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Three Dimensional Gravity Modeling Techniques
with Application to the Ennis Geothermal Area

by

Dave Semmens

Submitted December 1987

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NOTICE

Appendicies D,E,F, and G (pages 184-251) are not included in this version of the thesis. Data contained in these appendicies include output of the modeling programs pertaining to modeling of the Ennis hot spring. A complete version of this thesis is available at the library of the Montana College of Mineral Science and Technology, Butte, Mt., 59701.

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THREE DIMENSIONAL GRAVITY MODELING TECHNIQUES
WITH APPLICATION TO THE ENNIS GEOTHERMAL AREA

by

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A thesis submitted to the
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ABSTRACT

3-D gravity modeling was done in the area of the Ennis hot spring in an attempt to determine controlling structure of the Ennis hot spring. The modeling was done in a two-step process where:

- 1) The topography was modeled by modeling the valley fill from the highest elevation in the modeling area to some elevation below the lowest station elevation using Talwani and Ewing's (1960) method of modeling with vertically-stacked, horizontal, n-sided polygons. Once the gravity contributions of the valley fill included in this "topographic model" are calculated, they were removed from the original gravity data.
- 2) The remaining valley fill was modeled using blocks where the 3-D algorithm for modeling with blocks results from integrating the gravity formula in the X and Z directions and approximating the integration in the Y-direction using a quadrature formula.

Finally, an inverse 3-D gravity modeling program was written to automatically adjust the bedrock topography output from this two-step modeling process. The gravity data calculated from the adjusted bedrock topography, output from the inverse modeling program, should match the observed gravity data within the error of the survey.

This two-step forward gravity modeling procedure, used sequentially with the inverse gravity modeling program, was applied to modeling of the gravity data collected in Ennis hot spring area. The resulting bedrock topography model produces calculated gravity data that matches the observed gravity data, collected in the area of the Ennis hot spring, within the 0.5 mgal assumed error of the survey. In addition, this bedrock topography model matches reasonably with drilling and seismic refraction data.

Interpretation of this bedrock topography model indicates the presence of a wrench fault across which structural trends and/or bedrock depths change noticeably. This wrench fault is located approximately 500 meters north of the hot spring along a N-S line. A drill site, located approximately 750 meters NE of the hot spring and contained in the interpreted location of the wrench fault, is proposed based on an interpretation of the inverse 3-D gravity modeling results. If a well is drilled at this suggested site, a crude estimate indicates that 99°C water might be intercepted in the bedrock if the interpreted wrench fault controls the ascending geothermal water.

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INTRODUCTION

Purpose and Scope

Because of the search for alternate energy sources in the 1970's, the Ennis hot spring was the focus for an extensive amount of geophysical and hydrogeological work. This previous work provides a solid foundation on which new efforts are built, and this paper contains a discussion of the information gathered in the previous studies. In the summer and fall of 1985 new data was collected including controlled source audio-magnetotelluric (CSAMT) and gravity data. Collection, modeling, and interpretation of the CSAMT data are covered in a thesis presently being written by Gunnar Emilsson, and his final conclusions were not available at the time of this writing. Collection, modeling and interpretation of the gravity data are described in this paper.

The purpose of the gravity modeling is to map the bedrock topography in an attempt to locate faults, offsetting the bedrock, which might control the path of the geothermal water ascending to eventually emerge as the Ennis hot spring. The purpose of the gravity modeling, then, might be construed as an attempt to locate promising areas for a drilling program developed to use the geothermal resource. To satisfy this purpose, I wrote a 3-D forward gravity modeling program and also an inverse gravity modeling program which when used sequentially are designed to find a bedrock topography model

from which calculated gravity data will match the observed within the error of the gravity survey. I also devised a modeling sequence which increases the efficiency and accuracy of the modeling process.

The funding for this project was contingent on finding information pertaining to the Ennis hot spring. For this reason, the modeling programs and process discussed in this paper were applied to the gravity data collected near the Ennis hot spring. However, in general, these modeling techniques could be used to solve geological and hydrogeological (as well as geothermal) problems in any valley where a valley fill over bedrock geologic setting exists.

Geothermal Resources in SW Montana

There are 27 hot ($\geq 38^{\circ}\text{C}$) springs in Montana (NOAA-Key to Geophysical Records Documentation No. 12, 1980), and 25 (92%) of these hot springs are located in southwest Montana. Southwest Montana is defined as the area south of 47°N latitude and west of 110°W Longitude (Chadwick and Leonard, 1979). Of these 25 hot springs 19 (79%) issue either from crystalline rock adjacent to valley fill or from the valley fill itself (Chadwick and Leonard, 1979) as shown in Figure 1. The heat content of the springs in SW Montana appears to be controlled by deep circulation along fault and fracture zones in areas with average to above average thermal gradient. Heat derived from young magma bodies is not considered responsible for those thermal springs outside of the Yellowstone geothermal area because

Figure 1. Hot springs of southwestern Montana. Figure according to Chadwick and Leonard (1979).

"residual heat from all but the largest magma bodies is dissipated within one to two million years" (Williams, 1975). The youngest volcanism in southwest Montana (1-2 million years old) is associated with the Yellowstone Park area (Chadwick, 1978). In southwest Montana the crystalline rocks have an average thermal gradient of 30°C/Km, while the sediments average approximately 60°C/Km (Nathenson and others, 1983; Blackwell and Chapman, 1977). The approximate minimum depth of circulation necessary to attain the hot spring's surface temperature is found using the formula:

$$D = (T_R - T_S) / G$$

where:

D = Depth of Circulation

T_R = Reservoir Temperature

T_S = Mean Annual Surface Temperature

G = Thermal Gradient

Study Area and the Ennis Hot Spring

The Ennis hot spring is located in southwest Montana in the Madison Valley approximately 2 kilometers north of the town of Ennis, Montana along U.S. Highway 287 (see Figure 1). The spring issues from valley fill near the western edge of the Madison Valley at the base of a river terrace composed of fluvial cobbles and clay. The area studied for this paper

Figure 2. Study area (inner square, app. 9 Km²) shown as it would appear on the Ennis 15' quadrangle.

includes approximately 9 square kilometers surrounding the Ennis hot spring (see Figure 2).

Ever since the mid-1970's the Ennis hot spring has been studied intensively. This work has been focused on the Ennis hot spring for 3 general reasons:

- 1) At 83°C (NOAA-Key to Geophysical Records Documentation No. 12, 1980) the Ennis hot spring has the hottest surface temperature of any spring in Montana;
- 2) The Ennis hot spring is easily accessible in that it is located near highway #287 and the town of Ennis; and
- 3) The physical setting of the Ennis hot spring typifies the majority of hot springs in southwest Montana in that it issues from valley fill adjacent to crystalline rock.

The previous work in the area of the Ennis hot spring includes drilling, temperature and heat flow studies, hydrogeochemical, and geophysical surveys which will be discussed in the section to follow. I warn the reader here that the previous work section to follow is rather long because much of the previous work is incorporated (directly or indirectly) either in the gravity modeling or the interpretation of the gravity modeling results. If the modeling techniques and results are all that is of interest, skip directly to the gravity modeling section.

A Note on the Figures

In the figures of this text which display the locations of the observed gravity data and the gravity modeling results, spatial location units of both latitude and longitude and meters east and north of an assumed origin are used. I used units of meters north and east of an assumed origin because the gravity modeling programs I wrote require this orientation of the gravity stations; consequently, those figures which depict the modeling results contain spatial location units of meters. For those figures not showing the gravity modeling results, I followed the common convention of using spatial units of latitude and longitude.

The figures which display the locations of the gravity data and the gravity modeling results are not all shown at the same scale. Scale reductions or enlargements were necessary because I desired to keep all the figures within the text, rather than in plates located at the back of the thesis. I realize that these changes in scale make it difficult for the serious reader to compare results from figure to figure. However, I think it is generally more desirable to keep the figures within the text for ease of readability.

To reduce orientation problems which may arise because of unit and scale changes from figure to figure, on all figures containing units of latitude and longitude, I have also included a meter scale. Also on all figures showing spatial locations of features within the modeling area, I have included the

location of Ennis hot spring and the benchmark to which the gravity data has been referenced. The Ennis hot spring is shown as a ball with a tail while the benchmark is labeled as BM.

PREVIOUS WORK

Drilling

From October, 1977 to August, 1982, eleven test wells were drilled by the U.S. Geological Survey (TX-1 to TX-11) while one test well (TX-12) was drilled by the Montana Bureau of Mines and Geology in the area of the Ennis hot spring. A thirteenth well, MAC-1, was drilled in 1982 for Richard MacMillan and other investors. Along with these test wells, 4 shot holes (SH-1, 3, 4, and 5) and several domestic wells are also located in the area. See Figure 3 and Table 1 for the location of these wells. The shot holes were used for a seismic refraction survey, the results of which will be discussed in the geophysics subsection.

In each of the shot and test holes, variation of temperature with depth information was gathered. See Table 1 for maximum temperature with depth in each well. Three of the test wells, TX-9, 11, and 12, one shot hole, SH-3, and well MAC-1 penetrated the bedrock. Depth to bedrock information for these wells is also contained in Table 1. Drilling logs for the wells indicate that the valley fill is composed of a surficial layer of alluvium, five to ten meters thick, consisting of boulders and cobbles. Below this layer "poorly consolidated, commonly calcareous, tuffaceous clay, silt, and argillaceous sand -- all interbedded with lenticular layers of coarse sand and gravel" (Leonard, written communication) is found to compose the cores. In wells TX-9 and 11 a "well consolidated" silt and

Figure 3. Test wells (wells starting with TX), shot holes (wells starting with SH), and selected domestic wells in the area of the Ennis hot spring.

<u>Well</u>	<u>Elev.(m.)</u>	<u>Lat.^o</u>	<u>Long.^o</u>	<u>Max. Depth(m.)</u>	<u>Max. Temp ^oC/Depth (m.)</u>	<u>Dep. to Bedrock (m.)</u>
TX-1	1500	45.3675	111.7281	103	89.5 / 51.8	-----
TX-2	1500	45.4675	111.7300	116	73.0 / 116.1	-----
TX-3	1495	45.3616	111.7228	55	10.4 / 54.6	-----
TX-4	1495	45.3622	111.7275	84	17.9 / 83.5	-----
TX-5	1492	45.3669	111.7247	99	60.4 / 98.1	-----
TX-6	1496	45.3736	111.7381	60	19.1 / 60.4	-----
TX-7	1502	45.3636	111.7367	--	-----	-----
TX-8	1489	45.3803	111.7275	91	29.8 / 91.0	-----
TX-9	1502	45.3642	111.7333	120	42.0 / 120.0	107
TX-10	1480	45.3811	111.6969	258	26.9 / 258.0	-----
TX-11	1489	45.3706	111.7244	183	94.0 / 183.0	161
TX-12	1499	45.3678	111.7247	291	92.3 / 152.4	142
MAC-1	1489	45.3702	111.7253	372	97.0 / 335.0	216
SH-1	1489	45.3690	111.7213	15	62.1 / 15.4	-----
SH-3	1503	45.3567	111.7377	15	11.7 / 9.0	26
SH-4	1498	45.3678	111.7337	14	11.9 / 14.0	-----
SH-5	1488	45.3702	111.7185	14	18.9 / 14.0	-----

Table 1. Location, elevation, maximum depth, maximum temperature and depth in the well at which the maximum temperature was recorded, and depth to bedrock information for test wells and shot holes in the area of the Ennis hot spring. Data according to Leonard (written communication).

clay layer 10 to 15 meters thick is described in the well logs. Leonard (written communication) suggests that this clay layer is a "potentially extensive confining layer." In well TX-12 there is also a clay layer, 15 meters thick, above a 5 meter thick layer of cobbles and pebbles situated at the base of the valley fill. The driller's log for MAC-1 was not personally viewed by this author. The "bedrock" according to cores taken from wells SH-3, TX-9, 11, and 12 is composed of quartz-feldspathic hornblende and biotite gneiss.

Fracture Zone in Wells TX-12, MAC-1

The geologist's log of well TX-12 indicates that there is a zone of considerable fracturing in the gneiss at a depth of 150-187 m below the surface or approximately 8 meters below the base of the valley fill. The driller's log for well MAC-1 was not viewed by this author; nevertheless, Sonderegger and Zaluski (1983) indicate that well MAC-1 intercepted a fracture zone in an interval from 335 to 365 meters. The fracture zones in wells TX-12 and MAC-1 appear to be interconnected based on drawdown effects in TX-12 during a pump test of MAC-1 (Pump Test of the MacMillan well, Montana Bureau of Mines and Geology in-house report) as shown in Table 2. TX-12 is located 308 m south of MAC-1 on a nearly north-south line; consequently, if the fracture zone is assumed to be a linear feature of unknown strike between the two wells, then a dip component of 31°N is indicated for the fracture zone in this area. Well TX-11

intersected the bedrock yet did not cross a fracture zone. As TX-11 is located 80 meters southeast of MAC-1, it is possible that if TX-11 were deepened past its present 185 meters maximum depth, the fracture zone would be penetrated. During the pump test of MAC-1, the drawdown on TX-11 (80 meters away) was not as great as in well TX-12 (308 meters away). See Table 2.

The fact that drawdown in well TX-12 occurred during pumping of well MAC-1 indicates a lack of water supply from additional aquifers to well TX-12. Or, in other words, the fracture zone is possibly the major conduit by which water is supplied to well TX-12. Unfortunately the 3-D geometry of the fracture zone is unknown because a third well has not penetrated this zone. Nevertheless, the 2-D results of the pump test indicate a general northerly direction of supply to well TX-12 and possibly the Ennis hot spring. In well MAC-1 a sustained flow of $.0242 \text{ m}^3/\text{s}$ was achieved during the pump test of the MAC-1 well (Sonderegger, MBMG in-house report). For comparison surface flow at the Ennis hot spring was measured at $.0012 \text{ m}^3/\text{s}$ (Leonard, written communication). These numbers indicate the flow rate of the Ennis hot spring is ~20 times less than the pumped flow rate in the fracture zone below well MAC-1. For this reason the Ennis hot spring is assumed to represent only a minor portion of the Ennis geothermal system. Below the fracture zone flow drops off and the bedrock is described as "tight" to the bottom of well TX-12 at 291 meters according to the drilling log.

Table 2. Drawdown effects on wells TX-11, and TX-12 during a pump test of the MAC-1 well. Data taken from Pump Test of the MacMillan Well (Sonderegger, unpublished MBMG report).

<u>Well</u>	<u>Drawdown (m./1000minutes)</u>
TX-11	0.83
TX-12	1.19

Heat Flow and Thermal Gradients

Heat in the earth may be transported in the following three ways: 1) radiation, 2) conduction, and 3) convection. In geothermal systems radiative heat flow is negligible because of the temperature range pertinent to known geothermal systems (Rybach, 1981). Conductive heat flow/area (flux) is described by Fourier's Law (Rybach, 1981):

$$q = -k \nabla T$$

where:

$$q = \text{heat flux (mW/m}^2\text{)}$$

$$k = \text{thermal conductivity (mW/}^{\circ}\text{C}\cdot\text{m)}$$

$$\nabla T = \text{temperature gradient (}^{\circ}\text{C/m)}$$

The negative sign is used in Fourier's Law because heat flow is in a direction opposite to the thermal gradient.

Convective heat flow in a geothermal system (water is the medium by which heat is transferred) is given by the equation:

$$Q_{\text{conv}} = C_{p_{\text{H}_2\text{O}}}(\Delta T)(F)$$

where:

Q_{conv} = Convective heat flow (mW)

$C_{p_{\text{H}_2\text{O}}}$ = heat capacity of water [4.19×10^9
mW-s/($^{\circ}\text{C} \cdot \text{m}^3$)]

ΔT = difference between mean annual temperature
and temperature of the flow ($^{\circ}\text{C}$)

F = flow rate (m^3/sec)

In Figure 4, profiles of thermal gradients of selected shot holes and test wells are shown. This figure illustrates that in the high temperature wells (TX-1, 2, 5, and 11) large near surface gradients diminish to become near isothermal at depth. High thermal gradients which diminish to near isothermal at depth are common thermal profile features of many geothermal systems in the Basin and Range province (Chapman and others, 1978). To model these thermal profiles, Chapman and others (1978) achieved reasonable results by assuming the large near surface temperature gradients indicate a relatively large component of conductive heat flow while the deeper near isotherms suggest convective heat flow as the primary mechanism of heat transfer. Therefore, using Chapman and other's (1978) logic, an estimate the total heat flow (Q_T) of the Ennis geothermal system, based on a sum of the convective and conductive heat flows, was determined. For example,

$$Q_T = Q_{\text{conv}} + Q_{\text{cond}}$$

Figure 4. Thermal gradient profiles of selected wells located in the area of the Ennis hot spring (EHS). Figure according to Leonard (written communication).

where: Q_T = total heat flow.

Q_{cond} = conductive heat flow (found by multiplying the heat flux values by the area which they occupy).

Q_{conv} = convective heat flow.

To determine the thermal gradients for conductive heat flow calculations, slopes on the temperature gradient profiles for wells TX-1, 2, 5, and 11 were estimated in the near surface region of high temperature gradients. For the lower temperature wells, values for thermal gradients were taken from information given by Leonard (written communication). Some of the wells (SH-1, SH-5, CYPRUS 1 and 2, and GRIFFEN) had only bottom hole temperatures, and for these wells temperature gradients were estimated by the difference between mean annual surface temperature (6.6°C) and the bottom hole temperature. For the valley fill in the Ennis geothermal area an average thermal conductivity, determined from core samples of the valley fill (Leonard, written communication) of $1465 \text{ milliwatts/m} \cdot ^{\circ}\text{C}$ ($3.5 \text{ mcal/cm} \cdot \text{s} \cdot ^{\circ}\text{C}$) was used. Contours were then drawn for heat flux values of 200, 500, 1000, 5000, and 10,000 (mW/m^2). The area enclosed in each of these heat flux value contours was then estimated using a planimeter, and finally conductive heat flow values were estimated by multiplying the average heat flux value in each contour interval (trapezoidal rule) by the area contained in each contour interval. These results are summarized in Table 3, and the contours of heat flux values are shown in Figure 5. The total conductive heat flow (Q_{cond}),

Figure 5. Contours of conductive heat flux values (mW/m^2). The data was taken from wells located in the area of the Ennis hot spring.

estimated for the Ennis geothermal area in the area of drilling, is 33.764 kilowatts (see Table 3).

For the determination of convective heat flow in the Ennis geothermal system the following values were used:

$$C_{p_{H_2O}} = 4.19 \times 10^9 \text{ mW-sec}/(^{\circ}\text{C}\cdot\text{m}^3)$$

$$\Delta T = 83^{\circ}\text{C} - 6.6^{\circ}\text{C} = 76.4^{\circ}\text{C}$$

$$F = 1.2 \times 10^{-3} \text{ m}^3/\text{sec.}$$

The flow rate and temperature of the Ennis hot spring are those cited by Leonard (written communication). Because the flow rate used for these calculations is that of the Ennis hot spring, which apparently represents only a minor portion of the Ennis geothermal system, the conductive heat flow results represent a low-end value. Using these numbers, a value for convective heat flow of 384.139 KW is reached. Therefore, the low end approximation for the total heat flow ($Q_{\text{cond}} + Q_{\text{conv}}$) for the Ennis geothermal system in the area of drilling is approximately 420 kW (0.42 MW). The estimated "safe yield" (above this yield it is assumed the resource will be mined) of the Ennis

Table 3. Conductive Heat Flow Calculations.

<u>Heat flux (mW/m²)</u>	<u>Area(m²)</u>	<u>Avg. Heat Flux(mW/m²)</u>	<u>q(mW)</u>
200			
	25460	350	8.911 10 ⁶
500			
	12261	750	9.196 10 ⁶
1000			
	6231	1250	7.789 10 ⁶
5000			
	3685	7500	2.764 10 ⁶
10000			
	469	10822	5.104 10 ⁶
			33.764 kW

geothermal system is $3.15 \times 10^{-2} \text{ m}^3/\text{sec}$ (Sonderegger, personal communication). Using this value for the flow rate, a "safe yield" total heat flow value for the Ennis geothermal system is 10.5 MW.

Geothermometry of the Ennis Geothermal System

Geothermometers are "tools" used to estimate reservoir (maximum) temperatures in a geothermal system from geochemical measuring of geothermal waters. Their effectiveness is based on the fact that "the compositions of geothermal fluids are controlled by temperature-dependent reactions between minerals and fluids" (Fournier, 1981). There are several types of geothermometers each given a name based on the geochemical constituent(s) of the geothermal waters being used to estimate subsurface temperature conditions (see Table 4). All concentrations of cations for the equations given in Table 4 are in units of parts per million. The C on the right hand side of the equations for geothermometers a-f in Table 4 represents the concentration of silica in parts per million. Because temperature estimates for geothermometers a-f depend on the concentration of silica, they are collectively termed silica geothermometers. To determine the value of β to use for the Na-K-Ca geothermometer (equation 1 in Table 4), Fournier and Truesdell (1973) suggest the following:

- 1) Using $\beta = 4/3$ find the temperature using equation 1 of Table 4;

Table 4. Equations for temperature estimations using various geothermometers. Table taken from Fournier (1981).

Geothermometer	Equation	Restrictions
a. Quartz-no steam loss	$t^{\circ}\text{C} = \frac{1309}{5.19 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
b. Quartz-maximum steam loss	$t^{\circ}\text{C} = \frac{1522}{5.75 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
c. Chalcedony	$t^{\circ}\text{C} = \frac{1032}{4.69 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
d. α -Cristobalite	$t^{\circ}\text{C} = \frac{1000}{4.78 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
e. β -Cristobalite	$t^{\circ}\text{C} = \frac{781}{4.51 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
f. Amorphous silica	$t^{\circ}\text{C} = \frac{731}{4.52 - \log C} - 273.15$	$t = 0-250^{\circ}\text{C}$
g. Na/K (Fournier)	$t^{\circ}\text{C} = \frac{1217}{\log (\text{Na/K}) + 1.483} - 273.15$	$t > 150^{\circ}\text{C}$
h. Na/K (Truesdell)	$t^{\circ}\text{C} = \frac{855.6}{\log (\text{Na/K}) + 0.8573} - 273.15$	$t > 150^{\circ}\text{C}$
i. Na-K-Ca	$t^{\circ}\text{C} = \frac{1647}{\log (\text{Na/K}) + \beta [\log (\sqrt{\text{Ca/Na}}) + 2.06] + 2.47} - 273.15$	$t < 100^{\circ}\text{C}, \beta = 4/3$ $t > 100^{\circ}\text{C}, \beta = 1/3$
j. $\Delta^{18}\text{O}(\text{SO}_4^{2-} - \text{H}_2\text{O})$	$1000 \ln \alpha = 2.88(10^6 T^{-2}) - 4.1$ $\alpha = \frac{1000 + \delta^{18}\text{O}(\text{HSO}_4^{-})}{1000 + \delta^{18}\text{O}(\text{H}_2\text{O})}$ and $T = ^{\circ}\text{K}$	

- 2) If this calculated temperature is less than 100°C and the $\log [(\sqrt{\text{Ca/Na}}) + 2.06]$ is positive, then keep the temperature indicated in step 1; or
- 3) If the indicated temperature is greater than 100°C or $\log [(\sqrt{\text{Ca/Na}}) + 2.06]$ is negative, then use $\beta = 1/3$ and recalculate the temperature.

Various assumptions are made when using any geothermometer.

These assumptions are summarized by Fournier and others, 1974, as follows:

- 1) Temperature dependent reactions occur at depth;
- 2) All constituents involved in a temperature-dependent reaction are sufficiently abundant (that is supply is not a limiting factor);
- 3) Water-rock equilibration occurs at the reservoir temperature;
- 4) Little or no re-equilibration or change in composition occurs at lower temperatures as water flows from the reservoir to the surface; and
- 5) The hot water coming from deep in the system does not mix with cooler shallow groundwater (if this assumption is violated, "mixing models" as described later can be used).

The theory behind each of these geothermometers is amply discussed in the literature, and useful references include the following:

- 1) Silica Geothermometer:
 - Theory: Arnorsson (1975), Muffler (1979), Brook and others (1979).
 - Mixing model corrections: Fournier and Truesdell (1974), Truesdell and Fournier (1977).
- 2) Na/K and Na-K-Ca Geothermometers:
 - Theory: Fournier and Truesdell (1973)
 - Mg Correction: Fournier and Potter (1978)
- 3) Oxygen Isotope, Sulfate-Water Geothermometer:
 - Theory: Lloyd (1968)
 - Mixing model corrections: McKenzie and Truesdell (1977)
- 4) For an overall view of the applicability of each geothermometer to a given geothermal area and the assumptions involved in the use of each geothermometer, Fournier (1981) is an excellent reference.

