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COMPUTATION OF SUPERCONDUCTOR CRITICAL CURRENT DENSITIES AND MAGNETIZATION CURVES

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Using a constitutive law for superconductor hysteresis and the ideas of the critical state model, a system of equations is developed for the critical current density J_c , the equilibrium magnetization M_e , the upper and lower portions, M_U and M_L , of the major hysteresis loop, and their derivatives with respect to the applied field. The constitutive law consists of a differential equation relating the time rate of change of the flux density B to that of the applied field, $\dot{B} = \alpha(H)|\dot{H}||f(H) - B| + \dot{H}g(H)$, and a rule governing the abrupt changes observed at the turning points of H . For the case presented here in which f , g , and α have analytic forms, the constitutive law yields an easily differentiable expression for the major loop. Hysteresis curves computed from this constitutive law are shown to be in good agreement with experiments. Calculations of the critical current densities and equilibrium magnetization for the case in which the derivatives are neglected, i.e., the Bean model with $J_c \propto M_U - M_L$ and $M_e \propto M_U + M_L$, are compared with J_c and M_e computed using a system of equations that takes into account the first two derivatives of all quantities. Exponential approximations are given for the critical current densities of several samples of Y-Ba-Cu-O at temperatures between 5 K and 60 K.

I. INTRODUCTION

The critical state model^{1,2} provides a relation between the critical current density, i.e., the maximum current density that a superconductor can carry, and the major magnetic hysteresis loop. In the critical state, currents of the critical density, J_c , are flowing throughout the material, and under an assumption of the model, Ampère's law holds. If the dependence of the critical current density on the flux density B or, alternatively, on the applied field H is known, one may integrate Ampère's law to obtain the flux density and the magnetization. In a related problem, Ampère's law can be used to compute J_c from the magnetization.

The bulk magnetization exhibited by a material in its critical state is that of the major hysteresis loop. Let us consider a one dimensional case of a long slab of superconducting material of half width w , in an applied field H . In this case, Ampère's law can be written as $dB/dx = kJ_c$, and the bulk magnetization is the average over the sample,

$$4\pi M(H) = \frac{1}{w} \int_0^w B(x) - H dx \quad . \quad (1)$$

Let us assume that J_c is an analytic function of position and, hence, may be expressed as a series expansion about the edge of the sample. Integration of the one dimensional form of Ampère's law, taking into account the additional superconductor magnetization, M_e , due to surface currents, yields

$$B(x) = H + 4\pi M_e(H) + k \sum_{n=0}^{\infty} \frac{1}{(n+1)!} J_{c,n} \Big|_{x=0} x^{n+1} \quad . \quad (2)$$

Here $J_{c,n}$ denotes the n th spatial derivative.

The major loop and the corresponding current and flux densities may be divided into two portions, an unloading section, M_U , B_U , J_U , corresponding to decreasing values of H , and a loading section, M_L , B_L , J_L , for increasing H . Using this notation and Lenz's law, which states that the leading terms of J_U and J_L are

of opposite sign, the substitution of (2) into (1) yields

$$4\pi [M_U(H) - M_L(H)] = kwJ_c(H) + \sum_{n=1}^{\infty} \frac{kw^{n+1}}{(n+2)!} [J_{U,n}|_{x=0} - J_{L,n}|_{x=0}] \quad ; \quad (3a)$$

$$4\pi [M_U(H) + M_L(H)] = 8\pi M_e(H) + \sum_{n=1}^{\infty} \frac{kw^{n+1}}{(n+2)!} [J_{U,n}|_{x=0} - J_{L,n}|_{x=0}] \quad . \quad (3b)$$

Similar equations were derived by Fietz, Beasley, Silcox & Webb³, who used several forms for $J_c(H)$ and the first few derivatives to compute magnetization curves. In this paper, I will use an expression for the magnetization curve from a constitutive law for the hysteresis to calculate J_c and M_e and determine a functional form for $J_c(H)$.

II. THE CONSTITUTIVE RELATION

The constitutive law proposed by Coleman and Hodgdon⁴ for hysteresis in ferrites and ferromagnets is, with suitable modifications, useful for hysteresis in superconductors⁵. For superconductors, the constitutive relation consists of a differential equation, a jump condition at the turning points of H , and a set of constraints. At values of H for which $dH/dt \neq 0$, the hysteresis curves are well represented by the solutions of the equation

$$\frac{dB}{dH} = \alpha(H)s(\dot{H})[f(H) - B] + g(H) \quad , \quad (4)$$

where s is the signum function, and α , f , and g are piecewise continuous functions of H . α must be positive valued and even; f is required to be monotonically increasing and odd, with a finite derivative, $f'(\infty)$, for large H ; and g must be an even function of H with, for all H , $g(H) < f'(H)$, and in the limit of large H , $g(\infty) = f'(\infty)$. At the turning points of H , the flux density remains constant, $B(t^+) = B(t^-)$, while H exhibits a temperature dependent jump:

$$H(t^+) = \begin{cases} \min\{H(t^-) + \Delta, H_*\} & , \quad \text{for } \dot{H}(t^-) > 0 \\ \max\{H(t^-) - \Delta, H_*\} & , \quad \text{for } \dot{H}(t^-) < 0 \end{cases} \quad (5)$$

Δ is the largest change observed for the material at a fixed temperature, and H_* is a bound determined by the major loop.

