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SIMPLE MODEL FOR DECAY OF LASER GENERATED SHOCK WAVES

R. J. Trainor

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R. J. Trainor
University of California, Lawrence Livermore National Laboratory
Livermore, California 94550

ABSTRACT

A simple model is derived to calculate the hydrodynamic decay of laser-generated shock waves. Comparison with detailed hydrocode simulations shows good agreement between calculated time evolution of shock pressure, position, and instantaneous pressure profile. Reliability of the model decreases in regions of the target where superthermal-electron preheat effects become comparable to shock effects.

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24

1. INTRODUCTION

Hydrodynamic decay is one of the most important phenomena controlling the properties of laser-generated shockwaves. Its importance stems from the fact that targets used in laser-generated shockwave experiments are usually designed to be substantially thicker than the range of the superthermal electrons produced by the resonance absorption of the laser light. This permits the study of shock compression in a region of the target which has not been significantly preheated. A consequence of this design principle is that the shock transit time through the target is typically longer than the duration of the laser pulse, so that the rarefaction wave created when the pulse terminates may have time to overtake and attenuate the shock before it has traversed the target.

In this paper I show that a simple model of hydrodynamic decay can reproduce many of the features of laser-generated shocks calculated by detailed hydrocode simulations. The model should be reliable when superthermal electron preheat is not dominant and when laser pulse length is not so long that possible two-dimensional and decoupling phenomena may be important.

2. BACKGROUND AND APPROACH TO THE PROBLEM

Consider a planar target irradiated by a laser pulse of intensity I_0 and duration τ . During the time $0 < t < \tau$, ablation of target material by the pulse produces a shockwave, which propagates into the target. The pressure behind the shock is approximately related to I_0 by a simple power law

$$P_s = BI_0^\alpha \quad (1)$$

where P_s is the shock pressure, and I_0 is the incident laser intensity. When P_s is measured in Mbar and I_0 in units of 10^{14} W/cm², $B \approx 6-8$ and $\alpha \approx 0.67-0.82$. This result is obtained from both analytic¹ and hydrocode² studies and has some experimental support³ in the intensity range of interest, $I_0 > 10^{14}$ W/cm².

The shock propagates into the target with velocity D and imparts a velocity U to material behind the shock front. An ablation front follows the shock into the target with velocity U_A . At $t = \tau$, the pressure at the ablation front, and thus U_A , begins to drop rapidly. I assume that they suddenly go to zero at $t = \tau$. At this time, a rarefaction fan will propagate from the ablation front into the shock-compressed target. Since the leading edge of the fan travels at speed $c + U > D$ for a stable shock,⁴ where c is the local sound speed, the rarefaction will eventually overtake the shock, and the shock pressure will decrease.

The time t_c at which the leading edge of the rarefaction catches the shock front is found from

$$(c + U)(t_c - \tau) = Dt_c - U_A \tau$$

Solving for t_c gives

$$t_c = \left[\frac{c + U - U_A}{c + U - D} \right] \tau \quad (2)$$

It can be easily shown that $U_A \simeq U$; ⁵ thus, Eq. (2) reduces to

$$t_c \simeq \left[\frac{c}{c + U - D} \right] \tau \quad (3)$$

For most materials and pressures, Eq. (3) predicts $(t_c/\tau) \sim 2$; that is, after the shock has propagated a distance of about $2D\tau$ into the target, the pressure will begin to decrease. Note that for a shock generated by an impact whose duration is τ (as opposed to an ablation), $U_A = 0$, so that $(t_c/\tau) \sim 4$. Thus, an impact-generated shock will travel roughly twice as far as an ablation-generated shock before it begins to decay.