Hydrogeochemical data for test wells (TX-1 to TX-12), shot holes (SH-1, 3, 4, and 5), and several domestic wells (see Figure 3 for locations of these wells) in the Ennis geothermal area are shown in Table 5. This hydrogeochemical data is taken from Leonard (written communication) and Project Milestone Report which was prepared for the Montana Department of Natural Resources and Conservation by Geoproducts Corporation (June, 1984). Leonard (written communication) determined reservoir temperature estimates using the silica geothermometers, the Na-K-Ca geothermometer modified by the addition of a magnesium correction, and the oxygen isotope, sulfate-water geothermometer. Based on the geothermometry results, Leonard assumes a reservoir temperature of approximately 165°C for the Ennis geothermal system. Juncal (Project Milestone Report) also used quartz and chalcedony (with mixing model corrections), and Na-K-Ca geothermometers to determine the reservoir temperature of the Ennis geothermal system. Juncal (Project Milestone Report) concludes that reservoir temperatures calculated using the Na-K-Ca geothermometer are the most reliable, and these temperatures average approximately 160°C . It should be noted that Juncal and Leonard used the same data set; therefore, similar temperature estimates by Juncal and Leonard should be viewed as a confirmation of each others work rather than as unique results.

Using the 160°C reservoir temperature, end-value estimates of the depth of this reservoir can be attempted based on what is known of the Madison Valley near Ennis and applying the $30^{\circ}\text{C}/\text{km}$

<u>Well</u>	<u>Sampling Depth</u>	<u>pH</u>	<u>Ca++</u>	<u>Mg++</u>	<u>Na+</u>	<u>K+</u>	<u>Cl-</u>	<u>SiO2</u>	<u>B-tot</u>	<u>TDS</u>
EHS	0.0 (m.)	8.4	5.5	0.1	320	15.0	120.0	100	0.62	992
MAC-1	371.9	7.8	5.0	0.2	314	14.9	111.0	108	0.62	966
TX-1	104.2	8.4	3.5	0.3	340	15.0	120.0	45	0.71	945
TX-4	86.6	8.2	3.7	0.9	75	5.1	10.0	42	0.05	246
TX-5	98.5	8.3	3.3	0.2	130	4.3	20.0	44	0.20	374
TX-6	60.4	7.2	2.1	0.4	180	9.4	54.0	43	0.17	516
TX-7	15.2	7.4	58.0	17.0	25	7.4	16.0	39	0.09	---
TX-8	64.0	8.9	9.6	0.8	110	4.7	8.5	34	0.10	348
TX-9	122.8	9.4	5.1	0.4	110	7.3	18.0	43	0.13	340
TX-10	33.5	8.9	44.0	16.0	10	2.0	6.7	19	0.03	242
TX-11	157.0	----	4.8	0.0	340	16.0	120.0	160	0.61	1120
TX-12	152.4	7.7	5.2	0.2	331	15.2	116.0	107	0.68	1030
SH-1	16.1	7.1	5.8	0.6	330	18.0	110.0	90	0.64	985
SH-3	15.2	7.8	32.0	11.0	21	8.2	15.0	30	0.08	226
SH-4	14.0	7.5	66.0	20.0	26	6.6	21.0	28	0.07	371
CYPR.1	24.7	7.6	4.4	0.5	290	13.0	73.0	72	0.39	845
CYPR.2	11.6	8.9	5.6	1.3	140	7.1	24.0	23	0.09	398
GRIFF.	54.9	8.2	9.1	0.9	240	7.8	68.0	46	0.38	712
PRAY	41.2	7.5	5.0	1.0	330	12.0	110.0	75	0.54	945
TOTH	63.1	7.8	9.3	2.0	310	9.3	100.0	47	0.38	859
YENNY	37.8	7.4	28.0	5.6	75	5.8	32.0	43	0.12	342

Table 5. Hydrogeochemical data for wells in the Ennis geothermal area. Data according to Project Milestone Report prepared for the Dept. of Natural Resources and Conservation by Geoproducts Corporation (June, 1984). Concentrations in parts per million.

bedrock gradient and the 60 C/km thermal gradient for the valley fill. For example, by drilling it is known that the depth of valley fill below the Ennis hot spring is approximately 150 meters while the maximum depth of valley fill in the Madison Valley near Ennis is approximately 2 kilometers (Young, 1984). Using these valley fill depths and bedrock and valley fill thermal gradients, end values for the depth to the reservoir are 5 and 3.3 kilometers, respectively. The actual depth of the reservoir is probably somewhere between this range of 3.3 to 5 kilometers.

Structural Geology of the Madison Valley Near Ennis

Because the gravity modeling for this paper was done to determine controlling structure of the Ennis geothermal system, only the structural geology of the Madison Valley near Ennis will be discussed in this section. For information on the geomorphology and stratigraphy of the Madison Valley, see, for example, Paul and Lyons (1960), Shelden (1960), Montagne (1960), and Young (1984). Unfortunately, there are no mapped structures discussed in the literature to date (see, for example, Vitaliano and Cordua (1979) and Figure 6) which obviously control the present location of the spring. Because of this, major structures, apparently important in the structural development of the Madison Valley near Ennis, located a few to tens of kilometers away from the Ennis hot spring will be discussed. The purpose of this section, therefore, is to examine the

Figure 6. Regional geology of a portion of SW Montana. Figure adapted from Schmidt and Garihan (1983b).

structural history of the Madison Valley near Ennis. Based on this structural setting, the controlling structure of the Ennis geothermal system, indicated by the 3-D gravity modeling, can be interpreted.

The Madison Valley is a north-south trending basin and range style depression (Pardee, 1950) located approximately 75 kilometers east of the frontal fold and thrust zone of southwest Montana (see Figure 6 adapted from Schmidt and Garihan, 1983b). Near Ennis, the Madison Valley is bounded to the west by the Tobacco Root Mountains, to the east by the Madison Range, and to the north by the Norris hills. Major documented faults in the Madison Valley are exposed to the east of the Ennis hot spring in the Madison Range. These faults include the Spanish Peaks fault (Figure 6, number 3), the Jack Creek thrust fault (Figure 6, number 2) and the Madison Range front fault (Figure 6, number 1). Bounding the northern edge of the upper Madison Valley is the North Meadow Creek fault (Figure 6, number 4).

Faults of the Madison Valley Near Ennis:

Spanish Peaks Fault

The Spanish Peaks fault is a reverse fault trending N55-60°W and dipping steeply to the northeast (see Figure 7). Archean metamorphic rocks have been uplifted northeast of the fault, and Young (1984) estimates a minimum vertical displacement of 2200 meters for the portion of the Spanish Peaks Fault shown in Figure 7. Slikenside data indicate approximately

Figure 7. Major documented faults located in the N. upper Madison valley. Mapping according to Swanson (1950), Garihan and others (1983), and Young (1984).

equal reverse-dip-slip and left-lateral motion resulting in a net slip of 3100-4200 meters along the fault plane (Young, 1984).

Jack Creek Fault

The Jack Creek fault is a gently westward ($\sim 30^{\circ}$) dipping thrust fault that has placed archean metamorphic rock on top of overturned Paleozoic rocks (see cross section A-A', Figure 7). These overturned Paleozoic rocks are contained in the overturned limb of a syncline called the Mill Creek Syncline. This syncline is discussed by Swanson (1950), Schmidt and Garihan (1983), and Young (1984). It is uncertain whether the Jack Creek thrust continues west of the Madison Range front fault and into the Madison Valley. However, Young (1984) and Garihan and others (1983) propose that seismic reflection data shot in the Madison Valley indicates a $30-35^{\circ}$ west-dipping reflection which they interpret as the Jack Creek thrust system continuing west into the valley.

Tear faults

Three west-northwest trending left-lateral tear faults associated with the Jack Creek fault and the Spanish Peaks fault are located in the Jack Creek thrust area (Swanson, 1950; Young, 1984). Two of these tear faults are shown in Figure 7. Young (1984) gives left lateral displacements on these faults as follows:

- 1) Northernmost tear fault (not shown in Figure 7 but located parallel to the SPf and approximately a kilometer north of the middle tear fault) - 25 meters offset;
- 2) Middle tear fault (shown as northernmost tear fault in Figure 7) - 300 meters offset; and
- 3) Southernmost tear fault (shown as southernmost tear fault in Figure 7) - at least 200 meters offset.

The two northernmost faults are defined by offset of the Paleozoic rocks. The southernmost fault is described by Young (1984) as "a series of enechelon faults identified by shear zones in the Archean rocks and juxtaposition of Archean and Cambrian rocks."

Madison Range Front Fault

The Madison Range front fault bounds the uplifted Madison Range to the east from the valley floor to the west (see Figure 7). This fault is a major normal fault with displacement on the fault in the range of 1830 meters (Pardee, 1950; Shofield, 1981) for this area of the Madison Valley.

North Meadow Creek Fault

The North Meadow Creek fault is roughly parallel to, though offset to the north from, the Spanish Peaks Fault (see Figure 7). The fault dips steeply to the southwest and is downthrown on the southwest side (Garihan and others, 1983). From seismic

reflection and gravity data, Young (1984) estimates dip slip movement along the fault plane at 640 to 680 meters. Schmidt and Garihan (1983) also determine a component of left-lateral motion along the fault plane based on slickenside data.

Dating of Faults:

Precambrian Movement

Schmidt and Garihan (1983) have suggested that many of the northwest trending faults in southwest Montana which document substantial movement in the Cretaceous, because of Laramide tectonic activity, are reactivated Precambrian faults. Young (1984) states that the Spanish Peaks fault possibly had a Precambrian ancestry based on a change in metamorphic fold and foliation trends along the Spanish Peaks fault.

Cretaceous-Eocene (Laramide) Movement

Young (1984) brackets the age of the Spanish Peaks fault (with Precambrian ancestry) and the Jack Creek fault between mid-Cretaceous and Late Eocene. This dating according to Young (1984) is based on the following:

- 1) Andesite sills, 86 million years old, are not localized along the fault surfaces and, therefore, predate them;
- 2) The Scarface thrust fault, of the same system as the Jack Creek Thrust but 30 kilometers south,

offsets the Sphinx conglomerate (late Paleocene to early Eocene); and

- 3) Approximately 40 kilometers southwest of the Jack Creek thrust fault, the Spanish Peaks fault is overlain by middle Eocene pyroclastics.

This mid-Cretaceous to late Eocene dating indicates the association of the Spanish Peaks and Jack Creek fault systems with Laramide deformation.

Tertiary (Basin and Range) Movements

Young (1984) states that the presence of fresh water diatoms in deposits southwest (down side) of the North Meadow Creek fault indicates that this fault is Miocene or younger. Carl (1970) says that this late Cenozoic movement (down to the southwest) on the North Meadow Creek fault "ponded somewhat" the ancestral Madison river. This is based on approximately 30 meters of unconsolidated gravels deposited in the mouth of the Beartrap canyon where the Madison River leaves Ennis Lake. The Madison Range front fault documents a history of movement from Miocene to present (Paul and Lyons, 1960; Sheldon, 1960). The most recent movement of documented displacement along the fault was from the August 17, 1959, Hebgen Lake earthquake.

Formation and Chronology of Laramide Structure:

Swanson (1950) gives a Laramide chronology of:

"Initial folding of the (Mill Creek) syncline, with stress from the west, was followed by folding and faulting along the Gardiner fault zone (Spanish

Peaks Fault of today), with stress from the northeast, and that in turn was followed by renewed stress from the west and the development of the Madison fault zone (Jack Creek thrust zone of today)".

He based this chronology on structures developed along a northeast trending tear fault mapped near the western edge of the Spanish Peaks fault.

Garihan and others (1983) give the following Laramide structural sequence:

- 1) Development of the Mill Creek syncline;
- 2) Reverse movement along the Spanish Peaks Fault;
- 3) Jack Creek thrust faulting; and
- 4) Renewed Spanish Peaks fault movement.

They place development of the Mill Creek syncline before reverse movement along the Spanish Peaks fault because in plan view, the overturned west limb of the syncline is cut off by the Spanish Peaks Fault. They explain the tear faults as resulting from the eastern movement of rocks, carried by the Jack Creek thrust, being blocked by the rigid buttress of Archean rocks brought up along the northwest trending Spanish Peaks fault. This requires movement along the Spanish Peaks Fault to predate thrusting. Finally, renewed movement along the Spanish Peaks Fault is used to explain numerous small, east trending folds in the allochthonous sedimentary rocks.

Young (1984) believes that displacement on the Jack Creek thrust and Spanish Peaks faults "occurred contemporaneously", and that the Mill Creek syncline is "formed by the intersection

of the N-S trending foreland fold of the Jack Creek thrust system and the west-northwest trending foreland fold of the Spanish Peaks fault" (A foreland fold is a basement-involved, fault-related fold located in the foreland of the frontal fold and thrust zone). She bases contemporaneous movement of the faults on the fact that neither fault "can be shown to cut the other." In fact, according to Young, displacement on the Jack Creek thrust diminishes near the Spanish Peaks fault because the Spanish Peaks fault accommodated the east-directed stress of the Jack Creek Fault by acting as a tear fault. This would explain the left-lateral movement along the Spanish Peaks fault according to Young.

Inferred Faults Buried in the Madison Valley Near Ennis:

After the Laramide compressional events, a tensional environment, producing the basin and range tectonic activity, presumably resulted in the creation of the Madison Valley. The question to address here is: Based on its structural history, what probable structure is hidden beneath the valley fill of the Madison Valley?

As shown in Figure 7, the tear faults and the Jack Creek fault terminate against the Madison Range front fault. As these faults predate the Madison Range front fault, it is possible that they extend west into the Madison Valley underneath the valley fill. If the tear faults are the product of some interaction between the Spanish Peaks fault and the Jack Creek fault as Young (1984) and Garihan and others (1983) suggest, it

is not probable that they would be propagated a great distance west into the Madison Valley. This is because the mapped displacements of the tear faults, close to the intersection of the Spanish Peaks and Jack Creek faults, are in the range of hundreds of meters (Young, 1984). In addition, stress was also apparently relieved by strike slippage along the fault planes at the intersection of the Jack Creek and Spanish Peaks faults.

As for the Jack Creek fault, Young (1984) and Garihan and others (1983) interpret seismic reflection data as indicating the Jack Creek fault continues west of the Madison Range front fault and into the Madison Valley. In my opinion it is certainly possible that the Jack Creek and/or related thrusts extend across the Madison Valley. Even though there are no mapped thrusts in the bedrock exposed west of the Ennis hot spring (see Figure 6), the fact that Archean bedrock is present at the surface west of the hot spring indicates the possibility of thrusting (at least some sort of tectonic activity) in this area.

Another possible fault inferred by Young (1984), but not discussed previously because of the tenuous nature of its existence, has a bearing on the structural development of the Madison Valley near Ennis. Young's evidence for this W-NW trending fault is the linearity and extent of erosion through the Archean rock carried by the Jack Creek fault and the underlying Mesozoic and Paleozoic rocks. This erosion has created the Jack Creek Canyon located roughly E-NE of the town of Ennis in the western slope of the Madison Range. This

inferred fault would have to predate the Laramide faulting (Young assumes it is a Precambrian fault), and the relatively rapid erosion along the strike of the fault may be related to a shear zone developed along the W-NW trending fault during the Laramide events. It is not certain if this inferred fault extends west into the Madison Valley.

Geophysics

Seismic Refraction:

In February, 1979, two seismic refraction lines were shot by students of the physics/geophysics department under the direction of Dr. Charles Wideman. Shot holes (SH-1, 3, 4, and 5) were drilled for a dynamite source, and the data was collected at stations 33.5 meters apart using a 36 trace analog recording system. One line trends in a northeast direction and wells SH-3, TX-7, TX-2, SH-1, and SH-5 were used as shot holes, the other line trends to the northwest and wells TX-6, SH-4, TX-2, and TX-3 were used as shot holes (see Figure 8).

Double coverage (reversed shot and receiver orientations) was obtained for each profile along each of the lines except between wells TX-2 and TX-3 and TX-6 to SH-4 on the northwest line. Because of cable problems "the time break was lost on several shots" (Mcrae, unpublished senior report). In this instance approximate arrival times were recovered by adding a correction time calculated by dividing the depth of the shot by the velocity of the nearest known low velocity layer. Mcrae's

Figure 8. Location of NE and NW trending seismic refraction lines. The seismic refraction data was collected by the physics/geophysics department of Montana Tech under the direction of Dr. Charles Wideman.

(unpublished senior report) interpretation of the seismic refraction information is shown in Figures 9 and 10.

Seismic Reflection:

East of the Madison River in the upper Madison Valley, eight seismic reflection lines were shot by Grant Geophysical, Ltd. for Canadian Hunter, Ltd. in February, 1982. Young (1984) was able to obtain this proprietary data and from this data interpreted structures which she has termed the Jordan Creek and Shelhamer Ranch grabens. Figure 11 shows the location of the seismic lines, and Young's (1984) structural interpretation including subsurface locations of the Jordan Creek and Shelhamer Ranch grabens, the Madison Range front fault, and the axis of the Madison Valley. The dotted lines shown on Figure 11 connect the deepest points along each structure, and Table 6 contains these maximum depth values for each structure. The lines with balls located along the seismic lines indicate interpreted normal faulting with the balls on the downthrown side of the fault. A velocity analysis was done along each seismic line, and based on this analysis interval velocities were computed by Young using the Dix equation. Using this velocity information Young converted two way times to depth. This is how the depths shown in Table 6 were calculated. Young estimates a maximum error of ± 10 percent in the depth estimates based on improper velocity picks.

Figure 9. Seismic refraction data and accompanying bedrock depth interpretation for the NE trending seismic refraction line (see Figure 8). Interpretation according to McRae (unpublished senior report).

Figure 10. Seismic refraction data and accompanying bedrock depth interpretation for the NW trending seismic refraction line (see Figure 8). Interpretation according to McRae (unpublished senior report).

Figure 11. Placement of the 8 seismic reflection lines located in the northern Madison valley. Young interpreted the following structure from this seismic reflection data: a) The Jordan Creek graben, b) The Shelhamer Ranch graben, and c) The axis of the northern Madison valley. The dotted lines connect the deepest points along these interpreted structures.

Table 6: Greatest depth of Jordan Creek and Shelhamer Ranch grabens, Madison Range Front Fault, and North Meadow Creek fault as interpreted from seismic reflection data. Depths (in meters) according to Young, 1984.

<u>Seismic Line</u>	<u>N.M.Cr. F</u>	<u>Jordan Cr.G.</u>	<u>Sh.R.G.</u>	<u>M. R. F.F.</u>
2		640	1066	1763
3			1066	2348
4				722
5				800
6				1600

Audio-Magnetotellurics (AMT):

Long and Senterfit (1979) collected data from 20 AMT stations surrounding the Ennis hot spring area (see Figure 12). The data was collected using north-south and east-west oriented dipoles at frequencies of 7.5, 27, 285, 6700, and 18600 hertz. Electromagnetic waves decay exponentially into the earth, and the depth of penetration of these waves for interpretation purposes is given by the formula:

$$\delta_a \approx \left(\frac{\rho_a}{f}\right)^{1/2}$$

where ρ_a is an apparent resistivity (ohm-meters) given by

$$\rho_a = \frac{1}{5f} \left| \frac{E}{B} \right|^2$$

where:

f = frequency (Hertz)

δ_a = Apparent Skin Depth (meters)

Figure 12. Contours of apparent resistivities (ohm-meters) based on AMT data collected by Long and Senterfit (1979) along NS and EW oriented dipoles at the 19 stations shown. This data is displayed at 100 and 300 meter skin depths for the EW and NS dipole orientations based on interpretations by Semmens and Emilsson.

E = Magnitude of the Electric Field
(volts/meter)

B = Magnitude of the Magnetic Induction
(gammas)

Long and Senterfit (1979) interpret the AMT data as indicating that thermal water alters the alluvium over an area of 1.5 by 4 kilometers surrounding the Ennis hot spring. They also state that a northwest trend in the scalar resistivities may delineate a northwest trending fault and conclude that the geothermal potential of the spring may extend 1 kilometer to the northwest and 3 kilometers southeast of the surface location of the Ennis hot spring.

Long and Senterfit's (1979) data was also studied by Gunnar Emilsson, a fellow student, and myself. An example of our work is shown in Figure 12 which contains contours of apparent resistivities for east-west and north-south dipole orientations at apparent skin depths of 100 and 300 meters. Contours in Figure 12 are shown at 20, 60, 100, 200, 300, and 400 ohm-meters because of the large range of apparent resistivities. These data clearly show a broadening of the low resistivity contours in all directions except west of the hot spring for the 300 meter skin depth relative to the 100 meter skin depth data. Also a north-south trend through the Ennis hot spring is shown for the low resistivity data in the modeling. Without modeling this data, interpretations are tenuous; nevertheless, I believe the broadening of the low resistivity data with depth indicates

the surface flow of the Ennis hot spring represents only a minor portion of the geothermal system, and the north-south trend of the less than 20 ohm-meter data indicates a possible north-south oriented fault or aquifer controlling the geothermal water.

Controlled Source Audio-Magnetotellurics (CSAMT):

In the summer of 1985, an extensive set of CSAMT data was collected by Gunnar Emilsson and myself under the direction of Dr. William Sill. Frequencies of 1, 2, 4, 8, 16, 32, 64, 128, 256, and 1024 hertz were used. The data was collected at 50 or 100 meter intervals along east-west lines of data from 900 meters south to over a kilometer north of the hot spring. These lines extended approximately one kilometer west of the hot spring and east to the Madison River. Emilsson, for a thesis in progress, is interpreting this data using 1-D inversion and 2-D forward modeling programs. His final interpretations of these data are not yet available.

Gravity:

Burfeind

One of the first geophysical studies in the Ennis area was done by Walter Burfeind (1967). For a doctoral thesis Burfeind undertook a large scale gravity survey covering the Jefferson, Beaverhead, Three Forks, north Boulder, Madison, Prickley Pear, and Ruby-Gravelly basins and some of the intervening mountain

ranges. The accuracy of this survey is considered by Burfeind to be from 1 to 3 milligals in general and at worst 15 milligals in the mountain ranges.

In the Madison Valley, Burfeind modeled an E-W line, placed approximately 900 meters south of the town of Ennis, through his gravity data. Burfeind's 2-D modeling indicated that the Madison Valley near Ennis is an asymmetric, deep to the east, valley with a maximum depth of approximately 1.5 kilometers. Because the eastern valley wall has a much steeper slope than the western wall, Burfeind concluded that "the Madison Valley is the result of block faulting that has tilted the area eastward." Burfeind noted that the more negative gravity values to the south probably indicated deepening of the valley to the south.

Montana Tech

In September and October, 1978, students of the physics/geophysics department of Montana Tech collected 67 gravity stations (unpublished data) north of the Ennis hot spring under the direction of Dr. Charles Wideman (see Figure 13). The gravity data was referenced to a Department of Defense gravity base station located at Three Forks, Montana, and a United States Geological Survey gravity base station located in Helena, Montana. A secondary base, United States Coast and Geodetic Survey bench mark Y145 (elevation 1,492.4 meters), was used as the local reference point for the gravity data collected in this survey. The gravity stations were surveyed for

Figure 13. Locations of the 67 gravity stations collected by the physics/geophysics department of Montana Tech under the supervision of Dr. Charles Wideman (unpublished data).

elevation, and closure on the survey loop was within 3 hundredths of a meter.

Elevation (free air and Bouguer), drift, latitude, and terrain corrections were applied. The assumed accuracy for this survey is in the several tenths of a milligal range (Wideman personal communication).

Senterfit

On November 4, 1978, Senterfit (1980) established 33 gravity stations on two east-west lines south of the Ennis hot spring (see Figure 14). The gravity data were collected using a LaCoste-Romberg gravity meter (G-235) and is referenced to the Department of Defense gravity base station located at Three Forks, Montana, and the United States Geological Survey (USGS) gravity base station in Helena, Montana. Elevations were surveyed and an error of $\pm .03$ mgals is assumed to result from elevation uncertainty. Drift, elevation, latitude, and terrain corrections were applied. Senterfit offered no interpretation of these two lines in his 1980 USGS Open File report.

Figure 14. Locations of the 33 gravity stations collected by Senterfit (1980).

