*Implementation and Assessment of the
Mechanical-Threshold-Stress Model
Using the EPIC2 and PINON Computer Codes*

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*P. J. Maudlin
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IMPLEMENTATION AND ASSESSMENT OF THE MECHANICAL-THRESHOLD-STRESS MODEL USING THE EPIC2 AND PINON COMPUTER CODES

by

P. J. Maudlin, R. F. Davidson, and R. J. Henninger

ABSTRACT

A flow-stress constitutive model based on dislocation mechanics has been implemented in the EPIC2 and PINON continuum mechanics codes. This model provides a better understanding of the plastic deformation process for ductile materials by using an internal state variable called the mechanical threshold stress. This kinematic quantity tracks the evolution of the material's microstructure along some arbitrary strain, strain-rate, and temperature-dependent path using a differential form that balances dislocation generation and recovery processes. Given a value for the mechanical threshold stress, the flow stress is determined using either a thermal-activation-controlled or a drag-controlled kinetics relationship.

We evaluated the performance of the Mechanical Threshold Stress (MTS) model in terms of accuracy and computational resources through a series of assessment problems chosen to exercise the model over a large range of strain rates and strains. Our calculations indicate that the more complicated MTS model is reasonable in terms of computational resources when compared with other models in common hydrocode use. In terms of accuracy, these simulations show that the MTS model is superior for problems containing mostly normal strain with shear strains less than 0.2 but perhaps not as accurate for problems that contain large amounts of shear strain.

I. INTRODUCTION

Continuum mechanics computer codes for large deformation problems contain a material strength formulation that typically can be subdivided into five parts: (1) a linear stress/ strain relationship for the elastic behavior of the material, (2) a yield surface that specifies the state of multiaxial stress corresponding to the start of plastic flow, (3) a flow rule that relates strain increments to stress increments during plastic flow after yielding, (4) a consistency statement that constrains the multiaxial stress to remain on the yield surface during plastic flow, and (5) a hardening rule that describes how the yield surface is modified during plastic flow. Part 5 typically is implemented in continuum mechanics codes (hydrocodes) such as HULL,¹ EPIC,² and PINON* with a rather simple and/or empirical *flow-stress model* that specifies the maximum stress level that can be supported by the material as a function of strain, strain rate, temperature, and pressure during plastic flow. The term "flow stress" is used in this report to refer to a measure of strength for a changing material microstructure during plastic flow; the term "yield stress" refers to a constant material microstructure measure of strength just at the onset of plastic flow. The yield stress can be interpreted as the initial value of the flow stress.

Several new flow-stress models have been developed recently^{3,4} that attempt to provide a better understanding of the plastic deformation process at the macroscopic level based on dislocation mechanics. One of these more physically based models, the Mechanical Threshold Stress (MTS) model,⁴⁻⁷ introduces an internal state variable called the mechanical threshold stress ($\hat{\sigma}$) that can be interpreted as the material flow stress at 0 K. This state variable describes the lattice structure evolution using a balance of dislocation generation and recovery processes that depend on strain rate, temperature, and pressure. Our assessment of the MTS model for practical use in hydrocodes used for armor/anti-armor problems is discussed in this report. Any new flow-stress model, even if it represents an improvement in the understanding of deformation processes, is suitable for hydrocode calculations only if it improves accuracy and does not create numerical instabilities or use excessive computer resources.

The purpose of our work is technology transfer, that is, promoting the use of new material models being developed by material scientists to an engineering community that hopefully will benefit from using these models in their hydrocode analyses. In this work, this means incorporating the MTS model into the Lagrangian EPIC2 code and the

*PINON is a proprietary computer code in use at Los Alamos.

Eulerian PINON code and then using these two hydrocodes to evaluate the performance of the MTS model in terms of accuracy and use of computational resources through a series of assessment problems. These problems were chosen based on (1) our desire to exercise the model over a large range of strains and strain rates and (2) the availability of associated experimental data for comparison with the computational results. We also compare the Johnson-Cook (JC)⁸ flow-stress model, a mature empirical model that currently is used in both EPIC2 and PINON, with the MTS model.

Before the computational assessment of the MTS model is presented, we will review the MTS model equations briefly (Sec. II). We then will discuss a sensitivity analysis of the model parameters (Sec. III). Unlike most approaches for obtaining parameter sensitivities, the analysis in Sec. III uses Differential Sensitivity Theory.⁹⁻¹¹ The implementation of the MTS model in the EPIC2 and PINON codes is discussed in Sec. IV. Section V presents the computational assessment of the MTS model using Taylor cylinder tests,¹² normal flyer-plate tests,¹³ torsion tests,¹⁴ an EFP formation,¹⁵ and a set of inclined flyer-plate tests.¹⁶ Section VI provides a summary and our conclusions.

II. THE MTS MODEL

As stated in Sec. I, the MTS model focuses on the calculation and subsequent use of an internal state variable called the mechanical threshold stress.⁴ This variable describes the evolution of the strength of the lattice structure as a function of equivalent plastic strain (ϵ), equivalent plastic strain rate ($\dot{\epsilon}$), temperature (T), and pressure (P):

$$\hat{\sigma} = \hat{\sigma}[\epsilon, \dot{\epsilon}, T, P] . \quad (\text{II-1})$$

It is used to determine the flow stress as illustrated with the following functional form:

$$\sigma = \sigma[\hat{\sigma}, \dot{\epsilon}, T, P] . \quad (\text{II-2})$$

Most empirical models for flow stress that are intended for use in continuum mechanics codes (for example, the JC model⁸) are constructed by fitting data from macroscopic material tests into a form in which the flow stress depends explicitly on strain:

$$\sigma = \sigma[\epsilon, \dot{\epsilon}, T, P] \quad . \quad (II-3)$$

The physical importance of using a form like Eq. (II-2) with the flow stress being a function of $\hat{\sigma}$ instead of ϵ is based on the argument that the threshold stress, rather than strain, provides the proper description of how the structure evolves; strain is not a state variable and has no relationship to material properties. More discussion on this point can be found in Ref. 4.

The MTS equation set represents a phenomenological model based on dislocation mechanics in the sense that it macroscopically models contributions to the flow stress using dislocation interactions with barriers (such as other dislocations, interstitial and solute atoms, grain boundaries, and so on). Each of these interaction contributions is quantified by an individual threshold stress, $\hat{\sigma}_i$. The MTS model also contains a mechanism for modifying these individual threshold stresses based on the controlling kinetics of deformation. Two kinetics regimes currently are included with the model: a thermal-activation-controlled process, where the time that a dislocation must wait to overcome a barrier is kinetically dominant, and a drag-controlled process, where the transport time between barriers is dominant.

A. Equation Set

The equation set presented here is not meant to be a detailed formulation of the model as developed in the literature by Kocks and Follansbee.⁵ It simply documents the equations actually implemented in the EPIC2 and PINON hydrocodes.

The MTS flow-stress relationship [Eq. (II-2)] can be expressed in general form as a superposition of $n+1$ threshold-stress contributions to σ , where each contribution represents a dislocation interaction with some barrier "i":

$$\sigma = \hat{\sigma}_a + \sum_{i=1}^n \hat{\sigma}_i s_i(\dot{\epsilon}, T, P) \quad . \quad (II-4)$$

The summed product in Eq. (II-4) separates the contribution from interaction i into a structure evolution term ($\hat{\sigma}_i$) modified with a constant-structure deformation term (s_i) that is mainly a function of temperature and strain rate. The athermal threshold stress ($\hat{\sigma}_a$) represents dislocation interactions with long-range barriers, such as grain boundaries, and typically is assumed to be constant; however, its value depends on grain size. As indicated in more detail below, s_i is equal to unity at 0 K, and Eq. (II-4) simplifies to a simple

sum of mechanical threshold stresses. Also, when a material is being subjected to constant-structure yield-stress measurements at various strain rates and temperatures, the $\hat{\sigma}_i$ are constants.

The complementary equation to Eq. (II-4) is a differential hardening law described by

$$\frac{\partial \hat{\sigma}}{\partial \epsilon} = \Theta_0 [1 - F(X)] \text{ for interaction } i, \quad (\text{II-5})$$

where

$$X = \frac{\hat{\sigma}}{\hat{\sigma}_s}. \quad (\text{II-6})$$

The subscript i has been dropped from the above equations for convenience, but it is implied. In Eq. (II-5), Θ_0 represents hardening as a result of dislocation generation, and the product $\Theta_0 \cdot F$ represents softening resulting from dislocation recovery. The threshold stress at zero strain hardening, $\hat{\sigma}_s$, is called the "saturation threshold stress." If the function F is taken to be linear in terms of X , then the so-called Voce⁷ behavior for strain hardening is assumed.

The equation for $\hat{\sigma}$ given by Eq. (II-5) is left in differential form because the practical application of the MTS model assumes that arbitrary strain-rate and temperature histories are to be used. Also, $\hat{\sigma}$ is interpreted as a flow stress at 0 K even though the structure evolution represented by Eq. (II-5) contains a thermal history. In principle, a differential equation like Eq. (II-5) must be solved for each dislocation-barrier interaction that requires evolution; in practice, most of these $\hat{\sigma}_i$ are taken as constants except for the dislocation-dislocation interaction.

