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THREE-BODY MOLECULAR DESCRIPTION OF ^9Be .

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ABSTRACT

A three-cluster model composed of two α -particles and a neutron is used to describe the low-lying spectrum of ^9Be . The resulting three-body problem is solved by the Born-Oppenheimer method for different two-body n - α and α - α potentials. Molecular energy curves are obtained that explain the binding mechanisms leading to both the ground state and first excited state of ^9Be .

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1. Introduction

It is well known¹⁾ that the low-lying spectrum of ^8Be can be explained within the α -particle model. Since the energy of dissociation of ^9Be into $\alpha+n$ is small compared to the energy required to breakup an α -particle, it is generally expected that the low energy levels of ^9Be can be described as bound states or resonances of a three-body system composed of two α 's and a neutron n . From the pure three-body point of view the $\alpha+n$ system is also worthwhile considering. In the absence of bound ^5He and ^8Be nuclei, the ground state of $\alpha+n$ results from a genuine three-body effect, and in view of the resonant structure of both the n - α and α - α interactions it may be possible to identify Efimov-type states²⁾ among the low-lying levels of ^9Be .

The variational method³⁾ and the Faddeev formalism⁴⁾ have been widely used in the past to obtain the bound states of the quantum mechanical nuclear three-body problem⁵⁾. On the contrary, in molecular physics, the large proton-electron mass ratio favors the use of Molecular Expansion methods⁶⁾ together with the Born-Oppenheimer approximation⁷⁾ to calculate the energy levels of three-body molecules such as $\text{H}_2^+(\text{p}+\text{e}^+ + \text{p})$ and other more complicated molecular systems. Nevertheless, in a model three-body calculation involving two heavy particles and a lighter one, interacting through short range s -wave potentials, it has been shown recently⁸⁾ that the Born-Oppenheimer (B.O.) method yields remarkably good results for the binding energies and wave functions even when the mass ratio m between the heavy and light particle is small ($m \geq 1$). Since the validity of the B.O. method for low mass ratios seems to be related to the short range nature of the two-body interaction between pairs⁸⁾, we explore the analogy between the $\alpha+n$ system and the H_2^+

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molecule and make use of the B.O. method to obtain the bound states of the relevant three-body problem. Since an exact solution of the two-center problem may be obtained if the potentials are nonlocal but separable⁹⁾, we represent the $n-\alpha$ interaction by a separable potential that is both spin and angular momentum dependent and whose parameters are fitted to the low energy $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ phase shift analyses. For the $\alpha-\alpha$ interaction both Ali-Bodmer¹⁰⁾ and Neudatchin, et al.¹¹⁾ local potentials are considered.

In sect. 2 we discuss the molecular approach and in sect. 3 the two-body $n-\alpha$ and $\alpha-\alpha$ interactions of our model are presented together with the resulting Born-Oppenheimer predictions for the low-lying levels of $\alpha+n+\alpha$. Some conclusions are given in sect. 4.

2. Molecular Approach

In the Jacobian coordinates $\vec{r}$ and $\vec{R}$ of fig. 1 the three-body Schrödinger equation for the $\alpha+n+\alpha$ system is given by ($\hbar^2 = 1; m = m_\alpha/m_n; 2m_n = 1$)

$$[-\nabla_{\vec{R}}^2/\mu_{\alpha\alpha} - \nabla_{\vec{r}}^2/\mu + V_{\alpha\alpha}(\vec{r} + \frac{1}{2}\vec{R}) + V_{n\alpha}(\vec{r} - \frac{1}{2}\vec{R}) + V_{\alpha\alpha}(\vec{R})]\psi(\vec{r}, \vec{R}) = E\psi(\vec{r}, \vec{R}) \quad (1)$$

$$\mu_{\alpha\alpha} = m/2$$

$$\mu = 2m/(2m+1)$$

where $\psi(\vec{r}, \vec{R})$ is the exact wave function and E is the three-body binding energy.

