitate a continuous insertion of the transient rod bank.

6. Examination of software for programming errors:

The programming errors referred to here are ones such as divide checks, overflow of a constant, COMMON block mismatches, undefined variables, and ambiguous use of variables. Errors of this type are subtle and are often not discovered when the software is compiled. They are identified by actually using the software in a systematic manner such that each of its functions is verified. The COMMON block mismatch is one of the most difficult errors to detect

and its consequences are always unpredictable. This type of error is easy to make in a complex code and compilers do not check for it. Should a mismatch occur, variables that are intended to be assigned the same numerical value will have differing ones. If not eliminated prior to on-line use, such errors can have serious consequences. For example, a mismatch in a COMMON block would occur if the number of variables in the COMMON block of a subroutine were greater than the number of variables in the same COMMON block in the calling routine. The extra variables in the larger COMMON block would be made numerically equivalent to whatever other variables happened to be stored at the end of the smaller COMMON block. These other variables could be any of the variables in memory. This is the reason for the totally unpredictable behavior of the controller were an error like this to exist in the software. (Note: This type of error was not found in the software brought from MIT to Sandia.)

7. Examination of software for errors created during transport:

Software errors could be introduced when code is transferred from computer to computer or from computer to floppy disk. In addition, x-ray machines at airport security check stations could conceivably ionize the magnetic media and cause data errors. Installing the software in its final form and testing each of its intended functions allowed a definitive verification that the software had been properly transferred from MIT to Sandia.

8. Verification of computer configuration:

The proper configuration of the computer was verified by observing the responses of the input/output interface, the main RAM memory, and the virtual memory during program execution. Also, the proper configuration of the graphics displays on the CRT terminal were verified.

9. Functional testing of the control law algorithm:

Certain of the runs scheduled for actual testing on the Annular Core Research Reactor were selected for evaluation on the simulator. Runs so chosen included at least one from each group of experiments. These tests provided comprehensive assessments of the entire system. Typical results were shown in Figure 9.3.2.2-1.

In summary, it is perhaps worth making a general observation about the relevance of simulation studies to the design and development of control systems for nuclear reactors or, for that matter, any process system. Simulation studies are not a substitute for actual experiments. They do not allow for actual instrument and signal noise, they do not force engineers to fully analyze the safety of their proposed activities, and they do not build confidence in the

technology such that industry and regulatory agencies will accept the results. Nevertheless, as shown above, such studies do fulfill an essential role. Finally, it should be recognized that the selection of a simulator is important. That constructed at Sandia was particularly useful because the reactor and the control system were realistically represented as separate entities.

9.3.2 Installation of the Experiment

Once the simulation studies were complete, the simulator was disassembled and the LSI-11/73, together with the appropriate peripheral equipment moved to the Annular Core Research Reactor. Once set up at the site, the process of calibrating both the data acquisition system and the necessary sensors was begun. These calibrations were the penultimate item required prior to performance of the actual experiments. Described here in some detail is the process of calibrating the computing system so as to be certain that the information obtained by the control laws in the software was accurate as compared to the actual output of the sensors. (Note: This report does not cover the calibration of each item of equipment as specified in the manufacturer's instructions. That was done by comparing selected readings to those of a standard that was more accurate than the equipment.)

9.3.2.1 Internal Calibration of the Data Acquisition System

The first step in the calibration procedure was to verify the internal calibration of the computer data acquisition system. Each of the analog-to-digital and digital-to-analog converter channels had a twelve bit resolution. This meant that each channel was capable of resolving $2^{12}-1$ or 4095 discrete (i.e., integer) levels of the input or output voltage. Each of these channels was configured to be unipolar with each signal conditioned over the range 0-10 volts.

Calibration checks of the analog-to-digital channels began with the selection of a channel and the shorting of its connections to ground in order to produce a true zero voltage. Software written specifically for this calibration procedure was used to sample the channel and to print out its reading on the video screen. That reading should be zero. A 10 volt signal from the voltage source was then connected to the selected channel and the channel scanned to ensure that the full value of 4095 was obtained. The zero and the span of each analog input channel was controlled by fixed precision resistors mounted on the circuit board in the computer. These were factory-set and were not adjustable. Hence, were a major calibration error to have been detected on these analog input channels, it would have required factory service. Minor calibration errors were noted and corrected through software.

The calibration of each of the four channels of the digital-to-analog output board was accomplished by using the variable resistors that were mounted on the circuit board for this purpose. A special calibration program was written to send out a zero integer value relative to the channel being calibrated. A precision voltmeter with a 4

digit or more representation of the decimal portion was used to measure the output of the channel. The "Zero" variable resistor was then adjusted, if necessary, to obtain a zero reading on the voltmeter. The program was next set to output a 4095 integer value. The "Span" variable resistor was then adjusted until a 10 volt reading was obtained on the voltmeter. Fine adjustment to the "Span" resistor could be made at this time. This procedure was repeated for all four of the channels.

9.3.2.2 Calibration of the Linear Input Channels

The transient rod position and the fuel temperature signals connected to the analog input ports were linear channels. These channels could be calibrated with the reactor in the shutdown condition. Each was individually calibrated using the following equation:

$$Y = m(X - X_0) + Y_0 \quad (9.3.2.2-1)$$

where:

Y is the actual reading of the instrument,

m is the slope or the scaling factor,

X is the integer value obtained by the computer,

X₀ is the integer value obtained by the computer with the instrument reading zero, and

Y₀ is the actual reading of the instrument with a zero signal connected to its input.

Equation (9.3.2.2-1) could be written as:

$$Y = mX + b \quad (9.3.2.2-2)$$

where the quantity 'b' is given by the expression:

$$b = Y_0 - mX_0$$

The quantity m is referred to as the scaling factor and the quantity b is referred to as the bias. Equation 9.3.2.2-1 was the easier to use because X₀ and Y₀ were direct readings from the computer and the instrument respectively. However, equation 9.3.2.2-2 was the version programmed in the software because it was more compact and required less execution time. Thus, for explanation purposes, 9.3.2.2-1 is preferable, but for production purposes, 9.3.2.2-2 was used.

As an example of the calibration procedure, the ACRR transient rod position indicator had a zero reading when the computer scan of the corresponding input channel also showed a zero integer reading. The transient rods were brought to the full out position and a reading

of 9060 on the ACRR control console and a computer reading of 3700 were obtained. A scaling factor of 2.448 and a bias of 0 were then calculated using equation 9.3.2.2-2. The transient rods were positioned at different heights to verify proper readings on the computer. The following readings were obtained when driving the transient rods in both the 'in' and 'out' directions:

<u>ACRR Console</u>	<u>Computer Input</u>	<u>Computer Calculated</u>
0	0	0
999	410	1004
2000	815	1995
4000	1633	3998
5998	2454	6008
8000	3269	8004
9060	3706	9073

It was important to check the computer results for both the 'in' and 'out' directions because these two sets of readings could have been different had the transducer hysteresis of the rod position indicator been appreciable.

9.3.2.3 Calibration of the Log Input Channels

Input channels for the three log neutron power amplifiers are more difficult to calibrate than the linear ones. For one thing, the reactor must be operating. For another, the functional relationship for these channels is more complex than for the linear channels. Specifically, it is of the form:

$$Y = 10^{(mX + b)} \quad (9.3.2.3-1)$$

where:

- Y is the actual output of the log amplifier,
- X is the integer value obtained by the computer,
- m is the slope or scaling factor, and
- b is the bias.

Equation (9.3.2.3-1) was rewritten as follows when performing calibrations:

$$\text{Log } Y = mx + b \quad (9.3.2.3-2)$$

$$\text{Log } \frac{Y_2}{Y_1} = m(X_2 - X_1) \quad (9.3.2.3-3)$$

where:

Y_1 corresponds to the reactor power at the lower power data point,

Y_2 corresponds to the reactor power at the higher power data point,

X_1 is the integer value obtained by the computer at the lower power data point, and

X_2 is the integer value obtained by the computer at the higher power data point.

For example, the reactor was placed at 1 kW, 10 kW, and 100 kW by the ACRR reactor operators. The readings obtained by the computer during these runs were 468, 820, and 1229 respectively. Using the data obtained from the 1 kW and 100 kW readings, equation 9.3.2.3-3 was used to obtain 2.63×10^{-3} as the scaling factor. The 100 kW data was used to calculate the bias as -1.23 using equation 9.3.2.3-2. The reactor power was subsequently varied to obtain the following readings for a calibration check:

<u>ACRR Console</u>	<u>Computer Input</u>	<u>Computer Calculated</u>
1 kW	463	0.97 kW
10 kW	829	8.89 kW
100 kW	1232	102 kW

9.3.3 Final Test of Safety Features

Once the computer system was installed and all sensors calibrated, it remained to test the various trips, the most important of which was the one on overspeed frequency. This was checked by causing the computer to demand a stepper motor frequency in excess of 1100 Hz and observing that the protection circuit generated an insertion signal. Tests were also made at this time for signal continuity between the sensors and the data acquisition system and between the ACRR's console and the digital controller.

Preparations for the on-line experimental evaluation of the MIT-SNL Period-Generated Minimum Time Control Laws under conditions of closed-loop, digital control on the Annular Core Research Reactor were now complete.

9.4 Control Device Integral and Differential Reactivity Worths

Fundamental to the proper functioning of both the MIT-SNL Minimum Time Laws and the MIT-CSDL Non-Linear Digital Controller is accurate knowledge of both the available rate of change of reactivity and the net reactivity. The former is the rate of change of reactivity that could be produced were a control device to be moved. For both the MIT

and the Annular Core Research Reactors, it was found by reference to premeasured tabulations of the differential reactivity worth of the control devices. The net reactivity is the total reactivity present in the core from all effects including movement of the control devices, changes in fuel and moderator temperatures, and xenon. It can be measured using either inverse kinetics [22] or balance calculations. Both techniques have been used successfully on the MIT Research Reactor in conjunction with closed-loop experiments on digital control. Relative to the Annular Core Research Reactor, balance calculations were used exclusively for the control experiments reported here. Hence, a prerequisite to the MIT-SNL experiments was the determination of both the integral and differential reactivity worths of the control devices.

The reactivity associated with a control mechanism is normally measured using the following procedure:

- (a) The reactor is made exactly critical at a power level that is sufficiently high so that source neutron contributions are negligible and yet still low enough so that power-dependent feedback effects are not significant.
- (b) The control device that is to be calibrated is withdrawn by a fixed amount thereby placing the reactor on a period.
- (c) Once prompt effects have died out, the time required for the reactor's neutronic power to double is measured. This is then converted to the 'e-folding' time or period.
- (d) Using the inhour equation, the reactivity is inferred from the period. Division of this quantity by the distance that the control device was moved gives the differential reactivity worth.
- (e) The reactor is reshimmed so that it is again critical at the original power level.
- (f) The above process is repeated. Summation of the reactivity associated with each incremental withdrawal of the control device gives the integral worth.

Reactivity worths of the MIT Research Reactor's control devices are measured using the above procedure every nine to twelve months. The reason for this rather high calibration frequency is that the MIT Research Reactor normally runs at full power and hence there is significant fuel depletion. As the fuel depletes, there are shifts in the core power density which in turn cause significant variation both in the integral reactivity worth of the shim blades and in the shape of the differential reactivity worth of those blades. The first of these effects is illustrated in Figure 9.4-1 which shows the effect of core radial power distribution on the integral shim blade worth. Additional information is given in [23]. Appendix C to this report shows the integral and differential reactivity worth curves that were

Figure 9.4-1 Effect of Radial Power Distribution on MITR Fuel Element and Control Blade Worths

in use when the preliminary MITR trials of the MIT-SNL laws were conducted.

The Annular Core Research Reactor is normally operated in a pulsed mode. Hence, its rate of fuel depletion is much less than that of the MIT Research Reactor. As a result, measurements of the reactivity worths of the ACRR's transient rod bank remain valid for many years. Shown in Figures 9.4-2 and 9.4-3 are the transient rod bank integral and differential reactivity worth curves that were used in 1987. (Note: The curves labeled 'original calibration' are the relevant ones. The curves labeled 'new calibration' reflect installation of a large experiment in 1988.) As was the case with the MITR curves, it is worth noting that the differential reactivity is a highly non-linear function of position.

9.5 Management of the ACRR Experiments

The experimental evaluation of the MIT-SNL minimum time laws was conducted by a five member experiment team working in close cooperation with the Staff of the Annular Core Research Reactor. Responsibility for the safe operation of the ACRR rested with its licensed operators. During the MIT-SNL experiments, the ACRR's control room was continuously staffed by a minimum of one licensed reactor operator and one licensed senior reactor operator. The individuals who formed the experiment team had the following responsibilities:

- a) Management of the overall program (SNL).
- b) Liaison between the experiment team and the ACRR Staff (SNL).
- c) Experiment planning and data analysis (MIT).
- d) Operation of the computer system (MIT).
- e) Hardware configuration and instrument maintenance (SNL).

Of the five persons who comprised the experiment team, four had had extensive experience operating reactors. Specifically, three had served in the United States Navy's Nuclear Propulsion Program, two were currently licensed senior operators on the MIT Research Reactor, and one other had held such a license on SNL's Annular Core Research Reactor. Thus, the members of the experiment team were well-versed in both digital control technology and reactor operations. It was this combination of talent that made the experiments a success.

The management of the MIT-SNL experimental program ran counter to two trends that are rapidly becoming dominant in regards to the practice of engineering in the United States. Accordingly, given that this joint MIT-SNL endeavor was extremely successful in terms of both theoretical advances and practical application, it is perhaps worth examining its management aspects a little further. Two observations are in order. The first concerns engineering education, the second relates to management of complex technologies.

As regards education, ever-increasing reliance is being placed on the use of computerized simulation in lieu of actual experiment. In part, this reflects the soaring cost of quality experimental facili-

Figure 9.4-2 ACRR Transient Rod Bank Integral Reactivity Worth:
Function REACTR(P)

Figure 9.4-3 ACRR Transient Rod Bank Differential Reactivity Worth:
Reactivity Worth: Function REACTD(P)

ties such as research reactors. But that is only part of the explanation. There is also a misunderstanding on the part of educators concerning the importance of experimentation. Progress in engineering disciplines is achieved through the continual testing of new ideas and the willingness to challenge the limits of known technologies. This can not be done solely through the use of models. For one thing, what is unknown can not be simulated. For another, individuals who are not trained in experimental techniques may lack the skills to distinguish problems that will require long-range solutions from mere aberrations in the current technology. Mastery of the present has always been necessary for anyone wishing to chart a new course to the future. The capability to plan and conduct well-conceived experiments is a necessity because genuine experimentation is crucial to the advancement of any science or engineering discipline. We, as a society, should endeavor both to make the necessary experimental facilities available and to insist that their use be part of standard engineering curricula.

As regards management of advanced technologies in which safety is of paramount importance, it is often argued that the person in charge should be a disinterested party. As regards the MIT-SNL experiments, this would have meant selecting a manager who had no vested interest in the outcome or even in the accomplishment of the experiments. So doing would have been a mistake both from the viewpoint of productivity and safety. At best, selection of someone without a thorough understanding of the experiment program would have resulted in indecision as that person tried to interpret requests in terms of the approved protocols. At worst, it would have produced an adversarial relation of the type that now stymies much technical progress in the United States. Both digital control and reactor operations are demanding specialities. Progress can only be achieved if engineers who are experts in both disciplines have the opportunity to apply their knowledge. Independent committee reviews are essential in the formative stages of a program. However, once actual operation is to be undertaken, it is preferable to have a conscientious, knowledgeable individual in a position of authority. Such a policy will assure both safety and productivity. This was the approach taken at MIT regarding the initial trials of the MIT-CSDL Non-Linear Digital Controller and it was the approach adopted at Sandia for the evaluation of the MIT-SNL Period-Generated Minimum Time Control Laws.

10. Experimental Evaluation of the MIT-SNL Period-Generated Minimum Time Control Laws: October 1987

Experimental evaluations of the MIT-SNL Period Generated Minimum Time Control Laws were conducted on Sandia's Annular Core Research Reactor (ACRR) during the period 27-28 October 1987 and again on 3-4 December 1987. A total of eighty-nine separate runs, most of them closed-loop, were performed during those four days. The MIT-CSDL Non-Linear Digital Controller was also demonstrated on the ACRR during these sessions. Reported here are the results of the forty-three runs conducted in October 1987. The primary focus of these trials was to demonstrate the feasibility of the control concepts. Described in Chapter Eleven of this report are the results of the December experiments and the demonstration of the MIT-SNL minimum time laws on the MIT Research Reactor with a variable-speed shim blade being used as the control device.

10.1 Objectives of the October ACRR Control Experiments

The principal objectives of the closed-loop experiments on digital control conducted on SNL's Annular Core Research Reactor in October 1987 were to verify operability of the hardware and software and to demonstrate feasibility of the various control concepts. It had been decided at the outset that achievement of the program's ultimate goal, which was to raise a reactor's neutronic power by five-six orders of magnitude both on a period of several tenths of a second and without overshoot, would be deferred until December 1987.

The October experiments comprised two distinct sets. First, a series of open-loop or preoperational runs were performed. For these tests, the digital system was recording data but actual control of the reactor was retained in its entirety by the licensed operators. Second, the actual closed-loop runs were performed using the digital control system. These entailed a gradual progression in which the various control methodologies (standard and alternate Non-Linear Digital Controller and standard and alternate Period-Generated Minimum Time Control Laws) were subjected to increasingly severe demands.

10.2 Experiment Options

Prior to the performance of the experiments reported here, neither the MIT-CSDL Non-Linear Digital Controller (NLDC) nor the MIT-SNL Period-Generated Minimum Time Control Laws had ever been demonstrated on a reactor other than the one at MIT. The NLDC had been used to perform power changes of a factor of five while on periods of 30 seconds or longer. The MIT-SNL minimum time laws had been used to achieve specified periods of 300 seconds with a variable speed motor and ones of 50 seconds with a fixed speed device. Thus, neither of these control concepts had ever been tested under conditions of extreme demand such as were specified for the control of spacecraft nuclear reactors. (Refer to section 4.1 of this report.) Hence, the experiments to be performed on the ACRR represented a unique opportunity not only to prove the feasibility of the overall control concepts

but also to confirm much of the physics and engineering from which those concepts had evolved. Were the prompt terms in the MIT-SNL laws represented properly? Would the numerical method devised to estimate the multi-group decay parameters be effective? Would control of a reactor's power by adjustment of the rate of change of reactivity be practical? In order to provide the capability to answer these and other questions, the software was written to be as flexible as possible. Specifically, it contained many user-specified options which would facilitate comparative testing. These are listed in Table 10.2-1. Prior to each run, the experimenter would select the desired options and then enter them as an input file. In addition to these options, the experimenter could also select the size of the desired power increase or decrease, the specified period (MIT-SNL laws) or the maximum allowed reactivity (NLDC), the available stepper motor frequency, and whether or not the power would be cycled by repeatedly performing the same transient. In the event of the latter, periods of different magnitude could be specified for the power increase and the power decrease portions of the cycle.

10.2.1 Nuclear Instrumentation

Relative to the nuclear instrumentation, the arrangement of overlapping detectors shown in Figure 8.3.2.1-2 was not available until December 1987. For the October runs, only one neutron sensor was used. This impacted the conduct of the experiments in two respects. First, it limited the range of power levels over which the control methodologies could be tested. Specifically, the available instrument spanned a range of about three and a half decades and therefore permitted the performance of experiments from 500 watts to 500 kilowatts. This was not a severe limitation. Also, it should be recognized that while 500 kW is a relatively low power level for the ACRR, reactivity feedback effects resulting from changes in the reactor fuel and moderator temperatures were nevertheless appreciable. The second problem was of more concern because reliance on a single neutron detector meant that signal validation would not be possible. The validation of signals is, of course, a major tenet in the MIT approach to the closed-loop, digital control of nuclear reactors. The decision not to employ a sufficient number of overlapping and redundant sensors so as to make the validation of the neutronic power possible was acceptable only because of the special design of the Annular Core Research Reactor. Specifically, lacking some means of validating signals, the possibility existed for a continuous withdrawal of the transient rod bank. For example, were the sensor to fail, the controller would continuously withdraw the rods in a vain effort to increase the output of the malfunctioning sensor. Most reactors should not be subjected to the effects of such a transient. However, SNL's Annular Core Research Reactor was specifically designed for such events. Its BeO-UO₂ fuel has such a strong Doppler coefficient that, as the fuel temperature rises, sufficient negative reactivity is generated so as to cause a reactor shutdown. In this respect, the reactor is inherently safe. Hence, for this one special case, operation without validated signals was acceptable.

Table 10.2-1

User-Specified Experiment Options

- Specification of controller type:
 - (a) MIT-CSDL Non-Linear Digital Controller
 - (b) MIT-SNL Period-Generated Minimum Time Control Laws
- Selection of either the standard or alternate form of dynamic period equation.
- Inclusion or exclusion of prompt terms from the MIT-SNL minimum time laws.
- Inclusion or exclusion of reactivity feedback due to changes in fuel and moderator temperature from the reactivity balance.
- Use of either a real-time model or a thermocouple reading to determine fuel temperature.
- Inclusion or exclusion of the dynamic effect of reactivity feedback (ρ_f) from the MIT-SNL minimum time laws.
- Selection of the feedback method for updating the specified period on the MIT-SNL minimum time laws:
 - (a) No update
 - (b) Continuous update per equation 5.2.1-1.
 - (c) Other (user supplied)
- Specification of number of delayed or photoneutron groups:
 - (a) Standard six delayed groups
 - (b) Six delayed plus six photoneutron
 - (c) Other (user supplied)
- Specification of an upper limit on reactivity for use with the MIT-CSDL Non-Linear Digital Controller.
- Specification of the maximum allowed frequency associated with operation of the ACRR's variable-speed stepper motors.

10.2.2 Timestep Mode

Another aspect of the October experiments that warrants general mention is that a fixed time step was used for all of the runs. The use of a variable time step, as described in section 8.3.1.1 of this report, was not instituted until December 1987. The use of the fixed time step resulted in some oscillatory behavior in the power signals during many of the runs. As described earlier, oscillations may result whenever the execution time of the software is increased to the point where the code is no longer operating in real time. That is, the execution time exceeds the size of the time step. For example, this could happen if the software contained print instructions that were triggered by infrequent events and if the time step had been selected assuming that such events would not occur. Hence, whenever one did occur, the time required to print the additional material would delay the overall execution of the code. (Note: Oscillations may be caused by other factors including limitations on stepper motor frequency and improper methods for updating the specified period.)

10.3 Open-Loop (Preoperational Tests)

Open-loop runs were performed for the purpose of confirming the allowed range of stepper motor frequencies, to verify operation of the data acquisition system, to determine the optimal time step size, and to validate both the reactivity balance calculations and the numerical method for estimating the effective, multi-group decay parameters. These are described below.

10.3.1 Allowable Range of Stepper Motor Frequencies

The first series of open-loop tests was performed to confirm observations made earlier relative to the available range of stepper motor frequencies. For these tests, the reactor was maintained in a shutdown condition and the control devices that formed the ACRR's transient rod bank were decoupled from their electromagnets. This meant that the variable-speed stepper motors could be driven as desired but that the control rods would always be fully inserted. The motors were then subjected to a variety of frequency changes. It was found that step changes of frequency from 0 to 550 Hz were acceptable but increases to 575 Hz or above would result in stalling of the motor. An upper limit of 500 Hz was selected for the allowable stepper motor frequency. Next, tests were conducted to determine the maximum possible step decrease in frequency with the motors initially at 500 Hz. It was found that step decreases to -600 Hz were acceptable and that decreases to -700 Hz would cause a stall. A lower limit of -500 Hz was placed on the frequency. That is, if the demanded frequency exceeded this figure it would be set to -500 Hz. The use of a symmetric range for frequency was acceptable because, as discussed in section 6.6 of this report, it appears that the rate of reactivity change needed to initiate a transient is always greater than the rate needed to terminate it.