At $t = t_c$, the shock profile should look similar to that shown in Fig. 1a. If $dP/dx|_{x=x_c}$, the pressure profile at $x = x_c = Dt_c$, is known, then the subsequent decay of the shock, DP/Dx , can be obtained from Taylor's well-known formula:

$$\frac{DP}{Dx} = \frac{dP}{dx} \cdot \left[\frac{U+c}{D} - 1 \right] \quad (4)$$

The difference between DP/Dx and dP/dx is illustrated in Fig. 1. As the pressure begins to decay, U , c , D , dP/dx and DP/Dx will change, so a time-dependent (or spatially dependent) version of Eq. (4) is needed. Cottet and Romain⁶ have shown that when time is advanced by Δt

$$\frac{1}{dP/dx|_{t+\Delta t}} = \frac{1}{dP/dx|_t} + \Delta t \left[\frac{dc}{dP} + \frac{dU}{dP} \right] \quad (5)$$

where dc/dP and dU/dP are evaluated at the pressure $P(t)$.

In order to apply eqs. (4) and (5), expressions for c and $\left. \frac{dP}{dx} \right|_{t_c}$ are required. These will be derived in the following two sections. Then, if the principal Hugoniot is known, the pressure of the decaying shock at any time (or position) can be found from

$$P(t) = P_s + \int_{t_c}^t D(t) \cdot \frac{DP}{Dx} \cdot dt \quad (6)$$

with

$$x(t) = x_c + \int_{t_c}^t D(t) dt$$

3. SOUND VELOCITIES IN SHOCK COMPRESSED MATERIALS

I assume a Gruneisen equation of state for the target material, so that the internal energy E at pressure P and density ρ can then be expressed as

$$E = E_H + (P - P_H)/\rho\gamma, \quad (7)$$

where E_H and P_H are the energy and pressure on the Hugoniot, and γ is a Gruneisen parameter. Considering an isentropic release from a point (P_H, E_H) on the Hugoniot, we can combine the differential of Eq. (7) with

$$dE = -PdV$$

for the release, and

$$dE_H = \frac{1}{2} (V_0 - V) dP_H - \frac{1}{2} (P_H + P_0) dV,$$

where $V = \rho^{-1}$, from the Rankine-Hugoniot equations to obtain

$$c^2 = \frac{dP}{d\rho} = \frac{dP_H}{d\rho} \left[1 - \frac{1}{2} (V_0 - V) \rho \gamma \right] + \frac{\gamma}{2\rho} (P_H - P_0) + V^2 (P - P_H) \left[\frac{\gamma}{V} + \frac{d \ln(V/\gamma)}{dV} \right] \quad (8)$$

This result is derived by McQueen et al.⁷ and is general except for the assumption of a volume dependent Gruneisen parameter. The first two terms give the sound speed c_H along the Hugoniot, while the addition of the last term allows calculation of c at pressure offsets from the Hugoniot. Thus, Eq. (8) could be written as

$$c^2 = c_H^2 + V^2 (P - P_H) \left[\frac{\gamma}{V} + \frac{d \ln(V/\gamma)}{dV} \right] \quad (9)$$

For the calculations that follow it is useful to express c_H in terms of D and U . To do this, I assume a linear relationship between D and U :

$$D = c_0 + AU \quad (10)$$

where c_0 and A are fitting parameters. This relationship is valid for many simple materials over a large range of pressures. Also, I will assume, for simplicity, that

$$\frac{\gamma}{V} = \frac{\gamma_0}{V_0} = \text{constant} \quad (11)$$

where γ_0 and V_0 are taken at ambient conditions. Then, the first two terms of Eq. (8) may be evaluated in terms of D and U , using two of the Rankine-Hugoniot equations:

$$P_H = \rho_0 U D \quad (12)$$

and

$$V = V_0 (1 - U/D) \quad (13)$$

I find

$$c_H = (D - U) \left[2 \left(\frac{D}{c_0} - 1 \right) \left(1 - \frac{\gamma_0}{2} \frac{U}{D} \right) + 1 \right]^{1/2} \quad (14)$$

I have checked this equation against sound speeds obtained by interrogating the LLNL tabular equations of state (using the code GATOR) for Cu over the pressure range 90-120 Mbar and Al between 8 and 16 Mbar. The GATOR calculations agreed with Eq. (14) to within 5% in all cases except near 8 Mbar in Al, where the tabular EOS exhibits a jump in the U vs D curve. Therefore, it appears that Eq. (14) is quite reliable for any material obeying Eq. (11). For the case where $\gamma_0 = 2$ and $D \gg c_0$ (strong shock limit), Eq. (14) reduces to a very simple result:

$$c_H^2 \approx \frac{2}{c_0} (D - U)^3 \quad [\text{strong shock limit}] \quad (15)$$

which should be good in the 100 Mbar range.