The molecular approach⁶⁾ is based on the assumption that the wave function $\psi(\vec{r}, \vec{R})$ can be approximately written as

$$\psi(\vec{r}, \vec{R}) = \sum_i \psi_i(\vec{r}, \vec{R}) \phi_i(\vec{R}), \quad (2)$$

where $\psi_i(\vec{r}, \vec{R})$ is an eigenfunction of the two-center problem

$$[-\nabla_{\vec{r}}^2/\mu + V_{n\alpha}(\vec{r} + \frac{1}{2}\vec{R}) + V_{n\alpha}(\vec{r} - \frac{1}{2}\vec{R})]\psi_i(\vec{r}, \vec{R}) = \epsilon_i(\vec{R})\psi_i(\vec{r}, \vec{R}), \quad (3)$$

and $\epsilon_i(\vec{R})$ is the corresponding negative energy eigenvalue that depends on the distance R between the α 's but not on the orientation in space of the $\vec{R}$ vector. Substituting (2) in (1) and using the orthogonality properties of the $\psi_i(\vec{r}, \vec{R})$

$$\int d^3r \psi_i^*(\vec{r}, \vec{R}) \psi_j(\vec{r}, \vec{R}) = \delta_{ij}, \quad (4)$$

we obtain a set of coupled equations for the $\phi_i(\vec{R})$

$$[-\nabla_{\vec{R}}^2/\mu_{\alpha\alpha} + V_{\alpha\alpha}(\vec{R}) + \epsilon_i(\vec{R})]\phi_i(\vec{R}) = E\phi_i(\vec{R}) - \sum_j A_{ij}(\vec{R})\phi_j(\vec{R}), \quad (5)$$

where the coupling coefficients are

$$A_{ij}(\vec{R}) = \frac{-1}{\mu_{\alpha\alpha}} \int d^3r \psi_i^*(\vec{r}, \vec{R}) \{2[\vec{\nabla}_{\vec{R}} \psi_j(\vec{r}, \vec{R})] \cdot \vec{\nabla}_{\vec{R}} + [\nabla_{\vec{R}}^2 \psi_j(\vec{r}, \vec{R})]\}. \quad (6)$$

In molecular physics the large proton-electron mass ratio is invoked as a justification for the Born-Oppenheimer approximation⁷⁾ which amounts to neglecting the coupling terms in eq. (5). Although in the present work the

relevant mass ratio m is small ($m \approx 4$), we make use of the conclusions drawn in ref. 8) and neglect the coupling coefficients to obtain the negative energy spectrum of $\alpha+n+\alpha$. Thus

$$\Psi(\vec{r}, \vec{R}) = \psi_1(\vec{r}, \vec{R}) \phi_1(\vec{R}) \quad (7)$$

where

$$[-\nabla_{\vec{R}}^2/\mu_{\alpha\alpha} + V_{\alpha\alpha}(R) + \varepsilon_1(R)] \phi_1(\vec{R}) = E \phi_1(\vec{R}). \quad (8)$$

The potential energy for the relative motion of the α 's is the sum of two terms. The first is the α - α potential that fits on-shell $\alpha\alpha$ scattering data and the second represents the effective interaction between the two α 's due to the presence of particle n .

Since the two-body n - α interaction of our model is nonlocal but separable, an exact solution of the two-center problem may be obtained⁹⁾. If the n - α potential is written in its most general form

$$V(\vec{q}, \vec{q}') = \sum_{l=0}^{l_{\max}} \sum_{j=l-1/2}^{l+1/2} \sum_{m=-j}^j \sum_n^{N_{lj}} g_{nlj}(q) y_{jl}^m \lambda_{nlj} \varepsilon_{nlj}^*(q') y_{jl}^{m*}, \quad (9)$$

where

$$y_{jl}^m = (-1)^l \sum_{m's} \langle l s m' | j m \rangle Y_{ls}^{m'}(q') x_{ls}^s, \quad (10)$$

