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P. Moller, J. Nix, U. Swiatecki

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CALCULATED FISSION PROPERTIES OF THE HEAVIEST ELEMENTS

P. MÖLLER

Lawrence Livermore National Laboratory, Livermore, CA 94550

J. R. NIX

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545

W. J. SWIATECKI

Nuclear Science Division, Lawrence Berkeley Laboratory, Berkeley, CA 94720

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Abstract

Experimental results on the fission properties of nuclei close to ^{264}Fm show sudden and large changes with a change of only one or two neutrons or protons. The nucleus ^{258}Fm , for instance, undergoes symmetric fission with a half-life of about 0.4 ms and a kinetic energy peaked at about 235 MeV whereas ^{256}Fm undergoes asymmetric fission with a half-life of about 3 h and a kinetic energy peaked at about 200 MeV. Qualitatively, these sudden changes have been postulated to be due to the emergence of fragment shells in symmetric fission products close to ^{132}Sn . Here we present a quantitative calculation that shows where high-kinetic-energy symmetric fission occurs and why it is associated with a sudden and large decrease in fission half-lives. We base our study on calculations of potential-energy surfaces in the macroscopic-microscopic model and a semi-empirical model for the nuclear inertia. For the macroscopic part we use a Yukawa-plus-exponential (finite-range) model and for the microscopic part a folded-Yukawa (diffuse-surface) single-particle potential. We use the three-quadratic-surface parameterization to generate the shapes for which the potential-energy surfaces are calculated. The use of this parameterization and the use of the finite-range macroscopic model allows for the study of two touching spheres and similar shapes. Since these shapes are thought to correspond to the scission shapes for the high-kinetic-energy events it is of crucial importance that a continuous sequence of shapes leading from the nuclear ground state to these configurations can be studied within the framework of the model.

We present the results of the calculations in terms of potential-energy surfaces and fission half-lives for heavy even nuclei. The surfaces are displayed in the form of contour diagrams as functions of two moments of the shape. They clearly show the appearance of a second fission valley, that leads to scission configurations close to two touching spheres, for fissioning systems in the vicinity of ^{264}Fm . Fission through this new valley leads to much shorter fission half-lives than fission through the old valley. The reason is that the inertia associated with motion in the new valley is much smaller than in the old valley. The large decrease in the inertia is the reason for the unexpectedly short half-life for ^{258}Fm . Earlier, it had appeared that the the

short half-life was due to the disappearance of the second peak in the barrier below the nuclear ground-state energy. However, we find that the second peak is still about three MeV above the ground state in the new valley for ^{258}Fm and that the short half-life is instead due mainly to the lower inertia in the new valley.

1 Introduction

The advent of the macroscopic-microscopic Strutinsky shell-correction method [1,2] about 20 years ago made possible detailed theoretical studies of the fission process. With this method the potential energy of a nucleus can be calculated for arbitrary shapes, within given shape parameterizations. Coupled with a wealth of new experimental results this has led to an enormous increase in our understanding of nuclear shape changes during fission and also to a better understanding of the stability of elements at the end of the periodic system. For an extensive review of some of these developments see [3]. During fission the nucleus changes its shape from a usually slightly deformed ground-state shape through saddle-point shapes and scission configurations into two separated fragments. Measurements of fragment mass asymmetries, fragment kinetic energies, fission half-lives, neutron emission, fission barrier heights and correlations between these quantities yield detailed information about the nucleus during various stages of the fission process. Here we apply our model to the ^{264}Fm region for which new and somewhat unexpected experimental data are available. Our goal is to understand the nature of the fission process for the nuclei for which these new data are available and then to make predictions of properties of other nuclei in the vicinity of ^{264}Fm and of fission half-lives for heavier even nuclei.

1.1 EARLIER THEORETICAL AND EXPERIMENTAL RESULTS

The first success of the macroscopic-microscopic method was the interpretation of fission isomeric states as secondary minima in the potential-energy surface, corresponding to very deformed, approximately spheroidal shapes with a ratio of about 2:1 of the major to minor axis. This interpretation was later confirmed by experiments that measured the actual deformation of the nucleus in its fission isomeric state.

The second minimum in the potential-energy surface splits the fission barrier saddle point into two saddles, a first and a second saddle. A next major step in the study of the fission process for heavy elements was the experimental determination of the heights of the first and second saddle points and the height of the second minimum relative to the ground state for a large number of nuclei throughout the actinide region. Calculations of these barrier heights were performed within the framework of several of the macroscopic-microscopic models. Results of the calculations usually agreed with the experimental data to within an MeV or so.

The successful theoretical description of the structure of the fission barrier involved considering axially asymmetric shapes at the first saddle point and mass-asymmetric

shapes at the second saddle. The long mystery of why actinide nuclei undergo asymmetric fission was resolved. Mass-asymmetric shape degrees of freedom lowered the outer saddle by up to 3 MeV or so for the lighter actinides. The calculated value of the mass-asymmetry coordinate at the second saddle and the actually measured asymmetry of the fission fragments were found to be closely correlated for elements throughout the actinide region.

Calculated fission barriers served as a starting point for subsequent calculations of fission half-lives. Basically two types of models were used. The first type of model determines a one-dimensional fission barrier from the multi-dimensional potential-energy surface and selects a model for a semi-empirical inertia for the motion in the fission direction. The penetrability of the barrier and the corresponding fission half-lives are then determined by use of the WKB method. The second type of model is more complicated. In this model the inertia is calculated from some microscopic model, usually a cranking model, and the problem is treated in several dimensions, in the sense that the penetrability is calculated along several paths in a multi-dimensional deformation space. Fission is then thought to take place along the path with the highest penetrability. The simpler model has succeeded in calculating fission half-lives for even nuclei throughout the actinide region with a root-mean-square deviation of less than 2 for $\log(\tau_{1/2})$. The more sophisticated models usually have larger discrepancies.

To determine the stability of heavy elements the half-lives with respect to α - and β -decay must also be considered. These half-lives can usually be calculated with greater accuracy than the fission half-lives, since they depend only on nuclear ground-state properties. One application of models of the macroscopic-microscopic type has been to predict the properties of elements in the superheavy region, beyond the heaviest presently known elements. The predicted properties may then serve as a guide to the design of reactions leading to these elements. Another application is to do calculations of astrophysical interest. In this case one often has to make calculations for situations that are not accessible to experiment, but whose results are crucial for the understanding of astrophysical processes. It is then desirable to have available models that describe well properties of nuclei in the known regions of nuclei and which one can expect work equally well also outside the regions where the model parameters were determined.

1.2 THE ^{264}Fm REGION

We shall here not go very far from the regions of known nuclei, but instead focus most of our attention on the very interesting region in the vicinity of ^{264}Fm . This nucleus, which has not been seen experimentally, is of particular interest since in symmetric fission it would divide into the two doubly magic ^{132}Sn nuclei. The ground-state microscopic energy for each of these ^{132}Sn nuclei is -12 MeV according to [4], giving a combined total microscopic energy of -24 MeV. It is clear from studying tables of the microscopic energy for Sn isotopes close to ^{132}Sn that the microscopic effects grow rapidly as one approaches ^{132}Sn . For example, the microscopic ground-state energy

for ^{126}Sn , the product in the symmetric fission of ^{252}Fm , is only -4 MeV. For two nuclei combined it is -8 MeV, which is 16 MeV higher than for the products in the symmetric fission of ^{264}Fm . One may ask how far from $Z = 100$ and $N = 164$ the effect of the magic or near magic fragment shells manifest themselves and also how and at what stage in the fission process the effects become important, that is, how far inside the scission point some of the shell effects that are present in fully separated fragments remain. We shall now address these questions.

1.2.1 New experimental results in the ^{264}Fm region

First let us briefly review the experimental situation. As one sweeps through the actinide region from uranium to fermium many fission properties vary fairly smoothly. With increasing proton number there is a decrease in fission-fragment mass asymmetry, an increase in fission-fragment kinetic energies and a decrease in fission half-lives. Except for the fission half-lives these quantities vary fairly slowly with neutron number. However, at ^{258}Fm there are sudden changes in all of these quantities. Fission becomes symmetric with a very narrow mass distribution, the kinetic energy of the fragments is about 35 MeV higher than in the asymmetric fission of ^{256}Fm and the fission half-life is 0.38 ms for ^{258}Fm compared to 2.86 h for ^{256}Fm . We take information about the experimental fission half-lives from [5,6,7,8,9]. The first observation of the onset of symmetric fission in the region at the end of the periodic system was the study of ^{257}Fm fission by [10]. Subsequently, more observations of symmetric fission have been made in this region by [11,12]. Later, more extensive measurements on ^{258}Fm and other elements in this region by [13] have shown that there often are two components in the kinetic-energy distribution. For ^{258}Fm most of the events are distributed around 235 MeV but a few are distributed around 200 MeV. This distribution of fission events has been characterized as "bimodal" fission. It has been suggested that in bimodal fission there are two distinct symmetric fission valleys separated by a ridge [13]. One valley leads to elongated scission shapes similar to the scission configuration for lighter actinides, with a low kinetic energy, and another valley leads to the very compact scission configuration of two touching spheres. We refer to [13] for an extensive discussion of the results but we reproduce here as our fig. 1 a figure from [13] that shows mass and kinetic-energy distributions for these elements.

1.2.2 Previous theoretical results for the ^{264}Fm region

One previous study of the effect of fragment shells in the ^{264}Fm region by use of a macroscopic-microscopic model is presented in the series of papers [14,15,16,17]. However, in all of these studies, the models used have several deficiencies when applied to the present problem. For the microscopic model the two-center oscillator model is used. This potential is spuriously high in the neck region of the nucleus [16,18], which leads to fragment shell effects that manifest themselves too early during the fission process. In addition, the parameterization used is incapable of generating shapes that are crucial for the study of compact scission shapes and the accompanying high

fragment kinetic energies. For instance, it cannot generate the important configuration of two touching spheres. For the macroscopic model the liquid-drop model with a modified set of constants [16] is used. This model severely overestimates the energies for nuclei with pronounced necks. For a configuration of two touching spherical ^{132}Sn nuclei the resulting liquid-drop energy is 35 MeV higher than the energy given by the more realistic Yukawa-plus-exponential finite-range macroscopic model. Thus, we feel that many features of the symmetric high-kinetic-energy processes cannot be studied within the framework of this model. It is also not clear how close to spherical shape the end bodies were kept in the above studies. As we will discuss below it is important to keep the end parts of the nuclear shape spherical to see the full fragment shell effect.

The results of calculations by [19,20] show some similarities but also large differences with the results we obtain below. However, in their shape parameterization no independent control is exercised over the shape of the ends of the nucleus. The ends of the nucleus are not kept spherical, and therefore it is not possible to see the full effect of the fragment shells. Their results will be discussed in more detail and compared to ours in section (4.1).

Some studies with the Nilsson modified-oscillator potential have also been performed for elements in this region. The focus of those calculations was mainly on fission barriers and fission half-lives [21,22,23,24,25,26,27]. In general, fission half-lives were quite well reproduced for elements throughout the actinide region. Even the complicated behaviour of the fission half-lives for the elements beyond einsteinium was quite well reproduced by the calculations. The sudden drop in fission half-life at ^{258}Fm was explained as due to the disappearance of the second saddle in the fission barrier below the energy of the nuclear ground state, as has also been suggested by [28,29]. However, in the Nilsson model the perturbed-spheroid (ϵ) parameterization does not permit the generation of nuclear shapes that are even close to two touching spheres. Since such shapes are expected to be of importance for at least some elements in this region one may ask why the model was so successful in this study, limited to fission barriers and half-lives. Since the investigation below will show that the short half-life of ^{258}Fm is due to the low inertia in a new fission valley and *not* to the disappearance of the second peak in the fission barrier, we are forced to conclude that the good agreement with experiment in the Nilsson-model calculation was somewhat fortuitous.

We would also like to mention the static fission model of [30]. In this model one assumes statistical equilibrium among the collective coordinates at the scission point. The energy is calculated for two nuclei with a fixed distance d between the ends of the nuclei. The model for the energy of the system consists of a macroscopic-microscopic model for each individual nucleus plus a macroscopic term giving the interaction energy between the two nuclei. The deformations that are studied are spheroidal shapes for the two fragments. We feel that it is hard to justify many of the assumption of this model. The model also fails to reproduce some of the fission features of ^{258}Fm . In addition the model does not take into account important properties of the fissioning nucleus that influence the fission process, such as the fission barrier structure.

The model we will study below, a macroscopic-microscopic model with a Yukawa-plus-exponential macroscopic model and a folded-Yukawa single-particle potential has also been used earlier for some studies in this region. We only mention here as one example an earlier, unpublished result, that a fission half-life of 2.7 y was obtained for ^{258}Fm when the model was used in a standard way. These earlier results will be discussed below together with the results from the present calculation, which differs from the older calculations only in the sense that we explore additional degrees of freedom of the model. The model itself is not changed.