4. THE SHOCK PROFILE AT THE ONSET OF PRESSURE DECAY

I approximate the rarefaction fan as a sequence of small discrete releases.⁸ In this spirit I assume that the first such release, which

catches the shock at $t = t_c$, decreases the pressure P_s by ΔP . A very short time later, a second release will decrease the pressure further by ΔP . The situation is shown in Fig. 2. We now calculate $\Delta P/\Delta x$ in the limit as Δx becomes small to obtain dP/dx near the shock front at $t = t_c$. The first incremental release travels in the shock compressed medium at speed $U(P_s) + c_H(P_s)$, whereas the second travels at speed $U(P_s - \Delta P) + c(P_s - \Delta P) \equiv U_1 + c_1$. Then, at $t = t_c$ the two release fronts are separated by

$$\begin{aligned}\Delta x &= (U + c_H)(t_c - \tau) - (U_1 + c_1)(t_c - \tau) \\ &= \left[(U - U_1) + (c_H - c_1) \right] (t_c - \tau)\end{aligned}\quad (15)$$

Using Eq. (10) and (12) it is seen that for small ΔP

$$U - U_1 \approx \frac{dU}{dP} \cdot \Delta P = \frac{\Delta P}{\rho_0(20 - \epsilon_0)} \quad (17)$$

From Eq. (9),

$$c_1^2 - c_H^2 = V^2 \epsilon_P \left[\frac{\gamma}{V} + \frac{d \ln(V/\epsilon)}{dV} \right]$$

or

$$c_H - c_1 \approx \frac{V^2 \Delta P}{2c_H} \left[\frac{\gamma}{V} + \frac{d}{dV} \ln \left(\frac{V}{\epsilon} \right) \right] \quad (18)$$

Using Eq. (11), this becomes

$$c_H - c_1 \approx \left[\frac{\epsilon_0 \gamma_0 V^2}{2c_H} \right] \Delta P \quad (19)$$

Putting Eqs. (17) and (19) into Eq. (16) yields

$$\left[\frac{\Delta x}{\Delta P} \right]_{t_c} = (t_c - \tau) \left[\frac{1}{\rho_0(2D - c_0)} + \frac{\rho_0 \gamma_0 v^2}{2c_H} \right]$$

Taking the limit as $\Delta P \rightarrow 0$ and inverting we get

$$\left[\frac{dP}{dx} \right]_{t_c} = \frac{1}{(t_c - \tau) \left[\frac{1}{\rho_0(2D - c_0)} + \frac{\rho_0 \gamma_0 v^2}{2c_H} \right]} = \frac{c_H + U - D}{\tau(D - U) \left[\frac{1}{\rho_0(2D - c_0)} + \frac{\rho_0 \gamma_0 v^2}{2c_H} \right]} \quad (20)$$

Two generalizations can be deduced from this equation. First, $dP/dx]_{t_c}$ decreases with increasing τ , indicating the desirability of using long pulses. Secondly, Eq. (20) predicts that the magnitude of $dP/dx]_{t_c}$ and hence $DP/Dx]_{t_c}$ increases with increasing pressure; i.e., stronger shocks decay faster. However, $|(1/P)(dP/dx)|$ decreases with increasing P .

