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RESEARCH AND DEVELOPMENT OF A UNIFIED APPROACH TO
OPERATIONS SCHEDULING FOR ELECTRIC POWER SYSTEMS
UNDER UNCERTAINTY

Technical Progress Report for the Period June 1979—June 1980

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June 1980

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MASTER

U. S. DEPARTMENT OF ENERGY

Division of Electric Energy Systems

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PREPARED FOR:
U.S. DEPARTMENT OF ENERGY
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I. INTRODUCTION

This report is the First Annual Technical Progress Report for the study entitled "Research and Development of a Unified Approach to Operation Scheduling for Electric Power Systems Under Uncertainty," DOE Contract DE-AC01-79-ET 29243. This report summarizes the findings of the first 12 month period of the total 36 month contract duration.

The objective of this contract is to develop a unified approach for operations scheduling in electric power systems that explicitly takes into account sources of uncertainty present at all levels in this problem. This objective requires formulating the operations scheduling problem as a single, integrated problem, and developing solution methodologies that start from this integrated formulation and progress to subproblem solution methods that are consistent with it. Only in this manner will the resulting solution methodologies be guaranteed to capture properly subproblem interdependencies and allow for consistent treatment of effects at all levels in the problem.

Existing methods for operations scheduling first decompose this problem into subproblems in rough accordance with time-scale. These subproblems include: (1) maintenance scheduling and nuclear refueling, (2) production scheduling, and (3) unit commitment (including economic dispatch). These subproblems span the area of responsibility between system planning and system operation. They apply to the periods ranging from hours to several years, and include a wide range of complex decisions that have significant impact on the cost effectiveness and reliability of electric power system operation. Although the decomposition of operations scheduling into subproblems has been justified on a heuristic basis from years of operating experience, this approach nevertheless neglects the interdependencies among these subproblems, and is often carried out without due regard to consistent treatment of uncertainties from one time-scale to the next.

The approach under development in this research effort begins with a single, unified problem formulation with stochastic variables treated explicitly, and with a consistent objective function (expected production cost). It is then shown that, in principle, this problem may be solved by stochastic dynamic programming. This solution method for this problem is computationally infeasible due to its high dimensionality.

The next step then is to decompose this overall problem into subproblems. Two general decomposition methods are employed: temporal decomposition and spatial decomposition. Temporal decomposition separates the problem according to the time-scales naturally present in operations scheduling, such as those associated with demand, hydro inflows, release rates, storage capacities, and refueling cycles. Temporal decomposition in general leads to a multi-year problem, a seasonal problem, and a short-term problem. Although these subproblems are in approximate agreement with the traditional decomposition discussed above, they differ in several respects:

- they are more aligned with the actual time-scales present in the problem
- they have the potential of being tailored to a specific system, where certain time-scales are of greater influence than in other systems
- they retain the effect of other subproblem time-scales in each subproblem, rather than simply neglecting them.

Following decomposition by time-scale, spatial decomposition is applied to reduce the computational burden further. The primary methods for investigation here are one-variable-at-a-time stochastic optimization techniques which converge to optimum or good suboptimum solutions. The choice of variables corresponds to spatially distinct elements in the operations scheduling problem -- reservoirs or groups of reservoirs, thermal

units, units scheduled for maintenance, etc. -- and thus are generally described as spatial decomposition methods.

After the application of these decomposition methods, solution methods tailored to each subproblem can be developed to take advantage of the particular structure of the subproblem, but properly account for interfaces and interactions.

The overall approach is illustrated in Figure 1.1. The subproblem shown essentially comprise a nested sequence of problems. This figure shows the problem being carried out on three time-scales, with a multi-year, seasonal, and weekly scheduler. For adjacent time-scales, a model is developed of how the performance of the faster time-scale problem depends on the policies/constraints provided from the slower time-scale problem. These performance models are then integrated into the problem formulation at the slowest time-scale, and this problem is solved using spatial decomposition techniques. The resulting policies for the units on the slowest time-scale are then communicated as constraints to the units on the next slowest time-scale. The problem at the second slowest time-scale is solved, with the performance model from the faster time-scale and the unit policy constraints from the slowest time-scale. This process is repeated for the fastest time-scale, and the complete solution thereby obtained.

Obtaining the performance models is, then, an important part of this overall approach. The stochastic production simulation method developed by SCI and Systems Europe, provides such a performance model without the necessity of deriving a full solution for the fast subproblem. This method is described later in this report.

The details of this method are described throughout this report. Section II begins with a unified problem formulation of the operations scheduling problem, which sets up the analysis discussed in subsequent chapters. Section III then examines time-scale decomposition. The method

OPTIMIZATION
MODELS

NOMINAL
HORIZON

3 YEARS

1 YEAR

1 WEEK

NOMINAL
STAGE
LENGTH

1 MONTH

1 WEEK

1 HOUR

Figure 1.1. Overview of Solution Approach to Operations Scheduling.

of analysis here is the theory of singular perturbation, developed and refined by Kokotovic and his co-workers. However, because of the inherent stochastic nature of the operations scheduling problem, previously developed deterministic methods do not apply. Furthermore, the natural formulation for operations scheduling involves discrete-time models for system dynamics, which is also a departure from the usual singular perturbation analysis approach. These considerations lead to an analysis based on the use of Markov chains; time-scale separation is modelled as high vs. low probabilities of transition between states, analogous to fast and slow time-scales in a deterministic setting. This method is developed primarily for the hydro-scheduling problem being carried out on several time-scales. The optimization problem is formulated and solved, and sub-problems defined from weakly-coupled Markov chains are shown to be asymptotically optimum. Quantitative examples with simulation results showing excellent agreement with the overall optimum are presented. Future work includes possible extensions of this method to other aspects of operations scheduling, such as unit commitment and/or maintenance scheduling.

Chapter IV then describes new results in stochastic spatial decomposition. The method of dynamic programming successive approximations (DPSA), developed by Larson and his co-workers at SCI, is extended to the stochastic case. First, stochastic DPSA for the case of parallel reservoirs is shown to be equivalent to a static team problem with a nested information structure. The resulting solution is shown to be optimum in the class of local feedback control laws. Since one-variable-at-a-time methods constrain us a priori to this class of methods, the outcome is extremely favorable to this method. Furthermore, simulation results comparing stochastic DPSA with stochastic dynamic programming, which provides the true centralized optimum, shows minor loss in performance by stochastic DPSA for an extremely high computational gain. Initial concepts for the serial reservoir case, involving downstream information flow, are also presented.

Section V describes research on extending the stochastic production simulation developed by Jamouille of Systems Europe. These extensions

take into account two important sources of uncertainty: random energy in storage units and random fuel costs. It is shown that these uncertainties can be treated within the existing stochastic production simulation framework at the expense of added complexity and computation.

Section VI formulates the maintenance scheduling problem. This discussion focuses on describing the nature of this problem, reviewing existing solution methods, postulating two alternative formulations, and outlining a method of solution. This method involves a combination of branch-and-bound and dynamic programming, initially developed by Yamayee. An in-depth literature survey on maintenance scheduling is presented in Appendix C. Appendix D also discusses the nuclear refueling problem, including differences and similarities with maintenance scheduling.

Section VII then formulates the unit commitment problem and discusses hybrid solution methods appropriate for this problem.

Finally, Appendix B describes some initial work on joint spatial - temporal decomposition methods for the deterministic case. This research involves applying both DP/SA and singular perturbation techniques to a multiple reservoir problem, and leads to some novel aggregation methods.

II. PROBLEM DEFINITION

2.1 INTRODUCTION

Planning may be defined as formulating, evaluating, and choosing between the various courses of actions being considered. Operations scheduling in an electric utility consists primarily of multi-year generation and maintenance scheduling, seasonal production scheduling, and short-term resource commitment. Operations scheduling should be determined in such a way that the system is in a position to meet the future electricity demands with adequate reliability, requiring the lowest possible operating costs, and satisfying existing constraints on the system.