2 Macroscopic-microscopic model

Our model is of the macroscopic-microscopic type and has been discussed extensively in [31,32,4,33]. We use the model with the parameter set that was determined in the investigation [32], which calculated ground-state masses for 4023 nuclei and fission barriers for 28 nuclei throughout the periodic system. The root-mean-square deviation between experimental and calculated ground-state masses was 0.835 MeV for a set of 1323 masses and 1.331 MeV for the 28 fission barriers. Many other properties such as ground-state deformations are also well described by the model, as is extensively discussed in [33]. The model represents a unified approach to the study of many features of nuclear structure, fission and heavy-ion reactions. Here we apply it to the study of fission of the heaviest elements, in particular to the ^{264}Fm region, where we focus on symmetric fission. Below we shall present some details of the model that are of particular relevance to the present study, but we refer to [31,32,4,33] for a more complete presentation.

In the macroscopic-microscopic model the nuclear energy, which is calculated as a function of shape, proton number Z and neutron number N , is the sum of a macroscopic term and a microscopic term. Thus the total nuclear potential energy can be written as

$$E(Z, N, \text{shape}) = E_{\text{macr}}(Z, N, \text{shape}) + E_{\text{micr}}(Z, N, \text{shape}) \quad (1)$$

There exist several different models for both the macroscopic and the microscopic term. We use a Yukawa-plus-exponential model for the macroscopic term and a folded-Yukawa single-particle potential as a starting point for calculating the microscopic term. They are briefly discussed below.

2.1 MACROSCOPIC MODEL

The complete expression for the Yukawa-plus-exponential macroscopic energy is

$$\begin{aligned} E_{\text{macr}}(Z, N, \text{shape}) = & + M_{\text{H}}Z + M_{\text{n}}N \\ & - a_v (1 - \kappa_v I^2) A \end{aligned}$$

$$\begin{aligned}
& + a_2 (1 - \kappa_2 I^2) B_1 A^{2/3} \\
& + c_0 A^0 \\
& + c_1 \frac{Z^2}{A^{1/3}} B_3 \\
& - c_4 \frac{Z^{4/3}}{A^{1/3}} \\
& + f(k_1 r_p) \frac{Z^2}{A} \\
& - c_a (N - Z) \\
& + W \left(|I| + \begin{cases} 1/A, & Z \text{ and } N \text{ odd and equal} \\ 0, & \text{otherwise} \end{cases} \right) \\
& + \begin{cases} + \left(\frac{\Delta}{A^{1/2}} - \frac{1}{2} \frac{\delta}{A} \right), & Z \text{ and } N \text{ odd} \\ + \frac{1}{2} \frac{\delta}{A}, & Z \text{ or } N \text{ odd} \\ - \left(\frac{\Delta}{A^{1/2}} - \frac{1}{2} \frac{\delta}{A} \right), & Z \text{ and } N \text{ even} \end{cases} \\
& - a_{el} Z^{2.39}
\end{aligned} \tag{2}$$

The quantities c_1 and c_4 are defined by

$$\begin{aligned}
c_1 &= \frac{3 e^2}{5 r_0} \\
c_4 &= c_1 \frac{5}{4} \left(\frac{3}{2\pi} \right)^{2/3}
\end{aligned} \tag{3}$$

In the above expression for the proton form factor f is given by

$$f(k_1 r_p) = -\frac{1}{8} \frac{r_p^2 e^2}{r_0^3} \left[\frac{145}{48} - \frac{327}{2860} (k_1 r_p)^2 + \frac{1527}{1209600} (k_1 r_p)^4 \right] \tag{4}$$

The relative neutron excess I is given by

$$I = \frac{N - Z}{N + Z} \tag{5}$$

The quantity B_1 represents the relative generalized surface or nuclear energy in a model that accounts for the effect of the finite range of the nuclear force. It is given

by

$$B_1 = -\frac{A^{-2/3}}{8\pi^2 r_0^2 a^4} \int \int_V \left(\frac{|\mathbf{r} - \mathbf{r}'|}{a} - 2 \right) \frac{e^{-|\mathbf{r} - \mathbf{r}'|/a}}{|\mathbf{r} - \mathbf{r}'|/a} d^3 r d^3 r' \quad (6)$$

The relative Coulomb energy B_3 is given by

$$B_3 = \frac{15}{32\pi^2} \frac{A^{-5/3}}{r_0^5} \int \int_V \frac{d^3 r d^3 r'}{|\mathbf{r} - \mathbf{r}'|} \left[1 - \left(1 + \frac{1}{2} \frac{|\mathbf{r} - \mathbf{r}'|}{a_{\text{den}}} \right) e^{-|\mathbf{r} - \mathbf{r}'|/a_{\text{den}}} \right] \quad (7)$$

It includes diffuseness corrections to all orders. For spheres one can calculate the quantities B_1 and B_3 analytically. With

$$x_0 = \frac{r_0 A^{1/3}}{a} \quad \text{and} \quad y_0 = \frac{r_0 A^{1/3}}{a_{\text{den}}} \quad (8)$$

one obtains

$$B_1 = 1 - \frac{3}{x_0^2} + (1 + x_0) \left(2 + \frac{3}{x_0} + \frac{3}{x_0^2} \right) e^{-2x_0} \quad (9)$$

and

$$B_3 = 1 - \frac{5}{y_0^2} \left[1 - \frac{15}{8y_0} + \frac{21}{8y_0^3} - \frac{3}{4} \left(1 + \frac{9}{2y_0} + \frac{7}{y_0^2} + \frac{7}{2y_0^3} \right) e^{-2y_0} \right] \quad (10)$$

The expression B_3 for the relative Coulomb energy yields the energy for an arbitrary shaped, homogeneously charged, diffuse surface nucleus to all orders in the diffuseness parameter a_{den} . The constants in front of B_1 and B_3 have been chosen so that B_1 and B_3 are 1 for a sphere in the limit the range constant a and the diffuseness constant a_{den} go to zero, in analogy with the definition of the quantities B_L and B_C in the standard liquid-drop model.

2.1.1 Values of constants

The constants appearing in the expression for the Yukawa-plus-exponential macroscopic model fall into three categories [32,4]. The first category, which represents constants that were taken from previous work with no adjustment whatsoever include

M_H	=	7.289034	MeV	hydrogen-atom mass excess
M_n	=	8.071431	MeV	neutron mass excess
e^2	=	1.4399764	M _e V fm	electronic charge squared
a_{den}	=	0.99/2 ^{1/2}	fm	range of Yukawa function used to generate nuclear charge distribution
a_{el}	=	1.433 × 10 ⁻⁵	MeV	electronic-binding constant
Δ	=	12	MeV	pairing-energy constant
δ	=	20	MeV	pairing-asymmetry constant
r_p	=	0.80	fm	proton root-mean-square radius

The second category of constants, representing those constants whose values were determined from considerations other than nuclear ground-state masses, includes

r_0	=	1.16 fm	nuclear-radius constant
a	=	0.68 fm	range of Yukawa-plus-exponential potential
a_s	=	21.13 MeV	surface-energy constant
κ_s	=	2.3	surface-asymmetry constant

The third category, representing those constants whose values were determined from a least-squares adjustment to nuclear ground-state masses, includes

a_v	=	15.9937 MeV	volume-energy constant
κ_v	=	1.927	volume-asymmetry constant
W	=	36 MeV	Wigner constant
c_0	=	4.4 MeV	A^0 constant
c_a	=	0.212 MeV	charge-asymmetry constant

2.1.2 Discontinuities at scission

The terms in eq. (2) that contain shape-dependent functions have the important and desirable property that the result obtained when they are evaluated is independent of whether the system is considered to be a single system or to be two separate nuclei. However, several terms that do not contain shape-dependent functions do not have this property, for instance the Wigner and A^0 terms. The significance of the absence of a shape dependence is apparent by considering the evolution of, for example, ^{258}Fm from a single sphere to a configuration of two touching spheres into two separated nuclei. Below we will present calculated potential-energy surfaces that include a path that leads to the configuration of two touching spheres. In all such calculations we consider the nucleus as a single system. We will later see that the result of those calculations is that ^{258}Fm has a maximum in the calculated barrier of about 3.5 MeV close to the location of the two-touching-sphere configuration.

Now if we instead consider the two-touching-sphere configuration as a system of two separate nuclei and evaluate eq. (2) under that assumption, a different barrier will be obtained, mainly as a result from the absence of a shape dependence in the Wigner and A^0 terms. For a single system the sum of these two terms is 12.5 MeV but for two separate ^{132}Sn nuclei their sum is 25 MeV. Clearly, somewhere beyond the two-touching-sphere configuration, outside the range of the nuclear force, the expression for two systems should be used. This would give an increase of 12.5 MeV in the fission barrier close to the point of two touching spheres compared to the result obtained if the system is treated as a single system. Since the fission data for ^{258}Fm clearly indicates fission through a configuration close to that of two touching spheres, the

actual barrier can hardly be 16 MeV here, which would be the resulting barrier if 12.5 MeV is added to the 3.5 MeV barrier that is obtained if the configuration is treated as a single system.

A possible resolution of this contradiction could be that there are missing terms in the model, with the opposite shape dependence compared to the Wigner term. For instance, an A^0 term with a negative coefficient -8.5 MeV would cancel the jump in the Wigner term at the transition from a single system to two separate systems. However, the adjustment of coefficients for the nuclear mass formula in [32,4] rules out the possibility of a negative A^0 coefficient, within the present formulation of the mass formula. This indicates that one has to search for other missing terms in the mass formula. Such an investigation is outside the scope of the present study. Since we know of no well-investigated and generally accepted shape dependencies for the Wigner, A^0 , pairing and other terms in eq. (2) we shall use the model in its present form, evaluated for a single system for the entire range of shapes we study, which consists of single, connected shapes up to scission configurations.

2.2 MICROSCOPIC MODEL

The microscopic energy term arises because of the non-uniform distribution of single-particle levels in the nucleus. It is the sum of a shell correction term and a pairing term:

$$E_{\text{micr}}(Z, N, \text{shape}) = E_{\text{shell}}(Z, N, \text{shape}) + E_{\text{pair}}(Z, N, \text{shape}) \quad (11)$$

Both terms are evaluated from a set of calculated single-particle levels, the shell correction by use of Strutinsky's method and the pairing correction by use of the BCS approximation. Since the model is extensively discussed in ref. [31] we present here only the major features of the model and some changes relative to that reference.

The single-particle potential felt by a nucleon is given by

$$V = V_1 + V_{\text{s.o.}} + V_C \quad (12)$$

The first term is the spin-independent nuclear part of the potential, which is calculated in terms of the folded-Yukawa potential

$$V_1(\mathbf{r}) = -\frac{V_0}{4\pi a_{\text{pot}}^3} \int_V \frac{e^{-|\mathbf{r}-\mathbf{r}'|/a_{\text{pot}}}}{|\mathbf{r}-\mathbf{r}'|/a_{\text{pot}}} d^3\mathbf{r}' \quad (13)$$

where the integration is over the volume of the generating shape, whose volume is held fixed at $\frac{4}{3}\pi R_{\text{pot}}^3$ as the shape is deformed. The potential radius R_{pot} and potential depth V_0 for protons and neutrons are given by

$$R_{\text{pot}} = R_{\text{den}} + 0.82 \text{ fm} - 0.56 \text{ fm}^2 / R_{\text{den}} \quad (14)$$

$$V_p = (52.5 + 48.7\bar{d}) \text{ MeV} \quad (15)$$

and

$$V_n = (52.5 - 48.7\bar{d}) \text{ MeV} \quad (16)$$

where

$$\bar{\delta} = \frac{(N - Z)/A + 0.0112Z^2/A^{5/3}}{1 + 3.15/A^{1/3}} \quad (17)$$

$$\bar{\epsilon} = -\frac{0.147}{A^{1/3}} + 0.330\bar{\delta}^2 + \frac{0.00248Z^2}{A^{4/3}} \quad (18)$$

and

$$R_{\text{den}} = 1.16A^{1/3}(1 + \bar{\epsilon}) \text{ fm} \quad (19)$$

The range

$$a_{\text{pot}} = 0.8 \text{ fm} \quad (20)$$

of the Yukawa function in eq. (13) has been determined from an adjustment of calculated single-particle levels to experimental data in the rare-earth and actinide regions. It is kept constant for nuclei throughout the periodic system.

The spin-orbit potential is given by the expression

$$V_{\text{s.o.}} = -\lambda \left(\frac{\hbar}{2mc} \right)^2 \boldsymbol{\sigma} \cdot \nabla V_1 \times \mathbf{p} \quad (21)$$

where λ is the spin-orbit interaction strength parameter, m is the mass of either a neutron or a proton, $\boldsymbol{\sigma}$ is the Pauli spin matrix and $\mathbf{p}$ is the nucleon momentum.