10.3.2 Operability Check of Data Acquisition System

Following the tests to verify the limits for step changes in

frequency to the motor drives, the control elements were reconnected to their electromagnets. Two runs were then made with the ACRR shut-down but operational. These were for the purpose of verifying the data acquisition and display system. Given that the planned closed-loop control trials were experimental in nature, eighteen variables were printed at each sampling interval. These are listed in Table 10.3.2-1.

10.3.3 Selection of Optimal Time Step Size

Relative to the size of the time step, the word 'optimal' is used to connote that the time step should be as small as possible and yet allow software execution to be completed within that interval. Otherwise the code would not operate in real time. The need for the small time step and a criterion for its selection were given in section 5.2.2 of this report. In that regard, it is worth noting that simulation studies had shown that use of a large time step (≈ 1.0 s) in conjunction with a short period (≈ 0.30 s) would result in substantial overshoots. As part of the preoperational tests, three manually controlled transients were performed in which the neutronic power was raised from 1.5 kW to 15 kW on periods of approximately 10 seconds. Time step sizes of 1.0, 0.1, and 0.03 seconds were specified. After reviewing the data, a time step size of 0.05 seconds was selected. This was sufficiently long for real-time operation provided that no special diagnostic messages were printed during a run.

10.3.4 Validation of Reactivity Balance and Multi-Group Decay Parameter Calculations

Two manually-controlled runs of two orders of magnitude each (2 - 200 kW and 1.5 - 150 kW) were performed on 10-second periods to assess the accuracy of the reactivity balance and the multi-group decay parameter calculations. The balance calculation was verified by having a member of the experiment team repeat the arithmetic involved. Lacking an exact simulation of these two runs, it was not possible to verify exactly the accuracy of the estimation of the multi-group decay parameters. However, the values obtained were consistent with those anticipated.

10.4 Preliminary Closed-Loop Trials

The open-loop tests were completed in mid-afternoon on 27 October 1987. Accordingly, it was decided to perform several preliminary closed-loop runs and then commence the closed-loop production runs on the following day. For the preliminary runs, it was decided to first demonstrate operation at steady-state and then perform one test each using the MIT-CSDL Non-Linear Digital Controller and the MIT-SNL Period-Generated Minimum Time Control Laws.

10.4.1 Closed-Loop Steady-State Operation and the Importance of Signal Validation

A power level of 200 kW was selected for this initial trial.

Table 10.3.2-1

Parameters Recorded by the Data Acquisition and Display System

1. Elapsed time [s]
2. Neutronic power level [kW]
3. Required rate of reactivity change [mbeta/s]
4. Rate of reactivity change associated with feedback effects [mbeta/s]
5. Net reactivity [mbeta]
6. Instantaneous reactor period [s]
7. Effective, multi-group decay parameter [s^{-1}]
8. Total energy produced [MJ]
9. Required speed of the control mechanism [pulses/s]
10. Transient rod bank position [position units]
11. Required time for the reactivity constraint [s]
12. Available time for the reactivity constraint [s]
13. Third term in the denominator of the dynamic period equation [mbeta/s]
14. Magnitude of the prompt terms [mbeta/s]
15. Required time for the energy constraint [s]
16. Available time for the energy constraint [s]
17. Controller decision for transient rod bank movement
18. Fuel temperature [$^{\circ}C$]

Notes: (1) If a parameter can be calculated using either the standard or the alternate form of the dynamic period equation, it is displayed according to whichever form has been specified by the user.

(2) This list was effective 10/27/87. It was frequently modified depending on the purpose of the experiment.

The alternate MIT-SNL minimum time law was specified together with a period of 100 seconds. Steady-state operation was to be achieved by setting the target power equal to the reactor's actual power. The minimum time law would therefore compute the rate of change of reactivity needed to hold that power. Presumably, this would be zero. However, were a deviation to exist, the neutronic power would be raised or lowered on a 100 second period until the reactor was again operating at 200 kW. The test did not go as planned because shortly after transfer of control from the ACRR console to the digital controller, the neutron detector began to malfunction and its signal drifted low. The detector's output no longer reflected the true reactor power. Sensing an apparent decrease in power, the controller withdrew the transient rod bank until the absolute limit on reactivity that was noted in section 7.3.3.1 of this report, was attained. The reactor's power (the true power) began to rise as a result of this reactivity addition and the ACRR's licensed operators initiated a manual shutdown. As noted earlier, the ACRR is specially designed to accommodate such transients and no challenge to any of its limiting safety system settings occurred. Much can be learned from this episode. In particular it should be realized that the control law functioned as intended. The neutronic power level, as sensed by the digital controller, was low. Accordingly, the control law added reactivity in a vain effort to cause an increase in the output of the failing sensor. There was no defect in the control law. Rather, the missing factor was the lack of a signal validation system. Such a system would, by comparing readings from redundant sensors, have identified this particular sensor as failing and would have excluded its output from those being used to form an estimate of the 'true' power level. Finally, it should be noted that the most common method for validating signals is to accept as true any sensor output which falls within the anticipated range of signals. That approach would have been of marginal value here because there was no abrupt failure. The sensor degraded slowly. (Note: An excellent survey of instrument failure detection and isolation methods is given in [24].)

Following replacement of the sensor, the same run was repeated with a specified power level of 150 kW. The alternate MIT-SNL Period-Generated Minimum Time Control Law performed successfully holding the power of SNL's Annular Core Research Reactor constant at 150 kW. The resulting power and reactivity profiles are shown in Figure 10.4.1-1 for a portion of this transient. (Note: The spread in the data is considerable. This problem remained through the October runs and was only partially resolved in December 1987 by use of the variable time step. A further improvement, which appears to have eliminated the oscillatory effect was made in September 1988 during a set of runs which are beyond the scope of this report. Details are given in section 12.1.3 of this report and in [25].)

10.4.2 Initial Demonstration of the Non-Linear Digital Controller

The standard version of the MIT-CSDL Non-Linear Digital Controller was used to raise the ACRR's neutronic power from 117 kW to 200 kW subject to a reactivity limit of 200 mbeta. As always with trials of

Figure 10.4.1-1 Initial Demonstration of the MIT-SNL Period-
Generated Minimum Time Control Laws for Steady-
State Operation of the Annular Core Research
Reactor

the NLDC, a poorly conceived control law was intentionally used so as to be certain of testing the constraint. In this case, the selected law was to withdraw the ACRR's transient rod bank continuously. Hence, it would be necessary for the constraint to override the control law. The transient was completed without overshoot in less than five seconds. This was the first demonstration of the 'reactivity constraint approach', which is the basis of the NLDC, on a facility other than the MIT Research Reactor. The resulting power profile is shown in Figure 10.4.2-1. The spread in the data for this run and for all NLDC runs is greater than that for any corresponding transient accomplished with one of the minimum time laws. This difference occurs because the NLDC tends to act as an 'on-off' controller with either the rods being withdrawn at full speed by the control law or being inserted at full speed by the constraint. This is especially true at steady-state. The Non-Linear Digital Controller is a supervisory methodology and it performed that role admirably.

10.4.3 Initial Demonstration of the Minimum Time Control Laws

The initial demonstration of the MIT-SNL Period-Generated Minimum Time Control Laws under transient conditions was not achieved until 28 October 1987. Figure 10.4.3-1 shows the resulting power profile. The ACRR's neutronic power was raised from 134 kW to 240 kW on a specified period of 50 seconds. The transient was completed in the allowed time of 28.9 seconds. The exponential shape of the power profile shows that the specified period of 50 seconds was maintained. Also, the sharp cornering upon termination of the transient is apparent. This indicates that the control law correctly computed the rate of reactivity insertion needed to halt the transient.

10.5 Closed-Loop Production Runs

Four sequences of production runs were performed on 28 October 1987. Each involved the comparative testing of the four possible control methodologies which were the standard and alternate MIT-SNL Period-Generated Minimum Time Control Laws and the standard and alternate versions of the MIT-CSDL Non-Linear Digital Controller. The demanded power increases were gradually extended from a mild factor of two on a 50 second period to a rather severe factor of a hundred on periods of less than a second. As was discussed in section 5.1.1 of this report, operation of the NLDC is valid only for reactivities that are less than prompt critical. Accordingly, it was decided to specify a limit on the net reactivity that could be present whenever the NLDC was being used. This quantity is referred to in the experiment descriptions given below as the 'externally-imposed reactivity limit'. In general, it was chosen for a given experiment so that the response of the NLDC would occur on a time scale similar to that of the MIT-SNL laws. In addition to this reactivity restriction on the NLDC, there was a limit of 0.20 seconds on the minimum allowed period. It applied to both the NLDC and the MIT-SNL laws.

10.5.1 Power Doubling

Each of the four possible control methodologies was used to

Figure 10.4.2-1 Initial Demonstration of the MIT-CSDL Non-Linear
Digital Controller on the Annular Core Research
Reactor

Figure 10.4.3-1 Initial Demonstration of the MIT-SNL Period
Generated Minimum Time Control Laws for
Transient Operation of the Annular Core
Research Reactor

double the ACRR's neutronic power by raising it from 140 kW to 280 kW. Difficulties in reestablishing identical conditions at the outset of each transient caused slight variations in the initial power levels. Also, the target power for the standard NLDC run was only 240 kW. Nevertheless, comparative analyses of the resulting power and reactivity profiles provide much insight relative to each control concept.

10.5.1.1 Performance of the MIT-SNL Minimum Time Laws

Figures 10.5.1.1-1 and 10.5.1.1-2 show the power and reactivity profiles obtained using the alternate and standard MIT-SNL minimum time laws. The specified reactor period was 50 seconds. For both the standard and the alternate laws, the instantaneous reactor period was stepped to this value at the outset of the transient, held at that value during the transient, and then stepped back to infinity upon transient termination. Evidence for this is that in both instances, the transient was completed within the allotted time (36 seconds for the alternate law and 35 seconds for the standard one) and that the shape of both power profiles was exponential. Note that there is little difference between the power profiles for the alternate and standard laws. In fact, had the initial conditions been identical, there would have been no difference. This is to be expected because the two laws are mathematically equivalent.

Given both the mild nature of these transients and the scatter in the data, it is difficult to discern the features in the reactivity profiles that are characteristic of the MIT-SNL laws. Nevertheless, upon fitting the data using a least-squares methodology, it is evident that there was a rapid insertion of reactivity upon transient initiation and a rapid removal of reactivity upon transient termination. Thus, prompt neutron effects were being used to drive the transients. It is also evident that it was necessary to increase reactivity during the course of each transient. This action was necessary in order to compensate for negative reactivity feedback effects associated with the rising fuel temperature.

10.5.1.2 Performance of the MIT-CSDL Non-Linear Digital Controller

The reactivity constraint approach and its implementing algorithm, the MIT-CSDL Non-Linear Digital Controller, were described in section 5.1.1 of this report. More detailed information is given in [2]. The basic concept is to preclude challenges to a reactor's safety system by restricting operation to only certain states (combinations of reactivity and available rate of change of reactivity) while transiting from some initial to some final power. This is accomplished by utilizing constraints that take the form of inequalities. The left side of the constraint is the time required to establish conditions under which the driving force can be completely negated. The right side is the time remaining to attain the limiting condition. Hence, by observing the inequality, it is assured that conditions under which the transient can be halted will exist upon attaining the limit. Relative to neutronic power, the constraint limits the net reactivity present in the core to that which can be offset by the dynamic effect associated with insertion of a control device.

Figure 10.5.1.1-1 Power and Reactivity Profiles Associated with the Alternate MIT-SNL Minimum Time Law: Power Doubling Experiment

Figure 10.5.1.1-2 Power and Reactivity Profiles Associated with the Standard MIT-SNL Minimum Time Law: Power Doubling Experiment

Figures 10.5.1.2-1 and 10.5.1.2-2 show the power and reactivity profiles obtained using the standard and alternate versions of the MIT-CSDL Non-Linear Digital Controller. An externally imposed reactivity limit of 200 mbeta was specified. The available rate of change of reactivity associated with the ACRR's transient rod bank was approximately 320 mbeta per second during these runs. (Note: This figure would normally be larger, but the stepper motor frequency was set at 100 Hz.) Assuming that the standard and alternate effective multi-group decay parameters were 0.08 and 0.40 inverse seconds respectively, then the reactivities for which control was deemed to be continuously feasible by the standard and alternate constraints were 4000 mbeta and 800 mbeta respectively. Hence, for both the standard and alternate forms of the Non-Linear Digital Controller, the 200 mbeta reactivity restriction was the limiting condition. As a result, no discernible difference was observed between the two transients. (Note: Had the bounds of either the standard or the alternate absolute constraint been reached or exceeded, then differences such as those evident in Figure 9.3.1.2-1 would have been apparent.) In regard to the conduct of the transients themselves, the fact that constraint boundaries were never approached meant that, for both forms of the NLDC, the transient rod bank was withdrawn until 200 mbeta of reactivity had been added. The bank's position was then adjusted to maintain the net reactivity constant at 200 mbeta until the desired power level was attained. The transient rod bank was then inserted so as to generate the negative rate of reactivity change necessary to offset the 200 mbeta of positive reactivity and thereby halt the power increase. Each of the features described above is clearly visible in the two figures. The reactivity profiles shown are those associated with the transient rod bank. They did not go to zero following completion of the transient because of the need to compensate for reactivity feedback effects associated with the rising fuel temperature.

10.5.1.3 Comparison of the MIT-SNL Laws and the NLDC

Figure 10.5.1.3-1 is a composite that facilitates comparison of the power profiles of each of the four control methodologies that were used in the power doubling experiment. Of significance, is that the runs accomplished with the Non-Linear Digital Controller were achieved in much less time than those performed with the MIT-SNL Period-Generated Minimum Time laws. The meaning of the word 'minimum' therefore warrants explanation. As with the use of any performance index, the MIT-SNL laws are optimal only in relation to that index. Thus, these laws result in the minimum possible transit time between two power levels for a reactor subject to a restriction on the specified period. In this case, the specified period was fifty seconds which, under asymptotic conditions, corresponds to a reactivity of about 165 mbeta. In contrast, the NLDC could add 200 mbeta and it did so at the very outset of the transient. Hence, its response was faster.

10.5.2 Power Increases of a Decade on Ten Second Periods

The second step in the production run sequence was to perform power increases of approximately a decade. For the MIT-SNL Period-Generated Minimum Time Control Laws, the specified period was 10

Figure 10.5.1.2-1 Power and Reactivity Profiles Associated with the Standard MIT-CSDL Non-Linear Digital Controller: Power Doubling Experiment

Figure 10.5.1.2-2 Power and Reactivity Profiles Associated with the Alternate MIT-CSDL Non-Linear Digital Controller: Power Doubling Experiment

Experimental Data
Annular Core Research Reactor
ACR015-ACR018 (10/28/87)

Figure 10.5.1.3-1 Comparison of Control Methodologies: Power Doubling Experiment

seconds. For the Non-Linear Digital Controller, the externally imposed limit on reactivity was set at 500 mbeta. For all four runs, the power was raised from 15 kW to 170 kW. As before, there were small differences in the initial conditions. For the four runs in this series, the stepper motor frequency was set at 100 Hz.

10.5.2.1 Performance of the MIT-SNL Laws

Figures 10.5.2.1-1 and 10.5.2.1-2 show the power and reactivity profiles for runs performed using the alternate and standard MIT-SNL minimum time laws. As is to be expected, the resulting profiles are quite similar. Both transients were completed without overshoot and in the allotted times of 24.0 seconds for the alternate law and 24.3 seconds for the standard one. (Note: The difference in allowed times reflects slightly different initial conditions.) Also, in both cases, the shapes of the power profiles were exponential. Of particular interest is that, given the magnitude of the power increase, the characteristic features of the power and reactivity profiles associated with the MIT-SNL laws are clearly visible. These include:

- (1) There were abrupt changes in the slope of the power profile upon both transient initiation and termination. These occurred because the period was first stepped from infinity to the specified value (10 seconds) and then from that value back to infinity. A step change in the period results in a discontinuity in the slope of the power profile.
- (2) The shape of the power profile during the transient was exponential. This resulted because the control law maintained a constant value for the period during the entire transient. (Note: The profile was recorded on a linear plot. It would have appeared as a straight line if it had been recorded on a logarithmic scale.)
- (3) At the outset of the transient there was a rapid insertion of reactivity reflecting the need to use a high rate of change of reactivity to generate the specified reactor period. For the transients shown here, the specified period was 10 seconds. Hence, the requisite rate of reactivity change upon transient initiation was about 100 mbeta per second.
- (4) Once the transient had been initiated, there was a gradual cessation of reactivity addition indicating that the reactivity was approaching its asymptotic value for the specified period.
- (5) During the intermediate stages of the transient, there was a continuous but steady withdrawal of the control rods. Yet the reactor period, as evidenced by the exponential shape of the power profile, remained constant. This indicated that sufficient energy had been generated so as to cause heating of the fuel and that negative reactivity was being inserted as a result of the Doppler effect.

Figure 10.5.2.1-1 Power and Reactivity Profiles Associated
with the Alternate MIT-SNL Minimum Time
Law: Power Increase of a Decade

Figure 10.5.2.1-2 Power and Reactivity Profiles Associated
with the Standard MIT-SNL Minimum Time Law:
Power Increase of a Decade

- (6) Upon transient termination, there was a rapid decrease in reactivity indicating that the dynamic effect associated with control device insertion was being used to halt the transient. Note that there was still positive reactivity present. Hence, the reactor period was driven to infinity only by decreasing the prompt neutron population (negative rate of change of reactivity) so as to exactly balance the still-rising delayed neutron population (positive reactivity).
- (7) Subsequent to the leveling of the neutronic power, it was necessary to further withdraw the control rods in order to hold the power constant. Again, this was necessary in order to counter the negative reactivity being added as a result of the gradual heating of the fuel.

10.5.2.2 Performance of the MIT-CSDL Non-Linear Digital Controller

Figures 10.5.2.2-1 and 10.5.2.2-2 show the corresponding information for the runs performed using the MIT-CSDL Non-Linear Digital Controller. As was the case with the power doubling experiment, the reactivity never exceeded the bounds of the standard or the alternate absolute constraints. Hence, both profiles are quite similar and the explanation for the features shown is the same as for the power doubling runs. There are of course some differences but these are the result of the increase in magnitude of the reactivity restriction rather than a transition from an absolute to a sufficient constraint. In particular, the length of time that the reactivity was held constant was quite short and the rate of decrease of reactivity needed to terminate the transient was appreciable.

10.5.3 Power Increases of a Decade on One-Second Periods

The third step in the production run sequence was to repeat the power increases of a decade using a period of 1.0 second instead of 10 seconds. For the Non-Linear Digital Controller, the externally imposed limit on net reactivity was set at 700 mbeta. As with the previous set of decade power increases, the objective was to raise the ACRR's neutronic power from approximately 15 kW to 170 kW.

10.5.3.1 Performance of the MIT-SNL Minimum Time Laws

Figures 10.5.3.1-1 and 10.5.3.1-2 show the power and reactivity profiles for the runs performed using the alternate and standard MIT-SNL minimum time laws. Given that the alternate and standard laws are mathematically equivalent, it would be expected that the profiles shown in the two figures would be quite similar and this is in fact the case. Both transients were completed in the allotted time of 2.43 seconds. The classic characteristics of the MIT-SNL laws were enumerated in section 10.5.2.1 of this report when power increases of a decade conducted on ten-second periods were described. Those same characteristics were present during these transients. However, because of the rapidity of the power adjustment and the time-scale used

Figure 10.5.2.2-1 Power and Reactivity Profiles Associated with the Standard Non-Linear Digital Controller: Power Increase of a Decade

Figure 10.5.2.2-2 Power and Reactivity Profiles Associated with
the Alternate Non-Linear Digital Controller:
Power Increase of a Decade

Figure 10.5.3.1-1 Power and Reactivity Profiles Associated with the Alternate MIT-SNL Minimum Time Law: Power Increase of a Decade on a One Second Period

Figure 10.5.3.1-2 Power and Reactivity Profiles Associated with the Standard MIT-SNL Minimum Time Law: Power Increase of a Decade on a One Second Period

in the figure, not all of the features are readily apparent. In fact, the only ones that stand out are the rapid insertion and removal of reactivity associated with transient initiation and termination. (Note: Refer to Figure 10.5.4.1-3 for a similar transient drawn with an expanded time scale.)

One concern that was raised upon analysis of these runs was that the scatter in the data for the power profiles was excessive, especially during transient termination. On average, there was no significant overshoot. However, the fluctuations in the neutronic power about the specified value were of substantial magnitude. This behavior reflected high frequency oscillatory movement of the control devices. Such movement is of course intolerable because, if sustained, it would cause premature failure of the motors and gears that drive the control devices. Accordingly, a major objective of the MIT-SNL research program that was pursued subsequent to the one described here, was the development of filtering techniques that would eliminate or at least greatly mitigate this problem. Further information is given in Chapter Thirteen of this report.

Figure 10.5.3.1-3 provides additional information on the alternate MIT-SNL law for the transient given by Figure 10.5.3.1-1. The upper portion of the figure gives the reactivity and power profiles for the first few seconds of the transient. The latter is plotted using a linear scale. The power increase from 15 kW to 170 kW was completed essentially without overshoot in the allotted time of 2.43 seconds. Note that the period was maintained constant at 1.0 s as evidenced by the exponential shape of the power profile. Also note that power was leveled in the presence of substantial positive reactivity by making the rate of change of reactivity negative. That is, the prompt neutron population was intentionally decreased in order to exactly balance the continuing rise in the delayed neutrons. The lower portion of the figure illustrates the behavior of the dominant terms that comprise the control law. Recall that the quantity $\dot{\rho}$ corresponds to the effect of a changing prompt neutron population while the terms $\lambda_e' \rho$ and $\sum \beta_i (\lambda_e - \lambda_e')$ represent the combined effects of a change in the precursor population and a changing distribution of precursors among the defined groups. Note the discontinuous changes in the control signal that correspond to the moments at which the period was stepped from infinity to 1.0 second and from 1.0 second to infinity. Also note that the required rate of change of reactivity was a maximum at the outset of the transient and was then gradually reduced as the terms representing the delayed effects grew in magnitude.

Figure 10.5.3.1-4 provides similar information for the standard MIT-SNL law for the transient depicted in Figure 10.5.3.1-2. The principal difference is in the lower portion of the figure where the quantities $\lambda_e \rho$ and $(\dot{\lambda}_e / \lambda_e)(\beta - \rho)$ are shown. The former corresponds to the effect of changes in the precursor population while the latter reflects the changing distribution of the precursors among the defined groups.

Figure 10.5.3.1-5 depicts the behavior of the alternate and stan-

Figure 10.5.3.1-3 Analysis of Control Action for Alternate
 MIT-SNL Law: Power Increase of a Decade on a
 One Second Period

Figure 10.5.3.1-4 Analysis of Control Action for Standard
MIT-SNL Law: Power Increase of a Decade on a
One Second Period

Figure 10.5.3.1-5 Behavior of Multi-Group Decay Parameters
During Power Increase of a Decade on a One
Second Period

dard, effective, multi-group decay parameters during the aforementioned transients. Of significance are the substantial variations in the magnitude of these quantities during the power increase. Hence, it is quite evident that mathematical simplifications such as the modeling of delayed neutron effects using a single representative group (one-group approximation) would be invalid.