GRAVITY DATA

Additional Gravity Data^{*}

In September, 1985, 32 gravity stations, spaced 50 meters apart, were set up on 2 east-west lines approximately 500 meters north of the Ennis hot spring. These lines begin just east of highway 287 (see the Ennis 15 minute quadrangle map) and extend 850 meters east into the Madison Valley. Another 15 gravity stations, spaced 100 meters apart, were located in a loop of roads approximately 1.5 kilometers northeast of the hot springs, east of the Madison River. Finally, in January, 1986, an additional 16 gravity stations were set up on a loop of roads east of the Madison River approximately 1.5 kilometers southeast of the hot spring. These 16 stations were spaced at approximately 300 meter intervals along the road. See Figure 15 for the location of these 63 gravity stations. This gravity work was done using a LaCoste-Romberg (model D) gravity meter accurate to tens of micro-gals. The data is referenced to the U.S. Coast and Geodetic Survey benchmark Y145, and as Senterfit's (1980) data was also referenced to this benchmark, his value for the benchmark was added to all 63 of the new gravity stations. In

* This gravity data was collected by Gunnar Emilsson, a fellow student, and myself.

Figure 15. Locations of the 63 gravity stations collected by Semmens and Emilsson in September 1985 and January 1986.

this way the new gravity data was referenced to the Department of Defense gravity base station in Three Forks, Montana and the USGS gravity base station in Helena, Montana.

The station elevations were determined by surveying and each of the survey loops for the three gravity surveys discussed previously closed within 6 hundredths of a meter. During each of the three gravity surveys, benchmark Y145 was reoccupied in a time period of under two hours to determine drift (tidal effects were considered to be included in the drift correction). In the two surveys east of the Madison River, drift in the gravity readings based on reoccupation of benchmark Y145 was under 5 hundredths of a milligal, and drift for each station on these loops is considered negligible. For the survey west of the Madison River, closure was in the tenth of a milligal range and drift corrections were applied. All stations had latitude, elevation, and terrain corrections (covering an area out to 21.95 kilometers) applied. Accuracy for these surveys is assumed to be in the range of half a milligal. The raw data and gravity corrections are contained in Appendix A.

Modeled Gravity Data

The gravity data used for the 3-D gravity modeling includes Senterfit's (1980) gravity data, gravity data collected by Montana Tech (unpublished), and the 63 gravity stations collected by Gunnar Emilsson and myself in September, 1985, and January, 1986 (see Appendix E.1). All of the gravity stations used for

the modeling were surveyed for elevation control, and all of the gravity data is referenced to benchmark Y-145 of the U.S. Coast and Geodetic Survey. Terrain corrections for the stations collected by Senterfit and students of Montana Tech were calculated using a USGS computer program written to correct for an area inside a radius of 166.7 kilometers around each station. The stations collected by Emilsson and myself were terrain corrected by hand using Hammer charts, and the area surrounding each station, contained in a radius of 21.9 kilometers, was included in the corrections. Terrain corrections (using the 166.7 kilometer radius) values for two USGS gravity stations contained in the state gravity data file differed by only two and three tenths of a milligal from similarly located stations for which the terrain corrections were calculated by hand. Considering an error of not greater than .15 milligals for elevation uncertainties and several tenths of a milligal for terrain correction and operator errors, an average error in the range of .5 milligals is assumed for the gravity data used in the gravity modeling.

3-D GRAVITY MODELING

Assumptions and Modeling Decisions

Gridding of the Gravity Data:

For 3-D gravity modeling, spatial (x,y) location of the observation points must be considered. To limit the amount of input and simplify the programming, I chose to write the programs using gravity stations confined to a rectangular grid. Therefore, a rectangular window (confining the grid) in the data has to be defined, and the gravity values assigned to the grid nodes have to be interpolated from the original field data (if the gravity data was not collected on a grid). In my work, I contoured the observed gravity data by hand, and the values assigned to the grid nodes were interpolated (by eye) from the hand contoured gravity data. To graphically display the gridded observed gravity data, I used the SURFACE II contouring package of Robert B. Sampson (1978).

Figure 16 shows the locations of the 158 gravity stations included as data points for the gravity modeling, and the gravity contours (c.i. = 1 mgal.) drawn by hand through this gravity data. The bold inner box of Figure 16 outlines the area (approximately 8.5 km^2) chosen to contain the modeling grid. A grid of 15 columns and 8 rows (120 grid nodes) was selected for this area because this grid provides even coverage in the north and east directions (with approximately 300 meters between columns and rows). The location of these grid nodes together

Figure 16. Locations of the 158 gravity stations used in the gravity modeling. Data includes stations collected by Montana Tech, Senterfit (1980), and Semmens and Emilsson. The contours shown are contours of the gravity data (c.i. = 1 mgal) drawn by hand and the inner rectangle represents the area containing the gridded gravity data.

with the gravity contours (c.i. = 1 mgal.) drawn by computer using the SURFACE II contouring package (Sampson, 1978) are shown in Figure 17. The gravity contours of Figure 17 were drawn using values assigned to the grid nodes that were interpolated from the original hand contoured gravity data set. It is these 120 interpolated gravity values that are used in the modeling, and as these values are only weighted averages of the original data set, I will use the phrase "gridded observed gravity data" when referring to the gridded gravity data set. See Appendix E.2 for the actual gravity values and station locations in this gridded gravity data set.

Density Determination for Modeling:

Leonard (written communication) cites an average density of 2.1 gm/cm^3 for the valley fill in the area of the Ennis hot spring and a 2.6 gm/cm^3 density for the bedrock of gneiss and schist. The 2.1 gm/cm^3 average density assigned to the valley fill is determined from averages of measured saturated densities taken from cores samples in wells TX-2,4,5,6, and 11 as is shown in Table 7. The 2.6 gm/cm^3 density of the bedrock is based on a measured density from a core originally situated in an interval located 190.5-191.4 meters below the land surface (approximately 30 meters below the base of the valley fill) in well TX-11.

As is indicated in Table 7, there is a range of densities which display no systematic correlation with depth because of the inhomogeneity of the valley fill. This inhomogeneity of the valley fill is also emphasized in the drillers logs for wells

Figure 17. Locations of the 120 (15 columns x 8 rows) gridded gravity stations. Contours are gravity contours (c.i. = 1 mgal) drawn by computer using the Surface II contouring package (Sampson, 1978). The values assigned to the grid nodes were interpolated from the hand contoured gravity data (see Figure 16).

Table 7. Saturated densities (gm/cm) of the valley fill sediments taken from cores of wells located near the Ennis hot spring according to Leonard (written communication).

<u>Test Well</u>	<u>Cored interval below land surface (m)</u>	<u>Saturated Density</u>	<u>Lithologic Description</u>
TX-2	128.9-129.8	1.79 g/cm ³	Clay: silty, tuffaceous, mottled grayish-green and pink.
TX-4	85.3-86.2	2.21 2.06	Sand: quartzose, argillaceous, fine-grained, olive with welded and non-welded tuffaceous material; about 50% argillaceous matrix.
TX-5	97.5-98.4	1.94 2.02	Sand: argillaceous, micaceous, fine-grained, angular to subangular grains, quartzose, tan to dark gray. Gravel: subrounded pebbles to 2 cm. diameter, predominantly biotite gneiss and quartz, calcareous, tuffaceous.
TX-6	41.4-42.4	2.11 2.11	Sand: argillaceous, dark gray to olive, micaceous, arkosic. Clay: hard, tuffaceous, gray. Silt: sandy, brown, tuffaceous, with scattered lithic grains.

TX-11 and TX-12 (both relatively deep wells which penetrate the bedrock) which record the presence of alternating layers of clay, silt, shales, and unconsolidated to well consolidated sands. Clearly, the density variations between these alternating layers of varying composition will contribute to the observed gravity values, but because the geometry and lateral extent of these layers is unknown, it is impossible to include these layers in a model. Therefore, I modeled the valley fill using the -0.5 density contrast between the 2.1 gm/cm^3 average density for the valley fill and the 2.6 gm/cm^3 bedrock density measured from the core of well TX-11. Using this -0.5 density contrast, I am assuming that the observed gravity data results solely from the density difference between "average" valley fill and the metamorphic bedrock.

Modeling With Blocks:

Block faulting and step-like normal faulting of the bedrock in the Madison Valley near Ennis is indicated from interpretations of seismic and gravity data (McRae, unpublished senior report; Young, 1984; and Burfeind, 1967). Based on these interpretations, I assumed that the gravity data of the Madison Valley could be efficiently modeled by summing the gravity contributions of blocks of earth of variable depth and surface area. Figure 18 shows a schematic of a block-faulted valley and how it might be modeled with blocks assuming an average density contrast between the bedrock and the valley.

Figure 18. Modeling a block-faulted valley with blocks

Modeling of Topography:

It is often assumed that once elevation and terrain corrections have been applied, the gravity stations have been reduced to a plane. However, this is generally not the case because the assumptions inherent in the corrections are not totally accurate. Therefore, for more reliable results (especially in areas of steep terrain), the gravity stations should retain their elevations in the modeling process; and this involves modeling of the topography. Because blocks generally can not be used to efficiently model the topography, I used a 3-D

gravity modeling program (program 3D), written by Richard Allan Payne (1986) and myself. This modeling program is based on Talwani and Ewing's (1960) algorithm which sums the gravity contribution of vertically stacked, horizontal, n-sided polygons. These polygons can be used to accurately describe irregular 3-D bodies and are ideally suited to the modeling of topography. Coding for program 3D is contained in Appendix B and information on input to and output of program 3D is contained in Appendix C.

To model the topography, I modeled the valley fill from the surface down to some convenient elevation below the lowest station elevation then, using program 3D, the gravity contribution attributed to each of the gridded gravity stations from this topographic model was calculated and removed from the gridded observed gravity values. Once this removal of the gravity contributions from the valley fill in the topographic model was accomplished, the remaining valley fill (below the valley fill contained in the topographic model) was modeled using blocks.

This modeling procedure is illustrated in Figures 19 and 20.

Figure 19a shows a schematic of a cross section along a row of the gridded gravity data located in a block faulted valley. Gravity stations along the row of the gridded gravity data are indicated by the circles on the surface of the valley, and the cross-hatched area, extending from the surface to the shallowest bedrock, represents the valley fill included in the topographic model. Figure 19b shows an example of how the valley fill in the topographic model might be modeled using program 3D. The

Figure 19. A) Representation of a cross-section along a row of gravity data in a block faulted valley. Sediments from the highest elevation in the modeling area to some elevation below the lowest station elevation are included in the topography model.

B) Schematic showing how the valley fill included in the topography model might be modeled using vertically stacked, horizontal polygons according to the gravity modeling method of Talwani and Ewing (1960).

horizontal lines in Figure 19b are cross sectional representations of five vertically-stacked, horizontal polygons. An effective way to determine the shape and vertical location of these polygons is to digitize elevation contours (a sufficient number to "adequately" reflect the topography) in the modeled area relative to the origin of the gridded observed gravity data. Using this method each of the polygons is assigned a vertical location based on the contour elevation and an outline determined by the contour shape. Once the gravity contributions of the topographic model are removed, it is (ideally) as if all the sediments in the topographic model have been removed as is shown in Figure 20.

Figure 20 shows that once the gravity contributions of the valley fill in the topographic model have been removed from the gridded observed gravity data, the gridded gravity stations have not been dropped to the plane representing the base of the topographic model. However, with this correction, forward modeling with blocks is now reasonable because the upper edge of each block can be assigned a uniform depth representing the base of the valley fill in the topographic model. Figure 20 also shows an example of how the remaining valley fill could be modeled with blocks assuming the blocks were assigned some average density contrast between the valley fill and the bedrock.

It should be noted that both program 3D and program GRAVBL (the forward modeling program using blocks) allow for individual densities to be assigned to each polygon or block so that vertical and lateral density distributions can be modeled.

Figure 20. Once the gravity contributions of the valley fill in the topography model have been removed, the remaining valley fill can be modeled efficiently using blocks. Note that with the removal of the gravity contributions of the valley fill included in the topography model, it is not as if the gravity stations have been dropped to the plane at the base of the topography model.

As noted before, the topography was not modeled using blocks because blocks, in general, can not be used to model the topography efficiently. Therefore, a separate modeling program was used to model the topography from the highest elevation to a convenient elevation below the lowest station elevation. Another reason for using this two-step modeling procedure is that the removal of the valley fill in the topographic model increases the distance between the upper edge of the blocks and the gravity stations (see Figure 20) and this numerically stabilizes the forward modeling algorithm (equation 3, Forward Modeling Theory section). For example, it was found that modeling of blocks buried a depth of 5 meters or less often resulted in taking the logarithm of a near zero number. By removing the topography to a depth greater than five meters below the lowest station elevation, this numerical instability can be eliminated.

Determination of Depth to Bedrock Using Inversion Theory:

The gravity data in the area of the Ennis hot spring was collected with the purpose of finding offsets in the bedrock which might be fault controlled and therefore might possibly control the path of the ascending geothermal water. Consequently, the principal goal of my gravity modeling was to accurately map the topography of the bedrock hidden beneath the valley fill in the modeling area. For this reason, I wrote an inverse modeling program to adjust the bedrock depths, output from the forward modeling program, in an attempt find a model of

the bedrock surface that reproduced the gridded observed gravity data within the assumed survey error. It should be reiterated that I assumed the observed gravity data to be solely the result of a density contrast between "average" valley fill and bedrock; and therefore, I modeled using blocks extending from the base of the topography model to the assumed bedrock depth. Consequently, the inverse modeling program I wrote adjusts the lower depth of each block where I assume the base of each block represents the depth to bedrock.

3-D Gravity Modeling Theory

Forward Modeling Theory:

In cartesian coordinates the vertical component of gravity in three dimensions is given in the following formula (see Figure 21):

$$g_z(x,y,z) = \int_{y_1}^{y_2} \int_{z_1}^{z_2} \int_{x_1}^{x_2} \frac{G\rho(x',y',z')(z-z')dx'dy'dz'}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}} \quad (1)$$

where x,y,z = coordinates of the observation point

x',y',z' = coordinates of the source

$\rho(x',y',z')$ = density of the source

G = universal gravity constant

x_1,x_2 = x limits of the block

y_1,y_2 = y limits of the block

z_1,z_2 = z limits of the block

With no loss in generality, the observation point may be assumed to be at the origin:

$$g_z(0,0,0) = \int_{y_1}^{y_2} \int_{z_1}^{z_2} \int_{x_1}^{x_2} \frac{G\rho(x',y',z')z'dx'dy'dz'}{[(x')^2 + (y')^2 + (z')^2]^{3/2}} \quad (2)$$

Integration of (2) is not difficult if the exact integration is carried out in two directions and the final integration is approximated. Because an inverse modeling program was to be written to invert on depths, I chose to approximate the integration in the y-direction to keep z involvement at a minimum.

The resulting formula after integration of (2) in the x and z directions for block i is:

Figure 21. Coordinate system with a unit volume located at (x',y',z') and an observation point at (x,y,z) .

$$\begin{aligned}
g_z(i)(0,0,0) = G \rho(i) \int_{y_1}^{y_2} \{ & \text{Ln}[x_1(i)+(x_1(i))^2+(z_2(i))^2+(y')^2] \\
& - \text{Ln}[x_2(i)+(x_2(i))^2+(z_2(i))^2+(y')^2] \\
& + \text{Ln}[x_2(i)+(x_2(i))^2+(z_1(i))^2+(y')^2] \\
& - \text{Ln}[x_1(i)+(x_1(i))^2+(z_1(i))^2+(y')^2] \} dy'
\end{aligned} \quad (3)$$

This is an integral of the form:

$$g_z(i)(0,0,0) = G \rho(i) \int_{y_1}^{y_2} A_1(y') dy' \quad (4)$$

where: $A_1(y')$ are varying constants (mgals/meter)
dependent on the value of y' of block i .

I approximated the integration in (4) by using a quadrature formula proposed by Talwani and Ewing (1960). With a quadrature formula, the gravity anomaly from a 3-D body is approximated by summing a number of anomalies from thin rectangular laminae defined by $x_1(i)$, $x_2(i)$, $z_1(i)$, $z_2(i)$, and a thickness Δy .

In order for the quadrature formula approximation to be accurate, enough laminae must be chosen to keep Δy "small." In the forward program I wrote, laminae are equally spaced along the block and a minimum of 10 are inserted. The actual number of laminae (>10) is controlled by the following formula:

$$\begin{aligned}
\text{NUMLAM}(i) = & \text{YLEN}(i) \times (\text{MIN}[\text{XLEN}(i), \text{YLEN}(i)] \\
& / \text{MAX}[\text{XLEN}(i)/\text{YLEN}(i)]) \times \text{IM}
\end{aligned}$$

where:

$\text{NUMLAM}(i)$ = number of laminae in block i .
 $\text{MIN}, \text{MAX}[\text{XLEN}(i)/\text{YLEN}(i)]$ = Minimum, Maximum of x, y lengths of block i .
 IM = input multiplier.

If the number of laminae is even another will be inserted because the chosen quadrature formula requires an odd number of laminae. For example, if a block has an x-length of 100 meters and a y-length of 200 meters with an input multiplier of 1, then 101 laminae will be placed at equally spaced intervals along the block.

The number of laminae used in the quadrature formula approximation is partly responsible for the amount of computer time involved in reaching a solution. Therefore, the increased accuracy of the quadrature formula approximation resulting from the use of more laminae must be weighed against the increase in computer time for a result. Because the quadrature formula sums anomalies from vertical rectangular laminae, less laminae need be inserted for blocks (assuming constant density) which are broad in the x and z directions and thin in the y-direction. Blocks which approach an infinite, thin horizontal slab take the most laminae to achieve an accurate result. In my modeling with blocks 500 meters wide, (x-direction), 400 meters long (z-direction) and hundreds of meters deep, the 17-80 laminae I used in modeling with a -0.5 gm/cc density contrast appear to give reliable results (within the .5mgal assumed accuracy for the observed gravity data).

As noted before, I chose to confine the gravity stations to a rectangular grid in the modeling. Therefore, the number of gridded gravity stations (and the number of times the forward problem must be done) is determined by the number of columns and rows in the grid. The number of gridded gravity stations, number

of observation points, number of blocks in the model, and the number of laminae per block largely determine the amount of computer time necessary for the program to run. For a model with 80 blocks, having 17-80 laminae per block, and 120 observation points a CPU time of approximately 1.5 minutes was necessary for the forward modeling program to run using a VAX 8500 mainframe computer. Coding of the 3-D forward gravity modeling program (GRAVBL) is contained in Appendix B.1 while examples of preparing the gravity data for use with program GRAVBL is contained in Appendix C.

Inverse Modeling Theory:

As stated before, the inverse 3-D gravity modeling program was written to better determine the depth to bedrock in the modeled area of the Madison Valley. Because blocks of uniform density contrast which extend from the surface to the bedrock were used in the modeling, the parameter defining the maximum depth of each block represents the depth to bedrock. This is parameter z_2 in equation (3), and as equation (3) indicates, $g_z(1)(0,0,0)$ is not linear with respect to parameter z_2 . Therefore, to solve for parameter z_2 , using least squares generalized inverse techniques, equation (3) had to be linearized with respect to parameter z_2 . This linearization was accomplished by rewriting equation (3) using a Taylor series expansion and including only the linear terms (see for example Marquardt, 1963). Once equation (3) has been approximated with the Taylor series expansion, parameters must be updated using an

iterative solution technique which requires an initial guess for each of the parameters. Because equation (3) has been approximated in the linearization process, a "bad" initial parameter guess may cause a parameter to lay outside of the area of assumed local linearity. If this occurs, successive iterations will result in divergence of this parameter from the "true" solution. Therefore, a judicious choice of the initial parameters is necessary to achieve reliable results. The initial parameters (maximum depths of each block) I supplied to the inversion program were those which, when output from the forward modeling program, resulted in a reasonable results based on matches with the drilling and geophysical data.

Once the initial parameters have been supplied, new parameters are determined by parameter jumps; the magnitude and direction of which control the path to the solution, for example:

$$m_{i,j+1} = m_{ij} + dm_{ij}$$

where: $m_{i,j}$ = Parameter i in iteration j .

dm_{ij} = Parameter jump in iteration j .

$m_{i,j+1}$ = New parameters for iteration $j+1$.

To calculate these parameter jumps, I used a weighted, scaled, damped least squares solution (Menke, 1984) using Marquardt's (1963, 1970) maximum neighborhood method. This least squares solution is of the form:

$$dm = D(DA^T W_e A D + \lambda I)^{-1} DA^T W_e \Delta d \quad (5)$$

where: $dm = M \times 1$ ($M =$ number of blocks in the model) matrix containing the parameter jumps used to alter z_2 (depth to bedrock parameter) in a direction which ideally reduces the error between the observed and calculated data.

$D = M \times M$ diagonal scaling matrix.

$A = N \times M$ ($N =$ number of observed gravity data points) matrix of partial derivatives of parameter z_2 with respect to the observed gravity data.

$W_e = N \times N$ data (noise) weighting matrix.

$\lambda =$ Constant, updated with each iteration according to Marquardt's algorithm.

$\Delta d = N \times 1$ matrix of differences between the observed and calculated gravity data.

Linear Algebra

All of the matrix multiplications and manipulations in the inverse gravity modeling program (program GINDEP) were determined using subroutines of the IMSL (1982) library. To invert the the matrix $(DA^T W_e AD + \lambda I)$ in equation (6), an IMSL subroutine called LINV2F was selected. In this subroutine the matrix x is solved for column by column in the following equation:

$$Ax = B$$

where B is set to the identity matrix. The IMSL manual does not explicitly describe the technique used to determine x ; however, a possible solution method is to upper triangularize the A matrix and then solve for x using a back-solving procedure (Menke, 1984).

Scaling Matrix

The diagonal scaling matrix D is used to better condition the MxM matrix to be inverted. The values of D were determined as follows:

If $i = j$, then:

$$D_{i,j} = \sqrt{\frac{1}{(ATWeA)_{i,j}}}$$

otherwise:

$$D_{i,j} = 0.0$$

Determining the Marquardt Damping Factor

To determine the Marquardt damping factor, λ , the following algorithm proposed by Marquardt (1970) was used:

- 1) Let $\lambda = 10^{-3}$ for the initial parameters.
- 2) Calculate the new parameters, and if these new parameters reduce the error between the observed and calculated data:
 - set $\lambda = \lambda/10$ --> iterate again.
 - otherwise:
 - set $\lambda = \lambda \times 10$ --> keep the previous parameters, recalculate the parameter jumps.
- 3) Repeat step 2.

A solution is found once the parameters of a given iteration reduce the error between the observed and calculated data below some input value.

Determining the Error

Of the multitude of ways to estimate the error between the observed and calculated data, I chose the following formula:

$$E = \sqrt{\frac{1}{N-M} \times (g_{\text{observed}} - g_{\text{calculated}})^2} \quad (6)$$

As formula (6) indicates, if the number of blocks (M) used in the model approaches the number of gridded gravity stations (N), the error approaches infinity. This formula, therefore, tends to keep the modeler "honest" by not entertaining complicated models unless they are substantiated with a large amount of data.

Weighting

My choice of a noise weighting matrix was of the form described by Last and Kubik (1983). In their article Last and Kubik discuss noise and parameter weighting matrices used in a 2-D inverse gravity modeling program which inverts on densities assigned to a model formed of a grid of rectangles. The noise weighting matrix they use (assuming normally distributed noise) is:

$$W_{eii} = (G_{ij} W_{jj} (Density)_{jj} G_{ij}^T)_{ii} \quad \begin{matrix} i = 1 \text{ to } N \\ j = 1 \text{ to } M \end{matrix}$$

where:

$G = N \times M$ data kernel matrix mapping densities to the observed gravity data.

$W_m(\text{Density}) = M \times M$ diagonal density weighting matrix.

By using Last and Kubik's form of a noise weighting matrix each time the parameters (densities) are updated, the noise weighting matrix is also updated. This is a reasonable approach in my opinion because, in my modeling, with every parameter (depth to bedrock) change, "geologic noise" inherent in the model is also changed. "Geologic noise" being defined as gravity sources which contribute to the observed gravity yet which are not accounted for in the modeling (i.e. gravity contributions produced from density contrasts in the valley fill). In my modeling I assumed that geologic noise increased in direct proportion to the depth of the block. In other words, I assume the deeper the valley fill the greater the probability of encountering gravity sources not accounted for in the modeling.

As stated before, I chose a weighting matrix of the form of that used by Last and Kubik (1983). My noise weighting matrix being:

$$W_{e_{11}} = (A_{1j} W_m(\text{Depth}) A_{1j}^T)_{11} \quad (7)$$

where:

$A = N \times M$ matrix of derivatives of observed gravity with respect to the depth of bedrock (parameter z_2).

$W_m(\text{Depth}) =$ Diagonal depth weighting matrix.

The depth weighting matrix I chose to use is:

$$Wm(\text{Depth})_{jj} = 1./[ABS(z_2)_j]$$

This noise weighting matrix (equation 7) is a preferentially shallow depth weighted matrix in that shallow blocks are assigned a greater weight. The individual values of this matrix map the magnitude of the sensitivity to which the jumps in parameter z_2 match the differences between the observed and calculated data.