The relationships for Θ_0 , F , and $\hat{\sigma}_s$ are material-specific and are listed in Table I for the several materials currently characterized for the MTS model. For completeness, the relationships for copper are used here to finish the discussion of Eq. (II-5). Thus, for copper, the model uses

$$F(X) = \frac{\tanh(2X)}{\tanh(2)} \quad (\text{II-7})$$

TABLE I

MTS MODEL MATERIAL-SPECIFIC RELATIONSHIPS

Material	Dislocation Generation Rate	F(X)	Saturation Threshold Stress
Copper	$\Theta_0 = a_0 + a_1 \ln \dot{\epsilon} + a_2 \dot{\epsilon}$	$F(X) = \frac{\tanh(2X)}{\tanh(2)}$	$\hat{\sigma}_s = \hat{\sigma}_{s0} \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{s0}} \right)^{\frac{kT}{A\mu b^3}} - \hat{\sigma}_a$
Ti-6Al-4V	$\Theta_0 = 2721 \text{ MPa}$	$F(X) = X$	$\hat{\sigma}_s = 538 \text{ MPa}$
Depleted Uranium	$\Theta_0 = \Theta_{0s} \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{s0}} \right)^{\frac{kT}{B}}$	$F(X) = X$	$\hat{\sigma}_s = \hat{\sigma}_{s0} \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{s0}} \right)^{\frac{kT}{A\mu b^3}}$

and a dislocation generation rate expressed as a function of strain rate,

$$\Theta_0 = a_0 + a_1 \ln \dot{\epsilon} + a_2 \sqrt{\dot{\epsilon}}, \quad (II-8)$$

where the a_j are constants and Θ_0 has a maximum value constrained to the shear modulus. The saturation threshold stress is next expressed as a function of strain rate, temperature, and pressure (the pressure dependence is from the shear modulus, $\mu[T,P]$):

$$\hat{\sigma}_s = \hat{\sigma}_{s0} \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{s0}} \right)^{\frac{kT}{\mu b^3 A}} - \hat{\sigma}_a, \quad (II-9)$$

where k is Boltzmann's constant, b is the magnitude of Burgers vector (the interatomic distance in the slip direction), $\hat{\sigma}_{s0}$ is the saturation threshold stress at 0 K, $\dot{\epsilon}_{s0}$ is a reference strain rate, and the quantity A is a material constant.

After determining the $\hat{\sigma}_i$, the MTS model evaluates the flow stress, depending on which of two deformation mechanisms is controlling the plastic flow. For thermal-activation-controlled deformation, σ is evaluated with s_i ($s_i \equiv \sigma / \hat{\sigma}$) determined through an Arrhenius rate equation:

$$\dot{\epsilon} = \dot{\epsilon}_0 e^{-\Delta G/kT} \text{ for interaction } i, \quad (\text{II-10})$$

where the activation energy ΔG is expressed as

$$\Delta G = \mu b^3 g_0 \left(1 - s^p\right)^q \text{ for interaction } i. \quad (\text{II-11})$$

The exponents p and q are material constants, and g_0 is a normalized activation energy for a given dislocation/obstacle interaction, which is assumed to be constant. Replacing ΔG in Eq. (II-10) with Eq. (II-11) and solving for s_i yields a closed-form relationship for the constant-structure deformation variable:

$$s_t = \left[1 - \left(\frac{kT \ln(\dot{\epsilon}_0/\dot{\epsilon})}{\mu b^3 g_0} \right)^{1/q} \right]^{1/p} \text{ for interaction } i, \quad (\text{II-12})$$

where the subscript t denotes thermal activation deformation. Although this form for s_{ti} is assumed to be valid for different barriers, the value of the parameters (q , p , $\dot{\epsilon}_0$, and g_0) in Eq. (II-12) can be different depending on the type of dislocation-barrier interaction (i). See Ref. 5 for more detail concerning the development of Eqs. (II-4) through (II-12).

For drag-controlled deformation, s is assumed independent of obstacle and is evaluated with a modified Arrhenius rate equation:

$$\dot{\epsilon} = \dot{\epsilon}_0 \left[\frac{MB(\sigma)\alpha\mu v_0}{\sigma \hat{\sigma}} + e^{\Delta G/kT} \right]^{-1}, \quad (\text{II-13})$$

where the first term inside the brackets describes the drag contribution. The new quantities appearing in this term include a drag coefficient $B(\sigma)$; M is the Taylor factor, α is a constant equal to $1/2$, and v_0 is the jump or attempt frequency. The drag coefficient is a function of flow stress:

$$B = \sqrt{B_0^2 + \left(\frac{b\sigma}{C_s M} \right)^2}, \quad (\text{II-14})$$

where B_0 is a drag constant and C_s is the shear wave velocity determined from the shear modulus and material density:

$$C_s = \sqrt{\mu/\rho} . \quad (II-15)$$

Replacing ΔG in Eq. (II-13) with Eq. (II-11) and solving for s yields a relationship that is a function of σ :

$$s_d = \left\{ 1 - \left[\frac{-kT}{\mu b^3 g_0} \ln \left(\frac{\dot{\epsilon}_0}{\dot{\epsilon}} - \frac{MB(\sigma)\alpha\mu v_0}{\sigma \hat{\sigma}} \right) \right]^{1/q} \right\}^{1/p} , \quad (II-16)$$

where the subscript d denotes drag-controlled deformation. Using this constant-structure deformation form in Eq. (II-4) to determine flow stress results in a transcendental equation in σ that formally requires iteration to obtain a solution. Refer to Ref. 6 for more details concerning the formulation of Eqs. (II-13) through (II-16).

The transition from thermal-activation-controlled deformation at lower strain rates to drag-controlled deformation at high strain rates is based on a strain-rate criterion. This criterion is obtained by setting σ equal to $\hat{\sigma}$ in Eq. (II-13) and solving for a transition strain rate:

$$\dot{\epsilon}_{tr} = \dot{\epsilon}_0 \left[\frac{MB(\hat{\sigma})\alpha\mu v_0}{\hat{\sigma} \hat{\sigma}} + 1 \right] . \quad (II-17)$$

Therefore, for problem strain rates less than $\dot{\epsilon}_{tr}$, the flow-stress calculation represented by Eq. (II-4) assumes the thermal activation regime, and Eq. (II-12) is used. For strain rates greater than $\dot{\epsilon}_{tr}$, the drag regime is assumed, and Eq. (II-6) is used. Note that these equations provide for a smooth transition between regimes rather than a discontinuous jump. Typical values for $\dot{\epsilon}_{tr}$ range from about 10^4 to $5 \times 10^5 \text{ s}^{-1}$ for copper. Table II contains copper values for all the MTS parameters appearing in the equations discussed above.

The two other materials currently characterized for the MTS model that also are included in the hydrocode libraries are the titanium alloy Ti-6Al-4V¹⁷ and a depleted alpha uranium.¹⁸ Tables III and IV list the MTS model parameters for these materials.

TABLE II

NOMINAL SET OF MTS MODEL PARAMETERS FOR COPPER

Parameter α	Parameter Description	Nominal Value α_0
$\hat{\sigma}_s$	Rate Independent Threshold Stress	40.0 MPa
g_0	Normalized Activation Energy	1.6
$\dot{\epsilon}_0$	Thermal Activation Equation Constant	10^7 s^{-1}
b	Magnitude of Burgers Vector	2.55 Å
A	Saturation Stress Equation Constant	0.312
ρC_p	Density-Heat Capacity Product	3.82 MPa/K
T_0	Initial Temperature	300 K
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress (dislocation)	0 MPa
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	900 MPa
$\dot{\epsilon}_{s0}$	Saturation Stress Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$
p	Free Energy Equation Exponent	$\frac{2}{3}$
q	Free Energy Equation Exponent	1
$[a_0, a_1, a_2]$	Hardening Function Constants	2370.7, 8.295, 3.506
ν_0	Attempt Frequency	10^{11} s^{-1}
B_0	Drag Constant	43 $\mu\text{Pa s}$
M	Taylor's Factor	3.1

TABLE III

NOMINAL SET OF MTS MODEL PARAMETERS FOR Ti-6Al-4V

Parameter α	Parameter Description	Nominal Value α_0
$\hat{\sigma}_a$	Rate Independent Threshold Stress	100. MPa
g_0	Normalized Activation Energy (dislocation)	1.6
$\dot{\epsilon}_0$	Thermal Activation Eqn Constant (dislocation)	10^7 s^{-1}
b	Magnitude of Burgers Vector	2.55 Å
ρC_p	Density-Heat Capacity Product	2.97 MPa/K
T_0	Initial Temperature	300 K
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress (dislocation)	0 MPa
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	538. MPa
$\dot{\epsilon}_{s0}$	Saturation Stress Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$
p	Free Energy Exponent (dislocation)	$\frac{2}{3}$
q	Free Energy Exponent (all barriers)	1
$\hat{\sigma}_i$	Initial Mechanical Threshold Stress (interstitial)	1050. MPa
$\hat{\sigma}_s$	Initial Mechanical Threshold Stress (solute)	873. MPa
g_i	Normalized Activation Energy (interstitial)	0.264
g_s	Normalized Activation Energy (solute)	0.8
$p_{i,s}$	Free Energy Exponent (interstitial and solute)	2
$\dot{\epsilon}_{i,s}$	Thermal Activation Constant (interstitial and solute)	10^{10} s^{-1}

TABLE IV

**NOMINAL SET OF MTS MODEL PARAMETERS FOR DEPLETED
ALPHA URANIUM**

Parameter α	Parameter Description	Nominal Value α_0
$\hat{\sigma}_a$	Rate Independent Threshold Stress	60. MPa
g_0	Normalized Activation Energy	0.23
$\dot{\epsilon}_0$	Thermal Activation Equation Constant	10^7 s^{-1}
b	Magnitude of Burgers Vector	2.85 Å
A	Saturation Stress Equation Constant	0.176
ρC_p	Density-Heat Capacity Product	2.14 MPa/K
T_0	Initial Temperature	300 K
$\hat{\sigma}_0$	Initial Threshold Stress (dislocation+Peierls barrier)	600 MPa
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	2840 MPa
$\dot{\epsilon}_{s0}$	Saturation Stress Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$
p	Free Energy Equation Exponent	$\frac{2}{3}$
q	Free Energy Equation Exponent	1
B	Hardening Function Constant	$4.78 \times 10^{-25} \text{ MNm}$
Θ_{0s}	Hardening Function Constant	8072 MPa

The flow-stress equation [Eq. (II-4)] and threshold stress differential equation [Eq. (II-5)] for depleted uranium are the same as the equations for copper discussed above, except that the drag-controlled kinetics regime in uranium has not been included and the material specific relationships are somewhat different. (See Table I.)