10.5.3.2 Performance of the MIT-CSDL Non-Linear Digital Controller

Figures 10.5.3.2-1 and 10.5.3.2-2 depict the power and reactivity profiles for the power increases performed using the standard and alternate versions of the MIT-CSDL Non-Linear Digital Controller. These profiles are strikingly similar to those obtained when using the MIT-SNL laws. This resemblance suggests that, as the specified period is made shorter, either the minimum time laws or the NLDC could be used to achieve the objective. Such an inference would be incorrect because the theoretical basis for the two control concepts is quite different. The MIT-SNL laws are closed-form expressions that cause the reactor period to assume a user-specified value. These laws are predictive in nature in that they can be used to project values of the neutronic power for a given trajectory. That is their strength. Their weakness is that they can not be used to determine if it will be possible to halt the transient upon attainment of the desired power level. The NLDC fulfills that function. It is a supervisory methodology that restricts reactor operation to those states (combinations of reactivity and rate of change of reactivity) for which control will be feasible in the sense that the transient can be halted. The NLDC uses present information to determine if there will be a challenge to a limiting condition at some future time. That is its strength. Its weakness is that it provides no information as to either the timing or the possible severity of that challenge. Hence, both the MIT-SNL laws and the NLDC are needed to form a complete controller. They should be used in concert. This is further discussed in section 12.4 of this report.

As was discussed in section 5.1.1 of this report, the standard and alternate versions of the Non-Linear Digital Controller are not mathematically equivalent because each uses a different definition of the effective, multi-group decay parameter. For the transients examined thus far, no differences have been apparent because the reactivity restrictions imposed on the conduct of those transients have been more limiting than the reactivity constraints associated with the NLDC. The present set of power maneuvers was an exception in that the reactivity restriction had now been relaxed so as to no longer be limiting. As a result, it was possible to discern the differences between the standard and alternate forms of the NLDC.

Figure 10.5.3.2-3 shows both the power and reactivity profiles obtained using the standard NLDC together with the required and available times. Figure 10.5.3.2-4 provides the same information for the alternate NLDC. Note that when the standard NLDC was used, the target power level was attained in less than 2.0 seconds. In contrast, the completion time for the alternate NLDC was approximately 4.1 seconds.

Figure 10.5.3.2-1 Power and Reactivity Profiles Associated with the Standard Non-Linear Digital Controller: Power Increase of a Decade with a Reactivity Limit of 700 mbeta

Figure 10.5.3.2-2 Power and Reactivity Profiles Associated with the Alternate Non-Linear Digital Controller: Power Increase of a Decade with a Reactivity Limit of 700 mbeta

Figure 10.5.3.2-3 Power Increase from 15 kW to 170 kW on the Annular Core Research Reactor Using the Standard Reactivity Constraint Approach

Figure 10.5.3.2-4 Power Increase from 15.2 kW to 170 kW on the Annular Core Research Reactor Using the Alternate Reactivity Constraint Approach

Also, use of the alternate constraint resulted in somewhat less oscillation of the neutronic power about the operating point once the target power level was attained.

Examination of Figure 10.5.3.2-3, the standard NLDC, shows that the required time remained negative throughout the transient. This meant that the reactivity bound established by the standard absolute reactivity constraint ($\rho < |\dot{\rho}_c|/\lambda_e$) was not approached. Under such circumstances, the controller would add reactivity until the externally imposed reactivity limit was attained, maintain that reactivity until the target power level had been achieved, and then reverse the direction of movement of those devices in order to halt the transient. This is essentially what occurred during the run shown in the figure. Reactivity was added until the 700 mbeta restriction was attained. This occurred almost simultaneously with the attainment of the specified power level. In theory, it should have been possible to level the neutronic power at 170 kW. In reality, the power overshoot, peaked at about 207 kW, and then settled out at the desired value. Reasons for the overshoot include the finite measurement times for signal acquisition, the processing time needed to generate the control signal, and the response time associated with the control mechanisms. Also, the fact that the quantity $(\dot{\lambda}_e/\lambda_e)(\beta-\rho)$ is neglected in formulating the standard reactivity constraints may be of significance for transients conducted on very short periods. (Note: Arguments justifying the deletion of this term from the constraint are given in [2,26]. However, these arguments may not be valid for transients conducted on short periods.)

The conclusion to be drawn from examination of Figure 10.5.3.2-3 is that the preferred mode of operation would be for the controller to cut back on the reactivity prior to attaining full power and thereby perhaps terminate the transient more smoothly. Examination of Figure 10.5.3.2-4, the alternate NLDC, reveals that this is exactly what occurred during that run. Specifically, the required time became positive indicating that the alternate sufficient reactivity constraint which is of the form:

$$[\rho(t) - |\dot{\rho}_c|/\lambda_e'(t)]/|\dot{\rho}_c| \leq \tau(t) \ln(P_F/(P(t))) \quad (10.5.3.2-1)$$

was the limiting condition. Under such circumstances, the controller would insert positive reactivity until the required and available times became equal. For the run shown, this occurred at about 1.8 seconds when about 680 mbeta were present. The ACRR's transient rod bank was then inserted in a stepwise manner so as to keep the required and available times essentially equal. The power was leveled smoothly at about 4.1 seconds. In contrast, when the standard NLDC was used, full power was achieved after 1.9 seconds and power had settled out at the desired value after 2.9 seconds. Hence, the standard methodology is the more responsive. Nevertheless, the alternate version of the NLDC may be preferable because it does result in a smoother transition and its 80% rise time is certainly comparable to that of the standard NLDC. A detailed comparison of the two approaches for transients involving periods of 50 seconds or longer was given in [11]. For fast transients, the issue warrants further study.

10.5.4 Power Increases of Two Decades

The fourth series in the production run sequence entailed the performance of power increases of two decades, 5 kW - 500 kW. Three runs were performed using the alternate MIT-SNL Period-Generated Minimum Time Control Law with specified periods of 10.0, 3.0, and 1.0 seconds respectively. Also, runs were performed using the standard and alternate versions of the MIT-CSDL Non-Linear Digital Controller subject to an externally-imposed reactivity restriction of 900 mbeta. For this series of runs, the frequency of the stepper motors used to drive the ACRR's transient rod bank was increased from 100 Hz to 200 Hz. This meant that the available rates of reactivity insertion and removal were doubled.

10.5.4.1 Performance of the MIT-SNL Minimum Time Laws

Figures 10.5.4.1-1 - 10.5.4.1-3 show the power and reactivity profiles for the three runs performed using the alternate MIT-SNL law. The specified periods were 10.0, 3.0, and 1.0 seconds respectively. For each run, a different time-scale was used in plotting the data. This was done so that it would be obvious that the characteristic features of the MIT-SNL laws, as described in section 10.5.2.1 of this report, were present. The expected completion times for each run were 47.2, 13.9, and 4.7 seconds respectively. In each case, these were achieved with essentially no overshoot in the neutronic power. As is to be expected, the scatter in the data increased as the specified period was decreased. The third run in the series, the one performed on a 1.0 second period, proved to be the most informative because examination of the data points clearly revealed a high frequency oscillation superimposed on the overall transient. This problem was subsequently traced to an inadequacy in the method used to correct for deviations between the observed and specified power levels. Specifically, the approach given by equation 5.2.1-1 in which the current values of these two quantities were compared was shown to be inherently unstable. A superior method, which was developed by Dr. Takashi Washio of the MIT Nuclear Reactor Laboratory, is to provide compensation by comparing the current value of the observed power to that of the specified power projected several time steps ahead. Thus,

$$\omega_s(t) = \ln[n_s(t+k\Delta t)/n(t)]/\Delta t \quad (10.5.4.1-1)$$

where k is the number of sample intervals of size Δt by which the calculation of the specified power should be advanced. It was shown from theory that this quantity should be at least two. The symbols $n_s(t)$ and $n(t)$ denote the specified and observed values of the reactor power respectively. The quantity $\omega_s(t)$ is the value of the inverse period that should be substituted into the MIT-SNL laws in order to cause the actual neutronic power to vary as desired. Using this approach, implementation of the MIT-SNL laws, as described in section 5.2 of this report, is modified as follows. The specified inverse period is now taken as meaning the average inverse period on which the overall transient should be conducted. If a perfect model of the reactor in question existed, then the quantity $\omega(t)$ in the MIT-

Figure 10.5.4.1-1 Power Increase of Two Decades on the Annular
Core Research Reactor Using the Alternate
MIT-SNL Law: Ten Second Period

Figure 10.5.4.1-2 Power Increase of Two Decades on the Annular Core Research Reactor Using the Alternate MIT-SNL Law: Three Second Period

Figure 10.5.4.1-3 Power Increase of Two Decades on the Annular
Core Research Reactor Using the Alternate
MIT-SNL Law: One Second Period

SNL laws could continue to be taken as this specified period. However, given the realities of modeling errors and signal noise, deviations between the specified and observed power levels will develop. Accordingly, the desired power trajectory, $n_s(t)$, is determined from the specified period and, at each time step, the quantity $\omega_s(t)$ is calculated using equation 10.5.4.1-1. This term is then substituted for $\omega(t)$ in the MIT-SNL laws. The net effect will be to smoothly eliminate any power deviations. Detailed discussions of both the stability analysis of the MIT-SNL laws and the design of a feedback methodology to facilitate implementation of these laws are given in [25, 27]. (Note: The feedback scheme given by equation 10.5.4.1-1 was not implemented until after completion of the control experiments conducted in December 1987. Refer to section 12.1.3 of this report for additional information.)

Figure 10.5.4.1-4 is a composite showing the power profiles of the above runs in both linear and logarithmic profiles. The capability of the MIT-SNL laws to cause reactor neutronic power to vary continuously on a constant period is evident from the straight slopes of the logarithmic profiles. Also, it is apparent that the power can be leveled essentially without overshoot. However, the oscillations caused by the improper choice of the feedback methodology (equation 5.2.1-1) are also evident.

10.5.4.2 Performance of the MIT-CSDL Non-Linear Digital Controller

Figures 10.5.4.2-1 and 10.5.4.2-2 depict the power and reactivity profiles obtained when using the standard and alternate versions of the Non-Linear Digital Controller. In neither case did the reactivity restriction, which was set at 900 mbeta, become limiting. The completion time for this two-decade power increase, 5 kW - 500 kW, was 1.5 seconds when using the standard NLDC. Examination of Figure 10.5.4.2-1 shows that reactivity was continuously added until slightly more than 700 mbeta was present. An absolute limitation on the minimum allowed value of the reactor period then caused a lessening in the rate of reactivity addition. (Note: This restriction on the reactor period was, like the one on reactivity, an externally applied safety consideration.) The controller continued to add reactivity at this lower rate until the specified power level of 500 kW was attained. The bounds of the standard absolute reactivity constraint were never exceeded. The controller had some difficulty leveling the power at 500 kW and there was a brief but substantial power overshoot. The reasons for this problem were the same as those enumerated when use of the standard NLDC for power increases of a decade on periods of 1.0 second were described. (Refer to section 10.5.3.2 of this report.) Specifically, because the bounds of the standard absolute constraint were not exceeded, reactivity was still being added when the target power level was attained. Also, insufficient allowance was made for the time required to measure the power level, process the signal, and implement the controller's decision to commence reactivity removal. Another factor was that, as explained earlier, the NLDC was intentionally paired with a simplistic control law that continuously withdrew the ACRR's transient rod bank. The resulting control action was

Figure 10.5.4.1-4 Closed-Loop Power Increases from 5 kW to 500 kW Using Alternate MIT-SNL Period-Generated Minimum Time Control Law: Experimental Data (ACR034-ACR036) Obtained 28 October 1987 on the Annular Core Research Reactor

Figure 10.5.4.2-1 Power and Reactivity Profiles Associated with the Standard Non-Linear Digital Controller: Power Increase of Two Decades with a Reactivity Limit of 900 mbeta

Experimental Data
Annular Core Research Reactor
ACR038 - 10/28/87

Figure 10.5.4.2-2 Power and Reactivity Profiles Associated with the Alternate Non-Linear Digital Controller:
Power Increase of Two Decades with a Reactivity Limit of 900 mbeta

therefore of the 'on-off' type. If the power level was below 500 kW, the rod bank was withdrawn. If not, it was inserted. Given the significant reactivity associated with even small movements of this bank, the effect was to cause significant scatter in the data. In assessing the experiment shown here, it should be realized that the objective was not to demonstrate a perfected controller. Rather it was to evaluate the supervisory capability of the NLDC on short periods. In this respect, it was successful.

Figure 10.5.4.2-2 provides the results when this power increase of two decades was performed using the alternate NLDC. The transient completion time was 2.9 seconds. The conduct of this transient differed significantly from that of the standard NLDC because the bounds of the alternate absolute reactivity constraint were exceeded. Hence, the alternate sufficient constraint became the limiting factor. As a result, reactivity removal was initiated prior to the attainment of full power, 500 kW. The result, which is apparent in the figure, was that there was significantly less overshoot of the reactor power. Other features of the transient are similar to those obtained when using the standard NLDC. In particular, once at steady-state, the scatter in the data was significant. The reason for this was, as described above, the 'on-off' nature of the control action.

It is instructive to compare the traces obtained when using the Non-Linear Digital Controller (Figures 10.5.4.2-1 and 10.5.4.2-2) with those generated by the MIT-SNL Minimum Time Law when power was increased on a 1.0 second period (Figure 10.5.4.1-3). Both the NLDC and the MIT-SNL law exhibited oscillatory behavior. In the case of the NLDC, this occurred because the supervisory constraint either was or was not satisfied. If the former, the control devices were withdrawn. If the latter, they were inserted. Note that the oscillations did not occur in the NLDC runs until the process of leveling the reactor power was begun. Prior to that time, the reactivity constraints had been continuously satisfied. In the case of the MIT-SNL law, the oscillatory behavior occurred because of the inappropriateness of the feedback methodology. Note that in this instance, the oscillations were present from almost the very start of the transient.

10.6 Significance of the October 1987 Experiments

The experiments conducted on the closed-loop, digital control of neutronic power during the period 27-28 October 1987 on SNL's Annular Core Research Reactor provided experimental verification of the utility of the MIT-SNL Period-Generated Minimum Time Control Laws for the rapid adjustment of reactor power. The experiments were remarkable in that they were performed only eleven months after the initial realization that closed-form expressions for the time-optimal control of reactor power could be obtained by using the rate of change of reactivity as the control signal. During those eleven months, the theory had been perfected, simulation studies had been performed, the required software had been written and validated, the data acquisition system had been designed and the hardware-to-software interface constructed, a simulator had been assembled, and the necessary regulatory

approvals had been obtained. Specific accomplishments associated with the October experiments were as follows:

- (1) The concept of raising a reactor's neutronic power by adjusting the rate of change of reactivity was shown to be both practical and advantageous.
- (2) The capability to accurately estimate the various parameters contained in the MIT-SNL laws in real time was demonstrated. In particular, the technique for estimating the effective, multi-group decay parameters was shown to be effective.
- (3) The time-optimal adjustment of reactor neutronic power was achieved using a closed-form control law.
- (4) The MIT-SNL Period-Generated Minimum Time Control Laws were shown to be effective for power increases of two decades on periods as short as 1.0 second.
- (5) The reactivity constraint approach and its implementing algorithm, the MIT-CSDL Non-Linear Digital Controller, were demonstrated on a facility other than the MIT Research Reactor. This proved the generic nature of the concept.
- (6) The Non-Linear Digital Controller was shown to be effective for power increases of two decades on periods as short as a few tenths of a second.
- (7) Both the MIT-SNL laws and the NLDC were tested under realistic conditions of signal noise and modeling problems. The resulting data provided significant insight for the design of more robust methods for implementing the two concepts.

The most significant accomplishment of the October 1987 experiments was that confidence was established in the technology. This provided the impetus for additional work which culminated two months later when the MIT-SNL laws were shown to be capable of raising reactor neutronic power by five orders of magnitude both on a period of 0.50 seconds and without overshoot.

11. Experimental Evaluation of the MIT-SNL Period-Generated Minimum Time Control Laws: December 1987

Forty-six separate experimental runs were performed under conditions of closed-loop digital control on Sandia's Annular Core Research Reactor (ACRR) during the period 3-4 December 1987. Tests were made both of the MIT-SNL Period Generated Minimum Time Control Laws and of the MIT-CSDL Non-Linear Digital Controller. Whereas the experiments performed in October 1987 had been designed to stress the feasibility of the various control concepts, the December experiments were chosen to illustrate the flexibility of the MIT-SNL control methodology. Selected results from those runs are given here. Also, reported is a demonstration of the MIT-SNL minimum time laws on the MIT Research Reactor with a variable-speed shim blade being used as the actuator.

11.1 Objectives of the December ACRR Control Experiments

The principal objective of the closed-loop experiments on digital control conducted on SNL's Annular Core Research Reactor in December 1987 was to demonstrate the capability to raise a reactor's neutronic power by five-six orders of magnitude both on a period of several tenths of a second and without overshoot. A related objective was to perform other power maneuvers such as power decreases and cyclic operation. A final goal of the December experiments was, in accordance with Department of Energy policy concerning space technology, to identify areas that would require further research. Of special importance were any that would impede the achievement of autonomous control of a spacecraft nuclear reactor.

The December experiments comprised seven distinct sets of runs. These were power increases of three orders of magnitude, preparatory runs for power increases of many orders of magnitude, power increases of four orders of magnitude, power increases of five orders of magnitude, power decreases, power cycling, and demonstration of the standard and alternate sufficient reactivity constraints. In the preceding chapter, the analysis of the experiments focused on the underlying physics. For example, emphasis was placed on the use of the rate of change of reactivity to alter the reactor period. Information of that type can of course also be obtained from examination of the December experiments. However, so doing would be repetitious. Instead, the focus of this chapter is largely on the effect of external factors on the performance of the control laws. For example, an extensive discussion is given of the impact of the initial position of a reactor's control devices on the capability of the control laws to accomplish a power increase of many orders of magnitude. As a result of this change in emphasis, extensive use is made of the strip chart recordings obtained during the experiments. The material presented here should be of use to those designing reactors for spacecraft operation. In particular, it should be apparent that the requirements of the control system should be addressed during the initial stages of the reactor's overall design.

11.2 Experiment Options

Relative to the December experiments, three changes of significance were made to the software. These were:

- (a) The single neutron sensor used in the October runs was replaced by a set of three detectors with overlapping ranges. Thus, the instrumentation was as shown in Figure 8.3.2.1-2.
- (b) A variable time step, as described in section 8.3.1.1 of this report, was used in lieu of a fixed one.
- (c) The average temperature of the ACRR's fuel was estimated using the analytic model described in section 8.3.2.2 of this report. This change was necessary because of the time delay associated with obtaining actual measurements of the fuel temperature.

The combined effect of the sensors with overlapping ranges and of the use of the analytic temperature model was to permit the reactor's neutronic power to be raised by many orders of magnitude despite the presence of negative reactivity feedback effects. The shift to a variable time step significantly reduced the scatter in the measured data.

11.3 Open-Loop and Preliminary Closed-Loop Trials

The equipment used in conjunction with the testing of the MIT-SNL control methodologies on the Annular Core Research Reactor was not permanently installed at that facility. Instead, both the computers and the associated interfaces to the ACRR's instruments and control devices were brought to the facility and assembled prior to each series of experiments. Accordingly, it was necessary to conduct open-loop trials to verify the operability of the system prior to the beginning of each set of experiments. Relative to the work conducted in December 1987, open-loop trials were performed to verify both the operability of the data acquisition system and the functioning of the software. (Note: The software had, of course, also been tested rigorously using the off-line simulator described in section 9.3 of this report.)

Following completion of open-loop testing, several closed-loop runs were performed in which the results of the experiments conducted in October 1987 were duplicated.

11.4 Power Increases of Three Orders of Magnitude

Power increases of three decades had been attempted as part of the experiments performed in October 1987. However, as a result of the poor quality of the neutronic power signal at low power, it had only been possible to demonstrate the Non-Linear Digital Controller in this capacity and even those tests had not been entirely successful. Signal quality had been much improved for the December experiments and

it was possible to use both the MIT-SNL minimum time laws and the Non-Linear Digital Controller for the purpose of raising the ACRR's neutronic power by three orders of magnitude.

11.4.1 Performance of the MIT-SNL Minimum Time Laws

Figure 11.4.1-1 shows the power and reactivity profiles for a power increase from 500 W to 500 kW accomplished using the alternate minimum time law on a ten second period. The transient was completed without overshoot within the allotted time of 68 seconds. Figures 11.4.1-2 and 11.4.1-3 give the same information for both the alternate and standard laws for the same transient, only this time conducted on a period of one second. Both transients were completed within the allotted time of 6.8 seconds and essentially without overshoot. It is of interest to compare the curves shown in these figures. The power profiles are essentially identical. This is to be expected given that the standard and alternate forms of the MIT-SNL laws are mathematically equivalent. That is, even though the component terms in each law are different, they sum to the same result. In contrast, the reactivity profiles, while similar in shape, do differ in magnitude. When using the alternate law, about 800 mbeta was added. However, with the standard law, 880 mbeta was inserted. Why the difference? The reactivity shown is that associated with the ACRR's transient rod bank. The two profiles differ because of the effects of negative reactivity feedback associated with the fuel temperature. The run using the standard law was performed last when the core and its surroundings were already at a somewhat elevated temperature and their capability to store additional energy limited. As a result, the fuel temperature rose more during this run and it was necessary for the ACRR's transient rod bank to be withdrawn further in order to establish the requisite period.

Apparent in Figures 11.4.1-1 to 11.4.1-3 are the characteristic features associated with use of the MIT-SNL laws. These include the abrupt changes in slope of the power profile upon both transient initiation and termination, the exponential shape of the power profile, the rapid insertion of reactivity upon transient initiation, the gradual cessation of reactivity addition once delayed neutron effects become appreciable, and the rapid removal of reactivity to terminate the transient. Also apparent in these figures is the impact of the Doppler effect. Specifically, the ACRR is not equipped for forced-circulation heat removal. Thus, even at 500 kW, the fuel temperature will increase significantly. This in turn causes Doppler broadening of the resonance absorption peaks in the U-238 that is present in the ACRR fuel. As a result, negative reactivity is inserted. Maintenance of steady-state power at 500 kW therefore requires the withdrawal of the control devices. This is evident in all three figures, particularly so in Figure 11.4.1-1 after the 75 second mark.

11.4.2 Performance of the MIT-CSDL Non-Linear Digital Controller

Figures 11.4.2-1 and 11.4.2-2 show the power and reactivity profiles obtained when using the standard and alternate versions of the

Figure 11.4.1-1 Power Increase of Three Decades on the Annular Core Research Reactor Using the Alternate MIT-SNL Law: Ten Second Period

Figure 11.4.1-2 Power Increase of Three Decades on the Annular
Core Research Reactor Using the Alternate
MIT-SNL Law: One Second Period

Figure 11.4.1-3 Power Increase of Three Decades on the Annular Core Research Reactor Using the Standard MIT-SNL Law: One Second Period

Figure 11.4.2-1 Power and Reactivity Profiles Associated with the Standard Non-Linear Digital Controller: Power Increase of Three Decades with a Reactivity Limit of 900 mbeta

Figure 11.4.2-2 Power and Reactivity Profiles Associated with the Alternate Non-Linear Digital Controller: Power Increase of Three Decades with a Reactivity Limit of 900 mbeta

Non-Linear Digital Controller to raise the ACRR's neutronic power by three decades, 500 W - 500 kW. Both transients were completed in about 5.6 seconds. For both runs, there was an externally-imposed restriction on the allowed reactivity of 900 mbeta and both forms of the controller caused the ACRR's transient rod bank to be continuously withdrawn until this limit had been attained. The behavior of the standard and alternate versions of the NLDC then differed. For the standard version, the reactivity was kept at 900 mbeta until the neutronic power was raised to 500 kW. The ACRR's transient rod bank was then inserted so as to generate a negative rate of change of reactivity that was sufficient to halt the transient. For the alternate version, the sufficient constraint became limiting prior to the attainment of full power. As a result, reactivity was already being removed when the specified power level of 500 kW was reached. Hence, there was less overshoot associated with the alternate NLDC than with the standard version. For both versions, the scatter in the data obtained during both runs was substantial. As previously explained, this was the result of the 'on-off' control action associated with use of a supervisory constraint in conjunction with a control law of intentionally poor design. If the constraint was satisfied, the transient rod bank was withdrawn. If not, it was inserted. The result was the high frequency fluctuations in the neutronic power that are evident in the figures. When the reactivity constraints are used with a well-designed control law, this problem does not occur.