Constraining the Solution Path

I constrained the solution path taken in my modeling program in three ways in an attempt to get physically possible and geologically reasonable results. First of all if during an iteration, a parameter jump caused z_2 (the bottom of a block) to have a value less than z_1 (the top of a block) then this parameter jump was divided by 10 until z_1 had a value less than z_2 . Secondly, maximum and minimum ($z_{\max}$ and $z_{\min}$) values of bedrock depths estimated for the modeling area may be input in the program. If during an iteration a block ends up out of this range of values, the depth of the block is set either to $z_{\max}$ or $z_{\min}$ (whichever extreme was crossed), and this block is damped so that during the next iteration the block is essentially fixed at $z_{\max}$ or $z_{\min}$. The damping is done based on Marquardt's finding that parameter jumps will decrease as the damping variable is increased. This constraining method is implemented as follows:

If the depth of a block (z_2) is less than $z_{\min}$ then:

- 1) Set $z_2(j) = z_{\min}$
- 2) Set $\lambda = 15$ = Marquardt's damping factor

If the depth of a block is greater than $z_{\max}$ then:

- 1) Set $z_2(j) = z_{\max}$
- 2) Set $\lambda = 15$

If during the next iteration, the parameter jumps for the heavily damped blocks are in the direction that brings the depth back into the allowable range, the damping variable is reset to the current Marquardt damping value. Damping the blocks with a value of 15 is based on my modeling experience which showed that this value was sufficiently large to constrain the blocks. I found that if the Marquardt damping factor reached 100 (the Marquardt damping variable increasing by a factor of 10 each time an iteration produced diverging results) during the modeling process, repeated divergence in the successive iterations occurred. These results are based on the numbers pertaining to the specific model used for the Ennis geothermal area.

A third constraint is that individual blocks can be taken out of the inversion process. This is done by assigning a large Marquardt damping value ($\lambda = 100$) to specified blocks in the model. In this way selected blocks may retain their initial guess depth throughout the modeling process.

Finally, in order to make the program more efficient, I did not allow another iteration to occur unless the new model

parameters reduced the error between the observed and calculated data by more than one percent (a meaningful improvement).

Gravity Models of the Ennis Geothermal Area

Topographic Model:

Figure 22 shows the area to be modeled relative to the observed data area. The modeled area extends approximately 3 km beyond the boundary of the observed data set in north, south, and east directions and only 1 km to the west because of the shallowing of the bedrock to the west. The extension of the modeling area past the gridded observed data area serves the purpose of taking into account contributions to the gridded observed gravity values from sediments beyond the gridded observed gravity area. It is an a priori assumption that sediments outside the boundaries of the modeled area contribute negligibly to the gridded observed gravity values.

Figure 23 shows selected elevation contours, at elevations of 1475, 1487, 1500, 1524, and 1585 m, taken from the Ennis 15 minute quadrangle in the modeled area. The 1475, 1487, and 1500 meter (4840, 4880, 4920 feet) elevation contours were selected because they are the only contours drawn through the valley floor in the modeling area, while the 1524 and the 1585 meter (5000 and 5200 feet) elevation contours were selected because they were easily traceable. Consequently, Figure 23 is a representation of the topography in the modeled area where I

Figure 22. Modeling area (outer square) relative to the gridded observed gravity data area (inner square) as they would appear on the Ennis 15' quadrangle.

Figure 23. Selected contour elevations (meters above sea level) taken from the Ennis 15' quadrangle map. These contours are assumed to give an adequate representation of the topography in the modeling area.

have assumed that this representation is adequate in defining the topography in the modeling area. For the gravity modeling program (using Talwani and Ewing's method) each of these contours was approximated by the horizontal polygons which are shown in plan view in Figure 24a and individually in Figure 24b. The five polygons, at the elevations shown in Figure 24, make up the model used to represent the topography. In the modeling program, the gravity contribution of each of these polygons is determined, and the total gravity anomaly is found by summing each of the contributions through the use of a quadrature formula described in Talwani and Ewing's (1960) paper. Because the 120 gravity "stations" of the gridded observed data set are located at different elevations, the model must be shifted relative to each station elevation so that the topography appears as it should from an observation point at the individual gridded station elevations. In effect what is being done is a local correction from which the gravity contribution of sediments from the highest to the lowest elevation in the modeled area is determined for each gridded gravity station. Once these contributions are removed from the gridded observed data set, it is as if all sediments in the modeled area to an elevation of 1475 meters have been stripped away as far as gravity contributions are concerned.

Figure 25b shows the contours of the gravity contributions from the valley fill contained in the topographic model ($c_1 = 0.2$ mgals). See Appendix E.3 for these calculated gravity values. Note the correlation of the topographic correction

Figure 24. A) A plan view of the 5 polygons which comprise the topography model. These polygons are digitized representations of the 5 contours shown in Figure 23.

B) Schematic of the individual polygons.

contours in Figure 25b with the elevation contours in Figure 23. Figure 25a shows the contours of the gridded observed gravity data ($ci = 1$ mgal) with the gravity contributions from the topographic model removed from the gridded observed gravity data set (shown in Figure 17). See Appendix E.4 for the actual gravity values.

Forward Model:

Once the gravity contributions of the valley fill in the topographic model are removed, the sediments in the modeling area are assumed to start from a plane at an elevation of 1475 meters. Consequently, for the forward modeling each of the blocks are modeled with tops starting at an elevation of 1475 meters. At each gridded gravity station, the entire model of the sediments below 1475 meters is depth-shifted so that the model appears as it should from an observation point at the station's elevation. Consequently, the forward model includes a variably thick cushion of "air", the cushion thickness being determined by the difference between each station elevation and the 1475 meter elevation plane.

In the forward model 80 blocks of earth, contained in the modeling area shown in Figure 22, were used. These blocks are shown in plan view in Figure 26 and the numbers on the blocks correspond to how they were numbered in the actual modeling of the Ennis hot springs area. Those blocks directly below the observed data set, blocks 1-54, were given a smaller cross-sectional area (in the x-y plane) because it is these

Figure 26. Plan view of the spatial locations of the blocks used in the gravity modeling. The blocks are numbered as they were in the actual modeling.

blocks that, by virtue of their closeness to the gravity stations, should exhibit the most control over the shape of the gravity anomalies in the gridded observed gravity data.

The forward modeling process consisted of manually altering the depths of the blocks until a reasonable match with the "topographically corrected" gravity values was obtained. The final model resulting from this modeling process is shown in Figure 27 (see Appendix F.1.1). The numbers on the blocks in Figure 27 represent the depth in meters of each block below the 1475 meter level to the bedrock. In Figure 28b the gravity values calculated from the model shown in Figure 27 are shown in contoured form ($ci = 1$ mgal, see Appendices F.1.2 and F.1.3). The "goodness" of these model results are shown in Figure 29 which is a contour plot ($ci = .5$ mgals) of the absolute value of the differences between the "topographically corrected" gridded observed gravity data (Figure 28a) and the model results. The normalized average root mean square error (equation 6) between the forward model results and the topographically corrected data is approximately 1.0 mgals. See Appendix D for input to and output of the forward modeling program.

Inverse Model:

In the inverse modeling process the depths of the blocks are automatically adjusted so that with each iteration of the computer, the error (see equation 6) between the gridded observed "topographically corrected" gravity data and the

Figure 27. Plan view of the final model output from the forward modeling process. The numbers in the blocks represent the depth (in meters) below a 1475 meter elevation datum to the bedrock.

Figure 28. A) Contours (c.i.=1mgal) of the "topography corrected" gridded observed gravity data.

B) Contours (c.i.=1mgal) calculated using the forward modeling program with the model shown in Figure 27 input.

Figure 29. Contours of the absolute value of the differences between the gridded observed "topography corrected" gravity data (c.i. = 0.5 mgals) - Figure 28a and the calculated gravity data from the forward model - Figure 28b. The average root mean square misfit between the observed and calculated data is app. 1.0 mgal.

calculated gravity data is reduced. The forward model shown in Figure 27 is used as a starting model for the first iteration in the inversion process. After three iterations in the inversion program, the model shown in Figure 30 was output (see Appendices F.2.1 and F.2.2). The numbers in Figure 30 again represent the depth to bedrock in meters below an elevation of 1475 meters.

The "goodness" of the inverse model is displayed in Figures 31 and 32. Figure 31b shows the contours of gravity ($c_1 = 1$ mgal) calculated from the model shown in Figure 30 (see Appendix F.2.3). Figure 32 shows contours ($c_1 = .5$ mgals) of the differences between the "topographically corrected" gridded observed gravity data (Figure 31a) and the calculated gravity data from the inverse model (Figure 31b). The normalized average root mean square misfit (equation 6) between the inverse model results and the "topographically corrected" data is 0.49 mgals (within the assumed error of the gravity survey of 0.5 mgals).

Approximately 20 minutes of CPU time on a VAX 8500 computer was necessary to complete the three iterations and obtain the inverse modeling results. See Appendix D.2 for input to and output of the inverse modeling program (program GINDEP).

Figure 30. Plan view of the model output after 3 iterations using the inverse gravity modeling program. The numbers in the blocks represent the depth (in meters) below the 1475 meter elevation datum to the bedrock.

Figure 31. A) Contours (c.i. = 1 mgal) of the "topography corrected" gridded observed gravity data.
 B) Contours (c.i. = 1 mgal) calculated from the model shown in Figure 30 output after 3 iterations of the inverse modeling program.

Figure 32. Contours (c.i. = 0.5 mgals) of the absolute values of the differences between the gridded observed "topography corrected" gravity values - Figure 31a and the calculated gravity data using the model shown in Figure 30 - Figure 31b. The average root mean square misfit between the observed and calculated data is 0.49 mgals.

Reliability of the Inverse Model:

The gravity calculated from the inverse model matches the gridded observed "topographically corrected" gravity data within the assumed error of the gravity survey; however, an infinite number of mass distributions can produce a given gravity distribution. Therefore, because the gravity model shown in Figure 30 is not unique (and incorporates a number of assumptions), a demonstration of how well this model matches other constraining data is necessary. As shown in Figure 30, 80 blocks of earth were used in the model, however only the 54 blocks, located directly beneath the gridded observed gravity data, shall be used to check the reliability of the inverse modeling results. It is assumed that the blocks located directly beneath the gridded observed gravity data are the most highly constrained by the gridded observed gravity data.

Comparison with Seismic Refraction

Two seismic refraction lines were shot in the area of the Ennis hot spring (see Figure 8). The data from these surveys was interpreted to determine the depth to bedrock in the area of the Ennis hot spring (see Figures 9 and 10). Comparisons of the depth to bedrock information from these seismic refraction lines and the inverse gravity model are shown in Figure 33. The dotted line in Figure 33 represents the seismic refraction results while the zones between the line segments indicate the depth to bedrock plus or minus the standard deviation of the

Figure 33. Comparison of the bedrock depths determined from the seismic refraction work and the inverse gravity modeling results. The dotted line shows the depth to bedrock interpreted (McRae, unpublished senior report) from the seismic refraction data, and the straight line segments represent the depth to bedrock $\pm$ a standard deviation determined from the inverse gravity modeling.

calculated bedrock depth determined using the inverse gravity modeling program.

The gravity and refraction interpretations match quite well considering that each of the blocks in the gravity modeling program received a bedrock depth based on gravity data that is averaged over the area of a block. In the SW-NE trending refraction line, the gravity modeling results indicate a significant bedrock drop not shown in the seismic refraction interpretation; however, this bedrock offset is located beyond the area where the seismic refraction data could have indicated this drop. In the NW-SE trending refraction line, the gravity modeling results indicate a drop in the bedrock located between wells TX-6 and SH-4 that is not included in McRae's (unpublished senior report) interpretation of the refraction data. However, the data from the unreversed refraction profile extending from SH-4 to TX-6 does indicate a drop in the bedrock in the area of offset implied by the gravity modeling (see Figure 10). Apparently McRae averaged the changes in slope of the time distance curve after the initial slope change (representing the refraction along the bedrock surface SE of the drop of the bedrock indicated in the gravity modeling) in his interpretation.

Comparison with Drilling

Three of the test wells (Tx-9, 11, and 12) and one of the shot holes (SH-3) penetrated the bedrock. Comparison of bedrock depths determined from the gravity modeling and

drilling are shown in Table 8. The middle depth to bedrock column in the gravity modeling portion of Table 8 indicates the depth to bedrock below the 1475 meter datum plus or minus a standard deviation. The matches between the depth to bedrock determined from the drilling and the gravity modeling are reasonable (in my opinion) again considering the fact that the depth to the bedrock determined from the gravity modeling is based on averages of the gravity data over the area of the block.

Comparison with Seismic Reflection

Susan Young (1984) in a masters thesis written for the University of Texas discusses eight seismic reflection lines located in the northern Madison valley. From the processed seismic data, Young picked and traced what she considered were "distinctive reflector horizons" on each line for interpretation purposes.

A section of one of the seismic lines, line #2, (see Figure 11) crosses the gravity modeling area along a N-S line through the easternmost row of interpreted blocks. This seismic line was migrated and one velocity analysis was done along the line. From this velocity analysis interval velocities were computed by Young using the Dix equation. These interval velocities for line #2 are shown in Table 9. Figure 34 shows two pictures; the upper picture shows the picked and traced distinctive reflector horizons included by Young for line #2 while the lower picture in Figure 34 shows

Figure 34. The upper picture shows "distinctive reflector horizons" picked and traced from seismic reflection line #2 by Young (1984). The lower picture shows Young's interpretation of the data shown in the upper picture. Superimposed on both of these pictures are the depths to bedrock indicated from the inverse gravity modeling results.

Young's interpretation based on the reflector horizons shown in the uppermost picture.

In both the upper and lower pictures of Figure 34, the depth to bedrock indicated from the gravity modeling in the appropriate section of line #2 is shown. The depths to bedrock indicated from the gravity modeling were converted to time using the velocity information contained in Table 9. 293 meters had to be added to the depth of each block because of the discrepancy between the 1768 m. datum elevation used in processing the seismic reflection data, and the 1475 m. datum used in the gravity modeling.

Because only the picked and traced "distinctive" reflectors are shown, it is difficult to say whether the seismic reflection data supports the gravity modeling results.

Table 8. Comparison of bedrock depths determined from gravity modeling and drilling.

1) Drilling

<u>Well</u>	<u>Surface Elevation(m.)</u>	<u>Depth to bedrock (m. from surf.)</u>
TX-9	1502	107
TX-11	1489	161
TX-12	1499	142
SH-3	1503	26

2) Gravity Modeling

<u>Well</u>	<u>Depth to bedrock</u>	<u>Depth to bedrock (m. from surf.)</u>
TX-9	51 +/- 26	52 - 104
TX-11	210 +/- 56	168 - 280
TX-12	129 +/- 46	107 - 199
SH-3	0 +/- 19	9 - 47

Table 9. Interval velocities determined by Young, 1984 using the Dix equation for seismic reflection line #2. Time represents two way travel time.

<u>Two Way Time (sec.)</u>	<u>Interval Velocity (m/s)</u>
0.0	----- 2439
0.2	----- 2530
0.4	----- 2854
0.6	----- 3259
0.8	----- 3279
1.0	----- 3257
1.2	----- 3347
1.4	

Nevertheless, the south dip of the picked reflector horizons (located immediately south, or to the left, of the bedrock high shown in the gravity modeling results) suggest the possibility of a bedrock high in the location the gravity modeling results indicate.

As for Young's interpretation, I believe the gravity modeling results cast doubt on Young's inclusion of Paleozoic and Mesozoic rocks in the area south of the bedrock high shown in the gravity modeling. If there was a thick sequence of Paleozoic and Mesozoic rocks in this area, the gravity data should indicate a shallower depth of valley fill because of a density contrast between the valley fill and the Paleozoic and Mesozoic rocks. Therefore, I believe that the reflectors Young

indicated as possibly being Paleozoic and Mesozoic rocks, in the area south of the modeled basement high, are probably reflectors contained in the Tertiary valley fill.

STRUCTURAL INTERPRETATION OF THE INVERSE 3-D GRAVITY MODELING RESULTS

Interpretation Considerations

As shown in Figure 26, 80 blocks of earth were used in the model, 54 of which were located directly underneath the gridded observed gravity data, and it is these 54 blocks of valley fill that are interpreted. Figure 36 shows the location of the 54 blocks as they would appear on the Ennis 15 minute quadrangle map (see Figure 35 for an explanation of symbols). As the numbers in the blocks represent the depth of valley fill above bedrock (relative to a datum elevation of 1475 meters), it should be clear that Figure 36 represents a map of the bedrock topography in the area of the modeling.

To structurally interpret this bedrock topography, I considered significant bedrock displacements (relative to the depth of the block), mappable along consistent trends, as faults. As I did not believe the resolution in averaging the gravity data over the area of the blocks justified the drawing of these interpreted faults as lines, I drew the faults as zones approximately 100 meters wide. It should also be noted that I considered all the faults to be associated with the tensional tectonics involved in the creation of the Madison Valley. Consequently, I depict all the faults as normal faults (except for a wrench fault); however, because gravity modeling can not detect whether high (or low) bedrock is the result of

LEGEND

- ▽ TX3 = test well 3
- ▽ SH4 = shot hole 4 (for refraction survey)
- 20 = section number - Ennis quadrangle

= normal fault (assumed)
down on bell side

= wrench fault (assumed)

= Madison River

= Ennis hot spring

BM = gravity benchmark

Figure 35. Explanation of symbols shown in Figures 36 and 37.

Figure 36. Plan view of the 54 blocks of the inverse gravity model included for interpretation. These blocks are shown as they would appear on the Ennis 15' quadrangle, and the numbers in the blocks indicate the depth (in meters) below the 1475 meter elevation to the bedrock.

tensional or compressional forces, these interpreted fault motions represent at best an educated guess. Further, high or low bedrock may represent emplacement or erosional features or may be associated with tectonic activity resulting in something other than faulting (i.e. folding or flexure of the bedrock). These points should be remembered because in the writing to follow I consider the interpreted faults as such even though faulting may not have occurred. Finally, these interpreted faults are drawn as they would be located at the valley fill/bedrock interface.

Interpretation

The interpretation of the bedrock topographic model shown in Figure 36 is illustrated in Figure 37. The major structural feature contained in this interpretation is an east-west trending feature depicted as the dotted zone located approximately 500 meters north of the Ennis hot spring on a north-south line. If this dotted zone is to be called a fault, it would have to be considered a wrench fault as displacement directions change along the strike of the dotted zone. For example, west of the Madison River the bedrock is dropped north of the dotted zone while the reverse is true east of the Madison River. As the NW to NE trending faults appear to terminate against this zone, I will assume that the east-west feature predates interpreted NW-NE trending faults. If the east-west feature does predate the primarily east-west extensional tectonics responsible for the creation of the

Figure 37. Interpretation of the inverse gravity modeling results shown in Figure 36 (see Figure 35 for an explanation of symbols).

Madison Valley, it is possible that shearing, during the period of extensional tectonics, occurred along the east-west trending feature. At any rate, I will refer to the east-west trending dotted zone shown in Figure 37 as a wrench fault (along which shearing has possibly taken place) for the discussion to follow.

Structure in the gravity modeling area appears to have been controlled by this wrench fault because: 1) Many of the interpreted Tertiary normal faults appear to terminate against the wrench fault; and 2) major changes in bedrock depths and faulting relationships occur across this zone. The changing structural features of the bedrock across this zone is clearly demonstrated by comparing E-W cross sections located north and south of this wrench fault. This comparison is shown in Figure 39 (see Figure 38 for the location of the cross section lines). Note that the vertical offsets of the bedrock shown in Figure 39 represent the drops across the individual blocks in the model; therefore, these vertical offsets need not represent locations of the interpreted faults shown in Figures 37 and 38. Remember I only interpreted a bedrock drop as a fault if I considered that: 1) the offset was significant relative to the depth of the block; and 2) the offset could be traced along a consistent trend. And as these consistent trends of significant offset generally did not follow the edge of the blocks (N-S or E-W trends), I have interpreted faults which cut across the modeled blocks or, in other words, I have not interpreted only E-W and N-S trending faults. Therefore, even significant bedrock

Figure 39. Comparison of modeled bedrock topographies located north and south of the wrench fault. See Figure 38 for the locations of the cross-section lines.

displacements shown in Figure 39 need not correlate with the fault locations shown in Figures 37 and 38.

North of the wrench fault, the gravity modeling indicates the bedrock drops off rapidly to the east reaching a maximum depth of over a kilometer underneath the Madison River (see Figure 39). East of the Madison River, however, the bedrock shallows forming an eastern wall to a northwest trending bedrock valley, a profile of which is shown in Figure 39. South of the wrench fault the basement remains relatively shallow west of the Madison River in contrast to the bedrock depths observed in the gravity modeling north of the cross fault (see figure 39). East of the Madison River, the shallowing of the bedrock, defining the eastern wall of the bedrock valley north of the wrench fault, is not indicated south of the wrench fault.

Interpretation Implications

The purpose of the 3-D gravity modeling in the area of the Ennis hot spring was to determine the location of faults which have offset the bedrock and which might possibly control the ascending thermal waters of the Ennis geothermal system. Figure 37 represents my attempt to satisfy this purpose; however, the surface location of the Ennis hot spring can not definitely be explained by the gravity modeling results. This is because aquifers in the valley fill control the flow of the thermal waters above the bedrock. Nevertheless, there are NW and NE trending faults interpreted in the area of the surface

location of the springs. If these fault planes are permeable, they may act as conduits for geothermal water flowing in the bedrock and may therefore bring geothermal water into the vicinity of the surface location of the Ennis hot spring.

To determine a promising place to drill in the Ennis geothermal area, the following question must be asked: What path(s) does the water follow from the reservoir to the hot spring on the ascending limb of the Ennis geothermal system? As the location of the reservoir relative to the hot spring is unknown, perhaps the best way to answer this question is to try and track the water path from the spring to the reservoir. Sonderegger and Zaluski (1983) discuss the north dipping fracture zone that is hydraulically connected between wells TX-12, TX-11, and MAC-1. Because the flow rate in the fracture zone ($.0242 \text{ m}^3/\text{s}$ - from the Pump Test of MacMillan Well - MBMG in house publication) is approximately 20 times greater than the flow rate of the Ennis hot spring ($.0012 \text{ m}^3/\text{s}$ - Leonard, written communication), the following is indicated: 1) The surface flow of the Ennis hot spring is only a minor portion of the Ennis geothermal system; and 2) the fracture zone is an important near surface aquifer in the Ennis geothermal system and probably supplies hot water to the Ennis hot spring. Based on the inferred north dip of the fracture zone, I assume the hot water comes to the spring from a general northerly direction, however, because only two wells definitely intercepted the fracture zone, the 3-D orientation of the fracture zone is unknown.

If the fracture zone results from an overthrust effect, as Sonderegger and Zaluski (1983) suggest, then perhaps the fracture zone is a continuous feature that can be found in the deeper bedrock north of the wrench fault. At any rate, I recommend that a well be drilled in the area of the wrench fault because if the zone is transmissive, thermal water ascending from a reservoir east or west of the hot spring could be controlled by this feature. The point along the wrench fault that I recommend for drilling is at shot hole SH-5 because my interpretation of the modeling indicates that two faults to south of the wrench fault intersect the wrench fault in this area. Also, SH-5 is reasonably close to well MAC-1 which intercepted the fracture zone. The modeled depth to bedrock north of the wrench fault in the area of SH-5 is 600 meters while south of the wrench fault the modeled depth to bedrock is approximately 200 meters (see Figures 36 and 37).

To make a rough estimate of the water temperature that a well could intercept if drilled to the modeled bedrock depth in the area of SH-5, I will assume that the 90°C temperature encountered at approximately 150 meters in well TX-12 represents the temperature of the water below any cold water mixing zone. This 90°C temperature is 70°C below the estimated 160°C reservoir temperature, and this reservoir is assumed to be located 3.3-5 kilometers below the land surface. These numbers indicate that the ascending geothermal waters lose 60°C in approximately 4 kilometers. Based on this 60°C per 4 kilometer gradient, geothermal water circulating in the

wrench fault approximately 600 meters below the land surface will have attained a temperature of 99°C (9 degrees above the 90°C "unmixed" surface temperature). I emphasize that this 99°C is a crude estimate. If one is an optimist, the wrench fault may represent a highly permeable aquifer by which the ascending geothermal water may travel with very little loss in temperature. On the other hand, a pessimist would point out that wrench fault may be impermeable and no water whatsoever would be intercepted by drilling the wrench fault.