The flow-stress equation for Ti-6Al-4V is more complicated because there are three barrier interactions [$n = 3$ for the summation in Eq. (II-4)]. Instead of just a single dislocation-dislocation interaction, dislocation-interstitial and dislocation-solute atom interactions are modeled as well. However, only the dislocation barrier has structure evolution, which means that only one threshold stress differential equation (DE) [Eq. (II-5)] is needed; the other two threshold stresses are assumed to be constants. The drag-controlled kinetics regime in Ti-6Al-4V also has not been derived. Table I contains the material-specific relationships.

B. Numerical Illustration

Using the equations described above and the material constants for copper shown in Table II, Fig. 1 shows a three-dimensional perspective view of flow stress plotted as a function of strain and strain rate with the temperature and pressure held constant at 300 K and 1 bar, respectively. Note that σ tends to plateau to perfectly plastic behavior at large strains and hardens rapidly with increasing strain rate at low strains. The sharp increase in flow stress observed in Fig. 1 at large strain rates and low strains is produced by the drag term in Eq. (II-16), which physically dominates the $\dot{\epsilon}_0/\dot{\epsilon}$ term. (See Ref. 6.)

The temperature dependence of the MTS model is shown in Fig. 2, where σ is plotted as a function of strain and temperature with the strain rate and pressure held constant at 1 s^{-1} and 1 bar, respectively. This surface shows the material softening to near-zero strength close to the copper melting point of 1356 K. Possible negative values of flow stress produced by the model close to the melting point are set to 0 in the hydrocode implementation.

The isothermal results shown in Fig. 1 assume that any internal energy increase in the material as a result of plastic work is removed instantaneously. A more realistic example for armor/anti-armor problems is an adiabatic stress-strain problem where the material temperature is estimated from the plastic work:

$$T = T_0 + \frac{1}{\rho C_p} \int_0^{\epsilon_f} \sigma \, d\epsilon \quad , \quad (\text{II-18})$$

Fig. 1.

Perspective of the MTS flow stress for isothermal Cu plotted as a function of strain and $\log_{10}$ of the strain-rate. The flow-stress spike appearing at small strains and large strain-rates is a result of the drag-controlled deformation regime.

Fig. 2.

Perspective of the MTS flow stress for Cu plotted as a function of strain and temperature with the strain rate held constant at 1.0 s^{-1} . Note that the flow stress drops to a near-zero value at the melting point of Cu.

where ϵ_f is the final strain realized in the problem, ρ is the material density, and C_p is the material heat capacity. This equation assumes that all of the plastic work is converted into internal energy. Solving Eq. (II-18) simultaneously with the above MTS model equations yields the adiabatic results shown in Fig. 3. Here σ again is plotted as a function of strain and strain rate with the pressure held constant at 1 bar. The effect of thermal softening on flow stress can be observed by comparing Figs. 1 and 3. The two surfaces are very similar at low strains (less than 50%). At higher strains, the adiabatic flow stress reaches

Fig. 3.
Perspective of the MTS model flow stress for an adiabatic Cu problem plotted as a function of strain and $\log_{10}$ of the strain rate. Again, note the drag-controlled deformation spike and the surface dropping off at large strains as a result of thermal softening.

a maximum and then decreases with increasing strain, deviating substantially from the isothermal flow-stress results. The effect of thermal softening in this adiabatic problem is to "wash out" the importance of strain-rate hardening in the isothermal flow-stress of Fig. 1. Figure 4 presents the Fig. 3 results in a more quantitative, two-dimensional form.

Figures 5 and 6 show adiabatic flow-stress surfaces for the titanium alloy (Ti-6Al-4V) and depleted uranium, respectively.

C. Comparison with the Johnson-Cook Model

To highlight some of the differences between the MTS model and other established flow-stress models, we compared the MTS model with the JC model for copper. The JC model currently exists either as the default flow-stress model or as an option in the

Fig. 4.
A family of MTS model flow stress vs strain curves at various strain rates for an adiabatic Cu problem. These curves provide a more quantitative representation of the Fig. 3 results.

Fig. 5.

Perspective of the MTS flow stress for an adiabatic Ti-6Al-4V problem plotted as a function of strain and $\log_{10}$ of the strain rate. Note the strong thermal softening trend at large strains compared with Cu in Fig. 3; the higher stress levels and lower value of ρC_p for Ti-6Al-4V produce higher temperatures for a given value of strain.

Fig. 6.

Perspective of the MTS model flow stress for an adiabatic depleted U problem plotted as a function of strain and $\log_{10}$ of the strain rate.

Again, the higher stress levels and the lower value of ρC_p for U compared with Cu in Fig. 3 produce high temperatures and strong thermal softening for a given value of strain.

EPIC codes, the DYNA3D code,¹⁹ and the PINON code. The simple (and computationally fast) algebraic form of the JC model is

$$\sigma = (A + B \epsilon^n) (1 + C \ln \dot{\epsilon}^*) (1 - T^{*m}), \quad (II-19)$$

where

$$\dot{\epsilon}^* = \left(\frac{\dot{\epsilon}}{1 \text{ s}^{-1}} \right) \text{ and} \quad (II-20)$$

$$T^* = \left(\frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}} \right). \quad (II-21)$$

The first factor in Eq. (II-19) represents strain hardening with A interpreted as the initial yield stress, B as a strain-hardening coefficient, and n as a strain-hardening exponent. The second factor accounts for strain-rate hardening effects, with C being a strain-rate hardening coefficient. The last factor represents thermal softening with m being a softening exponent.

Using Eqs. (II-19) through (II-21) and the JC model material constants for copper shown in Table V, Fig. 7 shows a three-dimensional perspective view of the flow stress plotted as a function of strain and strain rate with the temperature and pressure again held constant at 300 K and 1 bar, respectively. In comparing Figs. 1 and 7, note that the JC model continues to strain-harden at large strains as opposed to the more perfectly plastic behavior of the MTS model. The strain-rate hardening in Fig. 7 is linear on this log axis [see Eq. (II-19)] as compared with the more rapid hardening that is inherent in the MTS model and shown in Fig. 1. Based on the isothermal hardening characteristics of the two models, the MTS model can be expected to yield harder flow stress at small strains and large strain rates and softer flow stress at large strains relative to the JC model.

Turning now to the adiabatic stress-strain problem discussed in Sec. II.B, solving Eqs. (II-18) and (II-19) simultaneously produces the adiabatic JC model results shown in three-dimensional perspective in Fig. 8 and (more quantitatively) Fig. 9. Again, σ is plotted as a function of strain and strain rate with the pressure held constant at 1 bar. The effect of thermal softening on the JC model can be seen by comparing Figs. 7 and 8. At small strains, the two surfaces are very similar. At larger strains, the adiabatic flow stress assumes a near perfectly plastic behavior, whereas the isothermal flow stress continues

TABLE V

NOMINAL SET OF JC MODEL PARAMETERS FOR COPPER

Parameter α	Parameter Description	Nominal Value α_0
A	Initial Yield Stress	89.6 MPa
B	Strain Hardening Coefficient	291.6 MPa
C	Strain-Rate Hardening Coefficient	0.025
n	Strain Exponent	0.31
m	Temperature Exponent	1.09
ρC_p	Density-Heat Capacity Product	3.82 MPa/K
T	Room Temperature	294 K
T_{melt}	Melting Temperature	1356 K

to harden. Comparing Figs. 3 and 7 (or Figs. 4 and 9) shows that the MTS model produces harder flow stress at large strain rates and comparable values of flow stress at large strains relative to the JC model.

As a further illustration of the differences between models, Fig. 10 shows stress-strain data from a prestrain/reload Split-Hopkinson Bar/quasi-static uniaxial compression test series compared with flow-stress predictions from the MTS, JC, and Zerilli-Armstrong³ models. In these tests, copper samples (initially annealed at room temperature) are subjected to normal compressive stress and are strained to 15% at a strain rate of 10^4 s^{-1} . The samples then are unloaded and reloaded to a strain of 25% at a strain rate of 10^{-3} s^{-1} . These three model predictions were expected to compare well with the data for the

Fig. 7.

Perspective of the JC flow stress for isothermal Cu plotted as a function of strain and $\log_{10}$ of the strain rate. Note that the strain-rate dependence of the flow stress is linear as observed in Eq. (II-19).

Fig. 8.

Perspective of the JC flow stress for an adiabatic Cu problem plotted as a function of strain and $\log_{10}$ of the strain rate. Note that the surface drops off at large strains as a result of thermal softening.

Fig. 9.

A family of JC flow stress vs strain curves at various strain rates for an adiabatic Cu problem. These curves provide a more quantitative representation of the Fig. 8 results and are much softer than the Fig. 4 results.

prestrain; the MTS model was expected to do best for the reload because the prestrain history of the samples is recorded within the value of the mechanical threshold stress by solving Eq. (II-5). Figure 10 shows that the MTS model accurately predicts the prestrain and the reload parts of the problem. The JC model overpredicts the prestrain flow stress by 25 to 30% and underpredicts the reload by roughly the same amount. The Zerilli-Armstrong model overpredicts the prestrain by 5 to 10% and underpredicts the reload by 30%.

Fig. 10.

