MASTER

Inelastic Strong Interactions at High Energies

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Abstract

We have continued our study of Grand Unified Field Theories, concentrating on the symmetry breaking by Higgs bosons with the inclusion of single-gauge-loop corrections at both zero and nonzero temperature.

We have found that the possible metastable phases of an $SU(N)$ gauge theory broken by a Higgs boson of the adjoint representation are $SU(r) \times SU(N-r) \times U(1)$ only. We displayed the phase diagram of broken $SU(N)$ and $O(2N)$ theories as a function of two renormalization-group-invariant parameters. All phase transitions were found to be first order and, at least in the Coleman-Weinberg regime, the phase diagram is completely different from that of the tree approximation; for example, several phase transitions may take place if the mass parameter (effectively the temperature) of the theory is decreased.

We have obtained upper bounds for the phase-transition temperature in terms of the Higgs boson mass both for an $SU(N)$ field theory with a Higgs boson in the adjoint representation and for the Glashow-Weinberg-Salam theory of electroweak interactions.

Finally, we found upper bounds for the masses of Higgs bosons in both of the above-mentioned types of theories in terms of the supercooling ratio, provided the expansion rate of the universe is too large and the tunneling rate is too small to complete the phase transition before the symmetric phase becomes unstable.

Grand Unified Field Theories

Our prime activity during the current contract year was the study of various problems in the area of Grand Unified Quantum Field Theories (GUT's). The first GUT was proposed by Georgi and Glashow¹ and was subsequently studied by many authors². General theorems show that the originally proposed SU(5) symmetry group¹ must be at least an important link in the chain of broken symmetry groups unifying strong, electromagnetic and weak interactions, provided all "light" fermions have been observed already save the elusive top quark.³

One of the most important of the numerous problems to be investigated is the study of symmetry breaking mechanisms by which higher groups break down to SU(5) and then to SU(3)xSU(2)xU(1) and finally to SU(3)xU(1). The various symmetry groups can be broken either by dynamical symmetry breakdown⁴ or by the use of Higgs bosons.² We have concentrated on various outstanding problems in the latter approach. As SU(3)x SU(2)xU(1) seems to be the correct approximate symmetry group of the low-energy world, it should correspond to the ground state of the broken SU(5) theory. To decide whether it really is requires the determination of the phase diagram of the system as a function of certain relevant variables.

First we showed that the only possible stable (or for that matter metastable) phases of an SU(N) invariant theory broken by a Higgs boson of the adjoint representation are SU(r)xSU(N-r)xU(1).⁵ We shall label these phases by P_r . Naturally, it is impossible to plot the phase diagram exactly, and the simplest approximation one can use is the tree approximation. Tree-approximation phase diagrams were investigated first by Li.⁶ If we write the effective potential⁷ in the tree approximation as

$$V_0 = \frac{1}{2}m^2 \text{Tr} \Phi^2 + \frac{1}{4}\lambda_1 (\text{Tr} \Phi^2)^2 + \frac{1}{2}\lambda_2 \text{Tr} \Phi^4, \quad (1)$$

then spontaneous symmetry breaking occurs if and only if $m^2 < 0$, and the phase

is determined solely by the sign of λ_2 . If $\lambda_2 < 0$ the absolute minimum of the potential is in phase P_4 . Otherwise it is in P_3 (the physically expected phase.)

We have investigated the phase diagram in the single-gauge-loop approximation⁶. This approximation is valid if $\lambda \ll \alpha \ll 1$, where $\alpha = g^2/4\pi$ and g is the gauge coupling constant. This is the so-called Coleman-Weinberg regime.⁸ In this approximation only one combination of the coupling constants is renormalized and consequently it can be eliminated by an appropriate choice of the renormalization point. The surviving coupling constant and an other renormalization-group invariant (a combination of the mass parameters divided by α^2) comprise the set of variables in which the phase diagram can be plotted.

We showed that in this case in contrast to the tree approximation all phase transitions are of first order. This had already been known for some special cases^{8,9}. In a substantial part of the phase diagram the system may undergo several phase transitions if we change the mass (something equivalent to changing the temperature) at a fixed coupling constant. It is interesting and important to note that the tree-approximation phase diagram is incorrect even in the approximation m^2/α^2 , λ/α^2 large, m^2/λ fixed, the very limit in which the tree part of the potential dominates the gauge-loop part. The reason for this apparent contradiction is that with our choice of the renormalization constants the tree part has degenerate minima and the degeneracy is resolved by the gauge-loop contribution only.

