A Study of Flow and Loss Processes
at the Ends of a Linear Theta Pinch

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ABSTRACT

Experimental and analytical studies of end loss from a theta pinch have been carried out. Detailed diagnostic studies of a 25 cm long theta pinch operating with reversed trapped fields have been completed; spectroscopic studies, magnetic probe, pressure probe, double diamagnetic loop, luminosity studies and Thomson scattering studies of the plasma have been carried out over the 8 μ sec duration of the transient loss. Two new diagnostic techniques have been developed based on the available Thomson scattering laser source. A study of plasma loss from a 10.5 long theta pinch with an axial Twyman-Green interferometer has been completed and reported. The basic studies needed for subsequent experimental work on heat conduction loss being diagnosed by Thomson scattering data in the end region, with and without mirror coil, has been completed as a part of the mirror field studies. Analytical studies continue to emphasize computational modeling of experiments. Zero-dimensional code studies of both collisional and collisionless experiments have been completed. One-dimensional codes are under development, with the first code, without thermal conduction, successfully describing Scylla I-C behavior. Implosion models, to be used with the above codes, are under study.

I. INTRODUCTION

The studies of end loss from theta pinches to be discussed in this progress report follow the line of approach outlined in the original proposal, but they also show some changes which mirror the advances in understanding the general problem. These changes relate to improvements in the precision of and conduct of the overall experiment, and, in one study, to the revitalized importance of plasma-field interactions on the reversed trapped field mode of operation.

Specifically, referring to work that has been completed with the 25 cm theta pinch, the importance of gas purity on reverse trapped field levels and column flux exclusion was evident. This influence was realized only because of the precision and sensitivity of the diagnostics that were used. Operation with pump base pressures of $\sim 2 \times 10^{-6}$ T (Mode I) and a "clean" discharge tube resulted in both; (1) a larger flux exclusion within the plasma column and (2) a smaller amount of reverse trapped field convected out of the machine during the flow event, than was obtained with operation at pump base pressures of $\sim 6 \times 10^{-6}$ T or a "dirty" discharge tube (Mode II). The reverse trapped field originates during the theta preionization phase with its intensity at pinch time being critically dependent upon the precision of main bank ignition. Reproducible reverse trapped field levels were obtained only with the development of 0.1 μ sec precision main bank switching. This work will provide a supplement to transient loss studies of plasma from a 1 m long, hot device at Los Alamos Scientific Laboratory.

Two new diagnostics have been brought to total fruition. The development of a Twyman-Green axial interferometer to indicate loss time for plasma in the coil-tube system has been completed and used to study the plasma in a 10.5 cm long coil. This work has been reported in a separate document. Also, the development and use of a ruby Thomson scattering system is completed and has

been used to study the plasma being lost from the 25 cm pinch; this work will be reported to provide a comparison of spectroscopic techniques with this precise and reliable diagnostic. This diagnostic will be the crucial foundation for the first studies of the 50 cm long theta pinch plasma, which will concentrate on particle and energy loss mechanisms; electron thermal conduction will be studied in detail. Several other diagnostics including 0.1 μ sec resolution pressure transducers, have been developed and applied. Studies of two new Thomson scattering diagnostics have been completed: (1) low density resolution Thomson scattering with Nd. Glass, and (2) velocity measurements with the present Ruby system. However, unless there is a DOE directed and supported effort in this diagnostic development, the work will not proceed further.

The evaluation of magnetic mirror field generation has been completed and has served to define the base configuration for the next phase of loss studies. With a 7.62 cm discharge tube in a 10.5 cm coil, mirror fields of 2:1 will first be generated. Add-on coils will be used to supplement this configuration to produce 3:1 fields and an anticipated increase in loss time by 250 percent.

The discharge apparatus will shortly be increased in size to 50 cm with the addition of another large capacitor-switch rack. A new preionization system will be installed to overcome difficulties with the present theta preionization. The question of mirror supplements by crowbarred high frequency coils or free-ringing low frequency solenoids will be answered by testing.

The theoretical effort has covered three main areas: (1) computer modeling of theta pinch experiments; (2) evaluation of simple implosion models for use with the computer models; and (3) modeling and analysis of reactor systems. The primary effort was devoted to computer modeling. The development and use of a zero-dimensional code for describing both collisional and collisionless

experiments has been completed. A report on this subject has been submitted, and an article has been accepted for publication in the Physics of Fluids.

A one-dimensional Lagrangian code without thermal conduction has been developed and used to simulate the Scylla I-C experiment. Agreement is excellent. A code with thermal conduction - using an implicit coding scheme - is being tested. These codes will be used to study plasma behavior in the exhaust region with a variety of boundary conditions. Comparison with the PSU 25 cm pinch results will then be made.

Implosion studies, aimed at obtaining post implosion values of density and temperature, have been performed for both collisional and collisionless experiments. Finite beta and trapped field effects have been incorporated into the snowplow models, and agreement with the Scylla I-C experiment and the PSU 25 cm pinch experiment have been achieved. To describe collisionless experiments several different bounce models were developed including a hybrid snowplow-bounce model and a new multiple-bounce model which is still being analyzed. Use of the results of these bounce theories with various experiments are being evaluated. None of the theories has yielded satisfactory values for Scylla IV-P.

Two projects were completed in the relatively small fusion engineering effort. The scaling analysis of Ellis has been applied to a linear theta pinch with multiple mirror scaling, and machines with $B = 200$ kG have been studied. Tradeoffs on coil radius and length are found, and machines with length 1 km appear possible if thermal conduction is inhibited. The zero-dimensional code has been extended to include alpha particles via a multigroup model, and bremsstrahlung. A system energy balance model is being developed so that the code can be used for design studies.

Efforts that are exploratory in nature but have received some attention and are promising avenues for further consideration, are: (1) multiple mirrors

for a theta pinch system would provide significant improvements in loss time ($\times 10$); they also would provide a configuration for (2) turbulent heating by DC discharge in a mirror cell at lower (10^{13} - 10^{14} cm^{-3}) densities to study anomalous conduction and energy dissipation and transport; (3) direct velocity measurements with the Thomson scattering system.

During the conduct of this work, the two principal investigators have been receiving the invaluable cooperation and efforts of Professor James Robinson of the Electrical Engineering Department; Professor Richard Mollo of the Electrical Engineering Department has consulted on problems dealing with the laser diagnostics. Graduate Assistants, who have been responsible for the detailed conduct of much of the work, are acknowledged following the title of each research activity.

II. EXPERIMENTAL STUDIES OF TRANSIENT LOSS FROM A 25 CM THETA PINCH (Heidrich, Jacoby, Robinson)

1. Description of the Theta Pinch System

The machine parameters are listed in Table 1. The coil is 25 cm long and 10.5 cm in diameter; the pyrex discharge tube has an inside diameter of 9 cm. The discharge tube diameter matched the coil diameter so that strong coupling of the coil to the plasma was obtained.

The fill gas used throughout the experiments reported was 100 mT helium. Preionization was performed by R. F. coils followed by a theta discharge. The preionizer was actively discharging for 9 μ sec before main bank ignition. The 100 μ F, 18.5 kV main bank has a storage energy of 17.1 kJ; the bank is not crowbarred and has a half cycle time of 9.45 μ sec. Peak magnetic field is 19 kG as measured by a calibrated probe. In Figure 1, the current and driving magnetic field waveforms are presented for both the preionization and main bank.

A schematic of the theta coil and discharge tube illustrating the coordinates used in the remainder of this report are present in Figure 2. In this figure: z = axial position (+ inside the coil, - outside); r = radial position inside the discharge tube; and y = chord position.

2. Diagnostics

A wide variety of diagnostics has been utilized in diagnosing the 25 cm theta pinch. Those which will be used to provide quantitative information on the plasma column and endloss phenomena are listed in Table 2. The physical quantities that were measured are shown in the left hand column. The corresponding diagnostics used to measure these physical quantities are listed in the right hand column. Several varieties of electric probes were also studied

TABLE 1
25 cm Theta Pinch Parameters

Coil Length: 25 cm	Coil Inside Diameter: 10.5 cm	Discharge Tube Inside Dia.: 9 cm
--------------------	-------------------------------	-------------------------------------

Preionization System: RF Coils Followed by a Theta Discharge.

RF Coils: Number of coils: two, one on either side of the theta pinch coil.

Location: 5 cm outside the theta coil.

Ring Freq.: 20 MHz

Theta Discharge:

Max theta electric field:	69 volt/cm
---------------------------	------------

Half cycle time:	2.25 μ sec
------------------	----------------

Ring time before ignition of main bank:	9 μ sec
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Main Bank:

Charging Voltage:	18.5 kV
Total system capacitance:	100.1 μ F
Total system inductance:	90.4 nH
Coil inductance:	32 nH
Peak current:	515,000 amp.
Peak driving magnetic field	19 kG
Half cycle time:	9.45 μ sec
Maximum theta electric field	138 volt/cm
over first quarter cycle:	88 volt/cm

Fill Gas: Helium at 100 mT

Figure 1. Current and Driving Magnetic Field Wave Form for the 25 cm Theta Pinch

Figure 2. Schematic of the 25 cm Theta Pinch and Coordinates for Diagnostic Location

TABLE 2
Diagnostics Used for 25 cm Theta Pinch Studies

<u>Quantities Measured</u>	<u>Diagnostic Used</u>
Bank Current	Rogowski Coil
Local Magnetic Fields	Pickup Loop Probes
Local Impact Pressure	Specially Designed Piezoelectric Transducer Pressure Probe
Luminosity	Photography Using Kerr Cell Shutter
Plasma Diamagnetism	Balance Signal From a Rogowski Coil and a Flux Loop
Electron Density	Stark Broadening of He^+ 4686Å Line; Thomson Scattering in Conjunction with Continuum Measurements
Electron Temperature	Intensity Ratio of He^+ 4686Å Line to Underlying 100Å Continuum; Thomson Scattering; Absolute Intensity of He^+ 4686Å Line

All measurements performed have a temporal resolution of 0.1 usec.

in some detail but marginal information concerning plasma behavior was obtained; these are discussed later.

In the remainder of this section, the following topics will be discussed: (1) the rationale for choosing the main bank delay time of 9 μ sec; (2) the origin of the reverse trapped magnetic field; (3) a brief description of the two modes of operation; and, (4) a brief description of each diagnostic, followed by some of the data obtained from it which characterize the plasma column and endloss.

3. Main Bank Ignition Delay Following Preionization and the Reverse Field Mode

Experimentation was begun by measuring flux exclusion due to plasma at the coil midplane with a single diamagnetic loop around the coil. It was observed that for delays in bank ignition in which the preionization was allowed to ring through more than three of its cycle times, the maximum flux exclusion during collapse markedly decayed to values below that predicted by Green, Newton and Segre.¹ They show that the maximum flux exclusion occurs when the sheath has moved 1/5 of the tube radius and is given by;

$$S = A_T \{f M B_0^2 \omega^2\}^{1/4},$$

where

B_0 = peak driving magnetic field

$\omega = 2\pi \times$ (frequency of the main bank)

M = mass per unit length in motion after the plasma sheath has traversed 1/5 of the tube radius.

A_T = area of the discharge tube.

Green et al. argue that low values of S indicate incomplete trapping of the gas due to low preionization.

A local magnetic field probe with multiple turns and incased in pyrex was inserted radially at the theta coil's midplane to monitor axial field. The excluded flux loop indicated plasma formation on the second half cycle of the theta preionization discharge, and the local probe simultaneously indicated a trapping of field in a reverse direction to that of the main bank's driving field. The local probe showed reverse field diffusing out of the plasma during the preionization phase.

As it was not possible to satisfy the two desirable goals of high percent preionization and low trapped magnetic fields, the main bank ignition was delayed for two complete preionization cycles, as creating a reasonable trade off between minimizing the size of the reverse trapped field but maintaining a level of preionization sufficient to give a strong diamagnetic signal upon collapse. While this experiment was not initiated to study reversed trapped magnetic field behavior in theta pinches, trapped field is inherent in theta preionization.

With reference to Figure 1, main bank ignition was chosen at a time near a current (field) zero on the preionization cycle. The PI discharge does break down the gas in the theta discharge mode and does trap some of the breakdown field in the plasma.

The radially inserted magnetic probe measurements indicated a 200 G reverse trapped magnetic field at main bank ignition. It should be noted that this reverse field is significantly lower than that of typical reverse field theta pinches. For example, Eberhagen and Grossmann² used reverse bias fields of 0.8 kG and 1.4 kG in association with a peak driving field of 28 kG.

4. Mode I ("Clean") and Mode II ("Dirty")

In recent work at Los Alamos Scientific Laboratory, R. Linford has developed a quantitative ordering of the effects of vacuum and tube cleanliness

on several trapped field plasma confinement. The present work was carried out more or less simultaneously and independently.

Mode I is evident with operation using a clean discharge tube at Vac Ion pump base pressures between 1.6×10^{-6} Torr to 2.8×10^{-6} Torr. Operating at Vac Ion pump base pressures between 4×10^{-6} Torr and 8×10^{-6} Torr results in Mode II operation. It was noticed that when the inside of the discharge tube was "dirty" that even though the base pressure at the Vac Ion pump was below 2.8×10^{-6} Torr Mode II results were obtained.

Physical differences were found to exist between Mode I and Mode II operation. Mode I is characterized by a larger flux exclusion at the coil midplane ($z = +12.5$ cm) than is Mode II. Also, photographic and spectroscopic luminosity studies have indicated that light emission from the plasma is less in Mode I than in Mode II operation.

5. Luminosity Measurements

End on and side on luminosity measurements have been performed. Framing photographs were taken using a Kerr cell shutter in conjunction with a 4x5 framing camera with polaroid back.³ The exposure time was 100 nsec in all cases. End on photographs were all taken in Mode I to identify plasma exclusion from the axis; side on photographs were taken earlier in Mode II to explore plasma expulsion from the coil.

End region, side on luminosity studies were performed only in Mode II operation since plasma light emission was insufficient in Mode I operation. Various front trajectories have been plotted for Mode II operation and are presented in a later figure (Figure 16). The side on luminous front is reasonably coincident with the impact pressure front.

Typical end-on framing shots focused on the coil midplane are shown on Figure 3 a-h. The times listed are all after main bank ignition. The first

a $t = 1.2 \mu\text{sec}$ Pinch

Discharge Tube Radius (cm)

b $t = 1.6 \mu\text{sec}$ First Expansion

Discharge Tube Radius (cm)

c $t = 1.5 \mu\text{sec}$

Discharge Tube Radius (cm)

d $t = 1.6 \mu\text{sec}$

Discharge Tube Radius (cm)

Figure 3. End on Kerr Cell Photographs (Mode I)

Figure 3. End on Kerr Cell Photographs (Mode I)

photograph, a, shows the plasma column at pinch time. The next two photographs, b and c, show the subsequent maximal plasma radius attained during the column oscillation and the beginning of column fluting. The flutes, typically 6 to 8 in number, grow as the plasma column oscillation proceeds to a minimum radius at about $t = 1.8 \mu\text{sec}$ in photograph e. The plasma column expands again until $t = 2.0 \mu\text{sec}$ at which point the reverse field has completely dissipated. The flutes were seen to grow until about $2.5 \mu\text{sec}$ at which point they sometimes reached the discharge tube wall. By $t = 3 \mu\text{sec}$ the flutes disappeared, and typically by $t = 4 \mu\text{sec}$ an $m = 2$ type profile was evident and remained for the duration of the time in which photographs could be taken ($t = 5 \mu\text{sec}$).

The reverse trapped magnetic field can be calculated by assuming flux conservation during dynamic collapse and estimating the plasma radius at the end of collapse. From photographs a and b an estimated average (equilibrium) inner column radius $\bar{r}_{in}$, is found to be 0.75 cm. The trapped field at time t is found by assuming magnetic pressure balance on either side of the annulus. Then from flux conservation;

$$B_{in}(t = 0) = B_o \sin(\omega t) (\bar{r}_{in}^2 / R_o^2) ,$$

where

$B_{in}(t = 0)$ = reverse trapped field at the beginning of collapse

$\omega = 2\pi/\tau = 2\pi/(\text{bank cycle time}) = \pi/9.45 \mu\text{sec}$

t = time at which plasma annulus passes its equilibrium position ($\approx 1.3 \mu\text{sec}$)

$\bar{r}_{in}$ = equilibrium radius = 0.75 cm

R_o = discharge tube radius = 4.5 cm

B_o = peak magnetic field = 19 kG

$B_{in}(t = 0) = 221 \text{ Gauss}$ for the initial reverse trapped field, in agreement with calibrated field probe results.

The reverse field lifetime appears to be limited by classical field diffusion;

$$\tau_m = \mu r \delta \sigma / 2 \approx 0.8 \text{ } \mu\text{sec} \quad ,$$

where

δ = distance over which there is a field gradient = 0.65×10^{-2} m

r = radius of the plasma annulus = 1.0×10^{-2} m

σ = electrical conductivity of the plasma = 2×10^4 mho/meter (calculated using an electron temperature of 12 eV, as determined spectroscopically)

μ = 4×10^{-7} H/m

The 0.8 sec result is almost precisely the lifetime of the trapped field after collapse.

The flutes are thought to be due to a Rayleigh-Taylor MHD instability where the driving acceleration is "centrifugal" acceleration due to plasma column rotation. To support this view, it is noted that the Rayleigh-Taylor growth rate is given by $\sqrt{gk}^4$, where, for the present situation,

$$g = v_t^2 / \bar{r}_p \quad \text{and} \quad k = 2\pi / \lambda$$

with

v_t = plasma annulus tangential velocity

$\bar{r}_p$ = average plasma annulus radius ≈ 1 cm

$\lambda = 2\pi \bar{r}_p / n$ = circumference of annulus/number of flutes $\approx 2\pi$ cm/7

The measured growth rate from the end on photographs is approximately $(0.1 \text{ } \mu\text{sec})^{-1}$. Equating this experimentally determined growth rate to $\sqrt{gk}$ we find

$$v_t \approx 3.8 \times 10^6 \text{ cm/sec}$$

$$\omega = v_t / \bar{r}_p \approx 3.8 \times 10^6 / \text{sec}$$

This rotational velocity would lead to a Doppler shift of $\Delta\lambda = -\tilde{\lambda} v_t / c$, where $\tilde{\lambda}$ is the optically observed wavelength. For the $4686\text{\AA}$ He^+ line and $v_t = 3.8 \times 10^6$ cm/sec,

$$\Delta\lambda = \frac{-(4686\text{\AA})(3.8 \times 10^6 \text{ cm/sec})}{3 \times 10^{10} \text{ cm/sec}} = 0.59\text{\AA}.$$

When viewing the $y = +1$ cm chord at $z = +6.25$, an approximate $0.5\text{\AA}$ red shift in the $4686\text{\AA}$ line center was observed. The plasma rotation just after collapse is in the direction of the diamagnetic current, and the measured value of v_t is consistent with the interpretation that the flutes are associated with a Rayleigh-Taylor instability.

6. Excluded Flux and Effective Plasma Radius

The double loop system developed previously⁵ was used to measure the excluded flux at $z = +12.5$ cm. Briefly, the system consisted of two coils; a single turn large area coil responded to changing flux inside the discharge tube at $z = +12.5$ cm and a Rogowski coil responded to changing current through the theta coil. For vacuum discharges, the signals from the two coils were balanced by a passive differencing circuit. The integral of the difference signal between the two coils obtained with a plasma is proportioned to the flux excluded by the plasma. A complete circuit analysis description will be presented in the final report on this work.

In order to ensure a reliable calibration, a 1 in diameter copper bar was inserted into the discharge tube to simulate a plasma. This resulted in a calibration of 3.11×10^5 Maxwells/volt.

Excluded flux measurements have been performed at the coil midplane in both modes of operation. Corresponding effective radii of flux exclusion were calculated and are plotted on Figure 4. The effective plasma radius (radius

Figure 4. Effective Radius of the Plasma Column at the Coil Midplane ($z = +12.5$ cm)

of flux exclusion) is defined as $r_{\text{eff}} = \sqrt{\Delta\Phi/\pi B_{\infty}}$, where $\Delta\Phi$ = excluded flux and B_{∞} = driving external magnetic field. The figure has two noteworthy points. First, after 2 μsec the effective radius in Mode II is less than that corresponding to Mode I operation. The implications of this result have not been studied in detail, but one possibility is that less reverse field is trapped at pinch time in Mode II at $z = +12.5$ cm. The second point is that after 4 μsec in Mode II and 5 μsec in Mode I we have included flux; i.e., the plasma's internal magnetic field strength is greater than the external field strength. This phenomena occurs near peak driving magnetic field. Of course, the resulting pressure imbalance implies that the plasma column is expanded outward due to high internal pressure. If, in fact, this phenomena is real, a study will be done to determine the physical causes and their implications.

The effective plasma radius and the inner and outer plasma radii from the end on Kerr cell photographs are plotted in Figure 5. The effective radius data shows less pronounced oscillation than the luminous radii.

7. Axial Magnetic Field Measurements

Local magnetic field measurements of the axial component of magnetic field have been performed using small diameter probes⁶ housed inside 5 mm o.d. pyrex tubes. The probe signals were integrated using passive RC integrators and calibrated from a known source of field.

A probe was inserted radially at the coil midplane during Mode II operation. Field trapping during preionization and collapse phenomena were studied. The reverse field on axis was ~ 200 Gauss at main bank ignition. This value agrees with that obtained from the end on Kerr cell studies during Mode I operation. The collapse data will be analyzed to yield implosion velocities and mass/length in the current sheath⁷.

Figure 5. Plasma Radius Variation with Time Luminosity and Flux Loop Indication (Mode 1)

The local z component of magnetic field, B_z , was measured by axial insertion of a probe through an endflange. B_z profiles were obtained in both modes of operation and in the vacuum. The data is presented on Figures 6 - 10 as the ratio of B_z in the plasma to B_z in the vacuum. Mode II results are plotted above the centerlines with Mode I results plotted below the centerlines. The most immediate observation is that less reverse field is convected out of the machine during Mode I operation.

The trajectory of initial suppression of B_z , due to convection of reverse field out of the machine during Mode II operation, is plotted in Figure 16 along with trajectories of pressure and luminosity. B_z begins to be suppressed before arrival of the pressure front. This is not understood at present and will be studied further.

Flux loop measurements in the endloss region confirmed the local measurements of B_z ; namely, Mode I operation results in less flux suppression during endloss than there is in Mode II operation.

8. Impact Pressure

Impact pressure measurements have been performed in the end region during Mode II operation. A piezoelectric transducer probe⁸ was used to make these measurements. Similar measurements in Mode I operation are yet to be carried out.

Pressure profiles during Mode II operation are plotted in Figures 11-15. The 1.2 μ sec profile is at pinch time. The two pressure signals are of a collapsing sheath outside the coil. A weak collapse exists at $z = -5$ cm, in agreement with magnetic probe results. A well collimated endloss is in progress at $t = 1.6 \mu$ sec. At $t = 2 \mu$ sec well collimated endloss still exists but a smaller secondary pulse is noted on the profile at the coil's edge. The pulse

Figure 6. Ratio of Axial Plasma Magnetic Field, B_{zp} , Axial Vacuum Magnetic Field, B_{zv} , at $t = 1.2 \mu\text{sec}$ After Main Bank Ignition

Figure 7. Ratio of Axial Plasma Magnetic Field, B_{zp} , to Axial Vacuum Magnetic Field, B_{zv} , at $t = 1.6 \mu\text{sec}$ After Main Bank Ignition

Figure 8. Ratio of Axial Plasma Magnetic Field, B_{zp} , to Axial Vacuum Magnetic Field, B_{zv} , at $t = 2.0 \mu\text{sec}$ After Main Bank Ignition

Figure 9. Ratio at Axial Plasma Magnetic Field, B_{zp} , to Axial Vacuum Magnetic Field, B_{zv} , at $t = 3.0 \mu\text{sec}$ After Main Bank Ignition

Figure 10. Ratio at Axial Plasma Magnetic Field, B_{zp} , to Axial Vacuum Magnetic Field, B_{zv} , at $t = 4.0 \mu\text{sec}$ After Main Bank Ignition

Figure 11. Impact Pressure Profiles at $t = 1.2 \mu\text{sec}$ After Main Bank Ignition (Mode II)

Figure 12. Impact Pressure Profiles at $t = 1.6 \mu\text{sec}$ After Main Bank Ignition (Mode II)

Figure 13. Impact Pressure Profiles at $t = 2.0$ μ sec After Main Bank Ignition (Mode II)

Figure 14. Impact Pressure Profiles at $t = 3.0$ μ sec After Main Bank Ignition (Mode II)

Figure 15. Impact Pressure Profiles at $t = 4.0 \mu\text{sec}$ After Main Bank Ignition (Mode II)

may be the loss of cooler plasma not picked up by the collapse inside the coil. At $t = 3 \mu\text{sec}$ after bank ignition the main pressure pulse is well outside the coil with a trailing region of pressure extending out to $z = -3\text{cm}$. The main pressure signal spreads radially somewhat starting at $z = -7.5 \text{ cm}$ with a distinct decrease in amplitude at $z = -10 \text{ cm}$.

The on-axis trajectory of the main pulse of impact pressure - not including the trailing region - is illustrated in Figure 16. The dotted lines indicate relative amplitudes at the different axial positions. Notice that peak pressure amplitudes are almost constant out to $z = -7.5 \text{ cm}$ but decrease dramatically by $z = -10 \text{ cm}$. The distinct drop in pressure between $z = -7.5$ and -10 cm is probably due to radial spreading of the profile.

At $z = 0$, the rise and decay times of the main pressure pulse are $0.2 \mu\text{sec}$, while at $z = -7.5$ the rise time increases to about $0.4 \mu\text{sec}$. Finally at $z = -10 \text{ cm}$ there is a very gradual rise and subsequent decay in the pressure signal.

9. Spectroscopic Studies

a. Apparatus

The basis spectroscopic apparatus consists of a $1/2 \text{ m}$ Jarrell Ash spectrometer, a converging input lens, a 1P21 photomultiplier tube, an operational amplifier and a 556 Tektronix oscilloscope.

Two sets of openings at $z = +12.5$ and $+6.25 \text{ cm}$ defined chords across the plasma column. Each set consists of 12 - 1 mm diameter holes, each hole separated vertically, such that a 0.5 cm distance exists between the chords viewed across the plasma column. For axial positions at $z < 0 \text{ cm}$, a mask with slots 2 mm wide was placed on the discharge tube and define chords across the plasma column which were separated by 0.5 cm .

Figure 16. Magnetic Field, Pressure, and Luminous Front Trajectories (Mode II)

The converging input lens had a focal length of 7.5 inches and a diameter of 2 inches. The object distance (distance from the center of discharge tube to the lens) was chosen so that the set of holes or slits being imaged on the spectrometer's entrance slit were demagnified ~ 10 times. The chords defined by the lens in conjunction with the holes or slits were less than 2 mm wide. Hence, spatial resolution was limited by the 0.5 cm spacing between the chords.

The spectrometer and lens were set up viewing the set of chords at the particular axial position being studied. Because of the system's demagnification, all the chords of the set were focused onto the spectrometer's entrance slit. The holes or slits not being viewed were taped. In order to insure optical alignment, a pyrex tube, with a mercury source in it, was inserted through an endflange until the source was on the chord being viewed. With the spectrometer set on a mercury line the lens was then adjusted to maximize the photomultiplier tube output.

The operational amplifier, an RCA CA3015A, was reworked to give a risetime $\leq 0.1 \mu\text{sec}$ and a transresistance (output voltage/input current) of 101,000 ohms.

The instrument function of the spectrometer was measured using a HeNe laser source. A Gaussian of full half width $1/3\text{\AA}$ was found to fit the instrument function when the spectrometer slits were 30 μ wide:

b. Measurements

Continuum and $4686\text{\AA}$ He^+ intensities were measured in Mode I operation. The $4686\text{\AA}$ profiles had to be taken using shot-to-shot data collection, since only one data channel existed. At axial positions $x = +12.5$ and $+6.25$, $4686\text{\AA}$ line profiles could only be taken for the $y = 0$ chords due to excessive shot-to-shot scatter at other chords. At $z = -2.5$ cm, shot-to-shot scatter was less severe, and $4686\text{\AA}$ profiles were taken for chord positions $y = 0, +.5, +1, +1.5$,

+2.0 cm. Shot-to-shot scatter in the continuum was much less than for line intensity data; hence, continuum intensity measurements at all chords across the plasma were taken at each of the following axial positions; $z = +12.5$ cm, +6.25 cm, -2.5 cm, -5.0 cm, -7.5 cm, and -10.0 cm.

c. Data reduction techniques

The intensity as a function of chord position was Abel inverted to give intensity as a function of radius. The Abel inversion was performed numerically utilizing an inversion algorithm developed by Barr.⁹

Electron densities were determined from the Stark broadening of the 4686Å He^+ line. Stark broadened line widths are typically listed in terms of a reduced wavelength; $\alpha \equiv \Delta\lambda/F_0$, where $\Delta\lambda$ = line width and F_0 = the field strength. When $\text{He}^{++} - \text{He}^{++}$ correlation (perturber - perturber correlations) can be ignored, F_0 is given by the Holtsmark field strength¹⁰

$$F_0 = 2\pi \left(\frac{4}{15}\right)^{2/3} |z_p e| N_p^{2/3}$$

where, N_p = ion perturber (He^{++}) density

z_p = ion perturber level of ionization = 2

e = electronic charge = 4.8×10^{-10} stat coul.