SUMMARY

The following is a list of significant results gathered from the previous work in the Ennis geothermal area and discussed in this paper:

- 1) A north dipping (at approximately 30° assuming the fracture zone is linear between wells TX-12 and MAC-1) fracture zone, approximately 30 meters wide, was intersected during the drilling of wells TX-12 and MAC-1. The fracture zone is located in the Archean metamorphic bedrock 157 meters below the surface in well TX-12 and 335 meters below the surface in well MAC-1. Hydraulic connection between the two wells (and also well TX-11) was confirmed during a pump test of the MAC-1 well. The pumped flow rate in the fracture zone is approximately 20 times greater than the measured surface flow rate of the Ennis hot spring and is possibly an important local supplier of water to the Ennis hot spring;
- 2) A heat flow study, which accounts for conductive and convective heat flow, indicates a heat flow range in the area of drilling from 0.42 to 10.5 MW assuming end value flow rates (the measured surface versus the assumed "safe yield" flow rate) for the Ennis geothermal system;

- 3) A reservoir temperature of approximately 160 degrees Celcius is assumed by Leonard (written communication) and Juncal (Project Milestone Report) based on geothermometry of the Ennis geothermal system. Assuming temperature gradients of 30°C/Km for the bedrock and 60°C/Km for the valley fill, a reservoir depth between 3.3 and 5 kilometers is indicated;
- 4) No presently mapped fault(s), observable at the surface in the area of the Ennis hot spring, obviously controls the present surface location of the spring. Major mapped faults, apparently important in the structural development of the upper Madison Valley (located a few to tens of kilometers north and east of the Ennis hot spring), include: a) the Spanish Peaks reverse fault; b) the Jack Creek thrust fault; c) the North Meadow Creek fault; and d) the Madison Range front normal fault;
- 5) Audio magneto-telluric survey results show a definite broadening of low resistivity values in 300 m skin depth results relative to 100 m results, in every direction but west of the hot spring. Also, a north-south low resistivity trend surrounding the hot spring is shown in the data indicating a possible N-S oriented structure; and

- 6) Young's (1984) interpretation of seismic reflection data indicates the Jack Creek thrust fault possibly extends west of the Madison Range front fault and a complex intra-valley bedrock topography including two grabens and numerous normal faults.

These previous work results provide a foundation on which my 3-D gravity modeling work is built.

The 3-D forward gravity modeling involves a two-step process which includes modeling with vertically-stacked, horizontal, n-verticied polygons (based on Talwani and Ewing's 1960 method) and with blocks (based on an algorithm derived from integration of the gravity formula). This two-step forward modeling process is used to find a bedrock topography model from which calculated gravity data approximately matches the observed. Next, an inverse gravity modeling program is used to create an adjusted bedrock topography model that yields a normalized average root mean square error (see Equation 6), between the observed and calculated gravity data, which matches the observed gravity data within the error of the survey. Although these gravity modeling techniques were applied to finding offsets in the bedrock caused (presumably) by faulting which might control water of the ascending limb of a geothermal system, these gravity modeling techniques could also be used to solve geological and hydrogeological problems wherever a valley fill over bedrock geologic setting exists.

This two-step forward modeling process, used sequentially with the inverse modeling program, was applied to the gravity data collected in the area of the Ennis hot spring. The bedrock topography model (output using these modeling techniques) produced a normalized average root mean square error (see Equation 6) within the 0.5 mgal assumed error for the gravity data collected in the modeling area. This bedrock topography model also matched reasonably with seismic refraction data and depth to bedrock information from wells drilled in the area of the Ennis hot spring. My interpretation of this bedrock topography model indicates the following (see Figure 37):

- 1) An intersection of northwest and northeast faults in the bedrock which possibly control flow of thermal water in the bedrock in the area of the Ennis hot spring;
- 2) An east-west trending wrench fault upon which north-south, presumably Tertiary, normal faults terminate and across which structural relationships change; and
- 3) A northwest trending bedrock valley (graben?) having an eastern wall north of the wrench fault but not south of the wrench fault in the modeling area.

This interpreted complex bedrock topography, in general, is not surprising considering Young's (1984) interpretation of seismic reflection data shot to the east of my modeling area in the Madison Valley.

If further drilling is done in the area of the Ennis hot spring, I suggest drilling along the interpreted wrench fault because, if the wrench fault is transmissive, it could control recharge water from east or west of the hot spring. A possibly promising drilling site along the wrench fault in the area of shot hole SH-5 is suggested because: 1) SH-5 is reasonably close to well MAC-1 which intercepted an apparently important near surface fracture zone of the Ennis geothermal system; and 2) I interpret two faults to intersect the wrench fault in this area. A crude estimate indicates a 99°C temperature for the geothermal water in the bedrock beneath this drill site.

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APPENDIX A

GRAVITY DATA AND CORRECTIONS

All corrections are referenced to U.S. Coast and Geodetic Survey benchmark Y145 labeled as BM in the data. Three surveys were completed at separate dates, and these surveys are referred to as the VALLEY FLOOR, the JEFFERS, and the JACK CREEK RANCH surveys. The VALLEY FLOOR survey includes stations collected on two east-west lines located west of the Madison River, the JEFFERS survey includes stations located on the southernmost loop of stations, and the JACK CREEK RANCH survey includes stations located on the northernmost loop of stations. See Figure 15 in text.

See the next four pages for the survey data.

VALLEY FLOOR SURVEY

(LOCATIONS, ELEVATIONS, OBSERVED AND REDUCED GRAVITY)

STA	LAT (DEG)	LONG (DEG)	OBS. GRV. (MGALS)	ELEV. (M.)	RED. GRV.	
BM	45.3782	111.7315	196.558	1492.2	0.00	-186.65
1	45.3782	111.7301	195.980	1491.9	-0.56	-187.21
2	45.3782	111.7288	196.039	1492.0	-0.51	-187.16
3	45.3782	111.7274	195.796	1492.0	-0.74	-187.39
4	45.3782	111.7261	195.914	1484.9	-2.04	-188.69
5	45.3782	111.7247	196.215	1484.9	-1.77	-188.42
6	45.3782	111.7234	195.892	1484.8	-2.14	-188.79
7	45.3782	111.7220	195.409	1484.4	-2.72	-189.37
8	45.3782	111.7206	195.132	1484.3	-3.04	-189.69
9	45.3782	111.7193	194.704	1484.6	-3.44	-190.09
10	45.3782	111.7179	194.365	1484.5	-3.80	-190.45
11	45.3782	111.7166	193.999	1484.5	-4.19	-190.84
12	45.3782	111.7153	193.487	1484.5	-4.71	-191.36
13	45.3782	111.7139	193.150	1484.1	-5.14	-191.79
14	45.3782	111.7126	192.768	1484.0	-5.56	-192.21
15	45.3782	111.7112	192.375	1483.6	-6.04	-192.69
16	45.3782	111.7099	192.053	1483.2	-6.46	-193.11
17	45.3763	111.7099	191.962	1484.5	-6.18	-192.83
18	45.3763	111.7122	192.337	1484.8	-5.76	-192.41
19	45.3763	111.7126	192.745	1485.2	-5.29	-191.94
20	45.3763	111.7139	193.074	1485.3	-4.94	-191.59
21	45.3763	111.7153	193.537	1485.4	-4.48	-191.13
22	45.3763	111.7166	193.867	1485.8	-4.07	-190.72
23	45.3763	111.7179	193.990	1485.3	-4.04	-190.69
24	45.3763	111.7193	194.759	1485.4	-3.26	-189.91
25	45.3763	111.7206	195.053	1485.8	-2.90	-189.55
26	45.3763	111.7220	195.473	1486.1	-2.43	-189.08
27	45.3763	111.7234	195.742	1486.1	-2.15	-188.80
28	45.3763	111.7247	196.149	1486.1	-1.75	-188.40
29	45.3763	111.7261	195.194	1492.1	-1.53	-188.18
30	45.3763	111.7274	195.396	1491.6	-1.48	-188.13
31	45.3763	111.7288	195.822	1491.6	-1.06	-187.71
32	45.3763	111.7301	196.305	1492.2	-1.06	-187.71

VALLEY FLOOR SURVEY

(CORRECTIONS-MGALS)

STA	ELEV	TERRAIN	LAT.	CURV.	DRIFT
BM	0.00	1.48	-2.60	-1.40	-0.13
1	-0.09	1.48	-2.60	-1.40	-0.02
2	-0.07	1.47	-2.60	-1.40	-0.04
BM	0.00	1.48	-2.60	-1.40	0.00
3	-0.08	1.47	-2.60	-1.40	-0.02
4	-1.48	1.47	-2.60	-1.40	-0.04
5	-1.48	1.46	-2.60	-1.40	-0.06
6	-1.49	1.46	-2.60	-1.40	-0.09
7	-1.56	1.45	-2.60	-1.40	-0.11
8	-1.58	1.45	-2.60	-1.40	-0.13
9	-1.54	1.44	-2.60	-1.40	-0.14
10	-1.55	1.44	-2.60	-1.40	-0.15
11	-1.55	1.43	-2.60	-1.40	-0.16
12	-1.55	1.43	-2.60	-1.40	-0.17
13	-1.62	1.42	-2.60	-1.40	-0.18
14	-1.64	1.42	-2.60	-1.40	-0.20
15	-1.72	1.41	-2.60	-1.40	-0.20
16	-1.80	1.41	-2.60	-1.40	-0.21
17	-1.55	1.40	-2.43	-1.40	-0.25
18	-1.50	1.40	-2.43	-1.40	-0.26
19	-1.42	1.40	-2.43	-1.40	-0.28
20	-1.39	1.40	-2.43	-1.40	-0.29
21	-1.38	1.40	-2.43	-1.40	-0.30
22	-1.30	1.41	-2.43	-1.40	-0.31
23	-1.39	1.42	-2.43	-1.40	-0.32
24	-1.37	1.42	-2.43	-1.40	-0.33
25	-1.30	1.43	-2.43	-1.40	-0.34
26	-1.24	1.43	-2.43	-1.40	-0.35
27	-1.24	1.43	-2.43	-1.40	-0.35
28	-1.24	1.44	-2.43	-1.40	-0.36
29	-0.06	1.45	-2.43	-1.40	-0.38
BM	0.00	1.48	-2.60	-1.40	-0.40
30	-0.15	1.45	-2.43	-1.40	-0.44
31	-0.15	1.46	-2.43	-1.40	-0.45
32	-0.03	1.47	-2.43	-1.40	-0.48
BM	0.00	1.48	-2.60	-1.40	-0.52

JEFFERS LOOP SURVEY

(LOCATIONS, ELEVATIONS, OBSERVED AND REDUCED GRAVITY)

STA	LAT (DEG)	LONG (DEG)	OBS. GRV. (MGALS)	ELEV. (M.)	RED. GRV.	
BM	45.3782	111.7315	196.804	1492.2	0.00	-186.65
33	45.3792	111.6832	187.208	1493.3	-9.57	-196.22
34	45.3808	111.6832	187.607	1494.0	-9.15	-195.80
35	45.3825	111.6831	187.971	1496.3	-8.79	-195.44
36	45.3826	111.6857	188.464	1498.0	-7.87	-194.52
37	45.3826	111.6882	188.862	1499.8	-7.17	-193.82
38	45.3827	111.6908	189.420	1500.5	-6.30	-192.95
39	45.3824	111.6935	189.704	1500.0	-5.89	-192.54
40	45.3807	111.6943	189.209	1499.1	-6.36	-193.01
41	45.3790	111.6944	189.031	1498.6	-6.54	-193.19
42	45.3772	111.6943	188.660	1497.7	-6.86	-193.51
43	45.3754	111.6943	188.251	1497.1	-7.30	-193.95
44	45.3750	111.6922	188.131	1495.6	-7.49	-194.14
45	45.3749	111.6897	187.857	1494.9	-8.00	-194.65
46	45.3766	111.6863	187.718	1494.7	-8.41	-195.06
47	45.3784	111.6863	187.856	1494.4	-8.51	-195.16

JACK CREEK RANCH SURVEY

(LOCATIONS, ELEVATIONS, OBSERVED AND REDUCED GRAVITY)

STA	LAT (DEG)	LONG (DEG)	OBS. GRV. (MGALS)	ELEV. (M.)	RED. GRV.	
BM	45.3782	111.7315	185.201	1492.2	0.00	-186.65
48	45.3676	111.6942	174.550	1491.9	-9.90	-196.55
49	45.3676	111.6979	175.125	1492.4	-9.25	-195.90
50	45.3676	111.7013	176.017	1491.6	-8.57	-195.22
51	45.3641	111.7027	175.417	1494.5	-8.30	-194.95
52	45.3606	111.7033	174.461	1497.2	-8.47	-195.12
53	45.3569	111.7038	173.286	1499.4	-8.86	-195.51
54	45.3533	111.7046	171.949	1501.5	-9.41	-196.06
55	45.3494	111.7046	170.914	1503.3	-9.71	-196.36
56	45.3493	111.6992	168.263	1505.0	-12.01	-198.66
57	45.3494	111.6944	166.673	1505.6	-13.41	-200.06
58	45.3491	111.6887	165.324	1505.0	-15.15	-201.80
59	45.3532	111.6887	167.121	1501.6	-14.33	-200.98
60	45.3568	111.6884	169.483	1498.1	-12.35	-199.00
61	45.3604	111.6888	170.975	1495.7	-11.69	-198.34
62	45.3639	111.6890	172.653	1493.9	-11.00	-197.65
63	45.3644	111.6936	173.525	1493.7	-10.28	-196.93

JEFFERS LOOP SURVEY

(CORRECTIONS-MGALS)

STA	ELEV	TERRAIN	LAT.	CURV.	DRIFT
BM	0.00	1.48	-2.60	-1.40	0.00
33	0.04	1.57	-2.70	-1.40	D
34	0.19	1.58	-2.84	-1.40	R
35	0.32	1.59	-2.99	-1.40	I
36	0.78	1.57	-3.00	-1.40	F
37	1.10	1.55	-3.00	-1.40	T
38	1.45	1.51	-3.00	-1.40	
39	1.60	1.47	-2.98	-1.40	N
40	1.49	1.46	-2.83	-1.40	E
41	1.33	1.46	-2.68	-1.40	G
42	1.22	1.45	-2.51	-1.40	L
43	1.04	1.44	-2.35	-1.40	I
44	0.92	1.46	-2.32	-1.40	G
45	0.64	1.50	-2.31	-1.40	A
46	0.46	1.55	-2.46	-1.40	B
47	0.39	1.55	-2.62	-1.40	L
					E
BM	0.00	1.48	-2.60	-1.40	0.02

JACK CREEK RANCH SURVEY

(CORRECTIONS-MGALS)

STA	ELEV	TERRAIN	LAT.	CURV.	DRIFT
BM	0.00	1.48	-2.60	-1.40	0.00
48	-0.10	1.38	-1.65	-1.40	D
49	0.01	1.35	-1.65	-1.40	R
50	-0.16	1.30	-1.65	-1.40	I
51	0.43	1.27	-1.34	-1.40	F
52	0.94	1.24	-1.03	-1.40	T
53	1.38	1.26	-0.70	-1.40	
54	1.80	1.30	-0.38	-1.40	N
55	2.16	1.33	-0.03	-1.40	E
56	2.47	1.37	-0.03	-1.40	G
57	2.59	1.44	-0.03	-1.40	L
58	2.48	1.50	-0.37	-1.40	I
59	1.82	1.50	-0.69	-1.40	G
60	1.12	1.50	-0.37	-1.40	A
61	0.65	1.46	-0.69	-1.40	B
62	0.30	1.46	-1.33	-1.40	L
63	0.25	1.40	-1.37	-1.40	E
BM	0.00	1.48	-2.60	-1.40	0.08

APPENDIX B

FORTRAN CODE OF PROGRAMS GRAVEL, GINDEP, AND 3D

B.1 CODE FOR PROGRAM GRAVEL

B.2 CODE FOR PROGRAM GINDEP

B.3 CODE FOR PROGRAM 3D

B.1

FORTRAN CODE OF PROGRAM GRAVEL:

A 3-D FORWARD GRAVITY MODELING PROGRAM
USING BLOCKS (MAX.=100) OF EARTH

```

C
C
C... GRAVBL IS A FORWARD GRAVITY PROGRAM
C... THAT MODELS THE EARTH BY USING BLOCKS
C... (UP TO 500) OF VARIABLE DENSITY.
C
C
C*****
C
C..                                VARIABLE LIST
C
C
C..GEOMETRY VARIABLES
C   ** ALL DISTANCE UNITS IN METERS **
C-----
C
C.. NBLKS = NUMBER OF BLOCKS IN MODEL J=1,NBLKS
C.. XL(J) = LEFT (OR MIN. X) EDGE OF BLOCK J
C.. XR(J) = RIGHT (OR MAX. X) EDGE OF BLOCK J
C.. ZU(J) = TOP (OR MIN. Z, Z POS. DOWN)
C...      EDGE OF BLOCK J
C.. ZD(J) = BOTTOM (OR MAX. Z, Z POS. DOWN)
C...      EDGE OF BLOCK J
C   -WHERE ZU AND ZD ARE THE POSITIVE DISTANCE
C   DOWN FROM THE DATUM ELEVATION CONSIDERING
C   THE DATUM ELEVATION ZERO
C
C.. LAMNUM(J) = NUMBER OF LAMINAE USED TO FORM
C               BLOCK J
C.. Y(I,J) = Y POSITION OF LAMINAE I ON BLOCK J
C..         I=1,LAMNUM(J)
C.. DEN(J) = DENSITY OF BLOCK J
C
C
C..GRID VARIABLES
C   (PREDICTED GRAVITY AS OBSERVED FROM GRID)
C-----
C
C.. XSIZE = SIZE OF GRID IN X-DIRECTION
C.. YSIZE = SIZE OF GRID IN Y-DIRECTION
C.. COLS = NUMBER OF COLUMNS IN GRID
C.. ROWS = NUMBER OF ROWS IN GRID
C.. ELEV(I) = ELEVATION OF GRID POINT I
C   -- READ IN FROM SURFII SAVED FILE
C.. DATELV = DATUM ELEVATION
C.. XPOS,YPOS - CURRENT X,Y POS. OF OBSERVATION
C               ON GRID
C
C
C..                                CALCULATION VARIABLES
C-----
C
C.. ANOM(I,J) = ANOMALY CALCULATED FOR
C               LAMINAE I OF BLOCK J

```

```

C.. V(J) = ANOMALY OF BLOCK J RESULTING
C          FROM A QUADRATURE FORMULA
C          SUMMATION OF THE CONTRIBUTIONS
C          OF EACH LAMINAE IN BLOCK J
C
C.. TANOM(J) = VARIABLE USED FOR THE SUMMATION
C          OF THE CONTRIBUTIONS OF EACH BLOCK IN
C          THE MODEL AT OBSERVATION POINT XPOS,YPOS
C          ON THE GRID. TANOM(NBLKS) IS THEREFORE
C          THE TOTAL ANOMALY OF THE PRESENT MODEL
C          AT OBSERVATION POINT XPOS,YPOS ON THE GRID.
C
C
C..          MISCELLANEOUS VARIABLES
C          -----
C
C.. MP = VALUE THAT CONTROLS HOW MANY LAMINAE WILL
C          BE USED TO DEFINE A BLOCK.
C
C          COMMON /GRAV/MP,XSIZE,YSIZE,COLS,ROWS,XL(500),
C          &XR(500),ZD(500),ZU(500),DEN(500),Y(500,500),
C          &ELEV(500),LAMNUM(500),V(500),ANOM(500,500),
C          &NBLKS,XPOS,YPOS,TANOM(500),M1,YMAX(500)
C
C          CHARACTER*20 FILENAME
C
C...  MODEL INPUT
C
C          OPEN(22,FILE='GRAVBL.DAT',STATUS='OLD')
C          OPEN(23,FILE='GBL.DAT',STATUS='NEW')
C          OPEN(24,FILE='GEOM.DAT',STATUS='NEW')
C
C          WRITE(24,('( ' BLOCK      XL      XR      ZU      ZD
C          &      YMIN      YMAX      DEN '))')
C          WRITE(24,('( '-----
C          &-----' '))')
C          READ(22,*)NBLKS
C          DO J=1,NBLKS
C          READ(22,*)XL(J),XR(J),ZU(J),ZD(J),Y(1,J),Y(2,J),
C          &DEN(J)
C
C          YMAX(J)=Y(2,J)
C          WRITE(24,('(14,7F10.1')J,XL(J),XR(J),ZU(J),ZD(J)
C          &,Y(1,J),Y(2,J),DEN(J)
C          END DO
C
C          READ(22,*)XSIZE,YSIZE,COLS,ROWS
C          READ(22,*)MP
C
C...  INPUT FOR TOPOGRAPHY: IF WANT TOPOGRAPHY SET LL=1
C...          ELSE SET LL .NE. 1
C
C          READ(22,*)LL
C          IF(LL.EQ.1)THEN

```

```

C
C... READ DATUM ELEVATION
C
      READ(22,*)DATELV
C
C... ENTER FILENAME OF SAVED SURFII DATA FILE
C... CONTAINING GRID LOCATIONS AND GRID POINT
C... ELEVATIONS
C
C
      READ(22,'(A20)')FILENAME
C
      OPEN(27,FILE=FILENAME,STATUS='OLD',FORM='UNFORMATTED',
&ACCESS='SEQUENTIAL')
C
C
C... READ IN FIRST RECORD
C
      READ(27)NCOLS,NROWS,IZERO
C
C
C... READ IN SECOND RECORD
C
      READ(27)IROWS,JCOLS,DIFY,DIFX,XMN,XXM,YMN,
&YMX,DUMMY
C
C... READ IN ELEVATIONS (METERS)
C
      DO I=NROWS,1,-1
      IBEG=(I-1)*NCOLS+1
      IEND=IBEG+NCOLS-1
      READ(27)(ELEV(J),J=IBEG,IEND)
      END DO
C
      DO I=1,NROWS*NCOLS
      ELEV(I)=ELEV(I)-DATELV
      END DO
C
C
      DO J=1,NBLKS
      ZU(J)=ZU(J)+ELEV(1)
      ZD(J)=ZD(J)+ELEV(1)
      END DO
C
      END IF
C
      DO I=1,NBLKS
          CALL REORDER(I)
C
C... MAKE SURE LAMNUM(I) IS AN ODD NUMBER SO
C... THAT QUADRATURE FORMULA WILL WORK
C
      LI=LAMNUM(I)/2
      T1=(LAMNUM(I)/2.)-LI

```

```

      IF(LAMNUM(I).GT.2)THEN
      IF(T1.EQ.0.0)THEN
      Y(LAMNUM(I)+1,I)=Y(LAMNUM(I),I)
      Y(LAMNUM(I),I)=Y(LAMNUM(I),I)-1.
      CALL FORWARD(LAMNUM(I),I)
      LAMNUM(I)=LAMNUM(I)+1
      END IF
      END IF
      PRINT *,I,' LAMNUM=',LAMNUM(I)
      END DO

C
      M1=0
      M=1
      CHECK=0.0
      XDELTA=XSIZE/(COLS-1)
      YDELTA=YSIZE/(ROWS-1)
      PRINT *, ' '
      PRINT *, 'GRID INFORMATION (UNITS=METERS)'
      PRINT *, '-----'
      PRINT *, 'XSIZE=',XSIZE,'YSIZE=',YSIZE
      PRINT *, 'COLS','COLUMNS',' ',ROWS,'ROWS'
      PRINT *, 'DIST. BETWN. COLS.=',XDELTA
      PRINT *, 'DIST. BETWN. ROWS =',YDELTA
      PRINT *, ' '

C
10  IF(M.EQ.1)THEN
      XPOS=0.0
      YPOS=0.0
      TANOM(1)=0.0

C
      DO J=2,NBLKS+1

C
          CALL QUAD(J-1)
          TANOM(J)=TANOM(J-1)+V(((LAMNUM(J-1)-1)/2)+1)
          END DO
          WRITE(23,'(3F12.4)')XPOS,YPOS,TANOM(NBLKS+1)
          M=M+1
          GO TO 10
          ELSE
          DO I=1,COLS*ROWS-1
          CALL GRID(I,CHECK,XDELTA,YDELTA)
          DO J=2,NBLKS+1

C
C... IF CHOSE TOPOGRAPHY LL=1
C
          IF(LL.EQ.1)THEN
          ZU(J-1)=ZU(J-1)+ELEV(I+1)-ELEV(I)
          ZD(J-1)=ZD(J-1)+ELEV(I+1)-ELEV(I)
          END IF

C
          CALL FORWARD(1,J-1)
          DO K=2,LAMNUM(J-1)
          CALL FORWARD(K,J-1)
          END DO

