MASTER

METHOD OF VIBRATIONAL CONTROL IN
THE PROBLEM OF STABILIZATION
OF CHEMICAL REACTORS

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Abstract - In this annual progress report, results are described which were obtained during the first year of the three year study of vibrational control (VC) approach to the problem of stabilization of Continuous Stirred Tank Chemical Reactors (CSTR). The work was performed at the Department of Electrical Engineering, Illinois Institute of Technology and was sponsored by the Division of Engineering, Mathematical and Geosciences, Office of Basic Energy Sciences of the Department of Energy. The objective of the first year study was the analysis of the efficiency of the VC in increasing the area of stability of CSTR. This objective was achieved on the basis of the following development:

- 1). Equations describing the dynamics of CSTR under periodic oscillations of input flow rate were derived and linearized equations were obtained. These equations are characterized by a periodic matrix diagonal elements of which are not equal to zero. In this case, the traditional VC routine is not applicable.
- 2). A new routine of VC was developed applicable to the mentioned situation. Also, the basis of nonlinear VC theory was established.
- 3). The developed routine was applied to the equations of CSTR in order to show that VC allows to stabilize the steady states where the rate of conversion is of the order of 0.5. This implies that the productivity of the reactor can be increased, theoretically speaking, by a factor of $3 \div 5$.

I. INTRODUCTION

In this annual progress report, results are presented which were obtained during the first year of the three year study of Vibrational Control (VC) of Continuous Stirred Tank Chemical Reactors (CSTR). The goals of the first year study, as formulated in [1], were:

1. Analysis of vibratile parameters in the reactors.
2. Derivation of the CSTR equations with vibrating parameters.
3. Analysis of the effect of vibrational control in the reactor.

These goals were achieved and the results are presented below. In Section II, equations describing the dynamics of CSTR with vibrating input flow are derived, simplified and analyzed in nonlinear as well as linearized case. Section III presents an extension of vibrational control theory which makes it applicable to CSTR with vibrating inputs. The elements of nonlinear vibrational control theory are also described. Section IV describes the effect of vibrational control in the problem of stabilization of the reactors and an example is considered. Section V gives information on the invited talks given in the field of the reported research, paper published in the current year as well as on the percentage of time spent by the principal investigator on this project.

II. EQUATIONS OF CSTR WITH VIBRATING FLOW

Consider the first order exothermic irreversible reaction in CSTR. Assume that the fresh feed rate is changing according to the law:

$$F(t') = F_0 + \frac{\alpha_1}{\varepsilon} f(\omega t')$$

where F_0 is a constant part of a flow and $\frac{\alpha_1}{\varepsilon} f(\omega t')$ is a zero mean periodic function of time t' . Parameter α/ε is used to denote that the "amplitude" of $f(\omega t')$ could be larger than F_0 . For example $\frac{\alpha_1}{\varepsilon} f(\omega t')$ could be a set of large positive impulses added to the constant part of the form $F_0 - \frac{1}{T_1} \int_0^{T_1} \delta(t - kT_1) dt$, $k = 0, 1, \dots$, where $\delta(\cdot)$ is a δ -function.

Material and energy balances of the reaction with the vibrating flow $F(t')$ can be presented as ^{*)}

$$\begin{aligned} V \frac{dc_A}{dt} &= (F_0 + \frac{\alpha_1}{\varepsilon} f(\omega t')) (c_{Af} - c_A) - V k_0 \exp\{-E/RT\} c_A, \\ C_p V \rho \frac{dT}{dt} &= \rho C_p (F_0 + \frac{\alpha_1}{\varepsilon} f(\omega t')) (T_f - T) \\ &\quad + V(-\Delta H) k_0 \exp\{-E/RT\} c_A - hA(T - T_c), \end{aligned} \quad (1)$$

where

- A heat transfer area
- c concentration
- C_p specific heat
- E activation energy

^{*)} Without loss of generality we consider a case without the recycle.

h heat transfer coefficient
 ΔH heat of reaction
 k_0 reaction rate constant
 R universal gas constant
 T temperature
 V reactor volume
 ρ density

and subscripts denote

A species A
 c cooling medium
 f feed state.