11.4.3 Importance of Signal Validation

Figure 11.4.3-1 illustrates the importance of using validated signals. Shown is a strip chart recording of the logarithm of two power signals obtained during the power increase of three orders of magnitude that was accomplished on the ACRR using the standard MIT-SNL Period-Generated Minimum Time Control Law. (Refer to Figure 11.4.1-3.) The specified period was 1.0 second. Power was increased from 0.57 kW to 500 kW in 6.73 seconds. Initially, the sensor on the right was on-scale while that on the left was off-scale low with the reactor's neutronic power at 500 W. As the power increased, the sensor on the right saturated and failed while that on the left became functional. The software was programmed to recognize this. As a result, the power increase was completed properly. Had some means of signal validation not been available, the controller would have withdrawn the ACRR's transient rod bank continuously in a vain effort to raise the power as seen by the saturated sensor.

11.5 Preparatory Runs for Power Increases of Many Orders of Magnitude

The next step in the MIT-SNL experimental program was to perform preparatory studies that would lead to power increases of many orders of magnitude. Such runs involved much more than a mere extension of the work completed earlier in which use of the MIT-SNL laws had been gradually extended from power doubling to increases of a factor of a thousand. For example, the achievement of power increases of five orders of magnitude means that the ACRR's neutronic power would be raised to 50 MW and held at that level until a shutdown was necessi-

Experimental Data
Annular Core Research Reactor
ACR007 - 12/03/87

Figure 11.4.3-1 Importance of Signal Validation

tated by either energy or temperature considerations. (Note: Recall that, as discussed in section 7.3 of this report, ACRR operation is limited by the energy produced per run and by the fuel temperature. The power level itself is not a restriction.) At 50 MW, heating of the ACRR's fuel is significant and temperature rises of many hundreds of degrees are to be expected. Hence, negative reactivity feedback associated with the Doppler effect (section 3.4 of this report) is appreciable. This in turn complicates the implementation of the MIT-SNL laws. In particular, two issues arise. The first is to position the reactor's control devices so that the period can readily be stepped to its specified value while at the same time ensuring that there will be sufficient excess positive reactivity to compensate for the negative reactivity feedback associated with the rising fuel temperature. The second is to calibrate the analytic models that are used to estimate the time-dependent behavior of various parameters such as the fuel temperature. Presented in the ensuing two sections of this report are the results of experimental runs that illustrate the difficulties inherent in these two tasks. Most of these runs were only partially successful. They have been included here to emphasize the significance of the issues involved to the proper functioning of a reactor controller. In particular, the importance of integrating the design of a reactor's control system with that of its core should be evident. Also to be noted is the need for iterative experimental evaluations to calibrate plant models.

11.5.1 Importance of the Available Differential Reactivity Worth

The objective of the MIT-SNL laws is to determine the rate of change of reactivity that will cause a reactor's neutronic power to vary on a specified period. The decisions of these laws are most readily implemented if the differential reactivity worth of the associated control device is at a maximum. The term 'differential reactivity worth' refers to the change in reactivity per unit of travel of the control device. In effect it is a spatial derivative. The quantity determined by the MIT-SNL laws is of course a time-derivative. The two are related by the speed at which the control device can be moved. Thus, denoting the required rate of reactivity change as calculated by the MIT-SNL laws by the symbol $\dot{\rho}_c$, we have:

$$\begin{aligned}\dot{\rho}_c &= d\rho/dt \\ &= (d\rho/dx)(dx/dt) \qquad (11.5.1-1)\end{aligned}$$

Clearly, if the control device's differential reactivity worth, $d\rho/dx$, is large, then the speed at which the control device must be moved, dx/dt , can be minimized. So doing is desirable because it will reduce wear on the variable speed stepper motors. In contrast, if the differential worth is low, then the stepper motor must be operated at high frequency. Not only will this accelerate wear but it also risks the possibility of causing the motor to stall.

Given the above, it is apparent that one of the objectives in designing a reactor for use in space should be to maximize the

differential reactivity worth of the control devices over as wide a range of travel as possible. For example, one means of accomplishing this would be to design the core so as to have a uniform axial flux shape. This could be done by varying the enrichment of the fuel in the longitudinal direction. This concept of varying enrichments is routinely used to flatten the radial flux profiles of commercial reactors and thereby achieve a more uniform power distribution. However, the use of longitudinally-varying enrichments is not a common practice. Accordingly, it is likely that the differential reactivity worths of the control devices used on the reactors now being designed for space missions will peak at the core mid-plane. This occurs because, in the absence of any effort to make the flux distribution uniform, the axial flux profile will be cosine-shaped. Given that a control device's differential reactivity worth is roughly proportional to the square of the neutron flux, it too will be cosine-shaped and peak at the core center. Such behavior is evident in Figure 11.5.1-1 which shows the differential and integral reactivity worths of the ACRR's transient rod bank. Note that the differential curve does go through a maximum at 4500 units which is close to the center-line of the core.

Virtually every experimental run described thus far in this report was initiated with the ACRR's transient rod bank positioned in the range 4000 - 4500 units. This was done to take advantage of the large magnitude of the differential reactivity worth available at these positions. The performance of power increases of many orders of magnitude conflicted with this strategy for the initial placement of the transient rod bank. Specifically, further examination of Figure 11.5.1-1 shows that at 4500 units, the available integral reactivity worth of the ACRR's transient rod bank is only 2.40 beta. That is, withdrawal of the rod bank from 4500 units to the full-out position would only insert 2.40 beta. In contrast, were the bank initially positioned at 2000 units, then 4.40 beta would be available for insertion. Thus, the need to provide reactivity to compensate for the effects of rising fuel temperature suggests that the control devices should initially be positioned as low as possible in the core. Unfortunately, so doing will result in an available differential reactivity worth that is nearly zero. Hence, for rapid transients of many orders of magnitude, there is a conflict between the need for immediate attainment of the specified period and the capability to provide compensation for negative reactivity feedback effects.

Figures 11.5.1-2 and 11.5.1-3 illustrate the importance of the available differential reactivity worth to the overall conduct of a transient. Shown in the first figure are the power and reactivity profiles of a transient in which the power was raised from 0.51 kW to 500 kW on a 1.0 second period. The allotted time was 6.9 seconds. The transient was completed within this time and essentially without overshoot. The same information is shown for the second run, which was similar in objective to the first. The ACRR's neutronic power was to be raised from 1.6 kW to 500 kW on a 1.0 second period. The allotted time was 5.7 seconds. Note that the actual completion time was about 10.0 seconds and that the power settled out at 460 kW. What

Figure 11.5.1-1 ACRR Transient Rod Bank Integral and Differential Reactivity Worths

Figure 11.5.1-2 Power Increase from 0.51 kW to 500 kW on a One Second Period Using Alternate MIT-SNL Law: Transient Rods Positioned at Peak Differential Worth

Figure 11.5.1-3 Power Increase from 0.51 kW to 500 kW on a One Second Period Using Alternate MIT-SNL Law: Transient Rods Positioned at Low Differential Worth

caused the difference between the two runs? In regards to the former, the transient was initiated with the ACRR's transient rod bank positioned at 4500 units. As for the latter, the rod bank was at 3000 units. For both cases the maximum allowed stepper motor frequency was 300 Hz. Examination of Figure 11.5.1-1 shows that the available differential reactivity worths associated with bank positions of 4500 and 3000 units are 1.3 and 0.65 mbeta per position unit respectively. (Note: The units given in the figure are those used at the ACRR. A quantity expressed in 10000 cents/position unit and given in increments of a thousand is equivalent to a mbeta per position unit.) Given that there are 2.661 position units of travel per pulse and that the stepper motors were set to provide 300 pulses per second, the available differential reactivity worths were 1037 and 439 mbeta per second respectively. Were these rates adequate? Neglecting prompt effects which were small for these runs, the alternate MIT-SNL law is:

$$\dot{\rho}_c(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda'_e(t)\rho(t) - \sum \bar{\beta}_i(\lambda_i - \lambda'_e(t)) \quad (11.5.1-2)$$

If the reactor is initially at equilibrium, then the above expression further reduces to:

$$\dot{\rho}_c(t) = \bar{\beta}\omega(t) \quad (11.5.1-3)$$

Hence, the minimum rate of reactivity change needed in order to step a reactor period from infinity to 1.0 second is 1000 mbeta per second. For the first of the two runs under discussion, this rate of reactivity change was attainable. For the second, it was not. Thus, the transient shown in Figure 11.5.1-3 required almost 10 seconds for completion instead of the expected 5.7 because the initial position of the control rod bank was so low that the required rate of reactivity change could not be generated. Delays in attaining the targeted power level often exacerbate other problems such as errors in the system model. These in turn may cause, as happened here, the controller to settle out at the wrong power level.

11.5.2 Importance of the Available Integral Reactivity

The experiments reported in the previous section of this report showed the importance of initiating a transient with the control device positioned so as to achieve the maximum available differential reactivity worth. As discussed in the introduction to section 11.5, the counterargument is that the control devices should be placed near their 'full-in' position so as to be capable of providing as much compensation as possible for negative reactivity feedback effects. If this is not done, then it will not be possible to hold the reactor power at the targeted value for very long. No runs were performed for the specific purpose of illustrating this effect. However, it is apparent from the runs shown in section 11.7 of this report that the provision of reactivity to compensate for feedback effects is of importance.

11.5.3 Importance of Accurate System Measurements and Models

The MIT-SNL laws are model-based in the sense that the reactor to which they are being applied must be accurately characterized in order for the laws to be effective. In particular, information is needed on the net reactivity present in the core and on the distribution of the delayed neutron precursors. Lacking these, the rate of change of reactivity needed to maintain the specified period will not be calculated correctly.

Figure 11.5.3-1 shows the power and reactivity profiles obtained from the only run performed without any use of feedback. That is, for this run, the quantity $\omega(t)$ in the MIT-SNL laws was not updated at each sampling interval as described in section 5.2.1 of this report. Rather it was set to the period specified for the overall conduct of the transient and changed only upon attainment of the target power level. As a result, modeling deficiencies and measurement errors became glaringly apparent. (Note: The reactivity shown is the net reactivity calculated as present by the controller.)

The objective of the run was to raise the ACRR's neutronic power from 1.25 kW to 1000 kW on a 1.0 second period. The allotted time was 6.67 seconds. There were two shortcomings in the performance of this run. First, the actual completion time for the transient was 11.8 seconds. The ACRR's transient rod bank had been positioned at 4500 units, the region of peak differential worth, at the outset of the transient. So, inability to achieve the specified period was not the cause of the delay in completion. Rather, the origin of the problem was an inaccuracy in the determination of the fuel temperature. Two methods were available. One was to measure the temperature using a thermocouple. The other was to calculate it using an analytic model. The former involved a significant time delay. The latter was difficult to calibrate because the ACRR only operates in a pulsed mode when on rapid transients. As a result both the fuel temperature and the negative reactivity associated with the rise in that temperature were underestimated. Hence, the reactivity present in the core was overestimated. What effect did this have on the MIT-SNL law? Ignoring the prompt terms, the alternate law is of the form:

$$\dot{\rho}_c(t) = (\bar{\beta} - \rho(t))\omega(t) - \lambda'_e(t)\rho(t) - \sum \bar{\beta}_i(\lambda_i - \lambda'_i(t)) \quad (11.5.3-1)$$

The target power of 1000 kW was to be attained at 6.67 seconds. Substituting the numerical values recorded by the data acquisition system at that moment (6.67 seconds) shows that the control law was not signaling for further withdrawal of the control devices even though the ACRR's power was only 75 kW, far below the specified value of 1000 kW. Specifically,

Figure 11.5.3-1 Power Increase from 1.3 kW to 1000 kW on a One Second
Period Using Alternate MIT-SNL Law: No Use of
Feedback

$$\begin{aligned}
 \dot{\rho}_c(t) &= (1000-771)(1.0)-(0.86)(771)-(-424) \\
 &= \quad (229) \quad - \quad (663) \quad + (424) \\
 &= -10 \\
 &\approx 0
 \end{aligned}
 \tag{11.5.3-2}$$

where $\dot{\rho}_c(t)$ is in units of millibeta per second.

It is believed that the true value of the net reactivity was actually about 500 mbeta at this time. Had that figure been used in the control law, then the quantity $\dot{\rho}_c(t)$ would have been about 200 mbeta/second.

A second performance weakness was that the reactor's neutronic power was not held at the specified level of 1000 kW. Rather, as soon as the target power had been attained, the controller began to continuously insert the control devices thereby driving down the power. As before, the problem was an error in the estimation of the fuel temperature which caused the net reactivity to be overestimated. Specifically, once the target power level had been attained, the quantity $\omega(t)$ was set to zero. This was done because maintenance of power at a given level implies that the specified reactor period should be infinite. Hence, the control law became:

$$\dot{\rho}_c(t) = \lambda'_e(t)\rho(t) - \Sigma \bar{\beta}_i(\lambda_i - \lambda'_e(t))
 \tag{11.5.3-3}$$

Substituting the numerical values recorded at 11.95 seconds, which was the start of the power decrease, yields:

$$\begin{aligned}
 \dot{\rho}_c(t) &= -(0.837)(945)-(-395) \\
 &= -396 \text{ mbeta/second}
 \end{aligned}$$

Had the correct value of the reactivity (≈ 500 mbeta) been used, the appropriate zero rate of change of reactivity would have been requested.

Following completion of the run shown in Figure 11.5.3-1, the numerical values of the parameters used in the analytic model of the fuel temperature were revised and further evaluations conducted. It should be understood that the problem of model calibration is not specific to the MIT-SNL laws. The laws themselves are generic in that they are derived from the equations that describe reactor dynamics. However, their application to a particular facility requires the accurate characterization of that facility. This problem of model calibration is germane to the use of any model-based control law. Of significance, is that, after much effort, the MIT-SNL experiment team surmounted this problem and was able to demonstrate the use of the MIT-SNL laws for power increases of five orders of magnitude. Model calibration and the accurate, reliable measurement of signals are non-trivial issues that are too often overlooked in the development of control techniques. A valuable discussion of the need to design control systems that allow for imperfections in system models has recent-

ly been given by Astrom [28].

11.6 Power Increases of Four Orders of Magnitude

Figures 11.6-1 and 11.6-2 show the power and reactivity profiles obtained during power increases of four orders of magnitude, 2 kW - 20 MW. Both runs were conducted on a specified period of 1.0 second and, in each case, the allotted time was 9.3 seconds. The run shown in Figure 11.6-1 used the improved version of the analytic model of the fuel temperature while that given in Figure 11.6-2 used either the thermocouple measurement or the output of the model, whichever was of greater magnitude. Both runs exhibited reasonable completion times. However, in each case, the power was leveled at about 18 MW rather than at the specified value of 20 MW. Also, while in both cases, the 18 MW power level was held for several seconds, control device withdrawal was not continued sufficiently to avert an eventual power decrease. These problems were the result of imperfections still present in the analytic model of the fuel temperature. Following these two runs, the numerical values of the coefficients in that model were further revised and power increases of five orders of magnitude were successfully performed.

11.7 Power Increases of Five Orders of Magnitude

Power increases of five orders of magnitude were conducted using the MIT-SNL Period-Generated Minimum Time Control Laws and the MIT-CSDL Non-Linear Digital Controller on the ACRR on December 4th 1987.

11.7.1 Performance of the MIT-SNL Minimum Time Laws

Figure 11.7.1-1 is a logarithmic plot of the power profile obtained from a power increase of five orders of magnitude performed using the alternate MIT-SNL law. The ACRR's neutronic power was raised from 0.54 kW to 50 MW, a full five orders of magnitude, on a constant period of 0.50 seconds. The solid line depicts the actual transient while the broken one is the trajectory that would be expected from theory. The controller experienced some difficulty in establishing the requisite period at the outset of the transient. The cause of this problem was originally reported to have been an error in the reactivity balance [29]. This was suspected because achievement of a condition of exact criticality at low power on the ACRR is difficult because of the strong photoneutron source associated with the $\text{UO}_2\text{-BeO}$ fuel. However, further analysis has suggested that the problem may have been that the stepper motor frequency was limited to 400 Hz during this run. Given that the ACRR's transient rod bank was initially positioned at 4500 units and recalling that application of one pulse to an ACRR stepper motor results in travel of 2.661 position units, it is evident from Figure 11.5.1-1 that the available rate of change of reactivity was 1383 mbeta/second. Solving the MIT-SNL law with prompt effects included shows that the rate of reactivity insertion needed to step the reactor period from infinity to 0.50 seconds was 2038 mbeta/second. Hence, at the outset of the transient, the control devices were being withdrawn too slowly. Further calcula-

Figure 11.6-1 Power Increase of Four Decades on the Annular
Core Research Reactor Using the Alternate
MIT-SNL Law on a One Second Period: Fuel
Temperature Calculated Using Analytic Model

Figure 11.6-2 Power Increase of Four Decades on the Annular
Core Research Reactor Using the Alternate
MIT-SNL Law on a One Second Period: Fuel
Temperature Estimated Using Higher of Analytic
Model or Thermocouple

Figure 11.7.1-1 Closed-Loop Power Increase of Five Orders of Magnitude (540 W to 50 MW) Using Alternate MIT-SNL Period-Generated Minimum Time Control Law on a 0.50-Second Period in 5.83 Seconds: Experimental Data (ACR029) Obtained 4 December 1987 on the Annular Core Research Reactor

tions, similar to those in Chapter Six of this report, show that the requisite rate of reactivity change dropped below that available from the ACRR's stepper motors within a few tenths of a second. Of significance is that the controller was sufficiently robust to overcome this difficulty and, in all other respects, to conduct the transient in the prescribed manner. Specifically, a period of 0.50 seconds was established and maintained. Completion time for the transient was 5.83 seconds which compared favorably with the theoretically-expected value of 5.72 seconds. Also, there was no overshoot. The power peaked at 50.5 MW, well within the permitted 5% band. Upon leveling the power at 50 MW, the controller was able to hold that power for about 0.75 seconds. The power subsequently began to decrease as the result of negative reactivity feedback associated with the temperature rise of the ACRR's fuel. There was not enough reactivity available to the controller to provide the necessary compensation.

Figure 11.7.1-2 shows the profiles of the ACRR's neutronic power and transient rod bank position during this run. Bank position is shown instead of reactivity in order to emphasize the difficulty in providing sufficient positive reactivity to compensate for the negative reactivity feedback associated with the rising fuel temperature. Note that the control rod bank was being continuously withdrawn even before the attainment of 50 MW. Nevertheless, it was not possible to hold the power at 50 MW. Had the bank's initial position been lower in the core, perhaps 3000 units instead of 4500, then additional reactivity compensation could have been provided. However, to have done so would have exacerbated the problem experienced in achieving the requisite rate of reactivity insertion upon transient initiation.

Four other power increases of five orders of magnitude were performed using the MIT-SNL minimum time laws in order to be certain that the maneuver was repeatable. All were performed satisfactorily. Figure 11.7.1-3 is the strip chart recording of the power signals obtained during one of these runs. The starting position for the transient rod bank was 3000 units. Although there was some oscillation in the power level at 50 MW, it is evident from the chart recording, that the ACRR's neutronic power was raised on a constant period, that it was leveled at 50 MW essentially without overshoot, and that there was sufficient excess reactivity to allow the power to be held at 50 MW for 2.5 seconds. Note the rather sharp cornering evident in the recording when the power reached 50 MW. Also note the slight downward concavity in the power trace upon transient initiation. This curvature indicates that the power increase was being performed less rapidly than specified. The scale of the chart recording is logarithmic and if the period had been a constant, the trace would have been linear. The restriction on stepper motor frequency and the low (3000 unit) initial position of the rod bank precluded immediate attainment of the desired period. In addition to documenting the capability of MIT-SNL laws to raise neutronic power by five orders of magnitude on a 0.50 second period, this run also illustrated the importance of signal validation. As was the case with the strip chart recording shown in Figure 11.4.3-1, the sensor on the right was on-scale at the outset of the transient while that on the left was off-scale low. As the power

Figure 11.7.1-2 Power and Transient Rod Bank Position for
Power Increase of Five Decades Shown in
Figure 11.7.1-1

Experimental Data
Annular Core Research Reactor
ACR035 - 12/04/87

Figure 11.7.1-3 Strip Chart Recording for Power Increase of
Five Orders of Magnitude

rose, the sensor on the left became functional while that on the right saturated and failed. The signal validation routine was programmed to identify changes of this type and to supply the controller with an accurate estimate of the reactor power. Had it not done so, the controller would have continuously withdrawn the rod bank in a vain effort to raise the signal of the saturated detector.

These experimental runs showed that the MIT-SNL Period-Generated Minimum Time Control Laws fulfilled the prime requirement of the space nuclear effort. Namely, these laws were capable of adjusting reactor neutronic power by many orders of magnitude in a few seconds without overshoot.

11.7.2 Performance of the MIT-CSDL Non-Linear Digital Controller

One power increase to 50 MW was performed using the Non-Linear Digital Controller. The ACRR's neutronic power was raised from 1.6 kW to 50 MW. Ideally, the initial power level would have been 500 W as it had been for the experiments involving the minimum time laws. However, the time available for the conduct of the MIT-SNL experiments on the ACRR was limited and as a result of the strong photoneutron source produced by the ACRR's $\text{UO}_2\text{-BeO}$ fuel, considerable time is required to return the ACRR's neutronic power to 500 W following operation at 50 MW. Accordingly, this NLDC run was initiated at a power level of 1.6 kW. The starting position of the transient rod bank was 3000 units. The run was subject to an external restriction of 900 mbeta on the net reactivity. The completion time for the transient was 9.2 seconds. Figure 11.7.2-1 shows the profiles of the ACRR's neutronic power and rod bank position during this run. Power peaked at 56 MW and then oscillated about that level for about a second. While this run was successful, the results were not as satisfying as those achieved with the minimum time laws. For example, the completion time for the run shown in Figure 11.7.1-3 was 5.28 seconds and power was held at 50 MW for 2.5 seconds. In general, the NLDC's completion times are shorter than those of the minimum time laws. The discrepancy observed in this case is believed to have occurred because the ACRR's fuel was still at elevated temperature when the run began. This caused an error in the reactivity balance which resulted in less withdrawal of the transient rod bank than should have been the case. Another deficiency was the oscillatory behavior while at steady-state. Again, this was the result of the 'on-off' nature of the NLDC's supervisory constraint.

11.8 Power Decreases

Controlled power decreases are in many respects more challenging than increases. The difficulty arises because of the effect of the delayed neutron precursors on a reactor's dynamic behavior. Specifically, in the absence of control blade movement, a reactor's neutronic power can not be reduced at a rate faster than that at which the longest-lived delayed neutron precursor group decays. The half-life of these precursors is 55 seconds. This corresponds to a reactor period of negative 80 seconds.

Figure 11.7.2-1 Power Increase from 1.6 kW to 50 MW Using Standard Non-Linear Digital Controller

One of the advantages to the use of the MIT-SNL laws is that they permit the performance of controlled power decreases on periods faster than -80 seconds. This is accomplished by determining the rate of removal of reactivity needed to maintain the specified period. The result is that the prompt neutron population is being decreased at a rate faster than that specified so that when prompt and delayed neutron contributions are averaged, the net effect will be a power decrease at the specified rate. As was noted in section 6.5 of this report, the implementation of this approach for power decreases of many orders of magnitude would require enormous negative rates of change of reactivity. Reactor control systems capable of providing such rates would not be practical to build. Hence, while reactor power can be decreased in a controlled manner on periods faster than -80 seconds, the reality is that such rates can only be sustained for power decreases of a factor of five to perhaps a factor of ten.