The spin-orbit strength has been determined from adjustments to experimental levels in the rare-earth and actinide regions. It has been shown [33] that many nuclear properties throughout the periodic system are well reproduced with λ given by a function linear in A through the values determined in these two regions. This gives

$$\lambda_p = 28.0 + 6.0 \left(\frac{A}{240} \right) \quad (22)$$

and

$$\lambda_n = 31.5 + 4.5 \left(\frac{A}{240} \right) \quad (23)$$

Finally, the Coulomb potential for protons is given by

$$V_C(\mathbf{r}) = e\rho_c \int_V \frac{d^3\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \quad (24)$$

where the charge density ρ_c is given by

$$\rho_c = \frac{Ze}{\frac{4}{3}\pi R_0^3} \quad (25)$$

Above we discussed the fact that some shape-independent terms in the macroscopic part of the energy should be generalized to include a proper shape dependence, so that the model yields the same result for scission-like shapes independent of whether the system is considered as one single nucleus or two separate nuclei. Similar problems exist in the microscopic part of the model. They are discussed in some detail in [34]. Here we shall only briefly mention some of these difficulties. In the present formulation of the

microscopic model the spin-orbit interaction strengths in eqs. (22,23) and the average pairing strength Δ are shape-independent, which may give rise to discontinuities at scission, if a transition is made from treating the system as one nucleus to treating it as two separate nuclei. A discontinuity that is more severe and probably more difficult to correct for arises at mass-asymmetric scission configurations. The reason is that the Strutinsky method yields very different results depending on whether the system is treated as one or two systems. The underlying reason for this particular discontinuity, which can be as large as 10 MeV, is that the locations of the Fermi surfaces are different depending on whether the system is treated as one single nucleus or two separate nuclei. Since we do not consider mass-asymmetric shapes this particular discontinuity does not affect the results in this investigation.

3 Nuclear shapes

Our primary goal in this investigation is to search for two fission valleys, one leading to elongated scission shapes, the other leading to compact scission shapes. The existence of two different valleys of this type was proposed by Hulet *et al.* [13]. Their proposal was based on the observance of a high- and a low-energy component in the fission kinetic-energy spectrum of some nuclei close to ^{264}Fm . A second goal is to study the implications of the presence of two valleys on quantities other than the kinetic energies, particularly on fission half-lives. To search for the new valley, we have to select a set of shapes for which to calculate the potential energy.

Two shape parameterizations are at present implemented in the model. One is the three-quadratic-surface parameterization [35] and the other is the ϵ parameterization [36]. The latter is the more suitable one for investigating ground-state shapes. In the fission half-life calculations below we always take the ground-state energy from calculations using the ϵ parameterization. We use the three-quadratic-surface parameterization in the calculation of potential-energy surfaces that we perform to search for the two fission valleys, since it is the most suitable one for generating shapes beyond ground-state shapes that are of interest in fission. In the calculation of potential-energy surfaces it is of considerable importance to select shapes that are related to the processes that are studied. However, we have also performed calculations of potential-energy surfaces as functions of ϵ_2, ϵ_4 and mass-asymmetric ϵ_3 shape coordinates. It turns out that along the old path the lowest saddle-point energies are obtained in that parameterization. Therefore, we use those results below in the calculation of fission half-lives along the old path.

3.1 SHAPE PARAMETERIZATION

To be able to discuss our choices in detail we give some details of the three-quadratic surface parameterization. In it the shape of the nuclear surface is specified in terms of three smoothly joined portions of quadratic surfaces of revolution. They are completely

specified by

$$\rho^2 = \begin{cases} a_1^2 - \frac{a_1^2}{c_1^2}(z - l_1)^2, & l_1 - c_1 \leq z \leq z_1 \\ a_2^2 - \frac{a_2^2}{c_2^2}(z - l_2)^2, & z_1 \leq z \leq l_2 + c_2 \\ a_3^2 - \frac{a_3^2}{c_3^2}(z - l_3)^2, & z_1 \leq z \leq z_2 \end{cases} \quad (26)$$

Here the left-hand surface is denoted by the subscript 1, the right-hand one by 2 and the middle one by 3. At the left and right intersections of the middle surface with the end surfaces the value of z is z_1 and z_2 , respectively.

There are nine numbers required to specify the expressions in eq. (26) but the condition of constancy of the volume and continuous first derivatives at z_1 and z_2 eliminates three numbers. The introduction of an auxiliary unit of distance u through

$$u = \left[\frac{1}{2} (a_1^2 + a_2^2) \right]^{\frac{1}{2}} \quad (27)$$

permits a natural definition of two sets of shape coordinates. We define three symmetric coordinates σ_i and three mass-asymmetric coordinates α_i by

$$\begin{aligned} \sigma_1 &= \frac{(l_2 - l_1)}{u} \\ \sigma_2 &= \frac{a_3^2}{c_3^2} \\ \sigma_3 &= \frac{1}{2} \left[\frac{a_1^2}{c_1^2} + \frac{a_2^2}{c_2^2} \right] \\ \alpha_1 &= \frac{1}{2} \frac{(l_1 + l_2)}{u} \\ \alpha_2 &= \frac{(a_1^2 - a_2^2)}{u^2} \end{aligned}$$

and

$$\alpha_3 = \frac{a_1^2}{c_1^2} - \frac{a_2^2}{c_2^2} \quad (28)$$

The coordinate α_1 is not varied freely but determined by the requirement that the center of mass be at the origin.

3.2 SYMMETRIC MOMENTS

It is not very useful to display calculated results as functions of the shape coordinates defined in eq. (28) because their values are related in a very nonlinear way to the

actual shapes. For instance, when z_1 and $z_2 \rightarrow 0$ for symmetric shapes, that is, when the middle body disappears, then $\sigma_2 \rightarrow -\infty$. We therefore display the calculated results as functions of moments of the shape. This has the additional advantage that results from calculations using different shape parameterizations can be displayed as functions of the same quantities. The two most important symmetric moments of the matter distribution are defined by

$$r = 2 \int_{z \geq 0} z \rho(\mathbf{r}) d^3 r / \int_{z \geq 0} \rho(\mathbf{r}) d^3 r$$

and

$$\sigma = 2 \left[\int_{z \geq 0} \left(z - \frac{1}{2} r \right)^2 \rho(\mathbf{r}) d^3 r / \int_{z \geq 0} \rho(\mathbf{r}) d^3 r \right]^{\frac{1}{2}} \quad (29)$$

The following physical interpretation can be given to the definitions in eq. (29). The first moment r is the distance between the centers of mass of the two halves of the system, which is symmetric with respect to the $z = 0$ plane. The second moment σ is the sum of the root-mean-square extensions along the symmetry axis of the mass of each half of the system about its center of mass. Below we display calculated total potential energies as functions of r and σ . Both in the figures and in the paper we use units in which the equivalent sharp radius R_0 of the spherical nucleus is 1. One should note that although the coordinate σ_1 , which is the distance between the centers of the two end bodies in units of u as defined in eqs. (27,28) seems similar to the coordinate r , there are large differences, and the r coordinate is definitely to be preferred for displaying results of the calculations. We would also like to point out that the moments r and σ do not in general define the shape uniquely, but are functions of the shape. To define the shape precisely, one must define the underlying shape parameterization and either specify the values of the shape parameters or introduce higher moments so that the number of moments equal the number of shape parameters.

3.3 CHOICE OF NUCLEAR SHAPES

The three-quadratic-surface parameterization allows the variation of three symmetric and two asymmetric shape coordinates, according to eq. (28). Since we are primarily interested in nuclei for which fission is symmetric, we shall here not vary the asymmetric shape coordinates but limit our study to symmetric shapes. This leaves us with the three symmetric shape coordinates σ_1 , σ_2 and σ_3 [35]. As seen in eq. (28), σ_3 is related to the eccentricities of the end bodies. One realizes that only a small deviation from sphericity of the end fragments removes the influence on the shell effect by magic or near magic numbers. Consideration of shape changes for a single nucleus illustrates the extreme sensitivity of the shell correction to small shape changes. We note for instance that for ^{208}Pb the shell correction for the spherical shape is -12.35 MeV [4], but already for the deformation $\epsilon_2 = 0.20$ the shell correction is about 0.

In fig. 2 we illustrate how the shell correction evolves for two merging ^{132}Sn nuclei and also the effect of deviations from sphericity for the end fragments. The overall

elongation of the nucleus where we tested the effect of spherical versus deformed end bodies is best characterized by the value of the first moment, which is $r \approx 1.3$. In terms of r the second saddle is located in the region $1.3 \leq r \leq 1.4$ [25]. We see that for ^{264}Fm the shell correction changes from -11.80 MeV to 0.35 MeV when the deformation of the end fragments changes from spherical to slightly deformed. The value $\sigma_3 = 0.60$ corresponds to approximately $\epsilon_2 = 0.25$. Two points are clear. First, the fragment shells have a large effect already at the second saddle, at least for ^{264}Fm . Second, we note that to see the full effect of the magic fragment shells the end parts of the nuclear shape have to be kept spherical. We therefore calculate the nuclear potential energies for shapes with the end bodies kept at fixed spherical shapes, while we vary σ_1 (elongation) and σ_2 (neck size).

We have actually also varied the third and last symmetric shape coordinate σ_3 , but we will here, except for one or two brief references, discuss only results obtained with $\sigma_3 = 1$, that is, with spherical end bodies. There are several reasons for this limitation. First, let us observe that many shapes that are generated by varying σ_3 are also approximately generated by varying σ_1 and σ_2 with σ_3 fixed at 1. This is best understood by considering the pure spheroid. In our case, with the ends constrained to be spherical, it can be generated anyway, by letting the middle body grow so that the entire nuclear surface consists of just the middle body. If σ_3 were allowed to vary, a spheroid of a certain eccentricity could be generated in an infinite number of ways, for instance by letting the middle body shrink so that the nuclear shape consists of just the two end bodies of the desired eccentricity, meeting at the middle. In a similar manner, many shapes that have $\sigma_3 \neq 1$ can be approximated by $\sigma_3 = 1$ and suitable values of the other two parameters.

The above arguments also show a difficulty in interpreting the results of varying all three symmetric shape parameters. Major problems will be caused by the fact that several points in the three-dimensional space will correspond sometimes exactly and sometimes approximately to the same shape. Thus, a spheroidal ground-state corresponds in a three-dimensional deformation space to a tube or line running from one boundary surface of the calculation to another. In some investigations that display results of multi-dimensional calculations as contour maps of only two variables the energy has been minimized with respect to the additional shape variables [16]. Care must be taken in such procedures, since there may be several minima in the direction of the additional variables, in which case a minimization procedure is not sufficient to display all the features of the calculation.

Summarizing the above discussion, we have deduced that the main features in the potential energy of symmetric fission can be studied by keeping σ_3 fixed at 1 and varying σ_1 and σ_2 . In our most extensive earlier study of fission potential-energy surfaces, σ_1 , σ_2 and a mass-asymmetry coordinate were varied and σ_3 depended on these three variables in a way that was taken from liquid-drop model calculations [18,25]. Thus the important class of shapes with spherical ends was never studied. The precise shapes we study here consist of a grid in σ_1 and σ_2 . Because σ_2 approaches $-\infty$ in

some cases, we introduce

$$\lambda = 1 - \frac{1}{2} \frac{\sigma_1}{1 - \frac{1}{\sigma_2}} \quad (30)$$

This definition holds only for $\sigma_2 = 1$. In our calculation we shall consider the evolution of a single system from the nuclear ground state to scission configurations, but not post-scission configurations. There will be a lower bound of λ , which for $\sigma_1 \leq 2$ corresponds to the limit in which the middle body has disappeared, and for $\sigma_1 \geq 2$ corresponds to the scission line. Thus for each value of σ_1 there is a minimum value of λ , given by

$$\lambda_m = \begin{cases} 1 - \frac{\sigma_1}{2}, & \sigma_1 \leq 2 \\ 1 - \frac{2}{\sigma_1}, & \sigma_1 \geq 2 \end{cases} \quad (31)$$

Two touching spheres correspond to $\sigma_1 = 2$ and $\lambda = 0$. The grid we choose is densely spaced for $\sigma_1 < 2$. Here we consider $\sigma_1 = 0.38(0.20)1.98$. For larger values of σ_1 we select a less dense spacing and the corresponding σ_1 values are $\sigma_1 = 2.48(0.50)7.98$. For λ we choose ten values that are equidistant in the interval $(\lambda_m + 0.01)$ to 1.9, where λ_m depends on σ_1 and is given by eq. (31).

The above discussion defines the shapes precisely. Our results are displayed as functions of the two moments r and σ given by eq. (29). We show some examples of the actual shapes considered in fig. 3. The shapes are plotted at locations corresponding to the moment values. We note that, although in general the moments r and σ do not define the shapes uniquely, they do in our study here, since the the shapes depend on only two variables of the underlying three-quadratic-surface shape parameterization.