The general problem formulation presented in this chapter can be used as an instrument for find the "optimum" operation scheduling from the point of view of the utility system operations planning as a whole. However, the solution of the various sub-problems--maintenance scheduling, hydro and thermal generation, nuclear refueling, production scheduling and unit commitment--cannot be performed simultaneously. A single computer model which is capable of simultaneously coping with and representing all relevant functions and interdependence involved in operations scheduling in a sufficiently detailed and systematic manner is not realizable at the present time.

The principal objective of general problem formulation presented here is to provide for the development of a unified methodology for analyzing the overall, integrated problem by means of a single stochastic formulation that encompasses all aspects of the operation scheduling problem. Stochastic dynamic programming is applied to obtain a mathematically correct formulation to the overall problem. Then, decomposition methods are used to partition the problem into computationally tractable sub-problems. Using this approach, an iterative solution process is used to

find near optimum solutions, where sub-problem interdependencies are taken explicitly into account.

The operations scheduling problem involves several types of generating units, including fossil-fired (coal, oil, natural gas) thermal units, hydro units, nuclear units, pumped storage units, and interchange contracts. Demand is forecasted as a function of time over the scheduling interval, which may vary from one day to a few years. The cost to be minimized includes fuel costs and other operating costs. Uncertainty is present in the form of unit reliability, fuel supply, forced outage rates, streamflow uncertainties, and demand uncertainties.

There are three basic types of decision variables in operations scheduling problems. The first type of variable represents the output of power generators that can store energy. Generally, these generators are associated with hydroelectric plants and other energy storage devices, although certain interchange contracts and nuclear reactors also have this characteristic. In some cases interruptible loads can be modeled as generators of this type. The second type of variable represents the output of power generators that do not store energy; most thermal generating units fall into this category. The third type of variable is a binary variable that determines whether or not a plant is available; generally, only the plants that do not store energy are affected in this way, although it is straightforward to include the availability of energy-storing plants as well.

The outline of this chapter is as follows. Section 2.2 describes the dynamic equations for energy-storing units, with particular attention to interpretation of these equations for reservoirs. Section 2.3 deals with the dynamic equations for availability of non-storage units. Section 2.4 describes the physical and the operational constraints, and Section 2.5 describes the costs and other performance criteria that applied to the operations scheduling problem. Finally, Section 2.6 provides a summary of the operations scheduling formulation.

2.2 DYNAMIC EQUATIONS FOR ENERGY-STORING UNITS AND FUEL INVENTORIES

The state variables for units that can store energy are the amounts of energy stored, and the decision variables are the amounts of energy removed from the energy-storing units. Generally, these units are associated with hydroelectric plant and other energy-storage devices, such as nuclear plants and interchange contracts.

Hydro units in a system are of three distinct types--long-term storage reservoirs with generation capacity, long-term storage reservoirs without generation capacity, and downstream generation units with little or no reservoir capacity (the latter units are often called run-of-river units). In the general case, there are two types of inflows into each generation site--controlled and uncontrolled. The controlled inflows are the result of discharges from upstream long-term storage. The uncontrolled inflows enter the river system between the long-term storage sites and the generation site.

The general energy balance associated with unit i at a specific time increment can be written as follows:

$$x_i(k+1) = x_i(k) - u_i(k) - s_i(k) + \sum_{j \neq i} y_{ji}(k) + W_i(k) + w_i(k) \quad (2.1)$$

where

k = index of the time increments

$x_i(k)$ = state variable for the amount of energy stored in the i th unit at the beginning of time increment k

$u_i(k)$ = decision variable for the amount of energy removed from the i th unit to generate electricity during time increment k

$s_i(k)$ = amount of energy removed from the i th unit during time increment k that is not used to generate electricity

$y_{ji}(k)$ = amount of energy transferred from unit j to unit i
during time increment k

$W_i(k)$ = deterministic exogenous input of energy into
unit i during time increment k

$w_i(k)$ = random exogenous input of energy into unit i during
time increment k .

This description clearly models hydroelectric generation facilities. Figure 2-1 represents a hydroelectric scheme with downstream reservoir i and upstream reservoir, and it should help in understanding the practical aspects of the variables in equation (2.1).

The energy storage state variable $x_i(k)$ represents the amount of water in the reservoir; in some cases this variable is expressed in terms of the level of water in the reservoir, but it is straightforward to convert such a level into a quantity of water. The decision variable $u_i(k)$ represents the amount of release that is run through the turbines to produce electricity. The variable $s_i(k)$ accounts for spillage, evaporation, or other releases that do not generate electricity. The uncontrollable energy losses such as evaporation and seepage provide a minimum level to $s_i(k)$. Evaporation losses are a function of stochastic weather processes (air-water temperature difference and humidity) and reservoir-volume (surface area). Uncontrollable spills, though not generally desired, may also occur due to unexpected streamflow events. The energy transfer variable $y_{ji}(k)$ in Eq. (2.1) accounts for releases from upstream reservoirs that flow into reservoir i . In some cases there is a significant delay between the release of water from the upstream reservoir and its arrival at the downstream reservoir. If the delay from reservoir i to reservoir j is Δ_{ji} , then this transfer variable can be expressed as follows:

$$y_{ji} = \gamma_{ji}[u_j(k-\Delta_{ji}), s_j(k-\Delta_{ji})] \quad (2.2)$$

Figure 2-1 Representation of Hydroelectric Scheme with Two Reservoirs

where γ_{ji} is a function that accounts for evaporation losses and diversions of water during transit from reservoir j to reservoir i . If there are no losses or diversions, then

$$y_{ji}(k) = u_j(k - \Delta_{ji}) + s_j(k - \Delta_{ji}) \quad (2.3)$$

where the time delay Δ_{ji} is not a constant, but depends upon the flow rate between the two reservoirs.

$W_i(k)$ and $w_i(k)$ represent the deterministic and random elements, respectively, of the inflows to reservoir i that do not originate from an upstream reservoir. The deterministic exogenous inputs $W_i(k)$ account for tributary rivers and streams that flow into the main river and then into the reservoir, as well as for direct rainfall runoff; these inputs are assumed to be known. The random exogenous inputs $w_i(k)$ account for stochastic variations in these inputs; the probability density functions $p[w_i(k)]$ are assumed to be known. It is sometimes assumed that the $w_i(k)$ are independent in both i and k . However, there is usually correlation of streamflow rates between reservoirs on the same river (correlation in i) as well as correlation in time (correlation in k).

Pumped storage plants are similar to hydro units except that inflow can be scheduled by decisions to pump. In the day-to-day operation of pumped storage plants, the quantity of energy stored will vary. The change in storage decision variable $u_i(k)$ may be positive ($u_i(k) \geq 0$) for generation or negative ($u_i(k) \leq 0$) for pumping.

Similar interpretations of these equations for nuclear refueling, fuel allocation scheduling and interchange contracts can be made. Nuclear refueling scheduling is an important component of the operations scheduling process. Typically, ore must be ordered two years, enriched one year, and fuel bundles fabricated six months in advance of the start of refueling. The generally large capacity of nuclear units makes the outage cost high relative to other unit types. Also, the operational cost

component differs from a conventional thermal unit; most thermal units have incremental costs which are constant over time while the nuclear fuel costs are not. This is because the cost of extracting a marginal MBTU by extended operation differs from that of modifying the batch size or enrichment. Extended operation involves running the unit for a longer interval between refueling at a lower capacity.

In the simplest case, the nuclear units or the interchange contracts start with some initial amount of energy, corresponding either to the amount of energy contracted or to the amount of energy in the nuclear core that can be utilized before the next refueling. This energy can then be drawn upon during the operations planning period. In this case, Eq. (2.1) reduces to

$$x_i(k+1) = x_i(k) - u_i(k) , \quad k = K, K+1, \dots ; \quad (2.4)$$

$$x_i(K) = c$$

where $u_i(k)$ represents the energy utilized during time increment k and c represents the amount of energy available at time K , the start of the interchange contract period or the date at which the nuclear reactor becomes available after the refueling. Clearly, the other terms in Eq. (2.1) can be used as necessary to represent a variety of additional effects for these types of units.