Since the plasma is essentially fully doubly ionized, $N_p = N_e/z_p = N_e/2$. Thus, the field strength can be written as $F_0 = 2\pi \left(\frac{4}{15}\right)^{2/3} z_p^{1/3} |e| N_e^{2/3}$, where N_e = electron density.

Perturber - perturber correlations can be ignored if the mean Coulomb interaction energy, $z_p^2 e^2 N_p^{1/3}$, is below the thermal energy, kT ,¹¹ or

$$N_p \lesssim \left(\frac{kT}{z_p^2 e^2}\right)^3 \approx 4 \times 10^{21} / \text{cm}^3$$

for our plasma. Substituting the Holtsmark field strength for F_0 in $\Delta\lambda = \alpha F_0$, one obtains

$$\Delta\lambda = 2\pi \left(\frac{4}{15}\right)^{2/3} z_p |e| N_e^{2/3} \alpha.$$

Solving for N_e , one finds

$$N_e = \frac{15}{4} \frac{1}{(2\pi e)^{3/2}} \frac{1}{z_p^{1/2}} \left(\frac{\Delta\lambda}{\alpha}\right)^{3/2} = 1.6 \times 10^{13} \left(\frac{\Delta\lambda}{\alpha}\right)^{3/2}.$$

Here $\Delta\lambda$ = half of the Stark half width when α reduced wavelength's half-half width. Values for α were obtained from Kepple.¹²

In order to obtain electron density the Doppler and spectrometer broadening components had to be subtracted from the half-half widths of the observed lines. This can be accomplished utilizing Voigt profiles because the cores of Stark profiles are almost Lorentzian¹³ while to a good approximation, both Doppler and spectrometer broadening are Gaussian.

Electron temperatures were obtained from the intensity ratio of the 4686Å^o line to the underlying 100Å^o continuum¹⁴ and from a measure of the absolute intensity of the 4686Å^o line. The two methods gave temperatures which were in agreement just after collapse at $t = 1.4 \mu\text{sec}$, but by $t = 2.0 \mu\text{sec}$ the line-to-continuum gave absurdly high results. It appears that even in the "clean" mode high z impurity levels significantly increased the bremsstrahlung continuum and hence invalidate the line-to-continuum method of temperature determination. The absolute intensity of the 4686Å^o line is given by,¹⁵

$$I_{4686\text{Å}}^{\circ} = \frac{\bar{h}\omega^3 r_o}{4\pi c} g_3^+ f_{43} \left(\frac{2\pi\hbar^2}{mkT_e}\right)^{3/2} e^{E_4/kT_e} \int_{\text{chord length}}^N N^+ dx,$$

where f_{43} = absorption oscillator strength = 0.8420

g_3^+ = degeneracy of the lower state, $n = 3$, of the 4686Å^o line = $2n^2 = 18$

r_o = classical electron radius = 2.82×10^{-5} Å

$\bar{h}$ = Planck's constant/ 2π = 1.055×10^{-27} erg - sec

k = Boltzmann's constant = 1.38×10^{-16} erg/°K

T_e = electron temperature in °K

m = electron mass = 9.11×10^{-28} gm

E_4 = energy difference between the upper level of the line and the continuum = 3.4 eV

N_e = electron density

N^{++} = He⁺⁺ density = $N_e/z = N_e/2$

ω = angular frequency of 4686Å line = 4.02×10^{15} /sec

c = speed of light = 3×10^{10} cm/sec.

Hence, with a knowledge of the electron density from the Stark broadening results, length of plasma across the particular chord of interest, and the measured intensity of the 4686Å line, the electron temperature can be obtained. This electron temperature is then compared with that used in the electron density determination. Since α is a weak function of temperature, if the two temperatures are not in agreement, the temperature obtained from the absolute line intensity is used to recalculate the electron density. This density is then used to obtain a new electron temperature from the 4686Å intensity. Hence, in this self consistent electron temperature and densities were obtained.

Relative density profiles were obtained from the Abel inverted continuum measurements. The bremsstrahlung continuum is proportional to $Z^2 N_e^2 / T_e^{1/2}$ in the visible spectrum for electron temperatures in our plasma. Hence, if the electron temperature, T_e , is not a strong function of radius the electron density, N_e , is proportional to the square root of continuum intensity.

Absolute density profiles resulted by Abel inverting 4686Å line profiles taken at various chord positions. Abel inversion of these line profiles results in line profiles at various plasma radii. Using the method described in part c of this section, the half-half width of the 4686Å line at various plasma radii

yields absolute electron density as a function of radius. As stated previously 4686Å profile data was taken only on $y = 0$ at $z = +12.5$ cm and $+6.25$ cm due to excessive shot-to-shot scatter in the data at other chords. Hence, absolute density profiles could not be obtained at $z = +12.5$ cm and $+6.25$.

d. Data

Figures 17 and 18 present relative density profiles at $z = +12.5$ cm and $z = +6.25$ cm. The normalizations used in Figures 17 and 18 are independent, hence, the amplitudes of one graph cannot be compared with those of the other. $\overline{N_e}$ and $\overline{T_e}$ noted on the figures are the average electron density and temperature, across the plasma column. They were obtained from the 4686Å line data taken at the $y = 0$ chords by utilizing the procedure described in part c of this section.

Some comments can be made concerning the relative density profiles:

- i. At $t = 1.4$ μ sec a density suppression exists on axis at $z = +6.25$ cm but none exists at $z = +12.5$ cm.
- ii. The shapes at $z = +12.5$ cm for $t = 2$ and 3 μ sec are similar to those at $z = +6.25$ cm for $t = 3$ and 4 μ sec, respectively. Hence, profile shapes at $z = +6.25$ cm tend to follow the shapes at $z = +12.5$ cm by approximately 1 μ sec.
- iii. Using these profiles to calculate a column radius within which 90% of the plasma resides gives approximately 2.1 cm at $z = +12.5$ for all four times. Likewise, at $z = +6.25$ cm the column radius is 2 cm at $t = 1.4$ μ sec and drops to 1.65 cm at $t = 2, 3$ and 4 μ sec. This constancy of the plasma column radius during a period of rising driving field indicates significant field diffusion into the plasma.

Figure 17. Relative Density Profiles at $z = +12.5 \text{ cm}$ (Mode I)

Figure 18. Relative Density Profiles at $z = +6.25 \text{ cm}$ (Mode I)

Figure 19 presents an absolute density profile at $z = -2.5$ cm for $t = 1.6$ μsec . This profile was obtained by Abel inverting the $4686\text{\AA}$ line profiles as a function of chord position to give $4686\text{\AA}$ line profiles as a function of radius, then using the Stark broadened component of these profiles to give electron density as a function of radius. Average electron density and temperature across the column are again listed for $t = 1.6$ μsec and 3 μsec . The density profile at $z = -2.5$ for $t = 1.6$ μsec shows density suppression on axis.

The average cross column densities decline steadily from center coil ($z = +12.5$ cm) to outside the coil ($z = -2.5$ cm) at early times ($t = 1.6$ μsec); at later times densities inside the coil tend to change very little along the coil axis but still drop off outside the coil. Temperatures inside the coil are independent of both z and t for the times shown, remaining almost constant at ~ 11 eV. Temperatures outside the coil do not drop during the early flow but do drop by $t = 3$ μsec . The results noted here at $z = -2.5$ cm correspond reasonably well with those obtained using Thomson scattering which are discussed in the following section.

10. Thomson Scattering Data from 25 cm Theta Pinch

a. Density profiles

The general technique used here is the same as that evolved at Culham¹⁶: Thomson scatter data at a point in the plasma fix density and temperature while continuum radiation is used to provide a relative density profile. This method presumes that temperature remains relatively constant across the plasma, as was the case for the Culham group. Continuum data were taken at a wavelength of 473 nm after it was verified that no discernible line radiation existed around the wavelength. No temperature profiles were produced. Consequently, in the graphs appearing in Figures 20 and 21, the temperature at the Thomson scatter point is merely quoted in each graph.

Figure 19. Absolute Density Profile at $z \approx -2.5 \text{ cm}$ (Mode I)

Figure 20. Thomson Scatter Absolute Density Profiles at $z = -2.5$ cm (Mode I)

Figure 21. Thomson Scatter Absolute Density Profiles at $z = -7.5 \text{ cm}$ (Mode I)

b. Preionization

Another parameter that proved amenable to measurement with the Thomson scattering apparatus was the density and temperature of preionized plasma at the $z = -2.5$ cm position. This permitted a rough determination of the preionization level before main bank initiation. At a radius of 6.5 mm from the center and at a time of 8.62 μsec from the start of the preionization discharge the electron density was determined to be $3.77 \times 10^{14} \text{ cm}^{-3}$ and the electron temperature was determined to be 3.33 eV. The fill density is about $3.6 \times 10^{15} \text{ cm}^{-3}$, and, hence, a preionization level of about 10% is indicated for Mode I operation.

c. Comparison with spectroscopy

At a time 1.6 μsec after main bank initiation and at a position of $z = -2.5$ cm both Thomson scatter data and spectroscopic data were taken. The 25 cm theta pinch section under spectroscopic studies (section d). In particular, Figure 19 of that section and Figure 20 of this section are to be compared at a time of 1.6 μsec . Overall, the agreement is reasonable, with differences of about 20%. The temperature values can be seen to agree very well. These results permit confidence in the spectroscopic data quoted earlier.

11. Electric Probes Studies

Biased and unbiased floating double electric probes were placed in the endloss region. The two biased pairs of probes were of the same design as those used to determine particle velocities in z pinch loss studies,³ and are referred to as velocity probes. The unbiased probe was designed to measure electric fields associated with endloss from the z pinch,³ and is referred to as an electric field probe.

The velocity probes involved the use of displaced pairs of biased double electrostatic probes. When properly biased, the electrode pairs conduct ion

saturation current. Hence, the passage of local fluctuations in plasma number density are recorded as fluctuations in ion saturation current. Thus, two pairs, separated by a known distance, yield a flow velocity through the time of flight of these perturbations.^{18,19}

The current collected by a floating probe pair with voltage, V , across them is given by¹⁸

$$I = i_+ \tanh\left(\frac{eV}{2kT_e}\right),$$

where i_+ = ion saturation current and the other symbols have their usual meanings. From this formula, the probe current drawn from the endloss plasma ($T_e = 10$ eV) will be $0.46i_+$, $0.76i_+$, $0.96i_+$ as bias voltages, V , of 10 volts, 20 volts, and 40 volts, respectively.

The probe pairs were biased in the 10 to 40 volt range. No discernible density fluctuations were recorded by the pairs after passage of the plasma front. Hence, only the velocity of the plasma front was obtained. Two undesirable signals were obtained; a 5.5 Mhz electrostatically coupled bank oscillation and $\dot{B}$ pickup.

The original development of the electric field probe was done by Burkhardt and Lovberg.¹⁹ Burton²⁰ developed a theory describing the probe-plasma interaction for collisionless flow. Ion mean free paths in our endloss plasma are approximately 0.0016 cm, hence, flow over the conical electric field probe with electrodes having 0.2 cm separation cannot be considered collisionless. These collisional effects make quantitative interpretation of any signal difficult. Also Turchi²¹ has shown that if the insulating cone separating the front and rear electrodes interrupts any transverse current density associated with the plasma flow resulting measurements of electric field are seriously impaired.

No definitive electric field could be associated with the endloss by using the electric field probe. However, the 5.5 Mhz main bank resonance was detected as a 0.25 volt signal between the probe electrodes. The floating plasma potential with respect to the theta pinch's high voltage ground was measured by connecting a Tektronix P6013A probe to one of the electric field probes electrodes. It was found that the plasma floats at a potential of $3 \text{ kV} \cos \omega t$, where $\omega = 2\pi/(\text{main bank cycle time})$.

A complete description of the design of the biased and unbiased probes and their circuitry will be given in the final report of these studies.

III. THOMSON SCATTERING DIAGNOSTIC STUDIES (Jacoby)

1. Description of the Diagnostic System

The previous progress report, C00-4020-3, established that a Thomson scatter diagnostic had been designed and the major components ordered. That system has been procured, tested, and utilized for diagnosis of the plasma in the 25 cm theta pinch. The final system employed was changed slightly from that previously described, consequently, a clear statement of the present diagnostic will be presented. A diagram illustrating the layout is presented in Figure 22.

The laser source was supplied by Apollo Lasers, Inc. and provides an energy of 10 J into a divergence angle of 2 mrad. It consists of an oscillator-amplifier configuration and has proven to be reliable in operation. The input lenses have been described earlier. New additions here are two divergence selecting apertures and the laser energy monitor (the photodiode). The apertures proved necessary in eliminating high divergence laser light and in reducing the stray light level. The photodiode laser monitor is placed in the path of the incident beam because of space limitations at the end of the θ -pinch. The diode views the beam through the use of a clear thin plate-glass window. This window reflects 4% of the incident beam onto the photodiode monitor.

The method chosen to pass the laser beam in and out of the discharge tube was to use Brewster windows on both ends. The Brewster end flange is stainless steel with an aperture cut in the Brewster angle mounted plate. Plate-glass 0.25 in thick sits on this plate and seals with an O-ring. There are two apertures set in the input Brewster assembly. These apertures proved necessary to reduce stray light signals to tolerable levels. On the exit Brewster assembly, two 3 in square blue glass beam dumps are set on top of the plate-glass

Figure 22. Physical Layout and Optics Diagram

window to absorb the incident beam. This set-up is very convenient since the plasma flowing out the ends of the pinch has left deposits on glass end flanges that seem impervious to cleaning; a minimal cost is entailed when the windows are replaced.

The detection apparatus has not changed from its initial design with the exception of the use of 30Å filters in place of 10Å filters. An enclosure of 1/8 in thick aluminum has proven quite adequate for noise shielding. A 32 cm long tube which allows the light to enter the box is an inch in diameter and 1/4 in thick. Operating on the principle of a cut-off waveguide, this tube allows light to enter, but keeps out RFI from the capacitor bank.

This system as described above has been tested thoroughly on the 25 cm θ -pinch. It has produced data that correlate well with the spectroscopic results presented earlier.

2. Operation

a. Electronics

The electronic systems for the Thomson scattering apparatus consist of two subsystems: (1) control circuitry that determines temporal placement of the laser probe, and (2) detection circuitry. Both are relatively simple and have been thoroughly tested. The control circuitry shall be described first, followed by a description of the detection system.

The triggering of the laser is provided by two delay generators with 0.1 μ sec resolution. The connections are shown in Figure 23. The first delay generator is manually fired when the bank reaches full charge. When the manual button is depressed an initial output pulse is sent to the Master trigger input on the laser. The master trigger starts the internal clock and firing sequence in the laser. Some 600 μ sec after this master trigger is fired, the flashlamps

Figure 23. Triggering Diagram

Figure 24. Detector Electronics

on the laser rods fire and 400 μ sec after that the external Q-switch trigger should receive a pulse. Meanwhile after a suitable delay a pulse is sent from the first delay generator to a car coil unit which triggers the preionization system (P.I.). Once the P.I. fires, a pick-up loop senses the current and triggers the second delay generator. Here the amount of delay determines where in time the laser pulse appears and hence, permits full examination of the plasma discharge in time. Thus, after its delay, a pulse is sent to the Q-switch trigger on the laser control panel which opens the Q-switch and dumps the pumped-up rods.

The detection apparatus consists of two PMT's, their voltage divider network, and a dual beam Techtronix 556 oscilloscope. This arrangement is depicted in Figure 24. The voltage measured is developed across a 50 Ω resistor; this resistor value was chosen such that the cables are terminated properly. The laser pulse is of about 20 μ sec duration and thus a bandwidth of 50 Mhz is required; the system will pass this bandwidth.

b. Calibration for density

Two techniques of intensity calibration were employed: Rayleigh scattering²² on neutral N_2 , and absolute intensity calibration using a known tungsten lamp source. In the Rayleigh scatter method about 100 Torr of N_2 was leaked into the vacuum vessel. Then half a day was permitted to elapse so that dust could settle out in the vacuum vessel. The 100 Torr provided a scattered signal comparable to a density of slightly less than 10^{16} cm^{-3} of plasma. This procedure worked rather well and has the inherent convenience of calibrating in situ. For the other technique, it was necessary to calibrate the detection apparatus on a separate bench and not in the position used to take data. However, even here the two methods disagreed by only around 20%. In this method,

the laser energy was measured with a calorimeter while the detection system was calibrated absolutely using the known tungsten source. Then from equations of the scattered intensity,²³ the density could be determined. In the future only the Rayleigh calibration method shall be used.

3. Doubled Nd-Glass Laser System for Low Density Incoherent Thomson Scattering

In the study of end loss from a theta pinch it has been found that a range of densities from 10^{16} cm^{-3} down to 10^{+12} cm^{-3} are common. The higher densities can be diagnosed with the ruby system described above. However, difficulties are encountered when it is desirable to measure deviations below 10^{13} cm^{-3} . Due to the pulsed nature of the experiment, a synchronous detection technique is impossible, and accordingly there were two alternatives: (1) use of a ruby laser with high quantum efficiency ERMA detectors, or (2) use of an alternative laser source. With the best ERMA photomultipliers measurements around 10^{12} cm^{-3} with a ruby laser source are possible. However, the second alternative will be examined here. This method presumed the use of an OMA (optical multi-channel analyzer). With the quantum efficiency available on the SIT or ISIT²⁴ used as the detector, a ruby laser system would experience difficulty at such low densities. In these calculations, the use of a SIT with an S-20 photocathode response is presumed. Accordingly, a different photocathode response may change the magnitudes, but the idea remains the same throughout.

The type of laser system considered here was frequency doubled Nd-glass. Basically, three advantages accrue with the use of such a system. One principle advantage, that of improved detector efficiency, has already been put forth by G. S. Voronev.²⁵ To recapitulate his arguments: the quantum efficiency for an S-20 response increases from 0.02 at 694.3 nm for a ruby laser to 0.13 at 530 nm for a doubled Nd-glass laser. A second advantage comes from

the larger laser pulse energies that can be delivered. Ruby lasers can achieve a maximum of about 15 J for a 30 nsec Q-switched pulse. However, a doubled glass system is capable of output energies around 50 J. The value of 50 J is arrived at after consideration of the doubling crystal characteristics. Using CDA as the doubling media in a one pass configuration,²⁶ the following parameters²⁷ must be considered with respect to doubled output energies available: power conversion efficiency of 30%, damage threshold of 300 Mw/cm², and crystal face diameter of 6 cm. With the presumption of a 30 nsec pulse width, use of the limiting values of the CDA crystal permits an output of 76 J. However, 50 J will be used as a reasonable value. The last advantage inherent in a doubled glass system is that for the same incident energy there is less of a heating effect and hence perturbation on the plasma. This effect is given by²³

$$\frac{\Delta T_e}{T_e} = 3.24 \frac{n_e \ln \lambda}{f_i^2 A T_e^{5/2}} W_i \quad (1)$$

where T_e is electron temperature in eV, n_e is electron density in cm⁻³, $\ln \lambda$ is the Coulomb logarithm, f_i is the incident frequency in Hertz, A is the focused spot size in cm², and W_i is the incident energy in Joules. Thus, the heating effort of a ruby laser can be shown to be a factor of

$$\frac{(T_e/T_e)_{\text{ruby}}}{(T_e/T_e)_{\text{doubled glass}}} = \left(\frac{f_{i_{\text{glass}}}}{f_{i_{\text{ruby}}}} \right) = 1.7 \quad (2)$$

greater than that of a doubled glass laser.

With the gain in quantum efficiency coupled with the greater available pulse energies for a doubled glass laser, a comparison between scattered signals for a ruby laser and doubled glass laser can be made. In this case, the same plasma parameters and laser pulse width for both systems are presumed. Using a 15 J incident ruby energy and a 50 J incident doubled glass energy

and normalizing to the ruby result gives a scattered signal of 1 V for the ruby and 21.7 V for the doubled glass. This result includes the direct proportionality of both the quantum efficiency and incident energy with the scattered signal. This result indicates that with an S-20 photocathode response in the detector system, either the signal to noise ratio can be improved by a factor of 22 or densities can be resolved that are a factor of 22 lower when a doubled glass system is used. However, one point should be recognized in the above computation: with a doubled output of 50 J, the Nd-glass laser itself requires a 30 nsec Q-switched output of 170 J. Consequently, a more realistic comparison should be made with existing or available components. Currently, an Apollo ruby laser with a 7 J output is available. If this laser was converted to Nd-glass rods of the appropriate size, an output of 20 J is possible. Accordingly, performing the same analysis as above produces a value of 5.6 V of doubled-glass scattered signal for every 1 V of ruby scattered signal.

Nd-glass is not the only laser where doubling the frequency of the output places it in a more efficient regime of detection. The tabulation below shows other systems on a comparative basis, where the maximum reasonable output energy is taken for each system. Again the same S-20 response is presumed and the same analysis as above is used with the scattered signals all normalized to the ruby result:

TABLE 3
Laser System Comparison

<u>System</u>	<u>Scattering Energy</u>	<u>Scattered Signal</u>
Ruby	15 J	1 V
Doubled Glass	50 J	21.7 V
Doubled Ruby	4.5 J	2.1 V
Doubled Nd-Yag	1.8 J	0.78 V

The doubled Nd-Yag system presume a Nd-Yag oscillator and two Nd-glass amplifiers to bring the output up to about 6 J. Again using an existing system with the simple modification of replacing rods, the table of output comparison is repeated for the available Apollo laser:

Table 4
Comparison of Different Rods in the Apollo Laser

<u>System</u>	<u>Scattering Energy</u>	<u>Scattered Signal</u>
Ruby	7 J	1 V
Doubled Glass	6 J	5.6 V
Doubled Ruby	2.1 J	2.1 V
Doubled Nd-Yag	0.6 J	0.56 V

For this case only one amplifier is used, and so the Yag output drops to about 2 J before it is doubled. In both comparisons it becomes clear that a doubled glass laser system offers the highest gain in scattered signal, or offers the best resolution of low densities.

While a doubled glass system can provide certain benefits, there is also one difficulty with this system. In standard incoherent Thomson scattering systems the laser line is usually treated as a delta function in wavelength. Nominally, however, a glass laser has a bandwidth of about 5 nm. Once the beam has undergone frequency doubling, this bandwidth is halved to 2.5 nm. This bandwidth is on the same order as the Doppler broadened bandwidth of a scattered signal from a plasma with a few electron volts in temperature.

Accordingly, the laser bandwidth should be taken into account. To do this, presume that the electrons have a Maxwellian-velocity and that the laser line is Lorentzian.²⁸ Thus, the photocathode current generated by the scattered signal becomes²³

$$I_{pc} = \frac{r_0^2 d\Omega n_e L \eta e}{h a^2 \sin^2 \frac{\theta}{2} \sqrt{\pi}} |\hat{s} \cdot (\hat{s} \times \hat{E}_{i0})|^2 P_i d\lambda \int_0^\infty d\lambda' \frac{\exp\left[-\frac{c^2 (\lambda - \lambda')^2}{4 a^2 \lambda'^2 \sin^2 \frac{2\theta}{2}}\right]}{(\lambda' - \lambda_1)^2 + \Delta\lambda^2} \frac{\Delta\lambda}{\pi} \quad (3)$$

Performing a variable change of $x = \lambda - \lambda_1$ for λ' , Equation 3 becomes

$$I_{pc} = \frac{r_0^2 d\Omega n_e L \eta e}{h a^2 \sqrt{\pi} \sin^2 \frac{\theta}{2}} |\hat{s} \cdot (\hat{s} \times \hat{E}_{i0})|^2 P_i d\lambda \int_{-\lambda_1}^\infty dx \frac{\exp\left[-\frac{c^2 \{x - (\lambda - \lambda_1)\}^2}{4 a^2 \sin^2 \frac{2\theta}{2} (\lambda_1 + x)^2}\right]}{x^2 + \Delta\lambda^2} \frac{\Delta\lambda}{\pi} \quad (4)$$

where r_0 is the classical radius of an electron, $d\Omega$ is the solid angle viewed by the detection apparatus, n_e is the electron density, L is the length of the scattering volume, η is the detector quantum efficiency, e is the electronic charge, h is Planck's constant, a is the electron thermal speed or $\sqrt{kT_e/m_e}$, θ is the angle between incident and scattered directions, $\hat{s}$ is the unit vector in the scattering direction, $\hat{E}_{i0}$ is the unit vector in the direction of the incident electric field, P_i is the incident laser power, c is the velocity of light, $d\lambda$ is the width of the detection channel in wavelength, λ_1 is center of the laser line, $\Delta\lambda$ is the half bandwidth at half maximum of the laser line, and λ is the center of detection channel.

The integral in Equation (4) takes on a familiar form under the following conditions: first, if x can be ignored compared to λ_1 (this is reasonable since the range of x over which the integral is non zero is $\ll \lambda_1$) and second the lower limit can be extended from $-\lambda_1$ to $-\infty$. When this is done, Equation (4) reduces to

$$I_{pc} = \frac{r_0^2 d\Omega n_e L \eta e}{h a^2 \sin^2 \frac{\theta}{2} \sqrt{\pi}} |\hat{s} \cdot (\hat{s} \times \hat{E}_{i0})|^2 P_i d\lambda \int_{-\infty}^\infty dx \frac{\exp\left[-\frac{c^2 \{x - (\lambda - \lambda_1)\}^2}{4 a^2 \sin^2 \frac{2\theta}{2} \lambda_1^2}\right]}{x^2 + \Delta\lambda^2} \frac{\Delta\lambda}{\pi} \quad (5)$$

This integral is readily recognizable on the Voigt integral commonly found in spectroscopic work. Since the laser bandwidth can be measured and, hence, is known, the width of the Maxwellian contribution can be determined from the tabulations of this integral.²⁹ Accordingly, an accurate value of T_e can then be found with the laser bandwidth acceptably accounted for.

The approximations used above will now be discussed. It can be noted that the value of x that contributes to the integral is $\ll \lambda_1$, thus permitting expansions in x/λ_1 . So, letting α be $\lambda - \lambda_1$ and

$$\Delta\lambda_{1/e} = 2a \sin\frac{\theta}{2} \frac{\lambda_1}{c} \quad (6)$$

the integral becomes

$$\frac{\Delta\lambda}{\pi - \lambda_1} \int_{-\lambda_1}^{\infty} \frac{\exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2} \left(1 + \frac{x}{\lambda_1}\right)^{-2}\right]}{x^2 + \Delta\lambda^2} dx \quad (7)$$

Expanding to first order in x/λ_1 gives

$$\frac{\Delta\lambda}{\pi - \lambda_1} \int_{-\lambda_1}^{\infty} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right] \left(1 + \frac{2x(x - \alpha)^2}{\lambda_1 \Delta\lambda_{1/e}^2}\right) \quad (8)$$

This contains the Voigt term desired and the estimation of the first order error. Examining only the error term with the substitution of

$$y = \frac{(x - \alpha)}{\Delta\lambda_{1/e}} \quad (9)$$

yields

$$\text{ERROR} \approx \frac{2\Delta\lambda\Delta\lambda_{1/e}}{\pi\lambda_1} \int_{-\lambda/\Delta\lambda_{1/e}}^{\infty} \frac{e^{-y^2} (\Delta\lambda_{1/e}y + \alpha)y^2 dy}{(\Delta\lambda_{1/e}y + \alpha)^2 + \Delta\lambda^2} \quad (10)$$

Since $\lambda/\Delta\lambda_{1/e}$ is $\gg 1$, the lower limit of this integral can be extended to $-\infty$. Also, the y in the denominator will be set to zero and this will only serve to increase the value of the integral. Thus, Equation (10) becomes

$$\text{ERROR} \sim \frac{2\Delta\lambda\Delta\lambda_{1/e}}{\pi\lambda_1} \int_{-\infty}^{\infty} \frac{e^{-y^2} (\Delta\lambda_{1/e}y + \alpha)y^2 dy}{\alpha^2 + \Delta\lambda^2} \quad (11)$$

This integral can now be evaluated with the result

$$\text{ERROR} \sim \frac{1}{2\sqrt{\pi}} \frac{\Delta\lambda}{\lambda_1} \frac{\alpha\Delta\lambda_{1/e}}{\alpha^2 + \Delta\lambda^2} \quad (12)$$

This is a small term due to $\Delta\lambda \ll \lambda_1$. Also, α is on the order of $\Delta\lambda_{1/e}$.

This establishes the validity of the neglect of x compared to λ_1 .

Next, the error involved in setting the lower limit of Equation (4) equal to $-\infty$ can be shown to be even less than that of Equation (12). In this case,

$$\begin{aligned} \frac{\Delta\lambda}{\pi} \int_{-\lambda_1}^{\infty} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right] &= \frac{\Delta\lambda}{\pi} \int_{-\infty}^{\infty} \frac{dx \exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right]}{x^2 + \Delta\lambda^2} \\ &\quad - \frac{\Delta\lambda}{\pi} \int_{-\infty}^{-\lambda_1} \frac{dx \exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right]}{x^2 + \Delta\lambda^2} \end{aligned} \quad (13)$$

Thus, the error term to examine here is

$$\frac{\Delta\lambda}{\pi} \int_{-\infty}^{-\lambda_1} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[-\frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right] \quad (14)$$

Letting y be $(x + \lambda_1)/\Delta\lambda_{1/e}$ reduces Equation (14) to

$$\text{ERROR} \sim \frac{\Delta\lambda}{\pi\Delta\lambda_{1/e}} \int_{-\infty}^0 \frac{dx \exp\left[-\left(y - \frac{\lambda}{\Delta\lambda_{1/e}}\right)^2\right]}{\left(y - \frac{\lambda_1}{\Delta\lambda_{1/e}}\right)^2 + \left(\frac{\Delta\lambda}{\Delta\lambda_{1/e}}\right)^2} \quad (15)$$

Following Laplace's method³⁰ the error can be shown to be about

$$\text{ERROR} \sim \frac{\Delta\lambda\Delta\lambda_{1/e}}{\pi[\lambda_1^2 + \Delta\lambda^2]} \frac{\Delta\lambda_{1/e}}{2\lambda} e^{-(\lambda/\Delta\lambda_{1/e})^2} \quad (16)$$

Again since

$$\lambda \sim \lambda_1 \gg \Delta\lambda \sim \Delta\lambda_{1/e} \quad (17)$$

the error term shown by Equation (16) can be ignored. Consequently, the use of the Voigt profile in Equation (5) is justified.