and N_{lj} is the number of separable terms in each (jl) channel interaction, the two-center wave function $\psi(\vec{q}, \vec{R})$ is given by

$$\begin{aligned} \psi_{MA1}(\vec{q}, \vec{R}) = \sum_{nljm} \frac{g_{nlj}(q) y_{jl}^m}{\varepsilon_{MA1}(R) - q^2/\mu} [e^{-i\vec{q} \cdot \vec{R}} + \Lambda(-1)^l e^{i\vec{q} \cdot \vec{R}}] \\ \times D_{m1}^j(\varphi, \theta, 0) C_{nlj}^{MA1}(R). \end{aligned} \quad (11)$$

The quantum number M is the projection of the neutron total angular momentum along the $\vec{R}$ vector and Λ is the parity quantum number ($\vec{q} + -\vec{q}, \vec{r} + -\vec{r}$).

The index "1" accounts for the possibility of different eigenvectors with the same M and Λ . D_{m1}^j are the Wigner rotation matrices where ϕ and θ are the polar angles of the vector $\vec{R}$. The energy eigenvalue ε_{MA1} and the coefficients C_{nlj}^{MA1} are obtained from the solution of a set of coupled homogeneous equations

$$\sum_{n'l'j'} \{ \lambda_{n'l'j'}^{-1} \delta_{nn'} \delta_{ll'} \delta_{jj'} - M_{nlj, n'l'j'}^{MA1}(\varepsilon_{MA1}) \} C_{n'l'j'}^{MA1} = 0 \quad (12)$$

with

$$M_{nlj, n'l'j'}^{MA1}(\varepsilon) = \int d^3q \frac{g_{nlj}(q) y_{jl}^m g_{n'l'j'}(q) y_{j'l'}^{m*}}{\varepsilon - q^2/\mu} [1 + \Lambda(-1)^l e^{i\vec{q} \cdot \vec{R}}]. \quad (13)$$

For each value of R the energy eigenvalues ε_{MA1} are those values of ε for which the determinant of the homogeneous equation (12) vanishes, thus allowing for a solution C_{nlj}^{MA1} . The number of coupled equations depends on M and does not exceed the total number of separable terms in the n - α interaction. If simple expressions are chosen for the form factors $g(q)$, the matrix elements $M(\varepsilon)$ may be calculated analytically, otherwise a numerical integration can be easily performed.

3. Interactions and Molecular Results

3.1. TWO-BODY INTERACTIONS

As it has been previously pointed out the $n\alpha$ interaction is represented by a separable potential that is both spin and angular momentum dependent and whose parameters are fitted to the low energy $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ $n\alpha$ phase shifts. Unlike the p -wave phase shifts that are positive and present an unequivocal resonant structure¹²⁾, the s -wave phase shifts decrease monotonically from π or zero (depending on the absolute normalization) and seem to satisfy the relation $\delta_0^{1/2}(E_n = 0) - \delta_0^{1/2}(E_n = \infty) = \pi$. In the absence of a bound ^5He nuclei the modified Levinson's theorem¹³⁾ attributes the π discontinuity to the existence of a Pauli forbidden $s_{1/2}$ bound state in the $n\alpha$ system. Since a two-body representation of the many-body $n\alpha$ interaction has to simulate the presence of the forbidden state we follow the method proposed by Saito¹⁴⁾ and Kukulin, Neudatchin and Smirnov¹⁵⁾. A one term s -wave separable potential $V_{n\alpha}^{0,1/2}$ is chosen that is strong enough to allow for an $n\alpha$ bound state at 13.3 MeV[†] and that fits the s -wave phase shifts in the $E_n = 0 - 20$ MeV range. Subsequently this bound state is subtracted by redefining the $n\alpha$ interaction as

$$\bar{V}_{n\alpha}^{0,1/2} = V_{n\alpha}^{0,1/2} + \Gamma |\phi_0\rangle \langle \phi_0| \quad (14)$$