4 Calculated results

We now present calculated results for nuclei in the vicinity of ^{264}Fm . We have learned that it is extremely useful to consider fission half-lives in the interpretation of the calculated potential-energy surfaces. However, it is instructive to first discuss solely the structure of the potential-energy surfaces. We therefore start with a discussion of potential energies for Fm isotopes and then proceed to discuss the significance of fission half-lives, making additional comments in that context about the Fm isotopes. Then we present and discuss potential-energy surfaces for additional elements.

4.1 CALCULATED POTENTIAL-ENERGY SURFACES FOR Fm ISOTOPES

We display the calculated energies in the form of contour diagrams. First we show the smooth trends of the underlying macroscopic energy. As a representative nucleus we show ^{252}Fm in fig. 4. The spherical shape is not included in the set of shapes studied, but is located at $r = 0.75, \sigma = 0.4873$ (cf. [37]). The saddle-point energy is about 2

MeV and is located at $r = 1.07$, $\sigma = 0.68$. A prominent structure in the diagram is a mountain, centered around the configuration of two touching spheres, at $r = 1.5874$, $\sigma = 0.7099$. One should, as we have pointed out above, realize the importance of using a finite-range model for the macroscopic energy to get a realistic value for the energy of this configuration. In the investigation by [38], using the liquid-drop model, which is *not* of the finite-range type, the calculated energy obtained for this shape configuration, for a nucleus with approximately the fissility of ^{252}Fm , is about 50 MeV above the energy of the spherical shape. After passing over the saddle point, the dynamical path of the nucleus depends strongly on dissipation. With no dissipation the nucleus follows approximately the valley that can be seen beyond the saddle point and reaches the scission line at about $r = 3.5$ [39]. We shall now see how these features and conclusions are modified by taking into account the microscopic part in the model for the total potential energy.

In figs. 5a and 5b we show calculated total potential-energy surfaces for ^{252}Fm and ^{264}Fm . The contour map for ^{252}Fm in fig. 5a is quite different from the one for the macroscopic energy displayed in fig. 4. In fig. 5a we can see the deformed ground-state at about $r = 0.86$, $\sigma = 0.57$, a first saddle at $r = 1.0$, $\sigma = 0.63$ and a second minimum at $r = 1.15$, $\sigma = 0.68$. The area beyond the second minimum shows interesting structure. There are two mountains, one centered at the two-touching-sphere configuration, the other at $r = 1.34$, $\sigma = 0.78$. The latter mountain is surrounded by two saddle points that are of equal height to within 0.5 MeV. The energy of the peak of the mountain separating these two saddle points is about 2 MeV higher than the saddle-point energies. A nucleus passing over either of the two saddles would seem to evolve into shapes leading into a valley that is similar to the one that is present in the macroscopic case in fig. 4, which in this case reaches the scission line at between $r = 3.75$ and $r = 4.25$. Access to the scission configuration of two touching spheres is blocked by the other mountain, which is about 3 MeV higher than the saddle points.

In the study by [19] limited to the two nuclei ^{258}Fm and ^{259}Md part of this structure is seen, namely the mountain at $r = 1.34$, $\sigma = 0.78$ with the two surrounding saddle points. In a later study by the same group [20] a "super-short" valley towards scission is seen for ^{252}Cf . This is contrary to our results below. In [20] fig. 2 shows that the shapes of the fragments at the end of this valley are far from spherical. The results of [20] may therefore be somewhat spurious and may occur because of the particular constraints imposed by their shape parameterization on the shape of the end of the nucleus or because a liquid-drop model with a standard surface-energy term is used instead of a more appropriate finite-range model for the surface energy. In fig. 3a in [20] some point outside the figure and somewhat below the lower left corner corresponds to two touching spheres. This important shape is inaccessible to the shape parameterization used. In addition one can see the unphysical rise in energy at this location, which is due to the choice of an inadequate macroscopic model. The contour maps are displayed in terms of geometrical parameters of the shape, whereas we display the contour maps as functions of moments of the shape, which we feel is a more appropriate method. This difference may also lead to differences in the appearance of valleys and ridges,

since these structures are not invariant under coordinate transformations.

Also in other, earlier studies that featured more limited shape parameterizations and models for the macroscopic energy, that did not allow for the study of shapes in the vicinity of the two-touching-sphere configuration, very little of the type of structure which we see here has been present. Examples of such results are fig. 1c in [23], which presented a general survey of heavy-element fission barriers, and fig. 9 of [14], which was directed specifically towards the study of the influence of fragment shells.

Fig. 5b shows a corresponding potential-energy surface for ^{264}Fm . There are major differences in structure between this nucleus and ^{252}Fm shown in the previous figure. Here there is no second minimum. Instead, a short but deep valley starts at the first saddle and leads directly to the two-touching-sphere configuration. The mountain that is present at this configuration in the ^{252}Fm case has completely disappeared here. This short, deep valley is separated by a high ridge from another valley in the upper part of the diagram. On the ridge there is a mountain at $r = 1.40$, $\sigma = 0.83$ and above this mountain there is a slightly lower saddle leading into the upper valley. The upper valley is similar to the one found in the plot of the macroscopic energy only, in fig. 4. The lower valley has clearly been created by fragment shell effects. Using terminology from [13], the lower valley is fragment-shell-directed and the upper one is liquid-drop-like. The upper valley is the old valley and the lower valley is the new valley.

We have displayed results for ^{264}Fm because it is the nucleus for which one would expect the fragment shell effects to be maximum and it is of value to see the theoretical results for this case. However, this nucleus has not been observed experimentally. Were the fission of this nucleus ever to be observed experimentally, one would expect all fission to follow the lower valley, since it is separated from the valley higher in the diagram by a high ridge. The cases that have been experimentally observed [13] often seem to lie in a transition region between fission in one valley or the other. In particular, for ^{258}Fm most of the events have a kinetic energy peaked around 235 MeV, but the skewed kinetic-energy distribution indicates the presence of a second smaller peak at 200 MeV. To show both the emergence of the new valley and the transition region from fission in the old valley to fission in the new valley, we display contour maps for 8 even nuclei from ^{252}Fm to ^{266}Fm in figs. 6a-h. To see more detail we expand the diagrams by plotting only out to $r = 1.70$ and by plotting contours at intervals of 1 MeV. The solid dots in the figures indicate local minima and the plus signs indicate local maxima. Also very shallow maxima and minima have been marked by these symbols.

It would be desirable to deduce the dynamical evolution of nuclear shapes from the last saddle from a dynamical model. Since shell effects are important in the region we study, such a model should also incorporate shell effects. A survey of a large number of nuclei with such a model would require an enormous effort. We therefore base some simple arguments about the dynamical evolution from saddle to scission mostly on the structure of the static potential-energy surfaces we present here. However, we relate the results of our arguments to experimental data. This will show if our approach

is a useful one. The choice of coordinates in terms of which the potential energy is displayed is suitable for arguments of this type, since the mass and dissipation tensors for separated fragments is diagonal in r and σ [37]. It should therefore be close to diagonal also for connected shapes close to scission, the region in which we are most interested.

Scanning through figs. 6a to 6h we see that ^{258}Fm appears to be a transition nucleus. Here the saddle point on the lower side of the mountain centered at $r = 1.34$, $\sigma = 0.78$ is the lower saddle. The plus sign in this saddle region indicates the presence of a small hill. Motion to either side of this hill seems about equally possible. Although there is a low hill at the configuration of two touching spheres, a valley just above this hill leads to compact scission just beyond the two-touching-sphere configuration. It also seems possible that the shape evolution proceeds from the lower saddle to the old valley in the upper part of the diagram. There is a considerable amount of experimental data available on the fission of this nucleus [40]. Fission is predominantly mass-asymmetric. However, by correlating the available data in different ways one finds that there are an unusual number of high-kinetic-energy events and also that the mass distribution of the high-kinetic-energy events is peaked at symmetry. This is entirely consistent with our discussion above of the structure of the potential-energy surface of ^{258}Fm . We note that the upper saddle is lowered somewhat by mass-asymmetric shape degrees of freedom [25], which provides a mechanism for the predominantly mass-asymmetric fission that is observed experimentally. The low-kinetic-energy mass-asymmetric fission into the old valley could proceed entirely along the upper valley and its mass-asymmetric saddle or initially follow the lower valley below the mountain peak and then turn back into the old valley. In the latter case the mass asymmetry would have to develop at the lower saddle or at a later stage beyond the last saddle in the older valley.

For the nucleus ^{258}Fm the lower saddle is about 1.5 MeV lower than the higher saddle. There is a tiny hill at the configuration of two touching spheres, but fission across the saddle just above this hill leads to the new valley and scission just beyond the two-touching-sphere configuration. There is a 1.2 MeV high ridge separating this valley from the old valley above. Since the ridge is low, some trajectories may penetrate this ridge and lead to the old valley. This structure is compatible with the observed fission characteristics of this nucleus. It is undergoing fission with a very narrow symmetric mass distribution. The kinetic energy is peaked around 235 MeV, but the skewness of the distribution indicates the presence of a second set of events whose kinetic energies are peaked around 200 MeV.

For ^{260}Fm the ridge separating the old and new valleys has grown to a height of 3 MeV. The considerable height of the ridge should block all access to the old valley.

4.2 FISSION HALF-LIVES

It has been proposed that the rapid change in half-life when going from ^{258}Fm to ^{259}Fm is due to the disappearance of the second saddle in the barrier below the ground-state energy. Fission through only one barrier, the first, gives very good agreement with the

observed short half-life of ^{258}Fm [24,26]. However, one may ask if and how the fission half-life is connected to the change in the other fission properties at this transition point, namely to the change to symmetric fission and to high kinetic energies. We show that the old interpretation that the barrier of ^{258}Fm has disappeared below the ground state is inconsistent with results from the present calculation and propose a new mechanism for the short half-life. In addition, we show that the connection between quantities like kinetic energies, fission half-lives and old and new fission valleys is complex, but that there is an interpretation that is consistent with the experimental data observed up to now.

To calculate fission half-lives it is necessary to know the potential energy, the inertia associated with the motion through the barrier and the path from the ground state through the barrier. As we mentioned in the introduction, mainly two models have been used for studies of this type. The first type is a microscopic model for the inertia, and the second type is a semi-empirical model. Since we base the calculation of the nuclear potential energy on a microscopic model it would seem desirable and consistent to base also the calculation of the inertia on some type of microscopic model. However, the microscopic models for the inertia seem more uncertain than the macroscopic-microscopic model for the potential energy. Different microscopic models yield very different results for the inertia, as can be seen from fig. 3 in [24], for instance. Usually the calculated inertia varies in a somewhat periodic manner with a peak-to-valley ratio of about two in some typical cases. One should also note that the peak-to-valley ratio and the magnitude of the calculated inertia are extremely sensitive to small changes in the model assumptions. However, we may observe from fig. 4 in [26] and fig. 2 in [41] that in some recent results from microscopic models, the calculated inertia shows a rapidly fluctuating behaviour around a mean which is approximately equal to the more simple semi-empirical model for the inertia. We may argue that the effect of the fluctuations averages out, since one evaluates an integral over the product of the fission barrier height and the inertia. It is also clear that when more effects are taken into account in the determination of the microscopic inertia the fluctuations become smaller. Because of the uncertainties in and the complexities of the microscopic models for the inertia, we here use the semi-empirical approach.

As a first step in the semi-empirical approach we select a one-dimensional path through the multi-dimensional potential-energy surface. This path starts at the nuclear ground state and includes saddle points and minima in the fission direction and a few points beyond the last saddle. Obviously, the choice of path between the saddle points and minima is open to some ambiguity. However, selecting somewhat different paths affects the calculated half-lives by less than an order of magnitude. But to remove the small ambiguity, we only use the energies and the values of position coordinate r at the saddle points and minima in the construction of the one-dimensional fission barrier. We then construct the fission barrier by connecting the ground-state to the first saddle with a third-degree polynomial whose derivatives are zero at the ground state and at the saddle. This completely defines the third-degree polynomial. The rest of the barrier is constructed in a corresponding manner, by connecting neighbouring

extremum points with third-degree polynomials. If there are no extrema beyond the exit point, which is the point where the barrier energy becomes lower than the ground-state energy, we construct the last part of the barrier by a straight line from the saddle point through a point somewhat beyond the exit point at the bottom of the valley that leads towards scission. An inspection of contour diagrams published in this paper shows that the contour lines are fairly equidistant in this valley, which means that it is much better to use a straight line in this region than a polynomial.

In a one-dimensional WKB spontaneous-fission model the fission half-life is connected to the penetrability by [22,42]

$$T_{sf} = 10^{-28.04} y/P \quad (32)$$

We have used the value $\omega_0 = 1 \text{ MeV}/\hbar$ for the frequency of assaults on the barrier. The probability P of penetrating the barrier $V(r)$ at the energy E_0 is given by [43]

$$P = \frac{1}{1 + e^K} \quad (33)$$

where

$$K = \int_{r_1}^{r_2} \left\{ \frac{2B_r(r)}{\hbar^2} [V(r) - E_0] \right\}^{1/2} dr \quad (34)$$

Here $V(r)$ is the barrier energy along the selected path. The penetration energy E_0 is the ground-state energy plus the zero-point energy in the fission direction at the ground state.