A comparison of flow-stress model predictions (the MTS, JC, and Zerilli-Armstrong models) with experimental data from uniaxial compression tests for a prestrain/reload problem. Here a Cu sample is strained to 15% at a high strain rate, unloaded, reloaded, and strained out to 25% at a low strain rate. Only the MTS model uses the prestrain deformation history to predict a flow-stress magnitude during the reload phase.

III. SENSITIVITY ANALYSIS OF THE MTS MODEL

The sensitivity analysis discussed here is an application of Differential Sensitivity Theory⁹ (DST) to the MTS model cast in a simple adiabatic stress-strain problem. Using DST for any physical problem involves deriving and solving equations that are adjoint to the physical equations differentiated with respect to an arbitrary parameter, α . The physical and adjoint equations then are integrated to form the sensitivities, $\partial R / \partial \alpha$, where R is an important output quantity or response (for example, a peak or average stress, strain, or

temperature). The main advantage of DST over statistical²⁰ and direct* methods for sensitivity analysis is that sensitivities to all the parameters in the physical equation set or computer code can be obtained with just two calculations, the physical base-case calculation and its corresponding adjoint calculation. Note that the sensitivities produced by DST are exact derivatives as opposed to the finite differences, $\Delta R/\Delta\alpha$, constructed from a statistical or direct method.

A. Physical Problem Equations

Four equations are used to describe the time-dependent adiabatic stress/strain problem used for this MTS sensitivity study. First, an arbitrary strain-rate history is specified with time t being the independent variable:

$$\frac{d\epsilon}{dt} = \dot{\epsilon} = f(t). \quad (\text{III-1})$$

Second, the mechanical threshold stress differential equation discussed in Sec. II.A is expressed in terms of time using the chain rule:

$$\frac{\partial \hat{\sigma}}{\partial t} = \frac{\partial \hat{\sigma}}{\partial \epsilon} \frac{d\epsilon}{dt} = \Theta_0 [1 - F(X)] \dot{\epsilon}. \quad (\text{III-2})$$

Third, the flow-stress equation for the thermal-activation kinetics regime from Sec. II.A is used without modification:

$$\sigma = \hat{\sigma}_a + \hat{\sigma} \left[1 - \left(\frac{kT \ln(\dot{\epsilon}_0/\dot{\epsilon})}{\mu b^3 g_0} \right)^{1/q} \right]^{1/p}. \quad (\text{III-3})$$

Fourth, the temperature-work relation from Sec. II.B is expressed in differential form and in terms of time again using the chain rule:

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \epsilon} \frac{d\epsilon}{dt} = \frac{\sigma \dot{\epsilon}}{\rho C_p}. \quad (\text{III-4})$$

*Direct Method: perturb a parameter by some $\Delta\alpha$ and recalculate using the computer code or model to obtain the change in the response ΔR .

It should be obvious from the above equation set that only the thermal-activation regime is included in this analysis. Note that the copper results given by Fig. 4 represent a family of solutions to the above system of equations for the case of $f(t)$ equal to a constant.*

The next item of definition is a response function (R) for the sensitivity coefficients. A general form can be cast as an integral over the time domain 0 to t_f of a function F that depends on the physical variables' equivalent plastic strain, threshold stress, flow stress, and temperature:

$$R = \int_0^{t_f} F(\epsilon, \hat{\sigma}, \sigma, T) dt. \quad (III-5)$$

An example of a response (which will be used in Sec. III.B) is the value of the flow stress at some final time t_f :

$$R = \int_0^{t_f} F(\epsilon, \hat{\sigma}, \sigma, T) dt = \int_0^{t_f} \delta(t - t_f) \sigma(t) dt = \sigma(t = t_f), \quad (III-6)$$

where $\delta(t - t_f)$ is the Dirac delta function.²¹

B. Sensitivity Results

The mechanics of applying DST to the Sec. III.A physical equation set will not be shown here; they are given in Ref. 22 for the interested reader. Instead, we concentrate on the presentation of the resulting sensitivity information for a numerical integration of Eqs. (III-1) through (III-4) from time zero out to a final time determined from the strain-rate history:

$$\epsilon_f = \int_0^{t_f} f(t) dt, \quad (III-7)$$

where ϵ_f is chosen arbitrarily.

As mentioned above, the sensitivity coefficients obtained with DST are exact derivatives of the response of interest with respect to the parameter α , $\partial \ln(R) / \partial \ln(\alpha)$.

*This statement is not precisely correct because the Fig. 4 results do include the drag regime.

Tables VI–IX present sensitivities for four different strain-rate histories, where $f = 10, 10^3$, and 10^5 s^{-1} for Tables VI, VII, and VIII and f is given by the EFP strain-rate history shown in Fig. 11 for Table IX. The response of interest is given by Eq. (III-6), which can be interpreted as the value of the flow stress at a final time determined from Eq. (III-7). (ϵ_f for each history is given at the bottom of each corresponding table.)

The information given in Tables VI through IX includes the adiabatic stress-strain problem parameters, nominal values for these parameters, and DST sensitivity coefficients. A few direct sensitivity coefficients are included as verification of the DST coefficients, and an "importance" ranking of the parameters is made based on the relative magnitudes of their sensitivity coefficients. The sensitivities in column 4 of these tables should be interpreted as the per cent change in the flow-stress response for a 1% change in the parameter α . For example, Table VIII indicates that a 0.486% increase in flow stress occurs when the shear modulus is increased by 1%. Table VI is the only table that includes some Direct Method sensitivities for comparison with the DST sensitivities; these two procedures should produce values that are close, but they should not be identical because DST produces exact derivatives and the Direct Method is a ratio of finite differences.

Although the sensitivities and parameter ranking change somewhat with strain rate, Tables VI through VIII indicate that Burgers vector, the shear modulus, the saturation stress constant (A) from Eq. (II-9), and the saturation stress at 0 K are the most important parameters in the MTS model. These tables also indicate that the initial mechanical threshold stress and the rate-independent threshold stress have vanishing effect on numerical evaluations of the MTS model. Trends in parameter importance are also apparent in the tables. Note that the constant " A " decreases in importance as the strain rate is increased, and the reference strain rate for Eq. (II-9) gains importance as the strain rate is increased. The information contained in these sensitivity tables should guide the experimentalist in allocating his resources when quantifying MTS model parameters for new materials.

The Table IX sensitivities are perhaps more realistic for armor applications because a strain-rate history for an element of material from an EPIC2 EFP formation calculation (see Sec. V.D) was used when solving the adiabatic stress-strain problem. This history was introduced above with Fig. 11 and is representative of an interior material particle roughly halfway between the nose and tail of the EFP. Comparing the Table IX sensitivities with the other sensitivity tables indicates that, except for the density-heat capacity product (ρC_p) and the hardening function (Θ_0), the ranking of the parameters changes only slightly. The density-heat capacity product rises to be the second most important

parameter with a sensitivity coefficient of 0.713, which implies that thermal softening in this EFP problem is more dominant than in the previously discussed problems where $f(t)$ function rise in ranking is likely a result of the strain-rates in excess of 10^5 s^{-1} present in the EFP strain-rate history.

IV. IMPLEMENTATION OF THE MTS MODEL INTO A HYDROCODE

Most of the discussion in this section is specific to the material strength formulation used in the EPIC2 and PINON hydrocodes. The model was implemented using the standard Method of Radial Return in conjunction with the von Mises yield function and

Fig. 11.

Strain-rate history for a Lagrangian element of liner material located roughly halfway between the nose and the tail of a Honeywell EFP. See Figs. 27 through 29 for the description of this EFP.

TABLE VI

**SENSITIVITY ANALYSIS OF THE MTS MODEL PARAMETERS FOR A
COPPER ADIABATIC STRESS-STRAIN PROBLEM**

Parameter α	Parameter Description	Nominal Value α_0	$\frac{\partial \ln(R)}{\partial \ln(\alpha)}$ ^a (DST)	$\frac{\Delta R/R}{\Delta \alpha/\alpha}$ (Direct)	Magnitude Ranking
$\hat{\sigma}_a$	Rate Independent Threshold Stress	40.0 MPa	0.0258		11
g_0	Normalized Activation Energy	1.6	0.105	0.0955	8
μ_{sf}	Shear Modulus Scale Factor	1	0.675		2
$\dot{\epsilon}_0$	Arrhenius Constant	10^7 s^{-1}	-0.00759		13
b	Magnitude of Burgers Vector	2.55 Å	2.02	2.14	1
θ_{osf}	Hardening Function Scale Factor	1	-0.0418		10
A	Saturation Stress Constant	0.312	0.570		3
ρC_p	Density-Heat Capacity Product	3.82 MPa/K	0.527	0.524	4
T_0	Initial Temperature	300 K	-0.399	-0.399	6
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress	0 MPa	-0.00652		14
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	900 MPa	0.470		5
$\dot{\epsilon}_{s0}$	Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$	-0.0253		12
$1/p$	Flow Stress Eqn Exponent	$\frac{3}{2}$	-0.0975		9
$1/q$	Flow Stress Eqn Exponent	1	0.207	0.214	7

a: $R = \sigma \left[\epsilon_f = 3, \dot{\epsilon} = 10^1 \text{ s}^{-1}, \vec{\alpha}_0 \right] = 296.419 \text{ MPa}$