The application of the Voigt profile to the temperature determination is unnecessary when the thermal broadening is large enough to ensure that a 2.5 nm width appears relatively narrow. In quantitative terms, this means that

$$\Delta\lambda_{1/e} > 10 \frac{\Delta\lambda}{2\sqrt{\ln 2}} \quad (18)$$

Accordingly, this means that the temperature should satisfy

$$T_e > 100 \text{ eV} \quad (19)$$

in order to ignore the width of the laser line. On the other hand, the separation of the Maxwellian from the Lorentzian certainly becomes questionable when

$$\Delta\lambda_{1/e} < \frac{\Delta\lambda}{20\sqrt{\ln 2}} \quad (20)$$

or when

$$T_e < .01 \text{ eV} \quad (21)$$

Thus, from .01 eV up to 100 eV, the analysis using a Voigt integral holds and should be used in evaluating the electron temperature with a doubled glass system. Above 100 eV the usual analysis with a Gaussian profile in temperature determination can be applied, while below .01 eV it is not possible to get at the temperature information in a doubled glass scattering system.

The advantages for low density determination that a doubled Nd-glass laser system holds can be clearly identified. Using the maximum energies for both laser systems and an S-20 photocathode response on the detectors, the ruby

laser can resolve densities as low as $\sim 10^{12} \text{ cm}^{-3}$ while the doubled glasss can resolve densities down to $\sim 5 \times 10^{10} \text{ cm}^{-3}$. Again it should be emphasized that a different photocathode response will affect these values. However, even if the two quantum efficiencies at 694.3 nm and 530 nm could be made equal, the doubled glass laser scattered signal still is about a factor of 4 above the signal from a ruby system. With the improvement in scattered comes the slight complication of using a Voigt profile for the electron temperature determination in the range of $0.01 \text{ eV} < T_e < 100 \text{ eV}$.

4. A Proposed Method for Measurement of Directed Electron Velocities

With an involvement in theta pinch endloss studies, a diagnostic indicating directed plasma velocities would be very useful. Following preliminary analysis, it appears that a velocity diagnostic is capable of following the full evolution of the endloss process. As well become evident below, the aspect of endloss that permits one to follow the velocity is that the flow velocity remains on the order of the ion thermal speed all along the flow.

To develop a set of general equations illustrating this diagnostic, it is useful to refer to Figures 25 and 26. A general expression for the scattered signals P_s , that holds for electron temperatures up to around 1 keV is²⁴

$$P_s d\lambda d\Omega = \frac{P_i r_o^2 n_e d\Omega L}{2\sqrt{\pi} \sin^2 \frac{\theta}{2}} \frac{c}{a} \frac{d\lambda}{\lambda_i} \left[1 - \frac{3.5\Delta\lambda}{\lambda_i} + \frac{c^2 \Delta\lambda^3}{4a^2 \lambda_i^2 \sin^2 \frac{\theta}{2}} \right] \exp \left[-\frac{c^2}{4a^2 \sin^2 \frac{\theta}{2}} \left(\frac{\Delta\lambda}{\lambda_i} + \frac{V_{oa} \pm V_{ob}}{c} \right)^2 \right] \quad (1)$$

where the plus sign refers to detector #2 and the minus sign to detector #1. Here $d\lambda$ is the wavelength passband of the filter or monochromator behind which the detector sits, $d\Omega$ is the solid angle viewed by the detector, P_i is the

Figure 25. Geometrical Arrangement

Figure 26. Detector Electronics

incident laser power, r_0 is the classical radius of an electron, n_e is the electron density, L is the scattering volume length, c is the speed of light, θ is the angle between incident and scattered directions, a is the electron thermal speed or $\sqrt{T_e/m_e}$, λ_1 is the incident laser wavelength, $\Delta\lambda$ is $\lambda_1 - \lambda_s$ with λ_s being the center wavelength of the detection passband, and v_{oa} and v_{ob} are the electron directed velocities along the a and b axes, respectively. Equation (1) also presumes that a shifted Maxwellian adequately describes the electron population in the scattering volume. For the analysis that follows, a 90° scattering arrangement is used. Returning to Equation (1) and performing the subtraction of detector #2's output from that of detector #1 produces

$$(P_{s_1} - P_{s_2})d\lambda d\Omega = A \exp\left[-\frac{c^2}{2a^2}\left(\frac{\Delta\lambda}{\lambda_1} + \frac{v_{oa} - v_{ob}}{c}\right)^2\right] \left\{ \exp\left[-\frac{c v_{ob}}{a^2}\left(\frac{\Delta\lambda}{\lambda_1} + \frac{v_{oa}}{c}\right)\right] - 1 \right\} \quad (2)$$

where

$$A = \frac{P_i r_0^2 n_e d\Omega L}{\sqrt{2\pi}} \frac{c}{a} \frac{d\lambda}{\lambda_1} \left[1 - \frac{3.5\Delta\lambda}{\lambda_1} + \frac{c^2 \Delta\lambda^3}{2a^2 \lambda_1^2} \right] \quad (3)$$

Ensuring that the laser beam is along a direction where the flow velocity is small, it can reasonably be expected that

$$\frac{v_{oa}}{c} < 0.1 \frac{\Delta\lambda}{\lambda_1} \quad (4)$$

and, hence, this term will be dropped. For simplification, presume that

$$\frac{c v_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} < 0.1 \quad (5)$$

such that expansion of the exponential is permitted. This need not be the case, but it does allow a certain transparency in the argument, and with the exception of high velocities compared to the electron thermal speed, this condition will not be violated. Accordingly, Equation (2) reduces to

$$(P_{s1} - P_{s2})d\lambda d\Omega = A \exp\left[\frac{-c^2}{2a^2}\left(\frac{\Delta\lambda}{\lambda_1}\right)^2 + \frac{V_{oa} + V_{ob}}{c}\right] \frac{cV_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (6)$$

Finally, if Equation (6) is normalized with respect to detector #2 output, the velocity can be isolated as

$$\frac{P_{s1} - P_{s2}}{P_{s2}} = \frac{cV_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (7)$$

Also, since the current, I_s , produced by the detectors is directly proportional to P_s , Equation (7) can be rewritten as

$$\frac{I_{s1} - I_{s2}}{I_{s2}} = \frac{cV_{ob}}{a^2} \frac{\Delta\lambda}{\lambda_1} \quad (8)$$

Thus, the velocity in the $\hat{b}$ direction can be arrived at when the electron temperature, T_e , at the scattering volume is known. The temperature can be obtained either from previously described diagnostics or through a simultaneous measurement of the scattered wavelength spectrum from the same plasma volume. One thing that can already be noted is the dependence of the difference signal on the electron thermal speed.

Since the signal has been established, it is necessary to determine the signal to noise ratio for this experimental arrangement. To avoid a profusion of symbols it is useful to note that in most applications $\Delta\lambda \ll \lambda_1$, and, hence, the terms with $\Delta\lambda$ can be ignored in Equation (1) when compared to unity. There are two basic noise sources in this system: (1) noise resulting from the photoemissive process of the photomultiplier, and (2) bremsstrahlung. The photomultiplier noise is first examined. The current, I , developed in such a tube with a gain, G , and quantum efficiency of η sitting behind a narrow band optical filter of half width $d\lambda$ centered on wavelength λ_s is

$$I_{sig} = \frac{P_i r_o^2 n_e L d \Omega d \lambda e T}{\sqrt{2\pi} h a} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i}\right)^2\right] \quad (9)$$

where T is the overall transmission of the system and the flow velocity has been ignored. Due to the Poisson statistics of the photoemissive process, a noise term of

$$I_n = \sqrt{I_{sig} e G / \tau} \quad (10)$$

arises with τ being the length of time of the incident laser pulse. After being processed by the differential amplifier, this noise is doubled. Therefore, the signal to noise ratio is

$$\frac{S}{N} = (I_{sig_1} - I_{sig_2}) / 2 \sqrt{I_{sig} e G / \tau} \quad (11)$$

$$\frac{S}{N} = \frac{c(\lambda_i - \lambda_s) V_{ob}}{2a^2 \lambda_i} \sqrt{\tau I_{sig} / e G} \quad (12)$$

This ratio can be maximized through suitable adjustment of λ_s . When this is done, the value of λ_s to use is found to be

$$\lambda_s = \lambda_i \left(1 - \frac{a}{c\sqrt{2}}\right) \quad (13)$$

This reduces Equation (12) to

$$\frac{S}{N} = \frac{V_{ob}}{a\sqrt{2}} \sqrt{w_i r_o^2 n_e L d \Omega T d \lambda \eta / \sqrt{2\pi} h a} e^{-1/2} \quad (14)$$

Note that it is now possible to return to Equation (5) and use Equation (13) to reduce the condition permitting expansion of the exponential to

$$\frac{V_{ob}}{\sqrt{2} a_e} < 0.1$$

Returning again to noise considerations, the bremsstrahlung contribution is now examined. Both detectors view the same plasma volume, and hence, the

bulk of the plasma noise signal should cancel out, but there are still the fluctuations of the bremsstrahlung again produced by the photoemissive process left to contend with. In this analysis the bremsstrahlung contribution shall be handled such that it can be simply appended to Equation (14). Accordingly, the ratio of the bremsstrahlung signal to the laser signal can be shown to be²³

$$R = \frac{2.09 \times 10^{-36} \tau_n e^{2\Delta\lambda_{1/e} V_p}}{W_i r_o^2 \lambda_i^2 T_e^{1/2} 4\pi L} \quad (15)$$

where V_p is the volume of plasma viewed by the detector, W_i is the incident laser energy and

$$\Delta\lambda_{1/e} = \frac{\sqrt{2a}}{c} \quad (16)$$

Taking the square root of this and adding the factor of 1 to this result gives an estimate by which Equation (14) should be divided to account roughly for the bremsstrahlung noise addition. The end result becomes

$$\frac{S}{N} = \frac{V_{ob}}{a\sqrt{2}} e^{-1/2} \sqrt{\frac{P_i r_o^2 n_e L d R T d \lambda_i}{\sqrt{2\pi} h a (1/(1+R))}} \quad (17)$$

It is interesting at this point to compare this system with a technique used in previous velocity determinations: that of subtracting the outputs of two detectors where one sees the full spectrum on the high side of 694.3 nm and the other sees the low side of 694.3 nm. This is shown in Figure 27. For ease of analysis, a square response is presumed for the filters. Accordingly, the result after subtracting the two signals is

$$I_{sig_1} - I_{sig_2} = \frac{P_i r_o^2 n_e L d R e T d \lambda_i}{\sqrt{2\pi} h a} \left[\int_{\lambda_i}^{\infty} e^{-(c^2/2a^2)[(\lambda_i - \lambda_s)/\lambda_i + (v_{oa} - v_{ob})/c]^2} d\lambda_s \right. \\ \left. - \int_{-\infty}^{\lambda_i} e^{-(c^2/2a^2)[(\lambda_i - \lambda_s)/\lambda_i + (v_{oa} + v_{ob})/c]^2} d\lambda_s \right] \quad (18)$$

Figure 27. Alternative Method of Velocity Determination

Figure 28. End Loss Velocity Experimental Arrangement

Presuming that

$$\frac{v_{oa} + v_{ob}}{c} \ll 1 \quad (19)$$

Equation (18) can be reduced to

$$I_{sig_1} - I_{sig_2} = \frac{P_1 r_o^2 n_e L d \Omega e T \eta G}{\sqrt{2\pi} h a} \frac{\sqrt{2} a \lambda_1}{c} \frac{2v_{oa}}{c} \quad (20)$$

Accordingly, a signal to noise ratio just accounting for the photoemissive process is

$$\frac{S}{N} = \frac{2v_{oa}}{2c\sqrt{\pi}} \sqrt{2\pi P_1 r_o^2 n_e L d \Omega T \eta \lambda_1 / hc} \quad (21)$$

Now for the same velocity, laser, and detection parameters a ratio of the two signal to noise ratios can be made in order to compare the two techniques:

$$\frac{(\frac{S}{N})_{two\ halves}}{(\frac{S}{N})_{two\ detectors}} = .00694 T_e^{3/4} \quad (22)$$

where a filter passband, $d\lambda$, of 1.0 nm has been used in Equation (14). This indicates that for electron temperatures up to 755 eV the technique described in this paper (illustrated by Figure 1) has a better signal to noise ratio and accordingly can resolve lower directed velocities than the alternate method shown above (illustrated by Figure 3). However, above an electron temperature 755 eV, the latter technique possesses the advantage.

For the application considered here: that of measuring the endloss velocity in the Penn State 50 cm theta pinch - the former technique is superior. The physical layout of the proposed experiment is presented in Figure 28. The detectors to be used are two RCA C70042K photomultiplier tubes with the following parameters:

Current Gain	8×10^4
Quantum Efficiency	.06 at 694.3 nm
Maximum Pulse Current	50 ma

The plasma is presumed to have the following characteristics:

Electron Temperature	100 eV
Electron Density	10^{16} cm^{-3}
Ion Temperature	100 eV
Plasma Column Radius	1.0 cm

Lastly, the laser and associated optics are presumed to have the following properties

Wavelength Passband	1.0 nm
Overall Transmission	0.1
Laser Energy	10 J
Laser Pulse Length	20 nsec
Focused Beam Radius	0.1 cm
Scattering Length	1 cm
Solid Angle of Detectors	2×10^{-2} ster

The detector placement shown is necessary, since they would be overloaded with bremsstrahlung if placed along the axis of the machine. Two practical considerations should be noted from the above parameters. First, with the pulse length of 20 nsec, only a 50 Mhz bandwidth is required of the differential amplifier. Second, the solid angle viewed by the detectors is to be set by the input lens of the detection scheme. This can be seen in the optical arrangement shown in Figure 29. The signals from the two tubes are

$$I_{sig1} = \frac{W_1}{\tau} \frac{d\lambda d\Omega n_e G Tr_o^2 n_e L}{\sqrt{2\pi} h a} \exp\left[\frac{-c^2}{2a^2} \left(\frac{\lambda_1 - \lambda_s}{\lambda_1} + \frac{\sqrt{2} V_{o2}}{c}\right)^2\right] \quad (23)$$

$$I_{sig_2} = \frac{W_i}{\tau} \frac{d\lambda d\Omega \eta e G Tr_o^2 n_e L}{\sqrt{2\pi} h a} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i} - \frac{\sqrt{2} V_{oy}}{c}\right)^2\right] \quad (24)$$

In taking measurements around the throat region of the pinch it is safe to presume that $V_{oy} = 0$. Then:

$$I_{sig_1} - I_{sig_2} = 61.3 \text{ ma} \exp\left[-\frac{c^2}{2a^2} \left(\frac{\lambda_i - \lambda_s}{\lambda_i}\right)^2\right] \exp\left[-\frac{c V_{oz}}{a^2} \left(\sqrt{2} \frac{\lambda_i - \lambda_s}{\lambda_i} - \frac{V_{oz}}{c}\right)\right] - 1 \quad (25)$$

Using Equation (13) to determine λ_s , Equation (24) can be shown to give a current of

$$I_{sig_2} = 22.6 \text{ ma} \quad (26)$$

with the Poisson noise current associated with it of

$$I_{noise} = 0.12 \text{ ma} \quad (27)$$

Presuming the flow is roughly $V_{oy} \sim \sqrt{T_1/M_1}$, an expected difference signal of

$$I_{sig_1} - I_{sig_2} = 1.49 \text{ ma} \quad (28)$$

can be derived. Accordingly, with just the Poisson noise, a signal to noise ratio of

$$\frac{S}{N} = 4.4 \quad (29)$$

is expected. Finally, adding in the bremsstrahlung contribution reduces the signal to noise ratio to

$$\frac{S}{N} = 4.13 \quad (30)$$

Overall, this is a quite acceptable noise level.

These calculations indicate that with the plasma parameters used above, it is possible to investigate velocities below the electron thermal speed and still maintain a reasonable signal to noise ratio. The lowest velocity that can be examined for given plasma and laser parameters when bremsstrahlung is neglected is given by

$$v_{ob} = 2 \frac{S}{N} \sqrt{\pi/2} (ha) / (W_{10}^2 n_e L d T d \lambda \eta) \quad (31)$$

Here the interplay between the lowest velocity measurable and the plasma density and temperature is clearly visible. Using Equation (27), it can be quickly ascertained whether this type of diagnostic would measure a given velocity.

Figure 29. Optical Arrangement

IV. MAGNETIC MIRROR FIELD STUDIES (Veglia, Robinson)

1. Introduction

As discussed in the original research proposal, a considerable number of unresolved physics questions remain regarding the effect and effectiveness of magnetic mirrors on the loss at the ends of theta pinches.³² Recent work on this project has concentrated on the development of appropriate mirror supplements to the basic theta pinch discharge coil. There are three viable possibilities: use of a discharge tube of smaller bore than the coil and reduction of the coil area at the ends so that the discharge tube has the same O.D. as the coil I.D.; addition of a (single turn) coil at each end, driven by a bank with rise time equal to the main bank and with a crowbar; addition of (multi-turn) coils at each end, driven by a bank with cycle time long compared to the main discharges. Each of these alternatives has been considered and will be discussed below.

With the assembled 50 cm pinch facility, a maximum magnetic field of 30 kG is anticipated with a risetime of 5 μ sec; with a crowbar the magnetic field will be 25-30 kG over the duration of the loss event. With a minimum fill pressure of 10 mT, density of 2×10^{16} and temperature of 290 eV are anticipated; under those conditions the ion-ion mean free path is 5.0 cm. Since $\lambda_{ii} \sim T^2/n$, a temperature on the order of 100 eV would indicate $\lambda_{ii} \approx 0.5$ cm. As the effectiveness of mirrors in the collisional or collisionless regions is related to the mirror interaction length, $B(dB/dz)^{-1}$, or coil length, as compared to the mean free path,³³ it would appear that mirror coil lengths for the present experiment should be on the order of 2-5 cm in length. The plasma column will be on the order of 2 cm diameter, allowing the possibility of rather small bore tubes and mirror coils to be used on the ends, within the constraints of vacuum fittings and gas purity.

First, the simplest alternative of a discharge tube with bore small compared to the coil, and with a reduced coil area at the ends will be specified. Again, the coil length is 50 cm with a bore of 10.5 cm. Present experiments have been carried out with 4 in (10.16 cm) I.D. discharge tubes. However, alteration of the discharge tube to 3 in (7.62 cm) I.D. and the insertion of 2-5 cm long rings into the coil and to produce a coil I.D. of 7.75 cm would produce a mirror ratio of 2:1 on the basis of flux conservation. The properties of the plasma in the column following compression with a 50-100 mT fill would still be on the order of 100 eV and 2×10^{16} so that Thomson scattering diagnostics remain useful. This procedure would allow careful diagnosis of a loss configuration without mirrors, with 2:1 ratio, and possibly higher ratios with add-on elements discussed below.

2. Order-of-Magnitude Estimates of Mirror Coil Performance

The magnetic fields that could be generated by coils of short length with their own capacitor energy storage and switching added at the ends of the 50 cm long coil were estimated. Such coils could be as generators of fields integral to the end mirror or used to generate cusp fields at the coil ends.

Considering single turn coils of radius r_m , length h_m , driven by available capacitors, the following magnitudes are presented:

r_m (cm)	2.54
h_m (cm)	2.0
Capacitance ea @ 14 (μ F)	28.0
Charge Voltage (kV)	18.0
Coil Inductance (nH)	53.0
Total Inductance (nH)	100.0
K_L	0.53

Coil inductance is given by³⁴

$$L = \frac{\mu_o \pi r_m^2}{h_m} K_L \quad (1)$$

where $K_L (h_m/2r_m)$ accounts for field fringing. The quarter cycle time $\tau_{1/4}$ is given by

$$\tau_{1/4} = \tau/4 = \frac{\pi}{2} \sqrt{LC} \quad (2)$$

and for this system is 2.75 μsec . Current can be evaluated from

$$I = \frac{2\pi V_o}{\tau} e^{-\alpha t} \sin\left(\frac{2\pi}{\tau} t\right) \quad (3)$$

where α is a constant which is characteristic of system damping and is estimated to be $5 \times 10^4 \text{ sec}^{-1}$; $I_{\text{max}} = 2.8 \times 10^5$ amperes. The magnetic field generated is expressed as:

$$B = \frac{\mu_o I}{h_m} \cos \delta \text{ where } \delta = \arctan \frac{2r_m}{h_m} \quad (4)$$

indicating a maximum value of 61 kG.

Considering multi-turn coils with both single and double layer solenoid construction, the self inductance is given by

$$L = \frac{\mu_o N^2 \pi r_c^2}{h} \quad (5)$$

where N is the number of turns, r_c is the radius to the center of the conductor, and h is the length. If the coil is double layered:

$$L_{\text{total}} = L_{\text{in}} + L_{\text{out}} + 2M \quad (6)$$

where L_{in} and L_{out} are the respective self inductances of the inner and outer layers, and M is the mutual inductance of the layers given by:

$$M = K \sqrt{L_{\text{in}} L_{\text{out}}} \quad -1 \leq K \leq 1 \quad (7)$$

The value of k is dependent on the coupling of the two layers and is assumed to be 1 in the following calculations.³⁵

Typically, these coils would be constructed of 8 gauge or heavier copper wire in order to provide for reasonable heat dissipation at the currents used. A typical coil has the following dimensions:

Size Wire	8 ga
Insulation Thickness (cm)	0.20
No. Turns per Layer	3
r_c (cm)	2.54
h (cm)	2.0

The inductance is found to be 70 μ H, giving a cycle time of 88 μ sec using the discharge system described previously. Peak current would be 2.9×10^4 amperes giving a magnetic field of 42 kG. This field is determined from

$$B = \frac{\mu_0 NI}{h} (\cos \delta_{in} + \cos \delta_{out}) \quad (8)$$

where $\delta_1 = \arctan r_{c1}/L$ and has a slightly different value for each layer. To aid in the design of the wire wrapped coils, that would be experimentally evaluated, a numerical study was undertaken to characterize the fields produced by various coil configurations. The various input parameters in a computer program are: coil length, coil inside radius, numbers of layers to the coil, wire diameter and insulation thickness, and system capacitance. Incremental variation of coil length is allowed along with capacitance increments of 14 μ F. Output includes: number of turns per layer, expected cycle time, maximum current, maximum magnetic field at the coil center, self inductance of the layers, mutual inductance, and total inductance. Cycle time, current and magnetic field are determined for each capacitance increment. There are two limitations to the program. One is that the value of α in Equation (3) for the current is undetermined. A value of $2 \times 10^3 \text{ sec}^{-1}$ is used as this order of magnitude was found to be characteristic of the coils used in test shots. The second limitation

is that the coil is assumed to be the dominant circuit inductance. In practice this is not too restrictive as the coil inductance is typically two orders of magnitude larger than the system inductance.

3. Experimental Evaluation of Mirror Coil Performance

A number of coils were constructed and tested. The discharge system parameters are:

Charging Voltage (kV)	18
Total Capacitance (μF)	28
Maximum Energy (J)	4500
Total No. Capacitors Used	2
Capacitance per Capacitor (μF)	14
Inductance of 2 Capacitors, Switch, and Transmission Lines (nH)	48

Most tests were carried out at 18 kV as this is the expected main bank charging voltage.

The first three wire wrapped coils were constructed as single layers, and were used to test the firing circuit and the mechanical properties of the coils themselves. Specifics investigated were: containment of coil expansion, breakdown of wire insulation, and heat dissipation in the wire. Following a series of material failures a coil was built with improvements in radial and hoop stress containment. Radial containment was provided by a 3 in diameter schedule 40 ABS pipe with a wall thickness of 6 mm. Screws were inserted through the pipe at the ends of the coil to prevent longitudinal expansion. The coil itself consisted of 11 turns of 8 ga wire with 2 mm thick TW insulation. The diameter was 6.8 cm with a length of 8.3 cm. There were 28 discharges at 18 kV with no apparent containment or insulation failure. A passive pickup loop placed 20

cm from the coil was used to monitor the current waveform (see Figure 30a). Cycle time was approximately 80 μsec while the indicated α is $2.08 \times 10^3 \text{ sec}^{-1}$. This gives a current of 36.500 Amps producing a magnetic field at 47 kG at coil center.

The next phase of the experimental development involved precise field measurement of three double layered, wire wrapped coils, which were estimated to have higher field production capability than single layers. The containment vessel developed for these coils is shown in Figure 31. The tubes were 1/4" (wall thickness) plexiglass while the retainers were made of heavy laminated cloth. Fabrication of the coil began by bolting the end retainers to the inner tube. Wire was then wrapped on the inner tube to form the coil, and this entire assembly was slid into the outer tube and bolted in place. The radial retainers were pressed onto the outer tube in order to give the plexiglass mechanical strength. This type of containment proved to be durable, as there was no structural failures in any of the tests. The physical dimensions of the three test coils are:

	<u>Coil 1</u>	<u>Coil 2</u>	<u>Coil 3</u>
Wire	8 ga Cu	8 ga Cu	8 ga Cu
Wire Insultation Thickness	2 mm	2 mm	2 mm
Inner Tube OD	11.43 cm	11.43 cm	6.35 cm
Outer Tube ID	13.97 cm	13.97 cm	8.89 cm
Inner Wire Layer Radius	6.02 cm	6.02 cm	3.43 cm
Outer Wire Layer Radius	6.63 cm	6.63 cm	4.09 cm
Number Turns Each Layer	8	3	3
Coil Length	5 cm	2 cm	2 cm

It should be noted that Coils 1 and 2 were designed to be slipped over the outside of the existing 10.16 cm O.D. discharge tube, with no alteration of

(a) i_θ Waveform for Third Structural Test Coil

(b) B_z and i_θ Waveforms for Wire Wrapped Mirror Coils

(c) B_z and i_θ Waveforms for Single Turn Mirror Coil

Figure 30. Mirror Coil B_z and i_θ Waveforms

Figure 31. Container for Wire Wrapped Mirror Coils

the system. All three coils were discharged a number of times at 19 kV. This was done both to test the structural integrity of the coils and the reproducibility of the diagnostics.

Two diagnostics were used in these tests. The first was a passive flux pickup coil made using RG58-U coaxial cable with the ground braid stripped off around the loop section and the end of the center conductor soldered to the braid. This loop was placed approximately 20 cm from the end of the coils and the emf set up by the coil's flux was recorded as indicative of $\dot{I}$. From this trace, the damping constant α , cycle time τ , maximum current $I_{\max}$ and ringing frequency ω can be determined.

The second diagnostic used was an axial magnetic field probe consisting of a small, multi-turn coil enclosed in a pyrex tube. The output of this probe was fed into an RC integrator which provides a signal proportional to magnetic field. Calibration of the probe-integrator combination was done using a wire loop to generate a magnetic field at its center given by:

$$B = \frac{\mu_0 I}{2a} \quad (9)$$

where a is the radius of the loop.

Magnetic field measurements were made with the probe on axis at the mid-plane of the coils. Figure 30b presents a typical record. Table 5 presents the average experimental results for the three coils.

Table 5
Field Parameters for Solenoidal Coils

$B(I)$ was calculated from current and the known geometry.

	<u>B(Probe)</u> <u>(kG)</u>	<u>$I_{\max}$</u> <u>(Amps)</u>	<u>B(I)</u> <u>(kG)</u>	<u>τ</u> <u>(usec)</u>
Coil 1	21.2	1.4×10^4	21.6	195
Coil 2	21.7	3.4×10^4	20.2	95
Coil 3	46.9	4.8×10^4	46.8	62

From the above results, it does not appear possible to reach a 3/1 mirror ratio with a wire wrapped solenoidal coil (one with a 4" I.D. to slip over the present pinch tube) driven by two 14 μ F capacitors. The capacitance of the mirror system must be increased and/or the diameter of the coils reduced. A smaller diameter coil (Coil 3) was tested and is shown to generate larger fields of 46 kG.

The third phase of the experimental effort involved fabrication and testing of several single turn coils. These coils have much smaller inductances and cycle times than the wire wrapped solenoids, however; the fields generated are not necessarily greater, due to the single turn. In fact, initial calculations indicated that no significant increase in field would be realized from using this type of coil. Six coils were tested, three being made of aluminum plate having considerable strength for handling the magnetic loads and the remaining three being formed from copper sheet. The data for these coils is as follows:

<u>Aluminum Coils</u>	<u>Inside Diam. (cm)</u>	<u>Length (cm)</u>	<u>Outside Diam/Bore Diam.</u>
1	7.62	2.54	2.6
2	7.62	5.08	2.6
3	2.77	2.54	5.5
<u>Copper Coils</u>			
1	5.08	5.87	-
2	2.54	5.87	-
3	2.54	2.6	-

All of these coils were discharged numerous times at 18 kV and none of the aluminum coils suffered any noticeable deformation. This was not true of the thin sheet copper coils which tended to compress into an elliptical shape following the electrical discharge. The experimental results are given in the following table. Figure 30c is a typical magnetic field record.