A natural extension of our work to nonzero temperature has also been initiated.¹⁰ Such an extension has important cosmological consequences. The early thermal history of the universe strongly influences the abundance of monopoles and the baryon to photon ratio at the present time. Since all phase

transitions are of first order the normal way to proceed would be to calculate T_c , the critical temperature at which the symmetric vacuum becomes degenerate with one of the vacua belonging to a broken symmetry. The system does not immediately undergo a phase transition but rather it supercools and remains in the metastable state. The metastable state can either tunnel to the true vacuum state in a finite time (scenario 1)¹¹ or if the tunneling rate is too small and the expansion rate is too large then tunneling may never complete the phase transition and one has to wait until the phase collapses (ceases to be a metastable state) (scenario 2).¹² The tunneling rates at nonzero temperature are calculated with an extension of Coleman's technique of Euclidean instantons.¹³ At nonzero temperature these instanton solutions satisfy a four-dimensional equation with three spatial dimensions and one dimension corresponding to the inverse temperature. As Linde¹⁴ has pointed out, if the tunneling temperature $T_0 \gg m_H$, the Higgs mass, the temperature dependence of the solutions can be treated in a simple way. Assuming $O(3)$ symmetry in the coordinates the calculation of tunneling rates reduces to the numerical solution of a one dimensional problem. A similar approximation has been applied in all subsequent investigations¹¹. The validity of such an approximation seems to be self-evident because a multiplier g always appears when one tries to calculate T_c , namely $T_c \sim gm_H$. On the other hand if one investigates this question a little more deeply the situation is not so simple. We have used the single-gauge-loop approximation potential $V(\phi, T)$ to investigate the relation of T_c to the other parameters of the theory, including the Higgs boson mass.¹⁰ It turns out that there is a relation among the vector boson mass, the Higgs boson mass and the critical temperature of the following kind in a broken $SU(N)$ theory;

$$m_H^2 = \alpha \pi N T_c^2 G(y)$$

where $y = m_V^2/T_c^2$, in agreement with the assertion that roughly speaking

$m_H \sim gT_c$. We have found numerically that the function $G(y)$ has a minimum at

$y = 7.00$ and $G_{\min} = 1.51$. These numbers imply for an SU(5) theory with the currently accepted value of the coupling constant, $\alpha \approx 1/45$, that $m_H \geq .73 T_c$. Since in the case of a substantial supercooling (which may be needed to avoid an excessive number of monopoles) $T_0 \ll T_c$, not only can the relation $T_0 \gg m_H$ never be valid but rather the inverse relation must be true. Consequently the calculations of tunneling rates¹¹ should be reconsidered.

A further consequence of the bound we have obtained can be derived if we accept scenario 2 for the transition from the symmetric to the broken state. Then the real transition temperature satisfies the relation

$$m^2 + \alpha \pi N T_0^2 = 0, \quad (3)$$

with $m^2 < 0$ and the Higgs mass is larger than the Coleman-Weinberg value $m_{\text{HCW}}^2 = 3\alpha N m_V^2/2\pi$. After expressing m^2 in Eq. (3) by the Higgs mass we obtained an upper bound on the Higgs boson mass as a function of the supercooling ratio, T_0/T_c , as follows

$$m_{\text{HCW}}^2 \leq m_H^2 \leq \frac{m_{\text{HCW}}^2}{1 - \frac{2T_0^2}{G_{\min} T_c^2}} \quad (4)$$

providing tight bounds for the mass of the Higgs boson if a substantial supercooling is required.

We showed that similar bounds can be derived for the Weinberg-Salam theory¹⁵ of electroweak interactions as well. A relation similar to Eq. (2) can also be derived in the single-gauge-loop approximation at nonzero temperature provided the symmetry is broken by a doublet of Higgs bosons.

We obtained

$$m_H^2 = \frac{3}{32\pi} \alpha \frac{2 + \sec^4 \theta}{\sin^2 \theta} T_c^2 G_W(y), \quad (5)$$

where $y = M_W^2/T_c^2$ and θ is the Weinberg angle. The function $G_W(y)$ is bounded

from below and it has a minimum at $y = 6.17$ providing an upper bound on the phase transition temperature as follows,

$$m_H \geq .35 T_c ,$$

where the value $\sin^2 \theta = 0.23$ was used for the Weinberg angle. Again we arrive at the conclusion that the relation $T_0 \gg m_H$ cannot be satisfied and tunneling rates calculated using that assumption¹¹ have to be reconsidered.

We also found inequalities similar to Eq. (4) for $SU(2) \times U(1)$ broken by a doublet provided we accept scenario 2 for the phase transition, namely

$$m_{HCW}^2 \leq m_H^2 \leq \frac{m_{HCW}^2}{1 - \frac{G_W(0)T_0^2}{G_{min}T_c^2}} \quad (6)$$

where $m_{HCW} = 9.18 \text{ GeV}$ and $G_W(0)/G_{min} = 1.32$. Again, we have a connection between the supercooling ratio and the mass of the Higgs boson.

The natural extension of our work to the study of phases of the theory at nonzero temperature including the calculations of abundance of monopoles and the magnitude of entropy production (important for the calculation of the baryon/photon ratio) is in progress, and will be described in the accompanying proposal.

Our investigations related to the new convergent perturbation expansion of the anharmonic oscillator have not been completed yet. At present our investigations are concentrated on the proof of the convergence of an expansion for coupled anharmonic oscillators similar to the one obtained for a single anharmonic oscillator.¹⁶

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