Introduce parameter

$$\tau = \frac{V}{F_0}$$

which describes in our case *average* residence time of the reactor. Assume that

$$\frac{2\pi}{\omega} \ll \frac{V}{F_0} \quad (2)$$

which means that the input flow rate has many oscillations during the average residence time. Condition (2) is easily implementable in practice. Introduce the dimensionless "reactor" time

$$t = \frac{F_0}{V} t' = \frac{t'}{\tau}$$

in order to obtain

$$F_0 \frac{dc_A}{dt} = (F_0 + \frac{\alpha_1}{\varepsilon} f(\frac{\omega V}{F_0} t)) (c_{Af} - c_A) - V k_0 \exp\{-E/RT\} c_A$$

$$C_p F_0 \rho \frac{dT}{dt} = \rho C_p (F_0 + \frac{\alpha_1}{\varepsilon} f(\frac{\omega V}{F_0} t)) (T_f - T) + V(-\Delta H) k_0 \exp\{-E/RT\} c_A - hA(T - T_c).$$

From (2) it follows that

$$\frac{\omega V}{F_0} \gg 1.$$

Therefore, denoting

$$\varepsilon = \frac{F_0}{\omega V} \ll 1,$$

we have

$$F_0 \frac{dc_A}{dt} = (F_0 + \frac{\alpha_1}{\varepsilon} f(\frac{t}{\varepsilon})) (c_{Af} - c_A) - V k_0 \exp\{-E/RT\} c_A$$

$$C_p F_0 \rho \frac{dT}{dt} = \rho C_p (F_0 + \frac{\alpha_1}{\varepsilon} f(\frac{t}{\varepsilon})) (T_f - T) + V(-\Delta H) k_0 \exp\{-E/RT\} c_A - hA(T - T_c).$$

Introducing now the following traditional notations

$$x_1 = \frac{c_{Af} - c_A}{c_{Af}} ; \quad x_2 = \frac{T - T_f}{T_f} \gamma ; \quad \gamma = E/RT_f ;$$

$$Da = k_0 e^{-\gamma} \tau ; \quad B = \frac{(-\Delta H) c_{Af}}{\rho C_p T_f} \gamma ; \quad (3)$$

$$x_{2c} = \frac{T_c - T_f}{T_f} \gamma ; \quad \beta = \frac{hA}{V \rho C_p} ,$$

we obtain the following dimensionless equations of CSTR with the vibrating fresh feed:

$$\frac{dx_1}{dt} = -(1 + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_1 + Da(1-x_1)\exp\{\frac{x_2}{1+x_2/\gamma}\}, \quad (4)$$

$$\frac{dx_2}{dt} = -(1 + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_2 + BDa(1-x_1)\exp\{\frac{x_2}{1+x_2/\gamma}\} - \beta(x_2-x_{2c}),$$

where $\alpha = \alpha_1/F_0$.

In order to simplify the computations, we assume without loss of generality that 1). activation energy is large ($\gamma \rightarrow \infty$), and 2). cooling does not take place ($x_{2c} = 0$). In this case,

$$\frac{dx_1}{dt} = -(1 + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_1 + Da(1-x_1)\exp\{x_2\} \quad (5)$$

$$\frac{dx_2}{dt} = -(1 + \beta + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_2 + BDa(1-x_1)\exp\{x_2\}.$$

Equation (5) belongs to the class of equations of the form

$$\frac{dx}{dt} = X(x) + \frac{\alpha}{\varepsilon} B(\frac{t}{\varepsilon})x \quad (6)$$

where $x \in E_n$, $X = \{X_1, \dots, X_n\}$, $B(\cdot)$ is a periodic, zero mean matrix and $0 < \varepsilon \ll 1$. Nonlinear equation (6) describes the dynamics of a wide class of chemical and biochemical processes. Analysis of this equation is presented in Section III.