TABLE 6
Field Parameters for Solid Coils

B(I) was calculated from $I_{\max}$ and known geometry.

<u>Aluminum Coils</u>	<u>B(Probe) (kG)</u>	<u>$I_{\max}$ (Amps)</u>	<u>B(I) (kG)</u>	<u>τ (usec)</u>
1	24.9	1.82×10^5	25.0	16
2	27.1	1.95×10^5	28.9	16.2
3	64.8	1.95×10^5	65.5	15.6
<u>Copper Coils</u>				
1	32.5	1.98×10^5	32.3	15
2	40.7	2.09×10^5	40.9	14.8
3	59.7	1.79×10^5	62.2	16.4

The results of tests indicate that these coils have no advantage over the wire wrapped coils as far as the generation of field strength is concerned. However, these cycle times are quite short and so could rise with main bank speed. However, this use would require crowbarring as for the main bank. Typically, the coil inductance in the solid coil tests is 1/4 of the system inductance, making the energy transfer system the ultimate limitation in field production. These tests indicate that the inductance of the discharge system is about 140 nH, while the coil inductance is 30 nH. Here again system capacitance would need to be increased, and its inductance reduced to reach higher fields.

4. Summary of Efforts

The considerations and results presented above indicate that the following sequence of efforts would be most productive. With the intent of relating to the theoretical models, for mirrors,³² collisional loss would be of primary interest. The effects of mirrors on the particle loss have been recently restated by Freidberg and Weitzner³² where for a mirror ratio of 2.0 with β

$\approx .75$, particle loss time would be increased 60% and with a mirror ratio of 3 there is an anticipated increase in loss time of 225%. Accordingly, the most direct experiment with mirrors would utilize the smaller bore (3 in) diam. discharge tube with 2 cm long coils inside the ends. With a mirror ratio of 2, significant alteration of loss is anticipated.

It is also of interest to pursue the alternatives of add-on coils with this same (3 in) diam. to enhance the 2:1 mirror ratio. Specifically, the single turn coils with fast ($< 5 \mu\text{sec}$) rise and crowbar circuitry should boost the mirror ratio to 3:1 at the mirror coil. However, there are further detailed considerations relating to the distortion of the plasma column under the influence of the mirror field with variable initiation time.³⁶

V. STUDIES OF END LOSS USING A TWYMAN-GREEN INTERFEROMETER* (Freeman, Mollo)

The general purpose of this theta pinch research effort is to investigate the particle loss phenomenon by developing and using a unique diagnostic for making local measurements to determine plasma properties in the vicinity of the plasma column ends.^{36,37} In addition, there is a need to determine the change in the total number of particles contained within the plasma column as a function of time for reference purposes. This measurement has normally been made using interferometry, which averages properties over the column to yield an average line density.³⁸ This line density can then be used to evaluate the total number of particles remaining in the plasma column, which in turn can be used to determine the $1/e$ particle loss time.

In the work being reported here, an interferometric diagnostic was designed, fabricated and used to obtain an estimate of the $1/e$ particle loss time in a very short (10 cm), highly transient ($\tau \approx 8$ μ sec) theta pinch. This experimentally determined loss time was then compared to loss times obtained from $2/(T_e/m_e)^{1/2}$ scaling of larger present-day machines, from the use of a zero dimensional computer code being developed by Stover³⁷ and also from other experimental methods.

1. Experimental Approach

In this work, plasma particle end loss and plasma column behavior were studied using the results from several plasma diagnostics in a linear theta pinch device. From these diagnostics, the following information was obtained: current sheath trajectory; percent mass pick up during collapse; column length; the frequency and damping rate of radial plasma oscillations; and the $1/e$ particle loss time. Plasma column properties such as electron and ion temperature,

*The details of this work have been presented and reported in document C00-4010-4, Nov. '77.

plasma column radius and average number densities were either calculated using a simple snowplow implosion model³⁹ or determined using the zero dimensional computer code mentioned earlier.

The $1/e$ particle loss time was determined experimentally using two independent methods. In the first method, a Twyman-Green type interferometric diagnostic, was developed as a major part of this experimental study. In the second method, an estimate of the $1/e$ particle loss time was determined by the use of a magnetic field probe positioned inside the plasma column. This probe was used to monitor radial plasma oscillation frequency, which can be used to evaluate the total number of particles in the column with respect to time.⁴⁰

The Twyman-Green interferometer used was a double pass, two beam device. One beam served as a reference beam and the other served as the test beam which traversed the plasma column. Since the plasma electron density was continually varying during the column lifetime, the plasma refractive index also was changing.³⁸ This caused variation in the optical path length of the test beam as compared with the reference beam. Therefore, when the reference beam was recombined with the test beam, there was a phase difference between the two. The phase difference varied in both space and time causing a spatially and temporally varying fringe pattern. This fringe pattern was monitored by placing two fast silicon photodiode detectors at appropriate positions in the path of the recombined beam. By appropriate interpretation of the fringe movement at these positions, the $1/e$ particle loss time was evaluated.

There are two important advantages that the interferometric set-up used here has over other more commonly used interferometers.³⁸ The first advantage is the minimal cost of the laser, detection system, and optics resulting from the use of a He-Ne CW laser light source ($\lambda = 6328\text{\AA}$). Secondly, the most

important advantage is that by expanding the laser beam to approximately the radial size of the plasma column both temporal and spatial resolution can be obtained simultaneously. This cannot be done using any of the commonly used interferometers without great sophistication.⁶¹

2. Discharge Apparatus

The discharge apparatus consisted of four fundamental components: the discharge chamber, the theta pinch electrical discharge coil, an energy storage capacitor bank and an open-air gas-triggered switch which connected the compression coil to the capacitor bank.

The current waveform was determined to be a damped sinusoid, which is characteristic for a lumped LRC network. The equation which represents this waveform is:

$$I(t) = V_0 \omega C e^{-\alpha t} \sin \omega t$$

where α is the damping coefficient, experimentally determined to be $4.01 \times 10^4 \text{ sec}^{-1}$. All other parameters are defined and their values can be found in Table 7 (Theta Pinch Characteristics).

Hydrogen gas at several fill pressures was used throughout the investigation; hydrogen has the low atomic weight of interest in fusion studies, and it was readily preionized by the relatively low energy (595 Joule) storage system. The first half cycle of the discharge current was used to preionize the gas while the second half cycle generated a current sheath which imploded to form the plasma column on axis. There was no indication of trapped magnetic fields from the half-cycle of the discharge. While discharge conditions with fill pressures from 30 to 200 millitorr were examined, fill pressures of 50 and 65 millitorr of hydrogen gas were investigated in detail. These particular fill pressures were chosen because higher fill pressures yielded loss times large

TABLE 7
Theta Pinch Characteristics

Voltage (kV)	11.0
Capacitance (μ f)	9.85
Total Source Inductance (nH)	79.2
Coil Length (cm)	10.5
Coil Diameter	11.5
Coil Inductance	81.3
Inside Radius of Discharge Chamber (cm)	10.16
Maximum Current (amps)	
1st Half Cycle	7.85×10^4
2nd Half Cycle	6.69×10^4
Maximum Magnetic Field (kG)	
1st Half Cycle	6.36
2nd Half Cycle	5.42
Maximum E_θ (At Tube I.D.) (V/cm)	
1st Half Cycle	137.3
2nd Half Cycle	107.9
Half Cycle Time (μ sec)	4.0

compared to the half cycle time of the discharge current making interpretation of the interferometric data difficult.

3. Experimental Layout of Twyman-Green Interferometer

The Twyman-Green interferometer setup (Figure 32) consisted of five major components: a laser light source; an expander and collimating lens apparatus; an isolation system; a set of optical components used to direct both interferometer beams; and two fast silicon photodiode detection systems. Each set of components will be discussed briefly below.

The light source was a He:Ne CW mode, model 3224H-PC laser fabricated by the Hughes Aircraft Corporation. The useable power output was measured at 6.9 milliwatts (mW) on a Spectra Physics model 401B power meter. Tests showed no visible drift in power output over a three-hour testing period. The power of the beam after passage through the isolation system was found to have been 2.9 mW.

The optical components used were a beamsplitter, two optical windows, three totally reflecting mirrors and a line filter, all of which were designed for use with a He:Ne 6328Å light source.

The laser beam was linearly polarized and had a divergence of approximately 1 mrad. The beam diameter, given by twice the radius where the intensity drops from the maximum value by e^{-2} , was 0.81 mm.

Connected directly to the laser head was a Spectra Physics collimator and expander, which consisted of a model 332 spatial filter and a model 331 collimating lens separated by a 15 micron aperture. This apparatus was used to expand and collimate the laser beam to a diameter of 19 mm, a dimension approximately equal to the radius of the plasma column being investigated. The spatial filter consisted of a converging lens with a focal length of 12.8 mm, which

Figure 32. Twyman-Green Interferometer

focused the laser beam to a spot size of approximately 13 microns at the 15 micron aperture. The aperture was used to cut off the radial edge of the beam's Gaussian profile, thereby making the emerging beam more uniform in intensity. After passing through the aperture, the beam was expanded to a diameter of 19 mm and then passed through the collimating lens for focusing to minimize beam divergence. The useable power output of the beam emerging from the collimator-expander was determined to have been approximately 4.8 mW.

A linearly polarized laser source was used so that an inexpensive optical system could be fabricated to isolate the laser from light reflected back by the external mirror cavities created by the two legs of the interferometer.

The detection system consisted of circuitry supporting two EGG SGD-040-A fast silicon photodiodes each of which had a 0.815 mm^2 photosurface. The output of each diode was amplified by an RCA CA-3015-A operational amplifier. The load resistance seen by the diodes was designed to be 1.47 k Ω , which gave each diode a band width of approximately 40 MHz. Amplifier gain was approximately 10 dB with the 3 dB cutoff occurring at approximately 18.3 MHz. The entire circuit was fabricated inside a 0.13 cm thick aluminum box wrapped with two layers of 3.81×10^{-3} thick aluminum foil to minimize electromagnetic noise pick up by the circuit components. The active surfaces of each diode extended out of the box with the centers spaced 8.5 mm apart.

The detector outputs were monitored by a Tektronix type 556 dual beam oscilloscope in which types 1A5 and 1L preamplifiers were used. The oscilloscope was used to record the passage of fringes with respect to time. Total spatial resolution was achieved by repositioning the dual detector between discharges, at different plasma column radii. Data were recorded at three sets of radial positions; $r = 0 \text{ mm}$ and 8.5 mm, 2 mm and 10.5 mm, and 4 mm and 12.5 mm with both positions in each set recorded simultaneously.

4. Theory of Operation

As previously mentioned, when the two beams of light within the interferometer recombine, they interfere with each other in such a way that a fringe pattern is formed which varies temporally and, with position of the detector, spatially. The reason for this interference, is that when the total optical path length of the test leg changes by a factor of λ_0 ($\lambda_0 = 6328\text{\AA}$), at any special position, one fringe shift occurs at that position.³⁷ However, since the test beam passes through the plasma column twice, one fringe shift occurs when the optical path length of the plasma at that radial position changes by a factor of $\lambda_0/2$.³⁷ The total number of fringes to be seen at any radial position is

$$X_r(t) = \frac{\text{number of fringes passing a position in a given amount of time}}{\text{difference in optical path length between a vacuum and the plasma. wavelength of the light source.}} = \frac{\text{difference in optical path length between a vacuum and the plasma.}}{\text{wavelength of the light source.}} \quad (\lambda_0 = 6328\text{\AA})$$

The optical path length is equal to μl_t where μ is the refractive index of the medium and l_t is the total distance traveled by the laser beam in passing through the medium. For a vacuum, $\mu = 1$, but for a plasma, it has been shown³⁷ that $\mu_p \approx 1 - \frac{1}{2}(\omega_p^2/\omega_l^2)$ where;

$$\omega_p = \text{plasma frequency} = [(4\pi e^2/m_e)n_e]^{1/2}, \text{ (rad/sec),}$$

$$e = \text{electronic charge, (esu),}$$

$$m_e = \text{mass of electron, (gm),}$$

$$n_e = \text{electron density, (cm}^{-3}\text{),}$$

$$\omega_l = \text{laser frequency} = 2\pi c/\lambda_0, \text{ (rad/sec),}$$

$$c = \text{speed of light, (cm/sec),}$$

$$\lambda_0 = \text{wavelength of laser light source being used} = 632.8 \times 10^{-7} \text{ cm.}$$

Making the appropriate substitutions, the refractive index for a plasma medium is:

$$\mu_p = 1 - 4.477 \times 10^{-14} n_e \lambda_o^2.$$

The optical path length can be written as $\mu l_T = \int_0^{2l_p} \mu dl$, where l_p is the length of the plasma column. The upper limit of the integration is $2l_p$ instead of l_p because the system was a double pass type. The number of fringes expected can be written as:

$$X_r(t) = \frac{\int_0^{2l_p} \mu_{\text{vacuum}} dl - \int_0^{2l_p} \mu_{\text{plasma}} dl}{\lambda_o} \quad (1)$$

or

$$X_r(t) = \frac{\int_0^{2l_p} P(l) dl - \int_0^{2l_p} P[1 - 4.477 \times 10^{-14} n_e \lambda_o^2] dl}{\lambda_o}$$

therefore,

$$X_r(t) = 4.477 \times 10^{-14} \lambda_o \int_0^{2l_p} P n_e dl. \quad (2)$$

By rearranging terms, the total change in the line density corresponding to a complete fringe was calculated to be,

$$\int_0^{2l_p} P n_e dl = \frac{1.117 \times 10^{13}}{\lambda_o} \text{ electrons cm}^{-2} \quad (3)$$

Since $\lambda_o = 632.8 \times 10^{-7}$ cm, one fringe represents a change in average line density of 1.765×10^{17} electrons cm^{-2} . The net change in average number density represented by one fringe can be obtained by dividing the change in average line density by the length of the plasma column, therefore, per fringe,

$$\frac{\Delta n_e}{l_p} = \frac{1.765 \times 10^{17}}{l_p} \text{ electrons cm}^{-3}. \quad (4)$$

5. Dynamic Current Sheath Collapse Phase of the Discharge

The implosion phase of the discharge was investigated using several diagnostic devices described previously. These devices were the axial magnetic-

field probe and the Twyman-Green interferometer. A radial survey of the time rate of change in the axial magnetic field, $\dot{B}_z$, was performed so that current sheath position with respect to time could be identified. The Twyman-Green interferometer was used to establish the time of mass arrival at the center of the discharge chamber. From these studies percent mass pick up by the imploding current sheath, and pinch time, t_p , the time for the current sheath to implode into the center of the discharge chamber were obtained for fill pressure of 50 and 65 millitorr of hydrogen gas.

The nonintegrated output of the magnetic field probe, proportional to $\dot{B}_z$, was monitored at five radial positions: $r = 3.81$ cm, 3.175 cm, 2.54 cm, 1.90 cm, and 1.59 cm. Typical data are presented in Figure 33; typical Twyman-Green data are presented in Figure 34.

6. Post-Implosion Plasma Column Behavior

This detailed analysis was performed using both theoretical and experimental results. Initial values of plasma column temperature, average density and radius were calculated using a snowplow implosion model. Both plasma column length and time varying particle inventory, $N(t)$, were estimated from axial magnetic field probe data. Plasma radius as a function of time, $r_p(t)$, was estimated using both the magnetic field probe data and a zero dimensional computer code. Also, five different estimates - three based on experimental results - are presented for the $1/e$ particle end loss time. The first analytical method uses the zero dimensional computer code; while another estimate is based on $k(T_1/m_1)^{3/2}$ end loss scaling from Scylla I-C,^{39,42} a larger collision dominated theta pinch experiment conducted at the Los Alamos Scientific Laboratory. Experimental loss times were determined from magnetic field probe data obtained by probing internal to the plasma column, and, using two different

$z = 38.10 \text{ mm}$ $\dot{B}_z$

arrival of current
sheath

$z = 31.75 \text{ mm}$ $\dot{B}_z$

start of second
half cycle

$z = 25.40 \text{ mm}$ $\dot{B}_z$

$z = 19.00 \text{ mm}$ $\dot{B}_z$

20 volts/div

(Note: data taken at discharge compression coil midplane:
 $z = 5.25 \text{ cm}$)

Figure 33. Radial Survey of Axial Magnetic Field Probe for a 65 Millitorr
Fill Pressure of H_2

Figure 34. Typical Twyman-Green Interferometric Data

analysis methods, from the Twyman-Green interferometric data. A brief comparison of the five loss time estimates is presented in Table 8.

7. Subsequent Efforts

Further experimental efforts have been carried out to quantitatively establish the accuracy of the assumption of the Gaussian distribution of density and to identify the on-axis value of density for the analysis of the fringe data. The Thomson scattering diagnostic was installed on this pinch apparatus and aligned for measurements on the center line at the mid-coil position. During the first microsecond of pinched column lifetime, there was little scattered signal, indicating a value of density on-axis that was below the level of identification ($\sim 10^{13} \text{ cm}^{-3}$). Such behavior could be due to field trapping and exclusion of the plasma from the axis until it could diffuse and establish its distribution. Photography of luminous patterns by a Kerr Cell shutter system did in fact show an annulus of light at a radius of about 1 cm, at 1 μsec after pinch; this annulus disappeared after about 2 μsec leading to a uniform circle on axis after this time. The Thomson scattering diagnostic will again be used in the near future to identify the time history of the density distribution in this pinch device, prior to a journal reporting of the diagnostic and pinch behavior.

TABLE 8
Comparison of $1/e$ Particle End Loss Times

<u>Method of Calculation</u>	τ_p (μ sec)	
	Fill Pressure <u>50</u>	(Millitorr) <u>65</u>
1. Computer Simulation	2.26	2.47
2. Radial Plasma Oscillations	2.04	2.45
3. Twyman-Green		
a. Use of First Method for Calculation of $n_r(0)$	2.85	2.81
b. Use of Second Method for Calculation of $n_r(0)$	1.82	2.37
4. $1/(T_i/m_i)^{1/2}$ Scaling for Scylla I-C	2.31	2.45

VI. BANK ENGINEERING
(Robinson, Heidrich, Jacoby, Veglia)

1. Thyratron Triggering

In the process of consistently generating the same plasma conditions, it was found necessary to provide a method for triggering the main bank discharge that was accurate to within ± 0.1 μsec . This ability proved especially crucial in maintaining the same condition of reversed trapped field. The method used in providing the needed accuracy in bank triggering involved the development of a thyratron triggering circuit.

Basically the thyratron unit discharges a length of RG8-U cable that is normally held at 3 kV. The circuit presently in use on the 25 cm theta pinch is shown in Figure 35.

When installed, the unit was found to provide the needed accuracy in main bank triggering. It was used during all data taking described above, and will continue in service on the 50 cm theta pinch.

2. Preionization System Triggering

Precise timing of the Thomson scattering laser pulse required the development of electronically controlled preionization gap. The method employed in triggering the preionization capacitor utilizes an SCR with an autotransformer. The circuit diagram is shown in Figure 36. This circuit produces a slowly rising pulse that reaches peak voltage in around 55 μsec . Thus, there is about a 15 μsec delay from the time the BNC delivers its pulse to the time the spark gap triggers. Inherent in the slow rise time of this circuit is a ± 2 μsec jitter in the timing of the firing of the preionization spark gap. This jitter, while unacceptable for the timing requirements of the main bank, is quite adequate for P.I. system timing requirements and in particular for these timing requirements that are demanded by the Thomson scattering diagnostic.

Figure 35. Circuit Diagram of Thyatron Trigger Unit

Figure 36. Preionization Trigger Circuit

3. Improvements in Vacuum System

Due to impurity level considerations, for operation with several trapped magnetic field, the vacuum system was changed from a diffusion pump-roughing pump combination to an ion getter pump-roughing pump combination. This clearly improved the impurity situation, since no foreign substance of any kind is involved in the better pump action. Secondly, in an effort to improve the base pressure, larger diameter glass pipe was introduced in connecting the vacuum vessel to the pump. This feature, in concert with the slightly greater pumping capacity of the ion getter pump, succeeded in producing a factor of about 10 improvement in the base pressure.

4. Capacitor Bank Doubling

During the present reporting period, construction of a 112 μF , 20 kV capacitor bank unit has been completed. This unit, when integrated with the bank now in use, will double the capacity of the theta pinch discharge system. The new bank consists of eight 14 μF , 20 kV Maxwell capacitors arranged into four rows. In each row the capacitors are connected in parallel by 1/4" aluminum feed plates. A dry air spark gap as well as the associated charging and triggering electronics have been installed for each row.

The spark gaps consist of two hemispherical brass electrodes with a triggering pin inserted midway between them. This assembly is enclosed in a heavy laminated cloth and plexiglass container with feedthroughs inserted to allow dry air to flow through the gap. Each gap is triggered through a delay line from a common switch. This triggering system is expected to provide better precision in igniting each gap than the type presently in use. The energy of the new bank will be fed to the theta pinch coil via 40 RG8/U cables.

Fabrication of the second 25 cm theta pinch coil has been completed. This coil will be mounted adjacent to the original 25 cm coil so as to effectively create a 50 cm coil. A small gap between the two coils will serve as a diagnostics port. The coils are to be mounted on 1/2" aluminum feed plates which have also been fabricated. These plates have areas allotted for connection of power feed cables and a pair of vacuum crowbar switches.

VII. COMPUTER MODELING OF PLASMA COLUMN BEHAVIOR (Stover, Awwad)

1. Introduction

The goal of this part of the research program is to develop computational models which are suitable for comparing theory with experiment. The first phase of this work was the development of a time dependent zero-dimensional code to model the average column behavior, primarily from peak compression to the end of the experiment. This work has been presented in a report⁴² and has been accepted for publication in the Physics of Fluids. A summary of this work is presented in Part 2 of this section, along with new results which have been obtained since the report was prepared.

The main effort during the past six months has been the development of one-dimensional codes to describe axial, as well as time dependent plasma behavior. This will enable us to model the flow behavior in detail, and to determine the influence of various physical phenomena in the exhaust region on the plasma behavior. Comparisons with the zero-dimensional code will also be possible so that various assumptions made in the zero-dimensional model can be tested.

The details of the one-dimensional models and the initial results when applied to the Scylla I-C experiment⁴¹ are presented in Part 3 of this section.

2. Zero-Dimensional Modeling

a. Behavior without mirrors and with classical resistivity

A zero-dimensional, time-dependent computer code has been developed to simulate plasma column behavior in linear theta pinch machines.⁴² Key features include: the use of a radially diffuse plasma profile; magnetic field diffusion and the ohmic heating associated with it, both using the Spitzer

classical conductivity; particle end loss modeled phenomenologically by an e^{-1} particle end loss time, $\lambda_{m_i}^{1/2} (T_e + T_i)^{-1/2} \chi$, with χ determined from experiment; and axial thermal conduction, modeled classically and incorporating a thermal sheath length which is specified intuitively. By comparing the predictions of this code with a collisional experiment and several collisionless experiments, it is felt that the agreement is sufficiently good to indicate that the proper physics has been incorporated into the model.

Over the range of parameters considered, χ varied from $\chi = 2.5$ for collisional Scylla I-C to $\chi = 1.8$ for collisionless Linear Scyllac.³⁵ Also, it was found satisfactory to treat χ as constant in time, although it is uncertain whether χ would remain constant in time for longer, hot experiments.

For electron thermal conduction, the data are not sufficiently complete to identify a thermal sheath length, λ_1 . For Scylla IV-P data, the time behavior of $T_e + T_i$ was compared with simulations for $\lambda_1 = 0.2 L$ and $\lambda_1 = 1.0 L$. Both gave reasonable agreement, indicating that the quantity $T_e + T_i$ is not sensitive enough to make a clear identification of λ_1 . Data for $T_e(t)$ are needed to reduce the uncertainty. Scylla I-C data are not helpful because thermal conduction was not very important for this experiment. There was no indication in any of these experiments of anomalously large values of electron thermal conduction.

The treatment of ion thermal conduction is largely uncertain because the detailed experiments have been either strongly collisional or collisionless, and ion thermal conduction was unimportant in both cases. It has been found that erroneous simulation results will be obtained if classical ion thermal conduction is used in the modeling of collisionless experiments. This led to the introduction of a cutoff to eliminate classical ion thermal conduction for $\lambda_{ii} \geq 0.25 L$. While this leads to satisfactory simulations, no experimental

data exists to check the cutoff value or to determine whether a cutoff model is generally valid. A machine much longer than the 5 meter Scylla IV-P machine would be required to obtain information on ion thermal conduction at keV ion temperatures.

Magnetic field diffusion was found to be important for simulation of the collisional Scylla I-C experiment. The results indicated that $\eta_{\perp}$ was within a factor of 2 of being classical. For the collisionless experiments, magnetic field diffusion was not indicated as being important.

b. Effects of mirrors and of anomalous resistivity

Two extensions of the above work have been completed. First, we modeled the Linear Scyllac results³⁵ with the magnetic mirrors included. Previously, we found that, without mirrors, the end loss parameter χ was $\chi = 1.8$. In Figure 37 we show the results when the magnetic mirrors are present. The best agreement with the one-dimensional code is achieved when we choose $\chi = 3.26$. This is an increase of only 1.8 whereas the mirror ratio was 2.5.

The other extension was to study the difference in predicted plasma behavior if we used anomalous resistivity instead of classical resistivity. For the anomalous resistivity we have adopted the expression developed by Liewer and Davidson⁴³ for the lower hybrid drift instability:

$$\eta_{an} = \frac{4.9 \times 10^{-12}}{A^{1/2} B} \left(\frac{r_{\perp}}{n} \frac{\partial n}{\partial x} \right) \alpha \quad (1)$$

where A is the ion mass number in a.m.u., B is the magnetic field in kilogauss, $r_{\perp}$ is the ion cyclotron radius in cm, and

$$\alpha = \frac{2\epsilon_F}{nm_e v_E^2} = \left[2 \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right) \right]^{-1} \quad (2)$$

Figure 37. Number of Electrons as a Function of Time for Linear Scyllac with $x = 3.26$

with $\epsilon_F = \langle \delta E^2 \rangle / 8\pi$ and $V_E = cE/B$ being the $\underline{E} \times \underline{B}$ drift velocity. Assume a Gaussian density profile,

$$n = n_0 \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (3)$$

and quasi-equilibrium pressure balance

$$B(x) = B_W [1 - \beta \exp(-\frac{x^2}{2\sigma^2})]^{1/2} \quad (4)$$

where B_W is the magnetic field at the wall and $\beta = 8\pi n_0 (T_e + T_i) / B_W^2$. To obtain a single value for n_{an} , the evaluation is performed at $x = \sigma$.

The analysis was applied to both Scylla IV-P,⁴⁴ a hot collisionless experiment, and Scylla I-C, a cold collisional experiment. For Scylla IV-P we took $\beta = 1$ and $\sigma^2 = 0.53$ to obtain

$$\begin{aligned} \frac{n_{an}}{n_{cl}} &= 1.5 \times 10^3 \frac{T_e^{3/2} (T_i + T_e)}{B_W} \left(\frac{r_i}{\sigma}\right)^2 \\ &= 3 \times 10^5 \frac{T_e^{3/2} T_i (T_i + T_e)}{B_W^3} \end{aligned} \quad (5)$$

For peak field conditions ($B_W = 48$ kG, $T_e + T_i = 3.2$ keV)⁴⁶ we estimate $T_i = 2.6$ keV and $T_e = 0.6$ keV, and obtain $n_{an}/n_{cl} = 10.5$.

Previous calculations using n_{cl} indicated that, for the lifetime of Scylla IV-P, resistivity was not important. Even with the factor of ten increase, it was found that when n_{an} was used instead of n_{cl} the effect was still very small. Consequently, for Scylla IV-P we are unable to distinguish between classical and anomalous resistivity.

For Scylla I-C we used $\beta = 0.4$ and $\sigma^2 = 0.18$. This results in

$$\frac{n_{an}}{n_{cl}} = 1.6 \times 10^6 T_e^{3/2} \frac{(T_i + T_e) T_i}{B_W^3} \quad (6)$$

For peak field conditions ($B_W = 33$ kG and $T_e = T_i = 0.055$ keV) we obtain

$$\frac{\eta_{an}}{\eta_{cl}} = 3.5 \times 10^{-3}$$

Thus, η_{an} is equivalent to having zero resistivity. As shown in our earlier work, the best agreement between theory and experiment was achieved when $\eta = 2\eta_{cl}$ where η_{cl} is the classical value. We conclude that the resistivity for Scylla I-C is probably classical and is certainly not given by Equation (1).

3. One-Dimensional Modeling

a. Introduction

In the past we have used zero-dimensional computer models^{5,42} to successfully predict transient plasma column behavior in linear theta pinch machines. These models are inherently limited, however, in that the most important mechanisms contributing to particle and energy end loss result from axial gradients in both pressure and temperature, neither of which can be rigorously included in the one-dimensional formulation. This requires that scaling laws be adopted to model these end loss processes. Experimental evidence to date^{35,41,44,45} indicates that the rate of particle end loss can be scaled simply as N/τ_p where N is the total particle inventory in the plasma column and τ_p is a characteristic particle confinement time

$$\tau_p = \frac{L}{2} (m_i / (T_e + T_i))^{1/2} \chi$$

with the factor χ being dependent upon the particular experiment. This scaling law, which guarantees satisfaction of both momentum conservation and particle conservation, was used to describe particle loss in our zero dimensional models.⁴² The rate of heat transfer was expressed as $(3/2)NT/\tau_{Th}$ where the characteristic thermal conduction time, τ_{Th} , was determined from an electron thermal conduction analysis by Morse.⁴⁶ Here, as with particle end loss, an

unknown parameter which is machine dependent must be specified. These difficulties are not encountered in the z dependent one-dimensional computer codes which have been developed during this reporting period. In addition, the one-dimensional formulations will allow plasma properties to be spatially resolved in both the hot column and the expanding exhaust. In particular details of wave propagation and throat formation and development can be analyzed in detail.