Figure 11.8-1 shows the power and reactivity profiles from an experimental run in which the alternate MIT-SNL minimum time law was to be used to reduce the ACRR's power from 55 kW to 5 kW on a -10 second period. The transient rod bank was initially at 4500 units. The allotted time for completion of the transient was 23.8 seconds. As is evident from the figure, the power was decreased on the specified period for the first 20 seconds. At that time, further insertion of the ACRR's transient rod bank was stopped by an in-limit switch. As a result, it was not possible to maintain the -10 second period and the period settled out, as expected, at -80 seconds. Of significance in this figure is the curvature of the reactivity profile. It is concave downwards indicating that an ever-increasing rate of reactivity removal was needed to sustain the power decrease.

Following the unsuccessful completion of the run shown in Figure 11.8-1, the ACRR's transient rod bank was reshimmed so that the reactor was critical at 55 kW with the bank positioned at 5300 units. The alternate control law was then used to lower power from 55 kW to 10 kW on a -10 second period. The resulting power and reactivity profiles are shown in Figure 11.8-2. The transient was successfully completed in the allotted time of 17.1 seconds. Of interest here is the rapid withdrawal of the control devices that was necessary to hold the power constant at 10 kW. Figure 11.8-3 is the strip chart recording (logarithmic scale) that corresponds to this figure. Note the sharp corners corresponding to transient initiation and termination. Also note that the power trace is a straight line showing that the period was in fact maintained at a constant value.

11.9 Power Cycling

Figure 11.9-1 shows the strip chart recording (logarithmic scale) of the power signal obtained during an experimental trial in which the ACRR's power was repeatedly cycled between 5.8 kW and 25 kW. Power increases were performed on 1.0-second periods and decreases on ones of negative 10.0 seconds. A hold time of 30.0 seconds was specified between each power change. The features apparent from the earlier trials are also evident here. The transients were completed within

Figure 11.8-1 Power Decrease from 55 kW to 5 kW Using Alternate MIT-SNL Law: Transients Rods Initially at Mid-Core Position

Figure 11.8-2 Power Decrease from 55 kW to 10 kW Using Alternate MIT-SNL Law: Transients Rods Initially at Upper-Core Position

Experimental Data
Annular Core Research Reactor
ACR042 - 12/04/87

Figure 11.8-3 Strip Chart Recording for Power Decrease Shown
In Figure 11.8-2

Experimental Data
Annular Core Research Reactor
ACR043 - 12/04/87

Figure 11.9-1 Strip Chart Recording Showing Power Cycling

the appropriate times, there were no overshoots or undershoots, and there was no oscillatory behavior. Figure 11.9-2 shows the power and reactivity profiles obtained during a portion of this run. The special significance of this trial was that the capability of the MIT-SNL period-generated minimum time control laws to continuously cycle the power in a controlled manner was established.

11.10 Demonstration of Sufficient Reactivity Constraints

The primary purpose of the MIT-SNL control experiments conducted in December 1987 was to evaluate the MIT-SNL minimum time laws. However, some time was allocated for testing of the MIT-CSDL Non-Linear Digital Controller (NLDC). This was used to demonstrate the effectiveness of the standard and alternate sufficient reactivity constraints in precluding power overshoots.

Extensive trials of the NLDC had been conducted during October 1987. However, as discussed in Chapter Ten of this report, only a few of those runs had resulted in a transition from an absolute to a sufficient reactivity constraint. Moreover, of those runs that did exhibit the transition, all were of the alternate form of the NLDC. Accordingly, it was decided to design an experiment in which the efficacy of the standard sufficient reactivity constraint would be apparent.

The standard absolute and sufficient reactivity constraints are of the form:

$$\rho(t) \leq |\dot{\rho}_c|/\lambda_e(t) \quad (11.10-1)$$

$$[\rho(t) - |\dot{\rho}_c|/\lambda_e(t)]/|\dot{\rho}_c| \leq \tau(t)\ln(P_F/P(t)) \quad (11.10-2)$$

where all symbols are as previously defined. (Refer to section 5.1.1 of this report.) The quantity $|\dot{\rho}_c|$ is the maximum rate of change of reactivity that could be obtained were a control device to be moved. As such, it is always a non-zero quantity regardless of whether or not a control device is actually being moved. If, as is the case with the ACRR, the available differential reactivity worth of the control devices is of large magnitude, then the quantity $|\dot{\rho}_c|$ will also be large. The two are related by the maximum available speed at which the control devices can be moved. That is,

$$|\dot{\rho}_c| = (d\rho/dt)(dx/dt)|_{\max} \quad (11.10-3)$$

For all of the ACRR control experiments described thus far in this report, the quantity $|\dot{\rho}_c|/\lambda_e(t)$ has far exceeded the net reactivity present in the reactor. Hence, control has always remained within the bounds of the standard, absolute constraint as given by equation 11.10-1. In fact, for only a few experiments did the reactivity exceed the bounds of the alternate, absolute constraint which is of the form $|\dot{\rho}_c|/\lambda_e'(t)$. Accordingly, in an effort to force a demonstration of the standard and alternate sufficient forms of the reactivity con-

Specifications	
-	Power Cycling: 5.8-25-5.8-25 kW
-	Specified Periods: 1 s/-10 s
-	Alternate MIT-SNL Law

Figure 11.9-2 Power and Reactivity Profiles for Power Cycling Run

straints, it was decided to severely restrict the maximum available speed of the control devices. This was done by limiting the frequency of the stepper motor drives to 25 Hz.

Figure 11.10-1 is the strip chart recording of a power increase from 1.3 kW to 15 kW performed using the standard version of the Non-Linear Digital Controller. The transient rod bank's stepper motor frequency was limited to 25 Hz and the bank was positioned at 3000 units. Both actions had the effect of reducing the maximum available rate of change of reactivity. Figure 11.10-2 provides the corresponding power and reactivity profiles. Note that the transient was completed without overshoot in 31 seconds. Examination of the computer generated data associated with this run shows that a transition to the standard, sufficient constraint did occur at about 12 seconds. The quantity $|\dot{\rho}_c|/\lambda_e(t)$ was approximately 500 mbeta at this time which, as shown in Figure 11.10-2, equaled the reactivity present in the core. The transition had, as expected, no visible effect on the power or reactivity profiles. At about 15.5 seconds, the inequality that forms the standard sufficient constraint was no longer satisfied and reactivity removal was initiated. The effect of this change is clearly visible in both figures. There is a discontinuity in the slope of the power profile evident in the strip chart recording and in the profile itself. The decision to commence reactivity removal at 15.5 seconds illustrates the unique advantage of the Non-Linear Digital Controller. It provides a means for determining the appropriate time during a transient for the reversal of rod motion so that there will not be a power overshoot at some future time. Moreover, it does this without the use of predictive models. Once reactivity removal was initiated, it was continued in a stepwise manner so as to allow the power to continue to increase while at the same time ensuring that the transient would be slow enough so that it could be halted by the dynamic effect of inserting the control device. Note that upon attainment of 15 kW, there was substantial positive reactivity still present. Physically, the delayed neutron population was still rising because it had not yet achieved equilibrium at 25 kW. The power was leveled by inserting the control rods thereby making negative the contribution of the prompt neutrons to the power increase.

Figures 11.10-3 and 11.10-4 provide the same information for a power increase from 1.4 kW to 15 kW conducted using the alternate version of the Non-Linear Digital Controller. Stepper motor frequency was again restricted to 25 Hz and the rod bank positioned at 3000 units. The transition from an absolute to a sufficient constraint occurs at a much lower reactivity when using the alternate form of the NLDC because the alternate effective multi-group decay parameter (λ'_e) is about four times that of the standard one (λ_e). Hence, the transition occurred about 2.5 seconds into the run when about 100 mbeta was present. Again, there was no effect on the power trace. At 23 seconds, with 460 mbeta present, the alternate sufficient constraint was no longer satisfied and reactivity removal was initiated. This action did cause a change in the slope of the power profile. Subsequently, reactivity was gradually removed so as to allow the power to continue to rise towards the target level of 15 kW but nevertheless at

Figure 11.10-1 Power Increase from 1.3 kW to 15 kW Using Standard Non-Linear Digital Controller with Rod Bank Motor Frequency Restricted so as to Activate Sufficient Constraint

Specifications	
-	Power Increase: 1.3-15 kW
-	Limit on Motor Frequency: 25 Hz
-	Standard NLDC

Figure 11.10-2 Power and Reactivity Profiles for NLDC Transient Illustrating Role of Standard Sufficient Constraint (Figure 11.10-1)

Experimental Data
Annular Core Research Reactor
ACR040 - 12/04/87

Figure 11.10-3 Power Increase from 1.3 kW to 15 kW Using Alternate Non-Linear Digital Controller with Rod Bank Motor Frequency Restricted so as to Activate Sufficient Constraint

Figure 11.10-4 Power and Reactivity Profiles for NLDC Transient
Illustrating Role of Alternate Sufficient
Constraint (Figure 11.10-3)

a slow enough rate so that the transient could be halted by insertion of the control device upon attainment of that power. Unfortunately, for reasons unrelated to the experiment, the run was terminated prior to completion. However, it is apparent that the power was asymptotically approaching 15 kW. Comparison of this run with the one performed using the standard NLDC shows that both controllers exhibited reasonable rise times (time to attain 80% of full power) but that the alternate version was much slower to complete the transient. This is expected from theory [11].

11.11 Demonstration of the MIT-SNL Minimum Time Laws on the MIT Research Reactor

Experimental evaluations of the MIT-SNL Period-Generated Minimum Time Control Laws had begun on the 5 MWt MIT Research Reactor, MITR-II, in December 1986, only a few weeks after the realization that a time-optimal response could be achieved by using the rate of change of reactivity as the control signal. However, those and subsequent runs were performed using either the MITR-II's variable speed regulating rod or a shim blade driven by a fixed speed motor. Approval to operate one of the MIT Research Reactor's shim blades with a variable speed motor was not obtained until almost a year later.

Figure 11.11-1 shows the power and reactivity profiles that were obtained during a trial of the alternate MIT-SNL period-generated minimum time control law on the MITR-II. Also, the strip chart recording of this transient is shown as an inset. The reactor's neutronic power was increased from 1 MW to 2 MW under closed-loop, digital control using a variable speed stepper motor to drive one of the reactor's shim blades. The specified period was 100 seconds. The period was stepped to this value by initially requiring a rate of reactivity insertion of about 10 mbeta/second ($0.0078\% \Delta K/K/\text{second}$). Thus, a changing prompt neutron population as represented by the ρ term was used to initiate the transient. Then, as delayed terms grew in magnitude, the required rate of change of reactivity decreased. On reaching the specified power level, the period was stepped back to infinity by making the rate of change of reactivity negative. A decreasing prompt neutron population was therefore used to offset the effect of the still rising delayed one. The power increase then stopped. Note that the actual transit time of 70 seconds is in excellent agreement with the theoretically expected one of 69 seconds. Another feature worth noting is that as the transient neared completion, the reactivity approached its asymptotic value which is about 100 mbeta for a 100-second period on the MITR-II.

Certain of the features characteristic of the MIT-SNL laws are apparent in this figure. These include the abrupt changes of slope in the power profile that occur upon transient initiation and termination because the period is being changed discontinuously, the exponential shape of the power profile because the power is rising on a constant period, the initial rapid insertion of reactivity reflecting the need for a high rate of reactivity change to step the period to 100 seconds, the gradual cessation of reactivity during the intermediate

Figure 11.11-1 Closed-Loop Power Increase from 1 to 2 MW Using
Alternate MIT-SNL Period-Generated Minimum Time
Control Law on a 100-Second Period in 70 Seconds:
Experimental Data Obtained 30 December 1987 on
the MIT Research Reactor

and final stages of the transient indicating that delayed effects have become appreciable, and the rapid decrease in reactivity that terminates the transient by using a negative rate of change of the prompt neutrons to balance the still rising delayed neutron populations.

11.12 Demonstration of the Energy Constraint Approach

The energy constraint approach was described in section 7.3.2.1 of this report. It was used as a supervisory routine during all of the ACRR control experiments to ensure that the reactor's energy output would not exceed the limiting value of 350 MJ. No experiments were performed on the ACRR for the specific purpose of illustrating the energy constraint concept. However, several runs of that type were performed on the MITR-II and one of those is described here. This material is preceded by a brief explanation of the possible utility of the energy constraint concept to the operation of spacecraft reactors.

A major concern in the operation of any reactor is that the maximum allowable fuel temperature not be exceeded. For a terrestrial reactor, the limiting condition is usually the capability to remove heat from the fuel itself. Accordingly, these reactors are subject to a limitation on the allowed power level. The situation with regard to reactors in space is likely to be quite different. There, the limiting condition will not be heat removal from the core but rather heat rejection from the spacecraft. The problem is that the only means of heat rejection is by radiation to deep space. This process depends on temperature to the fourth power and is only appreciable at elevated temperatures. As a result, space-based reactors must be designed to function at extremely high temperatures. One consequence is that temperature-insensitive ceramic fuels will be used. This in turn means that for a reactor operating in space, any power level might be acceptable provided that the total energy production over time (i.e., the integral of the power) is less than a certain value. This mode of operation would be particularly advantageous in that the reactor could be designed so that its steady-state power level equaled the anticipated housekeeping loads. This would minimize system mass. Yet, brief evolutions that demanded significantly more power, such as changes of orbit, could also be accommodated under the provisions of the energy restriction.

Figure 11.12-1 depicts a power increase conducted on the 5 MWt MIT Research Reactor for the purpose of demonstrating the energy constraint approach. For this run, the required time was not defined as the time needed to make the reactor subcritical but rather as that required to reduce the neutronic power to the permitted steady-state level. The maximum allowed value of the energy was specified to be 150 MJ and the authorized steady-state power level was taken as 1 MW. A transient was then requested in which power was to be raised from 1 MW to 3 MW on a 50 second period using the alternate minimum time law. Shown in the upper portion of this figure are both the power and energy profiles. Shown in the lower portion are the required and available times. Initially, the power level was equal to the steady-state

Figure 11.12-1 Demonstration of Energy Constraints for Closed-Loop Reactor Control

value and the reactor period was infinite. Hence, the required time was zero and the available time was infinite. The constraint was therefore satisfied and the decision of the MIT-SNL minimum time law was implemented. The control rods were withdrawn so as to cause the power to rise on a 50 second period. The required time then began to increase because the reactor power was rising. Also, given that the reactor period was now finite and energy was being produced, the available time began to decrease. Forty-four seconds into the transient, the required time became equal to the available time. Note that this occurred when the power level was approximately 2.42 MW, well below the target value of 3.0 MW. The control law was therefore still signaling for a continued power increase. Its decision was overruled by the energy constraint and rod insertion was initiated. This caused the reactor period to lengthen and the constraint to be momentarily satisfied. However, energy continued to accumulate with the result that the constraint was again violated and the rods were further inserted. The net result was a stepwise insertion of the control rods so that the power was reduced to the steady-state value (1 MW) upon attainment of the energy limit (150 MJ). This is shown in the upper portion of the figure. This example illustrates the complementary roles of the supervisory and predictive components of the control law. The former's contribution is that it reviews the decision of the predictive component in terms of current reactor conditions and intervenes as necessary. Its deficiencies are that it lacks both the ability to project values of energy and power forward in time and the capability to cause the shape of the transient power curve to conform to a particular set of specifications. The predictive portion of the controller can provide those functions in which the supervisory program is deficient. However, it can not guarantee the safety of its own actions unless the reactor is operating within the envelope of conditions for which it was designed. Both components are necessary.

11.13 Significance of the December 1987 Experiments

The experiments conducted on the closed-loop, digital control of neutronic power during the period 3-4 December 1987 on SNL's Annular Core Research Reactor and later that month on the MIT Research Reactor proved conclusively that the MIT-SNL Period-Generated Minimum Control Laws did fulfill the primary requirement of the space-nuclear effort as regards the design of reactor control systems. Specifically, these laws were capable of adjusting a reactor's neutronic power by many orders of magnitude both on periods as short as a few tenths of a second and without overshoot. In addition, the efficacy of these laws for performing controlled power decreases and power cycling was established. Specific accomplishments associated with the December experiments were as follows:

- (1) The MIT-SNL Period-Generated Minimum Time Control Laws were shown to be effective for power increases of five decades on periods as short as 0.50 seconds.
- (2) The MIT-SNL laws were shown to be capable of performing controlled power decreases on periods as short as -10.0 seconds.

- (3) The capability of the MIT-SNL laws to repeatedly cycle the reactor's neutronic power was established.
- (4) The sufficient reactivity constraints of the Non-Linear Digital Controller were demonstrated to be an effective means of raising reactor power on a facility other than the MIT Research Reactor.
- (5) Both the MIT-SNL laws and the NLDC were shown to function properly in the presence of strong negative reactivity feedback associated with a rising fuel temperature.
- (6) The energy constraint concept was demonstrated.
- (7) It was demonstrated that a reactor's neutronic power could be safely controlled under conditions of closed-loop digital control with reactivities of as much as 950 mbeta (95 cents) present in the reactor.
- (8) The generic nature of the MIT-SNL minimum time laws was established by demonstrating them on both SNL's ACRR and the MITR-II.
- (9) The need to integrate control system design with that of core design was documented.
- (10) The need for robust methods of feedback and for the accurate modeling of reactor parameters under dynamic conditions was identified as an area requiring significant research.

As was the case with the October experiments, the most significant accomplishment of the December 1987 experiments was an increase in confidence in the technology. The December experiments constituted the culmination of the joint MIT-SNL program on reactor control as originally funded through the U.S. Department of Energy's Multi-Megawatt Program. Work in this area is now continuing with grants from other agencies as well as from other groups within the Department of Energy. Under development are methodologies for performing automated reactor startups, for enhancing the use of feedback, and for measuring reactivity.

12. Contribution of the MIT-SNL Period-Generated Minimum Time Laws to the Control of Reactor Neutronic Power

The MIT-SNL Period-Generated Minimum Time Control Laws constitute a novel approach to the control of a reactor's neutronic power in that they are non-linear, generic, closed-form expressions that utilize the rate of change of reactivity, rather than the reactivity itself, for the control signal. It is as a result of these four unique features that the MIT-SNL laws have proven to be exceptionally advantageous. In particular, they can be utilized for the control of a wide range of transients because they preserve the non-linear aspects of a reactor's dynamic behavior. Moreover, their generic nature means that they can be applied to any type of reactor and the fact that they are closed-form expressions permits on-line corrections using feedback. Finally, utilization of the rate of change of reactivity facilitates the implementation of these laws because the desired control mechanism velocity, as opposed to merely information on its required position, is specified. These and other advantages associated with the MIT-SNL laws are described as they relate to the fulfillment of the requirements for space nuclear power systems. Also discussed are specific factors relevant to the implementation of these laws, the role of the MIT-SNL laws in a controller for the autonomous operation of a spacecraft nuclear reactor, and potential applications of these laws.

12.1 Fulfillment of the Space Nuclear Mission

The National Research Council recently issued a report entitled, "Advanced Power Sources for Space Missions" [1]. This report both delineates the requirements currently imposed on the design of power sources for spacecraft and provides a comparative analysis of the various options available for meeting those requirements. Included are assessments of photovoltaic, solar-dynamic, chemical, and magneto-hydrodynamic power systems as well as radioisotope thermoelectric generators, dynamic isotope power sources, nuclear reactors, and ground-based sources that beam power into space. The report divides power requirements for space missions into three categories: house-keeping, alert, and burst. The 'housekeeping' mode includes both military and civilian applications. Electric power ranging from several kilowatts to perhaps hundreds of kilowatts would be needed continuously for durations of as long as ten years. In addition to maintaining house loads on board the spacecraft, applications include radars for both air traffic control and surveillance, tracking, and communications. The 'alert' and 'burst' modes of operation involve electric power requirements ranging from tens of megawatts to several hundred megawatts. The objective is to provide the capability to either deter potential hostile action against the United States and its allies or, if necessary, to actually intercept and destroy enemy missiles. The report rates each of the various power options in terms of its capability to fulfill the requirements of these three modes of operation. For example, nuclear reactors were noted as being advantageous because they were of low mass, were capable of providing large amounts of power for extended intervals, and their use would not result in the generation of any effluents. However, balancing these

favorable factors was concern that it might not be possible to raise a reactor's power from the housekeeping to the burst mode at the requisite rate. Specifically, Conclusion No. 4 of this report states, "the rate of rise ('ramp-rate') from zero to full burst-mode power level appears to be a critical requirement". On p. 29 of the aforesaid report, it is stated that:

"Although activation of a chemical heat source can be rapid, it may be more difficult for a fission reactor to reach full power quickly. Consequently, substantial housekeeping power may be required to maintain the power conversion cycle components in a warmed-up condition ready for rapid start-up. Bimodal operation of a nuclear power plant to supply power for both the alert and burst modes would substantially alleviate its start-up problems. The reactor's increased power output in going from alert-mode operation to burst mode could then be achieved with only modest change in its operating temperature, in contrast to the significant changes in temperature and heat output that would occur if the reactor must be activated just prior to the burst mode."

Two issues are identified in the above paragraph. The first is whether or not the reactor's neutronic power can be raised by many orders of magnitude both in a few seconds and without overshoot. The second is whether or not the power conversion cycle that transforms heat from the reactor to electrical energy can similarly respond. The first of these concerns has clearly been resolved in the affirmative by the results of the theoretical and experimental research reported here. The second, which is a concern associated with both nuclear and non-nuclear energy sources, remains to be resolved. Enumerated below are the specific factors that enable the MIT-SNL laws to facilitate increases in a reactor's neutronic power that are both rapid and safe.

12.1.1 Incorporation of Non-Linear Dynamics

One of the central premises of the MIT approach to the design of a controller for reactor power is that reactor dynamics are non-linear and that reactor controllers should therefore be designed on a non-linear basis. The MIT-SNL laws were developed in accordance with this philosophy. Specifically, the model from which these laws were derived provides a rigorous description of a reactor's behavior during both mild transients and ones of many orders of magnitude conducted on short periods. In this regard, it is useful to review the importance of that model (the dynamic period equation) to the formulation of the MIT-SNL laws.

The basis of the MIT-SNL laws and of most other reactor control methodologies is the space-independent kinetics equations. One approach to controller design would be to linearize those equations about an intended operating point and then write the resulting relations in state-space form. (Refer to section 3.2.3 of this report.) This is an appealing option because it would facilitate the applica-

tion of the many excellent design techniques that have been developed for the control of linear systems. However, were this option to be exercised, much would be lost. In particular, linearized reactor models are valid for only small perturbations. For example, given in Figure 3.5-1 were the results of a simulation study in which power profiles obtained using linearized and non-linear models were compared. The transient shown was rather mild, involving a ramp reactivity addition of 100 mbeta over 20 seconds. Yet the error in the linearized system's response was substantial.

The MIT approach to model development was to combine the space-independent kinetics equations through a process of differentiation and substitution to obtain the dynamic period equation. Advantages to the use of this equation are that it preserves the non-linear aspects of a reactor's dynamics, that it is an exact relation rigorous within the limitations of space-independent kinetics, that it applies to all reactor operating regimes, and that it explicitly shows each of the physical processes that can affect the instantaneous reactor period. The dynamic period equation is, of course, not without its limitations. It is a form of space-independent kinetics and hence, as discussed in section 4.3.4 of this report, its use is restricted to transients for which the shape of the neutronic flux remains unchanged. However, that limitation appears minor for the transients studied here and in all other respects, the dynamic period equation provides a rigorous description of a reactor's non-linear behavior.