The function $B_r(r)$ is the inertia with respect to r associated with motion in the fission direction. An important aspect of the semi-empirical approach is to deduce asymptotic properties of the semi-empirical inertia from some general arguments about the expected properties of the inertia at small and large r values. Thus, at large distances we expect $B_r(r)$ to approach the value $\frac{1}{4}M$ appropriate to separated symmetric fragments. At small r values the inertia is expected to be considerably higher than what is given by a hydrodynamical irrotational-flow model, due to microscopic quantum-mechanical effects. In the semi-empirical model these asymptotic constraints are taken into account by relating the inertia B_r to the inertia B_r^{irr} corresponding to irrotational flow by [42]

$$B_r - \mu = k(B_r^{\text{irr}} - \mu) \quad (35)$$

where k is a semi-empirical constant and μ is the reduced mass of the final symmetric fragments. The k parameter accounts for the increase of the inertia above the hydrodynamical value due to quantum-mechanical effects.

The irrotational inertia has been numerically calculated for y -family shapes, which are defined in terms of the saddle-point shapes for an idealized uniformly charged liquid drop [42]. We approximate the numerical results by [44]

$$B_r^{\text{irr}} - \mu = \frac{17}{15} \mu \exp \left[-c \frac{128}{51} \left(\frac{r}{R_0} - \frac{3}{4} \right) \right] \quad (36)$$

where $c = 1.0$ yields the approximation to eq. (35). The parameter c determines how rapidly the semi-empirical inertia approaches the asymptotic limit $\frac{1}{4}M$. Randrup *et al.* [26] found that $c = 1.0$ yielded a root-mean-square deviation between calculated and experimental fission half-lives that was only marginally higher than the value obtained by letting c vary freely. He therefore chose $c = 1.0$ and determined $k = 11.5$ from an adjustment to experimental data. The approach [42,24] was referred to as semi-empirical with one adjustable parameter. Since the inertia, as discussed above and as shown in fig. 4 of [26] and fig. 2 of [41], agrees on the average with microscopic calculations for nuclei from ^{232}U to ^{254}Fm one may say that the model has been justified *a posteriori* by microscopic calculations.

In our case, for k we use the value $k = 16$, which was determined in [44] from an adjustment to five actinide fission half-lives. In that adjustment the root-mean-square deviation between the logarithms of the calculated and experimental half-lives was 2.5. If this method of calculating fission half-lives does not diverge outside the region of adjustment we may consequently expect a deviation between calculated and experimental fission half-lives of 2 to 3 orders of magnitude. We therefore call the agreement between calculated and experimental results *good* if the deviation is less than about three orders of magnitude.

The value $k = 16$ obtained in [44] is larger than the value $k = 11.5$ used in ref. [26]. The reason is that fission barriers calculated with the folded-Yukawa single-particle microscopic model are systematically thinner than those obtained with the Nilsson modified-oscillator single-particle potential.

In our study here we have calculated fission barriers also in the ϵ parameterization. We use those results to determine the barrier along the old valley. We have calculated two-dimensional symmetric potential-energy surfaces as functions of ϵ_2 and ϵ_4 , with single-particle parameters for $^{271}\text{110}$. These calculations yield slightly lower values for the saddle on the top side of the mountain around $r = 1.35$ than the results of the calculation in the three-quadratic-surface parameterization, which are presented in the form of contour diagrams in this paper. We also take mass-asymmetry into account when we determine this saddle-point height. The effects of mass-asymmetry we take from unpublished old calculations in terms of ϵ_3 and ϵ_5 at the second saddle. The shape parameters for that study were exactly as in [23], and the calculation used a slightly different set of parameters compared to the present calculation for the folded-Yukawa single-particle potential. It used the droplet model for the macroscopic energy, with a parameter set given by [25]. Although a slightly different model was used, which results in different barriers, we only take the *reduction* in the barrier height due to mass asymmetry from that calculation. The error in the value for the *reduction* in barrier height, in the old valley, that results from using the different model is very low, less than 200 KeV out of a total reduction of 1 to 2 MeV. We take the value for the zero-point energy from unpublished results from our 1981 ground-state mass calculation [32,4]. In table 1 we compare the predictions of our model for fission half-lives for a few heavy even elements for which data are available. We see that the calculated fission half-lives agree to within the expected two or three orders of magnitude with the experimental

TABLE 1
Comparison of fission half-lives
calculated for the normal path
with experimental values

<i>Z</i>	<i>N</i>	<i>A</i>	calc. log(<i>t</i> / <i>y</i>)	exp. log(<i>t</i> / <i>y</i>)
109	152	252	-1	2
100	154	254	0	0
100	158	258	0	-11
100	160	260	1	
102	154	256	-5	-4
104	154	258	-10	-9
106	154	260	-11	-10

values except as we approach neutron number $N = 164$. The disagreement for ^{258}Fm is 11 orders of magnitude. The calculated Fm fission half-lives remain roughly constant with increasing N .

Our first thought as to the reason for this huge disagreement was that the calculated barrier was wrong. In the old valley we had obtained a value of 4 MeV for the height of the second barrier. The reason for the short half-life of ^{258}Fm was thought to be the *disappearance* of the second peak in the barrier. One possible source of error in the second barrier height could be incorrect values of the parameters of the macroscopic model. We investigated this possibility within the framework of our macroscopic-microscopic model but with the finite-range droplet model [45] for the macroscopic energy. This model has nine adjustable parameters, which is four more than the model investigated in [32]. It was thought that it might be possible to readjust the parameters of the finite-range droplet model and retain the good agreement with the 28 fission barriers and 1488 masses to which the model parameters were originally adjusted and in addition obtain lower second barriers for elements in the vicinity of ^{264}Fm . In the original adjustment the rms deviation for the fission barriers was 1.13 MeV and for the ground state masses 0.676 MeV. In a typical attempt at readjusting the macroscopic-model parameters we obtained a 25% increase in the ground-state root-mean-square deviations. In that adjustment we still obtained a second barrier for ^{258}Fm that was 1.75 MeV high and calculated barriers for 20 other actinide barriers were consistently too low, by about 2 MeV on the average. Our conclusion was that the parameters had been well determined by the original adjustment to the data and that it was not possible to lower the calculated barrier by readjusting the macroscopic-model parameters.

Our second thought as to the reason for the ^{258}Fm disagreement was that the fission barrier in the proposed new valley might be lower and yield correspondingly lower half-lives, in better agreement with the experimental data. The results obtained in fig. 6d seem to partly bear this hypothesis out. The barrier here is 1.5 MeV lower at the lower saddle point than at the upper saddle point. However, the upper saddle point is lowered 1 MeV by mass-asymmetric shape degrees of freedom. This means that the lower saddle is just 0.5 MeV lower than the upper saddle and still 3.5 MeV above the ground state. We recall that we get the ground-state energy not from the contour maps published here but from the calculation in ref. [4], which is more exact near the ground state than the present calculation. When we evaluate the fission half-life along the new valley for ^{258}Fm , we obtain 10^5 y. We obtain a longer half-life although the barrier is slightly lower because the path through the barrier is longer. We have also investigated the stability of this lower saddle point with respect to variations of σ_3 and α_2 . The saddle height is stable with respect to variations of both of these shape parameters. That is, it is stable with respect to mass asymmetry and to deviations from the spherical shape that we have chosen for the end bodies in this study.

Since we obtain a barrier that is 3.5 MeV high along the new path and the error in the calculated height of the second peak in the fission barrier was never more than 1.6 MeV for actinide elements, and never more than 1.0 MeV beyond uranium (ref. [32]), it is unlikely that the source of the discrepancy for the fission half-life for ^{258}Fm is a huge error in the calculated barrier height. Leander pointed out to us [46] that one would expect the inertia B_r to be radically different in the new valley compared to the old valley, because in the new valley the single-particle level structure in the fissioning system approaches the structure of the final system very early in the fission process, and that effect of the magicity of the emerging fragments on the inertia is expected to be appreciable. Earlier, Mosel *et al.* [14] gave similar arguments for a much smaller inertia in the new valley, compared to the old valley. However, no calculation of fission half-lives was carried out, nor was any inertia for the new valley calculated or proposed.

We have investigated the single-particle level structures in the old and new valleys. In figs. 7a, 7b, 7c and 7d we show levels for shapes evolving from a spherical shape into the two valleys. Fig. 7a shows neutron levels for liquid-drop saddle-point shapes or y-family shapes [31], which are shapes corresponding to fission-barrier shapes along the old fission path. Fig. 7b shows neutron levels for intersecting spheres. These shapes correspond to shapes at the lower limit of σ in the contour diagrams exhibited in figs. 6a-6h. This limit is slightly lower than the new valley for ^{258}Fm , but we have also calculated the levels in the new valley for ^{258}Fm and they are practically indistinguishable from the levels for intersecting spheres. Since the new valley is slightly different for different nuclei and also because a valley is a slightly less well defined concept than intersecting spheres we display here only the levels corresponding to the intersecting spheres family of shapes. Figs. 7c and 7d are similar to figs. 7a and 7b but show proton single-particle levels.

It is immediately clear from inspecting figs. 7a-7d that the level structures in the two valleys are radically different. In the old valley the level crossings continue through

the entire deformation range studied, which for the heavy elements studied here corresponds to penetrating the entire fission barrier. Although gaps corresponding to ground-state and isomeric minima are present no truly large scale structures are present. The level structure appears fairly random. These diagrams together with the microscopic calculations discussed above explain the success of fission half-life calculations based on a smooth semi-empirical inertia [24,26].

For the new valley it is clear that our current form of the semi-empirical inertia is inadequate. In its present form the inertia reaches the limiting value $\frac{1}{4}M$ at infinity. However, since in the new valley the fragments separate already at $r = 1.59$ the asymptotic limit $\frac{1}{4}M$ should be reached already here. In addition we see by inspecting figs. 7b and 7d that the magic gaps $N = 82$ and $Z = 50$ extend far inside the point of two touching spherical nuclei. We discuss the consequences by focusing our discussion on the proton diagram 7d. The gap extends into such compact shapes as $r = 1.20$. To a somewhat lesser extent the presence of the gap is seen as far as $r = 1.07$, which is the deformation corresponding to the first saddle. Over this entire region the levels are parallel and there are very few level crossings compared to the situation in the old valley. Thus, in this region, already before separation, the inertia should have reached or be very close to its limiting value of $\frac{1}{4}M$.

Since we have, from general arguments been able to set some limits on the inertia in the new valley, we will investigate a generalization of the semi-empirical inertia for the old valley given by eq. (35), which allows for a natural and simple way of fulfilling the limiting conditions for the new valley. For the new valley we propose

$$B_r - \mu = f(r, r_{sc})k(B_r^{irr} - \mu) \quad (37)$$

where

$$f(r, r_{sc}) = \begin{cases} \left(\frac{r_{sc} - r}{r_{sc} - 0.75} \right)^m, & r \leq r_{sc} \\ 0, & r \geq r_{sc} \end{cases} \quad (38)$$

and r_{sc} is the r value where the new valley reaches scission, which in our investigation here is set equal to the r value for two touching spheres, that is $r_{sc} = 1.59$. The inertia in eq. (37) has the property that it is equal to the old inertia at the spherical shape and equal to $\frac{1}{4}M$ at the scission point, two limiting conditions we want to fulfill. In addition it approaches the limiting value horizontally for $m \geq 2$. That is, the derivative at this point is continuous, which is also a reasonable requirement. By varying m we change how fast the inertia approaches its limiting value of $\frac{1}{4}M$. The level structure indicates that the inertia is close to this value far inside the point of touching.

Also at the ground state one can expect the inertia to be lower for shape changes that evolve towards the new valley, compared to shape changes that lead to the old valley. This can occur because the inertia is not related to the values of the shape coordinates themselves but instead to their derivatives, or more precisely, to the direction of change of the shape coordinates. Quantitative support for this can be found in the level diagrams in figs. 7a-7d. Often the distance between levels at level crossings is

larger in figs. 7b and 7d than in figs. 7a and 7c, respectively. As a specific example we compare the level crossing of the proton states $[642\frac{3}{2}]$ originating from the $i\frac{11}{2}$ spherical state at -0.8 MeV and $[402\frac{3}{2}]$ originating from the spherical state $d\frac{3}{2}$ at -11.0 MeV. In fig. 7c, which corresponds to a path leading into the old valley, the crossing occurs at $r \approx 1.0$ and at an energy of about -6.0 MeV, with a splitting at the crossing of about 0.3 MeV. In fig. 7d, which corresponds to a path that leads to the new valley this crossing occurs at about the same location but the splitting at the level crossing is now 1.5 MeV. Thus, the contribution to the inertia from this level crossing should be considerably smaller in the latter case than in the former. The situation is similar at many other level crossings.