TABLE VII

**SENSITIVITY ANALYSIS OF THE MTS MODEL PARAMETERS FOR A
COPPER ADIABATIC STRESS-STRAIN PROBLEM**

Parameter α	Parameter Description	Nominal Value α_0	$\frac{\partial \ln(R)}{\partial \ln(\alpha)}$ ^a (DST)	$\frac{\Delta R/R}{\Delta \alpha/\alpha}$ (Direct)	Magnitude Ranking
$\hat{\sigma}_a$	Rate Independent Threshold Stress	40.0 MPa	0.0143		12
g_0	Normalized Activation Energy	1.6	0.0796		8
μ_{sf}	Shear Modulus Scale Factor	1	0.609		2
$\dot{\epsilon}_0$	Arrhenius Constant	10^7 s^{-1}	-0.00864		13
b	Magnitude of Burgers Vector	2.55 Å	1.83		1
θ_{osf}	Hardening Function Scale Factor	1	-0.0461		10
A	Saturation Stress Constant	0.312	0.529		3
ρC_p	Density-Heat Capacity Product	3.82 MPa/K	0.503		5
T_0	Initial Temperature	300 K	-0.344		6
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress	0 MPa	-0.00611		14
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	900 MPa	0.507		4
$\dot{\epsilon}_{s0}$	Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$	-0.0295		11
$1/p$	Flow Stress Eqn Exponent	$\frac{3}{2}$	-0.0756		9
$1/q$	Flow Stress Eqn Exponent	1	0.182		7

a: $R = \sigma \left[\epsilon_f = 3, \dot{\epsilon} = 10^3 \text{ s}^{-1}, \vec{\alpha}_0 \right] = 350.983 \text{ MPa}$

TABLE VIII

**SENSITIVITY ANALYSIS OF THE MTS MODEL PARAMETERS FOR A
COPPER ADIABATIC STRESS-STRAIN PROBLEM**

Parameter α	Parameter Description	Nominal Value α_0	$\frac{\partial \ln(R)}{\partial \ln(\alpha)}$ ^a (DST)	$\frac{\Delta R/R}{\Delta \alpha/\alpha}$ (Direct)	Magnitude Ranking
$\hat{\sigma}_a$	Rate Independent Threshold Stress	40.0 MPa	0.00713		13
g_0	Normalized Activation Energy	1.6	0.0468		8
μ_{sf}	Shear Modulus Scale Factor	1	0.486		3
$\dot{\epsilon}_0$	Arrhenius Constant	10^7 s^{-1}	-0.0101		12
b	Magnitude of Burgers Vector	2.55 Å	1.46		1
θ_{osf}	Hardening Function Scale Factor	1	-0.0199		11
A	Saturation Stress Constant	0.312	0.570		2
ρC_p	Density-Heat Capacity Product	3.82 MPa/K	0.439		5
T_0	Initial Temperature	300 K	-0.271		6
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress	0 MPa	-0.00230		14
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	900 MPa	0.481		4
$\dot{\epsilon}_{s0}$	Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$	-0.0329		10
$1/p$	Flow Stress Eqn Exponent	$\frac{3}{2}$	-0.0454		9
$1/q$	Flow Stress Eqn Exponent	1	0.134		7

a: $R = \sigma \left[\epsilon_f = 3, \dot{\epsilon} = 10^5 \text{ s}^{-1}, \vec{\alpha}_0 \right] = 416.709 \text{ MPa}$

TABLE IX

**SENSITIVITY ANALYSIS OF THE MTS MODEL PARAMETERS FOR A
COPPER ADIABATIC STRESS-STRAIN PROBLEM**

Parameter α	Parameter Description	Nominal Value α_0	$\frac{\partial \ln(R)}{\partial \ln(\alpha)}$ ^a (DST)	$\frac{\Delta R/R}{\Delta \alpha/\alpha}$ (Direct)	Magnitude Ranking
$\hat{\sigma}_a$	Rate Independent Threshold Stress	40.0 MPa	-0.0097		13
g_0	Normalized Activation Energy	1.6	0.0881		9
μ_{sf}	Shear Modulus Scale Factor	1	0.664		3
$\dot{\epsilon}_0$	Arrhenius Constant	10^7 s^{-1}	-0.0097		12
b	Magnitude of Burgers Vector	2.55 Å	1.99		1
θ_{osf}	Hardening Function Scale Factor	1	-0.212		7
A	Saturation Stress Constant	0.312	0.576		4
ρC_p	Density-Heat Capacity Product	3.82 MPa/K	0.713		2
T_0	Initial Temperature	300 K	-0.320		5
$\hat{\sigma}_0$	Initial Mechanical Threshold Stress	0 MPa	-0.0027		14
$\hat{\sigma}_{s0}$	Saturation Stress at zero Degrees K	900 MPa	0.288		6
$\dot{\epsilon}_{s0}$	Reference Strain Rate	$6.2 \times 10^{10} \text{ s}^{-1}$	-0.0321		11
$1/p$	Flow Stress Eqn Exponent	$\frac{3}{2}$	-0.0830		10
$1/q$	Flow Stress Eqn Exponent	1	0.182		8

a: $R = \sigma[\epsilon_f = 3.8, \dot{\epsilon}_{EFP}, \tilde{\alpha}_0] = 381.250 \text{ MPa}$

the assumption of isotropic hardening. This approach is illustrated in Eqs. (IV-1) through (IV-8) below. In these equations, total stresses and strains are denoted by σ_{ij} and ϵ_{ij} , respectively; deviatoric stresses and strains are indicated by s_{ij} and e_{ij} , respectively; the equations use indicial notation; and the superscript k designates previous time-step or cycle quantities.

First, the total strain-rate tensor, spin tensor, and deviatoric strain-rate tensor are constructed using velocity gradients:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right), \quad (IV-1)$$

$$\omega_{ij} = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} \right), \text{ and} \quad (IV-2)$$

$$\dot{e}_{ij} = \dot{\epsilon}_{ij} - \frac{1}{3} \delta_{ij} \dot{\epsilon}_{ll}, \quad (IV-3)$$

respectively, where

$$\dot{\epsilon}_{ll} = \dot{\epsilon}_{11} + \dot{\epsilon}_{22} + \dot{\epsilon}_{33} \quad (IV-4)$$

and δ_{ij} is the Kronecker delta function²¹. Trial deviatoric stresses (designated with asterisks) then are determined from deviatoric strain rates using elastic theory in a Lagrangian frame of reference:

$$s_{ij}^* = s_{ij}^k + \Delta t \left(2\mu \dot{e}_{ij} + s_{il}^k \omega_{lj} + s_{jl}^k \omega_{li} \right), \quad (IV-5)$$

and a flow stress is evaluated that is used in combination with a von Mises yield function and the trial stresses to form the ratio

$$R = \frac{\begin{pmatrix} s_{ij}^* & s_{ij}^* \\ \frac{2}{3} \sigma^2 \end{pmatrix}}{\frac{2}{3} \sigma^2}. \quad (IV-6)$$

Plastic flow is indicated if R is greater than unity, and hence the deviatoric stresses are returned to the flow surface via "radial return":

$$s_{ij}^{k+1} = \frac{1}{\sqrt{R}} s_{ij}^* \quad (IV-7)$$

For the case where R is less than or equal to unity, elastic flow is indicated, and the new deviatoric stresses are identical to the trial stresses. For the plastic flow case, the plastic strain then is evaluated with

$$\epsilon_{ij}^{p\ k+1} = \epsilon_{ij}^{p\ k} + \dot{\epsilon}_{ij}\Delta t - \left(\frac{s_{ij}^{k+1} - s_{ij}^k - s_{il}^k \omega_{lj}\Delta t - s_{jl}^k \omega_{li}\Delta t}{2\mu} \right) \quad (IV-8)$$

A. Implementation Scope

Implementing a new flow-stress model into the Radial Return formalism illustrated above is conceptually easy because all that is required is determining a value for the flow stress from the appropriate constitutive model. The Radial Return method does not need partial derivative information (that is, $\partial\sigma/\partial y_i$, where the y_i represents the dependencies of σ) as do other more classical approaches to strength modeling such as Prandtl-Reuss.²³ The difficulties that arise when implementing a new flow-stress model into a hydrocode are a result of a variety of items, most of which are listed below.

- (1) Path information (that is, the y_i) is not readily available where a flow-stress evaluation is required in the hydrocode.
- (2) Flow-stress evaluations are required in many parts (subroutines) of the hydrocode.
- (3) Internal variables associated with the new flow-stress model require advection (Eulerian formulations).
- (4) The data structure of the hydrocode needs to be expanded to accommodate the flow-stress model's new internal variables and possibly extra path variables.
- (5) The hydrocode's restart file needs to be expanded to include these new variables.
- (6) There are hydrocode logic problems associated with the new model. (For example, how should materials not yet characterized for the MTS model be handled during the strength calculation in the code?)

- (7) There are programming problems (for example, vectorization destruction and repair).
- (8) There are hydrocode numerical problems associated with using the new model (such as instabilities).
- (9) Large files of hydrocode modifications must be debugged (more of a Eulerian problem).
- (10) Both the code and the model must be maintained as new code versions are produced and new materials become available for the model.

A more specific discussion of the experience of implementing the MTS model into EPIC2 and PINON referencing the above list is given in Sec. IV.D.

B. Solution of the Threshold Stress Differential Equation(s)

Of the three materials currently incorporated into the codes, only the dislocation-dislocation interaction has evolution modeled in the form of a DE exemplified by Eq. (II-5). Formally, all of the "n" barrier interactions under the summation could experience evolution, so the equation set to be solved in the codes could expand to include these additional evolution DE's.