Relative to the theory of process control, the use of a single relation, such as the dynamic period equation, to describe a system's behavior is a significant departure from current practice which is to model an n th order system as a set of n first-order differential equations. This is referred to as a 'state-space' representation. As was described in section 3.2.3 of this report, its advantage is that the internal dynamics of the system become accessible. Thus, instead of feeding back merely the output of the process, each system parameter can be assigned a feedback coefficient. Hence, for an n th order system, n degrees of freedom are introduced. This provides the control engineer with enormous flexibility. In particular, by the judicious choice of the gain coefficients associated with each of the system parameters, the shape of the system's response as well as its stability can be specified. Disadvantages to the state-space approach are that it is restricted to linear systems and that physical understanding of the process dynamics is often lost. Use of the dynamic period equation avoids these drawbacks and, as was evident from the experimental evaluation of the MIT-SNL laws, also provides a means for shaping system response.

12.1.2 Generic Nature of the MIT-SNL Laws

The MIT-SNL minimum time laws were derived without approximation from the space-independent kinetics equations. As such, they are rigorous within the limitations of those relations. Moreover, they are generic in the sense that they can be applied to any reactor that can be described by space-independent kinetics. This includes test

and research reactors, spacecraft and propulsion reactors, and medium-sized commercial facilities. The generic nature of the MIT-SNL laws is of importance because designs for spacecraft reactors have not yet been finalized. The only reactor ever launched into space by the United States was the SNAP-10A. It was a NaK-cooled, 43 kWt reactor that used thermoelectric conversion to produce 0.56 kWe. It operated for only forty-three days, shutting down automatically following a malfunction of its voltage regulator. Presently under consideration are both liquid metal and gas-cooled reactors. The former is the SP-100 which, as the name implies, will produce 100 kW of electrical power. The latter is the Multi-Megawatt Reactor. Although still at the conceptual stage, it will probably use a gas-cooled pebble or particle bed core. Regardless of the designs that are ultimately selected and built, the MIT-SNL laws will be applicable. Their only restriction is that of the space-independent kinetics model which, as was described in section 4.3.4 of this report, assumes constancy of the shape of the neutron flux during transients.

The experimental evaluations of the MIT-SNL laws that were conducted on both SNL's Annular Core Research Reactor and the MIT Research Reactor provided clear evidence of the generic nature of the approach. These two reactors are of very different design. The MTR-II uses forced circulation to continuously remove the energy. The ACRR operates under essentially adiabatic conditions. The former uses fully-enriched UAl_x fuel. The latter uses 28% enriched $BeO-UO_2$ fuel. The MTR-II has a small negative reactivity coefficient associated with rising fuel and coolant temperatures. The ACRR's coefficient is substantial. Yet, except for changing the functional descriptions of the integral and differential reactivity worths of the control mechanisms and the temperature coefficient, no changes were made to the software that implements the MIT-SNL laws. Thus, these experiments both demonstrated the portability of the methodology's associated software and dramatically showed that a non-linear, digital control law designed in terms of general concepts could be used without modification on reactors of very different design.

12.1.3 Provision for Feedback

The MIT-SNL Period-Generated Minimum Time Control Laws are closed-form expressions for the time-optimal control of neutronic power in reactors subject to a limitation on the minimum allowed reactor period. The closed-form nature of these laws is unquestionably their most advantageous feature because use can then be made of feedback. The concept of feedback and its significance to process control was discussed at length in Chapter Three of this report. Through feedback, a controller can be designed that is insensitive to errors in the system model, to perturbations of unknown origin, and to signal noise. The incorporation of feedback in the design of controllers for linear systems has been extensively studied. One of the more sophisticated methodologies for so doing is the 'state-space' approach that was described in section 3.2.3 of this report. Using that technique, each parameter or 'state variable' is assigned a gain coefficient and then fed back to form the control signal. The desired shape of the

system's response to a demand change as well as its stability can be specified by an appropriate choice of the gains. For non-linear processes, the development of feedback methodologies is not as advanced. For example, as was noted in section 3.2.4 of this report, non-linear processes can be treated using optimal control techniques. The control engineer specifies a performance index, applies Pontryagin's maximum principle, and then solves for the control action that will maximize the index. Unfortunately, the mathematics of the approach are complex and the result is not a closed-form, analytic solution. Rather, one obtains a set of control signals that are to be applied sequentially in order to generate the trajectory that optimizes the performance index. No use can be made of feedback. This is unfortunate because the resulting sequence of signals is valid only for the initial conditions that existed when the problem was formulated. Should those conditions have changed while the solution was being calculated or should there be an unforeseen perturbation and/or a modeling error, then the actual trajectory will not be as desired. This was the state of controller design for the time-optimal control of reactor neutronic power prior to the elucidation of the MIT-SNL laws. The advent of these laws means that there now is a closed-form, analytic method for the achievement of time-optimal adjustments of a reactor's neutronic power. Moreover, feedback can be incorporated in that methodology.

The actual use of feedback in conjunction with the MIT-SNL minimum time laws was not a priority during either of the major experiment sessions conducted on SNL's Annular Core Research Reactor in October and December 1987. The objective there was to evaluate the physical basis of the MIT-SNL laws and the extensive use of feedback would have masked any deficiencies. Nevertheless, as was illustrated by the power and reactivity profiles that were shown in Figure 11.5.3-1, feedback was a necessity. As may be recalled, the transient reported in that figure was conducted without any form of feedback. The results were most unsatisfactory. A simple feedback scheme based on the observed error in the inverse of the reactor period was described in section 5.2.1 of this report. It was subsequently realized that this approach caused oscillatory behavior. A stable methodology that used proportional feedback was developed after the conclusion of the experiments conducted in December 1987. It is described in section 10.5.4.1 of this report. More recently, the MIT-SNL laws have been implemented using proportional-integral-derivative feedback as given by the following expression:

$$\omega_s(t) = \frac{(K) \ln[P_s(t+\delta)/n(t)] + (K/T_i) \int_0^t \ln[P_s(t+\delta)/n(t)]}{\Delta t} + \frac{(KT_d) \frac{d}{dt} \ln[P_s(t+\delta)/n(t)]}{\Delta t} \quad (12.1.3-1)$$

where $\omega_s(t)$ is the inverse reactor period that is to be substituted into the MIT-SNL laws,

K is the gain,

$P_s(t+\delta)$ is the specified power trajectory,

δ is the increment in time by which the calculation of the specified power should be advanced,

$n(t)$ is the observed power trajectory,

T_i is the integral or 'reset' time,

T_d is the derivative time, and

Δt is the sampling interval or cycle time.

This feedback scheme contains three terms. Each has the same mathematical form as does the inverse of the reactor period. Each provides corrective action by comparing the current value of the observed power, $n(t)$, to that of the specified power projected several time steps ahead, $P_s(t+\delta)$. For the system to be stable, the projection should be at least two sample intervals. A projection of five intervals was found to give the best results. Longer projections resulted in corrective actions that were quite slow and shorter ones caused excessive oscillations.

An extensive evaluation of the feedback method given by equation 12.1.3-1 was conducted on the MIT Research Reactor during 1988 and part of 1989. Studies of the methodology were also conducted on SNL's Annular Core Research Reactor. The complete results of those experiments are beyond the scope of this report. However, in the interest of demonstrating the robust behavior that can be obtained by combining the MIT-SNL laws with a properly-designed feedback technique, information is provided from an experiment in which the effects of pure proportional, proportional-integral, and proportional-integral-derivative control were compared. Additional information is given in [27].

12.1.3.1 Proportional Feedback

Figure 12.1.3.1-1 shows the power profile obtained when using pure proportional feedback in conjunction with the alternate MIT-SNL law. The objective was to raise the reactor's neutronic power from 1 MW to 3 MW in 55 seconds and then to hold it at 3 MW despite a 0.0786% $\Delta K/K$ reactivity perturbation intentionally introduced at about 130 seconds into the transient. The controller's performance was marginal. It overshoot the target power level and then only gradually returned to that level. Also, it did not restore the reactor power to the specified value following the reactivity perturbation. These difficulties occurred because the MIT-SNL law is model-based and the proportional feedback scheme was not sufficient to compensate for errors in that model. For example, upon introduction of the reactivity perturbation, the reactor's neutronic power rose. Hence, the observed and specified power levels no longer matched and the proportional feedback error signal, which is given by the expression $K \ln[P_s(t+\delta)/n(t)]$, became negative. This caused the term $(\beta - \rho)\omega$ in

Figure 12.1.3.1-1 Power Increase from 1 MW to 3 MW Using Alternate MIT-SNL Law with Proportional Feedback

the MIT-SNL law to also become negative. That in turn caused the shim blade to be inserted. Once that occurred, the sum of the quantities λ'_p and $\Sigma \beta_i (\lambda_i - \lambda'_i)$ in the MIT-SNL law began to change, thereby offsetting the effect of the quantity $(\beta - \rho)\omega$. The net result was that the reactor's neutronic power oscillated about an incorrect setpoint, ≈ 3.5 MW in this instance. Had the control system been designed to measure the reactivity associated with all of the shim blades, then the sum of the quantities λ'_p and $\Sigma \beta_i (\lambda_i - \lambda'_i)$ would have been taken as having the proper sign once the perturbation began. Hence, the net effect of both those quantities and the term $(\beta - \rho)\omega$ would have been to restore power to the correct setpoint of 3 MW.

In summary, this experiment showed that proportional feedback by itself was insufficient to compensate for modeling errors. Similar results were obtained using proportional-derivative control.

12.1.3.2 Proportional-Integral Feedback

Figure 12.1.3.2-1 shows the power profile obtained when using proportional-integral (P-I) feedback. The objective was again to raise power from 1 MW to 3 MW in 55 seconds using the alternate MIT-SNL law. In this case, the controller's response was excellent. The transient was completed with only a minor overshoot, the power settled out at the setpoint of 3 MW, and the controller restored the power to the specified steady-state value following both insertion and removal of the reactivity perturbation.

Why did this P-I controller function properly when the proportional one did not? In both instances, the reactor models were the same. The difference was that in the case of P-I feedback, the error signal was cumulative because the integral component was given by the expression:

$$(k/T_i) \int_0^t \ln[P_s(t+\delta)/n(t)] \quad (12.1.3.2-1)$$

This continued to increase in magnitude until the observed power did in fact match that which was specified. Hence, even though the quantities λ'_p and $\Sigma \beta_i (\lambda_i - \lambda'_i)$ were positive when they should have been negative, their effect was ultimately overridden by the term $[(\beta - \rho)\omega]$ which reflected the integral error.

Sensitivity studies were conducted to determine the proper settings for the gain and reset time. The results were:

- The parameter T_i was set at 10 seconds. Shorter values caused excessive oscillations at steady-state. Longer values resulted in too slow a response to the perturbations.
- The preferred weightings of the derivative and integral actions were found to be 1.0 and 0.1 respectively. Additional integral action resulted in steady-state oscillations while less degraded the controller's overall response.

Figure 12.1.3.2-1 Power Increase from 1 MW to 3 MW Using Alternate MIT-SNL Law with Proportional-Integral Feedback

12.1.3.3 Proportional-Integral-Derivative Feedback

Figure 12.1.3.3-1 shows a power profile obtained using proportional-integral-derivative feedback during a power increase of 1 MW to 3 MW in 55 seconds using the alternate MIT-SNL law. The controller's performance was excellent. In general, the addition of the derivative action was found to reduce the minor overshoots noted with the use of P-I feedback. Also, it appeared to lessen the oscillations that resulted at steady-state from the integral action. The preferred value of the derivative time, T_D , was found to be one second and the relative weightings of the proportional, integral, and derivative control actions were set at 1.0, 0.1, and 1.0 respectively.

Figure 12.1.3.3-2 shows the component terms of the feedback law for the transient given in Figure 12.1.3.3-1. Shown are the inverse period as calculated using equation 12.1.3-1 and the proportional, integral, and derivative control actions.

The following should be noted:

- The magnitudes of the proportional and derivative terms changed almost immediately upon initiation of both the transient and the perturbations. In contrast, the integral term required time for its magnitude to become appreciable.
- The integral term varied smoothly while the derivative component exhibited noisy behavior.
- The rapid change that occurred in the integral term at 55 seconds was intentional. Specifically, the controller was programmed to reinitialize the integral error upon first achieving steady-state power.
- Even when the power trace shown in Figure 12.1.3.3-1 appeared to have achieved equilibrium, the component terms of the controller were still varying in magnitude. In general, the power trace indicated equilibrium within 5-6 seconds of first attaining the target power. Nevertheless, the component terms of equation 12.1.3-1 did not settle out until about 20 seconds after steady-state power was attained.
- During the time interval 55-100 seconds, the proportional component stabilized near zero while the integral quantity maintained a small negative value, the offset correction.

In summary, the combination of proportional-integral-derivative feedback and the MIT-SNL minimum time laws is a robust method for the control of neutronic power in a nuclear reactor. As regards the feedback itself, the essential feature is that the error signal should be computed so that it has the mathematical form of a reactor period. (Note: Error signals formulated in terms of deviations in the neutronic power do not give as satisfactory a result.) Experiments

Figure 12.1.3.3-1 Power Increase from 1 MW to 3 MW Using Alternate MIT-SNL Law with Proportional-Integral-Derivative Feedback

Figure 12.1.3.3-2 Proportional, Integral, and Derivative Control
Actions Associated with Transient Shown in
Figure 12.1.3.3-1

showed that pure proportional control was not sufficient to offset the effect of modeling errors. Both proportional-integral and proportional-integral-derivative were shown to be excellent. Finally, to reiterate the point made at the outset of this section, it is the closed-form nature of the MIT-SNL laws that makes it possible for feedback to be used. This is the enormous advantage of these laws relative to other methodologies for obtaining a time-optimal response.

12.1.4 Control Using the Rate of Change of Reactivity

If the capability to incorporate feedback is the most advantageous contribution of the MIT-SNL Period-Generated Minimum Time Control Laws, then the choice of the rate of change of reactivity as the control signal is certainly the most novel. There were both theoretical and practical reasons for this decision. Regarding the former, the rate of change of reactivity is a directly controllable quantity. Reactivity, which is normally used for the control signal, is not. Regarding the latter, use of the rate of change of reactivity as the control signal means that the controller will provide information on both the direction in which the control device should be moved and the speed at which it should be withdrawn or inserted. Were reactivity to be used as the control signal, the output of the controller would only be the increment by which the control device should be repositioned. The significance of each of these features is further discussed below.

12.1.4.1 Selection of the Control Signal

Reactivity appears explicitly in the neutron kinetics equation and it has been the traditional choice for the control signal in reactor power controllers. However, its selection ignores a major requirement in the design of controllers for safety-constrained processes such as nuclear reactors. Namely, it should be possible to alter the control signal on demand and thereby cause an immediate effect on the process in question. Reactivity does not fulfill this requirement because adjustments of control devices can only be made over finite intervals. In contrast, the rate of change of reactivity can be immediately altered by merely initiating movement of a control device. Examination of the terms in the denominator of the dynamic period equation provides further insight. Specifically, if the origin of each of those terms is traced through the derivation, then it becomes apparent the quantity $\dot{\rho}$, which is the rate of change of reactivity, corresponds to the effect of a changing prompt neutron population, while the quantities $\lambda'_p \rho$ and $\sum \beta_i (\lambda_i - \lambda'_i)$ reflect the combined effects of a changing delayed neutron precursor population and a changing distribution of delayed neutron precursors within the defined groups. Precursor populations are a function of the power history and therefore can not be altered on demand. In contrast, the prompt neutron population is essentially a function of only the current power level and is therefore immediately controllable. Hence, it is apparent that if an immediate change is required in the period, then an adjustment should be made in the rate of change of reactivity rather than in the reactivity itself. This is the reason for using the rate

of change of reactivity as the control signal. It is itself directly controllable and, upon being changed, it will have an immediate effect on the course of the transient.

12.1.4.2 Shaping of the Power Profile

Use of the rate of change of reactivity as the control signal is advantageous because the output of the controller is the velocity at which the control device should be moved. In contrast, controllers that utilize reactivity as the control signal provide only the incremental distance by which the control device should be withdrawn or inserted. The provision by the MIT-SNL laws of both the control device's speed and direction facilitates the shaping of the power profile. In fact, virtually any desired profile can be generated. The user need only specify the period associated with the desired power trajectory and the control laws will determine the rate of reactivity change needed to generate that period and hence the associated trajectory.

Shown in Figure 12.1.4.2-1 is an experiment that was performed to illustrate the flexibility of the MIT-SNL Period-Generated Minimum Time Control Laws. Presented are the power and reactivity profiles for a run performed on the MIT Research Reactor in which the neutronic power was made to vary as a sinusoid. The baseline power was 1.5 MW and the amplitude of the wave was 500 kW. Thus, the neutronic power varied from 1.0 to 2.0 MW. The frequency shown is one cycle every 180 seconds. Note both the offset between the reactivity and the power peaks as well as the asymmetry of the reactivity profiles. These effects are of course the result of delayed neutron precursor behavior.

12.2 Advantages Concerning the Implementation of the MIT-SNL Laws

Enumerated below are observations made on the implementation of the MIT-SNL Period-Generated Minimum Time Control Laws during the course of either the simulation studies or the experimental evaluations. Most of the items noted have been previously discussed. They are summarized here in order to provide a comprehensive list of the advantages associated with the use of these laws.

- The MIT-SNL laws are a practical method for the time-optimal control of a reactor's neutronic power. They function by altering the rate of change of reactivity so that the instantaneous period is stepped from infinity to its minimum allowed value, held at that value until the desired power level is attained, and then stepped back to infinity. As a result, the power adjustment is completed in the minimum possible time for a reactor subject to a restriction on the allowed period.
- These laws are general-purpose, closed-form expressions suitable for use on any reactor that can be described by space-independent kinetics.

Figure 12.1.4.2-1 Sinusoidal Power Variation on Frequency of 1 Cycle
Every 180 Seconds Using Alternate MIT-SNL Law:
Experimental Data Obtained 17 October 1988 on the
MIT Research Reactor

- They are mathematically tractable which in turn means that they can be applied in real time using state-of-the-art, but nevertheless, readily available computer systems.
- They are exact in that they rigorously describe the non-linear dynamics of a nuclear reactor. No approximations were made in their derivations.
- They can be applied to all reactor regimes including both subcritical and prompt-critical operation. However, two cautions are in order. First, the transition from subcritical to supercritical operation may entail such extensive movement of the control devices that constancy of the flux shape is not assured. Hence, the validity of the space-independent kinetics model is questionable. Second, while the MIT-SNL laws do provide the correct control signals for use while prompt-critical, such operation is not recommended because the mechanical actuators used to implement those signals may not be capable of providing the required rapid response.
- The control signal for the MIT-SNL laws is the rate of change of reactivity. This means that both the direction and speed of the control device is specified. As a result, virtually any desired power trajectory can be generated.
- There are two mathematically equivalent versions of the MIT-SNL laws. This means that an independent check can be made of the arithmetic portion of the control signal's determination.
- The MIT-SNL laws are self-correcting in the same sense that a nuclear reactor is self-regulating. For example, the quantities $\lambda' \rho$ and $\Sigma \beta_i (\lambda_i - \lambda')$ in the alternate law are both subtracted from the quantity $(\beta - \rho) \omega$. Suppose that the neutronic power were rising too rapidly because too much reactivity had been inserted. The absolute value of the quantity $\lambda' \rho$ would therefore be too large. This would have the effect of reducing the demanded rate of reactivity change which would in turn lower the rate of power increase. (Note: This self-correcting feature is of course valid only if the reactivity and decay parameters can be accurately observed.)
- The MIT-SNL laws are closed-form expressions which means that they are amenable to the use of feedback. This is a major advantage because through feedback they can be made insensitive to modeling errors and perturbations of unknown origin.
- The MIT-SNL laws are not dependent on any type of external feedback. For example, the presence of a large negative reactivity coefficient associated with fuel temperature is not essential to their implementation.

- The resulting power trajectory is readily determined because the period is known in advance. This should facilitate coordination of heat removal systems with energy production.
- For reactivities less than the delayed neutron fraction, the rate of change of reactivity needed to step the period from infinity to a given value will be less than that needed to step it back to infinity. Hence, assuming that the differential reactivity worth of the control device is approximately constant, it should be feasible to terminate any transient that it was possible to initiate. (Note: This is an empirical observation.)
- Use of the MIT-SNL laws permits the performance of controlled power decreases on periods of less than negative eighty seconds.
- The MIT-SNL laws are consistent with the Pontryagin approach to optimal control. Specifically, if the Pontryagin condition for time-optimal control is applied to a system that is subject to a constraint, then the solution will be to move the system along the trajectory established by that constraint. These control laws do exactly that. Only, they are closed-form expressions amenable to feedback and not, as is the result with the Pontryagin approach, a sequence of open-loop signals valid for only one particular set of boundary conditions.

The most important advantage associated with the MIT-SNL laws is that they have been used under closed-loop conditions to conduct power maneuvers on operating reactors in the presence of actual signal and instrument noise. The physical concepts on which these laws are based are therefore known to be valid. Moreover, valuable experience has been gained both on the selection of hardware and on the design of the interface between the digital computer and the reactor's mechanical actuators. Finally, the information being accumulated from the experiments on the MIT-SNL laws is forming the nucleus of a data base from which the confidence will develop to use closed-loop digital technology for the autonomous operation of both spacecraft and terrestrial reactors.

12.3 Requirements for the Implementation of the MIT-SNL Laws

The performance of the MIT-SNL Period-Generated Minimum Time Control Laws can be enhanced if the reactor, and especially the core, is designed with an understanding of the requirements of the controller. Arguments concerning the effect of the prompt neutron lifetime on the operational characteristics of the MIT-SNL laws were enumerated in section 6.6 of this report. Basically, a short neutron lifetime is desired because less reactivity will be needed to achieve a given reactor period. The counterargument is that the reactor may be more sensitive to perturbations than if the core were designed to have a long neutron lifetime. Other pertinent aspects of core and system design are as follows:

Control Device Geometry: The validity of the space-independent kinetics model and hence that of the MIT-SNL laws can be enhanced by the judicious placement of the control devices. If the reactor is equipped with only one or two devices of significant integral reactivity worth, then the achievement of a short period may require that these be moved substantial distances. This may in turn cause a major perturbation in the local flux shape. A better approach would be to use many control devices, each of relatively low integral reactivity worth. Homogeneity could then be approximated by distributing these throughout the core. Moreover, if moved in unison, the impact on the flux shape would at least be uniform and perhaps small.

Differential Reactivity Worth: Achievement of a time-optimal response necessitates that the instantaneous reactor period be stepped to its minimum allowed value upon initiation of the transient. This is done by creating a large positive rate of reactivity change which in turn causes a rapid rise in the prompt neutron population. Variable speed stepper motors are used to initiate the transient. The stress imposed on these motors could be greatly reduced if the differential reactivity worth of the control devices were both large and roughly constant.

Multiple Rod Banks: Transient termination is accomplished by creating a large negative rate of change of reactivity. The reliability of this process can be enhanced by using two banks of control devices: one for withdrawal and one for insertion. If only one bank were used, then that bank's stepper motors would perhaps be required to abruptly reverse direction. Such reversals could conceivably result in a system stall.

The preceeding list underscores the need to integrate controller and reactor design. Another requirement for the implementation of the MIT-SNL laws is the presence of a supervisory controller. In this regard it should be clearly understood that the MIT-SNL laws are a means of determining the required control signal. No verification is made as to the capability of the control devices to generate that signal. That is the function of a supervisory algorithm that embodies the concept of 'feasibility of control'. (Refer to section 5.1.2 of this report.) As an illustration of the importance of supervisory control, consider the following scenario. The MIT-SNL laws are used to determine the rate of change of reactivity required to achieve a specified period and the transient is initiated. As noted earlier, the maximum required rate of change of reactivity occurs at the outset of the transient. Hence, in principle, there should be no difficulty in halting this transient because the required rate of reactivity removal should always be less than that which was used to initiate the transient. Unfortunately, this may not be true under all circumstances. For example, the differential worth of the control mechanism may be a non-linear function of position and decrease as the mechanism is withdrawn. The MIT-SNL laws are merely a means of determining the appropriate control signal. Should the differential worth of the control mechanisms decrease, the control laws would blindly compensate

by signaling for a more rapid rate of withdrawal. The eventual result would be that the control devices would be so far withdrawn as to not be capable of removing reactivity at the rate needed to terminate the transient. Occurrences of this type can be prevented by employing reactivity constraints to guarantee 'feasibility of control'. These constraints continuously check the actual differential worth of the control mechanism against the magnitude of the terms that correspond to the effects of the delayed neutrons. Should the available differential worth of the control mechanisms become too low, these constraints will override the decisions of the control laws and force corrective action while there is still time for such action to be effective. Thus, it is these constraints, not the control laws, that guarantee the absence of a challenge to the safety system.