Two transient, one-dimensional (z dependent), two fluid computer codes have been developed. The fluid equations in both codes are expressed in a Lagrangian representation where the observer moves with tagged fluid elements. an explicit differencing scheme is used to described the axial momentum equation in both codes. When heat conduction effects are negligible an explicit differencing scheme is used to difference both electron and ion energy equations. When heat conduction effects are important, numerical stability requirements and the highly non linear nature of this term requires that the energy equations be differenced implicitly. This was accomplished using a Crank-Nicholson implicit differencing scheme⁴⁷ to represent the heat conduction term. Radial momentum is satisfied with a simple radial pressure balance equation.

Both one-dimensional codes include the following physics: different electron and ion temperatures with electron-ion energy equilibration included in the energy equations; work done on the plasma by the applied magnetic field; and pseudoviscosity pressure for modeling hydrodynamic shock waves. Both electron and ion thermal conduction are included in the implicit code. Plasma electrical conductivity is assumed to be infinite; thus, the magnetic flux is taken to be independent of time for a give Lagrangian element. Finite resistivity effects will be added at a later date. Also included in both formulations is an element splitting routine which is used to enhance spatial resolution

of the system by splitting the distorted Lagrangian elements after their length has exceeded some predetermined minimum value.

In Section b the governing equations, their boundary conditions and their initial conditions are discussed. In Section c, the finite difference equations are formulated and their method of solution is discussed for both cases. Finally, in Section d, simulation results are presented for the Scylla I-C experiment⁴¹ and compared with experimental results.

b. Governing equations, boundary conditions and initial conditions

The governing collisional two fluid equations are stated in this section. The plasma is considered collisional to the extent that the plasma pressure is isotropic, that axial gradients in the plasma occur over lengths much greater than the ion-ion free path, λ_{ii} , and that the characteristic problem time (i.e. particle end loss time or thermal conduction time) is much greater than the particle collision time. It is felt that the fluid theory presented here may be applicable for describing plasma column behavior even when λ_{ii} is much larger than the plasma column diameter. The confining magnetic field will reflect plasma particles incident on the plasma-vacuum interface and thus give the plasma a fluid character. Finite gyroradius effects in the sheath region separating plasma from vacuum are neglected here but they may be important in fusion temperature plasmas. Furthermore, the plasma is assumed to be quasi-neutral, $n \approx n_e \approx n_i$, thus, plasma characteristic lengths are assumed to be much larger than the plasma Debye length.

The fluid equations for particle conservation, axial and radial momentum, and an internal energy equation for both the ions and electrons were derived, assuming the plasma to be one-dimensional, for spatial variations in plasma properties constrained to occur only along the plasma longitudinal axis. Thus,

plasma number density, n , electron and ion temperature, T_e and T_i , and mass averaged plasma axial velocity, U , discussed in the following paragraphs all represent averages taken over the plasma column cross sectional area, A , as defined below:

$$n(z,t) = (\int_0^A n(r,z,t) dA) / A(z,t)$$

$$T_{e,i}(z,t) = (\int_0^A T_{e,i}(r,z,t) n(r,z,t) dA) / (n(z,t) A(z,t))$$

and

$$U(z,t) = (\int_0^A n(r,z,t) U(r,z,t) dA) / (n(z,t) A(z,t)) .$$

The set of governing conservation equations used in this study was derived from a modified version of the standard control volume analyses commonly found in basic fluid mechanics texts. The control volume element used was considered to have fixed ends across which plasma mass, plasma momentum and plasma energy can be transported; and movable sides, defining a plasma vacuum interface, across which neither particles, momentum nor energy may be transported. In performing the control volume analysis, the effects of electron and ion thermal conduction, electron-ion equilibration and pseudoviscosity for numerically dealing with hydrodynamic shocks are included. In addition, the plasma is assumed to be fully ionized and have an infinite electrical conductivity, thus, plasma magnetic flux is conserved. Furthermore, magnetic field curvature effects due to gradients in the radial magnetic field are neglected in formulating the momentum equations. Neglecting radiation losses and making the above assumptions, the resultant set of equations governing plasma behavior is found to be identical to the set used by Jones et al.,⁴⁸

$$\frac{d(nA)}{dt} = -nA \frac{\partial u}{\partial z} , \quad (1)$$

$$mn \frac{dU}{dt} = - \frac{\partial}{\partial z} (nkT_i + nkT_e + q) , \quad (2)$$

$$\frac{d(kT_e)}{dt} = \frac{(\gamma - 1)}{n} kT_e \frac{dn}{dt} + \frac{(\gamma - 1)}{nA} \frac{\partial}{\partial z} (K^e_A \frac{\partial T_e}{\partial z}) + \frac{k(T_i - T_e)}{\tau_{eq}}, \quad (3)$$

$$\frac{d(kT_i)}{dt} = \frac{(\gamma - 1)}{n} (kT_i + q) \frac{dn}{dt} + \frac{(\gamma - 1)}{nA} \frac{\partial}{\partial z} (K^i_A \frac{\partial T_i}{\partial z}) + \frac{k(T_e - T_i)}{\tau_{eq}}, \quad (4)$$

and

$$nkT_e + nkT_i + \left(\frac{\phi}{A}\right)^2 \frac{1}{2\mu_0} = \frac{B_e^2}{2\mu_0} \quad (5)$$

where q is the pseudoviscosity pressure defined by Von Neumann and Richtmyer⁴⁹

$$q = \begin{cases} -c^2 \rho \Delta^2 \left| \frac{\partial u}{\partial z} \right| \frac{\partial u}{\partial z} & , \frac{\partial u}{\partial z} < 0 \\ 0 & , \frac{\partial u}{\partial z} > 0 \end{cases}$$

with Δ equal to the computational mesh width and c a constant normally chosen between 0.1 and 2.0;⁴⁷ where K^e is the Spitzer⁵⁰ classical formulation for electron heat conduction along magnetic field lines,

$$K^e = 6.12 \times 10^{23} \frac{T_e^{5/2}}{\ln \Lambda} \left(\frac{1}{\text{cm} - \text{sec}} \right)$$

for T_e in keV; where the ion thermal conductivity K^i is related to the electron thermal conductivity from Braginskii⁵¹

$$K^i = .0407 \left(\left(\frac{T_i}{T_e} \right)^{5/2} \frac{1}{W^{1/2}} \right) K^e$$

with W equal to the ion atomic weight in amu; where τ_{eq} is the Spitzer⁵⁰ expression for energy equilibration between Maxwellian ions and Maxwellian electrons,

$$\tau_{eq} = 9.96 \times 10^{12} \frac{T_e^{3/2}}{n \ln \Lambda} \quad (\text{sec})$$

with n in cm^{-3} ; and where ϕ and B_e are the plasma magnetic flux and applied driving magnetic field, respectively.

Realistic boundary conditions must be placed on the plasma to conduct meaningful plasma dynamics simulations. The applied magnetic field and background gas properties are assumed to be symmetric about the compression coil midplane. Since all plasma column properties will be symmetric about the midplane, analysis of only one-half of the plasma column is required. Also, as a result of this symmetry, the midplane plasma will have zero velocity and zero axial gradients in n , T_e , T_i and A .

$$\begin{aligned}U|_{z=0} &= 0 \\ \frac{\partial T_e}{\partial z}|_{z=0} &= \frac{\partial T_i}{\partial z}|_{z=0} = 0 \\ \frac{\partial n}{\partial z}|_{z=0} &= \frac{\partial A}{\partial z}|_{z=0} = 0\end{aligned}$$

where $z = 0$ defines the plasma column midplane. System geometry and the coordinate system adopted are presented in Figure 38. Motion of the plasma front as it expands into the vacuum vessel is determined by the pressure distribution in the plasma column at the column end and the pressure exerted by the cold background gas on the front. This background pressure is assumed known. The plasma is assumed to freely expand along the magnetic field until the exhaust boundary either strikes the vacuum vessel inner wall or impacts with a vacuum vessel end wall. In the first case, when the plasma strikes the vacuum vessel inner wall, the plasma column cross sectional area is fixed at the vacuum vessel area. Plasma properties for all $z > z$ (impact) are then determined by this constraint and the radial pressure balance equation is not needed. In the second case, the plasma velocity is set equal to zero at the end wall.

The plasma front is assumed to be thermally insulated from the cool background gas because $K \sim T^{5/2}$, thus the net heat flow at the ends is taken to be zero,

Figure 38. Linear Theta Pinch Geometry

$$KA \frac{\partial T}{\partial z} = 0$$

or

$$\frac{\partial T}{\partial z} = 0$$

for each species. The hot plasma particles upon striking the cold walls will adjust to the wall temperature ($\approx .025$ eV) after a period of transient heat transfer. During this transient period the nearly uniform plasma temperature will readjust to a time independent radial profile shape. After the plasma profile has adjusted so that the plasma temperature at the wall adjusts to the wall temperature, ion and electron heat transfer to the wall will be small for $K \sim T^{5/2}/m$. The net heat loss during transient adjustment of the plasma profile has not been analyzed at this time. As a result, the following boundary conditions at wall interfaces are being used at the present time,

$$\frac{\partial T}{\partial z} = 0$$

and

$$q(\text{radial}) = -\frac{1}{r} \frac{\partial}{\partial r} [rK \frac{\partial T}{\partial r}] = 0$$

As plasma expands to the walls it will interact with the low temperature background neutrals. This will result in a transfer of energy from the plasma to the neutrals which in turn can conduct thermal energy. Furthermore, when the plasma reaches the wall it will tend to recombine forming a neutral sheath adjacent to the wall. For neutrals thermal conductivity has the functional relationship, $K \sim T^{1/2}/m$, where the neutral collision cross section is assumed to be constant. Note that as $T \rightarrow 0$, $K \rightarrow 0$ as it does for the charged particles. Also, $m/m_e \gg 1$, thus thermal conduction through the neutral sheath should not be as great as heat conduction through the electrons. The uncertainty in describing the thermal conduction process near the wall is further amplified in

the case of high energy pinches³⁵ where high Z wall material will ablate from the vacuum vessel walls. This material will reduce thermal conductivity since $K \sim Z^{-4}$. The above factors will be evaluated during the following reporting period.

The values for the driving magnetic field used in satisfying radial pressure balance, Equation (5), are discharge chamber on-axis vacuum values obtained from a simple analytic solution of the Biot-Savart law⁵² for a uniformly distributed compression coil current,

$$B_e(z,t) = \frac{B_o(t)}{2} (\cos \theta_1 + \cos \theta_2) \quad (6)$$

In this expression, $B_o(t)$ is the time varying part of the solution which is adjusted to give the correct $B(t)$ at midplane measured from experiment, and θ_1 and θ_2 are the spatially dependent parts defined by

$$\cos \theta_1 = \frac{(L/2) - z}{[(L/2 - z)^2 + r_c^2]^{1/2}}$$

and

$$\cos \theta_2 = \frac{(L/2) + z}{[(L/2 + z)^2 + r_c^2]^{1/2}}.$$

Vacuum magnetic fields measured at the compression coil edge⁵ were found to be about 40 percent higher than predicted by Equation (6), suggesting a redistribution of current toward the ends of the compression coil. Corrections to Equation (6) for this effect have been calculated and will be used in selected simulations. A fully self consistent solution for the magnetic field in the plasma and vacuum regions using the complete set of Maxwells equations falls outside the scope of the one-dimensional model presented here.

The plasma initial conditions selected for a given simulation depend specifically on the problem investigated. When attempting simulations of experiments

for which experimental data are available, the plasma post implosion initial condition is chosen which allows the simulation results to match experimental data at the earliest data time point. The plasma column is assumed to be at rest immediately following implosion and the axial distributions in plasma properties at this time are inferred from radial pressure balance and mass conservation. When experimental results are not available, the post implosion initial conditions are determined from an implosion analysis which is discussed in Section VIII. The initial plasma column length is taken to be equal to the compression coil length although experimental evidence⁷ suggests that the effective plasma column length is probably somewhat less, especially for short experiments.

c. Finite difference equations and their solution

Two representations of the governing plasma equations discussed in the previous section may be used to solve the end loss problem: they are the Eulerian representation and the Lagrangian representation. Each has its particular advantages and disadvantages, the relative strengths between these determining the best method for solving a particular problem.

In the Lagrangian method, the finite difference equations are set up to follow the individual fluid elements, thus, particle, momentum and energy convection terms are zero and total derivatives with respect to time are replaced by partial derivatives with respect to time. The resultant equations are significantly simplified and elimination of the convection terms eases the numerical stability restriction placed on the maximum allowable time step.⁵³ Furthermore, the computational mesh follows the individual mass elements thus plasma boundaries and medium interfaces are reasonably well defined. However, distortion of the individual field elements as time progresses limits applicability of the

Lagrangian method. In problems of more than one spatial dimension, relative distortion of the elements in at least two dimensions makes this method unattractive.⁴⁷ In one-dimensional problems, however, this problem can be dealt with by splitting the distorted elements. The Lagrangian representation has been frequently used in solving one-dimensional fluid plasma problems.^{48,54-56}

The Eulerian representation differs from the Lagrangian representation in that the computational grid is now considered fixed with respect to the moving fluid. As a result, boundaries and interfaces are normally not well defined unless a very small mesh is chosen, and complicating convection terms must now be introduced. Problems with more than one spatial dimension, however, are more effectively solved using the Eulerian representation thus accounting for its popularity in solving such problems in plasma physics.^{57,58} A generalized computational scheme using a moving computational mesh was developed by Brackbill⁵⁹ which incorporates the advantages of both Eulerian and Lagrangian representations.

In this work, the Lagrangian prescription is used in developing the finite difference expressions for Equations (2), (3) and (4) primarily due to its computational simplicity. How the equations are differenced is dependent upon the particular machine being simulated. For low temperature machines such as Scylla I-C, the electron thermal conduction model of Morse⁴⁶ was used to predict thermal conduction times which we found to be approximately one order of magnitude greater than the particle confinement time.⁴² For such experiments, where thermal conduction is negligible, and explicit differencing scheme is suitable to describe the above set of equations. Richtmyer⁵³ performed a Fourier mode stability analysis of his first order explicit, isentropic one fluid Lagrangian difference equations without the area constraining radial pressure equation, and found that the computational time step is limited by

$$\Delta t \leq \frac{\Delta z}{C_s}$$

where Δz is the width of the fluid element in equation and C_s is the local fluid speed of sound. Including the radial pressure constraint equation, Hamada et al.⁵⁴ found the computational time step to be limited by

$$\Delta t \leq \frac{\Delta z}{U_c} \quad (7)$$

where U_c is the "cusp" speed defined by

$$U_c = C_s U_A / (C_s^2 + U_A^2)^{1/2} \quad (8)$$

with U_A equal to the plasma Alfvén speed defined by

$$U_A^2 = \frac{B_i^2}{2\rho\mu_0}$$

where B_i is the plasma magnetic field. Using Scylla I-C parameters in Equation (7), the maximum time step varied between 0.2 μsec to 0.3 μsec for a typical simulation. This is considered acceptable for numerical simulation of an experiment in which the driving magnetic field has a quarter cycle of 2 μsec and plasma properties were measured for a period of approximately 8 μsec .

For experiments where electron thermal conduction is important, an explicit formulation of the energy equations may result in a prohibitively small time step requirement for stability.⁴⁷ The Scylla IV-P experiment is such an example where the thermal conduction time is approximately 5 μsec , less than the 10 μsec particle confinement time.⁴² A simple Fourier mode analysis of an explicit numerical representation of a simplified form of the electron thermal condition equation indicates that the maximum time step will be on the order of

$$\Delta t \sim \frac{3}{4} \frac{n(\Delta z)^2}{K_e} .$$

For $T_e \sim 400$ eV, $n \sim 10^{16} \text{ cm}^{-3}$, typical of Scylla IV-P, the maximum time step is

$$\Delta t \sim 10^{-11} (\Delta z)^2 . \quad (9)$$

The magnetic field quarter cycle time is approximately 3 μsec ; thus, considering the confinement times noted above, time steps on the order of 0.2 μsec would be desirable. From Equation (9) it can be seen that for $\Delta t \sim 0.2 \mu\text{sec}$ then Δz must be approximately 140 cm, much too long for meaningful plasma spatial resolution since Scylla IV-P has a length of only 500 cm. Decreasing Δz to 10 cm, a reasonable value for spatial resolution, the maximum computational time step must be less than $10^{-3} \mu\text{sec}$ which would be prohibitively costly. The energy equation may be cast into an implicit differencing form where the plasma temperature distribution at each time step is determined by solving a system of algebraic equations. The momentum equation, however, is still solved explicitly as in the non heat conducting case. The difference equations for both heat conducting and non heat conducting cases and their solution are discussed in detail below. Before this is done, however, Equations (1), (2), (3), and (4) will be transformed to the Lagrangian representation from which the differencing mesh will be defined.

We define a Lagrangian coordinate system as in References 54 and 55 in which the initial element is tagged by a Lagrangian coordinate, x . This coordinate is treated as an independent variable in defining the element coordinate z , at time t , $z = z(x, t)$. From this the particle conservation equation, Equation (1), can be written as

$$n \Delta z = n_0 A_0 dx \quad (10)$$

where n_0 and A_0 represent the initial plasma number density and plasma column area, respectively. Equations (2), (3) and (4) can thus be transformed into Lagrangian coordinates using

$$\frac{\partial}{\partial z} = \frac{nA}{n_o A_o} \frac{\partial}{\partial x} \quad (11)$$

Using this transformation, the momentum equation, Equation (2), becomes

$$\frac{\partial U}{\partial t} = - \frac{A}{mn_o A_o} \frac{\partial}{\partial x} (nkT_i + nkT_e + q) \quad (12)$$

Similarly, applying this transformation to the electron and ion energy equations, Equations (3) and (4) become

$$\frac{\partial T_e}{\partial t} = (\gamma - 1) \frac{T_e}{n} \frac{\partial n}{\partial t} + \frac{1}{k} \frac{(\gamma - 1)}{(A_o n_o)} \frac{\partial}{\partial x} \left(\frac{K_e n A^2}{A_o n_o} \frac{\partial T_e}{\partial x} \right) + \frac{(T_i - T_e)}{\tau_{eq}} \quad (13)$$

$$\frac{\partial T_i}{\partial t} = \frac{(\gamma - 1)}{n} (T_i + \frac{q}{kn}) \frac{\partial n}{\partial t} + \frac{1}{k} \frac{(\gamma - 1)}{(A_o n_o)} \frac{\partial}{\partial x} \left(\frac{K_i n A^2}{A_o n_o} \frac{\partial T_i}{\partial x} \right) + \frac{T_e - T_i}{\tau_{eq}} \quad (14)$$

The radial pressure balance equation remains unchanged. It can be seen that the above coordinate transformation has the advantage that the z variable has been eliminated but at the expense of introducing variables x and $n_o A_o$. Although we have introduced these additional variables at the expense of eliminating z , it should be noted that they are not time dependent as is z and are at worst dependent only on their location in the Lagrangian mesh. Thus, computationally, the equations are easier to deal with in the Lagrangian space coordinate. Evolution of z as a function of time is obtained from

$$z(t + \Delta t) = z(t) + \int_t^{t+\Delta t} U dt \quad (15)$$

where U is determined from Equation (12). We adopted the space-time mesh system used by Hamada et al.⁵⁴ sketched in Figure 39. This difference scheme can be physically visualized by considering the space mesh points $i - \frac{1}{2}$, $i + \frac{1}{2}$, $i + \frac{3}{2}$, etc. to represent pressure surfaces for mass elements $i - 1$, i , $i + 1$ on which the surface forces act and across which heat will flow. The mass surfaces are specified by the user for the particular problem under consideration from which the pressure surface locations are then defined at the corresponding

Figure 39. Space-Time Computation Mesh

midpoints. An initialized space computation mesh for a typical 8 pinch simulation is presented in Figure 40. The finite difference equations may now be written.

The equation for axial momentum, Equation (12) is differenced explicitly to first order for both heat conducting cases as

$$u_i^{n+1/2} = u_i^{n-1/2} + \frac{A_i^n}{m(n_o A_o)_i} \left((nT_i)_i^{n+1/2} + (nT_e)_i^{n+1/2} - (nT_i)_i^{n-1/2} - (nT_e)_i^{n-1/2} \right) + \frac{q_{i+1/2}^{n-1/2} - q_{i-1/2}^{n-1/2}}{\Delta_{1/2}} \Delta t, \quad (16)$$

where

$$A_i^n = (A_{i+1/2}^n + A_{i-1/2}^n)/2$$

$$q_{i+1/2}^{n-1/2} = \frac{c}{2} m(n_{i+1/2}^n + n_{i+1/2}^{n-1}) (u_{i+1}^{n-1/2} - u_i^{n-1/2})^2$$

for

$$\frac{\partial u}{\partial z} < 0,$$

$$(n_o A_o)_i = ((n_o A_o)_{i+1/2} + (n_o A_o)_{i-1/2})/2,$$

$$q_{i-1/2}^{n-1/2} = \frac{c}{2} m(n_{i-1/2}^n - n_{i-1/2}^{n-1}) (u_i^{n-1/2} - u_{i-1}^{n-1/2})^2,$$

for

$$\frac{\partial u}{\partial z} < 0$$

$$q_{i+1/2}^{n-1/2} = q_{i-1/2}^{n-1/2} = 0,$$

for

$$\frac{\partial u}{\partial z} > 0$$

$$\Delta_i = x_{i+1/2} - x_{i-1/2},$$

and the zero subscript denotes initial values. As a result of the explicit differencing, all variables on the right hand side of Equation (16) are known

Example: The U, z surface $I \equiv i = 5$ is bounded by pressure surfaces $i - \frac{1}{2}$ and $i + \frac{1}{2}$ denoted as $I = 5$ and $I = 6$, respectively, on the n, T, A mesh.

Figure 40. Typical Theta Pinch Space Computation Mesh

from the previous time step. Therefore, to first order, the average U for the new time step, $n \rightarrow n+1$, is easily found. With this new velocity, the corresponding new z location can then be found from

$$z_i^{n+1} = z_i^n + U_i^{n+1/2} \Delta t \quad (17)$$

Particle conservation is written as

$$(z_{i+1}^{n+1} - z_i^{n+1})(nA)_{i+1/2}^{n+1} = (x_{i+1} - x_i)(n_o A_o)_{i+1/2} \quad (18)$$

and radial pressure balance, Equation (5) becomes

$$n_{i+1/2}^{n+1} k(T_e + T_i)_{i+1/2}^{n+1} + \frac{\phi^2}{2\mu_o (A_{i+1/2}^{n+1})^2} = \frac{B_e^2(z, t)}{2\mu_o} \quad (19)$$

For the case of negligible heat conduction, the energy equations are differenced in first order explicit form

$$(n^{(1-\gamma)} T_e)_{i+1/2}^{n+1} = (n^{(1-\gamma)} T_e)_{i+1/2}^n + (\gamma - 1)(n^{(1-\gamma)})_{i+1/2}^n (T_i - T_e)_{i+1/2}^n \frac{\Delta t}{\tau_{eq, i+1/2}^n} \quad (20)$$

and

$$\begin{aligned} (n^{(1-\gamma)} T_i)_{i+1/2}^{n+1} = & (n^{(1-\gamma)} T_i)_{i+1/2}^n + (\gamma - 1)(n^{(1-\gamma)})_{i+1/2}^n (T_{e, i+1/2}^n \\ & - T_{i, i+1/2}^n) \frac{\Delta t}{\tau_{eq, i+1/2}^n} + \frac{(\gamma - 1)}{(n_o A_o)_{i+1/2}} q_{i+1/2}^{n+1/2} \frac{A_{i+1/2}^n}{(n^{(\gamma-1)})_{i+1/2}^n} \\ & \times (u_{i+1}^{n+1} - u_i^{n+1}) \frac{\Delta t}{\Delta_{i+1/2}} \quad (21) \end{aligned}$$

where

$$\Delta_{i+1/2} = x_{i+1} - x_i$$

It should be noted that the particle conservation expression, Equation (1) and the approximation $\partial n / \partial t \approx (\partial n A / \partial t) (1/A)$ was used in reducing the pseudoviscosity term to the form appearing in Equation (21). This approximation is not considered severe for the pseudoviscosity term is in reality a numerical artifice.

Solution of the above set of equations proceeds as follows. First the velocity of a given element is advanced through a time step Δt from Equation (16) giving $U_i^{n+1/2}$. Next the new element position, z_1^{n+1} , is determined from Equation (17) and the quantities $(n^{(1-\gamma)} T_e)_{i+1/2}^{n+1}$ and $(n^{(1-\gamma)} T_i)_{i+1/2}^{n+1}$ are calculated from Equations (20) and (21). With these results, Equations (18) and (19) can be combined to eliminate the plasma area yielding the following algebraic equation for $n_{i+1/2}^{n+1}$

$$\left[\frac{\phi^2}{2\mu_0} \frac{1}{(n_o A_o)_{i+1/2}^2} \left(\frac{z_{i+1}^{n+1} - z_i^n}{\Delta_{i+1/2}} \right)^2 \right] (n_{i+1/2}^{n+1})^2 + [(n^{(1-\gamma)} T_e)_{i+1/2}^{n+1} + (n^{(1-\gamma)} T_i)_{i+1/2}^{n+1}] (n_{i+1/2}^{n+1})^\gamma - \frac{B_e^2(z, t)}{2\mu_0} = 0 \quad (22)$$

which is numerically solved to find the root $n_{i+1/2}^{n+1}$. With $n_{i+1/2}^{n+1}$ known, then T_{i+1}^{n+1} and $T_{e,i+1/2}^{n+1}$ can be found since $n^{(1-\gamma)} T$ is known for the ions and electrons, and $A_{i+1/2}^{n+1}$ can be determined from Equation (18). This procedure is carried out for all mesh points.

The numerical stability condition used for this set of equations was dictated by the minimum time increment obtained by comparing the minimum time increment predicted by Hamada et al.⁵¹ for a one fluid plasma and the result obtained from a Fourier mode analysis of the two temperature energy equations where only the electron-ion equilibration term is retained on the right hand side. From references 53 and 54, the applicable stability condition is dependent upon whether or not the mesh point is in a region of significant pseudoviscosity. If pseudoviscosity is negligible, then the time step is limited by⁵⁴

$$\Delta t \leq \frac{\Delta z}{U_c} \quad (23)$$

where U_c is the cusp speed defined in Equation (8). If pseudoviscosity is important, then the stability condition is⁵⁴

$$\Delta t \leq \frac{\sqrt{\gamma}}{2c} \frac{\Delta z}{U_c} \quad (24)$$

If $\sqrt{\gamma}/2c > 1$ than Equation (23) would be the relevant stability condition. For simulations with $T_1 \neq T_e$, the Fourier analysis of the limited two fluid equations yields the stability condition

$$\Delta t \leq \frac{\tau_{eq}}{\gamma - 1} \quad (25)$$

A general statement cannot be made as to which of the above criteria is most severe; rather, each condition is checked at every mesh point to get the minimum value for Δt .