5. EVOLUTION OF THE SHOCK DECAY; COMPARISON WITH CODE SIMULATIONS

All of the tools are now at hand for calculating the pressure of the decaying shock as a function of time (or space). The time t_c is calculated from Eq. (3), using Eq. (14) to evaluate c_H . The shock decay DP/Dx at $t = t_c$ is then found by combining Eq. (4) and Eq. (20). $P(t)$, for $t > t_c$, is calculated from Eq. (6), using Eq. (4) and Eq. (5), with dc/dP and dU/dP calculated from Eq. (10) and Eq. (9), respectively. Equation (6) is not easily evaluated analytically since c , U , D , dc/dP , and dU/dP are all time dependent, but is easily solved numerically. I now describe two calculations for laser shocked aluminum and compare them with simulations of these problems using the LASNEX fluid physics code.⁹

The LASNEX calculations each used pulses which rise rapidly in time to a maximum intensity I_0 (30 ps), remain constant for a time $\tau = 300$ ps, then fall rapidly to zero (fall time 10 ps). Incident intensities of 3×10^{14} W/cm² and 9×10^{14} W/cm² were used which approximately span the usable range achievable with the JANUS laser. The LASNEX hot electron transport physics package was turned off, so that the preheat effects could be minimized. The absorbed fraction of laser light was about 0.3 in both problems. Target thickness was 50 μ m.

Figure 3 shows plots of the shock pressure calculated as a function of time for the two problems. LASNEX predicts maximum pressures, P_s , of 13 and 30 Mbar for the two intensities, consistent with the scaling law of Eq. (1). Also plotted in Fig. 3 are the curves calculated from the model described here. Equation (1) was used to calculate P_s , with $\alpha = 2/3$ and the coefficient of I_0 taken from the lower-intensity LASNEX calculation. No other normalization to the code has been done. Equation of state information used in the model was obtained by fitting the D-U curves obtained from the LLL tabular EOS libraries to Eq. (10). The agreement between the hydrocode calculation and the simple model is remarkably good. Note that in traversing the 50 μ m slab the pressure drops by more than a factor of two, demonstrating that shock decay is a dominant effect in short pulse problems.

Figure 4 is an x-t plot for the shock generated by the 3×10^{14} W/cm² irradiation. The trajectories obtained assuming a uniform shock, the LASNEX predictions, and the model's predictions are shown. Again, the LASNEX results are nicely reproduced by the simple model. The arrival times at the target's back surface predicted by LASNEX and the model differ by less than 50 ps out of a transit time of 2000 ps. Neglect of decay effects leads to an underestimate of the transit time by about 400 ps.

The shock profiles calculated by the model and the code are shown for the higher intensity case in Fig. 5. Despite the simplicity of the model, it does a surprisingly good job of reproducing the code-calculated predictions of $P(t)$, $x(t)$, and dP/dx near the head of the shock. Similar agreement between the model and the code are found for the lower intensity problem. The profile of the shock as it nears the rear surface is shown in Fig. 6. For comparison, the position and magnitude of the steady 27 Mbar shock at this same time is also shown.

6. APPLICATION TO LASER-DRIVEN SHOCKWAVE EXPERIMENTS

As we saw in the previous section, a simple model is capable of reproducing hydrocode calculations of laser driven shock decay with reasonable accuracy. For serious application to real target design, the model must also work for different pulse shapes and when superthermal electron preheating is present in the target. To evaluate the importance of these effects, it is useful to compare the model's predictions with a recent computational study by More.² In these calculations the LASNEX code was used to study 25- μ m-thick aluminum slabs irradiated by Gaussian pulses with full-widths at half-maximum of 300 ps. Incident intensity was varied between 1×10^{14} and 2×10^{15} W/cm². Superthermal electron preheat effects were included. The maximum pressure was found to obey a scaling law

$$P_s = 8.6 I_0^{0.82} \quad (21)$$

where P_s is in Mbar and I_0 is in 10^{14} W/cm². P_s and the pressure at the 25 μ m target depth, calculated from LASNEX, are shown in Fig. 7 as functions of I_0 .