12.4 Role of the MIT-SNL Laws in a Reactor Controller

Figure 12.4-1 shows current MIT-SNL thinking concerning the structure of a controller for the autonomous control of a spacecraft nuclear reactor. It could be adapted for terrestrial use by the inclusion of a proper man-machine interface.

Major features of this controller are:

- A separate safety system.
- A means for signal validation and instrument fault detection.
- A supervisory algorithm that precludes challenges to the safety system.
- A set of predictive control laws.
- A reconfiguration logic to select the most appropriate law.
- A means for the verification of signal implementation.
- A module for automated reasoning.

Of special significance is the hierarchical structure of this controller. The actions of the predictive control law chosen by the reconfiguration logic are reviewed by the supervisory algorithm so as to ensure the absence of a challenge to the safety system. This 'defense-in-depth' approach is desired even though the safety system is an independent entity. One reason for the need for supervisory control was noted in the preceding section of this report. Namely, the differential reactivity worths of the control devices often vary with the position of those devices. A second reason was discussed in section 5.1 of this report. Namely, given that reactor dynamics are both non-linear and time-delayed, it is necessary to restrict operation to those conditions under which the transient can be halted. A third reason is that as the theoretical basis for predictive control laws improves in capability so does the dependence of those laws on plant models. Supervisory algorithms are needed to preclude improper operation in the event of an inaccuracy in a model.

Figure 12.4-1 Structure of Autonomous Controller

The following features of the controller are of significance:

Separation of safety and control systems: The nuclear safety system is separate from the closed-loop controller. The word 'separate' is defined as meaning that the output of an instrument used in the safety system must not be influenced by interaction with the control system. Thus, if an instrument is common to both systems, its signal must be passed through an isolation device, such as an optical transformer, to preclude any possibility of feedback from the control system. The purpose of keeping the two systems separate is to ensure that the capability of the safety system to perform its intended function will never be compromised.

Signal validation and instrument fault detection: All sensor information is processed by signal validation and fault detection routines. There are several methods for accomplishing this. The simplest is to verify that each reading is within the range expected for a given plant condition. A more sophisticated approach is to identify the largest consistent subset of signals and to reject any that is not a member of that set. A further refinement is to incorporate a real-time system model that generates an analytic value for the measured parameter. Sensor readings are then checked for consistency both with each other and with the calculated value. This latter method has been demonstrated at MIT as part of a numerical technique known as the 'parity space approach' [3]. In addition to validating sensor readings, this methodology performs instrument fault checks in which the weighting factor for each sensor is adjusted in proportion to the frequency with which its readings are judged to be valid. Thus, reliance on a failing sensor is gradually reduced, thereby assuring a 'bumpless' transition when complete failure actually occurs.

Supervisory algorithm, reconfiguration logic, and predictive control laws: The controller contains a supervisory algorithm, a number of selectable control laws, and a reconfiguration logic. The supervisory algorithm establishes the limiting conditions within which control will always be feasible and thereby guarantees that there will not be a challenge to the safety system as the result of any automatic control action. It does this by requiring a change in the present value of the control signal if conditions are such that an overshoot could occur at some future time. Among the selectable control laws are ones suited for each operational mode. The MIT-SNL laws would be one option within this group of control laws. The reconfiguration logic selects the law that is most applicable to the current plant condition [9]. The decision of the selected control law and that of the supervisory algorithm are then compared and the more conservative is implemented. This approach has the advantage that it can combine a general-purpose, supervisory algorithm that precludes challenges to the safety system with a plant-specific predictive control law.

Verification of signal implementation: The implementation of the control signal should be verified because there could be a failure in either the computer interface or in the actuators. Verification should be performed in the broadest possible manner. For example, if the controller's decision was to reduce the reactivity, then it should be determined both that a control device was inserted and that the period did actually lengthen.

Automated reasoning: The purpose of the automated reasoning module is to replicate those functions that would normally be performed by a human operator. Studies of the process by which humans exercise control suggest that four tasks are involved [30-32]. These are planning, prediction, implementation, and assessment. Planning involves determining both the desired plant state and the most efficient means of achieving that state given the overall operational objectives and the confines of engineering limitations. The planning process is therefore basically a search procedure in which operational goals are matched against available options. Moreover, the search space is essentially closed. That is, the number of possible options is finite. Expert systems technology is now being used to assist the managers of terrestrial reactors with such planning functions as preparation of work orders, verification of technical specification compliance, and the determination of rod withdrawal sequences in boiling water reactors [33]. It is therefore anticipated that this technology could be incorporated in an autonomous controller to perform the planning function. Prediction, which is the second of the four control subtasks, entails formulating projections of the effect that any of the available control options will have on the plant's behavior. For a control system to perform the prediction subtask autonomously, accurate numerical models of the plant will be required. The third subtask, implementation, is generally already automated and is therefore not discussed here. The last of the four control subtasks is assessment. This is perhaps the most difficult because it involves first distinguishing abnormal from normal behavior and then, if the former is present, diagnosing it as to cause. The automated performance of the assessment subtask will probably require the use of several different approaches. For example, numerical models could be used to establish normal system behavior and pattern recognition techniques might serve to identify abnormalities. Also, real-time analysis is now being attempted with expert systems. For example, within the nuclear industry, several very successful systems are being used for turbine-generator diagnostics [34]. However, this success may not be typical. The problem is that, unlike the planning subtask, the field of search is extremely large and all possible rules may not be identified. The potential for utilizing expert systems for diagnostics has been previously discussed [35]. Relative to the MIT-SNL program, a new methodology, causal analysis, is currently being explored for use in automating the assessment function.

In summary, the MIT-SNL Period-Generated Minimum Time Control Laws are but one component of a controller for a reactor's neutronic power. Moreover, that controller would itself be part of a larger system that coordinated the control of the balance of plant as well as of the spacecraft itself. Much additional research and experimentation is needed, particularly for the task of on-line diagnostics.

12.5 Possible Applications of the MIT-SNL Laws

The MIT-SNL minimum time laws were developed to facilitate the rapid maneuvering of neutronic power on spacecraft nuclear reactors. However, given that these laws are generic, they can also be used to advantage for many other reactor control applications. These include:

- (1) Rapid Recovery of Research and Test Reactors: The space-independent kinetics equations provide a valid description of the dynamics of research and test reactors because such facilities usually have small, compact cores. This in turn means that the MIT-SNL laws can be directly applied to reactors of this type. For reasons of safety, there is usually little excess reactivity present in research and test reactors. That is, the fuel loading is limited to that needed for critical operation at power. This may present an operational problem in that, following a shutdown, the concentrations of short-lived fission product poisons, such as xenon and samarium, may rise to the point where it will not be possible to restart the reactor. Reactors in such a condition are referred to as being 'xenon-precluded'. Once this occurs, the reactor must be left shut down until the fission products dissipate as the result of radioactive decay. This may take more than a day because xenon has a half-life of 9.2 hours. Depending on the core design, reactors may become xenon-precluded in as little as thirty minutes following an unanticipated shutdown. For reactors in this situation, a rapid restart is essential. Yet, such a requirement may place undue stress on the operators. The MIT-SNL laws could be used to automate the recovery process and thereby permit the operator to monitor the facility without also having to manipulate the controls.
- (2) Power Increases Following the Attainment of Criticality: The power level at which most commercial reactors attain criticality is usually many decades below the point at which appreciable energy is generated. Accordingly, once criticality is achieved, procedures often specify that the neutronic power should be raised at a fixed rate until thermal feedback effects become evident. The MIT-SNL laws are ideally suited for such operation because it entails raising the neutronic power on a constant period.
- (3) Power Adjustments in the Presence of Thermal Feedback: Once operating at power, many reactors are constrained by the rate at which the plant temperature can be changed. Exceed-

ing the specified rate could cause thermal stress that might be damaging to the piping. The MIT-SNL laws could be used to adjust reactor power so that temperature changes would occur at the uniform, prescribed rate. This would minimize thermal stress while maximizing efficiency. For this to be a practical matter, it would be necessary to accurately model the dependence of plant temperature on neutronic power. Research of this type is currently being pursued as part of a joint program between MIT and the Oak Ridge National Laboratory.

- (4) Automated Startups: As noted during the derivation of the MIT-SNL laws in Chapter Four of this report, these laws are applicable to any reactor operating regime including subcritical, supercritical, and even prompt-critical behavior. As such they can be used for the performance of automated startups. The only restriction is that, as was discussed in section 4.3.4 of this report, the shape of the neutron flux profile must remain approximately constant. Research on the use of the MIT-SNL laws for the automated startup of spacecraft reactors is now in progress as part of a joint MIT-SNL program. The initial results are quite promising [36].

12.6 Comparison with ROVER and NERVA Programs

Both rapid increases in reactor neutronic power and automated startups were achieved during the 1960s as part of the NERVA and ROVER Projects which concerned the development of nuclear-powered rockets. Provided here is a comparison of the control techniques that resulted from those projects with the MIT-SNL laws. The material on the former is drawn from a 1965 paper entitled, "Automatic Startup for Nuclear Reactor Rocket Engines" [37].

The ROVER/NERVA control techniques and the MIT-SNL laws are similar in that both will cause a reactor's control mechanisms to be manipulated in the same manner. Specifically, both result in rapid withdrawals of the control devices upon transient initiation followed by a gradual lessening of the withdrawal speed as delayed effects become appreciable. This similarity is to be expected because both methodologies were successful in facilitating rapid increases in neutronic power. However, in virtually all other respects, the two approaches differ. These differences are summarized below:

- (1) The ROVER/NERVA designs were implemented using hard-wired analog circuitry. In contrast, the MIT-SNL laws are part of a software package that can be used in conjunction with a variety of digital computers.
- (2) The ROVER/NERVA methodology achieves control by positioning the control devices. For rapid increases of power, this is done by moving the rods in response to the difference between the observed and demanded power signals. For automated startups, this is done open-loop by moving the rods

along a preset trajectory. In both instances, reactivity is being used as the control signal. The MIT-SNL laws differ in that, for any application, they are closed-form expressions that determine the rate of change of reactivity needed to generate a specified power trajectory.

- (3) The ROVER/NERVA approach combines both open and closed-loop control. For the open-loop portion, the methodology is restricted to one particular set of initial conditions. For example, for an automated startup, it is assumed that rotation of the drums that serve as the control devices by a prespecified amount will cause the neutronic power to rise by about one decade, not create an exceedingly short period, and result in the reactor's being slightly subcritical. Control would then be transferred to a closed-loop power comparator circuit which would bring the system to full power. The utility of such a system is limited by the range of initial conditions to which it is applicable. If those conditions change substantially, then the predetermined trajectory used for the open-loop portion of the cycle may not produce the anticipated result. In contrast, the MIT-SNL laws, being closed-loop, can be applied in conjunction with any set of initial conditions.
- (4) The ROVER/NERVA design used empirical observation to determine the settings for the various components in the control circuits. In contrast, the MIT-SNL laws are derived from a theoretically rigorous description of a reactor's dynamics.
- (5) Diagrams of the ROVER/NERVA control system indicate use of only proportional feedback by direct comparison of the observed and demanded power. The MIT-SNL laws differ in that use is made of proportional-integral-derivative feedback and that each component in the P-I-D scheme has the same mathematical form as does the reactor period. Also, the MIT-SNL laws incorporate feedback during all facets of operation. The ROVER/NERVA design uses it only once the reactor is critical.
- (6) Both techniques use balance calculations to estimate the reactivity. However, being digital, the MIT-SNL approach also has the computing capability to determine reactivity using inverse kinetics with six delayed neutron groups. This was in fact demonstrated on the ACRR in July 1989.

In summary, the technical achievements of the ROVER and NERVA projects were outstanding. However, those successes should not be taken as reason for not now pursuing further research on reactor control. First, more than two decades have elapsed since rapid power increases were first demonstrated as part of the ROVER Project on 30 April 1964. During those years, advances in digital technology have rendered obsolete the analog designs used for those demonstrations. Second, a significant advance has recently occurred in the modeling of reactor

behavior. Specifically, the dynamic period equation is a theoretically rigorous description of the dynamics of reactors that are characterized by space-independent kinetics. As described in this report, advantage has already been taken of this equation to obtain the MIT-SNL laws which are closed-form expressions for the time-optimal control of reactor neutronic power. Other benefits from this improved mathematical description are still being elucidated. Third, advances in expert systems and related technologies are now reaching the point where it may be possible to develop autonomous controllers. That is, the controller would distinguish abnormal from normal behavior, identify any discrepancy as to cause, and reconfigure the available equipment and software so as to permit continuation of the mission. The realization of such a controller would be of benefit both to space nuclear power and to the process control industry as a whole.

13. Issues Relevant to the Realization of Digital Controls

At the outset of this report, it was argued that the development of a reactor controller and its associated instrumentation should be performed in concert with the design of the core itself. Past practice has often been to optimize a proposed reactor in terms of heat removal and fuel utilization while assuming that a functional control methodology could be devised for the already-designed core. The present work has provided a rigorous theoretical model of the dynamic behavior of reactors described by space-independent kinetics. Based on that model, closed-form laws for the time-optimal control of reactor neutronic power were derived and experimentally validated. In so doing, certain core characteristics were identified as being of benefit to the operation of a reactor controller. For example, as was discussed in sections 6.6, 11.5.1, and 12.3 of this report, a differential reactivity worth that is both large and roughly constant will permit a reduction in the speed at which the motors for the control devices must be driven. Other factors that may impact on controller performance are the physical layout of the control rods within the core, the use of opposing rod banks for transient initiation and termination, the available integral reactivity, and the magnitude of the prompt neutron lifetime. During the conduct of the research that has been reported here, much insight was gained relative to the optimization of controller performance in terms of these parameters. However, much undoubtedly remains to be learned. Given below are concise summaries of both research topics that should be pursued in conjunction with the control of space nuclear reactors and the status of the MIT program on reactor control.

13.1 Research Issues

Listed below are several broad areas in which research should be conducted in order to facilitate the use of closed-loop, digital control for the operation of both spacecraft and terrestrial reactors.

13.1.1 General Theories of Non-Linear Control

The dynamics of nuclear reactors are non-linear and controllers for nuclear reactors should be designed with full recognition of that reality. Most of the control methods that are currently available assume linear behavior or else use some form of gain scheduling so that linear techniques can be applied. This approach made sense when only analog equipment was available for controller implementation. However, the advent of inexpensive, reliable digital computers with enormous computational capability has obviated the need to limit designs to the linear model. There is no longer any justification for forcibly fitting a non-linear system to the linear mold. Extensive efforts should be undertaken to develop general theories for the control of non-linear systems. In this regard, it is worth noting that the Society for Industrial and Applied Mathematics (SIAM) recently issued a report entitled, "Future Directions in Control Theory: A Mathematical Perspective" [38]. One of that report's findings was:

"Many fundamental theoretical issues, such as control of nonlinear multivariable systems, or control of nonlinear partial differential equations, are not yet understood. Moreover, exciting new applications and technologies such as robotics, manufacturing, and space technology are providing a wealth of new control problems. Existing theories must be developed and expanded to meet these challenges."

13.1.2 Model Validation

As was noted in section 12.4 of this report, control systems design is becoming increasingly dependent on the existence of precise models of the plant that is to be controlled. One issue that is being neglected is the need to maintain these models. Specifically, how often should they be calibrated, how can that be done efficiently, and can the models be verified for accuracy on a frequent basis? This issue is particularly germane to nuclear reactors because conditions within the core of a reactor may change on a time scale of hours if strong xenon transients are present. Also, on a much longer time scale, allowance must be made for the effects of fuel and absorber depletion.

13.1.3 Autonomous Operation

One of the major requirements for a spacecraft controller is that it be capable of sustained, autonomous operation. This implies a capability for both diagnosis of failed components and for system reconfiguration. Much work has recently been reported in the literature on the development of 'intelligent' control methodologies that in some degree do address these topics. Nevertheless, the issue remains unsolved. The aforementioned paper by Astrom [28] provides an interesting summary of progress that the control community, as a whole, has achieved. Relative to the MIT-SNL research effort, a discussion has been given of both pattern recognition and expert system techniques as a means of distinguishing between normal and abnormal controller responses [39]. Neither was judged to be sufficient and efforts are now focused on quantitative causal analysis. At present, the design of controllers capable of recognizing and correcting both internal and external faults is a most challenging task and it is an area to which the space nuclear community should be devoting significant resources.

13.1.4 Integration of Controller and Core Design

The argument has been repeatedly made in this report that a controller's performance can be optimized if it is designed in conjunction with the core itself. A cost/benefit study should be undertaken in which the tradeoffs between altering core design and improving controller response are assessed.

13.1.5 Issues Involving the Balance of Plant

The MIT-SNL minimum time laws address the task of raising a reactor's neutronic power by many orders of magnitude in a few

seconds. The complementary problem is to achieve a similar rate of rise from the thermodynamic cycle that is used to convert the reactor's thermal energy to electricity. This may in fact be the more difficult problem and it warrants immediate attention.

13.1.6 Demonstration Projects

The nuclear industry in the United States currently lacks an extensive history of digital operation. As a result, there is no data base on which the industry can draw for information to aid in the design of digital systems. This deficiency is now placing the U.S. nuclear industry at a growing disadvantage relative to foreign vendors who either have or are accumulating the requisite information. For example, Atomic Energy of Canada has almost two decades of experience in this area. Also, the Japanese and French are designing and using advanced control rooms that are dominated by CRT displays. Relative to the exploration of space, the Union of Soviet Socialist Republics has a long history of deploying reactor-powered spacecraft, while the United States has used radioisotope thermal generators as a power source. It is suggested that digital controllers together with the appropriate signal validation, instrument fault detection, signal verification, and suitable man-machine interfaces be installed on a prototype basis at several of the larger university research reactors and at some of the reactors operated by the various national laboratories. Experience gained at these digitally-controlled reactors would form a much needed source of data for control system design. Also, the operating experience gained from using such integrated control systems would provide confidence in the technology to both prospective users and regulators.

A second reason for establishing demonstration projects is the growing importance of experiment to advances in control engineering. Again quoting from the report "Future Directions in Control Theory" that was prepared by the Society for Industrial and Applied Mathematics:

"Experimental research in control theory is now becoming increasingly important. It has the dual purpose of finding the best control-oriented models of physical processes, and testing the validity of new control paradigms. The kinds of experimental data required in control are often very different from other sources."

13.1.7 Dissemination of Information on Control System Research

Accompanying the establishment of prototypes for the evaluation of digital system technology, there should be a vehicle for the rapid dissemination of information on the performance of these controllers. Journal and conference publications are of use but in general those sources only report positive experiences. If the full benefit of the prototypic demonstrations is to be realized then both positive and negative findings should be disseminated.

13.2 Status of Control Research for Reactor Operation at MIT

The MIT research effort on the closed-loop, digital control of nuclear reactors remains an on-going, joint effort of the Nuclear Reactor Laboratory (NRL) and the Department of Nuclear Engineering (NED). Efforts are currently focused on the control of reactors characterized by spatial dynamics, methods for the rapid startup of spacecraft reactors, and the coordinated control of core temperature and power. The distinguishing feature of the program remains the experimental demonstration of each new concept. In January 1988, when the reactor control report prepared by MIT for the National Science Foundation was issued, it was noted that financial considerations were making the performance of reactor experiments increasingly difficult [2]. That situation has not changed. In fact, it has worsened. University research reactors are closing at the rate of two or three per year and many of the DOE-operated facilities are currently shut down. It is imperative that the nuclear community find the means to support its experimental facilities. Research and test reactors are needed not only for controls research but also for studies in such diverse fields as materials, nuclear medicine including cancer therapies, geologic research, quantification of acid rain sources, archaeology, and neutron physics. As for the engineering disciplines, reactor experiments are essential. Without them, there will be little or no confidence in the theoretical results and difficulties that would eventually impede application to both commercial and spacecraft reactors will not be anticipated and resolved.

In regards to the MIT-SNL experimental program, control experiments were performed on the MIT Research Reactor on a weekly basis during the latter half of 1988 and much of 1989. Also, three sessions of several days each have been held at SNL's Annular Core Research Reactor since December 1987. Among the accomplishments achieved to date are:

- Experimental verification of the stability analysis performed by Dr. Takashi Washio of the MIT-SNL laws [25].
- Demonstration of proportional-integral-derivative feedback developed by CDR Tuddenham and Mr. Shing Hei Lau [27].
- Performance of automated increases of reactor power from subcritical conditions [36,40].
- Demonstration of model-based filters developed by Dr. Washio for the reduction of signal and measurement noise [41].
- On-line use of inverse kinetics for the measurement of reactivity for use under conditions of closed-loop control.
- Use of the parity space approach to obtain an estimate of the reactivity for use under conditions of closed-loop control.

- Performance of power cycling such that the neutronic power varied as a sinusoid.
- Preliminary experimental testing of a controller for the core average temperature [42].
- Performance of robustness tests in which reactivity perturbations of unknown origin were intentionally introduced during closed-loop operation.

These experiments were performed not only in conjunction with the Sandia National Laboratories but also with the support of the Oak Ridge National Laboratory and the U.S. Department of Energy. Although, as indicated above, some results have been reported, most of the information obtained from these experiments is currently awaiting publication. A comprehensive but nonetheless concise account of the MIT work on reactor control is given in a paper that was presented at the Conference 'Fifty Years with Nuclear Fission' that was cosponsored by the American Nuclear Society and the National Institute of Standards and Technology in April 1989 [43]. This paper integrates the minimum time laws and reactivity constraints that are discussed in this report with material on rule-based control and the man-machine interface.

Figure 13.2-1 is the strip chart recording of the results of an experiment that is indicative of the capability of the MIT-SNL Period-Generated Minimum Time Control Laws as of July 1989. Shown are the transient rod bank height and power profiles of a run in which the Annular Core Research Reactor's neutronic power was raised from 3 kW to 3 MW, three orders of magnitude, on a period of 1.0 second. Both proportional feedback and a noise reduction filter were used. Reactivity was estimated using the parity space approach with inverse kinetics being the default in the event of indecision. The transient was completed in the allotted time of 6.7 seconds with essentially no overshoot. Note that the slope of the power profile is straight indicating that power was indeed rising on the specified period of 1.0 second. Also note that upon attaining the desired power level, 3 MW, the power was held at that value. There was some oscillatory movement of the control rods, but far less than was observed during the 1987 runs reported in Chapters Eleven and Twelve of this report. Of special significance is that at 16.3 seconds, one of the ACRR's three transient rods was deliberately dropped back into the core. This caused a reactivity insertion of -400 millibeta. The controller was not 'told' of this perturbation. Yet, excepting only a minor downward blip in the power trace, it held the power constant at 3 MW. Specifically, the remaining two rods were withdrawn to provide the necessary compensation. This aspect of the run was most impressive. Also apparent in the figure is the effect of the rising fuel temperature. It is evident that the control rods were being moved out both before and after the reactivity perturbation at 16.3 seconds in order to compensate for negative reactivity feedback associated with the Doppler effect. This run and several others like it have demonstrated the truly robust nature of the MIT-SNL minimum time laws.