The dislocation-dislocation interaction DE currently solved in EPIC2 and PINON is done so in an explicit sense; at each $k^{th}+1$ time cycle, the threshold stress is updated using path information from the previous k^{th} cycle:

$$\hat{\sigma}^{k+1} = \hat{\sigma}^k + \Delta\epsilon \Theta_0^k \left[1 - F \left(X^k \right) \right], \quad (IV-9)$$

where $\Delta\epsilon$ is the equivalent plastic strain increment for time cycle $k+1$. Explicit solution for the threshold stress works well in EPIC2 and PINON because of the small time steps realized in these hydrocodes.*

Although Eq. (IV-9) represents all the work needed for computing $\hat{\sigma}$ in a Lagrangian code, the threshold stress also needs to be advected in a Eulerian code, as demonstrated by the following Eulerian time derivative (written for two dimensions):

*A typical time cycle in EPIC2 or PINON ranges from 0.1 to 0.01 μ s long.

$$\left(\frac{\partial \hat{\sigma}}{\partial t}\right)_{\text{Euler}} = \left(\frac{\partial \hat{\sigma}}{\partial t}\right)_{\text{Lagr}} - \frac{\partial(\hat{\sigma}u)}{\partial x} - \frac{\partial(\hat{\sigma}v)}{\partial y}, \quad (\text{IV-10})$$

where u and v are material velocities in the x and y coordinate directions, respectively. Equation (IV-10) states that in addition to the Lagrangian time changes to $\hat{\sigma}$ represented by Eq. (IV-9), the threshold stress must be advected through the Eulerian computational mesh as indicated by the divergence terms on the right-hand side. In other words, the Eulerian time changes are a combination of Lagrangian time changes modified by material advection. Just such a two-step process is used in PINON to determine a new value for $\hat{\sigma}$.

C. Transcendental Complication for the Drag Regime

As discussed in Sec. II.A, the flow-stress relationship for the drag regime [represented by substituting Eq. (II-16) into Eq. (II-4)] is transcendental and is solved numerically for σ in the EPIC2/PINON implementations using a Newton-Raphson²⁴ iteration of the following equation:

$$F(\sigma) = \frac{\dot{\epsilon}_0}{\dot{\epsilon}} - \frac{MB(\sigma)\alpha\mu\nu_0}{\sigma \hat{\sigma}} - e^{\Delta G(\sigma)/kT} = 0. \quad (\text{IV-11})$$

The Newton-Raphson iteration can be expressed by the following two equations that solve for the $m+1$ estimate of σ based upon m^{th} information:

$$\sigma^{m+1} = \sigma^m + \delta\sigma^{m+1}, \quad (\text{IV-12})$$

where

$$\delta\sigma^{m+1} = \frac{-F(\sigma^m)}{\left(\frac{\partial F}{\partial \sigma}\right)^m}. \quad (\text{IV-13})$$

The iteration continues until convergence is realized; that is, $\delta\sigma^{m+1}$ goes to zero. This iteration is not well behaved in practice at small $\hat{\sigma}$ (or small strains) and high strain rates; it tends to "blow up" and give unrealistically large values of σ . Therefore, an upper limit must be imposed on σ when this occurs. Currently, an arbitrary $\mu/20$ is used as the

maximum flow stress that can be realized physically in the material; the "spike" in Fig. 1 and Fig. 3 demonstrates the behavior of the drag regime.

D. Specific Implementation into the EPIC2 and PINON Codes

Implementing the MTS model into the EPIC2 Lagrangian code was simple. All the path information was already available, and only one subroutine (STRESS) in the code requires a flow-stress evaluation. The code's data structure and restart file were easy to expand for old values of the threshold stress and equivalent plastic strain. Materials not yet characterized for the MTS model logically use a JC^8 flow stress. No numerical problems were experienced, and the FORTRAN corrections to EPIC2 were minor and easy to debug.

In addition to the numerical misbehavior associated with the drag regime equation discussed in Sec. IV.C, solving Eq. (IV-11) iteratively creates a Do-Loop vectorization problem (which degrades the code's grind time*) if the hydrocode needs a flow-stress evaluation inside of a vectorized "space-loop." This turns out to be the case for both EPIC2 and PINON. The "best" solution to this problem is to vectorize the iteration in conjunction with a gather/scatter technique to filter out the converged spatial elements or cells as the iteration proceeds. An implementation of the MTS model that contains only thermal activation kinetics would result in a closed-form set of equations with all the original Do-Loops remaining vectorized. The current EPIC2 implementation includes the drag regime, which is programmed as a vectorized iteration without gather/scatter.

Implementing the MTS model into the Eulerian code PINON was more difficult than the EPIC2 experience for several reasons.

1. Temperature information needed to be extracted from the equation of state.
2. Several subroutines in the code required flow-stress evaluations.
3. Advection of the threshold stress was needed.
4. Expanding the code's data structure and restart file was more complicated.
5. Materials not yet characterized for the MTS model use a Steinberg-Guinan²⁵ flow stress.

*Grind time is defined as the computer time used by the code for running a specific problem divided by the number of cells and time cycles used for that problem; typical units for grind time are $\mu s/cell/cycle$.

6. Some numerical problems were experienced that required a more robust flow-stress strain-rate iteration.
7. The large set of FORTRAN corrections to PINON required significant debugging time.

The same comments apply to PINON concerning the vectorization problem associated with the drag kinetics regime that was discussed above for EPIC2. The current implementation of the MTS model in PINON contains both thermal activation and drag kinetics cast in a vectorized iteration.

V. ASSESSMENT CALCULATIONS

As mentioned in Sec. I, a new constitutive model is suitable for incorporation into a hydrocode only if it improves simulation accuracy without creating numerical stability problems and using excessive computing resources. A set of assessment calculations that test the suitability of the MTS model in practical application in the context of the EPIC2 and PINON codes is presented here. The set of assessment problems described in this section has evolved over time to be the "best" set for assessing the MTS model. The selection of problems was constrained by the available material libraries, the type of problem that could be addressed realistically with a two-dimensional hydrocode, and available experimental data. Table X lists the current set of assessment problems and indicates which code was used for what calculations. For the most part, both EPIC2 and PINON were used for each problem. All of the assessment problems use OFHC copper test samples ($\rho = 8930 \text{ kg/m}^3$) because experimental information is abundant for copper (including shaped-charge warheads) and copper was the first characterized MTS material. These problems attempt to assess the MTS model in uniaxial normal or shear states of strain, in multiaxial combined states of strain, with static experimental data, and with dynamic experimental data. Future assessment of the MTS model will include materials and experiments that have even better application to armor problems. The problem discussed in this section that has the most direct armor application is a copper EFP formation (Sec. V.D).

The calculative results presented below have been tested for "mesh convergence." This means that calculations for a given problem have been duplicated using more mesh

TABLE X**CALCULATIVE MATRIX FOR THE ASSESSMENT OF THE MTS
FLOW-STRESS MODEL**

Experiment	EPIC2	PINON	Reference
Taylor Cylinder Tests	completed	completed	G. R. Johnson
Torsion Tests	completed		G. R. Johnson
Normal Flyer Plate Tests	completed	completed	C. E. Morris
Axisymmetric EFP Formation	completed	preliminary results	Honeywell Design
Inclined Flyer Plate Tests	preliminary results	completed	R. J. Clifton

cells (PINON) or finite elements (EPIC2), and the results have been checked for changes. Changes of 5% or less are interpreted in this work to designate mesh convergence. There are situations where computational economics limit the mesh convergence exercise; however, only the EFP formation problem had large enough run times to affect the exercise.

A. Taylor Cylinder Tests

A schematic of the geometry of the Taylor cylinder tests¹² is given in Fig. 12. These tests consist of 1-in. (2.54-cm) -long copper cylinders 1/3 in. (0.762 cm) in diameter impacting a rigid surface at various velocities (130, 146, and 190 m/s) in a normal configuration. Both the EPIC2 and PINON codes were used to simulate these tests in an axisymmetric (r-z) configuration.

Fig. 12.

Schematic of the Taylor cylinder test simulated with both the EPIC2 and PINON using an axisymmetric geometry.

Figures 13 through 15 show final cylinder center-line profiles in centimeters for the three velocity cases. These figures compare experimental data with EPIC2 results using both the JC and MTS flow-stress models; six calculations are presented in these figures. The experimental data are shown by the solid lines, and the MTS and JC calculations are indicated by different dashed lines. Note that the MTS simulation does a better job at predicting the profiles than the JC simulation for all three velocity cases. MTS does very well predicting the cylinder "toe" (radius of the cylinder at $z = 0$) and the cylinder "bulge" (radius at roughly $z = 1$ cm); the cylinder length predictions are not quite as good, but they are still superior to the JC model predictions. The main difference between the MTS and JC calculations appears to be strain (or strain-rate) hardening in the cylinder toes. The MTS model hardens earlier in the simulation in the toe region, and thus, more strain is realized in the upper portions of the cylinder, which allows the sides to bulge at $z = 1$ cm. This is most apparent in the 190-m/s case, where the MTS toe hardens to the extent that it slightly underpredicts the plastic flow shown by the data. The hardening in the JC simulations is significantly less as demonstrated by the observed concentration of strain in the toes and the more vertical cylinder sides. The early-time toe hardening/cylinder bulging is the main reason why the MTS simulations show superior accuracy.

Fig. 13.

Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with EPIC2 simulations using the MTS and JC flow-stress models. Initial cylinder velocity is 130 m/s.

Fig. 14.

Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with EPIC2 simulations using the MTS and JC flow-stress models. Initial cylinder velocity is 146 m/s.

Fig. 15.

Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with EPIC2 simulations using the MTS and JC flow-stress models. Initial cylinder velocity is 190 m/s.

Figures 16 through 18 show a similar set of final cylinder center-line profiles for the same three velocity cases that compare the experimental data with the PINON calculations using the JC, MTS, and Steinberg-Guinan (SG)²⁵ flow-stress models: nine calculations are presented in these figures, and the experimental data again are shown by the solid lines. Although these Eulerian results are more "busy" with lines, the MTS simulation still gives the most accurate prediction of the cylinder shapes for all the velocity cases. The MTS model again does very well predicting the cylinder "toe" and the cylinder "bulge" and does a better job in these Eulerian calculations at predicting the cylinder length. The SG model (the default flow-stress model in the PINON code) falls between the JC and MTS calculations in terms of accuracy. Again, the apparent early hardening of the MTS model in the cylinder toes allows more strain (bulge) in the upper cylinder and therefore gives a more accurate result.