```

```

      CALL QUAD(J-1)
      TANOM(J)=TANOM(J-1)+V(((LAMNUM(J-1)-1)/2)+1)
      END DO
      WRITE(23,'(3F12.4)')XPOS,YPOS
      &,TANOM(NBLKS+1)
      END DO
      END IF
C
      CLOSE(22)
      CLOSE(23)
C
      PRINT *, '
      PRINT *, ' OUTPUT IN FILE GBL.DAT!'
      PRINT *, '
      CALL EXIT
      END
C
C
C*****
C
C...      SUBROUTINES
C
C
C.. SUBROUTINE REORDER DETERMINES HOW EACH
C.. BLOCK WILL BE CUT UP IN THE Y-DIRECTION.
C.. ENOUGH LAMINAE WILL BE INSERTED SO THAT
C.. THE GRAVITATION ATTRACTION BETWEEN
C.. ADJACENT LAMINAE DOES NOT VARY BY MORE
C.. THEN AN INPUT PERCENTAGE (DEF. BY VAR. MP).
C
      SUBROUTINE REORDER(I)
C
      COMMON /GRAV/MP,XSIZE,YSIZE,COLS,ROWS,XL(500)
      &,XR(500),ZD(500),ZU(500),DEN(500),Y(500,500),
      &ELEV(500),LAMNUM(500),V(500),ANOM(500,500),NBLKS
      &,XPOS,YPOS,TANOM(500),M1,YMAX(500)
C
C
C.. CUT UP EACH BLOCK LAMNUM(I) TIMES
C
      K1=ABS(Y(2,I)-Y(1,I))
      K2=ABS(XR(I)-XL(I))
      LAMNUM(I)=(K1*MP*(JMAXO(K1,K2)/JMINO
      &(K1,K2)))/100
      IF(LAMNUM(I).LT.10)LAMNUM(I)=10
C
      IF(LAMNUM(I).GE.500)LAMNUM(I)=497.
C
      YDIST=K1/FLOAT(LAMNUM(I))
      CALL FORWARD(1,I)
      LAMNUM(I)=LAMNUM(I)+1
      DO K=2,LAMNUM(I)
      Y(K,I)=Y(K-1,I)+YDIST
      CALL FORWARD(K,I)

```



```

      END DO
C
      RETURN
      END
C
C
C... SUBROUTINE FORWARD CALCULATES THE GRAVITY
C... ANOMALY/UNIT WIDTH FOR EACH LAMINAE I IN BLOCK J
C
      SUBROUTINE FORWARD(I,J)
C
      COMMON /GRAV/MP,XSIZE,YSIZE,COLS,ROWS,XL(500),
&XR(500),ZD(500),ZU(500),DEN(500),Y(500,500),
&ELEV(500),LAMNUM(500),V(500),ANOM(500,500),
&NBLKS,XPOS,YPOS,TANOM(500),M1,YMAX(500)
C
C
      GO=6.67E-8
C
C... START CALCULATIN'
C
C... PREVENT DIVIDE BY ZERO
C
      IF(XR(J).EQ.0.0)XR(J)=1E-20
      IF(XL(J).EQ.0.0)XL(J)=1E-20
      IF(Y(I,J).EQ.0.0)Y(I,J)=1E-20
C
      A0=(ZD(J)*ZD(J))+(Y(I,J)*Y(I,J))
      B=(ZU(J)*ZU(J))+(Y(I,J)*Y(I,J))
      A1=XL(J)+SQRT((XL(J)*XL(J))+B)
      A2=XR(J)+SQRT((XR(J)*XR(J))+B)
      B1=XR(J)+SQRT((XR(J)*XR(J))+A0)
      B2=XL(J)+SQRT((XL(J)*XL(J))+A0)
C
      IF((A1.EQ.0.0).OR.(A2.EQ.0.0).OR.(B1.EQ.0.0)
&.OR.(B2.EQ.0.0))M1=M1+1
C
C... PREVENT TAKING LOG OF 0.0 OR NEGATIVE NUMBER
C
      IF(A1.LE.0.0)A1=.001
      IF(A2.LE.0.0)A2=.001
      IF(B1.LE.0.0)B1=.001
      IF(B2.LE.0.0)B2=.001
      IF((A1.EQ.0.001).OR.(A2.EQ.0.001).OR.(B1.EQ.0.001)
&.OR.(B2.EQ.0.001))THEN
        PRINT *,'HAVE HIT A ZERO OR NEGATIVE NUMBER;
&THIS NUMBER HAS BEEN CHANGED TO .001 SO THAT
&TAKING ITS LOG IS POSSIBLE.'
        PRINT *,'YOU MAY WANT TO CHANGE THE GEOMETRY
&OF BLOCK :',J
      END IF
C
C... THE 100000 IS TO GET THE ANOMALY IN UNITS OF MGALS
C

```

```

      ANOM(I,J)=-100000.*G0*DEN(J)*(LOG(A1)
&+LOG(B1)-LOG(A2)-LOG(B2))
C
      RETURN
      END
C
C
C...  SUBROUTINE GRID GIVES X,Y GRID POSITIONS
C
C
      SUBROUTINE GRID(KOUNT,CHECK,XDELTA,YDELTA)
C
      COMMON /GRAV/MP,XSIZE,YSIZE,COLS,ROWS,XL(500),
&XR(500),ZD(500),ZU(500),DEN(500),Y(500,500),
&ELEV(500),LAMNUM(500),V(500),ANOM(500,500),
&NBLKS,XPOS,YPOS,TANOM(500),M1,YMAX(500)
C
      NI=KOUNT/COLS
      XPOS=KOUNT*XDELTA-(NI*XSIZE+XDELTA*NI)
      IF(ABS(XPOS).LE.1.0E-01)XPOS=0.00E+00
      YPOS=NI*YDELTA
      IF(XSIZE.GT.CHECK)THEN
        CHECK=CHECK+XDELTA
        DO J=1,NBLKS
          XL(J)=XL(J)-XDELTA
          XR(J)=XR(J)-XDELTA
        END DO
      ELSE
        CHECK=0.0
        DO J=1,NBLKS
C
          XL(J)=XL(J)+XDELTA*(COLS-1)
          XR(J)=XR(J)+XDELTA*(COLS-1)
C
          DO I=1,LAMNUM(J)
            Y(I,J)=Y(I,J)-YDELTA
          END DO
        END DO
      END IF
      RETURN
      END
C
C
C...  SUBROUTINE QUAD CALCULATES THE ANOMALY
C...  ASSOCIATED WITH EACH BLOCK K BY DOING
C...  THE INTEGRATION IN THE Y-DIRECTION USING
C...  THE QUADRATURE FORMULA TALWANI USED.
C
C
      SUBROUTINE QUAD(K)
C
      COMMON /GRAV/MP,XSIZE,YSIZE,COLS,ROWS,XL(500),
&XR(500),ZD(500),ZU(500),DEN(500),Y(500,500),
&ELEV(500),LAMNUM(500),V(500),ANOM(500,500),

```

```

&NBLKS,XPOS,YPOS,TANOM(500),M1,YMAX(500)
C
  DIMENSION TEMP(500)
C
C...  FIND THE ANOMALY
C
  IF(LAMNUM(K)-1.GT.1)THEN
    I=1
    V(1)=0.0
    DO J=2,((LAMNUM(K)-1)/2)+1
C
      AO=ANOM(I,K)*((Y(I,K)-Y(I+2,K))/(Y(I,K)-
&Y(I+1,K)))*(3.0*Y(I+1,K)-Y(I+2,K)-2.0*Y(I,K))
C
      B=ANOM(I+1,K)*((Y(I,K)-Y(I+2,K))*3)/((
&Y(I+1,K)-Y(I+2,K))*(Y(I+1,K)-Y(I,K)))
C
      C=ANOM(I+2,K)*((Y(I,K)-Y(I+2,K))/(Y(I+2,K)
&-Y(I+1,K)))*(3.0*Y(I+1,K)-Y(I,K)-2.*Y(I+2,K))
C
      TEMP(J-1)=(AO+B+C)/6.0
C
      V(J)=V(J-1)+TEMP(J-1)
      I=I+2
    END DO
C
  ELSE
C
C...  TRAPEZOIDAL RULE FOR ONE AREA
C
    V(((LAMNUM(K)-1)/2)+1)=(Y(2,K)-Y(1,K))*0.5
    &*(ANOM(1,K)+ANOM(2,K))
    END IF
C
  RETURN
  END

```

B.2

FORTRAN CODE OF PROGRAM GINDEP:

A 3-D GRAVITY MODELING PROGRAM
THAT INVERTS ON MAXIMUM DEPTHS
OF BLOCKS OUTPUT FROM PROGRAM GRAVBL

```

C
C... PROGRAM GINDEP IS AN INVERSE GRAVITY
C... MODELING PROGRAM THAT PREDICTS LOWER
C... BLOCK DEPTHS THAT WILL GIVE A BEST FIT
C... TO OBSERVED GRAVITY DATA BASED ON FORWARD
C... MODELING THAT USES UP TO 100 RECTANGULAR
C... BLOCKS OF EARTH. RIDGE REGRESSION,
C... WEIGHTING AND SCALING ARE USED
C
C*****
C
C..          VARIABLE LIST
C
C
C.. GEOMETRY VARIABLES
C          ** ALL DISTANCE UNITS IN METERS **
C          -----
C
C.. NBLKS = NUMBER OF BLOCKS IN MODEL J=1,NBLKS
C.. XL(J) = LEFT (OR MIN. X) EDGE OF BLOCK J
C.. XR(J) = RIGHT (OR MAX. X) EDGE OF BLOCK J
C.. ZU(J) = TOP (OR MIN. Z, Z POS. DOWN) EDGE OF
C           BLOCK J
C.. ZD(J) = BOTTOM (OR MAX. Z, Z POS. DOWN) EDGE OF
C           BLOCK J
C           -WHERE ZU AND ZD ARE THE POSITIVE DISTANCE
C           DOWN FROM THE DATUM ELEVATION CONSIDERING
C           THE DATUM ELEVATION ZERO
C
C.. LAMNUM(J) = NUMBER OF LAMINAE USED TO FORM BLOCK J
C.. Y(I,J) = Y POSITION OF LAMINAE I ON BLOCK J
C..          I=1,LAMNUM(J)
C.. DEN(J) = DENSITY OF BLOCK J
C
C
C.. GRID VARIABLES (PREDICTED GRAVITY AS OBSERVED
C           FROM GRID)
C           -----
C
C.. XSIZE = SIZE OF GRID IN X-DIRECTION
C.. YSIZE = SIZE OF GRID IN Y-DIRECTION
C.. COLS = NUMBER OF COLUMNS IN GRID
C.. ROWS = NUMBER OF ROWS IN GRID
C.. ELEV(I) = ELEVATION OF GRID POINT I -- READ
C           IN FROM SURFII SAVED FILE
C.. DATELV = DATUM ELEVATION
C.. XPOS,YPOS - CURRENT X,Y POSITION OF OBSERVATION
C           ON GRID
C
C
C..          CALCULATION VARIABLES
C           -----
C
C.. ANOM(I,J) = ANOMALY CALCULATED FOR LAMINAE I

```

```

C              OF BLOCK J
C... V(J) = ANOMALY OF BLOCK J RESULTING FROM A
C              QUADRATURE FORMULA SUMMATION OF THE
C              CONTRIBUTIONS OF EACH LAMINAE IN BLOCK J
C
C... TANOM(J) = VARIABLE USED FOR THE SUMMATION OF
C              THE CONTRIBUTIONS OF EACH BLOCK IN
C              THE MODEL AT OBSERVATION POINT XPOS,
C              YPOS ON THE GRID. TANOM(NBLKS) IS
C              THEREFORE THE TOTAL ANOMALY OF THE
C              PRESENT MODEL AT OBSERVATION
C              POINT XPOS,YPOS ON THE GRID.
C
C
C... MISCELLANEOUS VARIABLES
C-----
C
C... MAXPER = VALUE THAT CONTROLS HOW MANY LAMINAE WILL
C              BE USED TO DEFINE A BLOCK.
C
C              COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
C              &DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
C              &NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
C              &XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
C              COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,KVAR
C
C              CHARACTER CHAR*1
C
C              DIMENSION A(500,500),DPRED(500,500),DAMP(100),
C              &NF(50)
C
C              DIMENSION WDEP(250,250),DELT(400,1),DOBS(400),
C              &SQE(20,1),WS(250,250),WKAREA(10500),C(250,250),
C              &GINV(250,250),DELTN(250,1),DELTM(250,1),SQEN(20,1),
C              &ERR(20,1),COVM(250,250),SIGMAM(250),CA(250,250),
C              &D(250,250),WSS(250,250),WERR(250,250)
C
C              CHARACTER*20 FILENAME,FILNAM
C*****
C
C              INPUT MODEL DATA FOR FORWARD PROGRAM
C              DATA IN COMAND FILE G.DAT
C
C              OPEN(26,FILE='G.DAT',STATUS='OLD')
C
C... READ FILENAME OF SAVED SURFII GRIDFILE
C
C              READ(26,'(A20)')FILENAME
C              READ(26,*)TDEPL
C              READ(26,*)TDEPH
C
C... READ NUMBER OF BLOCK(S) TO FIX AT DEPTH GIVEN
C... IN THE FOWARD MODEL

```

```

C
  READ(26,*)NUMFIX
  IF(NUMFIX.GT.0)READ(26,*)(NF(I),I=1,NUMFIX)
C
  READ(26,*)NBLKS
  DO J=1,NBLKS
    READ(26,*)XL(J),XR(J),ZU(J),ZD(J,1),Y(1,J),
&Y(2,J),DEN(J)
    YMIN(J)=Y(1,J)
    YMAX(J)=Y(2,J)
  END DO
  READ(26,*)MAXPER
C
C  INPUT DATA FOR INVERSION
C
  READ(26,*)RATIO
  READ(26,*)STDDEV
  READ(26,*)ITMAX
C
C... IF WANT TOPOGRAPHY SET LL=1, ELSE SET LL .NE. 1
C
  READ(26,*)LL
  IF(LL.EQ.1)THEN
C
C
C*****
C
C          INPUT FOR TOPOGRAPHY
C
C... INPUT DATAUM ELEVATION FOR SURVEY
C
  READ(26,*)DATELV
C
C*****
C
C  INPUT - ELEVATION OF OBSERVATION POINTS (GRAVITY
C          STATIONS) FROM SAVED SURFII GRIDFILE
C
C... ENTER FILENAME OF SAVED SURFII GRIDFILE
C
  READ(26,'(A20)')FILNAM
C
  OPEN(28,FILE=FILNAM,STATUS='OLD',FORM='UNFORMATTED'
& ,ACCESS='SEQUENTIAL')
C
C
C... READ IN FIRST RECORD
C
  READ(28)NCOLS,NROWS,IZERO
C
C... READ IN SECOND RECORD
C
  READ(28)IROWS,JCOLS,DIFY,DIFX,XMN,XXM,YMN,YMX,DUMMY

```

```

C
C... READ IN ELEVATIONS
C
    DO I=NROWS,1,-1
    IBEG=(I-1)*NCOLS+1
    IEND=IBEG+NCOLS-1
    READ(28)(ELEV(J),J=IBEG,IEND)
    END DO
C
    DO I=1,NCOLS*NROWS
    ELEV(I)=ELEV(I)-DATELV
    END DO
C
    END IF
C
C
C*****
C
C    INPUT-OBSERVED GRAVITY DATA
C    FROM SAVED SURFII GRIDFILE
C
C
C... ENTER FILENAME OF SAVED SURFII DATA FILE
C... IN FILE G.DAT
C
    OPEN(27,FILE=FILENAME,STATUS='OLD',FORM=
    &'UNFORMATTED',ACCESS='SEQUENTIAL')
C
C
C... READ IN FIRST RECORD
C
    READ(27)NCOLS,NROWS,IZERO
C
C... READ IN SECOND RECORD
C
    READ(27)IROWS,JCOLS,DIFY,DIFX,XMN,XX,YYN,YYX,
    &DUMMY
C
C... READ IN OBS DATA
C
    DO I=NROWS,1,-1
    IBEG=(I-1)*NCOLS+1
    IEND=IBEG+NCOLS-1
    READ(27)(DOBS(J),J=IBEG,IEND)
    END DO
C
    XSIZE=ABS(XX-XMN)
    YSIZE=ABS(YY-YYN)
C
    CLOSE(27)
    NDAT=NCOLS*NROWS
C
    NPARMS=NBLKS
C

```


C

```

OPEN(16,FILE='GINDEP.OUT',STATUS='NEW')
WRITE(16,('( '                                ' '))')
WRITE(16,('( '*****' ' '))')
&*****' '))
WRITE(16,('( '                                ' '))')
WRITE(16,('( '                                GRID INFO' '))')
WRITE(16,('( '                                ' '))')
WRITE(16,('( '-----' ' '))')
&-----' '))
WRITE(16,('( '$GRID SIZE IN X DIRECTION
&(METERS)=' ',E12.4)' '))XSIZE
WRITE(16,('( '$GRID SIZE IN Y DIRECTION
&(METERS)=' ',E12.4)' '))YSIZE
WRITE(16,('( '$NUMBER OF COLUMNS=' ',I3)' '))NCOLS
WRITE(16,('( '$NUMBER OF ROWS=' ',I3)' '))NROWS
WRITE(16,('( '-----' ' '))')
&-----' '))
WRITE(16,('( '                                ' '))')
WRITE(16,('( '-----' ' '))')
&-----' '))
WRITE(16,('( '                                ' '))')
WRITE(16,('( '                                INPUT MODEL' '))')
WRITE(16,('( '                                ' '))')
WRITE(16,('( '-----' ' '))')
&-----' '))
WRITE(16,('( '    BLOCK #    XMIN    XMAX
&YMIN    YMAX    ZMIN    ZMAX    DENSITY' '))')
DO J=1,NPARMS
WRITE(16,('( 3X,I3,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,
&2X,F7.1,2X,F7.1,2X,F7.2)' '))
&J,XL(J),XR(J),Y(1,J),Y(2,J),ZU(J),ZD(J,1),DEN(J)
END DO
WRITE(16,('( '*****' ' '))')
&*****' '))
WRITE(16,('( '                                ' '))')

```

C

```

WRITE(16,('( '                                ' '))')
WRITE(16,('( '*****' ' '))')
&*****' '))
WRITE(16,('( '                                ' '))')
WRITE(16,('( '    TABLE OF CONVERGENCE' '))')
WRITE(16,('( '                                ' '))')
WRITE(16,('( '-----' ' '))')
&-----' '))

```

C

CLOSE(28)

C

```

IF(LL.EQ.1)THEN
DO J=1,NBLKS
ZU(J)=ZU(J)+ELEV(1)
ZD(J,1)=ZD(J,1)+ELEV(1)
END DO
END IF

```

```

C
C
    IA=250
    IB=250
    IC=250
    CDAMP=.001
C
    KEPTRK=1
    KOUNT=0
    KVAR=0
C
    CALL FWRD(A,DPRED)
C
5   KOUNT=KOUNT+1
    IF(KOUNT.GT.ITMAX)THEN
        PRINT *, 'MAXIMUM NUMBER OF ITERATIONS EXCEEDED -
& CONVERGENCE NOT OBTAINED'
        GO TO 20
    END IF
C
C... FIND A MATRIX
C
        CALL DERIV(A)
C
C... FIND DEPTH WEIGHTING MATRIX
C
    IF(RATIO.EQ.0.0)RATIO=1E-04
    DO I=1,NPARMS
        DO J=1,NPARMS
            IF(I.EQ.J)THEN
                WDEP(I,J)=(1./(ABS(ZD(J,KEPTRK))+10E-12))+1.
                IF(ZD(J,KEPTRK).LT.TDEPL)THEN
                    ZD(J,KEPTRK)=TDEPL
                    DAMP(I)=15E+1
                ELSE IF(ZD(J,KEPTRK).GT.TDEPH)THEN
                    ZD(J,KEPTRK)=TDEPH
                    DAMP(I)=15E+1
                END IF
            ELSE
                WDEP(I,J)=0.0
            END IF
        END DO
    END DO
C
C... FIND A*WDEP*A(TRANPOSE) MATRIX FOR NOISE
C... WEIGHTING
C
    CALL VMULFF(A,WDEP,NDAT,NPARMS,NPARMS,500,
&IB,C,IC,IER)
    CALL VMULFP(C,A,NDAT,NPARMS,NDAT,IA,500,
&WERR,IC,IER)
C
    DO I=1,NDAT
        DO J=1,NDAT

```

```

        IF(I.NE.J)WERR(I,J)=0.0
        END DO
        END DO
C
        CALL VMULFM(A,WERR,NDAT,NPARMS,NDAT,500,
&IB,C,IC,IER)
        CALL VMULFF(C,A,NPARMS,NDAT,NPARMS,IA,
&500,WS,IC,IER)
C
C... SCALE THE A(TRANPOSE)WA MATRIX FOR ONES
C... ON THE DIAGONAL
C
        DO I=1,NPARMS
        DO J=1,NPARMS
        IF(I.EQ.J)THEN
        D(I,J)=1./((SQRT(WS(I,I)))
        ELSE
        D(I,J)=0.0
        END IF
        END DO
        END DO
C
        CALL VMULFF(D,WS,NPARMS,NPARMS,NPARMS,IA,IB,
&CA,IC,IER)
        CALL VMULFF(CA,D,NPARMS,NPARMS,NPARMS,IA,IB,
&WSS,IC,IER)
C
C... ADD THE "NOISE/SIGNAL" RATIO
C
        DO I=1,NPARMS
        WSS(I,I)=WSS(I,I)+RATIO*RATIO
        END DO
C
        DO I=1,NDAT
        DELT(I,1)=DOBS(I)-DPRED(I,KEPTRK)
        END DO
C
C... FIND ERROR
C
        CALL VMULFM(DELT,DELT,NDAT,1,1,400,400,
&SQE,20,IER)
        WRITE(16,(''$ITERATION #'',I3)')KOUNT
        WRITE(16,(''$STARTING DATA ERROR='',E12.4)')
&SQRT(SQE(1,1)/(NDAT-NPARMS))
C
C***** MARQUARDT LOOP *****
C
10      IF(CDAMP.GE.100000)THEN
        WRITE(16,(''$DAMPING FACTOR HAS REACHED 100000
& - WILL NOT CONVERGE TO PRESENT INPUT DATA STD.
&DEV.''))
        CALL EXIT
        END IF
C

```



```

C      IF(ZD(I,KEPTRK).EQ.TDEPL.AND.DAMP(I)
&.EQ.15E+1)THEN
      IF(DELM(I,1).GT.0.0)DAMP(I)=CDAMP
C
      ELSE IF(ZD(I,KEPTRK).EQ.TDEPH.AND.DAMP(I)
&.EQ.15E+1)THEN
      IF(DELM(I,1).LT.0.0)DAMP(I)=CDAMP
      END IF
C
      WRITE(16,'(1X,I3,3X,E12.4,1X,E12.4)')I,
&DELM(I,1),ZD(I,KEPTRK+1)
      END DO
C
      WRITE(16,'(''$DAMPING (MARQUARDT) VALUE='',E12.4)')
&CDAMP
C
C... NOW HAVE NEW PARAMS.
C
      KEPTRK=KEPTRK+1
      CALL FWRD(A,DPRED)
C
      DO I=1,NDAT
      DELTN(I,1)=DOBS(I)-DPRED(I,KEPTRK)
      END DO
C
C... DO NEW PARAMS REDUCE ERROR?
C
      CALL VMULFM(DELTN,DELTN,NDAT,1,1,IA,
&IR,SQEN,20,IER)
      WRITE(16,'(''$NEW DATA ERROR='',E12.4)')
&SQRT(SQEN(1,1)/(NDAT-NPARMS))
C
      IF(SQEN(1,1).GE.SQE(1,1))THEN
C
      DO I=1,NPARMS
      ZD(I,KEPTRK)=ZD(I,KEPTRK-1)
      END DO
      CDAMP=10.*CDAMP
      GO TO 10
      ELSE
12  PERCENT=((SQRT(SQE(1,1)/(NDAT-NPARMS))
&-SQRT(SQEN(1,1)/(NDAT-NPARMS)))/SQRT(SQE(1,1)
&/(NDAT-NPARMS)))*100.
C
C... ERROR MUST BE IMPROVED BY AT LEAST ONE PERCENT FOR
C... NEW ITERATION
C
      IF(PERCENT.LT.1.)THEN
      CDAMP=10.*CDAMP
      GO TO 10
      END IF
C
      SQE(1,1)=SQEN(1,1)