Load management can also be modeled, in part, by this type of equation. Let $X_m(k)$ represent the amount of energy stored in interruptible group m , $W_m(k)$ the amount of energy used by the customer and $U_m(X_m(k), W_m(k), Y_m(k))$ the amount of energy delivered to the group during time interval k . $Y_m(k)$ is the decision variable:

$$Y_m(k) = \begin{cases} 1 & \text{if group } m \text{ is connected at time } k \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, U_m will be 0 if $Y_m(k)=0$. The dependence of U_m on $X_m(k)$ and $W_m(k)$ denotes that the energy delivered to group m during time interval k is dependent on the energy stored in the group and the amount of energy used during that time interval. The dynamic equations for this case are

$$X_m(k+1) = X_m(k) - W_m(k) + U_m(X_m(k), W_m(k), Y_m(k)) \quad (2.5)$$

The constraints limiting the amount of time a group can be disconnected are of various forms. One type of constraint requires that the group must be connected for a minimum number of hours over a day, or other time period, i.e.,

$$\sum_{k \in D_j} Y_m(k) > H.$$

Other types of constraints limit the maximum time the groups can be disconnected and/or the minimum time the groups must be connected.

2.3 DYNAMIC EQUATIONS FOR AVAILABILITY OF NON-STORAGE UNITS

The next set of decision variables to be considered are those associated with the availability of units. As mentioned earlier, these variables are primarily associated with thermal units that do not store energy, but they can also be used in conjunction with units that do store energy. In the case of the unit commitment problem, these variables determine whether a unit is started-up or shut-down. In the case of maintenance scheduling, these variables determine when a unit is taken down for maintenance and when it is returned to operation. The availability of any specific unit j can be expressed as

$$A_j(k) = B_j(k) C_j(k) \quad (2.6)$$

where

$B_j(k)$ = Binary variable describing decision to make unit available this period

$$= \begin{cases} 1 & \text{decision to make unit available} \\ 0 & \text{decision to make unit unavailable} \end{cases}$$

$C_j(k)$ = Binary random variable describing forced outage of unit

$$= \begin{cases} 0 & \text{if unit out} \\ 1 & \text{otherwise} \end{cases}$$

The variable $A_j(k)$ serves as a state variable that determines whether or not the j th unit is available. The variable $B_j(k)$ serves as a decision variable that represents whether a unit is made available or made unavailable, either for unit commitment or for maintenance scheduling or for some other purpose. The variable $C_j(k)$ takes into account random changes in availability due to stochastic effects such as forced outages, random duration of the maintenance interval, or construction delays. The probability distribution $P[\underline{C}(k)]$ is assumed known.

For variations in start-up and shut-down costs and constraints for the unit commitment problem, and determination of the nominal maintenance and refueling times for the maintenance scheduling and refueling problems, it is important to know the time elapsed since the last change in availability. The dynamic equations for the time since last change in availability of non-storage devices can be expressed in one of two ways. The cumulative up time of unit j up to time $k+1$ satisfies the equation

$$XU_j(k+1) = (XU_j(k) + 1) A_j(k) \quad (2.7)$$

while the cumulative down time is given by

$$XD_j(k+1) = (XD_j(k) + 1) (1 - A_j(k)) \quad (2.8)$$

The third set of decisions is related to the output of the unit that cannot store energy. These decisions are clearly affected by the availability indices: unless a unit is available, it has zero output. The output of such a unit at time increment k , $g_j(k)$, therefore has the important condition that $g_j(k) = 0$ if $A_j(k) = 0$.

Because these units do not store energy, there are no dynamic equations associated with their output; if the unit is available, its output is assumed to be completely adjustable from time increment to time increment. For the unit commitment problem and other operations planning problems on longer time scales, this assumption is usually valid. If the response time of a unit is of concern, it can always be modeled by introducing an extra state variable to represent the last output of the plant and placing restrictions on the rate of change of this state variable; one possible implementation is to characterize the time-variation of the output of this unit as

$$g_j(k+1) = g_j(k) + v_j(k) \quad (2.9)$$

where $v_j(k)$ is the change of generation over the k th interval, and where $v_j(k)$ is constrained by

$$|v_j(k)| \leq \Omega_j \quad (2.10)$$

where Ω_j is the limit on rate of change of generation on unit j from one time increment to the next. More elaborate dynamic models can be established if necessary.

2.4 CONSTRAINTS

The state and decision variables of the general energy balance equations must satisfy various constraints and equality relationships which define the essential characteristics of the power system. These constraints ensure that the system's instantaneous power demand is satisfied at all times, that no generating unit is allowed to operate outside its operating capacity limits, that the amount of energy available may be restricted and that the system operates with adequate reliability. As shown in Figure 2-2, the model of the power system is represented as a combination of energy sources, generating plants, transmission networks and loads.

The constraints may be categorized into physical constraints, operational constraints and demand constraints.

Physical constraints may include the various limits on power and energy capacities at the power generation units to ensure that all generating units are operating within their higher and lower limits. These constraints may include the peak available capacity of units; the minimum recommended operating capacity of units, such as nuclear units; physical limitations of withdrawal from energy-storage units; maximum capacities of energy-storage units; maximum power capacities that can be transmitted in the transmission network without overloading lines (this may influence the dispatch of power at various generating units); etc.

Operation constraints include the various limits on the generation of electricity that ensure system operation with adequate reliability and safety for the operating personnel and the public as a whole. These constraints may include specification of spinning reserve above the maximum demand to ensure system security and stability, minimum start-up and shut-down times of various units, maximum rate of change of power, list of generating units that must be run at all times, etc.

Figure 2-2 Model of the Power System

A fundamental constraint that affects all the decision variables is that the demand must be met. If $D(k)$ is the total demand over time increment k , including the amount allocated to the interruptible customers, $\sum_{m=1}^M u_m(k)$, then this constraint can be written as

$$\sum_{i=1}^H f_i[x_i(k), u_i(k)] + \sum_{j=1}^T A_j(k) g_j(k) = D(k) \quad (2.12)$$

where

H = number of units that can store energy

T = number of units that cannot store energy.

Here it is assumed that the demand $D(k)$ is known.

Another system-wide constraint is the spinning reserve requirement. This constraint states that the capacity of all the available units must exceed the demand by a given amount. If it is assumed that all units that store energy are available, this constraint becomes

$$\Pr \left\{ \sum_{i=1}^H K_i + \sum_{j=1}^T A_j(k) L_j \geq D(k) + \Omega \right\} \geq 1 - \delta \quad (2.13)$$

where

K_i = capacity of i th energy-storing unit

L_j = capacity of j th unit that cannot store energy

Ω = spinning reserve requirement.

δ = specified probability value for acceptable level of spinning reserve

Clearly, many modifications of this constraint can be accommodated using these basic variables.

Considering energy-storage units, a typical hydro system consists of both uncontrolled and controlled hydro plants. The uncontrolled hydro plants (or run-of-river plants) utilize the stream flows to generate energy with the amount of energy produced depending on the flows. These plants are usually base loaded. The controlled hydro are plants associated with reservoir systems. One important type of constraint is the limit on the amount of energy that can be stored. In general, these constraints can be expressed as

$$x_i^-(k) \leq x_i(k) \leq x_i^+(k) \quad (2.14)$$

where $x_i^-(k)$ and $x_i^+(k)$ are upper and lower limits that can vary with time. For hydro units, these constraints reflect physical limitations on the size of the reservoir, as well as operational constraints imposed by other uses of the reservoir, such as flood control and recreation.

Another type of constraint is limits on the amount of energy that can be drawn for production of electricity. In general, these constraints can be written

$$u_i^-(k) \leq u_i(k) \leq u_i^+(k) \quad (2.15)$$

where $u_i^-(k)$ and $u_i^+(k)$ are again time-varying limits. In the case of hydro units, these constraints reflect physical limitations on the generators, as well as operational constraints imposed by other uses of the reservoir, such as irrigation, navigation, and water quality control. In the case of interchange contracts and nuclear reactors, these constraints can be very significant, reflecting both physical limitations and operations policies.