Experimental Data

- Power Increase: 3-3000 kW
- Specified Period: 1.0 s
- Alternate MIT-SNL Law w/
Proportional Feedback
- ACRO54 - 07/15/89

Figure 13.2-1 Strip Chart Recording Showing Power and Transient Rod Bank Position Profiles Obtained During Power Increase of Three Decades on a One Second Period in which Robustness of the Alternate MIT-SNL Law was Demonstrated: Dropped Rod Experiment

The MIT-SNL minimum time laws are a theoretically rigorous method for the rapid maneuvering of reactor neutronic power under conditions of closed-loop digital control. The fact that they are closed-loop makes them compatible with any initial conditions and amenable to feedback. This gives the MIT-SNL laws an enormous advantage relative to competing techniques which, because they are computation-intensive, must be implemented in an open-loop manner. A second unique feature of the MIT-SNL laws is that they utilize the rate of change of reactivity as the control signal. This is in sharp contrast to the traditional approach which is to formulate the control signal in terms of reactivity. Advantage results to the MIT-SNL approach because the rate of change of reactivity is a function of the withdrawal speed of the control devices and hence directly controllable. In contrast, the reactivity itself is proportional to the position of the control devices and can therefore only be altered over a finite period of time. The result is that the MIT-SNL laws offer extreme flexibility and, assuming no limitation on blade withdrawal speed, are capable of generating virtually any desired power trajectory. A third advantage that accrues to the MIT-SNL laws is that they have been the subject of extensive experimental evaluations conducted on actual reactors under conditions of closed-loop digital control. These laws are known to function properly and the confidence now exists to use them as part of a master control system. In summary, the MIT-SNL Period-Generated Minimum Time Control Laws fulfill the requirements imposed on the design of the neutronic control system for the operation of reactor-powered spacecraft and it is expected that they will be used in that capacity as part of the continuing space exploration program now being undertaken by the United States.

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Appendix A

The Relation of the Inhour and the Dynamic Period Equations (Ref. 16)

The standard formulation of the dynamic period equation is:

$$\omega(t) = \frac{[\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta} - \rho(t))]}{(\bar{\beta} - \rho(t)) + \lambda_e^*(\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)})} \quad (A-1)$$

where $\omega(t)$ is the inverse of the instantaneous reactor period and $\lambda_e(t)$ is the standard, effective, multi-group decay parameter.

The assumptions implicit to the inhour equation are that reactivity is constant and that sufficient time has elapsed since the reactivity addition so that asymptotic conditions have been attained. Applying these assumptions to the standard dynamic period equation yields:

$$\omega = \frac{\lambda_e \rho}{(\bar{\beta} - \rho) + \lambda_e^*(\omega + \lambda_e)} \quad (A-2)$$

This relation is equivalent to the inhour equation. This can be shown by deriving both (A-2) and the inhour equation from the point kinetics equations subject to the assumption of asymptotic behavior.

The instantaneous reactor period ($\tau = 1/\omega$) is defined as:

$$\dot{T}(t) = \omega(t)T(t) \quad (A-3)$$

where $T(t)$ denotes the amplitude function and is a weighted integral of all neutrons present in the reactor. The point kinetics equations may therefore be written as:

$$\dot{\omega}(t)T(t) = \frac{(\rho(t) - \bar{\beta})}{\lambda_e^*}T(t) + \sum \lambda_i C_i(t) \quad (A-4)$$

$$\dot{C}_i(t) = \frac{\bar{\beta}_i}{\lambda_e^*}T(t) - \lambda_i C_i(t) \text{ for } i = 1, N \quad (A-5)$$

where N is the number of delayed neutron groups. Asymptotic behavior implies that the neutron and precursor populations exhibit the same exponential behavior. Thus, the precursor kinetics equations become:

$$\omega C_i(t) = \frac{\bar{\beta}_i}{\lambda_e^*}T(t) - \lambda_i C_i(t) \text{ for } i = 1, N \quad (A-6)$$

Solving for C_i and substituting into (A-4):

$$\omega T(t) = \frac{(\rho - \bar{\beta})}{\ell^*} T(t) + \sum \frac{\lambda_i \bar{\beta}_i T(t)}{\ell^* (\omega + \lambda_i)} \quad (A-7)$$

Rearranging gives the inhour equation:

$$\rho = \ell^* \omega + \sum \frac{\omega \bar{\beta}_i}{(\omega + \lambda_i)} \quad (A-8)$$

Equation (A-2) can also be obtained from (A-4) and (A-6). First, the neutron kinetics equation is rewritten using the standard, effective multi-group decay parameter:

$$\omega T(t) = \frac{(\rho - \bar{\beta})}{\ell^*} T(t) + \lambda_e \Sigma C_i(t) \quad (A-9)$$

Differentiating (A-9) and noting that the quantities $\dot{\omega}$, $\dot{\rho}$, and $\dot{\lambda}_e$ are all zero as a result of the assumed asymptotic behavior:

$$\omega \dot{T}(t) + \frac{(\bar{\beta} - \rho)}{\ell^*} \dot{T}(t) = \lambda_e \Sigma \dot{C}_i(t) \quad (A-10)$$

Using (A-3) and performing a series of substitutions to eliminate precursor terms:

$$\begin{aligned} \omega^2 T(t) + \frac{(\bar{\beta} - \rho)}{\ell^*} \omega T(t) &= \lambda_e \omega \Sigma C_i(t) \\ &= \lambda_e \left[\frac{\bar{\beta} T(t)}{\ell^*} - \Sigma \lambda_i C_i(t) \right] \\ &= \frac{\lambda_e \bar{\beta} T(t)}{\ell^*} - \lambda_e \left[\omega T(t) + \frac{(\bar{\beta} - \rho)}{\ell^*} T(t) \right] \end{aligned} \quad (A-11)$$

Consolidating terms and dividing through by $T(t)$:

$$\omega \left[\omega + \frac{\bar{\beta} - \rho}{\ell^*} + \lambda_e \right] = \frac{\lambda_e \rho}{\ell^*} \quad (A-12)$$

or

$$\omega = \frac{\lambda_e \rho}{(\bar{\beta} - \rho) + \ell^* (\omega + \lambda_e)} \quad (A-13)$$

The inhour equation is therefore a special case of the dynamic period equation.

Appendix B

Safety Analysis Supporting General Use of the MIT-CSDL
Non-Linear Digital Controller (Ref. 2)

The safety evaluation that was prepared in support of the general use of the MIT-CSDL Non-Linear Digital Controller or NLDC on the 5 MWt MIT Research Reactor is reviewed here. As discussed in section 5.1 of this report, the NLDC is designed in terms of general principles and may therefore be used to control the neutronic power of many different reactors.

The extent of the Massachusetts Institute of Technology's request to the U.S. Nuclear Regulatory Commission regarding licensing of the NLDC should be clearly understood. Approval was requested for general use on the MIT Research Reactor of controllers that incorporate the concept of feasibility of control as specified by Equations 5.1.1-3 and 5.1.1-4. These are the standard absolute and sufficient reactivity constraints. It was further requested that there be no limitation in the reactivity worth of any absorber associated with these controllers. It was also specified by MIT that these controllers would be separate from the reactor's safety system and that control revert to manual if too short a period existed. Approval was not requested for any specific control law. In this regard, it should be remembered that the decisions of the specified control law are compared to those of the reactivity constraint and, should there be a disagreement, the decision of the latter has precedence. Use of a control law by itself will not guarantee the absence of challenges to the safety system. The approved approach is to combine use of a plant-specific control law with the appropriate supervisory constraint in a controller having a multi-tiered structure.

Listed below in outline form are the arguments contained in the safety analysis that was submitted to the U.S. Nuclear Regulatory Commission in support of extending use of the MIT-CSDL Non-Linear Digital Controller to shim blades. The argument shows that the controller is designed to prevent challenges to the nuclear safety system and that a "defense-in-depth" coverage of the reactor existed.

- (1) Simulation studies had been performed in which the NLDC successfully changed the power on a computer model of the MIT Research Reactor.
- (2) The NLDC had been experimentally tested using an absorber of low reactivity worth. All tests were successful with no need for safety system action.
- (3) The controller theory and the experimental results had been documented and published.
- (4) An investigation had been made of the "loss of regulation" problem. Methods for precluding continuous blade withdrawals had been implemented.

- (5) A full-scale mockup had been built of the blade drive to test both the position-indicator system and the speed control. The results had shown that excessive positive reactivity insertions would not occur.
- (6) An auxiliary period trip would be functional. It would cause a transfer to manual and sound an alarm should the instantaneous period become too short.
- (7) The position-indicator system and software would restrict the blade's travel to the allowed band about the bank height.
- (8) Surveillance would be performed of the differential and integral blade worths, blade position indication, and the auxiliary period trip at specified frequencies.
- (9) The controller interface was designed so that all existing interlocks and safety circuits would function normally.
- (10) Control would revert to manual and an alarm would sound if the circuits monitoring the computer, the interface, or the software sensed a fault.
- (11) If the operator wished to take control, he or she need only move the shim blade control switch. This action would transfer control to manual and give the operator direct control of the blades.
- (12) Only one shim blade could be withdrawn by the closed-loop controller at a time.
- (13) Simultaneous manual and automatic control would not be possible. Manual control would take precedence.
- (14) Simultaneous movement of two or more blades would not be possible because all control signals, either manual or automatic, would pass through the "blade-selector" switch and that switch could be manually positioned to only one blade at a time.
- (15) The computer system would be totally contained at the reactor site and continuously within the viewing area of the licensed console operator.
- (16) The MIT Research Reactor would remain under the direct supervision of a licensed operator at all times.
- (17) The controller is designed so that the property of 'feasibility of control' is maintained. This capability means that the safety system should not be challenged as the result of any action initiated by the control law.
- (18) The control system was and would remain completely separate from the nuclear safety system.

- (19) The capability of the nuclear safety system to protect the core from fuel damage would be unchanged because the nature of any possible challenge from the control system would not be altered by the use of digital automatic control.

As noted previously, approval was received on 2 April 1985 from the U.S. Nuclear Regulatory Commission for general use of controllers on the 5 MWt MIT Research Reactor that incorporate the concept of feasibility of control as defined by the standard absolute and sufficient reactivity constraints. Approval was later obtained under 10 CFR 50.59 for use of the alternate absolute and sufficient reactivity constraints.

The successful use of the MIT-CSDL Non-Linear Digital Controller for the adjustment of power on the 5 MWt MIT Research Reactor has demonstrated that digital systems can be used safely in the closed-loop control of reactor power. Furthermore, it has been shown that such control systems are licensable. The key issue in obtaining regulatory approval appears to be that the controller be separate from the safety system so that the latter is not in any way impeded in the performance of its intended function. Other factors are that the controller be designed so that its use will not result in a challenge to the safety system, that it be extensively tested, and that it be at least as reliable as existing manual or analog systems. It should also be noted that the reactivity worth of the absorber associated with a digital controller need not be a significant issue. Incorporation of the reactivity constraint approach in the controller's design is one means of accomplishing this objective.

A second important conclusion that is illustrated by the MIT experience in using a digital system for reactor control is that simulation studies are not by themselves sufficient for qualifying digital controllers. Ex-core mockups and on-line testing with absorbers of low worth such as are available on research and test reactors are necessary in order to identify potential malfunctions in the interface between the digital system and the blade drives, to assess the effect of instrument noise, and to gain confidence in the approach.

A final conclusion evident from the MIT work is that the existence of a closed-loop, digital controller has not diminished the operator's role. His, or her, responsibility for the safe operation of the reactor remains total. However, it is anticipated that his, or her, ability to fulfill that responsibility will be enhanced.

-C1-

Appendix C

MITR Integral and Differential Shim Blade and Regulating Rod
Reactivity Worth Curves

INTEGRAL REACTIVITY WORTH (β)

PM 6.5-16.2

BLADE #4

INTEGRAL WORTH CURVE

7/27/87

2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

BLADE HEIGHT (in)

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Associate Director, Engineering
Argonne National Laboratory
Building 207
9700 South Cass Avenue
Argonne, IL 60439-4841
312/972-3293

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SP-100 Program
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804/385-2810

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Los Alamos National Laboratory
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Los Alamos, NM 87545
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Frederick R. Best
Texas A&M University
Nuclear Engineering Department
129 Zachry Engineering Center
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409/845-4101

Billy Bingham
Babcock & Wilcox
3315 Old Forest Rd.
Lynchburg, VA 24506
804/385-3267

Harvey S. Bloomfield
NASA Lewis Research Center
MS 301-5
21000 Brookpark Rd.
Cleveland, OH 44135
216/433-6131

Dick J. Bohl
MMW Project Manager
Los Alamos National Laboratory
MS E-561
P. O. Box 1663
Los Alamos, NM 87545
505/667-5590

William U. Borger
U. S. A. F.
AFWL/POO
Wright Patterson AFB, OH 45433
513/255-6226

Maj. David R. Boyle
U. S. A. F.
AFWL/AWYS
Kirtland AFB
Albuquerque, NM 87117-6008
505/846-4987

Maj. Michael Brasher
U. S. A. F.
AFSTC/TPC
Kirtland AFB
Albuquerque, NM 87117-6008
505/846-4987

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THIS PAGE

Robert Brengle
Space Nuclear Power Atomics International
Rockwell International Corp.
Rocketdyne Division – MS-HA04
6633 Canoga Ave.
Canoga Park, CA 91303
818/718-3404

David Buden
Idaho National Engineering Laboratory
P. O. Box 1623
Idaho Falls, ID 83415-3511

Wade P. Carroll
U.S. Department of Energy, NE-52
19901 Germantown Road
Germantown, MD 20874
301/353-2898

Len Caveny
SDIO Innovative S&T Office
The Pentagon
Washington, DC 20301-7100
202/693-1530

Ed P. Coomes
Senior Development Engineer
Pacific Northwest Laboratory
MS K5-02
P. O. Box 999
Richland, WA 99352
509/375-2549

Roy Cooper, Jr.
Program Manager
Oak Ridge National Laboratory
Building 4500S
P. O. Box 2008
Oak Ridge, TN 37831-6079
615/574-4470

Richard C. Dahlberg
General Atomics
P. O. Box 85608
San Diego, CA 92138
619/455-2997

Nils J. Diaz
University of Florida
I. N. S. P. I.
College of Engineering
Gainesville, FL 32611
904/392-1427

Dale S. Dutt
Westinghouse Hanford Co.
MS L5-60
P. O. Box 1970
Richland, WA 99352
509/376-9336

Mohamed El-Genk
Institute for Space Nuclear Power Studies
Chemical and Nuclear Engineering Department
University of New Mexico
Albuquerque, NM 87131
505/277-5442

Dr. Colin R. Fisher
General Atomics
P. O. Box 85608
San Diego, CA 92138-5608
619/455-3821

Mr. C. Brad Greene
Garrett Fluid Systems
1300 W. Warner
Tempe, AZ 85282

Richard Hahn
Director of Office of Weapons Research
U. S. Department of Energy
Development and Testing
DP-24
Washington, DC 20545

Mr. J. K. Hartman
Director of Energy Research Division
U. S. Department of Energy
San Francisco Operations Office
1333 Broadway Ave.
Oakland, CA 94612

F. Eric Haskin
Institute for Space Nuclear Power Studies
Chemical and Nuclear Engineering Department
University of New Mexico
Albuquerque, NM 87131
505/277-8026

Robert Haslett
Project Director
Grumman Aerospace Corp.
MS B09-25
1111 Stewart Ave.
Bethpage, NY 11714
516/575-3924

DO NOT MICROFILM
THIS PAGE

Eugene Edward Hoffman
U. S. Department of Energy
P. O. Box 2001
200 Administration Road
Oak Ridge, TN 37831-8600
615/576-0735

Mr. J. L. Hooper
Sr. Program Mgr., Technology Management
Office
U. S. Department of Energy
Chicago Operations Office
9800 South Cass Avenue
Argonne, IL 60439

Mr. R. J. Hornung
Boeing Aerospace
MS 2E-98
P. O. Box 3999
Seattle, WA 98124-2499
206/251-4007

Lon Isenberg
Jet Propulsion Laboratory, MS 601-257
SP-100 Advanced Mission Integration
4800 Oak Grove Drive
Pasadena, CA 91109
818/354-1903

Eshan Kahn
BDM Corp.
1501 BDM Way
McLean, VA 22102
703/848-5619

Walter Y. Kato
Brookhaven National Laboratory
Upton, NY 11973
516/282-2444

Elliot B. Kennel
U. S. A. F.
AFWAL
AFWAL/POOS
Wright Patterson AFB, OH 45433-6563
513/255-6241

Walter Kirchner
Program Manager, New Production Reactor
Los Alamos National Laboratory
MS K-575
P. O. Box 1663
Los Alamos, NM 87545

Mr. W. E. Lambert
Acting Division Director
Advanced Concept Research Division
U. S. Department of Energy
San Francisco Office
1333 Broadway
Oakland, CA 94612

Nelson A. Hannan
Argonne National Laboratory
9700 South Cass Avenue
Argonne, IL 60439-4891

James H. Lee, Jr.
S-Cubed
2501 Yale Blvd. SE
Albuquerque, NM 87106
505/247-8688

Roger Lenard
SDIO
SDIO/TICK
Washington, DC 20301-7100
202/695-8839

Hans Ludewig
Brookhaven National Laboratory
Building 701
Upton, NY 11719
516/282-2624

Dr. Tom Mahefky
U. S. A. F.
WRDC/POO
Wright Patterson AFB, OH 45433-6563
509/255-6226

John A. Martinell
Idaho National Engineering Laboratory
P. O. Box 1625
Idaho Falls, ID 83415
208/526-0517

Alan T. Marriott
Jet Propulsion Laboratory
MS 122-108
4800 Oak Grove Dr.
Pasadena, CA 91109
818/354-1902

Don Mateo
General Electric Co.
Astro Space Division
P. O. Box 8555
Philadelphia, PA 19101
215/354-4120

DO NOT MICROFILM
THIS PAGE

R. Bruce Matthews
Los Alamos National Laboratory
MS E-505
P. O. Box 1663
Los Alamos, NM 87545
505/667-2358

John D. Metzger
Los Alamos National Laboratory
MS K-575
P. O. Box 1663
Los Alamos, NM 87545
505/667-9646

Jack F. Mondt
CAL TECH / JPL
4800 Oak Grove Dr.
Pasadena, CA 91109
818/354-1900

Ron E. Murata
General Electric Co.
6835 Via Del Oro
Box 530954
San Jose, CA 95153
408/365-6467

Lawrence A. Neimark
Argonne National Laboratory
Building 212
9700 South Cass Avenue
Argonne, IL 60439
312/972-5177

Don F. Nichols
U. S. A. F.
WL/AWYS
Kirtland AFB, NM 87117
505/844-0253

Monte B. Parker
Los Alamos National Laboratory
MS K-560
P. O. Box 1663
Los Alamos, NM 87545
505/667-2180

Philip R. Pluta
General Electric Co.
SP-100 Programs
6835 Via Del Oro
San Jose, CA 95153-5354
408/365-6305

Dr. James R. Powell
Brookhaven National Laboratory
Upton NY 11973
516/282-2440

William A. Ranken
Los Alamos National Laboratory
MS K-552
P. O. Box 1663
Los Alamos, NM 87545
505/667-0965

Ned S. Rasor
Rasor Associates
253 Humboldt Ct.
Sunnyvale, CA 94089
408/734-1622

Gregory M. Reck
NASA Headquarters
Code RP
Washington, DC 22043
202/453-2847

Dr. John Reuscher
Texas A&M University
Nuclear Engineering Dept.
129 Zachry Engineering Center
College Station, TX 77843-3133
409/845-4108

Richard E. Rice
Idaho National Engineering Laboratory
P. O. Box 1625
Idaho Falls, ID 83415-3511
208/526-1992

William H. Roach
S-Cubed
2501 Yale Blvd. SE
Albuquerque, NM 87106
505/764-3141

M. Frank Rose
Space Power Institute
Auburn University
231 Leach Science Center
Auburn, AL 36849
205/844-5894

Don Roy
Babcock & Wilcox
3315 Old Forest Rd.
Lynchburg, VA 24506
804/385-3297

DO NOT MICROFILM
THIS PAGE

Ronald E. Schmidt
General Electric Co.
P. O. Box 8555
Philadelphia, PA 19101
215/354-3089

A. Dan Schnyer
NASA Headquarters
Code RP
Washington, DC 20546
202/453-2855

Michael J. Schuller
Air Force Weapons Laboratory
AFWL/AWYS
Kirtland AFB
Albuquerque, NM 87117-6008
505/846-4987

Dr. Robert Seale
Nuclear Engineering Department
University of Arizona
Tucson, AZ

Dr. Duane C. Sewell
Consultant
4265 Drake Ct.
Livermore, CA 94500
415/447-1618

Lt. Col. Joseph A. Sholtis
U. S. A. F.
DET 1 AFISC/SNRA
Kirtland AFB
Albuquerque, NM 87117
505/846-9897

John M. Smith
NASA Lewis Research Center
MS 301-5
21000 Brookpark Rd.
Cleveland, OH 44135
216/433-6130

R. Joe Sovie
Deputy, Power Technology Division
NASA Lewis Research Center
MS 301-5
21000 Brookpark Rd.
Cleveland, OH 44135
216/433-6129

Marland L. Stanley
Idaho National Engineering Laboratory
P. O. Box 1625
Idaho Falls, ID 83415-3511
208/526-2041

Gerhard Stapfer
Jet Propulsion Laboratory
MS 122-108
4800 Oak Grove Dr.
Pasadena, CA 91109
818/354-3064

Bobby G. Strait
Los Alamos National Laboratory
MS K-551
P. O. Box 1663
Los Alamos, NM 87545
505/667-2192

Dr. Vincent C. Truscello
SP-100 GES Project Office
Jet Propulsion Laboratory
MS 122-108
4800 Oak Grove Dr.
Pasadena, CA 91109
818/354-1820

Bruce G. Varnado
ERC - EESCO
Suite 202
1717 Louisiana NE
Albuquerque, NM 87110
505/265-5592

Richard L. Verga
SDI Office / SLKT
The Pentagon
Washington, DC 20301-7100
202/693-1660

Walter L. Von Flue
Department of Energy
San Francisco Operations Office
1333 Broadway
Oakland, CA 94612
415/273-6445

Susan S. Voss
Los Alamos National Laboratory
MS K-551
P. O. Box 1663
Los Alamos, NM 87545
505/667-1660

Earl J. Wahlquist
Director, Defense Energy Projects
U. S. Department of Energy
MS NE-52
Germantown Building
Washington, DC 20545
301/353-3321

DO NOT MICROFILM
THIS PAGE

Carl E. Walter
Lawrence Livermore National Laboratory
MS L-197
7000 East Avenue
P. O. Box 808
Livermore, CA 94550
415/423-8822

John W. Warren
U. S. Department of Energy
NE-521 Germantown
Washington, DC 20545
301/353-6491

Joe Wetch
Space Power Inc.
621 River Oaks Parkway
San Jose, CA 95134
408/434-9500

Jack F. Wett
Space and Defense Programs
Westinghouse Electric Corp.
Advanced Energy Systems Division
P. O. Box 158
Madison, PA 15663-0158

Richard D. Widrig
Manager, Multimegawatt Program
Pacific Northwest Laboratory
MS K-134
P. O. Box 999
Richland, WA 99352
509/375-2220

Robert L. Wiley
Auburn University
Washington Office
6998 Camelback Lane
Columbia, MD 21045
205/826-5894

Alice Williams
U. S. Department of Energy
Idaho Operations Office
785 DOE Place, MS 1134
Idaho Falls, ID 83402
208/526-0972

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THIS PAGE**

Sandia National Laboratories

400 A. C. Marshall
5165 K. Boldt
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3151 W. I. Klein
8524 J. A. Wackerly

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W. I. Klein, 3151

NUCLEAR REACTOR LABORATORY

AN INTERDEPARTMENTAL CENTER OF
MASSACHUSETTS INSTITUTE OF TECHNOLOGY