```

```

      CDAMP=CDAMP/10.
      DO I=1,NDAT
      DELT(I,1)=DELTN(I,1)
      END DO
      END IF

C
C... CHECK TO SEE IF SOLN. IS WITHIN STD. DEV.
C... OF DATA
C
      CALL VMULFM(DELT,DELT,NDAT,1,1,400,400,ERR,
&20,IER)
      CHIERR=(1./(NDAT-NPARMS))*ERR(1,1)

C
C
C
      IF(SQRT(CHIERR).LE.STDDEV)GO TO 20
      WRITE(16,('( ' ' '))')
      GO TO 5

C
C***** OUTPUT *****
C
20  WRITE(16,('( '-----
&-----'))')
      WRITE(16,('( ' ' '))')
      WRITE(16,('( '*****
&*****'))')
      WRITE(16,('( ' ' '))')
      WRITE(16,('( '*****
&*****'))')
      WRITE(16,('( ' ' '))')
      WRITE(16,('( ' OUTPUT-PROGRAM GINDEP'))')
      WRITE(16,('( ' ' '))')
      CALL VMULFP(GINV,GINV,NPARMS,NDAT,NPARMS,IA,IB,
&C,IC,IER)
      DO I=1,NPARMS
      DO J=1,NPARMS
      COVM(I,J)=CHIERR*C(I,J)
      END DO
      END DO

C
      DO I=1,NPARMS
      SIGMAM(I)=SQRT(COVM(I,I))
      END DO

C
      WRITE(16,('( '-----
&-----'))')
      WRITE(16,('( ' ' '))')
&
      WRITE(16,('( 'I ZD(I) +/- STD.DEV.))')
      WRITE(16,('( '-----
&-----'))')
      DO I=1,NPARMS
      WRITE(16,('(1X,I2,3X,E10.3,2X,E10.3))')
&I,ZD(I,KEPTRK),SIGMAM(I)

```

```

      END DO
      WRITE(16, '(' '-----
&-----')')
C
      WRITE(16, '(' '-----
&-----')')
      WRITE(16, '(' '$INPUT STD. DEV. = ', E12.4)') STDDEV
      WRITE(16, '(' '$CALC. DATA STD. DEV. = ', E12.4)')
&SQRT(CHIERR)
      WRITE(16, '(' '$NUMBER OF ITERATIONS FOR SOLUTION =
& ', I3)') KOUNT
      WRITE(16, '(' '-----
&-----')')
C
      WRITE(16, '(' '-----
&-----')')
      WRITE(16, '(' ' MODEL RESOLUTION MATRIX')')
      CALL VMULFF(GINV, A, NPARMS, NDAT, NPARMS, IA, 500, C,
&IC, IER)
      WRITE(16, '(' '
                                ')')
C
      IF(NPARMS.GT.15)THEN
        WRITE(16, '(' (1X, A1, 11X, A10, 7X, A5, 7X, A10)')') 'I',
&'...(I, I-2)...', '(I, I)', '(I, I+2)...'
        WRITE(16, '(' (1X, I3, 1X, A1, 22X, 3F6.2, 10X, A1)')')
&1, '|', (C(1, J), J=1, 3), '| '
        WRITE(16, '(' (1X, I3, 1X, A1, 16X, 4F6.2, 10X, A1)')')
&2, '|', (C(2, J), J=1, 4), '| '
        DO I=3, NPARMS
          WRITE(16, '(' (1X, I3, 1X, A1, 10X, 5F6.2, 10X, A1)')') I, '|',
&(C(I, J), J=I-2, I+2), '| '
          IF(I+2.EQ.NPARMS)THEN
            WRITE(16, '(' (1X, I3, 1X, A1, 10X, 4F6.2, 16X, A1)')')
&NPARMS-1, '|', (C(NPARMS-1, J), J=NPARMS-3, NPARMS), '| '
            WRITE(16, '(' (1X, I3, 1X, A1, 10X, 3F6.2, 22X, A1)')')
&NPARMS, '|', (C(NPARMS, J), J=NPARMS-2, NPARMS), '| '
          GO TO 21
        END IF
      END DO
C
      ELSE
C
        DO I=1, NPARMS
          WRITE(16, '(' (1X, A1, <NPARMS>F6.2, 2X, A1)')') '|', (C(I, J)
&, J=1, NPARMS), '| '
        END DO
C
      END IF
C
21 WRITE(16, '(' '-----
&-----')')
      WRITE(16, '(' '-----
&-----')')
      WRITE(16, '(' ' DATA RESOLUTION MATRIX')')

```



```

&NPARMS,'|',(COVM(NPARMS,J),J=NPARMS-2,NPARMS),'|'
  GO TO 23
  END IF
  END DO
  ELSE
C
    DO I=1,NPARMS
      WRITE(16,'(1X,A1,<NPARMS>E10.2,2X,A1)')'|',
&(COVM(I,J),J=1,NPARMS),'|'
      END DO
C
    END IF
C
23  WRITE(16,'(''-----
&-----'')')
C
    WRITE(16,'(''-----
&-----'')')
    WRITE(16,'('' CORRELATION MATRIX'')')
    WRITE(16,'(''
DO I=1,NPARMS
DO J=1,NPARMS
  C(I,J)=COVM(I,J)/(SQRT(COVM(I,I))
&*SQRT(COVM(J,J)))
  END DO
  END DO
C
    IF(NPARMS.GT.15)THEN
      WRITE(16,'(1X,A1,11X,A10,7X,A5,7X,A10)')
&'I','...(I,I-2)...','(I,I)','(I,I+2)...'
      WRITE(16,'(1X,I3,1X,A1,22X,3F6.2,10X,A1)')
&1,'|',(C(1,J),J=1,3),'|'
      WRITE(16,'(1X,I3,1X,A1,16X,4F6.2,10X,A1)')
&2,'|',(C(2,J),J=1,4),'|'
      DO I=3,NPARMS
        WRITE(16,'(1X,I3,1X,A1,10X,5F6.2,10X,A1)')I,'|',
&(C(I,J),J=I-2,I+2),'|'
        IF(I+2.EQ.NPARMS)THEN
          WRITE(16,'(1X,I3,1X,A1,10X,4F6.2,16X,A1)')
&NPARMS-1,'|',(C(NPARMS-1,J),J=NPARMS-3,NPARMS),'|'
          WRITE(16,'(1X,I3,1X,A1,10X,3F6.2,22X,A1)')
&NPARMS,'|',(C(NPARMS,J),J=NPARMS-2,NPARMS),'|'
          GO TO 24
        END IF
      END DO
C
    ELSE
C
      DO I=1,NPARMS
        WRITE(16,'(1X,A1,<NPARMS>F6.2,2X,A1)')'|',(C(I,J)
&,J=1,NPARMS),'|'
      END DO
C
    END IF

```

```

24  WRITE(16, '(''*****'')
    &*****')')
    CLOSE(16)
    PRINT *, ' OUTPUT IN FILE GINDEP.OUT.'
50  CALL EXIT
    END

C
C*****
C  SUBROUTINES
C
C... SUBROUTINE FWRD IS A FORWARD GRAVITY PROGRAM
C... THAT MODELS THE EARTH BY USING BLOCKS (UP
C... TO 100) OF VARIABLE DENSITY.
C
C  SUBROUTINE FWRD(A,DPRED)
C
C  COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
    &DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
    &NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
    &XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
C  COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,KVAR
    DIMENSION A(500,500),DPRED(500,500),B(250,1),
    &G(500,500)
C
C  M1=0
C  M=1
C  CHECK=0.0
C  XDELTA=XSIZE/(NCOLS-1)
C  YDELTA=YSIZE/(NROWS-1)
C
C  IF(KEPTRK.EQ.1.AND.KVAR.NE.20)THEN
    DO I=1,NBLKS
      CALL REORDER(I)
C
C... MAKE SURE LAMNUM(I) IS AN ODD NUMBER SO THAT
C... QUADRATURE FORMULA WILL WORK
C
C  L1=LAMNUM(I)/2
C  T1=(LAMNUM(I)/2.)-L1
C  IF(LAMNUM(I).GT.2)THEN
    IF(T1.EQ.0.0)THEN
      Y(LAMNUM(I)+1,I)=Y(LAMNUM(I),I)
      Y(LAMNUM(I),I)=Y(LAMNUM(I),I)-1.
      CALL FORWARD(LAMNUM(I),I)
      LAMNUM(I)=LAMNUM(I)+1
    END IF
  END IF
  END DO
C
10  IF(M.EQ.1)THEN
    XPOS=0.0
    YPOS=0.0
C

```

```

DO J=2,NBLKS+1
CALL QUAD(J-1)
G(1,J-1)=V(((LAMNUM(J-1)-1)/2)+1)
END DO
M=2
GO TO 10
ELSE
30 DO I=1,NCOLS*NROWS-1
CALL GRID(I,CHECK,XDELTA,YDELTA)
DO J=2,NBLKS+1
C
IF(LL.EQ.1)THEN
ZU(J-1)=ZU(J-1)+ELEV(I+1)-ELEV(I)
ZD(J-1,KEPTRK)=ZD(J-1,KEPTRK)+ELEV(I+1)-ELEV(I)
END IF
C
CALL FORWARD(1,J-1)
DO K=2,LAMNUM(J-1)
CALL FORWARD(K,J-1)
END DO
CALL QUAD(J-1)
G(I+1,J-1)=V(((LAMNUM(J-1)-1)/2)+1)
END DO
END DO
C
C.. FIND THE A MATRIX IK KVAR=20
C
IF(KVAR.EQ.20)THEN
DO I=1,NDAT
DO J=1,NPARMS
A(I,J)=G(I,J)*DEN(J)
IF(A(I,J).EQ.0.0)A(I,J)=1E-10
END DO
END DO
GO TO 7
END IF
C
DO I=1,NPARMS
B(I,1)=DEN(I)
END DO
C
CALL VMULFF(G,B,NDAT,NPARMS,1,500,IB,DPRED,
&500,IER)
C
DO I=1,NDAT
DPRED(I,KEPTRK)=DPRED(I,1)
END DO
C
7 IF(KEPTRK.EQ.1)THEN
C
GO TO 40
END IF
IF(KK.EQ.2)GO TO 40
END IF

```

```

C      ELSE
C
C      KK=1
20    IF(KK.EQ.1)THEN
      XPOS=0.0
      YPOS=0.0
      DO J=2,NBLKS+1
C
C      IF(LL.EQ.1)THEN
      ZU(J-1)=ZU(J-1)+ELEV(1)
      ZD(J-1,KEPTRK)=ZD(J-1,KEPTRK)+ELEV(1)
      END IF
C
C      CALL FORWARD(1,J-1)
      DO K=2,LAMNUM(J-1)
      CALL FORWARD(K,J-1)
      END DO
      CALL QUAD(J-1)
      G(1,J-1)=V(((LAMNUM(J-1)-1)/2)+1)
      END DO
      KK=2
      GO TO 20
      ELSE
      GO TO 30
      END IF
      END IF
C
C
40    DO J=1,NBLKS
      XL(J)=XL(J)+XSIZE
      XR(J)=XR(J)+XSIZE
      IF(LL.EQ.1)THEN
      ZU(J)=ZU(J)-ELEV(NROWS*NCOLS)
      ZD(J,KEPTRK)=ZD(J,KEPTRK)-ELEV(NROWS*NCOLS)
      END IF
      DO K=1,LAMNUM(J)
      Y(K,J)=Y(K,J)+YSIZE
      END DO
      END DO
C
C      RETURN
      END
C
C
C...  SUBROUTINES
C
C
C      SUBROUTINE REORDER(I)
C
C      COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
&DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
&NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
&XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C

```

```

COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,
&KVAR
C
  K1=ABS(Y(2,1)-Y(1,1))
  K2=ABS(XR(1)-XL(1))
  LAMNUM(I)=(K1*MAXPER*(JMAXO(K1,K2)/
&JMINO(K1,K2)))/100
C
  IF(LAMNUM(I).LT.10)LAMNUM(I)=10
C
  IF(LAMNUM(I).GE.500)LAMNUM(I)=497.
C
  YDIST=K1/FLOAT(LAMNUM(I))
  CALL FORWARD(1,I)
  LAMNUM(I)=LAMNUM(I)+1
  DO K=2,LAMNUM(I)+1
    Y(K,I)=Y(K-1,I)+YDIST
  CALL FORWARD(K,I)
  END DO
C
  RETURN
  END
C
C
C... SUBROUTINE FORWARD CALCULATES THE GRAVITY
C... ANOMALY/UNIT WIDTH FOR EACH LAMINAE
C
  SUBROUTINE FORWARD(I,J)
C
  COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
&DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
&NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
&XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
  COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL
&,KVAR
C
  GO=0.0000000667
C
C... START CALCULATIN'
C
C... PREVENT INSTABILITIES
C
  IF(XR(J).EQ.0.0)XR(J)=1E-10
  IF(XL(J).EQ.0.0)XL(J)=1E-10
  IF(Y(I,J).EQ.0.0)Y(I,J)=1E-10
C
C.. FIND DERIVATIVE OF GRAVITY WRT ZD IF KVAR=20
C
  IF(KVAR.EQ.20)THEN
C
    C1=SQRT(XL(J)*XL(J)+ZD(J,KEPTRK)*ZD(J,KEPTRK)
&+Y(I,J)*Y(I,J))
    C2=SQRT(XR(J)*XR(J)+ZD(J,KEPTRK)*ZD(J,KEPTRK)

```

```

      &+Y(I,J)*Y(I,J))
C
      ANOM(I,J)=((1./((XL(J)*C1)+(C1*C1)))-(1./((XR(J)
&*C2)+(C2*C2))))*ZD(J,KEPTRK)*100000.*GO
C
      RETURN
      END IF
C
C
      A0=ZD(J,KEPTRK)*ZD(J,KEPTRK)+Y(I,J)*Y(I,J)
      B=ZU(J)*ZU(J)+Y(I,J)*Y(I,J)
      A1=XL(J)+SQRT(XL(J)*XL(J)+B)
      A2=XR(J)+SQRT(XR(J)*XR(J)+B)
      B1=XR(J)+SQRT(XR(J)*XR(J)+A0)
      B2=XL(J)+SQRT(XL(J)*XL(J)+A0)
      IF((A1.EQ.0.0).OR.(A2.EQ.0.0).OR.(B1.EQ.0.0)
&.OR.(B2.EQ.0.0))M1=M1+1
C
C... PREVENT TAKING LOG OF 0.0
C
      IF(A1.EQ.0.0)A1=.001
      IF(A2.EQ.0.0)A2=.001
      IF(B1.EQ.0.0)B1=.001
      IF(B2.EQ.0.0)B2=.001
C
      IF(A1.EQ.0.001.OR.A2.EQ.0.001.OR.B1.EQ.0.001.OR
&.B2.EQ.0.001)THEN
        PRINT *, ' HAVE HIT A ZERO OR NEGATIVE NUMBER;
&THIS NUMBER HAS BEEN CHANGED TO .001 SO THAT
&TAKING ITS LOG IS POSSIBLE.'
        PRINT *, ' YOU MAY WANT TO CHANGE THE GEOMETRY
&OF BLOCK : ',J
      END IF
C
C... THE 100000 IS TO GET THE ANOMALY IN UNITS OF MGALS
C
      ANOM(I,J)=-100000.*GO*(ALOG(A1)
&+ALOG(B1)-ALOG(A2)-ALOG(B2))
C
      RETURN
      END
C
C
C... SUBROUTINE GRID GIVES X,Y GRID POSITIONS
C
C
      SUBROUTINE GRID(KNT,CHECK,XDELTA,YDELTA)
C
      COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
&DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
&NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
&XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
      COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,

```

```

&KVAR
C
  NI=KNT/NCOLS
  XPOS=KNT*XDELTA-(NI*XSIZE+XDELTA*NI)
  IF(ABS(XPOS).LE.1.0E-01)XPOS=0.00E+00
  YPOS=NI*YDELTA
  IF(XSIZE.GT.CHECK)THEN
    CHECK=CHECK+XDELTA
    DO J=1,NBLKS
      XL(J)=XL(J)-XDELTA
      XR(J)=XR(J)-XDELTA
    END DO
  ELSE
    CHECK=0.0
    DO J=1,NBLKS
      C
      XL(J)=XL(J)+XDELTA*(NCOLS-1)
      XR(J)=XR(J)+XDELTA*(NCOLS-1)
      C
      DO I=1,LAMNUM(J)
        Y(I,J)=Y(I,J)-YDELTA
      END DO
    END DO
  END IF
  RETURN
  END

C
C
C...  SUBROUTINE QUAD CALCULATES THE ANOMALY
C...  ASSOCIATED WITH EACH BLOCK BY DOING THE
C...  INTEGRATION IN THE Y-DIRECTION USING TALWNI'S
C...  QUADRITURE FORMULA
C
C
C      SUBROUTINE QUAD(K)
C
C      COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
&DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
&NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
&XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
C      COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,
&KVAR
C
C      DIMENSION TEMP(500)
C
C...  FIND THE ANOMALY
C
C      IF(LAMNUM(K)-1.GT.1)THEN
        I=1
        V(1)=0.0
        DO J=2,((LAMNUM(K)-1)/2)+1
          C
          A1=ANOM(I,K)*((Y(I,K)-Y(I+2,K))/(Y(I,K)-

```

```

&Y(I+1,K)))*(3.0*Y(I+1,K)-Y(I+2,K)-2.0*Y(I,K))
C
      B=ANOM(I+1,K)*((Y(I,K)-Y(I+2,K))*3)/
&((Y(I+1,K)-Y(I+2,K))*(Y(I+1,K)-Y(I,K)))
C
      C=ANOM(I+2,K)*((Y(I,K)-Y(I+2,K))/(Y(I+2,K)
&-Y(I+1,K)))*(3.0*Y(I+1,K)-Y(I,K)-2.*Y(I+2,K))
C
      TEMP(J-1)=(A1+B+C)/6.0
C
      V(J)=V(J-1)+TEMP(J-1)
      I=I+2
      END DO
C
      ELSE
C
C...  TRAPEZOIDAL RULE FOR ONE AREA
C
      V(((LAMNUM(K)-1)/2)+1)=(Y(2,K)-Y(1,K))*0.5
&*(ANOM(1,K)+ANOM(2,K))
      END IF
C
      RETURN
      END
C
C
C..  SUBROUTINE DERIV IS USED TO SET KVAR TO 20
C... SO THAT DERIVITIVES OF GRAVITY WRT ZD WILL
C... BE CONTAINED IN THE A MATRIX.  THE A
C..  MATRIX IS USED TO LINEARIZE THIS NONLINEAR
C... PROBLEM SO THAT INCREMENTAL PARAMETER JUMPS
C... TOWARD THE SOLUTION CAN BE FOUND.
C
      SUBROUTINE DERIV(A)
C
      COMMON /GRAV/XL(100),XR(100),ZD(100,100),ZU(100),
&DEN(100),Y(500,500),NBLKS,XSIZE,YSIZE,NCOLS,
&NROWS,YMAX(500),YMIN(500),MAXPER,LAMNUM(500),
&XPOS,YPOS,ANOM(500,500),V(500),ELEV(500)
C
      COMMON /CBGINV/NDAT,NPARMS,KEPTRK,IA,IB,IC,LL,
&KVAR
C
      DIMENSION A(500,500),DPRED(500,500)
      KVAR=20
      CALL FWRD(A,DPRED)
      KVAR=0
      RETURN
      END

```


B.3

FORTRAN CODE OF PROGRAM 3D:

A 3-D GRAVITY MODELING PROGRAM

BASED ON TALWANI AND EWING'S

(1960) ALGORITHM

```

COMMON NUMPTS(150),DENLAM(150),DEPTH(150),
&LAMNUM,X(150,150),Y(150,150)
DIMENSION STAEV(500),ELEV(500)
INTEGER COLS,ROWS
CHARACTER*20 FILENAME
C
OPEN(1,FILE='3DGRV.DAT',STATUS='OLD')
C
C*****
C      MODEL INPUT
C
C...  FILENAME OF SAVED SURFII FILE CONTAINING
C...  STATION ELEVATIONS UNITS=METERS
C
C...  IF NUMB=1 THEN INPUT STATION ELEVATIONS FROM SAVED
C...  SURFII FILE
      READ(1,*)NUMB
      IF(NUMB.EQ.1)READ(1,'(A20)')FILENAME
C
      IF(NUMB.EQ.1)OPEN(10,FILE=FILENAME,STATUS=
&'OLD',FORM='UNFORMATTED',ACCESS='SEQUENTIAL')
C
      READ(1,*),LAMNUM
      L1=LAMNUM/2
      T1=(LAMNUM/2.)-L1
C
      IF(T1.EQ.0.0.AND.LAMNUM.NE.2)THEN
        PRINT *,'MUST HAVE ODD NUMBER OF LAMINAE
&FOR PROGRAM TO WORK - START OVER SLAMEEL!'
        CALL EXIT
      END IF
C
      READ(1,*), (NUMPTS(J),J=1,LAMNUM)
      READ(1,*), (DENLAM(J),J=1,LAMNUM)
      IF(NUMB.EQ.1)THEN
        READ(1,*), (ELEV(J),J=1,LAMNUM)
      ELSE
        READ(1,*), (DEPTH(J),J=1,LAMNUM)
      END IF
      READ(1,*),XSIZE,YSIZE,COLS,ROWS
      DO I=1,LAMNUM
        READ(1,*), (X(J,I),Y(J,I),J=1,NUMPTS(I)+1)
      END DO
C
C...  IF NUMB=1 READ IN ELEVATIONS
C
      IF(NUMB.EQ.1)THEN
C*****
C
C...  STATION ELEVATION INPUT
C
C...  READ IN FIRST RECORD
C
      READ(10)NCOLS,NROWS,IZERO

```

```

C
C... READ IN SECOND RECORD
C
      READ(10)IROWS,JCOLS,DIFY,DIFX,XMN,XXM,
      &YMN,YMX,DUMMY
C
C... READ IN ELEVATIONS (METERS)
C
      DO I=NROWS,1,-1
      IBEG=(I-1)*NCOLS+1
      IEND=IBEG+NCOLS-1
      READ(10)(STAEV(J),J=IBEG,IEND)
      END DO
C
C*****
C
      END IF
C
      XDELTA=XSIZE/(COLS-1)
      YDELTA=YSIZE/(ROWS-1)
      CHECK=0.0
      XPOS=0.0000E+00
      YPOS=0.0000E+00
C
      IF(NUMB.EQ.1)THEN
      DO J=1,LAMNUM
      DEPTH(J)=STAEV(1)-ELEV(J)
      END DO
      END IF
C
      CALL TDCALC(XPOS,YPOS)
      DO I=1,COLS*ROWS-1
C
      IF(NUMB.EQ.1)THEN
      DO J=1,LAMNUM
      DEPTH(J)=STAEV(I+1)-ELEV(J)
      IF(ABS(DEPTH(J)).LT.2.)DEPTH(J)=2.
      END DO
      END IF
C
      NI=I/COLS
      XPOS=I*XDELTA-(NI*XSIZE+XDELTA*NI)
      IF ( ABS(XPOS).LE.1.0000E-01)XPOS=0.0000E+00
      YPOS=NI*YDELTA
      IF (XSIZE.GT.CHECK)THEN
      CHECK=CHECK+XDELTA
      DO J=1,LAMNUM
      DO K=1,NUMPTS(J)+1
      X(K,J)=X(K,J)-XDELTA
      END DO
      END DO
      ELSE
      CHECK=0.0
      DO J=1,LAMNUM

```

```

DO K=1,NUMPTS(J)+1
Y(K,J)=Y(K,J)-YDELTA
X(K,J)=X(K,J)+XDELTA*(COLS-1)
END DO
END DO
END IF
CALL TDCALC(XPOS,YPOS)
END DO
PRINT *, 'OUTPUT IN FILE 3DGRVT.DAT!'
CALL EXIT
END

C
SUBROUTINE TDCALC(GRIDX,GRIDY)
C... SUBROUTINE 3DCALC CALCULATES THE GRAVITY
C... ANOMALY PER UNIT THICKNESS OF HORIZONTAL
C... LAMINAE OF UNIT THICKNESS(=V). TO GET THE
C... TOTAL ANOMALY OF THE 3D BODY, THE AREA
C... UNDERNEATH THE RESULTING V-Z CURVE IS
C... CALCULATED. THIS PROGRAM IS BASED ON THE 1960
C... TALWANI AND EWING PAPER FOUND IN GEOPHYSICS,
C... VOLUME 25, 1960.
C
C... SET UP MEMORY
C
COMMON NUMPTS(150),DENLAM(150),DEPTH(150),
&LAMNUM,X(150,150),Y(150,150)
REAL*8 R(100,100),R1(100,100),R2(100,100),
&XM(100,100),P(100,100),Q(100,100),F(100,100),
&GANOM(100),TEMP(100),TEMP1(100,100),
&TEMP2(100,100),V(100)
REAL*8 A,B,C,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
C
C... OPEN FILE TO WRITE GRAVITY ANOMALY AND SURFACE
C... OBSERVATION POINTS INTO - IN FORMAT USABLE FOR
C... SURFACEII AND DISSPLA GRAPHICS. FILE CALLED
C... 3DGRVT.DAT.
C
OPEN(10,FILE='3DGRVT.DAT',STATUS='NEW')
C
C... UNIVERSAL GRAVITY CONSTANT IN CGS UNITS
C
GO=0.0000000667
C
C... CALCULATE THE GRAVITY ANOMALY/UNIT THICKNESS FOR
C... EACH LAMINAE
C
DO J=1,LAMNUM
TEMP2(1,J)=0.0
IF(NUMPTS(J).EQ.1)GO TO 10
M=2
DO I=1,NUMPTS(J)
C
C... R,R1,R2 CANNOT EQUAL 0.0 (COMPUTER WILL NOT LIKE
C... IT)