In some hydro plants the amount of spillage may also be constrained, either separately or in conjunction with the releases for generation electricity. In general, such constraints may be written

$$\eta_i^-(k) \leq \eta_i[u_i(k), s_i(k)] \leq \eta_i^+(k) \quad (2.16)$$

where η_i is an appropriate function and $\eta_i^-(k)$ and $\eta_i^+(k)$ are time-varying limits.

The electrical output of the generators can now be written in terms of the state variables and decision variables associated with the unit. In general, $f_i(k)$, the generation of the ith plant during time increment k , can be written as

$$f_i(k) = f_i[x_i(k), u_i(k)]. \quad (2.17)$$

For a hydro plant, f_i is primarily a function of $u_i(k)$; however, if generation varies significantly with the head of the reservoir, then there will also be a dependence on $x_i(k)$. For interchange contracts and nuclear reactors, the generation usually takes the simpler form

$$f_i(k) = u_i(k). \quad (2.18)$$

Considering the availability of generating units, some constraints specify that certain units must, if possible, be made available at specific time increments. These constraints can be written

$$\begin{aligned} \beta_j(k) &= 1, \quad k = M, M+1, \dots, M+N \\ &= 0, \quad k \in \text{another set} \end{aligned} \quad (2.19)$$

where $M, M+1, \dots, M+N$ are the time increments at which unit j must be available.

Minimum up-time or down-times can also be expressed. The constraint that j must remain down for N time increments after first being made unavailable can be expressed as

$$XD_j(k) \geq XDMIN_j \quad \text{if } \beta_j(k) = 1 \quad (2.20)$$

where $\beta_j(k)$ is change in status of unit j given by

$$\beta_j(k) = A_j(k+1) - A_j(k) \quad (2.21)$$

Numerous other constraints on the availability of units or combinations of units can be conveniently expressed in terms of these variables. These constraints can be combined with availability variables for other types of resources, such as fuel of specific types and maintenance crews. More details on these constraints can be found in the sections on the individual problems of unit commitment, production scheduling, and maintenance scheduling.

Considering the output of the generation units, again, numerous constraints can be imposed. One very important type of constraint is the maximum and minimum generation from each unit, as determined by physical limitations, economic feasibility, and operating policies. These constraints can be expressed as

$$g_j^-(k) \leq g_j(k) \leq g_j^+(k) \quad (2.22)$$

where the upper and lower limits can be time-varying. Note that this minimum generation constraint, if non-zero, is applicable only if the unit is available, i.e., if $A_j = 1$; if the unit is not available, i.e., if $A_j = 0$, then the generation will be zero.

2.5 PERFORMANCE CRITERIA

Now that the basic dynamic equations and constraints on the system decision variables have been specified, it is possible to define a criterion for the optimality of these variables. This criterion represents the total system operating cost over the operations planning interval. This cost includes production cost, unit start-up and shut-down costs, maintenance costs, transmission losses, nuclear refueling costs, interchange contracts, controllable load costs, emergency energy costs,

unmet demand penalties, penalties for violating system reliability constraints (spinning reserve, network constraints), and other factors associated with system operation.

The predominant cost in operations planning is production cost. Generally, it is assumed that only the units that do not store energy have production costs; this is consistent with the interpretation that the units that store energy are hydro units, while the units that do not store energy are thermal units that burn fossil fuel. In this case the production cost can be written

$$\begin{aligned} \text{PRCOST} = \sum_{k=0}^{N-1} L[x_1(k), x_2(k), \dots, x_T(k), \\ u_1(k), u_2(k), \dots, u_T(k) \\ A_1(k), A_2(k), \dots, A_T(k), k] \end{aligned} \quad (2.23)$$

where

PRCOST = total production cost over the operations
planning interval

N = number of stages in the operations planning
interval

L(k) = production cost over the kth time increment.

Inherent uncertainties within the problem may call for the modification of this production cost function. Such random effects include the uncertainties in fuel availability and/or cost. The former uncertainty affects the availability of units that utilize this fuel, while the latter uncertainty affects the cost of generation of the units that utilize this fuel. If $\underline{e}(k)$ is the vector of uncertain variables associated with fuel supply, then

$$L(k) = E_{\underline{e}(k)} \left[\sum_{j=1}^T A_j[\underline{e}(k), k] \phi_j[g_j(k), \underline{e}(k), k] \right] \quad (2.24)$$

where ϕ_j = cost of producing $g_j(k)$ units of electricity from plant j during time increment k .

In some cases, particularly when the time increments represented by K are at long duration, such as a week or a month, the characterization of $D(k)$ as a single number is not sufficient. In these cases it is possible to use load-duration curve techniques and stochastic production costing techniques to obtain more accurate costing. Also, it is possible to use the latter technique to explicitly account for uncertainties in the demand, fuel costs, fuel availability, and hydro inflows. The details of these techniques will be considered in a later section; however, in terms of the problem formulation, even when these techniques are applied, the production of cost still takes the basic form of Eq. (2.23).

There are two types of costs associated with the change in availability of the thermal units. One set of costs is related to the unit commitment problem, and the other to the maintenance scheduling problem. Both types of costs can be expressed in terms of the availability variables delineated in Eq. (2.6).

In the case of the unit commitment problem, there is a cost associated with starting up a unit and another cost associated with shutting down the unit. Typically, these costs vary with time; they may also vary with other factors, e.g., the shut-down cost may vary with the current output of the generator, while the start-up cost may depend on how long the unit has been shut down. If the variations are only with time, the total starting costs can be expressed as

$$STCOST = \sum_{k=0}^{N-1} \sum_{j=1}^T STC_j(XD_j(k), \beta_j(k)) \quad (2.25)$$

where

$STCOST$ = Total cost for unit start-ups

$STC_j(XD_j(k), \beta_j(k))$ = Start-up cost for unit j

$XD_j(k)$ = Cumulative down time for unit j

$\beta_j(k) = A_j(k) - A_j(k-1)$

Similarly, the total shut-down costs can be written as

$$SHCOST = \sum_{k=0}^{N-1} \sum_{j=1}^T SHC_j(XU_j(k), \beta_j(k)) \quad (2.26)$$

where

$SHCOST$ = Total cost for unit shut-downs

$SHC_j(XU_j(k), \beta_j(k))$ = Shut-down cost for unit j

$XU_j(k)$ = Cumulative up time for unit j

The maintenance costs are also related to the change in availability of the unit. Typically, however, the decision to take a unit down for maintenance has an effect over a much longer period of time than does a unit commitment decision--weeks as opposed to hours. Also, when a unit is taken down for maintenance, there is a nominal period required to perform the maintenance; thus, once the decision is made to maintain the unit, the time at which it can be put back into service may have little or no flexibility. Finally, the costs for maintenance are complex functions of variables associated with maintenance resources, such as crews and equipment. Notwithstanding these complexities, it is generally possible to express the cost of maintenance as a time-varying function of the decision to perform the maintenance; thus,

$$MACOST = \sum_{k=0}^{N-1} \sum_{j=1}^T MAC_j(k) \cdot M\beta_j(k) \quad (2.27)$$

where

$MACOST$ = Total maintenance cost

$MAC_j(k)$ = Maintenance cost if unit j is taken down for maintenance for a nominal period beginning at time increment k

and where it is understood that $M\beta_j(k) = 1$ refers to a decision to start maintenance on a unit, not to a normal shut-down. If there is some flexibility in the length of the period required to perform the maintenance, e.g., if the maintenance period can be shortened by having the crews work overtime, then the change in maintenance costs (increase or decrease) can be expressed as a function of the time at which the unit is brought back into service. Also, there may be some uncertainty associated with the time duration of the maintenance interval; this uncertainty can be modeled by including a dependence of the cost on $C_j(k)$, when $C_j(k) = -1$ is understood to mean that maintenance is completed at a time increment k that is different from that expected on the basis of the decisions made so far. The total change in maintenance costs due to these effects can be expressed as

$$DMCOST = \sum_{k=0}^{N-1} \sum_{j=1}^T DMC_j[C_j(k), k] \cdot N\beta_j(k) \quad (2.28)$$

where

$DMCOST$ = Total change in maintenance cost due to a decision to change in end of maintenance period from nominal value

$DMC_j[C_j(k), k]$ = Change in maintenance cost of j th unit if maintenance period ends at time increment k rather than at the nominal time due to either a planned decision ($C_j(k) = 0$) or a random termination of the maintenance duration ($C_j(k) = -1$)

and where $N\beta_j(k) = 1$ is understood to be the end of the maintenance period, not a normal unit start-up.