We have calculated the fission half-life for ^{258}Fm along the new valley with the inertia given by eq. (37) and $m = 2$, the simplest choice. The calculated fission half-life under these assumptions is 10^{-2} y, or 7 orders of magnitude lower than with the inertia appropriate to the old valley. The calculated fission half-life is still about 9 orders of magnitude larger than the experimental value. A higher exponent m will make the inertia in the new valley approach the limiting value faster and also decrease the half-life for fission in the new valley. With the choice $m = 4$ we obtain a half-life of 10^{-7} y for ^{258}Fm in the new valley. Although this is 4 orders of magnitude larger than the experimental result it is nevertheless fairly close to the experimentally observed value of 10^{-11} y. One should also note that in the macroscopic-microscopic model it is often difficult to reproduce over a small range of neutron numbers the entire magnitude of an effect that experimentally occurs over a range of 2 neutron numbers. In the theoretical model these changes often develop over a slightly larger range of neutron number. Thus, it is not surprising that we in this particular case do not reach the millisecond range of half-lives until ^{260}Fm , as is seen in table 1. We also observe in table 1 that the calculations in the old valley completely fail to reproduce the experimental half-lives for the heavier Fm isotopes. Based on the above discussion we choose $m = 4$ for the inertia in the new valley. The semi-empirical inertias for the old and new valleys are plotted in fig. 8.

Our discussion above leads to a new proposal for the mechanism behind the short half-life for ^{258}Fm . The mechanism is *not*, as previously proposed, the disappearance of the second peak in the barrier below the energy of the nuclear ground state. Instead, it is the lower inertia associated with the new path to compact scission shapes. Thus, we find that it is not an accident that there is a sudden drop in fission half-life at the same time as highly energetic symmetric fission fragments appear, but instead that the two events are intimately connected. We now discuss calculated potential-energy surfaces for additional nuclei that focus both on fission-kinetic-energy and fission half-life properties.

The proposed inertia for the new valley is most appropriate for Z near 100 and N near 164. Below we see that the new valley remains for Z and N values rather far from these values. For such nuclei we expect that the inertia is higher than the one proposed here. Since we have shown here that a consistent explanation of fission barrier heights and fission half-lives requires radically different inertias for different nuclei, it is clearly

desirable to develop a model for the microscopic inertia in the different valleys. Such a project is a major undertaking and outside the scope of this investigation, and for the new valley we consistently use the simple prescription given by eq. (37). This leads to some underestimate of the fission half-lives far from ^{264}Fm .

4.3 ADDITIONAL POTENTIAL-ENERGY SURFACES

In figs. 9a-f, 10a-f, 11a-h, 12a-h, 13a-f and 14a-f we present contour diagrams similar to those for Fm shown in figs. 6a-h, but for Cm, Cf, No, Rf, $Z=106$ and $Z=108$, respectively. The contour diagrams have been selected to cover the transition from fission in the old valley to fission in the new valley.

4.3.1 The switchback phenomenon

According to our discussion above, if high kinetic energies are a signature of fission along the lower path and low kinetic energies a signature of fission along the upper path, then low-kinetic-energy events should have long half-lives and high-kinetic-energy events should have short half-lives. In addition to ^{258}Fm , data in the region of interest are available for only two other even nuclei, namely ^{258}No and ^{260}Rf [13]. The first nucleus, ^{258}No , undergoes symmetric fission with a fairly narrow symmetric mass distribution and low kinetic energies but with a small high-kinetic-energy component. Our argument above led us to expect a long half-life for low kinetic energies. However, a look at fig. 11d provides an understanding of the data for ^{258}No . The barrier is penetrated along the lower path, with its low inertia, and after the saddle at about $r = 1.35$, $\sigma = 0.73$ and the shallow minimum beyond, at $r = 1.47$, $\sigma = 0.75$, the nucleus can decide to go either into the new valley or into the old valley. This perhaps unexpected phenomenon of initially following the new valley and then returning to the old valley we call *switchback*. From the above discussion it is clear that the calculated potential-energy surface is consistent with the observed data. However, a consideration of fission half-lives will add to our understanding of the character of the fission process for this nucleus and other nuclei throughout the heavy region.

Since both high-kinetic-energy and low-kinetic-energy events are seen for ^{258}No , the half-lives for fission into the old valley and into the new valley are of comparable magnitude, experimentally. We have argued that the nucleus actually follows the new valley during most of the barrier penetration process and only after about $r = 1.47$ do the paths diverge. To study the switchback to the old valley would require a knowledge of the inertia for this part of the fission path, which we do not have. However, we do have a simple model for the inertia for fission along the new path into the new valley. With $m = 4$ we obtain a fission half-life of 420 ms. The half-life calculated along the old valley is 45 m. This can be compared to the observed half-life of 1.2 ms. Thus, along the old valley the calculated half-life is more than 6 orders of magnitude larger than the observed half-life. Since two components are observed experimentally in the kinetic-energy distributions in the fission of these two elements the fission half-lives corresponding to the two components cannot differ by more than one or two orders

of magnitude. We conclude that this indicates that the *same* barrier is penetrated in the two cases, except for a tiny portion at the end of the penetration process. As we said above, we cannot calculate the half-life along the switchback path since we do not know the inertia. The inertia is probably larger than the inertia along the new valley, since we expect more level crossings along the switchback path. This would tend to increase the half-life. However, the old valley is deeper than the new valley and the barrier will have been penetrated at a lower r value than along the new valley. The actual half-life will depend on the balance between these factors. Since only a few high-kinetic-energy events are observed experimentally the experimental indication is that the half-life along the switchback is shorter than the half-life for fission into the new valley.

Our argument for interpreting the bimodal fission as fission along a new valley followed by a switchback into the old valley is partly based on the fact that the fission half-lives calculated along the old path for ^{258}No and ^{258}Fm are 6 and 11 orders of magnitude larger than the experimental half-lives, respectively. Since we feel it is a reasonable assumption that the error in the model is smaller than this magnitude, it follows that it is fission along the switchback path that leads to scission in the old valley in most cases where bimodal fission is observed.

However, there is an additional argument for interpreting the switchback as the mechanism for bimodal fission that is independent of the accuracy of the theoretical fission half-life model. Experimentally, the fission half-lives change extremely rapidly from nucleus to nucleus in this region. For instance, from ^{256}Fm to ^{258}Fm the half-life changes by seven orders of magnitude. At a transition point one might expect the half-lives for fission through the two different barriers to be similar, as a general rule. However, when the change across the transition point is seven orders of magnitude it is unlikely that the two half-lives are equal to within two orders of magnitude. It might perhaps occur in one case but experimentally there are four cases of observed bimodality and it is extremely unlikely that the two barriers have approximately the same half-life in all four cases with such violent changes in half-lives across the transition points.

When we initially discussed the calculated potential-energy surfaces for ^{256}Fm we pointed out that our interpretation of the available experimental data and the calculated potential-energy surface was that this system too may follow the lower valley. At a late stage during barrier penetration the nucleus may either select to switchback to the old valley or to remain in the new valley and continue to compact scission. We also pointed out that the upper saddle would be lowered by mass-asymmetric shape degrees of freedom and by deviations of the ends of the nucleus from sphericity and that penetration across the upper saddle would provide a mechanism for the observed mass-asymmetric events. It may also be that the nucleus always follows the lower valley initially and that mass asymmetry is established as the old valley is entered from below or somewhat later. As we pointed out earlier a full-fledged study of mass-asymmetry is outside the scope of this study. As some high-kinetic-energy events are observed experimentally, the half-life for those events should be comparable to the

observed total fission half-life. The experimental fission half-life is 2.6 h. We find theoretically for penetrating along the lower valley a half-life of 7.7 m. The half-life along the upper valley is 190 d.

4.3.2 Fission half-lives from Cf to $Z = 110$

In table 2 we present calculated fission half-lives for even nuclei from Cf to $Z = 110$. The fission half-lives have been calculated for both the old and new paths. In some cases the compact scission shapes are not accessible because a mountain at this configuration blocks access and clearly pushes the nucleus back into the old valley. These cases are indicated in the table by (s) for switchback. In these cases we have calculated the fission half-lives by integrating along a path that leads back into the old valley. The inertia we have used in these few cases is the one that is appropriate to the new valley and, as pointed out above, the calculated half-lives along the switchback are therefore an *underestimate* of the half-life that would be obtained by use of an inertia appropriate for the switchback path. In principle it should be possible to determine where the transition from fission along the old and new paths takes place by comparing the fission half-lives along the two paths. In practice this is complicated because we do not know the inertia along the switchback path and our model for the inertia is inappropriate far from ^{264}Fm . However, we will now systematically discuss what we can learn from the contour maps and the results in table 2 and show that we have a good quantitative understanding of the fission process for these heavy elements. In this comparison we assume that the half-lives marked (s) are severely underestimated and that the shorter half-life in these cases is always obtained by fission along the upper path. For cases not marked by (s) we have calculated the half-life for fission into compact shapes. However, we should note that for nuclei without the (s) the fission path may divide and lead both into compact shapes and into the old valley according to our discussion above.

We should also note that the half-lives at $N = 152$ are always underestimated. This is because we have not included the effects of ϵ_6 deformations. They are maximum around ^{252}Fm , where they would lower the ground-state energy by about 1 MeV [32]. To include these corrections in a systematic manner would require a search for ground-state minima as functions of ϵ_2 , ϵ_4 and ϵ_6 shape coordinates and a slight readjustment of the parameters of the macroscopic model, which is outside the scope of the present investigation.

For Cm isotopes we do not have available potential-energy surfaces calculated in the ϵ parameterization, which we need to calculate fission half-lives along the old, upper path. We have therefore not calculated any fission half-lives for Cm. However, a transition from fission in the old valley to fission in the new valley should occur at ^{256}Cm or ^{258}Cm .

For Cf we find that the elements ^{254}Cf , ^{256}Cf and ^{258}Cf span the transition region. For ^{254}Cf we find that access to the lower path is blocked by a mountain. The calculated fission half-life is 1 y along the old path, which is in good agreement with the experimental value. As discussed above we call the agreement good if the difference

TABLE 2
Comparison of fission half-lives calculated for the normal
and new paths with experimental values

<i>Z</i>	<i>N</i>	<i>A</i>	Calc. for old path	Calc. for new path	Exp.
98	156	254	1.0 y		60.5 d
	158	256		1.5 d	12 m
	160	258		1.2 m	
100	152	252	42 d	(s) 10 s	150 y
	154	254	1.3 y	(s) 30 s	222 d
	156	256	190 d	7.7 m	2.86 h
	158	258	2.7 y	9.3 s	0.38 ms
	160	260	13 y	42 ms	
102	152	254	11 s		24 h
	154	256	3.9 m	(s) 17 ms	23 m
	156	258	45 m	420 ms	1.2 ms
	158	260	14 d	540 ms	
	160	262	85 d	1.7 ms	
	162	264	19 y	21 μ s	
104	152	256	46 μ s		5 ms
	154	258	2.6 ms	(s) 960 μ s	11 ms
	156	260	73 ms	(s) 520 μ s	20 ms
	158	262	3.3 s	300 μ s	63 ms
	160	264	2.8 d	2.0 ms	
	162	266	3.9 y	44 μ s	
106	152	258	1.3 ms		
	154	260	470 μ s	(s) 15 μ s	7 ms
	156	262	25 s	32 μ s	
	158	264	1.0 m	19 μ s	
	160	266	14 h	5.1 μ s	
	162	268	21 y	160 μ s	
108	154	262	170 μ s	7.2 μ s	
	156	264	1.2 ms	4.4 μ s	> 100 μ s
	158	266	57 ms	28 μ s	
	160	268	22 s	9.4 μ s	
	162	270	3.3 y	82 μ s	
	164	272	18 d	71 μ s	
110	158	268	190 ms		
	160	270	28 s		
	162	272	8.5 h		
	180	290	10 ¹³ y		

between calculated and experimental half-lives is less than about three orders of magnitude. Since a change of the ground-state energy by 0.5 MeV affects the half-life by three orders of magnitude and since variations in the inertia also affects the half-life, one realises the difficulty of achieving greater accuracy in fission half-life calculations. For ^{256}Cf and ^{258}Cf we find half-lives of 1.5 d and 1.2 m, respectively, for penetration along the new path. We do not have potential-energy surfaces in the ϵ parameterization available for these two cases. This is necessary for determining the half-life along the upper path. However, by comparing with the results for Fm we may fairly safely assume that such a calculation would result in half-lives along the upper path of about 1 y for both these elements. The nucleus ^{256}Cf may switchback, but for ^{258}Cf there is a well-developed ridge that separates the new valley from the old one. Thus, according to our calculations this nucleus should exhibit high-kinetic-energy symmetric fission. Our calculations of fission half-lives and the transition point are in agreement with the known experimental data for this nucleus.