Coming back to equation (5) we see that the "equilibrium" solution (ES) is defined by

$$\begin{aligned} -(1 + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_{1s}(t) + Da(1-x_{1s}(t))\exp\{x_{2s}(t)\} &= 0 \\ -(1 + \beta + \frac{\alpha}{\varepsilon} f(\frac{t}{\varepsilon}))x_{2s}(t) + BDa(1-x_{1s}(t))\exp\{x_{2s}(t)\} &= 0 \end{aligned} \quad (7)$$

from where we conclude that equation (5) has a periodic ES with a frequency $1/\varepsilon$, i.e., with a period $2\pi\varepsilon$ which is much smaller than the average residence time of the reactor. Denote this solution as

$$\begin{aligned} x_{1s}\left(\frac{t}{\varepsilon}\right) &= x_{1s}^0 + \sum_{i=1}^{\infty} \alpha_{1s}^i \sin\left(\frac{i}{\varepsilon}t + \phi_{1s}^i\right) \\ x_{2s}\left(\frac{t}{\varepsilon}\right) &= x_{2s}^0 + \sum_{i=1}^{\infty} \alpha_{2s}^i \sin\left(\frac{i}{\varepsilon}t + \phi_{2s}^i\right). \end{aligned} \quad (8)$$

In order to analyze the stability of this ES, consider a linear approximation of system (5) around (8):

$$\frac{dy_1}{dt} = -\left(1 + \frac{\alpha}{\varepsilon} f\left(\frac{t}{\varepsilon}\right)\right) \frac{1}{1 - x_{1s}\left(\frac{t}{\varepsilon}\right)} y_1 + \left(1 + \frac{\alpha}{\varepsilon} f\left(\frac{t}{\varepsilon}\right)\right) x_{1s}\left(\frac{t}{\varepsilon}\right) y_2 \quad (9)$$

$$\begin{aligned} \frac{dy_2}{dt} &= -\left(1 + \frac{\alpha}{\varepsilon} f\left(\frac{t}{\varepsilon}\right)\right) \frac{Bx_{1s}\left(\frac{t}{\varepsilon}\right)}{1 - x_{1s}\left(\frac{t}{\varepsilon}\right)} y_2 + \left\{\left(1 + \frac{\alpha}{\varepsilon} f\left(\frac{t}{\varepsilon}\right)\right) \right. \\ &\quad \left. [-1 + Bx_{1s}\left(\frac{t}{\varepsilon}\right)] - \beta\right\} y_2. \end{aligned} \quad (10)$$

These are linear differential equations with periodic coefficients which belong to the class of equations of the form:

$$\frac{dy}{dt} = \left(A + \frac{\alpha}{\varepsilon} B\left(\frac{t}{\varepsilon}\right)\right)y, \quad (11)$$

*) The values of x_{js} , α_{js}^i , $j = 1, 2$; $i = 1, 2, \dots$, can be determined on the basis of truncation of the equation for Fourier coefficients obtained from (7) with $x_{1s}(t/\varepsilon)$ and $x_{2s}(t/\varepsilon)$ of the form (8).

where $y \in E_n$, A is $n \times n$ constant matrix, $B(\frac{t}{\epsilon})$ is a periodic, zero mean matrix with a period $2\pi\epsilon$, $0 < \epsilon \ll 1$. Therefore, analysis of stability of ES (8) is reduced to the analysis of stability of the trivial solution $y = 0$ of equation (10) or (11). The theory of vibrational control reported in [2]-[5] contains the results on stability of the system of form (11) but only for the cases where $B(t/\epsilon)$ has a quasitriangular form:

$$B(t/\epsilon) = \begin{vmatrix} 0 & 0 & \cdots & 0 & 0 \\ b_{12}(t/\epsilon) & 0 & \cdots & 0 & 0 \\ \vdots & & & & \\ b_{n1}(t/\epsilon) & b_{n2}(t/\epsilon) & \cdots & b_{n,n-1}(t/\epsilon) & 0 \end{vmatrix}. \quad (12)$$