For simulations where thermal conduction is important, the energy equations are differenced implicitly to avoid inherently small time steps required in an explicit formulation of the thermal conduction diffusion terms. As can be seen from Equations (13) and (14), the thermal conduction terms are highly non linear for $K \sim T^{5/2}$, thus an iterative solution of the implicit equations is desirable and often necessary. This non linearity is treated by the unconditionally stable Crank-Nicholson implicit method.⁴⁷ In this method K is successively iterated during each time step until the T obtained from solving the energy equation approaches the value obtained from the previous iteration,

$$\lim_{p \rightarrow \infty} T^{p+1} = T^p$$

where p denotes the p^{th} iteration. The simple diffusion equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right)$$

is represented using the Crank-Nicholson method by

$$T^{p+1} - T^n = \frac{\partial}{\partial x} \left[\frac{\bar{K}}{2} \frac{\partial}{\partial x} (T^{p+1}) \right] \Delta t + \frac{\partial}{\partial x} \left[\frac{\bar{K}}{2} \frac{\partial}{\partial x} (T^n) \right] \Delta t$$

where

$$\bar{K} = \frac{1}{2} K(T^p) + \frac{1}{2} K(T^n)$$

with T^n representing the temperature at the previous time step. Using this method, the electron energy equation, Equation (13), can be written as

$$\begin{aligned} T_{e,i+1/2}^{p+1} = T_{e,i+1/2}^n &+ \frac{\gamma - 1}{2} \left[\left(\frac{T_e}{n} \right)_{i+1/2}^n + \left(\frac{T_e}{n} \right)_{i+1/2}^p \right] (n_{i+1/2}^p - n_{i+1/2}^n) \\ &+ \frac{(\gamma - 1)}{(n_o A_o)_{i+1/2}^2} \frac{1}{k} \frac{\Delta t}{\Delta_{i+1/2}} \left[\frac{1}{2} \left[(K^n A^2)_i^p + (K^n A^2)_{i+1}^n \right] \right. \\ &\times \frac{T_{e,i+3/2}^{p+1} - T_{e,i+1/2}^{p+1}}{\Delta_{i+1}} - \frac{1}{2} [(K^n A^2)_i^p + (K^n A^2)_i^n] \\ &\times \frac{(T_{e,i+1/2}^n - T_{e,i-1/2}^n)}{\Delta_i} + \frac{1}{2} [(K^n A^2)_i^p + (K^n A^2)_i^n] \\ &\times \frac{(T_{e,i+3/2}^n - T_{e,i+1/2}^n)}{\Delta_{i+1}} - [(K^n A^2)_i^p + (K^n A^2)_i^n] \\ &\times \frac{(T_{e,i+1/2}^n - T_{e,i-1/2}^n)}{\Delta_i} \left. \right] - \frac{(T_{e,i+1/2}^{p+1} + T_{e,i+1/2}^n)}{2\tau_{eq}^{np}} \Delta t \\ &+ \frac{(T_{i,i+1/2}^p + T_{i,i+1/2}^n)}{2\tau_{eq}^{np}} \Delta t \end{aligned} \quad (26)$$

where

$$\Delta_i = x_{i+1/2} - x_{i-1/2}$$

$$\Delta_{i+1} = x_{i+3/2} - x_{i+1/2}$$

$$\Delta_{i+1/2} = \frac{1}{2} (\Delta_{i+1} + \Delta_i)$$

and $\tau_{eq,i+1/2}^{np}$ indicates that τ_{eq} is calculated with $n = (n^n + n^p)/2$ and $T_e = (T_e^n + T_e^p)/2$. Note that the coefficient of the second term on the right hand side and the equilibrium term were written in an attempt to time center the energy equation. Defining

$$\bar{K}_{i+1}^e = \frac{1}{2}[(K^n A^2)_{i+1}^p + (K^n A^2)_{i+1}^n](\gamma - 1)\Delta t / [k\Delta_{i+1/2}(n_{oo}^A)_{i+1/2}^2],$$

$$\bar{K}_i^e = \frac{1}{2}[(K^n A^2)_i^p + (K^n A^2)_i^n](\gamma - 1)\Delta t / [k\Delta_{i+1/2}(n_{oo}^A)_{i+1/2}^2],$$

$$\bar{n}_{i+1/2}^{emp} = (\gamma - 1) \left[\left(\frac{T_e}{n} \right)_{i+1/2}^n + \left(\frac{T_e}{n} \right)_{i+1/2}^p \right] (n_{i+1/2}^p - n_{i+1/2}^n) / 2,$$

and

$$\Pi = (T_{i,i+1/2}^p + T_{i,i+1/2}^n - T_{e,i+1/2}^n),$$

Equation (26) becomes

$$\begin{aligned} T_{e,i+1/2}^{p+1} = & T_{e,i+1/2}^n + \bar{n}_{i+1/2}^{emp} + \frac{\bar{K}_{i+1}^e}{2\Delta_{i+1}} T_{e,i+3/2}^{p+1} - \frac{\bar{K}_{i+1}^e}{2\Delta_{i+1}} T_{e,i+1/2}^{p+1} \\ & - \frac{\bar{K}_i^e}{2\Delta_i} T_{e,i+1/2}^{p+1} + \frac{\bar{K}_i^e}{2\Delta_i} T_{e,i-1/2}^{p+1} + \Pi \frac{\Delta t}{2\tau_{eq,i+1/2}^{np}} \\ & - \frac{T_{e,i+1/2}^{p+1} \Delta t}{2\tau_{eq,i+1/2}^{np}} + \frac{\bar{K}_{i+1}^e}{2\Delta_{i+1}} (T_{e,i+3/2}^n - T_{e,i+1/2}^n) \\ & - \frac{\bar{K}_i^e}{2\Delta_i} (T_{e,i+1/2}^n - T_{e,i-1/2}^n) \end{aligned}$$

which can be rewritten as

$$A^e T_{e,i+3/2}^{p+1} + B^e T_{e,i+1/2}^{p+1} + C^e T_{e,i-1/2}^{p+1} = D^e \quad (27)$$

where we have defined

$$A^e = \bar{K}_{i+1}^e / 2\Delta_{i+1},$$

$$C^e = \bar{K}_1^e / 2\Delta_1 ,$$

$$B^e = -(1 + A^e + C^e + \Delta t / 2\tau_{eq_{i+1/2}}^{np}) ,$$

and

$$D^e = C^e (T_{e,i+1/2}^n - T_{e,i-1/2}^n) - (T_{e,i+1/2}^n + \bar{n}_{i+1/2}^{emp} + \frac{T_1 \Delta t}{2\tau_{eq_{i+1/2}}^{np}} + A^e (T_{e,i+3/2}^n - T_{e,i+1/2}^n))$$

which are known from plasma properties at the previous time step, n , or previous iteration of the latest time step, p . A similar expression can be written for the ions,

$$A_{i,i+3/2}^{i,p+1} + B_{i,i+1/2}^{i,p+1} + C_{i,i-1/2}^{i,p+1} = D^i , \quad (28)$$

where A^i , C^i , B^i and D^i have the same functional relationships as A^e , C^e , B^e and D^e , respectively, with the exception that T_e is replaced by T_i and D^i now contains the pseudoviscosity dissipation term in $\bar{n}_{i+1/2}^{-inp}$,

$$\bar{n}_{i+1/2}^{-inp} \equiv (\gamma - 1) \left[\left(\frac{T_1}{n} \right)_{i+1/2}^n + \left(\frac{T_1}{n} \right)_{i+1/2}^p + \frac{(\gamma - 1)}{(n^2)_{i+1/2}^n} q_{i+1/2}^n \right] (n_{i+1/2}^p - n_{i+1/2}^n) .$$

Note that no attempt was made to center the pseudoviscosity term coefficient because of the arbitrariness in its definition. Again A^i , C^i , B^i and D^i can all be calculated from plasma properties at the previous time step, n , or previous iteration of the latest time step, p .

Examination of Equations (27) and (28) reveals that each differential equation has been reduced to a system of algebraic equations in temperature which can be solved for the $p+1$ iteration on the temperature distribution. Each system of equations can be reduced to a matrix form

$$[M^S][T^S] = [D^S]$$

where $[M^S]$ is a tridiagonal matrix determined from the A, B and C coefficients defined above, $[T^S]$ is the temperature column matrix for temperature species S, and $[D^S]$ is the D^i or D^e column matrix. The dimension of the M matrix is $(NMAX + 1) \times (NMAX + 1)$ where NMAX is the defined number of mass elements. From symmetry, the first and second pressure faces have equal temperatures thus defining the first algebraic equation; and the heat conduction boundary condition placed at the plasma column end defines the $NMAX + 1$ equation. From the boundary conditions stated in the previous section, this equation is $T_e(i = NMAX) = T_e(i = NMAX + 1)$ for the electrons and $T_i(i = NMAX) = T_i(i = NMAX + 1)$ for the ions. The tridiagonal M matrix is inverted from which the temperature distributions are determined. Using the new element locations determined from Equations (16) and (17) with nth time step plasma properties, the plasma number density distribution, n^{p+1} , is determined by solving a simple quadratic equation obtained by combining Equations (18) and (19) to eliminate plasma area,

$$\begin{aligned} (n_{i+1/2}^{p+1})^2 + [k(T_{i,i+1/2}^{p+1} + T_{e,i+1/2}^{p+1})2\mu_0 \frac{(n_o A_o)_{i+1/2}^2}{\phi^2} (\frac{\Delta_{i+1/2}}{z_{i+1}^{n+1} - z_i^{n+1}})^2] n_{i+1/2}^{p+1} \\ - \frac{(n_o A_o)_{i+1/2}^2}{\phi^2} (\frac{\Delta_{i+1/2}}{z_{i+1}^{n+1} - z_i^{n+1}})^2 B_e^2(z, t) = 0 \end{aligned}$$

Using $n_{i+1/2}^{p+1}$ obtained above, the plasma area $A_{i+1/2}^{p+1}$ is then found from Equation (18). Convergence of the $p+1$ iteration is checked by making the test

$$\lim_{p \rightarrow \infty} \frac{F^{p+1}}{F^p} \rightarrow 1$$

for n , A , T_e and T_i . When the limit of the ratio is equal to or less than 1.005 for all of the above variables, the $p+1$ iteration is taken to be the solution. Convergence has been obtained in only 3 iterations for preliminary checkout runs conducted to date.

Although the stability limitation for an explicit formulation of the heat conduction terms has been removed as suggested in Reference 47 by adopting implicit Crank-Nicholson time differencing of these terms, the time step is still probably limited by the remaining energy terms and the remaining equations. As a result, the stability limitation on time is considered to remain limited by Equations (23), (24), and (25).

As noted earlier in this section, the accuracy of the Lagrangian representation is limited by distortion of the fluid elements as the flow evolves in time. This is particularly true in the study of end loss in a theta pinch where the plasma expands rapidly in a magnetic field which has sharp spatial gradients external to the compression coil. An element splitting procedure has been developed to alleviate this problem but preliminary results indicate that the success of this scheme is limited to splitting in regions of plasma flow where plasma properties do not change sharply over a typical element width. The splitting procedure developed here is outlined below.

A simple criterion is used to determine if a given difference element is to be split. If the distance between two adjacent z surfaces exceeds some predetermined value, say as an example twice the initial separation, then that element will be split. The manner in which the element is split is determined by where the element lies in the mesh. If the elements split involve either the $I = I$ or $I = NMAX$ surfaces (refer to Figure 40) then only two then only two elements are affected: all other elements split will affect both neighboring elements. This splitting procedure is depicted in Figure 41. All newly introduced z surfaces are inserted in the mesh at points bisecting the adjacent pressure surfaces in an attempt to center the z surfaces (mass surfaces) with respect to the pressure surfaces which ultimately determine the elements momentum. In all splitting cases presented in Figure 41, plasma properties n , T_i , T_e and A

Figure 41. Splitting Geometry

determined after split are related to those prior to split by requiring conservation of particles, conservation of internal energy for each species and conservation of magnetic flux over the affected elements during the splitting process,

$$\sum_{\text{before}}^{\text{after}} (nA)_{i+1/2} (z_{i+1} - z_i) = 0 ,$$

$$\sum_{\text{before}}^{\text{after}} (nAT_e)_{i+1/2} = 0 ,$$

$$\sum_{\text{before}}^{\text{after}} (nAT_i)_{i+1/2} = 0 ,$$

$$\phi = \text{constant} ,$$

where radial pressure balance was invoked to satisfy the ϕ constraint. These algebraic constraint equations are easily solved to obtain the desired plasma properties on the newly defined pressure surfaces. The old pressure surfaces defined inside regions which are split remain tagged with their pre split values. New element velocities are determined by a linear interpolation procedure based on their z location, thus momentum and kinetic energy are not necessarily conserved over the mesh during the split. Conservation of these quantities will be implemented during the next reporting period.

The current developmental status of the computer codes discussed in this section is as follows. The numerical formulation of both the non heat conducting and heat conducting difference equations has been coded on the University's IBM 370 computer system. The version of these programs in which pseudoviscosity terms are neglected have been checked out. Although a version of the non heat conducting code includes these terms, checkout remains to be completed. A preliminary simulation of the Scylla I-C experiment has been completed and will be discussed in the following section.

d. Scylla I-C simulation

The Scylla I-C consisted of a 1 meter long compression coil which enveloped a 3.6 cm i.d. quartz vacuum vessel. An initial fill pressure of 100 mTorr deuterium was chosen for this experiment so that the resulting plasma column would be fully collisional, $\lambda_{ii}/D \ll 1$ and $\lambda_{ii}/L \ll 1$. As a result, the one dimensional fluid plasma code developed here is particularly well suited for predicting measured plasma properties. As was noted in the previous section, the characteristic particle end loss time, twice as long as the time over which good data could be taken, was much larger than the characteristic electron thermal conduction time, thus thermal conduction effects were negligible during this experiment. As a result, the non heat conducting fully explicit version of the one dimensional computer code was used for simulating this experiment.

Computer simulations of Scylla I-C conducted earlier using our zero dimensional computer codes yielded good agreement with integrated on-axis number density and midplane plasma temperature results. However, using our one-dimensional code, specification of a phenomenological end loss scaling law is not necessary for the total particle inventory inside the compression coil can now be predicted as a function of time. Examination of Figure 42 indicates that the total electron inventory predicted agrees quite closely with the experimental results obtained from end-on laser interferometry. Simulation results fall nearly on top of the 14.4 μ sec confinement time least squares fit curve of the data.⁴¹

Additional Scylla I-C experimental data and computer simulation results are presented in Figure 43. Data presented include integrated on-axis number density, n_a , also obtained from interferometry; plasma temperature, $T_e = T_i = T_p/2$, obtained from n_a , midplane diamagnetic loop measurements and radial pressure balance; and the applied magnetic field waveform. Computer simulation

Figure 42. Time History of Scylla I-C Total Plasma Column Electron Inventory

137.

Figure 43. Time History of Scylla I-C O_n -Axis Electron Density, Single Component Temperature and Applied Magnetic Field.

results closely follow the magnetic field shape and generally compare well with the experimental results. The post implosion initial conditions for T_e and n_a were the same for both the one-dimensional and the zero-dimensional simulations. In the one-dimensional simulation, however, the axial distribution in number density was adjusted to satisfy radial pressure balance at every z location, thus $n(z)$ decreased monotonically with z as the coil edge was approached. Magnetic flux, plasma temperature and plasma column area were taken to be uniform at the initial condition. The one-dimensional and zero-dimensional predicted n_a histories of $\eta = 0$ exhibit excellent agreement, with both predicting higher n_a than measured experimentally. Comparing these results with the zero-dimensional predictions⁴² for $\eta \geq \eta_1$ suggests that $\eta \sim \eta_1$ in the Scylla I-C experiment. Finite electrical resistivity effects will be incorporated in both one-dimensional models for the next reporting period.

Plasma electron temperatures predicted by the one-dimensional model gives best overall agreement with the electron temperatures inferred from experimental data. Plasma temperatures predicted with the one-dimensional model are approximately 10 percent higher than predicted using the zero-dimensional code for this experiment. This is not surprising, however, for the effect of convective energy loss is uniformly distributed over the length of the plasma column in the zero-dimensional model, whereas, the midplane plasma sensed by the diamagnetic loop is compressed and expands essentially isentropically over the effective duration of the Scylla I-C experiment. This is modeled properly by the one-dimensional computer codes, thus the value of a one-dimensional code in predicting z dependent plasma behavior is apparent.

The behavior of $n(z,t)$ and $T(z,t)$ have not been evaluated but will be discussed in the next reporting period.

e. Future work

During the next several months, the following topics will be examined:

1. Simulation of the PSU 25 cm and 50 cm pinches with the one-dimensional code.
2. Comparison of the 25 cm simulations with the experimental results.
3. Simulation of Scylla IV-P, where electron thermal conduction is important.
4. Study the influence of boundary conditions in the end region on plasma flow.
5. Comparison of predictions of zero- and one-dimensional modeling.

VIII. IMPLOSION MODELING STUDIES (Stover, Jacoby, Awwad)

1. Introduction

It is desirable to simulate theta pinch dynamics beginning with capacitor bank discharge; consequently consideration has been given to the problem of implosion dynamics. In this aspect of the study, simplified models for implosion dynamics were used to generate plasma initial conditions for the zero-dimensional post-implosion computer code.⁴² The appropriate implosion model for a given experiment was identified by comparing predicted plasma parameters with experimental data. For the experiments considered, the earliest available data were taken approximately 2 μ sec following discharge initiation and, as implosion times are typically on the order of hundreds of nanoseconds, the zero-dimensional code was used to span this gap in time. Implosion dynamics for Scylla I-C⁴¹ were modeled by a simple snowplow implosion model. As discussed in the next section, it was found to be necessary, to introduce finite beta corrections in the snowplow analysis in order to achieve agreement between predicted snowplow values of n and T and required starting values for the post-implosion computer model.

In fact, two different snowplow models have been developed. In the first model, the snowplow equation does not include the trapped magnetic field but the energy and momentum equations at the end of implosion do. This enables us to use the approximate analytic solution of Artsimovich.⁶⁰ In addition, it is assumed that the post-implosion equilibrium consists of a uniform cylinder of plasma. This model is applied to the Scylla I-C experiment.⁴¹ A second snowplow model was developed for the PSU 25 cm pinch. The trapped magnetic field was included in the implosion equation which was solved on the computer. The reverse trapped field led to a plasma annulus at the end of implosion rather

than a uniform plasma cylinder. Also, the fill gas was He with a preionization of approximately 20% for first-electron ionization. Consequently, a substantial energy input goes into ionization. All of these effects are included in the second model. Good agreement with experimental measurements has been achieved.

The implosion phase of Scylla IV-P,⁴⁴ Linear Scylla³⁵ and Scylla IV-3⁴⁵ were also studied. Because of the low fill pressure the ion mean free path is long compared with the pinch tube radius so that a simple bounce model⁶¹ was employed initially. Several versions of the bounce model are presented in the third section, including an exact solution, an average-bounce-model solution,⁶¹ and a solution with oscillation energy. The post-implosion values of n_p and T_p predicted by these solutions is found to be in poor agreement with the values required for the numerical simulation. As a consequence, modifications to the bounce model are being investigated. A multiple-bounce model is discussed in the fourth section, and a hybrid snowplow bounce model⁶² is introduced in the fifth section.

2. Snowplow Modeling

a. Model I: snowplow equation without trapped field

An excellent discussion of the snowplow model is found in Artsimovich⁶⁰. The snowplow momentum equation for a linearly rising B field is

$$mn_0 \frac{d}{dt}[(b^2 - R^2)\frac{dR}{dt}] = - \frac{R}{\mu_0} \left(\frac{dB}{dt}\right)_{t=0}^2 t^2, \quad (1)$$

where b is the discharge radius, R is the sheath radius, m is the ion mass, and n_0 is the fill density. The sheath is presumed to be quite thin, and it is assumed that no trapped field is present. The implosion energy resides in both radially directed kinetic energy and in plasma thermal energy. This can be seen by multiplying Equation (1) by dR/dt and integrating over the discharge

chamber volume from sheath radius "a" to tube radius b. The left hand side of Equation (1) becomes

$$\frac{m_a}{2} \left(\frac{dR}{dt} \right)_a^2 + \frac{1}{2} \int_0^m a \left(\frac{dR}{dt} \right)^2 dm, \quad (2)$$

where m_a = mass per unit length

$$= \pi m_0 (b^2 - a^2),$$

and $dm = -\pi m_0 2R dR = -\pi m_0 2R \frac{dR}{dt} dt$. The first term in Equation (2) is E_{KE} , the radially directed kinetic energy, while the second term is E_{Th} , the thermal energy in the sheath, which is sometimes referred to as the sticking energy.

If dR/dt were set equal to a constant, then the sticking energy would equal the kinetic energy. However, if we use the approximate solution to Equation (1) given by Artisimovich⁶⁰

$$x(t) \equiv \frac{R(t)}{b} = \left(1 - \frac{\tau^2}{\sqrt{12}} \right), \quad (3)$$

where $\tau = t/t_0$ and $t_0 = (\nu_0 b^2 m_0)^{1/4} / (dR/dt)^{1/2}$,

then

$$\frac{E_{Th}}{E_{KE}} = \frac{[1 - (2\tau^2/3\sqrt{12})]}{2[1 - (1/2\sqrt{12})\tau^2]} \equiv \alpha. \quad (4)$$

For collapse to the axis (where $R = 0$), $\tau^2 = \sqrt{12}$ and $\alpha = 0.33$. The maximum value of α is 0.5.

At the end of the implosion phase, equilibration values for density, temperature and radius of the plasma are obtained using energy, pressure and particle balance. In this analysis we allow an internal field to be present. However, we do not include an internal field in the description of the snowplow implosion. Either the internal field is present at the beginning of implosion or it diffuses through the sheath during implosion. Letting R_T and R_p be the radii at the end of sheath implosion and after equilibration, B_i and B_e be the

internal and external magnetic fields, and T_p be the plasma temperature (electron plus ion), then the necessary equations are

$$E_{\text{imp}}(R_T) + \frac{B_i^2(R_T)}{2\mu_0} L\pi R_T^2 + \frac{B_e^2(R_T)}{2\mu_0} L\pi R_T^2 = \pi b^2 L n_0 \frac{3}{2} T_p + \frac{B_i^2(R_p)}{2\mu_0} L\pi R_p^2 + \frac{B_e^2(R_p)}{2\mu_0} L\pi R_p^2, \quad (5)$$

$$n_p T_p = \beta \frac{B_e^2}{2\mu_0}, \quad (6)$$

$$R_p^2 n_p = b^2 n_0, \quad (7)$$

where E_{imp} is the plasma implosion energy and is given by

$$E_{\text{imp}} = E_{\text{KE}} + E_{\text{Th}} = (1 + \alpha) E_{\text{KE}}, \quad (8)$$

and plasma magnetic flux is assumed conserved during equilibration.

In the thin sheath model, if $\beta = 1$ the sheath is allowed to collapse to the origin, i.e. $R_T = 0$. (Alternative choices are possible; e.g. Hoekzema et al.⁶³ set $R_T = R_p$.) We assume that when $\beta < 1$, the implosion is stopped when $B_e(R_T) = B_i(R_T)$. This is based on the argument that the sheath stops when the retarding pressure of the internal field equals the driving pressure of the external field. One other assumption is introduced; we assume $B_e(R_T) = B_e(R_p)$ which ignores any increase in B_e during the expansion from R_T to R_p . From radial pressure balance and flux conservation $B_i(R_p) = \sqrt{1 - \beta} B_e(R_p)$, so we obtain

$$R_T^2/R_p^2 = \sqrt{1 - \beta}$$

which reduces to zero when $\beta = 1$.

With the above assumptions, Equation (5) can be reduced to the form

$$\left(\frac{1 + \alpha}{2}\right) m_1 \left(\frac{dR}{dt}\right)^2 n_0 \sqrt{1 - \beta} \frac{b^2}{R_T^2} \left(1 - \frac{R_T^2}{b^2}\right) = \left[1 + \frac{\beta}{4} - \sqrt{1 - \beta}\right] \frac{B_e^2(R_T)}{\mu_0}. \quad (9)$$

During implosion phase, B_e is considered to rise linearly and can be approximated by

$$B_e^2(R_T) = B_o^2 \left(\frac{\pi}{2t_{1/4}} \right)^2 t_o^2 \tau_T^2 \quad (10)$$

where $t_{1/4}$ is the quarter cycle time, B_o is the maximum B field and τ_T^2 is obtained from Equation (3) to be

$$\tau_T^2 = \sqrt{12} \left(1 - \frac{R_T}{b} \right) = \sqrt{12} (1 - x_T) \quad (11)$$

From Equation (3) we also obtain

$$(dR/dt)_{R_T}^2 = (b^2/t_o^2) (\tau_T^2/3) \quad (12)$$

so that Equation (9) reduces to

$$(1 - x_T^2) = G x_T^2 \quad (13)$$

where, using the definition of t_o

$$G = \frac{6}{1 + \alpha} \frac{(1 + \beta/4 - \sqrt{1 - \beta})}{\sqrt{1 - \beta}} \quad (14)$$

This enables us to solve for x_T

$$x_T = \frac{1}{\sqrt{1 + G}} \quad (15)$$

From Equations (6), (7), (9), (10), (11) and (15),

$$T_p = K_1 \frac{\beta}{\sqrt{1 - \beta}} \frac{[1 - (1/\sqrt{1 + G})]}{1 + G}, \quad (16)$$

and

$$n_p = n_o \sqrt{1 - \beta(1 + G)}, \quad (17)$$

where

$$K_1 = \frac{\sqrt{3}}{4} \frac{B_o^2}{\mu_o} \frac{\pi^2 t_o^2}{n_o^2 t_{1/4}^2} \quad (18)$$

This model has been used to obtain post-implosion values for the Scylla I-C experiment⁴¹ which has the following parameters: $B_0 = 33$ kG; $t_{1/4} = 2$ μ sec; $b = 1.8$ cm; and $p_0 = 100$ mTorr deuterium. With these parameters, and with the choice $\alpha = 1$, we obtain the results shown in Figures 44 and 45. For $\beta = 0.7$, we obtain $T_p = 47$ eV and $n_p = 1.73 \times 10^{22} \text{ m}^{-3}$. These values enabled accurate simulation of the experimentally reported plasma parameters at maximum magnetic field (2 μ sec) with the zero-dimensional code. The total plasma temperature, T_p , was assumed to be equipartitioned between the ions and the electrons since the electron-ion equilibration time was only 0.2 μ sec.

The use of $\alpha = 1$ yields significantly better post-implosion values than would $\alpha \leq 0.5$, as required by Equation (4). This suggests that the straightforward application of the snowplow model described above is not satisfactory and additional thermal energy input is required. This additional energy may be associated with viscous heating accompanying column oscillation during equilibration. Calculations on this topic are planned for the near future.

The model of Hoekzema et al.⁶³ has also been considered, but it yields $n_p = 2.89 \times 10^{22} \text{ m}^{-3}$ and $T_p = 37.5$ eV, neither of which is satisfactory.

b. Model II: snowplow equation with trapped field

In order to model the implosion process in the 25 cm theta pinch, some method of accounting for the trapped field was deemed essential. To do this in a simple yet reasonably accurate way the basic snowplow mechanical equation with a term added to account for the trapped field was used in conjunction with energy, pressure, and particle balance equations. The result was quite satisfactory in its agreement with experimental results while requiring a minimum of assumptions.

The model itself consists of sharp boundaries between external driving field, plasma sheath, and internal trapped field. At the end of implosion

Figure 44. Post Implosion Density vs. β for the Parameters $B_0 = 33$ kG;
 $t_{1/4} = 2$ μsec ; $b = 1.8$ cm and $p_0 = 100$ mT Deuterium.

Figure 45. Post Implosion Temperature vs. β for the Parameters $B_0 = 33$ kG; $\tau_{1/4} = 2$ μ sec; $b = 1.8$ cm and $p_0 = 100$ mT deuterium.

the sheath is presumed to be stopped by the trapped field. Also, to determine values of the trapped field throughout the implosion process, conservation of flux is used, i.e., no trapped field diffusion occurs. Thus, the basic snow-plow equation becomes

$$\frac{d}{dt}[\pi m_1 n_0 (r_o^2 - r^2) \frac{dr}{dt}] = \frac{B_e^2 \sin^2 \omega t}{8\pi} 2\pi r - \frac{B_1^2}{8\pi} \left(\frac{r_o}{r}\right)^4 2\pi r, \quad (19)$$

where m_1 is the ion mass, n_0 is the fill density, r_o is the internal discharge tube radius, B_e is the peak driving field, ω is the angular frequency of the external field, and B_1 is the trapped reverse field. This equation is solved numerically using an explicit two-step difference technique.⁴⁷ From the numerical solution of Equation (19), a maximum sheath velocity, $v_{\max}$, pinch time, t_p , and internal field collapse radius, r_c , are determined.

With these values and the following equations:

$$n_0 (r_o^2 - r_c^2) = n_p (r_p^2 - r_s^2), \quad (20)$$

$$n_p k T_p = \beta \frac{B_e^2 \sin^2 \omega t_p}{8\pi} p, \quad (21)$$

$$n_0 m_1 \pi (r_o^2 - r_c^2) v_{\max}^2 = \frac{3}{2} n_0 (r_o^2 - r_c^2) \pi k T_p + \frac{B_e^2}{8\pi} \sin^2 \omega t_p \pi (r_o^2 - r_c^2) - \frac{B_1^2}{8\pi} r_o^4 \int_{r_c}^{r_o} \frac{2\pi r dr}{r^4} + (1 - f_1) E n_0 \pi (r_o^2 - r_c^2), \quad (22)$$

$$B_{1n} \left(\frac{r_o}{r_s}\right)^2 = B_e \sin \omega t_p, \quad (23)$$

the temperature, T_p , equilibrium internal field radius, r_s , plasma ion density, n_p , and plasma radius, r_p , can be calculated. Equation (20) represents particle conservation. Equation (21) is simply pressure balance. Equation (22) represents the energy balance of the plasma. On the left, the thermal energy is

assumed equal to the kinetic energy. The energy used in ionizing the gas as it implodes is included in the last term on the right. Here f_1 is the fraction of the fill gas that is preionized with E_1 representing the ionization energy of an individual atom. Lastly, Equation (23) represents the equalization of internal and external field. This arises out of the sharp boundary and no field diffusion assumptions since then the internal field must also support the pressure exerted by the external field. Finally, in order to arrive at a particle temperature, the quantity T_p is divided by the number of components an atom ionizes into. This of course presume thermalization of energy between all particles.

The results of the program compared to those same quantities that were experimentally measured are shown below in tabular form.

<u>Quantity</u>	<u>Program</u>	<u>Measured</u>
Collapse Time	1.23 μsec	1.2 μsec
Collapse Radius of Trapped Field	0.257 cm	0.20 cm
Equilibrium Radius of Trapped Field	0.744 cm	0.750 cm
Electron Temperature	13.3 eV	11.0 eV
Ion Density	$1.67 \times 10^{16} \text{ cm}^{-3}$	$1.8 \times 10^{16} \text{ cm}^{-3}$
Plasma Outer Radius	2.21 cm	2.1 cm

The measured plasma outer radius quoted is the r_{eff} value arrived at through a determination of the radius in which 90% of the plasma is contained. The computed values were arrived at using a 20% preionization factor for the outer electron of helium and a 0% preionization factor for the inner electron. This value was determined from Thomson scattering results from the preionized plasma outside the coil. Here a 10% factor for ionization was measured. Consequently, a 20% factor inside the coil was deemed to be reasonable. Also, the 200 G initial trapped field value used in the program was taken from

measurements. Lastly, a β value of 0.5 was used in the program. This value was chosen since the magnitude of field goes from reversed trapped through zero to forward external field when the plasma sheath is traversed. An approximate averaging of this type of profile yields a beta value of 0.5.