I have repeated these calculations using the analytic model, assuming that P_s is given by Eq. (21), approximating a Gaussian pulse by a square pulse, and ignoring superthermal electron preheat. The model-calculated pressures at the 25 μm depth are shown in Fig. 7. At intensities below about $3 \times 10^{14} \text{ W/cm}^2$ the model adequately reproduces More's calculations, indicating that below this intensity, preheat effects do not significantly perturb the shock decay process and that the square-pulse approximation is valid. However, above $I_0 \simeq 3 \times 10^{14} \text{ W/cm}^2$, a systematically increasing deviation between the analytic and hydrocode predictions is observed. This effect is attributed to preheat. The LASNEX calculations predict that at over the studied intensity range the preheat contribution to the total pressure increases much more rapidly with increasing intensity than does the shock contribution. For example, at 10^{14} W/cm^2 More finds that the preheat pressure at the 25 μm level is about 1% of the total pressure, but at $9 \times 10^{14} \text{ W/cm}^2$ it has increased to 20%. Deeper into the target, where the pressure maximum occurs, this fraction is much greater. Since the analytic model does not treat preheat, the use of Eq. (20) leads to an overestimate of the shock contribution, and thus spuriously high rear surface pressures at high intensity. Caution must therefore be employed in the application of this model when preheat effects are large.

Caution may also be necessary in applying this model to long laser-pulse problems, where two dimensional expansion effects¹⁰ and possible decoupling between the critical surface and ablation surface could lead to rarefaction effects not treated here.

In conclusion, it has been shown that a simple analytic model can reproduce the shock decay phenomena predicted by detailed hydrocode calculations. Applications of the model appears valid when preheat effects

are not significant (e.g., below intensities of $\approx 3 \times 10^{14}$ W/cm² in aluminum).

Efforts are now underway to combine the shock decay model discussed here with a recently developed analytic model for superthermal-electron preheat.¹¹ Preheat pressures could then be calculated and shock propagation through strongly preheated target material could be treated, at least approximately. Moreover, the combination of these models should provide a simple package for optimizing the design of laser driven shockwave experiments. Effects of variations in laser intensity, wavelength, pulse length, target composition, and target thickness could be quickly evaluated, thus giving guidance for subsequent more-detailed studies using hydrocode simulations.

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FIGURE CAPTIONS

- Fig. 1 (a) The qualitative appearance of the shock profile at the time t_c , when the leading edge of the rarefaction fan has just overtaken the shock. The shock has penetrated a distance $x_c = Dt_c$ into the target. The decay of the shock at times $t > t_c$ is sketched in (b). The spatial attenuation of the shock pressure, DP/Dx , is related to the instantaneous value of dP/dx through Eq. (4).
- Fig. 2 The approach used to calculate dP/dx at time $t = t_c$. The rarefaction fan is approximated by a sequence of small releases, each travelling at speeds $C + U$, the sum of the local sound speed and local particle velocity. As P decreases, $C + U$ decreases, giving the shock profile its characteristic concave shape.
- Fig. 3 Shock pressures calculated as functions of time for incident intensities of 3×10^{14} and 9×10^{14} W/cm². LASNEX calculations are shown as the solid lines, while the analytic calculations are shown as the dots. The target is a 50 μ m-thick aluminum slab.
- Fig. 4 Time versus position for the shock produced by an incident intensity of 3×10^{14} W/cm². Shown are calculated trajectories predicted by the analytic model (solid line) and LASNEX (open circles). The trajectory of steady 13 Mbar shock is shown as the dashed line.
- Fig. 5 Shock profiles at different times produced by a 9×10^{14} W/cm² laser intensity. The peak of the 300 ps pulse occurs at $t = 150$ ps. LASNEX calculated profiles are the solid lines and model-calculated profiles are the dots. Position is with respect to the rear surface. The pulse comes from the right, irradiating the target at $x = 50$ μ m.

Fig. 6 The shock profile near the rear surface ($x=0$) at 1500 ps after the beginning of the laser pulse ($I_0 = 9 \times 10^{14} \text{ W/cm}^2$). LASNEX calculations (solid line) and model calculations (dots) are shown. Also shown is the magnitude and position of a steady shock produced by a long pulse of the same laser intensity.

Fig. 7 Comparison of model predictions with computational results for aluminum from Ref. 2. The computational study used a hydrocode which includes suprathreshold electron preheat effects. Maximum pressures predicted by the code are shown as the solid line. The solid circles are the code-predicted pressures at a 25 μm depth (the dashed line is a best fit to these points). Model calculated pressures at 25 μm depth are shown as open squares. Note the close agreement between code and model below intensities of about $3 \times 10^{14} \text{ W/cm}^2$.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7