Since in the system (10) vibrations are present in the diagonal elements, results of [2]-[5] are not directly applicable to the analysis of the reactor with a vibrating flow. Section III below extends the vibrational control routine for the cases where $B(t/\epsilon)$ has a general form:

$$B(t/\epsilon) = \begin{vmatrix} b_{11}(t/\epsilon) & \cdots & b_{1n}(t/\epsilon) \\ \vdots & \ddots & \vdots \\ b_{n1}(t/\epsilon) & \cdots & b_{nn}(t/\epsilon) \end{vmatrix}.$$

III. GENERALIZATION OF VIBRATIONAL CONTROL

A). Linear Theory

Consider a system of the form (11):

$$\frac{dx}{dt} = (A + \frac{\alpha}{\epsilon} B(\frac{t}{\epsilon}))x. \quad (13)$$

Introduce fast dimensionless time

$$s = \frac{t}{\epsilon}$$

in order to obtain

$$\frac{dx}{ds} = (\epsilon A + \alpha B(s))x. \quad (14)$$

Introduce a Liapunov (preserving the stability property) transformation

$$\exp\{\alpha \int_0^s B(s)ds\}$$

which generates a substitution of the dependent variable $x(s)$ of the form:

$$x = \exp\{\alpha \int_0^s B(s)ds\}z.$$

In this case, from (14) we obtain:

$$\frac{dz}{ds} = \epsilon \exp\{-\int_0^s \alpha B(s)ds\} A \exp\{\int_0^s \alpha B(s)ds\} z. \quad (15)$$

This is an equation in so called *standard form* [6] which means that all coordinates $z_1, \dots, z_n$ are slow in time s (whereas in (14) $x_1, \dots, x_n$ are not slow in time s). An averaging principle [6] can be applied to equation (15) in order to obtain the equation with constant coefficients:

$$\frac{d\bar{z}}{ds} = \epsilon \frac{1}{T_1} \int_0^{T_1} [\exp\{-\int_0^s \alpha B(s)ds\} A \exp\{\int_0^s \alpha B(s)ds\}] ds \bar{z} \quad (16)$$

where T_1 is a period of $B(s)$.

Trajectories of $z(s)$ and $\bar{z}(t)$ defined by the identical initial conditions are related as [6]

$$||z(s) - \bar{z}(s)|| < \epsilon \quad \text{for } s \in [s_0, s_0 + \frac{1}{\epsilon}].$$

In case where (16) is asymptotically stable, $z(s)$ and $\bar{z}(s)$ are related as [7]

$$||z(s) - \bar{z}(s)|| < \epsilon \quad \text{for all } s \in [s_0, \infty).$$

Therefore, asymptotic stability of time invariant equation (16) guarantees the asymptotic stability of periodic system (15).

In order to analyse equation (16) assume that

$$\alpha < 1. \tag{17}$$

In this case considering only three first terms of matrix exponents in (16), we obtain:

$$\begin{aligned} \frac{d\bar{z}}{ds} = & \epsilon \{ A + \frac{\alpha^2}{2} \overline{A [\int_0^s B(s) ds]^2} - \alpha^2 \overline{\int_0^s B(s) ds} \overline{A \int_0^s B(s) ds} } \\ & + \frac{\alpha^2}{2} \overline{[\int_0^s B(s) ds]^2} A \} \bar{z}, \end{aligned} \tag{18}$$

where $\overline{R(s)}$ means the time average of $R(s)$. Comparing (18) with (14), we see that the vibration term $\alpha B(s)$ causes in the average description a change in matrix A which could ensure the desired localization of spectrum in the averaged equation.