As can be seen from the table the overall agreement is quite good. The assumptions required to arrive at these results consisted of those normally made in the mechanistic snowplow model plus the one additional assumption of 0.5 value for beta.

3. Bounce Model Calculations

A simple implosion model, presumed to apply when there are no internal fields, and when the ion-ion collision mean-free-path is long compared to the discharge tube radius, is the bounce model of Freidberg et al.⁶¹. Their analysis was presented for a constant implosion field B_s , since their calculations were directed toward cases where the rise time of the implosion field was short compared to the implosion time. When comparison of the model was made with Los Alamos experiments, where the implosion field is increasing linearly with time, they simply adopted average values for the implosion field and the implosion velocity.

In this section we first examine an exact solution of the implosion equation. When the solution is combined with energy, particle and pressure balance equations, predicted post implosion values for n_p , T_p and r_p result. Calculations for Scylla IV-P⁴⁴ are presented. These values are then compared to the values computed from the average bounce model.⁶¹ They are found to be quite different. We then introduce a revised bounce model to account for the difference. Finally, we note that none of the predicted values for n_p and T_p give good agreement with Scylla IV-P results.

a. Exact solution, no oscillations

Assume particles overrun by the imploding sheath are reflected at twice the sheath velocity. Then the momentum balance equation can be written

$$2m_i n_o \left(\frac{dR}{dt}\right)^2 = \frac{1}{2\mu_o} \left(\frac{dB}{dt}\right)^2 t^2, \quad (24)$$

from which we obtain

$$\frac{dR}{dt} = -\frac{b}{2t_1} t \equiv -\frac{b}{t_1} \tau = v_s(t) \quad (25)$$

and

$$x(\tau) = \frac{R(t)}{b} = 1 - \frac{\tau^2}{2}, \quad (26)$$

where

$$t_1 = (4\mu_o b^2 m_i n_o)^{1/4} / \left(\frac{dB}{dt}\right)^{1/2} = \sqrt{2} t_o. \quad (27)$$

Imploding ions pass through the center and hit the sheath at R_T , at which point they are assumed to thermalize. To compute the value of R_T we note that ions reflected at $R(t)$ with velocity $2v_s(t)$ reach the imploding sheath on the opposite side in time

$$\Delta t = \frac{2R(t) - \Delta R}{2|v_s|} \quad (28)$$

where $\Delta R = R(t) - R(t + \Delta t)$. Using Equations (25), (26) and (28) we obtain

$$(\Delta \tau)^2 + 6\tau(\Delta \tau) - 4(1 - \tau^2/2) = 0 \quad (29)$$

where $\Delta \tau = \Delta t/t_1$; consequently,

$$\Delta \tau = -3\tau + \sqrt{9\tau^2 + 4}. \quad (30)$$

The total time from the beginning of implosion until the ion passes through the center and reaches the imploding sheath is $t + \Delta t$. Minimizing the total time leads to the result $\tau_T = \tau + \Delta \tau$ where $\tau = 0.87$ and $\Delta \tau = 0.44$. The value $R_T =$

$R(t + \Delta t) = R(t_T)$ is determined to be $R_T = 0.15b$. This differs from the case of constant v_s , where $R_T = 0.33b$.

The energy balance equation at the end of implosion is given by

$$\frac{1}{2} n_o m_i L \int_{R_T}^b (2v_s)^2 2\pi R dR = \frac{3}{2} \pi b^2 \ln_o k T_p + \frac{B_e^2}{2\mu_o} \pi L (R_p^2 - R_T^2), \quad (31)$$

where R_p is the final plasma radius. Using Equations (25) and (26) in the left hand side, and pressure and particle balance on the right hand side, we obtain

$$\frac{\pi n_o m_i b^4}{2 \tau_1} \tau_T^4 (1 - \frac{\tau_T^3}{3}) = \pi b^2 \frac{B_e^2}{2\mu_o} [\frac{5}{2} x_p^2 - x_T^2], \quad (32)$$

Finally, we take $B_e = B_o (\pi/2 t_{1/4}) t_1 \tau_T$, $\tau_T = 1.31$ and $x_T = 0.15$ and solve for x_p .

We find

$$x_p^2 = 0.16; \quad x_p = 0.40.$$

This result is significantly different from the value $x_p^2 = 0.40$ obtained when v_s is a constant.

The density is obtained from

$$n_p = \frac{n_o}{x_p^2} = 6.25 n_o, \quad (33)$$

and the temperature is determined from

$$T_p = \frac{B_e^2}{2\mu_o n_p} = 1.685 \times 10^{24} \frac{t_1^2 B_e^2}{t_{1/4}^2 n_o} \text{ (eV)}. \quad (34)$$

For Scylla IV-P, with $B_o = 4.8$ T, $b = 4.4$ cm, $t_{1/4} = 2.8$ μ sec, and $p = 10$ mTorr deuterium, we find $t_1 = 0.24$ μ sec, $t_T = 0.31$ μ sec, $n_p = 4.42 \times 10^{21} \text{ m}^{-3}$ and $T_p = 395$ eV.

b. Average bounce model⁶¹

The results of the "average" bounce model are based on the bounce model expression of Freidberg et al.

$$\frac{B_s^2}{2\mu_0} = 2m_i n_o v_s^2, \quad (35)$$

$$\frac{n_o k T_p}{x_p^2} = \frac{B_{SH}^2}{2\mu_0}, \quad (36)$$

$$\pi b^2 n_o \left[1 - \left(\frac{1}{3}\right)^2\right] \left[\frac{1}{2} m_i (2v_s)^2\right] = \frac{3}{2} \pi b^2 n_o T_p + \frac{\pi b^2 B_{SH}^2}{2\mu_0} \left[x_p^2 - \left(\frac{1}{3}\right)^2\right], \quad (37)$$

where B_s is the value of B which gives the average implosion velocity v_s , and B_{SH} is the field at the end of implosion. Freidberg et al. chose $B_{SH} = 3B_o$ and obtained $x_p = 0.29$. Also

$$n_p = 11.9 n_o = 8.41 \times 10^{21} \text{ m}^{-3}, \quad (38)$$

and

$$T_p = 2.09 \times 10^{23} \frac{B_{SH}^2}{n_o} (\text{eV}). \quad (39)$$

where the numerical value for n_p applies to the 10 mTorr fill pressure at Scylla IV-P.

If we choose $B_{SH} = B_e = 0.83 \text{ T}$, as in the previous section, then $T_p = 171 \text{ eV}$. However, this is not the choice made by Freidberg et al.⁶¹. Their choice is based on the behavior of $x(t)$, as shown in Figure 1. The initial collapse stops at radius x_T at time t_T . (For Scylla IV-P this time is $0.31 \mu\text{sec}$.) The plasma then expands and contracts to a radius x_I , which is close to x_T but usually a little larger. At this point the plasma thermalizes and $x(t)$ expands to x_p . In the average bounce model theory, both x_T and x_I are set equal to 0.33-values obtained for the case where the magnetic field rise time is much shorter than the implosion time - and it is assumed that $t = 2t_I$.

Data from the Scylla toroidal sector experiment³ suggest that the value x_I is reached at $t_p = 0.52 \mu\text{sec}$. Using this value for the Scylla IV-P experiment,

we obtain $B_{SH} = 1.4$ T, so that $T_p = 579$ eV.

c. Revised bounce model including column oscillations

One of the main differences between the two bounce models presented was that, in the "average" bounce model, a column oscillation was included as part of the implosion process, while in the exact solution it was not. In this section we revise the first theory to include oscillation energy input. We replace Equation (31) by the expression

$$W_{os} + \frac{1}{2} n_{o1} m_i L R_T^b (2v_s)^2 2\pi R dR = \frac{3}{2} \pi b^2 L n_o k T_p + \frac{B_{SH}^2}{2\mu_o} \pi L (R_p^2 - R_I^2), \quad (40)$$

where W_{os} is the oscillation energy input and is given by

$$W_{os} = L \int_{t_T}^{t_p} dt \frac{B^2(t)}{2\mu_o} 2\pi R \frac{dR}{dt}. \quad (41)$$

To make an estimation of W_{os} , we choose oscillation parameters from Figure 46. Specifically, we take $R_I = R_T$, $t_T = 0.31$ μ sec, $t_p = 0.52$ μ sec, and

$$R(t) = R_T [1 + a \sin \omega(t - t_T)] , \quad t_T \leq t \leq t_p, \quad (42)$$

where $\omega = \pi/(t_p - t_T) = 5\pi \times 10^6$ rad/sec. For numerical evaluations we take $a = 1.5$. The oscillation energy term can now be evaluated to give

$$W_{os} = \frac{3\pi^4 B_o^2 R_T^2 L}{4\mu_o \omega^2 t_{1/4}^2} \left(1 + \frac{2\omega t_T}{\pi}\right) \left(1 + \frac{\pi a}{8}\right). \quad (43)$$

For Scylla IV-P parameters $W_{os}/L = 201$ J, while $W_{KE}/L = 625$ J. The oscillation energy is seen to be significant.

When the oscillation energy is added to Equation (32) and B_e is replaced by B_{SH} , where

$$B_{SH} = B_o \left(\frac{\pi}{2t_{1/4}}\right) t_1 t_T \left(1 + \frac{\pi}{\omega t_1 t_T}\right)$$

Figure 46. Sheath Radius Time History During Implosion

we obtain for x_p :

$$x_p = \left[\frac{0.16}{\left(1 + \frac{\pi}{1.31 \omega t_1}\right)^2} + 1.72 \times 10^{-2} \frac{\left(1 + \frac{2.62 \omega t_1}{\pi}\right) \left(1 + \frac{a\pi}{8}\right)}{\left(1 + \frac{1.31 t_1}{\pi}\right)^2} \right]^{1/2} \quad (44)$$

Using Scylla IV-P data we obtain $x_p = 0.28$, $T_p = 522$ eV and $n_p = 9.3 \times 10^{21} \text{ m}^{-3}$. These results are in close agreement with those obtained from the average bounce model.

d. Comparison with Scylla IV-P data

Scylla IV-P data⁴⁴ indicate that at peak B field, $T_p = 3.1$ keV and $n_{\text{axis}} = 1.5 \times 10^{22} \text{ m}^{-3}$. Using adiabatic compression from post-implosion to peak B field, the peak B field conditions predicted by each of the theories is shown in Table 9. The theories predict approximately the same values for n_p and T_p . While the product $n_p T_p$ is in satisfactory agreement with experiment, the theories predict a value of n_p which is a factor of two too large and a temperature, T_p , which is a factor of two too low.

At the present time agreement between bounce theory and experiment are not good. Modifications to the theory appear to be in order. One deficiency in the theory is that particles which bounce from the sheath at early times will be overrun. We, however, do not allow multiple bouncing. Two possible ways of correcting this are discussed in the next two sections.

4. Multiple Bounce Analysis

Collisionless implosion dynamics should be reasonably described by the simple bounce model of Freidberg et al.⁶¹ in cases where the driving magnetic field rise time is short in comparison to the implosion time. The sheath velocity in such cases would be essentially constant thus particles reflected earlier by the sheath would not be overtaken by the sheath at a later time. Such

TABLE 9
Peak B Field Values of n and T for Scylla IV-P
Data and Three Bounce Models

	<u>Scylla IV-P data⁴⁶</u>	<u>Bounce Model 1</u>	<u>Average Bounce Model</u>	<u>Revised Bounce Model</u>
n m^{-3}	1.5×10^{22}	3.63×10^{22}	3.69×10^{22}	4.08×10^{22}
T keV	3.1	1.61	1.51	1.41

an assumption cannot be made, however, in simulating collisionless dynamics in situations where the magnetic field rise time is longer than the implosion time.⁶⁴ In this case, the imploding sheath will accelerate and overtake particles reflected from the sheath at earlier times. A relatively simple model for including this multiple overtaking effect in describing bounce implosion dynamics is presented below. A computer formulation of this model will begin in April 1978.

The imploding sheath momentum balance with multiple bounce effects included can be written as

$$2v^2 + 2\sum_i \frac{n_i(r)}{n_0} (v - v_i(r))^2 = \frac{B^2 t^2}{8\pi n_0} \quad (45)$$

where v is the sheath velocity at time t , $v_i(r)$ is the velocity of the i th reflected element at location r which is overrun by the sheath in time dt at time t , and $n_i(r)$ is the number density of the reflected particles in the i th reflected element. All particles which were reflected by the sheath at times earlier than t are tracked and if the element Δr in which they are located is overtaken by the sheath at r in time dt when they are included in $\sum_i$ summation of Equation (45). By summing over these reflected elements we arrive at a quadratic equation in v which can be solved in terms of t

$$v = \frac{\frac{1}{n_0} \sum_i n_i(r) v_i(r) - \sqrt{\left[\frac{1}{n_0} \sum_i n_i(r) v_i(r)\right]^2 - \left[1 + \frac{1}{n_0} \sum_i n_i(r)\right] \left(\frac{1}{n_0} \sum_i v_i^2(r) - \frac{B^2 t^2}{8\pi n_0}\right)}}{1 + \frac{1}{n_0} \sum_i n_i(r)} \quad (46)$$

Knowing $v(t)$ from this equation, then if the radial displacement of the sheath, Δr , is specified, an implicit expression for the required time increment $\Delta t = t_2 - t_1$ can be found from

$$\Delta r = \int_{t_1}^{t_2} v dt$$

to be

$$\Delta r = \alpha \Delta t - \beta \{ (t_1 + \Delta t) \sqrt{(t_1 + \Delta t)^2 + \gamma} + \gamma \log_{10} (t_1 + \Delta t + \sqrt{(t_1 + \Delta t)^2 + \gamma}) + \Gamma \} \quad (47)$$

where

$$\alpha \equiv \frac{\sum_i n_i(r) v_i(r)}{n_o + \sum_i n_i(r)},$$

$$\beta \equiv \frac{\dot{B}}{[16\pi n_o (1 + \sum_i n_i(r)/n_o)]^{1/2}},$$

$$\gamma = \frac{\dot{B}^2}{16\pi n_o} \left[1 + \frac{1}{n_o} \sum_i n_i(r) \right] \left\{ \frac{1}{\left[\frac{1}{n_o} \sum_i n_i(r) v_i^2(r) \right]^2} - \frac{1}{n_o} \left(\sum_i v_i^2(r) \right) \left(1 + \frac{1}{n_o} \sum_i n_i(r) \right) \right\},$$

and

$$\Gamma = -(\gamma \log_{10} (t_1 + \sqrt{t_1^2 + \gamma}) + t_1 + \sqrt{t_1^2 + \gamma}).$$

The average sheath velocity v over $\Delta t = t_2 - t_1$ defines newly reflected elements consisting of an element created from overrun background gas and elements created based on reflection of allowable elements reflected at earlier times, i.e. elements lying in Δr when the sheath passes through the location. These elements are tagged and tracked as time progresses.

The number density of i th element reflected particles, $n_i(r)$, is not equal to n_o for there will be a geometric focusing of these particles as the element approaches the discharge chamber centerline. These element locations are always known for the following time increment, $\Delta t = t_2 - t_1$, because Equation (47) is solved for fixed space elements Δr to determine the required time increment. Thus, $n_i(r)$ is a known quantity in Equation (47) and can be written from simple geometry considerations as

$$n_i(r) = n_o \frac{2r_{oi} - \Delta r}{2r_{ot} - \Delta r}$$

where r_{oi} is the outside radius of element i at reflection and r_{ot} is the outside radius of element i at time t . Each element width is fixed at Δr to simplify

numerical formulation of the multiple bounce terms. If previously reflected elements fall within the sheath Δr at time t in dt then all of those particles within Δr are assumed to interact with the sheath uniformly over Δr . The grid Δr will be fixed at a value which will be sufficiently small to give acceptable spatial resolution and to minimize if not eliminate multiple overtaking in Δr of particle reflected in Δr .

The plasma sheath is assumed to implode until ions reflected from the sheath pass through the chamber centerline and interact with the sheath. This radius defines the sheath implosion radius, r_T . The implosion energy is calculated by summing the kinetic energy of all reflected particles. Equilibrium plasma properties are then calculated from Equations (5), (6) and (7) of Section 2 where $B_i = 0$ and $\beta = 1$.

The multiple bounce concept could be adapted to the free bounce hybrid model discussed in Section 5 but the relatively simple expression for sheath velocity, Equation (47) would be replaced by a relatively complex differential equation in which the snowplow inertial terms would be included. The resultant numerical complexity introduced by allowing snowplowing of a certain percentage of overrun particles has not been evaluated. Experiments and more sophisticated theoretical models^{65,66} suggest that only a fraction of the particles interacting with the sheath are bounced.

5. Hybrid Implosion

Implosion studies conducted at low fill pressures⁶⁴ (10 mTorr) had revealed that the current sheath does not exhibit pure snowplow behavior, but rather exhibits snowplow behavior early in the discharge and particle bounce behavior at later times. An attempt is made to adapt a high magnetic field rise rate hybrid implosion model developed by Oliphant⁶² to model experiments where the

magnetic field quarter cycle time is almost one order of magnitude greater than the implosion time.

Current sheath implosion is divided into two phases. The first phase is identified by plasma particles being successively reflected and overtaken by the accelerating current sheath. Consistent with Oliphant, we approximate this multiple reflection interaction by a snowplow interaction until the plasma sheath reaches a given radius - which we designate as the crossover radius - where the reflected particles have sufficient velocity so that they cannot be overtaken by the imploding sheath and will be able to just pass through the discharge chamber axis and interact with the sheath imploding from the other side. For radii less than the crossover radius, the bounce fraction, f , i.e. fraction of particles interacting with the sheath which bounce, can be programmed to increase from 0 to M as the sheath approaches the axis. Different bounce fraction functions will be investigated and M will be varied 0 to 1.

The snowplow calculations for the first phase of implosion were made using the Artsimovich⁶⁰ snowplow model. The terminal conditions for this phase - sheath radius, sheath velocity and mass pickup - were then used as initial conditions for the second phase. The radial momentum equation for the imploding sheath during the second phase can be written for a linear magnetic field approximation as

$$-mn_o \frac{d}{dt}[(R_I^2 - R^2)\frac{dR}{dt}] - mn_o(b^2 - R_I^2)\frac{d^2R}{dt^2} + mn_o 2R \frac{dR}{dt}(2 \frac{dR}{dt}) = \frac{R}{\mu_o}(\frac{dB}{dt})_{t=0}^2 t^2 ,$$

where R_I is the crossover radius and all other symbols were defined earlier. This equation can be cast in the non-dimensional form

$$-\frac{d}{d\tau}[(x_I^2 - x^2)\frac{dx}{d\tau}] - (1 - x_I^2)\frac{d^2x}{d\tau^2} + x(\frac{dx}{d\tau})^2 = x^2 \quad (48)$$

where $x \equiv R/b$ and $\tau \equiv t/t_0$. The initial condition for Equation (48) will be iteratively determined to satisfy the overtaking conditions stated above.

Particles reflected at subsequent times achieve higher velocities due to sheath acceleration and pass through the discharge axis to interact at some $x > 0$ with the sheath imploding from the other side. The final dimensionless pre-equilibrium radius, i.e., where the plasma begins to thermalize, was identified at that time when the incoming sheath made first contact with reflected particles coming through the center from the opposite side of the discharge chamber. This radius, x_T , is evaluated during the simulations.

From the detailed implosion results from Phase 1 and Equation (48), the total plasma energy per unit column length just prior to thermalization can be calculated from

$$E_{\text{imp}} = (SE)_{x_I} + (SE)_{x_T} + (KE)_{x_T} + L \int_{x_T}^{x_I} \frac{1}{2} m (2v)^2 f(R) n_0 2\pi R dR \quad (49)$$

where the sticking energy terms are;

$$(SE)_{x_I} = \int_0^{m(x_I)} \frac{1}{2} \left(\frac{dR}{dt} \right)^2 dm,$$

and

$$(SE)_{x_T} = \int_{m(x_I)}^{m(x_T)} \frac{1}{2} \left(\frac{dR}{dt} \right)^2 dm,$$

where;

$$(KE)_{x_T} = \frac{1}{2} (b^2 - R_I^2) L n_0 m \left(\frac{dR}{dt} \right)^2_{x_T} + \int_{x_T}^{x_I} \frac{1}{2} m \left(\frac{dR}{dt} \right)^2 (1 - f(R)) n_0 2\pi R dR$$

is the total radially directed kinetic energy of snowplowed plasma at x_T ; and the last term in Equation (49) represents the total kinetic energy of reflected particles, $x < x_I$. With E_{imp} known, equilibration values for density, tempera-

ture and radius of the plasma can be obtained from energy, particle and pressure balance as outlined in Section 2.

The hybrid model, assuming a constant bounce fraction, predicts a total implosion energy which is approximately 90 percent the implosion energy predicted using the $\beta = 1$ snowplow model. Consequently, the hybrid model will predict plasma properties comparable to those obtained using either simple implosion models discussed in Section 2 and 3. It is anticipated that programming the bounce fraction will not significantly change the above results. Therefore, it is felt at this time that the hybrid model will not be able to adequately predict post-implosion plasma properties for low fill pressure experiments.

6. Future Work

The following topics will be examined:

1. Simulation of the PSU 50 cm pinch for various fill pressures, including ionization of the partially preionized fill gas.
2. Examination of the importance of column oscillations in implosion modeling.
3. Finite sheath width modeling for low to moderate beta plasmas.⁵³
4. Comparison of predictions of bounce, hybrid and multiple bounce models.

IX. FUSION ENGINEERING (Klevans, Mattis, Dagan, Werly)

1. Introduction

The fusion engineering effort is aimed at better defining and determining the parameters needed to achieve power with a linear theta pinch system. The effort has been accomplished primarily through the participation of outstanding undergraduate students who take three credits of project work. The three students involved in the work this past year are listed above. Two of these students are going to graduate school for further studies in fusion.

Two projects are described below. In the first, we employed the simple scaling methods of Ellis,⁶⁷ but used the multiple mirror confinement law of Makhijani et al.⁶⁸ Machines with lengths of about 1 km were found if thermal conduction was negligible.⁶⁸

In the second project, we developed a time dependent zero-dimensional computer code to describe a fusion machine. This is an extension of the code developed for modeling theta pinch experiments,⁴² and includes alphas. The alphas are analyzed by use of a multigroup model. The model is presented. We are still debugging the program, which is essentially complete. It will then be used to study the influence of various confinement schemes on machine parameters.

2. Multiple Mirror Confinement

The system being analyzed consists of a straight central section of length L , two mirror sections, each of length $L_m/2$, and a direct converter. The initial transient response which occurs during plasma heating and the rapid filling of the mirror sections is ignored. Rather, we treat the plasma as being in a quasi-steady state with endloss being governed by the multiple mirror confinement law developed by Makhijani et al.⁶⁸ This confinement is diffusive rather

than free streaming, so particle confinement is substantially improved. The scaling law is of the form $\tau_{MM} \sim LT_i^{-5/2}$ in contrast to the free streaming confinement time $\tau \sim LT^{-1/2}$. Note, however, that the greatest improvement is achieved if T_i can be held to a low value, since $\tau_{MM} \sim T_i^{-5/2}$. In conventional theta pinch reactors,⁶⁹ T_i is expected to rise to values as high as 17 keV; Ellis chose an average burn temperature of 10 keV. We examine both cases and find that the use of $T_i = 5$ keV leads to an unacceptably large coil radius. Consequently, a trade-off between coil radius and coil length is observed.

It is also assumed that thermal conduction is negligible. This is based on the belief that if a direct converter is present, thermal conduction paths to the walls can be inhibited. This assumption requires further study for pulsed machines, for thermal conduction will be the limiting energy loss if it is not effectively stopped.

Two other assumptions have been made. First, energy production and loss are only considered for the center section. This assumption is based on a calculation we performed which showed that the energy produced in the mirror coils was approximately that amount needed to operate the mirror coils. Second, the implosion model followed the work of Klevans and Ankney,⁷⁰ and used a simple bounce model rather than the programmed bounce model. This issue is still controversial and needs to be reexamined. The model we have chosen is more pessimistic in that it leads to larger coil radii.

We have also employed the efficiency model outlined by Steiner,⁷¹ and used by Klevans and Ankney, except that a direct converter is added. An efficiency of 0.75 was chosen for the direct converter and 0.40 for the thermal converter.

The most significant results are shown in Table 10. To obtain these results it was assumed that $E_0 = 2$ kV/cm, the wall loading factor is $\bar{I}_w = 2$ MW/m², the

TABLE 10
Reactor Parameters for Linear Theta Pinch
with Multiple Mirror Confinement

Case No.	ϵ	$\bar{\epsilon}$	T (keV)	B (kG)	a (cm)	b (cm)	$10^{16} n_{cm^{-3}}$	τ (msec)	τ_c (sec)	L (m)	L_m (m)	L_{tot} (m)	E_M (MJ)	P_{th} (MW)	η %	r %	R
1	0.1	0.8	5	200	8.77	73.1	7.94	37.8	5.13	318	192	510	1.11E5	4675	21.4	58.8	2.7
2	0.1	0.8	10	200	6.63	39.1	4.0	75.6	19.1	336	918	1254	3.35E4	2644	37.8	26.2	2.7
3	0.1	0.8	9	150	11.6	59.2	2.48	121	18.7	223	956	1179	3.23E4	2654	37.7	25.6	3.6
4	0.1	0.8	9	250	4.54	29.3	6.89	43.5	15.9	467	579	1046	4.66E4	2765	36.2	29.7	2.2
5	0.1	0.9	9	250	4.04	26.2	7.75	38.7	16.0	519	515	1034	4.12E4	2740	36.5	28.9	2.2
6	0.25	0.8	5	200	5.55	46.3	7.94	37.8	3.24	867	192	1059	1.21E5	8063	12.4	78.4	2.7
7	0.25	0.8	10	200	4.19	24.7	4.0	75.6	12.1	697	918	1615	2.82E4	3471	28.8	49.7	2.7
8	0.25	0.8	9	250	2.87	18.6	6.89	43.5	10.1	980	579	1559	3.92E4	3675	27.2	52.7	2.2

mirror ratio $R = 2.7$ for $B = 200$ kG, and $\epsilon = 0.1$, where $\epsilon \equiv E_j/E_{\text{tot}}$, with E_j the joule heating loss in the main coil, and E_{tot} the total thermonuclear energy production in the coil. The choice $\epsilon = 0.1$ is arbitrary and was employed by Ellis with the belief that the fraction of energy which is recirculated is approximately 2ϵ . We have considered two values of ϵ , but have not optimized this parameter. Finally, we have chosen $n\tau = 3 \times 10^{15} \text{ sec-cm}^{-3}$. This is larger than the value $n\tau = 10^{15} \text{ sec-cm}^{-3}$ used by Ellis, but the smaller value did not achieve breakeven. Further calculations to optimize $n\tau$ should be performed.

In the table, a is the plasma column radius; b is the coil inner radius, τ is the burn time, which equals the confinement time; τ_c is the cycle time; L , L_m and L_{tot} are the lengths of the main coil, the mirrors and the reactor; P_{th} is the thermal power produced; E_M is the magnetic energy store for the compression coil; η is the plant efficiency and r is the percentage of gross energy which is recirculated. The results with $\epsilon = 0.1$ are more favorable than those with $\epsilon = 0.25$, having higher efficiency, shorter length, and a smaller energy recirculation. However, smaller ϵ results in larger coil radii.

Cases 1 and 2 show the effect of increasing burn temperature. Case 1 has a very large coil radius and a low efficiency. It is also the shortest device. For Case 2, the coil radius is 40 cm, which is still quite large. A radius of 20 cm is considered large for a field of 200 kG, and 40 cm may be very difficult to achieve. In Case 3, we decrease the field to 150 kG, and this leads to an increase of b to 60 cm. Cases 4 and 5 show that, even at 250 kG, the machine exceeds 1 km, and an increase of β from 0.8 to 0.9 has only a small effect.

These machines are certainly in an interesting parameter range, and further study is indicated.

3. Computer Model for Theta Pinch Reactor

We present here a description of the numerical model which will be used to describe a linear theta pinch reactor. The model used is a zero dimensional, collisional, point kinetic model assuming a uniform density profile in the plasma. The code includes Bremsstrahlung radiation and a multigroup analysis of the alpha particles. The endloss is taken as free streaming. This will be a base model to compare the computer calculations with the calculations of Ellis⁶⁷ and Klevans and Ankney.⁷⁰ When this is completed, calculations with other scaling laws for energy and particles will be analyzed.

a. Alpha multigroup analysis

The alpha particles are considered to exist in six groups, the lowest or thermal group having the same energy as the nonalpha plasma ions. It is assumed that the alphas are born into the top group from D-T reactions in a 50-50 mix and then slow down into the lower groups, contributing their energy to the plasma. Alpha leakage is included.

The alpha group energy boundaries, E_g , used are:

$$E_1 = 3500 \text{ keV}$$

$$E_2 = 2300 \text{ keV}$$

$$E_3 = 1100 \text{ keV}$$

$$E_4 = 500 \text{ keV}$$

$$E_5 = 150 \text{ keV}$$

$$E_6 = 3T_i/2 ,$$

with T_i in energy units (keV).

To determine the average energy of each group, leakage is ignored, and the following analysis is introduced as an alternative to solving the rigorous equations which include leakage and area changes.