Equations (4) and (5) provide expressions for important points and loops of the hysteresis curves, such as the remanence, the coercivity, the initial magnetization curve, and the minor loops, as well as the flux density and magnetization the result from arbitrary changes in the applied field.^{4,5} This constitutive relation also yields expressions for the two portions of the major loop:

$$4\pi M_U(H) = f(H) - H + e^{\tilde{\alpha}(H)} \int_H^{H_*} [f'(\eta) - g(\eta)] e^{-\tilde{\alpha}(\eta)} d\eta, \quad (6a)$$

$$4\pi M_L(H) = f(H) - H - e^{-\tilde{\alpha}(H)} \int_{-H_*}^H [f'(\eta) - g(\eta)] e^{\tilde{\alpha}(\eta)} d\eta, \quad (6b)$$

where $\tilde{\alpha} = \int^H \alpha$.

III. CALCULATION OF J_c and M_c

Using Ampère's law, the expansion of J_c in (3) about $x = 0$ can be written in terms of the derivatives of J_c with respect to B , evaluated at $B(0) = H$. The expression for the major loop in (6) can also be differentiated with respect to H ; for example, $4\pi M'_U = g(H) - 1 + \alpha(H)B_U(H)$. Such differentiation yields a system of equations for M_U , M_L , J_U , J_L , and M_c and their derivatives. For cases in which all derivatives of order greater than two can be neglected, this system is

$$\begin{aligned} M_U - M_L - J &+ \frac{1}{12} [J(J')^2 + J^2 J''] = 0, \\ M_U + M_L - 2M_c &+ \frac{1}{3} J J' = 0, \\ M_U' - M_L' - J' &+ \frac{1}{12} [(J')^3 + 4J J' J''] = 0, \\ M_U' + M_L' - 2M_c' &+ \frac{1}{3} [(J')^2 + 4J J''] = 0, \\ M_U'' - M_L'' - J'' &+ \frac{1}{12} [7(J')^2 J'' + 4J (J'')^2] = 0, \\ M_U'' + M_L'' - 2M_c'' &+ J J'' = 0, \end{aligned} \quad (7)$$

where $\mathbf{M} = 4\pi M(H)$, $\mathbf{J} = kwJ_c(H)$, and the primes denote differentiation with respect to H .

I will present results for the following choices of the material functions f , g , and α :

$$f(H) = \begin{cases} A_1 \arctan A_2 H & , \quad \text{for } |H| \leq H_{bp} ; \\ A_1 \arctan A_2 H_{bp} + (H - H_{bp}) & , \quad \text{for } H > H_{bp} ; \\ -A_1 \arctan A_2 H_{bp} + (H_{bp} - H) & , \quad \text{for } H < -H_{bp} ; \end{cases} \quad (8)$$

$$g(H) = \begin{cases} f'(H) \left[1 - A_3 \exp\left(\frac{-A_4 |H|}{H_x - |H|}\right) \right] & , \quad \text{for } |H| \leq H_x ; \\ f'(H) & , \quad \text{for } |H| > H_x ; \end{cases} \quad (9)$$

and

$$\alpha(H) = 0.02 \operatorname{sech}\left(\frac{1.2H}{A_5}\right) + 0.00001 \quad , \quad (10)$$

H_x is the closure point of the major loop. Discussion of the evaluation of the constants A_1 , A_2 , A_3 , A_4 , and A_5 in terms of the remanence, coercivity and closure point of the experimental hysteresis curves is described in [5]. Hysteresis curves for two samples of 123 Y-Ba-Cu-O are shown in Figs. (1) and (2). The critical current densities and equilibrium magnetizations compute from (7) for the cases of neglecting and including the derivatives are shown in Figs. (3)-(6).

IV. EXPONENTIAL FORM OF J_c

Several functional forms, $J_c(H)$, for the dependence of the critical current density on the applied field have been proposed.^{1,2,3,6,7} A measure of the validity of these forms is the accuracy of the major loop magnetization curves that result from the substitution of the assumed form for J_c into an equation similar to Eq. (7).^{3,6,7} In the work presented here, computations proceed in the opposite direction, from the magnetization curves to the critical current density. Hence, in this case, the problem of selecting a function form for J_c is not that of verifying an assumption, but the simpler task of fitting the computed values of J_c with a useful expression. I have found that the exponential form suggested by Fietz, Beasley, Silcox & Webb,³

$J_c(H) = k_1 \exp(-k_2|H|) + k_3$, where k_1 , k_2 , and k_3 are positive valued and independent of H , yields curves in excellent agreement with those computed from Eq. (7) for $|H|$ less than H_z . For example, the values $k_1 = 1.772$, $k_2 = 0.125$, and $k_3 = 0.4004$ provides a good fit for the curve for the critical current density drawn with a solid line in Fig. 3 for the nonirradiated sample shown in Fig. 1. Similarly, the values $k_1 = 4.41$, $k_2 = 5.165 \times 10^{-2}$, and $k_3 = 1.003$ yield a curve in good agreement with the solid curve in Fig. 4 for the irradiated sample.