In order to further simplify equation (18) assume that

$$B(s) = \phi(s)L \tag{19}$$

where $\phi(s)$ is a zero mean, periodic function and L is a constant $n \times n$ matrix.

Substituting (19) in (18) we obtain an equation

$$\frac{d\bar{z}}{ds} = \epsilon \left\{ A + \frac{\alpha^2}{2} \overline{\int_0^s \phi(s) ds}^2 (AL^2 - 2LAL + L^2 A) \right\} \bar{z} \quad (20)$$

analysis of asymptotic stability of which is a trivial problem.

Thus, we have proven the following theorem:

Theorem 1. Assume that conditions (17), (19) are met. In this case the trivial solution $y \equiv 0$ of the equation (11) is asymptotically stable if and only if equation (20) is asymptotically stable.

Corollary 1. Vibrational stabilization of a first order linear system is not possible.

This corollary follows directly from (20) with A in diagonal form, which yields

$$\frac{d\bar{z}}{ds} = \epsilon A \bar{z}.$$

This equation coincides with the original not perturbed equation (11):

$$\frac{dy}{ds} = \epsilon A y.$$

As we will see later, this is not a case for nonlinear systems of the form (5) or (6) which describe the nonlinearized dynamics of CSTR.

B). Nonlinear Theory

Vibrational control theory developed in [2]-[5] deals exclusively with linear systems. There are, however, examples of vibrational stabilization of nonlinear systems (for instance, the inverted pendulum of [6]) but the analogous results can be achieved in the linearized model as well (see, for instance, [3]). Consequently, the known nonlinear examples have a linear nature, which implies that nonlinear vibrational control theory is "untouched".

On the other hand, equations of CSTR (5) give a good model where nonlinear vibrational control phenomena could be approached. Below we describe the first

results obtained in this direction.

Consider a vector differential equation of type (6):

$$\frac{dx}{dt} = X(x) + \frac{\alpha}{\epsilon} B\left(\frac{t}{\epsilon}\right)x ,$$

where, as before, $B\left(\frac{t}{\epsilon}\right)$ is a zero mean, periodic matrix and $0 < \epsilon \ll 1$.

Introduce fast dimensionless time

$$s = \frac{t}{\epsilon}$$

to obtain

$$\frac{dx}{ds} = \epsilon X(x) + \alpha B(s)x. \quad (21)$$

Employing once more the Liapunov transformation $\exp\{\alpha \int_0^s B(s)ds\}$, we introduce a substitution:

$$x = \exp\{\alpha \int_0^s B(s)ds\}z$$

which yields

$$\frac{dz}{ds} = \epsilon \exp\{-\alpha \int_0^s B(s)ds\} X(\exp\{\alpha \int_0^s B(s)ds\}z). \quad (22)$$

This is again an equation in the standard form [6], averaging of which gives:

$$\frac{d\bar{z}}{ds} = \epsilon \frac{1}{T_1} \int_0^{T_1} [\exp\{-\alpha \int_0^s B(s)ds\} f(\exp\{\alpha \int_0^s B(s)ds\} \bar{z})] ds , \quad (23)$$

$$||z(s) - \bar{z}(s)|| < \epsilon \quad \text{for all } s \in [s_0, \infty)$$

if the equilibrium position of time invariant nonlinear equation (23) is asymptotically stable on the whole [7].