We have earlier discussed the Fm isotopes. There is good agreement in all cases if we in accordance with the calculations assume a transition from the upper to the lower valley at $N = 156$. Although we cannot calculate the exact details of the switchback for this nucleus the potential energy is entirely consistent with fissioning along the lower valley and switching back to the old valley. One may also assume that fission into the old valley in this case proceeds along the upper valley, since the discrepancy between calculated and experimental half-life in that case is no larger than about three orders of magnitude, which we consider to be good agreement.

For ^{258}Fm fission should occur along the new valley. The low-kinetic-energy events that are observed experimentally should result from a switchback from the new valley to the old valley. In fig. 15 we show shapes along various fission paths for ^{258}Fm . The first three shapes are shapes along a single path from the ground-state minimum to the first saddle. At the first saddle the paths divide into an upper path along the old valley and a lower one along the new valley. At about $r = 1.45$ we show how a third path splits off from the new, lower path. This is the proposed switchback trail along which we show two shapes. Beyond these two shapes the switchback trail joins up with the last shape shown in the old valley at about $r = 1.66$. We feel that the present investigation shows that shapes along the initial part of the upper valley are not involved at all in the fission of ^{258}Fm . Instead, fission involves shapes along the lower new valley and along the switchback trail.

For No, there is also good agreement between the calculations and experiment. As discussed above we find a transition point at ^{258}No .

As we move to Rf elements we see that for the lower neutron numbers there is not only a mountain at the two-touching-sphere configuration but also a ridge in the lower part of the contour diagram. Up to about ^{260}Rf , fission along the lower valley is diverted back to the old valley by the mountains at the two-touching-sphere configuration. For ^{262}Rf , the model would indicate fission along the new valley with a half-life of 300 μs . This is slightly more than two orders of magnitude shorter than the experimental value of 63 ms. However, for this nucleus the scission point may not be

at $r=1.59$ since there is a 3 MeV high ridge below the valley. Thus, our assumptions about the inertia in eq. (37) should probably be modified. However, it is difficult to develop unambiguously a more sophisticated semi-empirical model. We just mention, to show the sensitivity of fission half-life calculations to various assumptions, that if we assume r_{∞} in eq. (37) to be 1.99 instead of 1.59 we find a half-life for ^{262}Rf of 400 ms, an increase of about three orders of magnitude over the tabulated value. However, even with our original assumptions we find that the calculated half-lives agree with experiment to within about two orders of magnitude, for all Rf elements where data are available.

Also, our results for elements $Z = 106$ and $Z = 108$ are consistent with experiment to within the 3-order-of-magnitude accuracy where experimental data are available. One can expect that the true half-life along the new path is considerably longer than the half-life calculated here, since we are now far from the ^{264}Fm region and since the two-touching-sphere configuration is blocked by a ridge.

For $Z = 110$, table 2 includes some predicted fission half-lives corresponding to the old path. However, we do not have potential-energy surfaces appropriate to the new path available for this Z value. The half-lives close to $N = 162$ are severely overestimated since we feel that in the vicinity of $N = 164$ fission proceeds along the new path. However, we expect the fission half-life for $^{290}110$ to be a more nearly correct estimate.

It has been predicted [47] that there is a rock of stability in the vicinity of the element $^{272}110$. However, since $N = 162$ is close to $N = 164$, where the new fission valley leads to considerably shorter fission half-lives than was expected earlier, this rock may be less stable than previously thought, when α decay was thought to be the dominating mode of decay on the rock. In table 2 we find that the predicted fission half-lives for $^{268}106$ and $^{270}108$ are both about 100 μs . Since we expect that we underestimate the inertia in the new valley this far from ^{264}Fm it is perhaps reasonable to expect a fission half-life of about 10 ms or 100 ms for these nuclei and also for $^{272}110$. We draw a similar conclusion from comparing the predicted and experimental half-life for ^{262}Rf . Unfortunately, this means that the expected fission half-lives for elements on the rock are uncomfortably close to the predicted α half-lives. This may complicate the design of experiments to reach the rock.

4.4 ADDITIONAL IMPLICATIONS OF THE NEW VALLEY

4.4.1 Effect of magic numbers on heavy-ion fusion

The potential-energy surfaces for those nuclei where the fragment-shell effects lead to a new valley illustrate the anticipated beneficial influence of magic target-projectile combinations on fusion and evaporation-residue cross sections in heavy-ion reactions.

First, the magicity of the fragments lowers the energy of the two-touching-sphere configuration relative to what it would be in the absence of shell effects. It follows that in cases where a compound nucleus is formed by a dynamical descent from the two-touching-sphere configuration, the compound nucleus will be formed with less

excitation energy than if fragment shell effects were absent. The resulting relatively cold compound nucleus has then a better chance of surviving, without fissioning, the subsequent stage of de-excitation by particle emission. The result is an enhancement of evaporation-residue cross sections.

The second, more subtle effect, has to do specifically with the appearance of a valley in the potential-energy surface and the effect this has on minimizing the need for an extra push to fuse heavy nuclei [48,49]. This extra push, or additional collision energy in excess of the energy of the two-touching-sphere configuration is needed when, roughly speaking, the electric repulsion after contact exceeds the nuclear attraction between the two fusing nuclei. As will be argued presently, this attraction is considerably greater for nuclei that manage to preserve approximately their spherical shapes than for nuclei that allow a neck to form between them. As a result, magic nuclei that resist as far as possible the growth of a neck, beyond the minimum defined by the geometry of overlapping spheres, experience a stronger compactifying force and can evolve towards fusion in the face of a stronger electric repulsion than if shell effects were absent.

The semi-quantitative argument for this mechanism is as follows. According to the proximity force theorem [50] the force between two nuclei in the form of portions of slightly intersecting spheres is approximately $4\pi\bar{R}\gamma$, where γ is the specific surface energy and $\bar{R}$ is the reduced radius of the two spheres, equal to $R_1R_2/(R_1+R_2)$. On the other hand, the attraction that is provided by a cylindrical neck of radius c is only $2\pi c\gamma$, the rate of increase of the cylinder's surface energy with the cylinder's length. Now the condition that the stronger attraction $4\pi\bar{R}\gamma$ should persist is that the configuration of the fusing system should stay as close as possible to that of portions of intersecting spheres, corresponding to the lower boundary, to the left of $r = 1.5874$, of the contour maps in this paper. This means that there should be a force resisting the growth of the neck. In other words, there should be a valley running close to that boundary. So long as such a valley is present, the attractive nuclear force will remain relatively large and the need for an extra push in fusion will be minimized or avoided entirely. The new valley in a potential-energy surface such as in fig. 6g demonstrates that fragment shell effects such as those in ^{132}Sn can survive in configurations with even a fairly large window between the two halves, thus providing a mechanism for mitigating the extra-push hindrance in reactions between near magic nuclei. We believe it is quite likely that this mechanism may be responsible for the anomalously low hinderance factors in fusion reactions such as $^{48}\text{Ca} + ^{208}\text{Pb}$ [51,52,53].

4.4.2 The new valley and nuclear molecules

So long as fragment shell effects can be counted on to confine the nuclear configuration to the vicinity of portions of overlapping spheres, the possibility arises of a straightforward balancing of the forces acting in the separation degree of freedom in such a way that a nuclear molecule would result. Roughly speaking, this requires that the electric repulsion should balance the nuclear proximity force, resulting in a potential energy with a meta-stable minimum. That such a minimum can, in fact be formed

under suitable conditions is illustrated in fig. 6d for ^{258}Fm . Here the dot near $r = 1.3$ and $\sigma = 0.7$ can reasonably be described as corresponding to a molecular configuration of two ^{129}Sn nuclei in fairly intimate contact, stabilized against neck growth by the fragment shell effects and against fusion or reseparation by a balance between nuclear and electric forces.

In cases where angular momentum is present, a similar situation may obtain if the size of the system is reduced, so that the sum of electric and centrifugal forces continues to balance the nuclear forces. Molecules consisting of more than two magic or near-magic nuclei might also prove meta-stable when their sizes and the amount of angular momentum are similarly adjusted.

5 Summary

We have presented potential-energy surfaces for elements from Cm to $Z = 108$ appropriate to the study of symmetric fission leading to compact scission shapes. Relative to earlier studies, the model, used here is more appropriate to the study of compact scission shapes. The model also represents a more unified approach to the study of fission properties of the heaviest elements than other studies, since the present model has been used to calculate ground-state masses and shapes and fission barriers for nuclei throughout the periodic system [32,4]. In addition, it has been used to calculate α half-lives and Q-values for α and β decay for heavy elements [47]. In the present investigation we have focused on fission properties in the heavy-element region.

An extremely interesting feature of the new experimental fission data is that they imply that scission occurs "under the fission barrier" and not 20 MeV or so below the point where the barrier has been penetrated. This emphasizes an unphysical property of all current macroscopic-microscopic models, namely that a different result is obtained when a scission configuration is treated as one nucleus compared to when it is treated as two separate nuclei, even if nuclear and Coulomb interaction energies are taken into account. The difficulties are partly related to the neglect of the shape dependence of such terms as the Wigner term. The new experimental results should stimulate further study of these problems and hopefully lead to their resolution.

Another area that should be subjected to further theoretical study is a microscopic calculation of the inertia along the new fission path and along the switchback trail.

The use of a better macroscopic-microscopic model that includes the shape dependence of such additional terms and the use of a microscopic model for the inertia may somewhat modify the results we have presented here. However, the general appearance of the inertia we have proposed for the new valley should remain the same. It is clear from the very general and model-independent arguments that we have used above that the inertia in the new valley is considerably lower than the inertia in the old valley.

Despite some deficiencies in the scission region, the macroscopic-microscopic model remains a powerful model for studying nuclear fission phenomena. In particular, our

investigation here shows that we now have a quantitative understanding of many of the fission phenomena in the heavy-element region.

On the experimental side, a more extensive study of fission properties for nuclei with $N = 158, 160$ and 162 would be extremely valuable for increasing our understanding of the new fission valley and its properties. Such data would also be an important guide for further developing the theoretical models.

In summary, the most important new results that we have found are:

- For elements close to ^{264}Fm a deep valley leading to compact scission shapes is a very prominent feature in the calculated potential-energy surfaces.
- From a study of single-particle level diagrams and a calculation of fission barriers and fission half-lives we conclude that there is a much lower inertia associated with fission in the new valley than in the old valley.
- Fission may initially proceed along the new valley and *switchback* to the old valley at a later stage during the process.
- We propose that the short half-life of ^{258}Fm is due to the low inertia in the new valley and *not* to the disappearance of the second peak in the fission barrier.
- The new valley is present up to at least $Z = 108$ and lowers fission half-life predictions for elements on the rock centered around $^{272}110$, relative to earlier predictions.
- Calculated fission half-lives agree to within three orders of magnitude with experimental data for elements from Cf to $Z = 108$ for which $N \geq 152$.

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FIGURE CAPTIONS

- Fig. 1** Fragment mass and kinetic-energy distributions for fission of nuclei close to ^{264}Fm . The structure of these distributions reflect the valleys, ridges, minima and saddle points of the underlying nuclear potential-energy surface.
- Fig. 2** Effect of fragment eccentricities and window size on shell effects. For a fissioning ^{264}Fm nucleus the fragment shell effects manifest themselves at an early stage during the fission process, if the ends of the nucleus are kept spherical. A deviation from sphericity, of a magnitude corresponding to the ground-state deformation of a typical rare-earth nucleus completely removes the manifestation of the fragment shell effects.
- Fig. 3** Nuclear shapes corresponding to potential-energy surfaces. The end parts of the shape are kept spherical. The upper boundary corresponds approximately to spheroids and the lower boundary to two overlapping spheres for $r \leq 1.59$ and to scission configurations for $r \geq 1.59$. The important configuration of two touching spheres is included in our study.
- Fig. 4** Saddle to scission region in a macroscopic model. The configuration of two touching spheres is blocked by a 15 MeV high mountain. The saddle location can not be seen exactly in this figure due to the large spacing between contour lines, but it is about 2 MeV high and located at $r = 1.06$ and $\sigma = 0.68$. Beyond the saddle there is a valley leading to scission. In a non-viscous dynamical model the scission configuration is reached at $r = 3.5$.
- Fig. 5a** Potential-energy contours from ground state to scission in a macroscopic-microscopic model. The deformed ground state is barely visible in the lower left-hand corner of the figure. Beyond the first saddle and the second minimum two saddles separated by a mountain descend into the old fission valley which leads to scission at about $r = 4$. The two-touching-sphere configuration is blocked by a mountain slightly higher than 6 MeV. The region below $r = 1.7$ is shown in greater detail in fig. 6a.
- Fig. 5b** Valleys to compact and normal scission shapes for ^{264}Fm . For this nucleus a short valley leads from the first saddle to a scission configuration of two touching spheres. Access to the normal valley to scission is blocked by an 8 MeV high ridge. The region below $r = 1.7$ is shown in greater detail in fig. 6g.
- Fig. 6a** Mountains, saddle points, valleys and minima for ^{252}Fm . The plus signs indicate maxima and the solid dots indicate minima. The tangent sphere configuration is blocked by a mountain, slightly higher than 6 MeV. The initial part of the normal valley to scission is seen in the upper right-hand part of the figure. There are two saddles separated by a mountain that lead into this valley.