```

```

C
  R(I,J)=(X(I,J)**2. + Y(I,J)**2. )**0.5
  IF(R(I,J).EQ.0.0)R(I,J)=0.00000000005
  R1(I,J)=(X(I+1,J)**2. + Y(I+1,J)**2. )**0.5
  IF(R1(I,J).EQ.0.0)R1(I,J)=0.00000000005
  R2(I,J)=((X(I,J)-X(I+1,J))**2+(Y(I,J)
&-Y(I+1,J))**2)**0.5
  IF(R2(I,J).EQ.0.0)R2(I,J)=0.00000000005
C
  C1=X(I,J)/R(I,J)
  C2=Y(I,J)/R(I,J)
  C3=X(I+1,J)/R1(I,J)
  C4=Y(I+1,J)/R1(I,J)
  C5=(X(I,J)-X(I+1,J))/R2(I,J)
  C6=(Y(I,J)-Y(I+1,J))/R2(I,J)
C
  XM(I,J)=C2*C3-C4*C1
  P(I,J)=(C6*X(I,J))-(C5*Y(I,J))
C
  IF(P(I,J).GE.0.0)THEN
    S=1.0
  ELSE
    S=-1.0
  END IF
  IF(XM(I,J).GE.0.0)THEN
    W=1.0
  ELSE
    W=-1.0
  END IF
  C7=(P(I,J)*P(I,J)+DEPTH(J)*DEPTH(J))**0.5
C
  Q(I,J)=C5*C1+C6*C2
  F(I,J)=C5*C3+C6*C4
C
  C8=C1*C3+C2*C4
  C9=(DEPTH(J)*Q(I,J)*S)/C7
  C10=(DEPTH(J)*F(I,J)*S)/C7
C
  C... PREVENT ROUNDOFF ERROR IN COMPUTER FROM GIVING
  C... NUMBER G.T. 1.0 OR L.T.-1 - COMPUTER WILL NOT
  C... LIKE THIS NUMBER (DEALING WITH SINES AND
  C... COSINES)
C
  IF(C8-1.0.GT.0.0)C8=1.0
  IF(C9-1.0.GT.0.0)C9=1.0
  IF(C10-1.0.GT.0.0)C10=1.0
  IF(C8.LT.-1.0)C8=-1.0
  IF(C9.LT.-1.0)C9=-1.0
  IF(C10.LT.-1.0)C10=-1.0
C
  TEMP1(I,J)=W*ACOS(C8)-ASIN(C9)+ASIN(C10)
C
  TEMP2(M,J)=TEMP2(M-1,J)+TEMP1(I,J)
  M=M+1

```

```

      END DO
C
C... THE 100000 IS TO PUT V IN UNITS OF MGALS/METER
C
      V(J)=G0*DENLAM(J)*TEMP2(NUMPTS(J)+1,J)*100000.0
C
      GO TO 20
10  V(J)=0.0
20  END DO
C
C... FIND THE ANOMALY (AREA UNDER V-Z CURVE)
C
      IF(LAMNUM-1.GT.1)THEN
        I=1
        GANOM(1)=0.0
        DO J=2,((LAMNUM-1)/2)+1
C
          A=V(I)*((DEPTH(I)-DEPTH(I+2))/
            &(DEPTH(I)-DEPTH(I+1)))*(3.0*DEPTH(I+1)
            &-DEPTH(I+2)-2.0*DEPTH(I))
C
          B=V(I+1)*((DEPTH(I)-DEPTH(I+2))**3)/
            &((DEPTH(I+1)-DEPTH(I+2))*(DEPTH(I+1)-
            &DEPTH(I)))
C
          C=V(I+2)*((DEPTH(I)-DEPTH(I+2))/
            &(DEPTH(I+2)-DEPTH(I+1)))
            &*(3.0*DEPTH(I+1)-DEPTH(I)-2.0*DEPTH(I+2))
C
          TEMP(J-1)=(A+B+C)/6.0
          TEMP(J-1)=TEMP(J-1)
C
          GANOM(J)=GANOM(J-1)+TEMP(J-1)
          I=I+2
        END DO
C
      ELSE
C
C... TRAPEZOIDAL RULE FOR ONE AREA
C
        GANOM(((LAMNUM-1)/2)+1)=(DEPTH(2)-
          &DEPTH(1))*0.5*(V(1)+V(2))
        END IF
C
C... PRINT THE 3D ANOMALY TO FILE 3DGRVT.DAT
C
        WRITE(10,'(E12.4,2X,E12.4,2X,E12.4)')GRIDX,
          &GRIDY,GANOM(((LAMNUM-1)/2)+1)
        RETURN
      END

```

APPENDIX C

PREPARING OBSERVED GRAVITY DATA FOR USE WITH
PROGRAMS GRAVBL AND GINDEP

Preparing Observed Gravity Data

1) Saving gridded data on a file:

Once an observed data set has been gridded using SURFACE II, grid information may be stored in a file of the form FORO_.dat (where __ represents a number from 11 to 99) using the SAVE command of the SURFACE II contouring package. The following is an example of a SURFACE II program used to grid and save the observed gravity data for my modeling area in file FOR015.DAT:

```
TITLE SAVING GRID INFORMATION IN FILE FOR015.DAT
DEVICE 5,'CJWIDEMAN'
ROUTINE 39,1,'(2F12.7)'
EXTREMES -111.74222,-111.68778,45.36077,45.37962
IDXY 158,11,3,2,1,3,0,0,0,9999,'(3F12.4)'
BOX .01,2,.005,1,3,-111.74222,45.36077,1,.2
GRID 0,15,8,0,0,1,0
SAVE 15
PERFORM
STOP
```

File FOR015.DAT will have a specific format (see SAVE command in the SURFACE II manual) and GRAVBL and GINDEP were written to read in a data grid based on the format of a saved SURFII file.

2) Changing units from latitude, longitude to meters north, east:

If the saved SURFACE II file has grid point locations based on units of latitude and longitude, the units must be converted to meters east and meters north before GINIDEP and GRAVBL can be used. Also, the first point in the grid [(0,0) in units of meters] must be located in the upper left hand corner of the grid because of the way GRAVBL and GINDEP expect to receive the data. I have written a program (program RSURFILE) to read the saved SURFACE II file, output

pertinent grid information, and convert latitude and longitude to meters east and meters north based on the scale of the Ennis 15 minute quadrangle map. The program will write the data to a file called GRID.DAT in the following order:

xpos on grid, ypos on grid, data value
in 3F12.4 format. GRID.DAT will contain the number of columns (NCOLS) times the number of rows (NROWS) lines of data. Code for program RSURFILE is as follows:

```

      DIMENSION A(10000), DUMMY(12), VAR(4)
      CHARACTER TOPOFILE*20,BLOCKFILE*20,OUTPUTFILE*20,CHAR*1
C
      WRITE(*,(''$TYPE IN SAVED SURFII FILE>''))
      READ(*,'(A20)')TOPOFILE
      OPEN(21,FILE=TOPOFILE,ACCESS='SEQUENTIAL',STATUS='OLD',
&FORM='UNFORMATTED')
C
C   READ IN FIRST RECORD OF THE SAVED SURFII FILE
C
      READ(21)NCOLS,NROWS,IZERO
      PRINT *,'      '
C
C   READ IN SECOND RECORD
C
      READ(21)IROWS,JCOLS,DIFX,DIFY,XMIN,XMAX,YMIN,YMAX,DUMMY
C
C   CONVERT TO METERS, SCALE OF ENNIS 15' TOPO
C
      WRITE(*,(''$CONVERT TO METERS(Y/N)>''))
      READ(*,'(A1)')CHAR
C
      IF(CHAR.EQ.'Y'.OR.CHAR.EQ.'y')THEN
        DIFY=DIFY*111840.
        DIFX=DIFX*79040.
        XSIZE=ABS((XMAX-XMIN)*79040)
        YSIZE=ABS((YMAX-YMIN)*111840)
        XDELTA=DIFX
        YDELTA=DIFY
      ELSE
        XSIZE=ABS(XMAX-XMIN)
        YSIZE=ABS(YMAX-YMIN)
        IF(DIFY.LT.0.0)THEN
          YDELTA=-DIFY
        ELSE
          YDELTA=DIFY
      
```

```

END IF
END IF
C
PRINT *, '-----'
PRINT *, '
PRINT *, '          MATRIX INFO'
PRINT *, '
PRINT *, 'NCOLS,NROWS',NCOLS,NROWS
C
PRINT *, '
PRINT *, 'DIFY(METERS)=' ,DIFY,
&'DIFY(METERS)=' ,DIFX
PRINT *, 'XMIN=' ,XMIN, '      XMAX=' ,XMAX
PRINT *, 'YMIN=' ,YMIN, '      YMAX=' ,YMAX
C
PRINT *, 'XSIZE(M)=' ,XSIZE,
&'      YSIZE(M)=' ,YSIZE
PRINT *, '-----'
C
C
C
C
NOW READ IN THE GRID MATRIX VALUES
C
DO I=NROWS,1,-1
  IBEG=(I-1)*NCOLS+1
  IEND=IBEG+NCOLS-1
  READ(21)(A(J),J=IBEG,IEND)
END DO
DO I=1,NROWS*NCOLS
  IF(A(I).LE.-9.9999E+33)A(I)=0.0
END DO
C
C
C
PRINT GRID VALUES TO FILE GRID.DAT
C
WRITE(*,('$PRINT GRID VALUES IN FILE
&GRID.DAT(Y/N)>''))
READ(*,('A1'))CHAR
IF(CHAR.EQ.'N')CALL EXIT
OPEN(10,FILE='GRID.DAT',STATUS='NEW')
XPOS=0.0
YPOS=0.0
WRITE(10,'(3F12.4)')XPOS,YPOS,A(1)
DO I=1,NCOLS*NROWS-1
  NI=I/NCOLS
  XPOS=I*DELTA-(NI*XSIZE+XDELTA*NI)
  IF(ABS(XPOS).LE..1)XPOS=0.0
  YPOS=NI*YDELTA
  WRITE(10,'(3F12.4)')XPOS,YPOS,A(I+1)
END DO
CLOSE(10)
CLOSE(21)
CALL EXIT
END

```

Example of using program RSURFILE

Say you have used SURFACE II to create a grid of four columns and three rows where the distance between columns is 25 meters and the distance between rows is 30 meters (the data was collected in units of meters east and north), and this grid information is saved in file FOR019.dat. IF RSURFILE is used to read FOR019.dat and output the grid values in file GRID.DAT, these are the input steps and output displayed when RSURFILE is run:

```
RUN RSURFILE
TYPE IN SAVED SURFII FILE>FOR019.DAT
CONVERT TO METERS(Y/N)>N
```

MATRIX INFO

NCOLS, NROWS 4,3

DIFY(METERS)=30

DIFX(METERS)=25

XMIN=0.0

XMAX=75.0

YMIN=0.0

YMAX=60.0

XSIZE=75.0

YSIZE=60.0

PRINT GRID VALUES IN FILE GRID.DAT(Y/N)>Y

Now file GRID.DAT has been created where the grid points are numbered as follows:

*1	*2	*3	*4
*5	*6	*7	*8
*9	*10	*11	*12

And the output contained in GRID.DAT will be:

0.0000	0.0000	GRID VALUE 1 (mgals) top left of grid
25.0000	0.0000	GRID VALUE 2
50.0000	0.0000	.
75.0000	0.0000	.
0.0000	30.0000	.
25.0000	30.0000	.
50.0000	30.0000	.
75.0000	30.0000	.
0.0000	60.0000	.
25.0000	60.0000	.
50.0000	60.0000	.
75.0000	60.0000	GRID VALUE 12 bottom right of grid

4) Removal of Topography

Program 3D (coding is in Appendix B) is based on Talwani and Ewing's (1960) method of describing 3-D bodies with horizontal, n-sided polygons. Because n-sided polygons accurately describe elevation contours, program 3D was used to model the topography in the survey area (see text). In general, topography from the highest elevation in the modeled area to the lowest station elevation should be included in the topography model. A model may be created by digitizing points of contours taken from a topographic map in the survey area. Enough points on each contour and enough contours should be chosen so that the topography is described reasonably. Input to program 3D is read from a data file called 3DGRV.DAT, and the input must be ordered as follows:

<u>Line</u>	<u>Input</u>
1	1 if the station elevations are to be read from a saved SURFII grid file any other integer if the stations are assumed to be collected on a plane.
2	Saved SURFII file of elevations (i.e. FOR032.DAT).
3	Number of laminae (horizontal polygons) used in the model. For example, if five elevation contours are digitized, then 5 would be input on this line.

- 4 Number of vertices in each laminae of the model separated by a comma. For example, if three laminae were used in the model where:
- top laminae: 5 vertices,
middle laminae: 16 vertices, and
bottom laminae: 8 vertices,
- then input here would be: 5,16,8
- 5 Density of each laminae (gm/cm^3) separated by a comma.
- 6 Elevation (meters above sea level) of each laminae separated by a comma.
- 7 X-size of grid(m), Y-size of grid(m), number of columns in grid, number of rows in grid. These numbers should match the grid parameters used for programs GINDEP and GRAVBL.
- 8 - EOF X,Y location in meters of each point on each laminae entered in clockwise order. The starting point must be repeated as the final point so that the laminae is closed. For example, consider a model with three laminae:

top laminae - 5 vertices: (0,0),(100,200),
(200,300),(100,20), and (-50,20)

middle laminae - 3 vertices: (-100,20),(100,20),
and (50,-20)

bottom laminae - 3 vertices: (-40,60),(50,10),
and (-10,-10).

Then input would be ordered as follows:

Line	Input
8	0.0,0.0
9	100.0,200.0
10	200.0,300.0
11	100.0,20.0
12	-50.0,20.0
13	0.0,0.0
14	-100.0,20.0
15	100.0,20.0
16	50.,-20.
17	-100.0,20.0
18	-40.0,60.0
19	50.0,10.0
20	-10.0,-10.0
21	-40.,60.

Note: Those repeat points used to close the laminae are not counted as extra points for the input of line #4. For the model above the input for line #4 would be 5,3,3.

Output of program 3D is written to a file named 3DGRVT.DAT which will contain (NCOLS x NROWS) lines of data (NCOLS = number of columns in the grid, NROWS = number of rows in the grid), and each line will contain the X-position on the grid, the Y-position on the grid, and the gravity value of the grid point (in mgals). This information is written in 3F12.4 format, and the first data point output is the one located in the upper left hand corner of the grid. The data contained in file 3DGRVT.DAT can best be explained using an example. Say you create a grid of data with 4 columns and 3 rows with the distance between columns = 30 meters and the distance between rows = 25 meters. Assume the values of the 12 grid points in this grid are as follows:

Grid point	Grid value (mgals)
1	12
2	15
3	16
4	20
5	7
6	9
7	12
8	17
9	5
10	7
11	10
12	15

The output in file 3DGRVT.DAT for this example would be:

0.0000	0.0000	12.0000 (upper left corner of grid)
30.0000	0.0000	15.0000
60.0000	0.0000	16.0000
90.0000	0.0000	20.0000
0.0000	25.0000	7.0000
30.0000	25.0000	9.0000

60.0000	25.0000	12.0000
90.0000	25.0000	17.0000
0.0000	50.0000	5.0000
30.0000	50.0000	7.0000
60.0000	50.0000	10.0000
90.0000	50.0000	15.0000 (bottom right corner of grid)

The gravity contributions of the topography to each grid point should be removed from the gridded observed gravity set contained in file GRID.DAT output from program RSURFILE. To do this the grid parameters and locations of the grid used for the topography model must identically match that of the grid determined for the observed gravity data.

4) Removing a DC (constant gravity value from the data

If the observed gravity data is tied into a gravity "net" consisting of base stations where actual gravity values are known, a constant must be removed from each gridded gravity value so that the gravity values generated by the modeling program will match the observed gravity values. For example, the gravity data that I collected is tied to base stations at Helena and Three Forks at which actual values of gravity are known. Therefore, the gravity values in my data set represent actual gravity values which range from $-(980)178$ mgals to $-(980)198$ mgals (the 980 mgals is constant for all the stations and is not recorded in my data set). However, the gravity modeling program duplicates the relative changes in the gravity data (a maximum of 20 milligals in my data area) not the actual gravity values. Consequently, I removed a constant value of

-178 mgals from each of the gridded observed data values before using the modeling programs because these programs should generate numbers in a range from 0-20 milligals to match my data set.

Summary

In summary, the following four steps should be used to prepare the data used in the gravity modeling programs:

- 1) Grid the gravity and elevation data with SURFII and save the gridded data using the SAVE command of SURFII;
- 2) Run RSURFILE for both the gridded gravity and elevation data. RSURFILE will convert station locations in units of latitude and longitude to meters east and meters north (for the Ennis 15' quadrangle), display grid information, and write grid locations and values to file GRID.DAT in 3F12.4 format;
- 3) Run 3D to remove the topography from the highest elevation in the modeled area to the lowest station elevation; and
- 4) Remove a constant value from each of the gravity stations so that the gravity values represent the relative changes of gravity values across the survey area.

APPENDIX D

INPUT TO AND OUTPUT OF PROGRAMS GRAVBL AND GINDEP

- D.1 INPUT TO AND OUTPUT OF PROGRAM GRAVBL
- D.2 INPUT TO AND OUTPUT OF PROGRAM GINDEP

D.1

INPUT TO AND OUTPUT OF PROGRAM GRAVEL

1) Input to program GRAVBL:

Program GRAVBL is a 3-D forward gravity modeling program in which up to 100 rectangular blocks of earth may be used. GRAVBL reads input out of a data file called GRAVBL.DAT, and the input in file GRAVBL.DAT must be ordered as follows:

<u>LINE</u>	<u>INPUT</u>
1	Number of blocks in the model (max=100)
2	XL(1),XR(1),ZU(1),ZD(1),YMIN(1),YMAX(1), DEN(1)
where:	
	XL(1) = Minimum X-edge of block 1
	XR(1) = Maximum X-edge of block 1
	ZU(1) = Upper edge of block 1 (Z pos. down)
	ZD(1) = Lower edge of block 1
	YMIN(1) = Minimum Y-edge of block 1
	YMAX(1) = Maximum Y-edge of block 1
	DEN(1) = Density of block 1
	distance units = meters
	density = gm/cc
3	XL(2),XR(2),ZU(2),ZD(2),YMIN(2),YMAX(2), DEN(2)
4	XL(3),XR(3),ZU(3),ZD(3),YMIN(3),YMAX(3), DEN(3)
.	.
.	.
.	.
NBLKS + 1	XL(NBLKS),XR(NBLKS),
NBLKS + 2	XSIZE,YSIZE,NCOLS,NROWS

where:

XSIZE = X-size of grid (meters)
 YSIZE = Y-size of grid (meters)
 NCOLS = number of columns in the grid
 NROWS = number of rows in the grid.

NBLKS + 3 MAXPER

where:

MAXPER = Multiplier which directly effects the number of laminae used in the quadrature formula. For example if MAXPER=5, then 5 times as many laminae will be used for the quadrature formula then if MAXPER was equal to one.

NBLKS + 4

NUMB: If NUMB = 1, then include topography in the model. If NUMB is not equal to one then assume the data is collected on a plane.

Last two lines used only if NUMB = 1.

NBLKS + 5

Datum elevation (meters above sea level) of the tops of the blocks after the removal of the gravity contributions of the valley fill included in the topography model.

NBLKS + 6

Saved SURFACE II file of elevations (see Appendix C).

2) Output

The output of program GRAVBL is written to a data file called GBL.DAT. GBL.DAT will contain NCOLS x NROWS lines of data. Each of these lines of data will contain the following:

XPOS , YPOS , GRIDDED GRAVITY VALUE (mgals)

Where:

XPOS = X-position of the grid point (meters east of origin)

YPOS = Y-position of the grid point (meters south of origin).

The data is written in 3F12.4 format and the first and last grid point represent the upper left and lower right grid values, respectively.

D.2

INPUT TO AND OUTPUT OF PROGRAM GINDEP

1) Input

Program GINDEP is an inverse 3-D gravity modeling program that uses the forward modeling results of program GRAVBL to determine depths of the bedrock which reduce the error between the gridded observed gravity data and the calculated gravity data. Therefore, the model output using the forward program GRAVBL is used as the input model for program GINDEP. In order for program GINDEP to work, the blocks used in the forward modeling process must extend from the elevation that was used as the base elevation for the removal of topography to the assumed bedrock depth (see text for an explanation). The input to program GINDEP is read from a command file called G.DAT, and the input in the file G.DAT must be ordered as follows:

<u>Line</u>	<u>Input</u>
1	Filename of saved SURFII data file containing the prepared (see Appendix C) gridded observed gravity data.
2	TDEPL,TDEPH

where:

TDEPL = minimum expected depth (meters below the surface, z positive down) of valley fill. The program will constrain the parameter jumps so that each block's minimum depth will not go below this value during the inversion process.

TDEPH = maximum expected depth (meters below the surface, z positive down) of valley fill. The program will constrain the parameter jumps so that each block's maximum depth will not exceed this value in the inversion process.

3 NUMFIX

where:

NUMFIX = total number of blocks to be held fixed to the initial maximum depth throughout the inversion process.

If NUMFIX is not equal to zero:

- 4 Individual block numbers whose maximum depths are to be held fixed separated by a comma. For example, if blocks 2,6,12, and 24 are to have their initial maximum depths held fixed throughout the iterations toward a solution, the following would be entered on line #4:

2,6,12,24.

For this example, NUMFIX = 4.

If NUMFIX is equal to zero, the following would still be input; however, the above line is not input. Therefore, the following line numbers have values as if NUMFIX = 0. If NUMFIX is not equal to zero, add one to each of the following line numbers.

- 4 Number of blocks in the model (NBLKS).
5 XL(1),XR(1),ZU(1),ZD(1),YMIN(1),YMAX(1),DEN(1)

where:

XL(1) = Minimum X edge of block 1
XR(1) = Maximum X edge of block 1
ZU(1) = Upper edge of block 1 (Z positive down)
ZD(1) = Lower edge of block 1
YMIN(1) = Minimum Y edge of block 1
YMAX(1) = Maximum edge of block 1
DEN(1) = Density of block 1

Distances in meters, Density in gm/cm³

- 6 XL(2),XR(2),ZU(2),ZD(2),YMIN(2),YMAX(2),DEN(2)

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NBLKS + 4 XL(NBLKS),XR(NBLKS),ZU(NBLKS),ZD(NBLKS),
YMIN(NBLKS),YMAX(NBLKS),DEN(NBLKS)

NBLKS + 5 MAXPER

where:

MAXPER = Multiplier which directly effects the number of laminae used in the quadrature formula. For example, if MAXPER = 5, then five times as many laminae will be used for the quadrature formula then if MAXPER was equal to one.

NBLKS + 6 RATIO

where:

RATIO = Number which weights the overdetermined part of the solution relative to the underdetermined part. If RATIO = 0, then the standard overdetermined solution is used (see text for an explanation).

NBLKS + 7 STDDEV

where:

STDDEV = An input error value (mgals). When the error between the gridded observed and calculated data is less than this value the program will output the depth to bedrock values.

NBLKS + 8 Maximum number of iterations allowed (i.e. 10)

Last three lines:

NBLKS + 9 NUMB

where:

If NUMB = 1, then topography is included in the model. If NUMB is not equal to 1, the data is assumed to be collected on a plane.

Last two lines included only if NUMB is not equal to one:

- DATELV

where:

DATELV = Datum elevation (meters above sea level) at which the tops of the blocks are located (elevation to which topography has been removed).

- Filename of saved SURFII file of elevations

2) Output

Output from program GINDEP will be written to a file called GINDEP.OUT. GINDEP.OUT will contain the input model, a table of convergence (showing the parameter jumps, the new parameters, the Marquardt damping factor, and the error between the gridded observed and calculated data for each iteration), and finally a listing of the block along with the maximum depth of each block (meters) if the error between the observed and calculated data is less than the value of STDDEV as input.