Comparing Figs. 13 through 15 with Figs. 16 through 18, respectively, indicates that the MTS predictions are more accurate in the Eulerian calculations than in the Lagrangian calculations. Also it is interesting to note that the JC model produces a more accurate result in the PINON code than it does in EPIC for the cylinder length and bulge; the toe size is nearly identical in both the EPIC2 and PINON calculations.

Fig. 16.
Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with PINON simulations using the MTS, SG, and JC flow-stress models. Initial cylinder velocity is 130 m/s.

Fig. 17.
Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with PINON simulations using the MTS, SG, and JC flow-stress models. Initial cylinder velocity is 146 m/s.

Fig. 18.

Comparison of the final shapes of Cu cylinders from Taylor cylinder tests with PINON simulations using the MTS, SG, and JC flow-stress models. Initial cylinder velocity is 190 m/s.

The grind times for the above calculations are as follows. The EPIC2 calculations for the 190-m/s case required 27 and 31 $\mu\text{s}/\text{cell}/\text{cycle}$ for the JC and MTS models, respectively. The PINON calculations for the same case required 283 $\mu\text{s}/\text{cell}/\text{cycle}$ and 382 $\mu\text{s}/\text{cell}/\text{cycle}$ for the JC and MTS models, respectively. These grind times represent a 15% and a 35% increase in the amount of computational work per cell-cycle when using the MTS model in the EPIC2 and PINON codes, respectively. The larger value for the PINON code is consistent with the extra computation needed in a Eulerian calculation (for example, advection of the threshold stress, mixed-material cells,* etc.). Although these EPIC2 and PINON grind times are quoted for a specific Taylor cylinder calculation, they are "representative" of all the calculations discussed below for the other assessment problems.

Even though the MTS code versions may have larger grind times, this does not necessarily translate into longer computer run times for a specific problem. For example, the EPIC2-MTS calculations of the Taylor cylinders had smaller run times than the EPIC2-JC calculations (96.7 s vs 128.9 s for the 190-m/s case) because the more accurate MTS shapes had less distortion of the Lagrangian cells in the toes of the cylinders, which

*A Eulerian computational cell that contains more than one material is called a mixed-material cell.

meant that larger time steps (and consequently fewer cycles) were required for the calculations.

B. Torsion Tests

A schematic of the geometry for the torsion tests¹⁴ is given in Fig. 19. These tests were modeled with a 0.649-cm-i.d. copper tube fixed at one end, an applied torque at the other end, and a 0.079-cm-thick, 0.312-cm-long test section notched into the tube wall to concentrate the shear strain. PINON cannot simulate these tests. Only EPIC2 is capable of simulating them for the following reasons.

- (1) The axisymmetric with spin option²⁶ (r - z - θ), which produces a fully three-dimensional solution for this specialized problem, was needed for these torsion tests.

Fig. 19.

Schematic of the dynamic torsion test simulated with the EPIC2 code using an axisymmetric geometry with spin. The shear strain rates indicated are average values realized in the Cu test section.

- (2) The duration of these torsion tests is rather long for explicit hydrocodes like PINON and EPIC2; therefore, assuming that wave propagation and inertial effects are small, the time-compression, β , feature¹⁴ of EPIC2 also was needed to make the calculations practical.
- (3) Given the duration of these tests, heat conduction is expected to be significant, and therefore, the conduction modeling option in EPIC2 also was used in these calculations.

Two strain-rate cases were simulated with the EPIC2 code, 9.6 s^{-1} and 330 s^{-1} , using both the MTS and JC flow-stress models. Figures 20 and 21 show results from these four calculations cast in terms of shear stress vs shear strain* averaged over the test section shown in Fig. 19. Figure 20, which shows the low-strain-rate case, shows the calculations overpredicting the data for strains > 1 by roughly 10% for the JC calculation and 20% for the MTS calculation. Both calculations have a "bump" in the results at a strain of 2; we have no physical explanation for it but assume it to be associated with the numerical modeling. The 1981 calculation published by Johnson in Ref. 14 also shows this anomaly. Figure 21, which presents the higher strain-rate case, again shows both calculations overpredicting the data for strains greater than 0.5. The data for this case show a thermal softening trend starting at a strain of 4 with the shear stress dropping off by 30% at a strain of 7. The MTS calculation duplicates this trend nicely and maintains a roughly constant 50 MPa (20%) offset from the data over most of the transient. The JC calculation initially is closer to the data but misses this thermal softening trend. The calculation was expected to compare well with the test data because about half¹² of the data base for characterizing JC model parameters consists of torsion tests; in fact, the two tests presented here are part of that data base.

It was observed above that the MTS simulations overpredict both tests by roughly 20% for strains greater than 0.5. It was demonstrated in Ref. 27 that metal samples subjected to large amounts of shear strain will experience grain texturing and macroscopically soften to lower magnitudes of flow stress. Depending on the initial texture of the test samples, this phenomenon can account for a flow-stress decrease ranging from 10% to 20%. Because the JC data base contains torsion tests, some amount of textured material with inherently softer flow stress is present empirically in the JC model, whereas the MTS

*The shear strain shown in Figs. 20 and 21 is γ_{ij} as opposed to ϵ_{ij} , where $\gamma_{ij} = 2 \epsilon_{ij}$.

Fig. 20.

Torsion test comparison of experimental data (shear strain rate = 9.6 s^{-1}) with two EPIC2 simulations that use the MTS and JC flow-stress models.

Fig. 21.

Torsion test comparison of experimental data (shear strain rate = 330 s^{-1}) with two EPIC2 simulations that use the MTS and JC flow-stress models. Note the thermal softening trend in the data.

model (which is based on Split-Hopkinson Bar normal compression tests) is expected to overpredict the flow stress in torsion tests because of material texturing.

C. Normal Flyer-Plate Tests

A schematic of the geometry of the normal impact flyer-plate tests¹³ is given in Fig. 22. In this test, a copper impactor disc 51 mm in diameter, 2.9 mm thick, and backed

Fig. 22.
Schematic of the normal Cu flyer-plate impact test simulated with both the EPIC2 and PINON codes using a one-dimensional geometry.

with foam is mounted on an aluminum alloy projectile. The projectile is fired into a copper target disc 39.7 mm in diameter and 18.3 mm thick at two different velocities (285 m/s and 645 m/s). The impactor disc produces a square-top pulse of about 1.5- μ s wide (full width at half the maximum) that is recorded by a Manganin gage embedded halfway through the target. This information then is compared with calculative results from both the EPIC2 and PINON codes used in a one-dimensional mode.

Figures 23 and 24 show pressure wave profiles in units of GPa vs μ s for the two velocity cases, respectively. These figures compare experimental data with EPIC2 results using both the JC and MTS flow-stress models; four calculations are presented in these figures. The experimental data are shown by the solid lines, and the MTS and JC calculations are indicated by different dashed lines.

In Figs. 23 and 24, the MTS simulations do only a slightly better job at predicting the pressure pulse shape. For the 285-m/s case, both EPIC2 simulations underpredict the maximum pressure of 5.4 GPa by 4% and do a poor job of modeling the unloading. For the 645-m/s case, both EPIC2 simulations hit the maximum pressure of 12.3 GPa exactly but again do a poor job of modeling the unloading. All of the EPIC2 simulations have elastic precursors for both the loading (the copper Hugoniot Elastic Limit is about 0.2 GPa) and unloading transients. The data for the 285-m/s case are not good enough to show a loading precursor, but the 645-m/s case does show the loading precursor; neither case shows unloading precursors. This shock unloading of the copper target is a complicated physical process that may not be modeled correctly by the hydrocode calculations; for example, there may be a significant Bauschinger²⁸ effect present.

Figures 25 and 26 present a similar comparison showing PINON calculations that feature the JC, MTS, and SG flow-stress models; six calculations are presented in these figures. Here the MTS simulations do not show an accuracy improvement predicting the pressure-pulse shape. For the 285-m/s case, all the PINON simulations closely predict the maximum pressure of 5.4 GPa and do a poor job of modeling the unloading. The JC calculation best simulates the entire problem. For the 645-m/s case, all the PINON simulations overpredict the maximum pressure of 12.3 GPa by 5%, and the unloading again does not seem to be modeled very well. Some of the difference between experiment and calculation also could be a result of the difficulties of making pressure measurements *in situ* in the metal target. Similar observations can be made of these PINON results concerning elastic precursors and the unloading process as made above for the EPIC2 results.

Fig. 23.

Pressure-time profiles from a copper flyer-plate test for a 285-m/s impactor velocity. The EPIC2 JC and MTS simulations are compared with experimental data.

Fig. 24.

Pressure-time profiles from a copper flyer-plate test for a 645-m/s impactor velocity. The EPIC2 JC and MTS simulations are compared with experimental data.

Fig. 25.

Pressure-time profiles from a copper flyer-plate test for a 285-m/s impactor velocity. The PINON JC, MTS, and SG simulations are compared with experimental data.

Fig. 26.

Pressure-time profiles from a copper flyer-plate test for a 645-m/s impactor velocity. The PINON JC, MTS, and SG simulations are compared with experimental data.