Further details on the costs for unit commitment and maintenance scheduling can be found in the sections discussing these problems.

Another type of cost that is of concern in the operations planning problem is system reliability. Generally, system reliability is related to system loss-of-load probability, to the expected unserved energy for the system, or to some other indicator. System reliability is primarily a function of the available capacity relative to demand and of the outage probabilities of the available units. Thus, this portion of the criterion can be expressed as

$$\text{SECOST} = E \left[\sum_{k=0}^{N-1} \text{SEC}[A_1(k), A_2(k), \dots, A_T(k), \right. \quad (2.29)$$

$$\left. C_1(k), C_2(k), \dots, C_T(k), K_1, K_2, \dots, K_T, D(k)] \right]$$

where

SECOST = Total cost associated with system security

SEC = Cost associated with system security during
time increment k

K_j = Capacity of j th unit.

There may also be dependence on the availability of energy-storing plants (if they are not always available) and on specific generator outputs of both energy-storing and non-energy-storing plants. Indicators such as loss-of-load probability and expected unserved energy can be conveniently calculated using stochastic production costing techniques.

Another part of the system operating cost is related to the flexibility (if any) in interchange contracts and nuclear refuelings. It may be possible to vary the amount of energy in an interchange contract or a nuclear refueling, but at some cost. A term can be added to account for this change in cost; the effect on production cost will be seen by changing the amount of energy in the state variable x_1 corresponding to this contract of nuclear reactor.

Still another cost is transmission losses. With the development of integrated power systems and the interconnection of operating companies for purposes of economy interchange, it is necessary to consider not only the incremental fuel costs but also the incremental transmission losses for optimum economy.

It is possible to enumerate still other costs associated with operations planning. However, it is felt that this list is sufficient for the general problem formulation. More details on these costs and some additional costs will be determined in the formulation of the unit commitment, production scheduling, and maintenance scheduling problems.

2.6 SUMMARY OF FORMULATION

The prior description provided for the development of a unified methodology for analyzing the overall, integrated problem, by means of a single stochastic formulation that encompasses all aspects of the operations scheduling problem. In the global formulation all variables are classified as state variables, decision variables and exogeneous variables. Further, the decision variables are classified as those representing the output of energy-storage units, the output of non-storage units and binary variables that determine the availability of generating units.

In the general case, the dynamic equations for energy-storing units can be written as:

$$\underline{x}(k+1) = \underline{x}(k) - \underline{u}(k) - \underline{s}(k) + Y[\underline{u}(k), \underline{s}(k)] + \underline{W}(k) + \underline{w}(k) \quad (2.30)$$

where

$\underline{x}(k)$ = state vector with components $x_i(k)$, $k=1,2,\dots,H$, the amount of energy in unit i at the beginning of time increment k

$\underline{u}(k)$ = decision vector with components $u_i(k)$, $i=2, \dots, H$, the amounts of energy removed from unit i to generate electricity during time increment k

$\underline{s}(k)$ = vector with components $s_i(k)$, $i=2, \dots, H$, the amount of energy removed from unit i during time increment k that is not used to generate electricity

Y = $H \times H$ energy transfer matrix, where component Y indicates the fraction of energy release $u_j(k)$ and other releases $s_i(k)$ that become available for storage in unit i (typically, $Y_{ji} = 0$ or 1)

$\underline{W}(k)$ = deterministic exogenous input vector with components $W_i(k)$, $i=1, 2, \dots, M$, the deterministic input into unit i during time increment k

$\underline{w}(k)$ = random exogenous input vector with components $w_i(k)$, $i=1, 2, \dots, M$, the random input into unit i during time increment k .

The dynamic equations for the availability of non-storage units can be expressed as

$$\underline{A}(k) = \underline{B}(k) \odot \underline{C}(k) \quad (2.31)$$

where

$\underline{A}(k)$ = state vector with components $A_j(k)$, $j=1, 2, \dots, T$, availability of unit j ; $A_j(k) = 0$ or 1

$\underline{B}(k)$ = decision vector with components $B_j(k)$, $j=1, 2, \dots, T$, change in availability of unit j ; $B_j(k) = 0$ or 1

$\underline{C}(k)$ = random input vector with components $C_j(k)$, $j=1, 2, \dots, T$, change in availability of unit j , due to stochastic effects such as forced outage, random duration of maintenance interval, or construction delay; $C_j(k) = 0$ or 1

$\odot$ = component by component multiplication.

The dynamic equations for time since last change in availability of non-storage devices are given by:

$$\underline{XU}(k+1) = (\underline{XU}(k) + 1) \odot A(k) \quad (2.32)$$

$$\underline{XD}(k+1) = (\underline{XD}(k) + 1) \odot (1 - A(k))$$

where

$\underline{XU}(k)$ = state vector with components $XU_j(k)$
 $j=1,2,\dots,T$, the cumulative up time of unit j

$\underline{XD}(k)$ = state vector with components $XD_j(k)$
 $j=1,2,\dots,T$, the cumulative down time of unit j .

The constraints on the state and decision variables are represented as

$$\underline{x}(k) \in X \quad (2.33)$$

$$\underline{u}(k) \in U$$

$$\underline{s}(k) \in \mathcal{P}$$

$$\underline{B}(k) \in \mathcal{B}$$

$$\underline{XU}(k) \in \Gamma$$

$$\underline{XD}(k) \in \Delta$$

where X , U , $\mathcal{P}$, $\mathcal{B}$, Γ , and Δ are appropriate constraint sets that can vary with k and include dependence across both types of state and decision variables.

The constraint for satisfying the demand is given by

$$EH[\underline{x}(k), \underline{u}(k)] + ET[\underline{A}(k), \underline{G}(k)] + LC(k) + UL(k) = \bar{D}(K) + d(k) \quad (2.34)$$

where

$\underline{G}(k)$ = decision vector with components $g_j(k)$, $j=1,2,\dots,T$,
the output of non-energy-storing unit j at time k

$\bar{D}(k)$ = expected system demand at time k

$d(k)$ = uncertainty in system demand at time k , and probability
distribution of which is $P[d(k)]$

EH = total generation from energy-storing units at time k

ET = total generation from non-energy-storing units at time k

LC = total controllable load not supplied at time k

UL = total unmet load at time k .

The criterion for the performance optimization is the total system operating costs over the entire operations planning period, T . These costs are summarized as follows:

PRODUCTION COST:

$$\text{PRCOST} = E \sum_{k=0}^{N-1} L[\underline{x}(k), \underline{u}(k), \underline{A}(k), \underline{G}(k), \bar{D}(k), d(k), \underline{e}(k), k] \quad (2.35)$$

where

E = expected value operator

$\underline{e}(k)$ = uncertainties in fuel price, the probability
distribution of which is $p[\underline{e}(k)]$

and where $\underline{G}(k)$ is determined by the economic dispatching operation for each $\underline{x}(k)$, $\underline{u}(k)$, $\underline{A}(k)$, $\bar{D}(k)$, and $\underline{e}(k)$.