The temperature dependence of J_c is included by allowing k_1 , k_2 , and k_3 to vary with temperature. The general form, $J_c(H, T)$, is

$$J_c(H, T) = k_1(T)e^{-k_2(T)|H|} + k_3(T) \quad . \quad (11)$$

Measured and calculated magnetic hysteresis curves in a sample of 123 Y-Ba-Cu-O at the ambient temperatures of 5 K, 10 K, 20 K, 30 K, 40 K, and 60 K are given in Ref. [5]. In Fig. 7 are shown the critical current densities at five of these temperatures calculated from Eq. (7) with the first two derivatives of all quantities taken into account. Values for k_1 , k_2 , and k_3 in Eq. (9) are given in Table I. For the critical current densities shown in Fig. 7, the temperature dependence these parameters can be written as

$$k_1(T) = 3.88 - 0.18T + 3.13 \times 10^{-3}T^2 - 2.02 \times 10^{-5}T^3,$$

$$k_2(T) = 0.025 + 7.87 \times 10^{-4}T + 7.56 \times 10^{-5}T^2 - 6.08 \times 10^{-7}T^3 \quad ,$$

and

$$k_3(T) = 0.27 + 3.5 \times 10^{-3}T - 3.9 \times 10^{-4}T^2 + 4.36 \times 10^{-6}T^3 \quad ,$$

for T between 5 K and 60 K.

Table I. Values of the parameters k_1 , k_2 , and k_3 in the form $J_c(H) = k_1 \exp(-k_2|H|) + k_3$ for several superconducting samples.

	k_1	k_2	k_3
nonirradiated (Figs. 1,3,5)	1.772	0.125	0.4004
irradiated (Figs. 2,4,6)	4.410	0.0516	1.003
5 K (Fig.7)	2.99	0.0289708	0.1868
10 K (Fig.7)	2.528	0.043326	0.26619
20 K (Fig.7)	1.349	0.06585	0.2136
30 K (Fig.7)	0.761	0.09741	0.1384
40 K (Fig.7)	0.5142	0.140826	0.0598
60 K (not shown)	0.08097	0.213	0.0125

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FIGURE CAPTIONS

Fig. 1. Magnetic hysteresis in a sample of nonirradiated 123 Y-Ba-Cu-O. The curve is calculated from the constitutive law in eqs. (4), (5), (8)-(10) with $A_1 = 8176$, $A_2 = 1.223 \times 10^{-4}$, $A_3 = 0.3867$, $A_4 = 19.18$, $A_5 = 10$, and $\Delta = 0$. The material is initially in the demagnetized state. The field increases to 55 and then cycles between 55 and -55. Experimental values are shown as solid dots.

Fig. 2. Magnetic hysteresis in a neutron irradiated sample of 123 Y-Ba-Cu-O. The curve is calculated using Eqs. (4), (5), (8)-(10) with $A_1 = 9789$, $A_2 = 1.022 \times 10^{-4}$, $A_3 = 0.8485$, $A_4 = 8.302$, $A_5 = 17$, $\Delta = 0.8$, and with field variations similar to those in Fig. 1. Experimental values are shown as solid dots.

Fig. 3. Critical current density calculated using Eq. (7) and the magnetic hysteresis data shown in Fig. 1. In the curve drawn with the solid line, the first two derivatives of all quantities are included in the calculation. The curve drawn with the broken line gives the Bean approximation, i.e., Eq. (7) with the derivatives neglected. Here, k is a scaling constant that depends in part on the units and w is the half width of the sample.

Fig. 4. Critical current density calculated using Eq. (7) and the magnetic hysteresis data shown in Fig. 2. The Bean approximation is shown in the curve with the broken line. The results of Eq. (7) with the first two derivatives of all quantities included are given in the curve drawn with the solid line. The constants k and w are as in Fig. 3.

Fig. 5. Equilibrium magnetization calculated from Eq. (7) and the magnetic hysteresis data in Fig. 1. Results for the Bean approximation are denoted by the broken line. The results from Eq. (7) with the first two derivatives included are shown as the solid line.

Fig. 6. Equilibrium magnetization calculated from Eq. (7) and the magnetic hysteresis data in Fig. 2. The broken line gives the Bean approximation; the solid line shows results of Eq. (7) with the first two derivatives included.

Fig. 7. Critical current densities in a sample of 123 Y-Ba-Cu-O at five temperatures: — 5 K; ——— 10 K; 20 K; - - - - 30K; and — - — - 40 K. The constant k depends on units, and w is the half width of the sample. The values were calculated using Eq. (7), the first two derivatives of all quantities, and the hysteresis data given at these temperatures in [5].