Fig. 6b Initial part of approach towards ^{264}Fm . Already 10 neutrons away we see the changing structure of the second minimum and a weak indication of an emerging new valley. The mountain at the two-touching-sphere configuration is melting away.

Fig. 6c Transition nucleus. During fission this nucleus probably initially follows the lower, new valley and switches back into the old valley at the large, flat saddle region at $r = 1.5$ and $\sigma = 0.80$. A low mountain blocks the two-touching-sphere configuration, but a saddle somewhat above leads to compact scission shapes.

Fig. 6d Ridge separating old and new fission valley. Here fission proceeds along the lower valley across a low second saddle to compact scission shapes. Access to the old valley is blocked by a ridge. Although the penetrability through the ridge is higher than through the low saddle, penetration into the old valley can still occur in a small number of cases, in accordance with the experimental observation of some low-kinetic-energy fission events.

Fig. 6e Higher ridge, deeper new valley and changing location of second minimum. Access to the old valley is effectively blocked by a 3 MeV high ridge. The structure of the surface raises the possibility of the existence of a molecular state at this new location of the second minimum, which corresponds to slightly intersecting spheres.

Fig. 6f Disappearing second minimum and rising ridge. Here the second minimum and second peak in the barrier have almost disappeared along the new valley.

Fig. 6g Maximum effect of fragment magicity. In symmetric fission this nucleus divides into two doubly magic ^{132}Sn nuclei, which each has a ground-state microscopic correction of about ~ 12 MeV. Along the new valley the second minimum has entirely disappeared and there is only a single peak in the barrier.

Fig. 6h Indication of reemergence of second minimum beyond ^{264}Fm . There is still a deep short valley starting at the first saddle and leading to compact scission shapes, but a second minimum shows signs of reappearing.

Fig. 7a Neutron single-particle levels corresponding to fission along the old path. Gaps in the level spectra are present at certain deformations and neutron numbers. For actinide nuclei the gaps give rise to deformed ground-states and deformed second minima. The spherical $N = 126$ gap is centered around -9.4 MeV.

Fig. 7b Neutron single-particle levels corresponding approximately to fission along the new valley. The fragment gaps extend through about the outer half of the barrier. Since there is hardly any rearrangement of internal structure beyond $r \approx 1.2$, we expect the inertia to be close to the value for the reduced mass of the final fragments for larger values of r , in particular close to $N = 164$ and $Z = 100$.

Fig. 7c Proton single-particle levels corresponding to fission along the old valley. The $Z = 82$ gap is centered at -9.2 MeV. No truly large scale structure is seen. On a smaller scale there are gaps in the level spectra that for actinide nuclei give rise to deformed ground-states and secondary minima.

Fig. 7d Proton single-particle levels corresponding approximately to fission along the new valley. The $Z = 2 \times 50$ gap is very prominent and is not completely damped out until slightly outside the *first* peak in the barrier. The structure suggests that the inertia is close to the reduced mass early during the barrier penetration process. We also note the additional feature that already close to the ground state the inertia along the new path can be expected to be lower than along the old path, since the distances across level crossings often are substantially larger here than along the old path.

Fig. 8 Comparison between the semi-empirical inertia along the old path, the semi-empirical inertia along the new path, the irrotational inertia along the old path and the reduced mass μ . The inertia along the new path may be lower than the inertia along the old path already at the ground state, even though the deformation here is the same, because the direction of the shape change is different.

Fig. 9a Conventional fission landscape. For this nucleus only the old fission valley exists. A high mountain is located at the two touching-sphere configuration and the second minimum is located at its conventional position.

Fig. 9b Conventional fission landscape with some reduction of energy at the two-touching-sphere configuration. A small mountain seems to form inside the beginning of the old fission valley.

Fig. 9c Changing structure in the region of two tangent spheres. The mountain at this location has decreased in height relative to lighter Cm isotopes.

Fig. 9d Two valley structure emerging. A new symmetric saddle can be seen leading towards compact scission shapes. It is still very high. Since fission half-lives have not been calculated for Cm elements no judgement can be made in borderline cases of which path has the shorter half-life.

Fig. 9e High ridge between the two valleys. With an inertia that is lower along the new path than along the old path we expect fission of this nucleus to follow the lower path. The structure of the ridge between the two valleys is somewhat different for Cm compared to the Fm isotopes in figs. 6a-h. Here the high point on the ridge is located at larger r values.

Fig. 9f High ridge and changing location of second minimum. Although we have reached $N = 164$ there is still a shallow second minimum at this neutron number. Fission should be highly symmetric with high kinetic energies.

Fig. 10a A conventional fission landscape, but with a minor trend towards development of a new fission valley. There are two saddles leading to the old fission valley. The energy at the two-touching sphere location is high.

Fig. 10b Elongation of second minimum in direction of tangent-sphere configuration apparent. The energy in the region of two touching spheres is still high.

Fig. 10c Saddle leading to compact scission present. Some events may follow this path and some may switchback into the old valley. A calculation of the relative distribution of fission in the old and new valleys would require a very accurate knowledge of the inertia for this transition nucleus.

Fig. 10d Well-developed new fission valley. The new valley is separated from the old valley by a 2 MeV high ridge and we expect fission to exclusively follow the lower valley to compact scission shapes.

Fig. 10e Landscape with high ridge and well-developed second minimum. At this neutron number, $N \approx 162$, the second minimum is still 2 MeV deep, suggesting a possible candidate for a molecular state.

Fig. 10f One-peak barrier structure. For this nucleus with $N = 2 \times 82$ the second minimum has disappeared and only a single barrier exists between the ground state and the configuration of tangent spheres.

Fig. 11a Multi-saddle structured potential-energy landscape. Already at this low neutron number $N = 150$ there are two saddles leading to the old fission valley. In contrast to elements with the lower Z values that were shown in previous diagrams the contour lines are more densely spaced at the configuration of touching spheres for this nucleus.

Fig. 11b Two saddles at the beginning of the old fission valley. The two saddles at the beginning of the normal, old fission valley are of equal height and separated by a mountain. The mountain at the configuration of two touching spheres rises to slightly more than 4 MeV above these saddle points.

Fig. 11c Signs of new fission valley emerging. The saddle on the lower side of the mountain is the lower saddle. The nucleus follows the lower valley initially and may in the second saddle region either switchback into the old valley or continue into the emerging new valley. The half-life we have calculated in table 2 is for a path leading to the new valley, but the half-life for a switchback into the old valley is probably shorter since the energy decreases much faster in this direction.

Fig. 11d Transition nucleus. The second saddle region along the lower path is very flat and extended over a large region. Initially the nucleus follows the lower, new path. Beyond the second saddle it seems about equally probable that the nucleus descends into either valley.

Fig. 11e Low ridge between old and new valley. A long and narrow path leads from the *first* saddle into the new fission valley. Some events may cut across the ridge into the old valley.

Fig. 11f Well-developed ridge between old and new valley. The ridge should block any switchback into the old valley. The descent from the first saddle has become steeper.

Fig. 11g Steep descent to two-touching-sphere configuration. Although a shallow pool exists along the new path at $r \approx 1.20$ the barrier is effectively a one-peak barrier.

Fig. 11h Maximum effect of 2×82 gap. The height of the ridge between the two valleys depends on at what r value it is measured. At the r value corresponding to two tangent spheres the ridge is about 8 MeV high.

Fig. 12a Single fission valley structure. There are two mountains in the second saddle region. Paths across all saddles lead to the old fission valley.

Fig. 12b Single fission valley structure. Paths across three saddle points lead into the old fission valley.

Fig. 12c Lowest saddle along bottom, new fission path. The structure of the potential-energy surface seems to indicate that the nucleus would switchback into the old valley, although signs of an emerging new valley can be found.

Fig. 12d Signs of emerging ridge between new and old fission valley. The new valley actually seems to change into a ridge for the largest r values. The fission half-life in table 2 has been calculated for a path that leads back into the old valley. An inspection of the experimental data in fig. 1 indicates that fission takes place along the old valley and perhaps even proceeds along the upper saddle point which is the lowest saddle when additional shape-degrees of freedom are taken into account.

Fig. 12e Stronger signs of a new valley. However, since the ridge between the two valleys is low, fission along the lower path may still switchback to the upper path.

Fig. 12f Well-developed new valley. Only a shallow second minimum remains.

Fig. 12g Steep descent from first saddle into the new fission valley. The ridge between the valleys is fairly well-developed.

Fig. 12h Maximum effect of 2×82 gap. The new valley extends to the first saddle and a high ridge separates the new valley from the old valley.

Fig. 13a Two saddles and low mountain at beginning of fission valley. Both the upper and lower valley seem to lead into the old fission valley.

Fig. 13b New valley changing into ridge. This structure makes it difficult to decide into which valley the nucleus evolves.

Fig. 13c More developed new valley. However, the fact that the new valley changes into a ridge makes it difficult to judge whether the nucleus evolves into the old or new valley.

Fig. 13d More well-developed ridge separating old and new valley. In this case it seems clear that the nucleus descends into the new valley at the first saddle point and remains there.

Fig. 13e Steep descent into new valley from first saddle. In contrast to the potential-energy surfaces for the lower Z values there is a low ridge present in the lower boundary of the contour diagrams at these high Z values.

Fig. 13f Maximum effect of 2×82 gap. There is a very steep descent from the first saddle into the new valley. There remains somewhat of a ridge at the bottom boundary of the diagram.

Fig. 14a High ridge at lower boundary and initially very broad old fission valley. This nucleus seems to be evolving towards a narrowing old fission valley in the upper right-hand corner of the potential-energy surface.

Fig. 14b New valley changing into ridge. The new valley changes into a ridge and the valley below this emerging ridge is not very well developed. There is a high ridge at the lower boundary of the diagram and the nucleus probably evolves into the old valley.

Fig. 14c Somewhat more developed new fission valley. The rather smooth features of this potential-energy landscape makes it difficult to how the dynamical path develops solely from the features of this potential-energy surface.

Fig. 14d Emerging ridge between new and old fission valley. The new fission valley is fairly well-developed here. The fact that the ridge towards the old valley is only weakly developed leaves the possibility that the nucleus may evolve in this direction.

Fig. 14e Strongly developed new fission valley. Also the ridge is better developed in this case, but less pronounced than for lower Z values.

Fig. 14f Maximum effect of 2×82 gap. A fairly strong ridge exists here, between the old and new fission valley. It is of interest to see that the effect of the fragment magic neutron gap remains with Z so far from 100. The presence of this valley will lower the fission half-lives of these elements relative to earlier estimates.

Fig. 15 Shapes corresponding to various fission paths. Our results in this paper indicate that shapes along the upper path are not involved in fission of ^{258}Fm .

The lower path leads to compact scission shapes and high-kinetic-energy events. The shapes that break off from this path at about $r = 1.5$ and evolve towards the old valley represent shapes along the *switchback* path. This path leads back to the old valley. Our interpretation is that it is evolution along the switch-back path that is responsible for the low-kinetic-energy events that are observed experimentally.

Fig. 1

Fragment shell effects

Energy (MeV)		Shape		
^{254}Fm	^{264}Fm	r	σ_3	
-10.47	-18.24	1.41	1.00
-10.33	-11.80	1.27	1.00
-3.69	0.35	1.31	0.60
-6.60	-3.66	1.15	1.00

Figure 2

Figure 3

Figure 4

Figure 5a

Figure 5b

Figure 6a

Figure 6b

Figure 6c

Figure 6d

Figure 5e

Figure 6f

Figure 6g

Figure 5h

Figure 7a

Figure 7b

Figure 7c

Figure 7d

Figure 8

Figure 9a

Figure 9b

Figure 9c

Figure 9d

Figure 9e

Figure 9f

Figure 10a

Figure 10b

Figure 10c

Figure 10d

Figure 10e

Figure 10f

Figure 11a

Figure 11b

Figure 11c

Figure 11d

Figure 11e

Figure 11f

Figure 11g

Figure 11h

Figure 12a

Figure 12b

Figure 12c

Figure 12d

Figure 12e

Figure 12f

Figure 12g

Figure 12h

Figure 13a

Figure 13b

Figure 13c

Figure 13d

Figure 13e

Figure 13f

Figure 14a

Figure 14b

Figure 14c

Figure 14d

Figure 14e

Figure 14f

Figure 15