where $|\phi_0\rangle$ is the original bound state wave function and $\Gamma \rightarrow \infty$. Although $V_{n\alpha}^{0,1/2}$ allows for no negative energy bound state as required in the absence of a bound ^5He nuclei, the calculated phase shifts are identical to the ones resulting from $\bar{V}_{n\alpha}^{0,1/2}$ and satisfy the condition $\delta_0^{1/2}(E_n = 0) - \delta_0^{1/2}(E_n = \infty) = \pi$ suggested by the phase shift analyses. Therefore the final $s_{1/2}$ $n\alpha$ interaction to be used in the molecular calculation is a two-term separable potential where the second term is a projection operator on the bound state allowed by the first term. Since in most previous works¹⁷⁻¹⁸⁾ where the $n\alpha$

[†]The Wood-Saxon potential that fits $n\alpha$ scattering data allows for a 1s bound state at 13.3 MeV¹⁶⁾.

interaction is represented by a separable potential, the forbidden state is simulated by an effective s -wave repulsion, we also consider another set of parameters for $V_{n\alpha}^{0,1/2}$ that make the $n\alpha$ potential repulsive. This has the advantage that there are no $n\alpha$ bound states to be subtracted but the resulting phase shifts do not satisfy the π discontinuity. For the p -wave interaction a simple one-term separable potential is adopted that is sufficiently attractive to reproduce the resonant structure of the $p_{1/2}$ and $p_{3/2}$ phase shift analyses in the 0 - 20 MeV energy region. Among the several possible form factors that have been used in the past¹⁷⁻¹⁸⁾ for $V_{n\alpha}^{1/2,j}$ we chose

$$g_{1j}(q) = \frac{q^2}{(q^2 + \beta_{1j}^2)^{2+1}} \quad (15)$$

that leads to two-body solutions that have the correct threshold and asymptotic behavior. The parameters of the $V_{n\alpha}^{1/2,j}$ interaction in each $(1j)$ channel are shown in table 1.

The $\alpha\alpha$ interaction is represented by a phenomenological local potential whose parameters are fitted to the s -, d -, and g -wave $\alpha\alpha$ phase shifts. Both Ali-Bodmer¹⁰⁾ and Neudatchin, et al.¹¹⁾ potentials have been adopted in the molecular calculation. Since the Neudatchin, et al. potential is strong enough to allow for 1s, 2s and 1d bound states of $\alpha\alpha$, the relevant states to $\alpha+n\alpha$ emerging from eq. (8) are 3s, 2d and 1g.

3.2. MOLECULAR RESULTS

With the two-body $n\alpha$ interaction described above we solve first eq. (12) to obtain the eigenvalues $\epsilon_{M\Lambda}(R)$ of the two-center problem. The results of this calculation are shown in fig. 2 for $V_{n\alpha}^{0,1/2}$ attractive and in fig. 3 for $V_{n\alpha}^{0,1/2}$ repulsive. Since the $p_{3/2}$ $n\alpha$ potential is solely responsible for $\epsilon_{3\Lambda}(R)$,