In order to analyze (23), assume, as before, that

$$\alpha < 1 \quad (24)$$

$$B(s) = \phi(s)L, \quad L - \text{constant matrix,}$$

which yields (retaining three terms in matrix exponents and expanding $f(z + \cdot)$ in Taylor series) with accuracy to α^2 :

$$\frac{d\bar{z}}{ds} = \epsilon \{ X(\bar{z}) + \frac{\alpha^2}{2} \left[\int_0^s \phi(s) ds \right]^2 \left[L^2 X(\bar{z}) + \frac{\partial X}{\partial \bar{z}} L^2 \bar{z} - 2L \frac{\partial X}{\partial \bar{z}} L \bar{z} + \frac{\partial^2 X}{\partial \bar{z}^2} L^2 \bar{z}^2 \right] \} \quad (25)$$

This is a time invariant nonlinear equation stability analysis of which can be performed by the same method as that of the original not perturbed equation (6):

$$\frac{dx}{ds} = \epsilon X(s). \quad (26)$$

Therefore, we obtain the following theorem:

Theorem 2. Equilibrium solution $x^0(s)$ of the equation (21) [defined by the unique solution of the equation $\epsilon X(x(s)) + \alpha B(s) = 0$] is asymptotically stable if and only if the equilibrium position $z^0 = \text{const}$ of the equation (25) is asymptotically stable on the whole.

Corollary 2. Vibrational stabilization of a first order nonlinear system is possible.

Indeed, in the case of the first order system, we obtain from (25):

$$\frac{d\bar{z}}{ds} = \epsilon \{ X(\bar{z}) + \frac{\alpha^2}{2} \left[\int_0^s \phi(s) ds \right]^2 L^2 \left[X(\bar{z}) - \frac{\partial X}{\partial \bar{z}} \bar{z} + \frac{\partial^2 X}{\partial \bar{z}^2} \bar{z}^2 \right] \}. \quad (27)$$

Evidently, the right hand side of this equation differs from the right hand side of the generating equation (26) which could ensure different dynamic properties of (26) and (27). This opens additional perspectives to the vibrational control applications.

The development of the nonlinear vibrational control theory will be continued during the second year of the study.

IV. ESTIMATION OF AREA OF VIBRATIONAL STABILIZATION OF CSTR

The linear theory presented in Section III shows that the vibrational stabilization could be achieved if $\text{Tr } A < 0$. Therefore, relying on the linear theory, the area of possible stabilization of CSTR is defined by

$$\text{Tr } A < 0$$

$$\text{Det } A < 0,$$

since the sign of $\text{Det } A$ can be changed by vibrations.

Hence, consider a linearized equation (10) of CSTR with $\alpha = 0$:

$$\frac{dy_1}{dt} = -\frac{1}{1-x_{1s}} y_1 + x_{1s} y_2 \quad (28)$$

$$\frac{dy_2}{dt} = -\frac{Bx_{1s}}{1-x_{1s}} y_1 - [(1+\beta) - (Bx_{1s})] y_2$$

where x_{1s} and x_{2s} are constants defined by the relationships:

$$x_{1s} = Da(1-x_{1s})\exp\{x_{2s}\}$$

$$(1+\beta)x_{2s} = BDa(1-x_{1s})\exp\{x_{2s}\}.$$

Since usually $x_{1s} \ll 1$, let us rewrite (28) in the form:

$$\begin{vmatrix} \dot{y}_1 \\ \dot{y}_2 \end{vmatrix} = \begin{vmatrix} -1 & x_{1s} \\ -Bx_{1s} & -(1+\beta)+Bx_{1s} \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}$$

Evidently,

$$\text{Tr } A = -2-\beta+Bx_{1s}$$

$$\text{Det } A = (1+\beta)-Bx_{1s} + Bx_{1s}^2.$$

Therefore, the range of parameters of β and B which give

$$\text{Tr } A < 0$$

$$\text{Det } A < 0$$

is defined by the inequalities:

$$\frac{1+\beta}{(1-x_{1s})x_{1s}} < B < \frac{2+\beta}{x_{1s}} \quad (29)$$

The range of x_{1s} where these inequalities are satisfied is defined by

$$\frac{2+\beta}{x_{1s}} - \frac{1+\beta}{(1-x_{1s})x_{1s}} > 0$$

which gives

$$x_{1s} < \frac{1}{2+\beta}.$$

Since $\beta \in [0, 2.5]$ (see [8]), the vibrationally stabilizable x_{1s} are up to the order of $0.3 \div 0.5$.