START-UP/SHUT-DOWN COSTS:

$$STCOST + SHCOST = \sum_{k=0}^{N-1} \{h_1[\underline{x}(k), \beta(k)] + h_2[\underline{x}(k), \beta(k)]\} \quad (2.36)$$

MAINTENANCE COSTS:

$$MACOST = E \left[\sum_{k=0}^{N-1} \{h_3[\underline{B}(k), \underline{C}(k), \underline{T}(k)]\} \right] \quad (2.37)$$

SYSTEM RELIABILITY COST:

$$SRCOST = E \left[\sum_{k=0}^{N-1} h_5[\underline{A}(k), \underline{x}(k), \underline{C}(k), \underline{D}(k), d(k)] \right] \quad (2.38)$$

UNMET LOAD COST:

$$ULCOST = E \left[\sum_{k=0}^{N-1} h_6[UL(k), d(k)] \right] \quad (2.39)$$

CONTROLLABLE LOAD COST:

$$LCOST = \sum_{k=0}^{N-1} h_7[LC(k)] \quad (2.40)$$

CHANGE IN INTERCHANGE CONTRACT PROVISION COST:

$$ICCOST = \sum_{k \in K^*} h_8[\underline{x}(k), \underline{W}(k)] \quad (2.41)$$

CHANGE IN NUCLEAR REFUELING ENERGY COST:

$$NRCOST = \sum_{k \in K^*} h_9[\underline{x}(k), \underline{W}(k)] \quad (2.42)$$

TRANSMISSION LOSSES COST:

$$\text{TRCOST} = \sum_{k=0}^{N-1} h_{10}[\underline{u}(k), \underline{G}(k), k] \quad (2.43)$$

It is possible to enumerate still other parameters associated with operations scheduling. However, it is felt that this list is sufficient for the problem formulation. More details will be developed in the next phase of the project.

III. TIME-SCALE DECOMPOSITION

3.1 BACKGROUND

Operations scheduling in electric power systems involves decisions made over time-scales from hours (short-term hydro scheduling, unit commitment) to years (long-term hydro reserve management, nuclear refueling, maintenance scheduling). As discussed earlier, this problem can nonetheless in principle be solved by stochastic dynamic programming. However, the computational intractability of this approach has led practitioners to partition the operations scheduling problem along heuristic lines, resulting in a set of subproblems treated as though each were independent of the others. The result of this approach is the traditional definition of subproblems in operations scheduling, where, for example, the costs computed for the seasonal hydro problem are assumed independent of the commitment of units on the shortest time-scale. The use of this assumption in the operations scheduling problem leads to suboptimal solutions; furthermore, the structure of this ad hoc solution methodology does not permit easy introduction of correction terms to reduce the degree of suboptimality achieved following the partitioning of subproblems.

The objective of the time-scale decomposition research effort is to develop rigorous procedures for separating the overall operations scheduling problem into subproblems, and for recomposing subproblem solutions into an overall solution. Bounds on the loss of optimality from this method, and the proper introduction of error correction terms are also important goals of this research. Furthermore, the determination of the time-scales on which the decomposition is itself to be based is of interest here; the resolution of this issue will rest more on the exploitation of natural time-scales in the operations scheduling problem, rather than being prejudiced by existing conventional approaches. It is in fact anticipated that different types of reservoir systems will lead to different specifications of subproblems, according to the natural periodicities associated with that system.

Table 3.1 shows state variables, decision variables, and exogenous deterministic and random variables associated with the operations scheduling problem, together with the time-scale with which these variables are normally associated. Note that hydro reservoir scheduling, demand, and hydro inflows are present in all time-scales; this observation led us to select the hydro scheduling problem as a starting point for the development of time-scale decomposition methods.

The basic conceptual tool for time-scale decomposition is the singular perturbation method [1,2]. This theory has reached a highly developed state for deterministic problems wherein phenomena occur on distinct time-scales. Examples of applications include trajectory optimization, electric power plant analysis and jet engine control [3,6]. With the accommodation of uncertainty in operations scheduling as a major focus of this research, however, a corresponding theory for singular perturbation in stochastic systems is required. This need has led to the use of model structures involving Markov chains, where now "fast" states are represented by high probabilities of transition, while "slow" states are represented by low probabilities of transition. This formalism allows one to capture the usual advantages of singular perturbation analysis, including prescriptions for decomposing systems into weakly coupled subsystems, and asymptotic results concerning the associated optimization problem. The main accomplishment of the work reported on in this chapter concerns the establishment of this model structure, and the proof of its properties.

Ref. [7] treats the hydro scheduling problem in a manner similar to that presented here. A multiple time-scale stochastic optimal control problem with discounted cost function is formulated and discretized to yield a Markov chain problem. The results described here treat the same problem with a non-discounted cost function (a minimum average cost per unit time problem). Theoretical background in weakly coupled Markov chains is contained in [8, 9]. A discussion relating results in weakly coupled Markov chains to those obtained for discrete-time singularly perturbed systems is contained in [10].

TABLE 3.1 TIME-SCALES IN OPERATIONS SCHEDULING

	TREATMENT IN SUBPROBLEM		
	LONG-TERM	MEDIUM-TERM	SHORT-TERM
<u>STATE VARIABLES</u>			
Maintenance or Refueling Status	✓	-	-
States of Nuclear Core	✓	-	-
Hydro Reservoir Contents	✓	✓	✓
Status of Energy-Limited Inter- Change Contracts	-	✓	✓
Unit Commitment Status	-	-	✓
Status of Controllable Load	-	-	✓
Contents of Energy Storage Devices....	-	-	✓
<u>DECISION VARIABLES</u>			
Maintenance Schedule	✓	-	-
Nuclear Refueling Schedule	✓	-	-
Hydro Release Schedule	✓	✓	✓
Interchange Schedule	-	✓	✓
Unit Start-up and Shut-down	-	-	✓
Generation Dispatch	-	-	✓
Switching of Controllable Load	-	-	✓
Operating Schedule of Energy Storage	-	-	✓
<u>EXOGENOUS DETERMINISTIC AND RANDOM VARIABLES</u>			
Load (Deterministic and Random)	✓	✓	✓
Hydro Inflows (Deterministic and Random)	✓	✓	✓
Power Plant Availability	✓	✓	-
Fuel Supply	✓	✓	-
Fuel Cost	✓	✓	-
Maintenance Duration	✓	✓	-
Maintenance Crew Availability	✓	-	-
Construction Delay	✓	-	-
Demand Uncertainty on Controllable Load	-	-	✓

NOTES: ✓ = Present in Subproblem

- = Can Generally be Excluded from the Subproblem

This chapter is organized as follows: Section 3.2 formulates the multiple time-scale hydro scheduling problem. The sufficient conditions for optimality for this problem are discretized to yield the optimization problem for weakly coupled Markov chains presented in Section 3.3. Section 3.4 contains theoretical results for this class of problems. Section 3.5 contains proofs of the theorems in Section 3.4. Simulation results and directions for future research are described in Sections 3.6 and 3.7.

3.2 PROBLEM FORMULATION: HYDRO SCHEDULING WITH MULTIPLE TIME SCALES

As a starting point for our research, we concentrate on the optimization of hydro resources. The time-scales under consideration range from hours to years. The fast time-scale problem involves determination of hour-by-hour releases over a week as a function of weekly releases and fast variations in load. The slowest time-scale problem involves determination of week-by-week releases over a year or more as a function of random inflows and the seasonal variation in load. These two problems are not independent; the cost function for the long term problem is a function of how water is released in the short term.

The natural periodicities associated with the hydro scheduling problem include daily, weekly and yearly periodicities in load as well as seasonal or yearly periodicities in reservoir inflows. In addition, the variations in reservoir capacities (both storage and release) introduce a multiple time-scale behavior which may be exploited through decomposition.

Before formulating the optimization problem associated with hydro scheduling on multiple time-scales, the reservoir dynamics, inflow and demand models are described and the cost function reviewed. In contrast to Chapter II, these models are presented in continuous time format; discretization will lead to a discrete time problem after formulation of the conditions for optimality.