the $\frac{1}{2}^+$ molecular curves are identical in both figures (since $\epsilon_{\frac{1}{2}^+}(R) \leq \epsilon_{\frac{1}{2}^+}(R=\infty)$ for all R the $\frac{1}{2}^+$ curve has been omitted for clarity). Unlike the $\frac{1}{2}^-$ solution that exists for all R , the $\frac{1}{2}^+$ emerges around $R = 7.4$ fm and joins the $\frac{1}{2}^-$ at $R = \infty$. Although there is no $p_{\frac{1}{2}}$ bound state in the $n\alpha$ system, the $\frac{1}{2}^-$ curve converges to a very small value of the energy at $R = \infty$ ($\epsilon_{\frac{1}{2}^-}(\infty) = 0.48$ MeV). This is an inherent feature of the molecular treatment that results from using in eq. (3) the reduced mass μ of the "united athom" ($R=0$) for all values of R including the "separated athom" limit ($R \rightarrow \infty$) (see ref. 8)). The asymptotic behavior has little or not effect in a bound state calculation but may be crucial in the scattering region where it leads to an incorrect scattering threshold¹⁹⁾. In fig. 2 the dashed lines correspond to $\Gamma=0$ in eq. (14) ($s_{\frac{1}{2}}$ no bound state not subtracted) and the solid lines to $\Gamma=\infty$. For $R > 7.4$ fm the lowest $\frac{1}{2}^-$ curves in both figures behave like $\epsilon_{\frac{1}{2}^+}(R)$ and for that reason have not been drawn in that region. Although all $\epsilon_{\frac{1}{2}^+}(R)$ eigenvalues depend on the way the $s_{\frac{1}{2}}$ Pauli forbidden $n\alpha$ state is treated by the two-body $n\alpha$ interaction the $\frac{1}{2}^-$ eigenvalues are the most affected. The striking feature in fig. 2 is that in the limit $R \rightarrow 0$ the lowest $\frac{1}{2}^-$ and $\frac{1}{2}^+$ eigenvalues (solid curves) stay undegenerate while the corresponding states in fig. 3 approach each other to form the $p_{\frac{1}{2}}$ bound state of the "united athom." This indicates that the spherical symmetry is not restored in the limit $R \rightarrow 0$ whenever the Pauli excluded $s_{\frac{1}{2}}$ $n\alpha$ state is simulated by an attractive $n\alpha$ interaction with the bound state subtracted[†]. It is interesting to note [†]At $R=0$ the s -wave components of the $n\alpha$ interaction has no effect on the $\frac{1}{2}^-$ eigenvalues (see eq. (11)). Therefore $\epsilon_{\frac{1}{2}^-}(R=0) = \epsilon_{\frac{1}{2}^+}(R=0)$ and the spherical symmetry is locally recovered. For $R \neq 0$ but infinitesimally close to zero the limit $\Gamma \rightarrow \infty$ takes over and creates a spherical asymmetry that leads to a non-infinitesimal difference between eigenvalues corresponding to different M values.

that in the two-center Hartree-Fock calculation of Okabe, Abe and Tanaka²⁰⁾ where the Pauli principle is fully accounted for (they use nine-particle Slater determinants) the lowest $\frac{1}{2}^-$ and $\frac{1}{2}^+$ energy curves are also nondegenerate in the limit $R \rightarrow 0$. Although in their work the dynamics is treated in a completely different way the energy curves they obtain have essentially the same meaning as ours. Therefore it may be appropriate to conclude that in the interaction between composite particles the subtraction method of Kukulin, et al.¹⁴⁾ provides a better representation of Pauli forbidden states than a simple effective repulsion. The remaining molecular curves to be discussed are the second $\frac{1}{2}^-$ energy eigenvalue curve and the lowest positive parity $\epsilon_{\frac{1}{2}^+}(R)$. While in fig. 3 there is a second $\frac{1}{2}^-$ energy curve for $0 \leq R < 0.8$ fm that coincides at $R=0$ with the $p_{\frac{1}{2}}$ bound state of the "united athom," the corresponding state in fig. 2 only exists for $\Gamma=0$. Once the $s_{\frac{1}{2}}$ no bound state is subtracted ($\Gamma=\infty$) the second $\frac{1}{2}^-$ curve disappears from the negative energy region. On the contrary the $\frac{1}{2}^+$ molecular curve is little affected by the nature of the $n\alpha$ interaction. It emerges at $R=2.5$ fm and for $R > 4.5$ fm is the lowest eigenvalue solution of the two-center problem. Since there is no $s_{\frac{1}{2}}$ bound state of $n\alpha$ (except for $\Gamma=0$) the $\frac{1}{2}^+$ curve cannot subside all the way until $R=0$. Its existence is mainly due to the p -wave $n\alpha$ interaction that is allowed to contribute with increasing weight for larger values of R . It is again worthwhile noting that the relative position of the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ curves is consistent with the results of the Hartree-Fock calculation of Okabe, et al.²⁰⁾ where for $R > 5$ fm the $\frac{1}{2}^+$ is the lowest energy solution.