Taking into account that the stable steady states of CSTR occur for x_{1s} up to the order of 0.1, we see that the vibrational control increases the productivity of the reactor by a factor from 3 to 5.

Consider now an example. Take up linearized equations (9), (10) of CSTR with a vibration flow. Assume, for simplicity of calculations that $x_{1s}(\frac{t}{\epsilon}) \ll 1$, which gives

$$\frac{dy_1}{dt} = -(1 + \frac{\alpha}{\epsilon} f(\frac{t}{\epsilon}))y_1 + (1 + \frac{\alpha}{\epsilon} f(\frac{t}{\epsilon}))x_{1s}(\frac{t}{\epsilon})y_2 \quad (30)$$

$$\frac{dy_2}{dt} = -(1 + \frac{\alpha}{\epsilon} f(\frac{t}{\epsilon}))Bx_{1s}(\frac{t}{\epsilon})y_1 - \{(1 + \frac{\alpha}{\epsilon} f(\frac{t}{\epsilon}))[-1+Bx_{1s}(\frac{t}{\epsilon})]-\beta\}y_2.$$

The ES $x_{1s}(\frac{t}{\epsilon})$ defined by (7) can be represented as

$$x_{1s}(\frac{t}{\epsilon}) = x_{1s}^0 + \frac{\alpha}{\epsilon} \tilde{x}_{1s}(\frac{t}{\epsilon})$$

where x_{1s}^0 is an average value of periodic function $x_{1s}(\frac{t}{\epsilon})$ and $\tilde{x}_{1s}(\frac{t}{\epsilon})$ is a zero mean, periodic function with a period $2\pi\epsilon$. In this case equations (30) can be rewritten as

$$\frac{dy}{ds} = (A + B(s))y \quad (31)$$

where $s = t/\epsilon$, $y = \{y_1, y_2\}$,

$$A = \begin{vmatrix} -1 & x_{1s}^0 + \alpha^2 \overline{f(s)\tilde{x}(s)} \\ -Bx_{1s}^0 & -(1+\beta) + Bx_{1s}^0 + \alpha^2 \overline{f(s)\tilde{x}(s)} \end{vmatrix}, \quad (32)$$

$$B(s) = \begin{vmatrix} -\alpha f(s) & \alpha \tilde{x}(s) + x_{1s}^0 \alpha f(s) \\ -\alpha f(s) Bx_{1s}^0 - \alpha \tilde{x}(s) B & +\alpha f(s) (-1 + Bx_{1s}^0) + B\alpha \tilde{x}(s) \end{vmatrix}.$$

It is easy to see that with $\alpha = 0$ system (31) coincides with the linearized equations of CSTR with a constant flow. Assume that x_{1s}^0 is such that the reactor without vibrations ($\alpha = 0$) is unstable in this steady state (due to $\text{Det } A < 0$). Let us show that with $\alpha \neq 0$, system (31), (32) could be asymptotically stable.

Indeed, from (32) we see that

$$\text{Tr } A = -2 - \beta + Bx_{1s}^0 + \Delta$$

$$\text{Det } A = (1+\beta) - Bx_{1s}^0 + B(x_{1s}^0)^2 + \Delta(Bx_{1s}^0 - 1),$$

where $\Delta = \alpha^2 \overline{f(s)\tilde{x}(s)}$. Therefore, if

$$Bx_{1s}^0 > 1,$$

introduction of vibrations increases the value of $\text{Det } A$. Furthermore, substituting $B(s)$ of the form (32) into equation (18), we can see that additional terms appear in nondiagonal elements which also increase the value of determinant of the averaged equation, whereas trace remains unchanged. Thus, introduction of flow vibrations stabilizes the unstable steady states of the reactor.