Reservoir Dynamics Model

The dynamic equations for the reservoirs can be written in the form

$$dx' = (w' - u' - s' + B'u' + C's')dt \quad (3.1)$$

where τ is in units of weeks and

x' = state vector with components $x'_i(\tau)$, $i=1, \dots, N$ which are the amount of water stored in reservoir i at time τ .

u' = decision vector with components $u'_i(\tau)$, $i=1, \dots, N$ which are the rates of water release from reservoir i for electric generation at time τ .

s' = decision vector with components $s'_i(\tau)$, $i=1, \dots, N$ which are the rates of water spillage at time τ .

B' = $N \times N$ water transfer matrix, where the component b'_{ij} denotes the fraction of the rate of release u'_j which becomes available to reservoir i (typically, $b'_{ij} = 0$ or 1).

C' = $N \times N$ water transfer matrix, where the component c'_{ij} denotes the fraction of the rate of spillage v'_j which become available to reservoir i (typically, $c'_{ij} = 0$ or 1).

m' = inflow vector with components $m'_i(\tau)$, $i=1, \dots, N$, denoting the rate of inflow into reservoir i at τ .

Note that this model neglects delays between reservoirs; these can be accommodated by appropriate modifications to (3.1).

The release rates for electric generation and spillage are constrained by

$$0 \leq u_i' \leq u_{imax} \quad (3.2)$$

$$0 \leq u_i' \leq s_{imax} \quad i=1, \dots, N. \quad (3.3)$$

The maximum spillage constraint is for flood control, fish habitat maintenance, adherence to water laws, etc.; this constraint is not normally active. The states are constrained by

$$0 \leq x_i' \leq x_{imax} \quad i=1, 2, \dots, N \quad (3.4)$$

The reservoir water level is often constrained to be greater than some minimum value. This constraint is accounted by defining x_i' to be the amount of water above this minimum value.

Inflow Model

A number of inflow models are possible. Here we assume that the reservoir inflows take the form

$$dw_i' = f_i(\tau, w_i'(\tau))dt + \sqrt{\rho_i(\tau, w_i'(\tau))} dB_\tau^i \quad i=1, \dots, N \quad (3.5)$$

where f_i and ρ_i may be obtained by identification [7] and the B_τ^i are independent Brownian motions.

Load Model

The load may be modeled as the diffusion process [7]

$$dD = \epsilon \mathcal{B}(\epsilon\tau, \tau, D)dt + \sqrt{\alpha\epsilon} dB_\tau \quad (3.6)$$

where D has periods, dependent on $\mathcal{B}$, in $\epsilon\tau$ (years) and τ (weeks) where $\epsilon = 1/52$ is a time scaling parameter. The load may also be modeled as a Markov chain as described in [11].

Cost Function

For the hydro scheduling problem formulation treated here, the value of water is the cost of the displaced thermal generation; the associated optimization problem is to minimize production cost. Let $e_i'(x_i', u_i')$ be the amount of electricity generated by release rate u_i' when the amount of water in reservoir i is x_i' . Then given that the demand is D , the thermal part of demand the power system must generate

$$T = D - \sum_{i=1}^N e_i'(x_i', u_i') \quad (3.7)$$

and the production cost is

$$c(T) = c\left(D - \sum_{i=1}^N e_i'(x_i', u_i')\right) \quad (3.8)$$

where $c(\cdot)$ is usually a convex function of the required thermal generation T .

The optimization problem below will require computing $c(T)$. This cost may be obtained in a number of different ways. Given a thermal unit loading order a total expected incremental cost (TEIC) curve may be used. Alternatively, the unit commitment may be solved in parallel with the problem and cost computed from the solution.

Optimization Problem

The problem examined here is that of minimizing $J(u', s')$, the expected value of the average production cost per unit time given by

$$J(u', s') = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} E \left\{ \int_0^{\tau} c\left(D - \sum_{i=1}^N e_i'(x_i', u_i')\right) d\tau \right\} \quad (3.9)$$

Hence the problem is

$$\begin{array}{ll} \text{Minimize} & J(u', s') \\ \{u_i', s_i' \mid i=1, \dots, N\} & \end{array}$$

subject to (3.1) - (3.9) where $u'_i(x', w', D)$, $s'_i(x', w', D)$ are stationary feedback policies.

3.3 TIME-SCALE DECOMPOSITION APPROACH: FORMULATION OF MARKOV CHAIN MODEL

Sufficient conditions for this minimization are given by the stochastic version of the Hamilton Jacobi equation (HJE). Prior to setting up these sufficient conditions we scale the variables in (3.1) - (3.9) to bring out the natural time-scales present in the problem. The HJE for these scaled equations is then derived. In order to obtain an approximate solution to the HJE, it is discretized by the method of stochastic approximations [12] to yield an optimization problem for a discrete time Markov chain. This Markov chain is weakly coupled, because of the scaling of the equation prior to derivation of the HJE. Time-scale decomposition techniques are applied to the weakly coupled Markov chain optimization problem.

Scaling Variables

Defining

$$\left. \begin{aligned} x_i &= x'_i / x_{imax} \\ u_i &= u'_i / u_{imax} \\ s_i &= s'_i / u_{imax} \\ w_i &= w'_i / u_{imax} \\ b_{ij} &= b'_{ij} \frac{u_{jmax}}{u_{imax}} \\ c_{ij} &= c'_{ij} \frac{u_{jmax}}{u_{imax}} \end{aligned} \right\} i=1,2,\dots,N \quad (3.10)$$

$$H' = \text{diag} \left[\frac{u_{1\max}}{x_{1\max}} \quad \frac{u_{2\max}}{x_{2\max}} \quad \dots \quad \frac{u_{N\max}}{x_{N\max}} \right] \quad (3.11)$$

we obtain the transformed reservoir dynamics equation

$$dx = H'(w-u-s + Bu + Cs)dt \quad (3.12)$$

Assume that the components of x' in (3.1) or x in (3.12) are ordered so that

$$x_{1\max} \geq x_{2\max} \geq \dots \geq x_{N\max} \quad (3.13)$$

and that the first N_S reservoirs have sufficient capacity to be considered seasonal or annual reservoirs and that the last $N_F = N - N_S$ reservoirs are weekly reservoirs, with k defined such that

$$x_{(N_S + 1)\max} = \epsilon k x_{N_S\max} \quad (3.14)$$

where k is a constant and $k\epsilon \ll 1$, equation (3.12) may be written

$$dx = \begin{bmatrix} \epsilon k I_{N_S} & 0 \\ 0 & I_{N_F} \end{bmatrix} H(w-u-s + Bu + Cs)dt \quad (3.15)$$

I_{N_S} and I_{N_F} are identity matrices of appropriate order, and

$$H = \text{diag} \left[\sigma \frac{u_{1\max}}{x_{1\max}} \quad \dots \quad \sigma \frac{u_{N_S\max}}{x_{N_S\max}} \quad \frac{u_{(N_S+1)\max}}{x_{(N_S+1)\max}} \quad \dots \quad \frac{u_{N\max}}{x_{N\max}} \right] \quad (3.16)$$

$$\sigma = \frac{x_{N_S\max}}{x_{(N_S+1)\max}} \quad (3.17)$$

where the diagonal elements of H are of the same order. Equation (3.15) shows that the first N_S reservoirs change levels much slower than the levels of the last N_F reservoirs. For this reason, (3.15) is referred to as a model of the reservoir dynamics in a fast time-scale. This model may be translated into the slow time-scale by setting $t = \epsilon \tau$. Then (3.15) becomes

$$\begin{bmatrix} 1/k I_{N_S} & 0 \\ 0 & \epsilon I_{N_F} \end{bmatrix} dx = H(w-u-s + Bu + Cs) d\tau. \quad (3.18)$$

Setting $\epsilon=0$ in (3.18) yields the equilibrium condition for the last N_F reservoirs

$$w_i - u_i - s_i + \sum_{j=1}^N b_{ij} u_j + \sum_{j=1}^N c_{ij} s_j = 0 \quad i=N_S+1, \dots, N.$$

Under these conditions, the inflows into the small (weekly) reservoirs are equal to the outflows. This is referred to as a slow time-scale model, where small reservoirs have no storage capacity over a long period.