As it has been previously pointed out the next step of the B.O. method involves the solution of eq. (8) to obtain the three-body binding energy. The potential energy for the relative motion of the α 's is the sum of the α - α two-body interaction with the lowest $\epsilon_{MA}(R)$ of a given parity. In fig. 4 we show the effective

$$V_{\alpha\alpha}^{MA}(R) = V_{\alpha\alpha}(R) + \epsilon_{MA}(R) \quad (16)$$

for the case of Ali-Bodmer α - α potential. The dotted line is the Ali-Bodmer potential alone and the full lines correspond to adding the $\frac{1}{2}^-$ and $\frac{1}{2}^+$ energy curves of fig. 2 to $V_{\alpha\alpha}$. While $\epsilon_{\frac{1}{2}^-}(R)$ essentially deepens the α - α effective potential the main effect of $\epsilon_{\frac{1}{2}^+}(R)$ is to increase its range. As shown in table 2 both the $\nu_{\alpha\alpha}^{\frac{1}{2}^-}$ and $\nu_{\alpha\alpha}^{\frac{1}{2}^+}$ allow for an $L=0$ eigenstate of (8) in contrast to no bound states when the Ali-Bodmer potential is used alone. $\nu_{\alpha\alpha}^{\frac{3}{2}^-}$ also allows for an $L=2$ three-body bound state that is less bound than the $L=0$ eigenvalue resulting from $\nu_{\alpha\alpha}^{\frac{1}{2}^+}$. The same calculation was done with Neudatchin, et al. Wood-Saxon potential¹¹⁾ which by itself allows for two $L=0$ and one $L=2$ bound states. Since these states are meant to simulate Pauli forbidden states in the $\alpha+\alpha$ system it is not a priori clear whether the forbidden states in $\alpha+\alpha$ should be eigenstates of

$$V_R^2/\mu_{\alpha\alpha} + V_{\alpha\alpha}(R) + \epsilon_{MA}(R) \quad (17)$$

or

$$V_R^2/\mu_{\alpha\alpha} + V_{\alpha\alpha}(R) \quad (18)$$

In the first case the calculation is simple and the bound states of $\alpha+\alpha$ are the 3s and 2d eigenvalues of the operator (17) that are shown in table 2 for two different $\epsilon_{MA}(R)$. If the forbidden states of $\alpha+\alpha$ are eigenvalues of (18) then we have to proceed as in eq. (14) and redefine the α - α potential in (8) as

$$\bar{V}_{\alpha\alpha} = V_{\alpha\alpha} + \Gamma_1 |1s\rangle\langle 1s| + \Gamma_2 |2s\rangle\langle 2s| + \Gamma_3 |1d\rangle\langle 1d|. \quad (19)$$

The bound state of $\alpha+\alpha$ are the low energy eigenvalues of the resulting inhomogeneous differential equation and the corresponding eigenstates are orthogonal to the eigenstates of (18). Compared to the values shown in table 2 this method lowers the binding energy of the ground state by 0.7 MeV and leaves all the other states unchanged. Although the three-body energy levels of $\alpha+\alpha$ we obtain are lower than the known experimental values, the ones obtained with the Ali-Bodmer potential have the correct relative position. The first excited state is the $\frac{1}{2}^+$, $L=0$ state which is 1.95 MeV above the ground state. Since the resulting B.O. wave functions are not eigenfunctions of the total three-body angular momentum $\vec{J}$ we abstain from making a direct comparison with experiment. In a subsequent publication we will present the results of a full molecular calculation where some of the coupling coefficients will be retained (see eq. (5)-(8)) and the three-body wave function is constructed to be a pure state of the total $\vec{J}$.