The particle balance equation for the gth group is⁷²

$$\frac{\partial n_E}{\partial t} = \frac{1}{\tau_E} \frac{\partial}{\partial E} [n_E(E) F(E)] \quad (1)$$

where

$$F(E) = E + \frac{r_a^3}{E^{1/2}},$$

$$r_a^3 = 3\sqrt{\pi} m_\alpha^{3/2} T_e^{3/2} / 4m_{r_i} m_e^{1/2},$$

$$m_{r_i} = \frac{m_\alpha m_i}{(m_\alpha + m_i)},$$

n_E = number of energetic alphas per volume per energy interval,

and

$$\tau_E = \frac{\tau_s}{2.5}.$$

For $m_\alpha = 4$ amu, $m_i = 2.5$ amu, $m_e = 0.00055$ amu,

$$r_a^3 = 29.53 T_e^{3/2} \text{ with } T_e \text{ in keV.}$$

Integrating Equation (1) from E_g to E_{g-1}

$$\frac{\partial n_g}{\partial t} = \frac{1}{\tau_E} [n_E(E_{g-1}) F(E_{g-1}) - n_E(E_g) F(E_g)] .$$

where

$$n_g = \int_{E_g}^{E_{g-1}} dE n_E(E) = \text{number density of gth group}$$

and defining $\tau_g^t = \frac{n_g}{n_E(E_g) F(E_g)} \tau_E$ = transition time from the (g-1)th group to the gth group results in

$$\frac{\partial n_g}{\partial t} = \frac{n_{g-1}}{\tau_{g-1}^t} - \frac{n_g}{\tau_g^t} .$$

To solve for τ_g^t , the steady state solution to Equation (1) is used

$$n_E(E)F(E) = \text{constant} = n_E(E_g)F(E_g)$$

then

$$n_E(E) = \frac{n_E(E_g)F(E_g)}{F(E)}$$

and

$$n_g = \int_{E_g}^{E_g-1} n_E(E) dE = n_E(E_g)F(E_g) \int_{E_g}^{E_g-1} \frac{dE}{F(E)}$$

so

$$\frac{\tau_g^t}{\tau_E} = \frac{n_g}{n_E(E_g)F(E)} = \int_{E_g}^{E_g-1} \frac{dE}{F(E)} = \frac{2}{3} \ln \frac{r_a^3 + E_g^{3/2}}{r_a^3 + E_g^{3/2}}$$

Solving for τ_g^t gives

$$\tau_g^t = \frac{2}{3} \tau_E \ln \frac{r_a^3 + E_g^{3/2}}{r_a^3 + E_g^{3/2}} = \frac{4}{15} \tau_s \ln \frac{r_a^3 + E_g^{3/2}}{r_a^3 + E_g^{3/2}}$$

where¹

$$\tau_s = 1.25 \times 10^{18} \frac{T_e^{3/2}}{n_i}$$

with T_e in keV and n_i in m^{-3} .

The mean value theorem is used to solve for $\langle E_g \rangle$.

$$\frac{\tau_g^t}{\tau_E} = \int_{E_g}^{E_g-1} \frac{dE}{F(E)} = \frac{E_{g-1} - E_g}{F(\langle E_g \rangle)} = \frac{E_{g-1} - E_g}{\langle E_g \rangle + \frac{r_a^3}{\langle E_g \rangle^{1/2}}}$$

so that

$$\langle E_g \rangle^{3/2} - \frac{\tau_E}{\tau_g^t} (E_{g-1} - E_g) \langle E_g \rangle^{1/2} + r_a^3 = 0 \quad (4)$$

which can be solved for $\langle E_g \rangle$.

The actual equation used to describe the time dependent alpha group number density is a differential equation which includes the D-T reaction source, the transfer of alphas from higher to lower groups, alpha leakage and the effect of cross sectional area change. The alphas, being charged particles in a magnetic field, have cyclotron orbits and do not spend the entire time in the plasma. Since alphas slow down only while in the plasma, a residence factor, R_g , which is the fraction of time that an alpha spends in the plasma, must be considered in the transfer process.

$$\frac{dn_g}{dt} = \frac{1}{4} n_1^2 \langle \sigma v \rangle \delta_{g1} + \frac{R_g n_{g-1}}{\tau_{g-1}} (1 - \delta_{g1}) - \frac{n_g}{\tau_g} - \frac{n_g}{A_p} \frac{A_p}{dt} \quad (5)$$

(alphas born in group 1) (transfers to gth group from (g-1)th group) (loss from gth group) (effect of cross-sectional area changes)

where

$$\frac{1}{\tau_g} = \frac{R_g}{\tau_c} + \frac{1}{\tau_g^l}$$

The leakage time, τ_g^l , is taken to be

$$\tau_g^l = \frac{L}{2} \frac{1}{v_{g11}} = \frac{L \sqrt{m_g/T_g}}{2} \quad \text{where } T_g = \frac{2}{3} \langle E_g \rangle \quad (6)$$

If τ_g^l is greater than the particle confinement time, τ_p , then τ_p is used for τ_g^l . For the thermal group $\tau_g = \tau_p$.

The residency factor, R_g , is described as⁷³

$$R_g = \frac{1 + \frac{9}{5} \rho_g^2 e^{-2\rho_g}}{1 + \frac{3}{8} \pi^2 \rho_g^2}$$

where

$$\rho_g = \frac{v_{g\perp}}{\omega_c a}$$

a = plasma column radius and

$$v_{g1} = \left[\frac{4}{3} \frac{\langle E_g \rangle}{m_\alpha} \right]^{1/2}$$

and

$$\begin{aligned} \omega_c &= \frac{eB}{m_\alpha} \\ &= 9.58 \times 10^3 \frac{zB}{\mu} \end{aligned}$$

with B in gauss, $z = 2$ for alphas and μ = ion mass ratio = 4 for helium.

In slowing down, the alphas transfer part of their energy to the electrons and part to the non-alpha ions. This energy transfer is represented by $\dot{w}_{ae}$, the energy transfer rate of alphas to electrons, and $\dot{w}_{ai}$, the energy transfer rate of alphas to ions. These quantities are the summation of the transfer rate of each alpha group to the electrons or to the ions.

$$\dot{w}_{ae} = \sum_g \dot{w}_{ge}$$

$$\dot{w}_{ai} = \sum_g \dot{w}_{gi}$$

where¹

$$\dot{w}_{ge} = \frac{n_g R_g}{\tau_g} (E_{g-1} - E_g - F_g^i) \quad (8)$$

$$\dot{w}_{gi} = \frac{n_g R_g}{\tau_g} F_g^i \quad (9)$$

F_g^i = energy transfer from the energetic alphas in the g th group to the ions

$$= r_a^3 \int_{E_g}^{E_{g-1}} \frac{dE}{E^{3/2} + r_a^3}$$

$$F_g^1 = \frac{2}{3} r_a^3 \left\{ \frac{1}{2} \ln \left[\left(\frac{E_g^{3/2} + r_a^3}{E_g^{3/2} + r_a^3} \right) \left(\frac{E_g^{1/2} + r_a}{E_g^{1/2} + r_a} \right)^3 \right] + \sqrt{3} \left[\tan^{-1} \left(\frac{2E_g^{1/2} - r_a}{\sqrt{3}r_a} \right) - \tan^{-1} \left(\frac{2E_g^{1/2} - r_a}{\sqrt{3}r_a} \right) \right] \right\} \quad (10)$$

with the group energy bounds, E_g , in keV. There is no transfer of energy from the thermal group to the ions or electrons.

The reaction rate parameters used in the model are

$$\langle \sigma v \rangle_{DT} = \frac{3.8 \times 10^{-12} \exp\{-19.02/T_i^{1/3}\}}{T_i^{2/3}} \text{ (cm}^3/\text{sec) for } T_i \leq 10 \text{ keV} \quad (11)$$

or

$$\langle \sigma v \rangle_{DT} = \frac{3.41 \times 10^{-14} \exp\{(-27.17/T_i^{2/3}) + (3.688/T_i^{1/3})\}}{T_i^{2/3}} \text{ (cm}^3/\text{sec) for } T_i > 10 \text{ keV} \quad (12)$$

b. Thermal analysis

The energy and particle balance equations are a modified form of those appearing in Reference 42. The modifications include the addition of a multi-group treatment of the alpha particles, Bremsstrahlung radiation and the pressure and particle gains and losses associated with D-T fusion reactions.

$$\frac{dw_e}{dt} = \frac{w_i - w_e}{\tau_{eq}} + \dot{w}_{ae} - \frac{w_e}{\tau_{Th}} - \epsilon_e \Gamma_e - P_e \frac{dV}{dt} - \dot{w}_{Br} \quad (13)$$

$$\frac{dw_i}{dt} = -\frac{w_i - w_e}{\tau_{eq}} + \dot{w}_{ai} - \frac{w_i}{\tau_{Th}} - \epsilon_i \Gamma_i - P_i \frac{dV}{dt} - n_i^2 \langle \sigma v \rangle \omega_i A_p L \quad (14)$$

where w_s = thermal energy of species s

$$= \frac{3}{2} n_s T_s A_p L, \quad (15)$$

$\frac{w_i - w_e}{\tau_{eq}}$ = rate of energy transfer from ions to electrons,

$P_s \frac{dV}{dt}$ = work done by species s in expanding the plasma column
 $= n_s T_s L (dA_p/dt)$,

(16)

$\epsilon_s \Gamma_s$ = flow rate of species s thermal energy across the column ends,

w_s / τ_{Th}^s = rate of heat transfer by species s by conduction across the column ends

and w_i = average energy of a fusing ion⁷⁴

$$= 6.3 T_i^{2/3} \text{ (keV/ion).}$$

(17)

For the electrons, the thermal conduction time is given as⁴⁶

$$\tau_{Th}^e = 4.29 \times 10^{-31} \frac{n_e l_1 L \ln \lambda}{T_e^{5/2}} \quad (18)$$

with T_e in keV, l_1 and L in m and n_e in m^{-3} , where $\ln \lambda$ is the Debye logarithm⁷⁵

$$\ln \lambda = 37.82 - \ln \left(\frac{n_e^{1/2}}{T_e} \right) \quad (19)$$

with T_e in keV and n_e in m^{-3} . If the sheath thickness, l_1 , is taken to be $0.2L$ as done by Morse,⁴⁶ Equation (18) becomes

$$\tau_{Th}^e = 0.858 \times 10^{-31} \frac{n_e L^2 \ln \lambda}{T_e^{5/2}} \quad (20)$$

For the ions, the ion thermal conduction time is given as

$$\tau_{th}^i = 0.5729 \tau_{th}^e \left(\frac{n_e}{n_i} \right) \left(\frac{T_e}{T_i} \right)^{5/2} \left(\frac{m_i}{m_e} \right)^{1/2} \quad (21)$$

with T_s in keV and n_s in m^{-3} . The thermal equilibration time is given by⁷⁹

$$\tau_{eq} = 9.96 \times 10^{18} \frac{A T_e^{3/2}}{n_e \ln \lambda} \quad (22)$$

with T_e in keV, n_e in m^{-3} and A is the ion mass in amu.

For the flow rate of species s thermal energy across the column ends, $\epsilon_e \Gamma_e$, a collisional model provide $\epsilon_i = \frac{5}{2} T_i$, and $\epsilon_c = \frac{5}{2} T_e$. The equations used for Γ_s are

$$\Gamma_i = A_p L \frac{n_i}{\tau_p} \quad (23)$$

$$\Gamma_e = A_p L \left(\frac{n_i}{\tau_p} + 2 \sum_s \frac{n_s}{\tau_s} \right) \quad (24)$$

where τ_p = particle confinement time⁴²

$$= \frac{L}{2} \left[\frac{m_i}{2(T_e + T_i)} \right]^{1/2} \psi \quad (25)$$

and m_i is the ion mass and ψ is an end loss parameter chosen to match end on interferometry measurements.

The Bremsstrahlung radiation loss from the electrons is given by⁷⁵

$$\dot{w}_{Br} = A_p L C_1 n_e^2 z T_e^{1/2} \quad (26)$$

where $C_1 = 3.34 \times 10^{-21}$ with T_e in keV and n in m^{-3} gives $C_1 n_e^2 z T_e^{1/2}$ in keV/ m^3 and

$$\frac{z}{z} = \frac{z_i^2 n_i}{z_i n_i} = \frac{n_i + 4n_\alpha}{n_i + 2n_\alpha} \quad (27)$$

The modified particle continuity equation accounts for ions being lost in fusion and by leakage.

$$\frac{dN_i}{dt} = -\frac{N_i}{\tau_p} - 2f \frac{2Vn_1^2 \langle \sigma v \rangle DT}{4} \quad (28)$$

where N_i = total number of non alpha ions

n_i = number density of non alpha ions.

To convert Equation (22) from dN_i/dt to dn_i/dt , the time derivative of

$N = n A_p L$ where $V = A_p L$ gives

$$\frac{\partial N}{\partial t} = LA_p \frac{\partial n}{\partial t} + Ln \frac{\partial A_p}{\partial t},$$

and Equation (22) becomes

$$\frac{dn_i}{dt} = -\frac{n_i}{\tau_p} - \frac{n_i}{A_p} \frac{dA_p}{dt} - \frac{1}{2} n_i^2 \langle \sigma v \rangle. \quad (29)$$

$$\text{Assuming quasineutrality, } n_e = n_i + 2n_a. \quad (30)$$

From Equation (15), $w_s = (3/2)n_s T_s A_p L$ and

$$\frac{1}{w_s} \frac{\partial w_s}{\partial t} = \frac{1}{T_s} \frac{\partial T_s}{\partial t} + \frac{1}{n_s A_p} \frac{\partial n_s A_p}{\partial t}. \quad (31)$$

From the energy conservation Equations (13) and (14) and the particle continuity Equations (29) and (30) along with Equations (16), (23), (24), (26) and (31), one can obtain the following temperature equations:

$$\frac{1}{T_e} \frac{dT_e}{dt} = \frac{n_i T_i - n_e T_e}{n_e T_e \tau_{eq}} + \frac{\dot{w}_{ae}}{w_e} - \frac{1}{\tau_{Th}} - \frac{2}{3n_e} \left[\frac{n_i}{\tau_p} + 2 \frac{n_g}{g \tau_g} \right] - \frac{2}{3} \frac{1}{A_p} \frac{dA_p}{dt} - \frac{2}{3} \frac{C_1 n_e^2}{T_e^{1/2}} \quad (32)$$

$$\begin{aligned} \frac{1}{T_i} \frac{dT_i}{dt} = & -\frac{n_i T_i - n_e T_e}{n_i T_i \tau_{eq}} + \frac{\dot{w}_{ai}}{w_i} - \frac{1}{\tau_{Th}} - \frac{1}{\tau_p} \left[\frac{5}{3} - 1 \right] - \frac{2}{3} \frac{1}{A_p} \frac{dA_p}{dt} \\ & + \left[1 - \frac{2}{3} \frac{w_i}{T_i} \right] \frac{n_i}{2} \langle \sigma v \rangle. \end{aligned} \quad (33)$$

c. Rate of change of area

To obtain the rate of change of the cross sectional area, A_p , with time, a radial pressure balance is considered

$$p(r) + \frac{B_{in}^2}{2\mu_0} = n_e T_e + n_i T_i + \sum_g n_g T_g + \frac{B_{in}^2}{2\mu_0} = \frac{B_e^2}{2\mu_0}. \quad (34)$$

Multiplying Equation (34) by A_p and taking its time derivative, setting $B_{in} = \phi/A_p$ where ϕ is constant, defining

$$\beta = \frac{n_e T_e + n_i T_i + \sum_g n_g T_g}{B_e^2 / 2\mu_0},$$

using Equation (13) and (14) for dw_e/dt and dw_i/dt , considering $\Sigma_g \frac{dw_g}{dt}$ and utilizing a considerable amount of algebra, the following result is obtained:

$$\begin{aligned} \frac{1}{A_p} \left(\frac{dA}{dt} \frac{6-\beta}{6} \frac{B_e^2}{\mu_0} \right) = & \frac{1}{6} \langle \sigma v \rangle n_i^2 E_\alpha - \frac{1}{3} \langle \sigma v \rangle n_i^2 \omega_1 - \frac{B_e}{\mu_0} \frac{dB_e}{dt} - \frac{n_e T_e}{\tau_{Th}} - \frac{n_i T_i}{\tau_{Th}} - \frac{2}{3} \Sigma_g \frac{\langle E_g \rangle n_g}{\tau_g} \\ & - \frac{5}{3} \left(T_e \left(\frac{n_i}{\tau_p} + 2 \Sigma_g \frac{n_g}{\tau_g} \right) + T_i \left(\frac{n_i}{\tau_p} \right) \right) - \frac{2}{3} C_1 n_e^2 T_e^{1/2} \quad (35) \end{aligned}$$

where

$$\frac{1}{6} \langle \sigma v \rangle n_i^2 E_\alpha = \text{expansion due to alpha heating,}$$

$$\frac{1}{3} \langle \sigma v \rangle n_i^2 \omega_1 = \text{loss due to fusing ion energy (neglected in program since } \omega_1 \ll E_\alpha),$$

$$\frac{B_e}{\mu_0} \frac{dB_e}{dt} = \text{compression due to change in applied magnetic field,}$$

$$\frac{n_e T_e}{\tau_{Th}} + \frac{n_i T_i}{\tau_{Th}} = \text{compression due to energy loss associated with thermal conduction,}$$

$$\frac{5}{3} \left(T_e \left(\frac{n_i}{\tau_p} + 2 \Sigma_g \frac{n_g}{\tau_g} \right) + T_i \left(\frac{n_i}{\tau_p} \right) \right) = \text{compression due to energy loss associated with ion and electron particle loss,}$$

$$\frac{2}{3} \Sigma_g \frac{\langle E_g \rangle n_g}{\tau_g} = \text{compression due to energy loss associated with alpha particle leakage}$$

$$\text{and } \frac{2}{3} C_1 n_e^2 T_e^{1/2} = \text{compression due to energy loss associated with Bremsstrahlung radiation.}$$

d. Program solution

The initial conditions for the plasma were obtained using an area, temperature and number density for ions and alphas at the end of compression and calculating

back using adiabatic confinement relations. The shock field was found from Oliphant⁶²

$$B_s = 2.32(1 + f)^{5/14} \frac{E_0^{5/7} B_0^{2/7}}{T_0^{5/14}} \quad (36)$$

with B in kgauss, E_0 in kV/cm and $T = \text{keV}$, where $f = 1$ for the bounce model.

Using the values of $E_0 = 2 \text{ kV/cm}$, $T_0 = 5 \text{ keV}$ and $B_0 = 200 \text{ kg}$ from Ellis,⁶⁷ a shock field of $B_s = 12.5 \text{ kg}$ is calculated. Adiabatic confinement relations give n_s , T_s , and a_s if values of $a_0 = \text{plasma radius after compression}$ and n_0 are assumed.

The magnetic field has a one millisecond rise time.

The program uses these initial values and simultaneously solves Equations (5), (29), (32), (33) and (35) to give the number densities of the alpha groups, ions and electrons, the ion and electron temperature and the plasma cross sectional area. The calculations are performed using a Hamming Predictor Corrector Method.

e. System energy balance

In order to determine plant efficiency and Q value, and to find out how the energy is produced and circulated through the system, a system energy balance model is being developed. As a first step we have computed and integrated each of the terms in Equations (13) and (14). These will be incorporated into a system energy balance model similar to that used by Krakowski et al.⁶⁹

The above program has been completed and debugging is in progress.

4. Future Work

This work will continue with students not funded by the contract. The three main tasks for the next year are: (1) evaluation of the simple scaling models of Ellis, and Klevans and Akney by comparison with results from the energy

balance code; (2) incorporation of a system energy balance model to compute plant efficiency and Q ; and (3) begin an evaluation of particle and energy end stoppering methods.

X. COOPERATION WITH OTHER LABORATORIES AND EXPLORATORY WORK
(Sabbatical activities of T. M. York)

Professor York is on Sabbatical leave from The Pennsylvania State University for the period September 1977 through May 1978. From September through December, he was a Visiting Staff Member to CTR-3, Los Alamos Scientific Laboratory. During that visit, he cooperated with several on-going efforts in the CTR-Division at LASL; those cooperative efforts are continuing, do relate to the contracted research, and so will be briefly reviewed here.

1. Position and Radius Detectors on Scylla IV-P: Instability Theory
(This work was done in cooperation with K. F. McKenna and R. J. Comisso.)

--Abstract of Research Memorandum--

The determination of position and displacement of the plasma column as a function of time and space in the Scyllac torus and sector machines was of considerable importance. Position detectors were developed⁷⁶ and used on the Scyllac sector.⁷⁷ The identification of the dynamics and stability of the plasma column is also of importance on the Scylla IV-P machine. A recent theoretical work by Freidberg and Pearlstein⁷⁸ clarified the understanding of instabilities. In the experimental evaluation of plasma column behavior by streak photography of luminosity, there has been clear evidence of the $m = 1$ "wobble" at 10 mT fill pressures and increasingly earlier onset of a shredding (probably $m = 2$) instability with increasing fill pressure.⁷⁹ Accordingly, in order to provide more precise information on the dynamic history of the plasma column, the position detectors have been emplaced on Scylla IV-P (Figure 1); some of the sensing units were designed to simultaneously sense column radius. Since the placement of the radius detection units is carried out so that both a horizontal view and vertical view detector are at the same axial location,

at the same point in time and space a different radius indication from each view would provide $m = 2$ instability information.

The purpose of this effort is twofold: (1) To review the Friedberg and Pearlstein paper and to clarify or plainly state its significance for the Scylla IV-P experiment; and (2) to describe an evaluation and calibration procedure that has been carried out for the radius detection circuitry.

2. Plasma Loss Measurements on RFTP Using Pressure Sensing Diagnostics
(This work was done in cooperation with R. K. Linford.)

--Abstract--

The transient sequence of loss events on the 1 meter long RFTP facility was further investigated by the design, testing, and utilization of a local pressure sensing diagnostic to map the plasma flow from the reversed trapped field configuration. Following successful preliminary noise and perturbation evaluation, a specific piezoelectric probe system was designed and tested. A 2 mm diameter sensor was encased in an 8 mm diameter quartz tube; the risetime was 1 μ sec, and the output was 0.4 V/atm. The entire surface of the structure in contact with the plasma is quartz so as to avoid contamination of the discharge tube and thus affect containment. The piezoelectric sensor is coupled to an oscilloscope through 60 feet of RG/64U cable preceded by a follower amplifier or line driver. Following a standard design with a Burr-Brown 3553 Buffer Amplifier, a series of test refinements resulted in noise signals on the order of several millivolts with signal levels on the order of tens of millivolts. The unit was shock tube tested for calibration. It was installed in the RFTP-A device with a special fitting at an end plate so that the probe could be rotated to survey the exhaust region, and a series of data surveys were taken. A spurious drift of the signal base line of variable magnitude was detected, but appears to be a heat conduction effect and can be corrected for.

The compatible interpretation of this data with other diagnostics used by Dr. Linford will be pursued; the results will be presented in appropriate LASL reports and journal articles.

It will be noted that the present work being completed at Penn State on the 25 cm pinch with reversed trapped field is intended to precisely determine the highly transient initial phase (0-5 μ sec) of a similar loss event. In that work there is 0.1 μ sec diagnostic resolution; it should serve as a complement to the work at LASL which reaches containment time of 40 μ sec in the 100 cm long device.

3. Multiple Mirror Studies

--Abstract--

The problem of containing particles and energy at the ends of a linear theta pinch has taken on a new perspective with the recent results on Scylla IV-P. Enhanced particle confinement was evident with solid end plugs. However, the advances in understanding energy loss to the plugs do not generate optimism for their eventual usefulness in energy stoppering. Boundary effects do seem to provide the ultimate constraint.

Consideration has been given to the identification of the confinement characteristics of a theta pinch system with multiple mirrors. The results of Lichtenberg, Logen et al.⁸⁰ at Berkeley and Livermore has been considered in detail. While their work is providing substantial new information on one plasma regime (low density, short-pulse, diffuse), the stability and confinement characteristics of higher β and higher density plasma is very much of interest but poorly understood. Also, detailed diagnosis of the plasma in present experiments could be improved.

Preliminary calculations with a 500 eV plasma in Scylla IV-P indicate that four 50 cm mirror cells at each end of the device would reduce particle loss by about one order of magnitude. The energy loss problem could be seriously researched at the ends of such a device because of the low density conditions; specifically, the possibility that sheath formation will inhibit energy loss has been advanced by Hobbs and Wesson and appropriate experiments could be conducted. There is also the possibility of carrying out several unique physics experiments to understand energy loss. The possibilities of heating the plasma in one of the reduced density mirror cells deserves attention for several reasons. First, several hundred eV increases in electron temperature are easily generated by DC turbulent or EM wave heating in such a plasma. The generation of anomalous thermal conductivity could be studied; also, the distribution of the energy added to the electrons between the pinch column plasma and that plasma in the end vessel is of interest. Further, the present theoretical solutions for thermal loss from a system are limited by the constraint of zero temperature at the end wall. A heating experiment could significantly alter this boundary condition for the plasma in the confined column, and allow the realistic exploration of alternative axial temperature profile models that could lead to enhanced energy confinement.

XI. REVIEW OF FACULTY AND STUDENT PARTICIPATION

T. M. York	Aug 1977 FT 3 weeks
Associate Progress of Aerospace Engineering	Sept-May (sabbatical)
E. H. Klevans	June 1977 FT 1 week
Professor of Nuclear Engineering	July 1977 FT 4 weeks
	Sept 1978-May 1978 25%
John Heidrich	June 1977-May 1978 50%
Graduate Asst., Physics	
Elmer Stover	June 1977-May 1978 50%
Graduate Asst., Nuc. E.	
Barry Jacoby	June 1977-May 1978 50%
Graduate Asst., Nuc. E.	
Robert Freeman	June 1977-Aug 1977 50%
Graduate Asst., Nuc. E.	
Amhad Awwad	Sept. 1977-May 1978 50%
Graduate Asst., Nuc. E.	
Vincenzo Veglia	June 1977-Aug 1977 50%
Graduate Asst., Nuc. E.	
ERDA Traineeship	Sept 1977-May 1978 0%

XII. PROJECT REFERENCES

1. Reports

- a. T. M. York, "Scaling of End Loss Times in Linear Theta Pinches," ERDA C00-4020-1, Sci. Rept. 77-1, The Pennsylvania State University, January 1977.
- b. E. H. Klevans and R. Ankney, "Machine Parameters for Linear Theta Pinch Fusion Reactors," ERDA C00-4020-2, Sci. Rept. 77-2, The Pennsylvania State University, January 1977.
- c. T. M. York and E. H. Klevans, "A Study of Flow and Loss Processes at the Ends of a Linear Theta Pinch," Progress Report for the period June 1, 1976-May 31, 1977, ERDA C00-4020-3, Sci. Rept. 77-3, The Pennsylvania State University, Feb. 1977.
- d. R. S. Freeman, "Studies of End Loss From a Theta Pinch Using a Twyman-Green Interferometer," ERDA C00-4020-4, Sci. Rept. 77-4, The Pennsylvania State University, November 1977.
- e. E. K. Stover, E. H. Klevans and T. M. York, "Computer Modeling of Linear Theta Pinch Machines," ERDA C00-4020-5, Sci. Rept. 77-5, The Pennsylvania State University, November 1977.

2. Publications

- a. E. H. Klevans and R. Ankney, "Machine Parameters for Linear Theta Pinch Reactors," Proceedings of the Second A.N.S. Topical Meeting on the Technology of Controlled Fusion, Richland, Wash., 1976, p. 987.
- b. E. K. Stover, E. H. Klevans and T. M. York, "Computer Modeling of Linear Theta Pinch Machines," accepted for publication in Phys. Fluids.

3. Presentations

- a. E. H. Klevans with R. Ankney, "Machine Parameters for Linear Theta Pinch Fusion Reactors," paper presented at Second Topical Meeting on the Technology of Controlled Nuclear Fusion, September 21-23, 1976, Richland, Washington.
- b. T. M. York with E. H. Klevans, "Scaling of Particle and Energy End Loss in Linear Theta Pinches," presented at 18th Annual Meeting of the American Physical Society, Division of Plasma Physics, November 15-19, 1976, San Francisco, California.
- c. E. Stover, E. H. Klevans and T. M. York, "Computer Modeling of Linear Theta Pinch Machines," presented at IEEE International Conference on Plasma Science, May 23-25, 1977, Troy, New York.

- d. T. M. York, R. Freeman and R. Mollo, "Evaluation and Scaling of Plasma Loss from a Very Short Theta Pinch," presented at IEEE International Conference on Plasma Science, May 23-25, 1977, Troy, New York.
- e. E. K. Stover, E. H. Klevans and T. M. York, "Simple Theta Pinch Implosion Dynamics Models," presented at Division of Plasma Physics meeting of the American Physical Soc., Nov. 7-11, 1977, Atlanta, GA.
- f. J. E. Heidrich, T. M. York, J. W. Robinson and E. H. Klevans, "Early Transient Flow at the End of a Linear Theta Pinch," presented at 19th Annual Meeting of the American Physical Society, Division of Plasma Physics, November 7-11, 1977, Atlanta, Georgia.
- g. B. A. Jacoby, T. M. York and R. A. Mollo, "A Doubled Nd-Glass Laser System for Low Density Incoherent Thomson Scattering," presented at 2nd Topical Conference on High-Temperature Plasma Diagnostics, March 1-3, 1978, Sante Fe, New Mexico.
- h. B. A. Jacoby and T. M. York, "Proposed Method of Measuring Directed Electron Velocities Using the Incoherent Regions of Laser Scattering," presented at 2nd Topical Conference on High-Temperature Plasma Diagnostics, March 1-3, 1978, Sante Fe, New Mexico.

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