D. Formation of an Explosively Formed Projectile (EFP)

A schematic of the initial geometry of a Honeywell EFP¹⁵ is given in Fig. 27. This warhead has a copper liner, an Octol charge 5.6 in. in diameter and 4.5 in. long, a 1/4-in. steel case, and at least 1.5 in. of polyurethane foam in front of the liner. The axisymmetric liner geometry is generated with the following two relationships for the top and bottom elevations of the liner as a function of its radius (r):

$$z_{\text{top}} = 0.16951 + 0.011877 r + 0.06790 r^2, \text{ and} \quad (\text{V-1})$$

$$z_{\text{bot}} = 7.16535 - \sqrt{51.3423 - r^2}, \quad (\text{V-2})$$

where the thickness of the liner (τ) is

$$\tau = z_{\text{top}} - z_{\text{bot}}. \quad (\text{V-3})$$

All displacements in Eqs. (V-1) through (V-3) are in units of inches.

Fig. 27.

Initial geometry of a Honeywell EFP simulated with the EPIC2 code using an axisymmetric geometry. The velocity indicated is the final formation velocity.

Experimental information for this EFP formation is given in gross form (length, diameter, and thickness) in column two of Table XI, and an outline of the radiograph of the final shape is shown in Fig. 28. Outlining the original radiograph is a complicated procedure with an estimated uncertainty equal to about one line width of the Fig. 28 outline. This uncertainty is in addition to whatever experimental uncertainty can be associated with warhead construction and formation.

Table XI also compares two EPIC2 calculations using the JC and MTS models with the gross shape of the EFP. Both calculations predict a final velocity very close to the experimental result and come very close to the dimensions of the EFP (given the uncertainty in experimental shape), with the JC calculation producing a longer, more slender EFP by 7 to 8% compared with the MTS calculation.

Figure 29 compares the two calculated shapes with the Fig. 28 outline; all three shapes are to scale. A more detailed analysis of the calculative results reveals that the MTS calculation is producing slightly larger values of flow stress when compared with the JC calculation, with the expected result that the Fig. 29 MTS shape appears to be slightly shorter and larger in diameter than the JC calculative result.

From the EPIC2 calculation, the amount of shear strain realized by the liner material is significant. Figure 30 shows the shear strain, ϵ_{rz} , as a function of time for an element of liner located midway along the side of the EFP as observed in a material (unrotated) frame of reference. The figure shows this element shearing to -20% at about 70 μ s

TABLE XI
FORMATION OF A HONEYWELL EFP

	Experimental Data ^a	EPIC2 (JC)	EPIC2 (MTS)
Length	135 mm	142 mm	131 mm
Diameter	30 mm	32 mm	34 mm
Thickness	10 mm	8.5 mm	9.0 mm
Final Velocity	2.4 km/s	2.47 km/s	2.47 km/s

a: 5.6 Inch Honeywell EFP (shot number H-538)

Fig. 28.
Outline of a radiograph of the final Honeywell EFP shape. The uncertainty in position of this outline is roughly one line width.

Fig. 29.
EPIC2 results from two Honeywell EFP formation simulations using the JC and MTS flow-stress models being compared with the experimental radiograph outline from Fig. 28.

Fig. 30.

Shear strain history as realized in a material (unrotated) frame of reference for a Lagrangian element of liner material located roughly halfway between the nose and the tail of the formed Honeywell EFP.

into the problem and then back to -4% at about 250 μ s. The material texturing resulting from this shear strain will soften the actual flow stress significantly.* Based on a sensitivity analysis of the change in shape with respect to the magnitude of the flow stress, the final EFP shape is known to be moderately sensitive to the flow-stress magnitude. Therefore, softening the flow stress would move the calculated MTS shape closer to that of the radiograph. More investigation of the influence of texturing on the shape of the Honeywell EFP is needed.

E. Inclined Flyer-Plate Tests

A schematic of the geometry of the inclined flyer-plate tests¹⁶ is given in Fig. 31. In this test, a tool-steel (1.93-GPa yield strength) flyer disc 38 mm in diameter and 4.75 mm thick has a 0.287-mm-thick copper sample mounted in front of it. This flyer is fired at 183 m/s into a tool-steel anvil disc 38 mm in diameter and 2.85 mm thick. The contacting surfaces of the impactor and target are inclined at 23° to produce both normal and shear stress waves.

*Private Communication with A. D. Rollett (August 1989).

Fig. 31.
Schematic of the inclined Cu flyer-plate impact test simulated with the PINON code using plane-strain two-dimensional geometry.

During impact, the free-surface velocity at the back of the anvil is measured as a function of time as shown by the results given by Fig. 32 (from Ref. 29). This transverse velocity history is compared in Fig. 33 with two-dimensional plane-strain results from the PINON code using the JC and MTS models. The experimental velocity from Fig. 32 is shown in Fig. 33 shifted in time by $0.92 \mu\text{s}$, which is roughly the transit time for the shear wave to cross the steel anvil. The comparison shows that the MTS calculation accurately predicts the magnitude of the velocity step and then oscillates about the data. The JC calculation overpredicts the velocity step and then also oscillates about the data. Comparison of these calculations with the experiment is not valid after about $1.6 \mu\text{s}$ because release waves from the free surfaces of the calculative domain decelerate the transverse velocity.

Shot No. 01 (OFHC Copper)

Fig. 32.

Experimental transverse velocity history measured at the back of the anvil for an initial inclined flyer-plate speed of 183 m/s and an obliquity of 23°.

Inclined Flyer Plate (Copper)

Fig. 33.

Comparison of the transverse velocity history from the inclined flyer-plate test with PINON simulations using the MTS and JC flow-stress models. The experimental history has been shifted forward in time by an amount that corresponds to the shear wave transit time of the anvil.

VI. SUMMARY, CONCLUSIONS, AND FUTURE WORK

In this report, the equation set for the MTS flow-stress model that was implemented in the EPIC2 and PINON hydrocodes is described in some detail. Basically, the MTS equations are partitioned into kinematics (path-dependent) and kinetics parts. The kinematics part is the material structure evolution description represented by Eq. (II-5). The kinetics part is the deformation regime represented by either Eq. (II-12) or Eq. (II-16) for the thermal-activation-controlled or drag-controlled deformation regimes, respectively. These parts then are combined in a summation over dislocation interactions through Eq. (II-4) to predict a value of flow stress. The reader can compare the complexity of this equation set with that of the simpler JC flow-stress model by inspecting the above-mentioned equations with Eq. (II-19). Because the MTS model is a more complicated, physically based model, it should be expected that more computational resources will be required to use it in a hydrocode.

Numerical comparisons between the MTS model and the JC model also were presented in Sec. II for a simple adiabatic stress-strain problem and a prestrain/reload uniaxial compression test. The adiabatic comparisons show the MTS model predicting harder values of flow stress than does the JC model for small-to-moderate strains and strain rates greater than 1. The prestrain/reload comparisons show the MTS model to produce very accurate results for materials that contain some deformation history where knowledge of this history is needed for predicting accurate values of flow stress.

A sensitivity analysis of the MTS model cast in an adiabatic stress-strain problem containing an arbitrary strain-rate history was presented in Sec. III. This sensitivity analysis was performed using an analytical technique called Differential Sensitivity Theory, which produced sensitivity coefficients, $\partial \ln(R)/\partial \ln(\alpha)$, for all the parameters that appear in the physical equation set given by Eqs. (III-1) through (III-4). The results from this analysis are given in Tables VI through IX for various strain-rate histories and show which MTS model parameters are the most important in determining the flow stress. The results should guide the experimentalist in allocating his resources when quantifying MTS model parameters for new materials.

In Sec. IV we reviewed the equation set for the Method-of-Radial-Return strength formulation used in both the EPIC2 and PINON codes and discussed the scope of implementing the MTS model into these hydrocodes. Some of the main implementation items discussed include the numerical solution of the differential threshold stress equation, advection of the threshold stress in an Eulerian code, solution of the transcendental

drag regime equation and associated vectorization difficulties, and itemization of all the other "minor" problems that constitute the majority of the implementation effort.

A practical assessment of the MTS model was presented in Sec. V using experimental data from Taylor cylinder tests, torsion tests, normal flyer-plate tests, an EFP formation, and an inclined flyer-plate test. We performed EPIC2 and PINON simulations of these experiments using the MTS and JC models. These calculations indicate that the more complex MTS model is "reasonable" in terms of computational resources when compared with JC calculations. A few additional arrays and a 15% to 35% increase in run times were observed.

In terms of accuracy, these simulations show the MTS model to be superior for problems containing mostly normal strain with shear strains less than 0.2 (ϵ) (for example, the Taylor cylinder tests or the inclined flyer-plate tests) but perhaps not as accurate for problems that contain large amounts of shear strain (for example, the torsion tests or the EFP formation). As mentioned in Sec. V.D, we believe the explanation for this to be material texturing. Because the experimental data base used to characterize parameters for the MTS model consists exclusively of normal compressive tests and because the MTS model does not have a texturing feature, it is expected that the MTS flow-stress predictions will be too hard for problems containing large shear strains. Conversely, for problems dominated by mostly normal strain, it is expected that the MTS predictions should be accurate, as we observe for the Taylor cylinders. The JC model has been designed to better address problems with large shear because the experimental data base used to characterize JC parameters includes a number of torsion tests with its softer textured material. However, even for problems that have large shear strain, the MTS model still can produce a more accurate result as demonstrated by the torsion test comparison in Fig. 21, which contained thermal softening. Work is under way to better understand the effect of texturing on the assessment problems discussed in Sec. V.

The MTS model currently has three materials available in the hydrocode material libraries (Cu, Ti-6Al-4V, and depleted U). More materials are being characterized for the model; Nitronic 40 stainless steel, 4340 steel, a U-Ti alloy, and a W alloy soon will be added to code libraries. Methods to obtain MTS material constants from existing databases¹² also are being developed. These additional materials will allow the MTS model to be applied seriously in realistic armor/anti-armor calculations. Future assessments planned for the MTS model will use simple penetration and perforation test results when the appropriate materials become available.

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