4. Discussion

In the previous sections we have used the Born-Oppenheimer method to obtain the low-lying levels of ^9Be as a three-body molecular system composed of two α -particles and a neutron n . The molecular approach provides a very simple and transparent explanation of the level structure of ^9Be . The presence of the neutron n is crucial to the existence of a bound ^9Be nuclei. It creates an effective potential between the α 's that when added to the two-body α - α interaction generates sufficient attraction to allow for three-body bound states. In the model the p_3 n - α interaction is solely responsible for the ground state energy (its absolute value depends also on the two-body α - α potential that has been chosen) and the corresponding wave function has no nodes (see eq. (11)). The low-lying positive parity state is essentially due to a long range force between the α -particles that is mediated by the neutron. This long range force is particularly sensitive to the strength of the p -wave n - α interaction and for that reason the resulting state may turn out to be an Efimov state. This will be further studied in a forthcoming Faddeev calculation. In view of the simplicity of the B.O. method it is our hope that the molecular approach may be useful in studying three-body cluster structure in heavier nuclei such as ^{25}Mg ($^{12}\text{C}+n+^{12}\text{C}$), ^{29}Si ($^{12}\text{C}+n+^{16}\text{O}$) and others that can be dissociated in two closed shell nuclei and a nucleon. Another interesting subject of research is to study the best way to simulate Pauli forbidden states in the two-body interaction between composite particles. The B.O. method was able to show that some components of the effective α - α interaction resulting from the presence of particle n are extremely sensitive to the nature of the s -wave n - α interaction. Nevertheless because the difference was on the $\frac{1}{2}$ molecular energy

curve we cannot make a definite statement as how this change will affect the three-body observables. This will be studied in a subsequent publication where a coupled channel calculation based on both $\frac{1}{2}^-$ and $\frac{3}{2}^-$ two-center eigenfunctions will be performed.

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Table Captions

Table 1: Parameters of the two-body n - α interaction. The $p_{\frac{1}{2}}$, $p_{\frac{3}{2}}$ and $s_{\frac{1}{2}}$ (repulsive) parameters are from ref. 18) and μ' is the n - α reduced mass, $\mu' = m/m + 1$.

Table 2: Born-Oppenheimer spectrum of $\alpha+n+\alpha$.

Table 1

	$s_{\frac{1}{2}}$ (attractive)	$s_{\frac{1}{2}}$ (repulsive)	$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$
β	1.494 fm^{-1}	0.7 fm^{-1}	1.177 fm^{-1}	1.449 fm^{-1}
$2\mu'\lambda$	-9.283 fm^{-3}	3.770 fm^{-3}	-13.873 fm^{-5}	-60.708 fm^{-5}

Table 2

		Ali-Bodmer		Kukulin, et al.	
		Fig. 1	Fig. 2	Fig. 1	Fig. 2
$\frac{1}{2}^-$	$l=2$	-1.05	-1.05	-2.67	-2.67
$\frac{1}{2}^+$	$l=0$	-1.70	-1.65	-1.59	-1.56
$\frac{3}{2}^-$	$l=0$	-3.85	-3.85	-6.10	-6.10

Figure Captions

- Fig. 1: Jacobian coordinates of the three-body problem that are most convenient to molecular expansion methods.
- Fig. 2: Two-center binding energy $\epsilon(R)$ versus R for an attractive $s_{1/2}$ n - α interaction and different values of M and Λ . The dashed lines correspond to $\Gamma = 0$ while the solid lines to $\Gamma = \infty$.
- Fig. 3: Two-center binding energy $\epsilon(R)$ versus R for a repulsive $s_{1/2}$ n - α interaction and different values of M and Λ .
- Fig. 4: Potential energy $v(R)$ for the relative motion of the α 's versus R . v^{AB} is Ali-Bodmer α - α potential alone.

Fig. 1

Fig. 2

Fig. 3

Fig. 4