V. PRESENTATION OF THE RESULTS OBTAINED DURING THE FIRST YEAR OF RESEARCH

The results reported in this paper were discussed in two invited Seminars given at the

1. Decision and Control Laboratory, University of Illinois at Urbana-Champaign (November 20, 1980);

2. Center of Control Sciences, University of Minnesota, Minneapolis (February 5, 1981).

Announcements of these Seminars are appended.

One paper was published during this year (August, 1980) of research in IEEE Transactions on Automatic Control intitled "Vibrational Control: Theory and Applications". This paper was submitted for publication before the beginning of the Research Project sponsored by DOE. Another paper reporting the above presented results is now in preparation.

During the period of time July - August 1980, 100% of time was spent on this Project; September - November 1980 - 20% of time; December 1980 - January 1981 - 50% of time; February - March 1981 - 20% of time. Projected activity in April - May 1981 is 20% of time; June - 100% of time.

VI. BIBLIOGRAPHY

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5. S.M. Meerkov, "Principle of Vibrational Control: Theory and Applications", *IEEE Trans. Automat. Contr.*, vol. AC-25, No. 4, pp. 755-762, Aug. 1980.
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CONTROL AND OPTIMIZATION SEMINAR

Speaker: Professor S. Meerkov
Illinois Institute of Technology
Chicago, Illinois

Topic: "Method of Vibrational Control"

Date: Thursday, November 20, 1980

Time: ~~3:00 p.m.~~ 2:00 p.m.

NOTE CHANGE OF TIME

Place: 4-122 Coordinated Science Laboratory

UNIVERSITY OF MINNESOTA
TWIN CITIES

Institute of Technology
Center for Control Sciences
Minneapolis, Minnesota 55455

CONTROL SCIENCE AND DYNAMICAL SYSTEM CENTER

UNIVERSITY OF MINNESOTA

SEMINAR - WINTER QUARTER 1981

3:15 PM THURSDAYS IN AERO ENG. 319

THEME - DYNAMICAL SYSTEMS: OSCILLATION, BIFURCATION AND CHAOS

- JAN. 8 M. LEVI (DUKE UNIVERSITY)
"RANDOM BEHAVIOR IN DETERMINISTIC SYSTEMS"
- JAN. 15 E. B. LEE AND L. MARKUS (CSDS CENTER, UNIV. OF MINNESOTA)
"CONCEPTS OF NONLINEAR DYNAMICS - DISCUSSION"
- JAN. 22 D. ARONSON (UNIVERSITY OF MINNESOTA)
"BIFURCATIONS FROM AN INVARIANT CIRCLE"
- JAN. 29 A. K. BIAJ (PURDUE UNIVERSITY)
"BIFURCATIONS IN SYSTEMS WITH SYMMETRY"
- FEB. 5 S. M. MEERKOV (ILLINOIS INST. TECH.)
"VIBRATION CONTROL: THEORY AND APPLICATIONS"
- FEB. 12 P. HOLMES (CORNELL UNIVERSITY)
"BIFURCATION OF SUBHARMONICS AND FORCED OSCILLATIONS"
- FEB. 19 S. N. CHOW (MICHIGAN STATE UNIVERSITY)
"INTERVAL MAPPINGS AND SINGULARLY PERTURBED DELAY EQUATIONS"
- FEB. 26 E. B. LEE (CSDS CENTER, UNIV. OF MINNESOTA)
"ALGEBRAIC METHODS FOR LINEAR SYSTEMS"
- MAR. 5 J. GUCKENHEIMER (UNIV. OF CALIFORNIA, SANTA CRUZ)
"CO-DIMENSION 2 BIFURCATIONS"

STUDENTS MAY REGISTER FOR 1-3 CREDITS AS EITHER MATH 8531 (L. MARKUS) (373-2586) OR EE 8291 (E. B. LEE) (373-4895) AND INFORMAL TUTORIAL SESSIONS WILL SUPPLEMENT THE SEMINAR PROGRAM.