Sufficient Conditions for Optimality

The minimization problem can be restated

$$\begin{array}{l} \text{Minimize } J(u, s) \\ u, s \end{array}$$

subject to equations (3.2) - (3.9), (3.15). The sufficient conditions for this problem are given by Hamilton-Jacobi equation:

$$\frac{\delta J^\epsilon}{\delta \tau} + \epsilon \beta(\epsilon \tau, \tau, D) \frac{\delta}{\delta D} J^\epsilon + \frac{1}{2} \epsilon \alpha \frac{\delta^2 J^\epsilon}{\delta D^2}$$

$$\begin{aligned}
& + \min_{u,s} \left[\epsilon k \sum_{i=1}^{N_S} \sigma \frac{u_{imax}}{x_{imax}} [w_i - u_i - s_i + \sum_{j=1}^N (b_{ij} u_j + c_{ij} s_j)] \frac{\partial J^\epsilon}{\partial x_i} \right. \\
& + \sum_{i=N_S+1}^N \frac{u_{imax}}{x_{imax}} [w_i - u_i - s_i + \sum_{j=1}^N (b_{ij} u_j + c_{ij} s_j)] \frac{\partial J^\epsilon}{\partial x_i} \left. \right] \quad (3.19)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^N [f_i(\tau, u_{imax} w_i(\tau)) \frac{\partial J^\epsilon}{\partial w_i} + \frac{1}{2} \rho_i(\tau, u_{imax} w_i(\tau)) \frac{\partial^2 J^\epsilon}{\partial w_i^2}] \\
& + c \left(D - \sum_{i=1}^N e_i(x_i, u_i) \right) = 0
\end{aligned}$$

where J^ϵ is the optimal value function and

$$e_i(x_i, u_i) = c_i'(x_i, x_{imax}, u_i, u_{imax}) \quad (3.20)$$

An approximate solution to the optimization problem may be obtained by discretizing the HJE by the method of stochastic approximations to obtain an optimization for Markov chains.

Discretization

In the stochastic approximation the states x_i , w_i and D and time τ are discretized. Time is discretized in increments of $\Delta\tau$. The quantity of water in reservoir i , $i=1,2,\dots,N$ is discretized into n_i+1 values

$$x_i \in (0, \frac{1}{n_i}, \frac{2}{n_i}, \dots, 1),$$

and the inflows and load are similarly discretized into k_i+1 and $k_{N+1}+1$ levels, respectively. The state of the Markov chain is the $(2N+1)$ tuple $\{x_1, x_2, \dots, x_N, w_1, w_2, \dots, w_N, D\}$ where the values of x_i, w_i and D are any

of the possible discretized values. The total number of states is $\ell = (h_1+1)(h_2+1)\dots(h_N+1)(k_1+1)\dots(k_{N+1})$. A given state of the Markov chain is occupied if the quantities of water in the N reservoirs are given by the first N values, the inflows by the next N values and the demand by the last value of the $(2N+1)$ tuple.

In order to obtain the Markov chain formulation from the HJE, the partial derivatives are approximated by

$$\begin{aligned} \frac{\partial J^\varepsilon}{\partial \tau}(\tau, x_1, x_2, \dots, x_N, w_1, \dots, w_N, D) &\approx \frac{J^\varepsilon(\tau+\Delta\tau, x_1, \dots, D) - J^\varepsilon(\tau, x_1, \dots, D)}{\Delta\tau} \\ m(\tau, x_1, \dots, r, \dots, D) \frac{\partial J^\varepsilon}{\partial r} &\approx \left[m^+(\tau+\Delta\tau, x_1, \dots, r+\Delta r, \dots, D) \frac{J^\varepsilon(\tau+\Delta\tau, \dots, r+\Delta r, \dots, D) - J^\varepsilon(\tau+\Delta\tau, \dots, r, \dots, D)}{\Delta r} \right. \\ &\quad \left. - m^-(\tau+\Delta\tau, x_1, \dots, r+\Delta r, \dots, D) \frac{J^\varepsilon(\tau+\Delta\tau, \dots, r, \dots, D) - J^\varepsilon(\tau+\Delta\tau, \dots, r-\Delta r, \dots, D)}{\Delta r} \right] \\ \frac{1}{2} m(\tau, x_1, \dots, r, \dots, D) \frac{\partial^2 J^\varepsilon}{\partial r^2} &\approx \left[\frac{1}{2} \frac{m}{2} \frac{J^\varepsilon(\tau+\Delta\tau, \dots, r+\Delta r, \dots, D) + J^\varepsilon(\tau+\Delta\tau, \dots, r-\Delta r, \dots, D) - 2J^\varepsilon(\tau+\Delta\tau, \dots, r, \dots, D)}{(\Delta r)^2} \right] \end{aligned}$$

where

$$\Delta x_i \triangleq 1/h_i \quad (3.21a)$$

$$\Delta w_i \triangleq (w_{i\max}/u_{i\max})/k_i \quad (3.21b)$$

$$\Delta D \triangleq D_{i\max}/k_{N+1} \quad (3.21c)$$

$$m^+(\cdot) \triangleq \max(m(\cdot), 0) \quad (3.22a)$$

$$m^-(\cdot) \triangleq \min(m(\cdot), 0) \quad (3.22b)$$

and m and r are any of the functions and parameters in the partial derivatives of the HJE. By properly arranging the terms in this discretization the following optimization problem is obtained:

$$\begin{aligned} & \text{Minimize} \quad J_\epsilon(u, s) = \frac{1}{1+T} E \sum_{\tau=0}^T c(D - \sum_{i=1}^N e_i(x_i, u_i)) \Delta\tau \\ & \{u_i, s_i \mid i=1, \dots, N\} \end{aligned} \quad (3.23)$$

subject to

$$0 \leq u_i \leq 1 \quad (3.24)$$

$$0 \leq s_i \leq s_{\max}/u_{\max} \quad (3.25)$$

$$p(\tau + \Delta\tau) = p(\tau) [A(u, s) + \epsilon B(u, s)] \quad (3.26)$$

where the ℓ -dimensional row vector $p(\tau)$ has elements $p_n(\tau)$ which represent the probability that state n is occupied at time τ , the matrix $A(u, s)$ is block diagonal,

$$A(u, s) = \text{diag}[A_{11}(u, s) \ A_{22}(u, s) \dots A_{KK}(u, s)], \quad (3.27)$$

and the elements $a_{ij}(u, s)$ and $b_{ij}(u, s)$, $i, j=1, \dots, \ell$, satisfy

$$b_{ij}(u, s) \geq 0 \text{ if } a_{ij}(u, s) = 0 \quad (3.28)$$

$$\sum_{j=1}^{\ell} b_{ij}(u, s) = 0 \quad (3.29)$$

This chain is referred to as a Markov chain with strong and weak interactions or as a weakly coupled Markov chain. The strength of the coupling is dependent on the small parameter ϵ .

An example of the solution to this optimization problem is given in Section 3.5. The solution takes the form of a feedback policy $u(n)$, $s(n)$ for n being each of the states $\{x_1, x_2, \dots, x_N, w_1, \dots, w_N, D\}$.

Two Reservoir Example

The following two reservoir example shows how the Markov chain optimization problem is set up in accordance with the general approach described above. It shows how a Markov chain can even be obtained for a deterministic problem.

Consider the scaled reservoir dynamic system

$$dx_1 = \epsilon(m_1 - u_1 - s_1)d\tau \quad (3.30)$$

$$dx_2 = (u_1 + s_1 - u_2 - s_2)d\tau \quad (3.31)$$

where τ is in units of weeks,

$$0 \leq u_i \leq 1 \quad i=1,2 \quad (3.32)$$

$$0 \leq v_i \quad i=1,2 \quad (3.33)$$

$$0 \leq x_i \leq 1 \quad i=1,2 \quad (3.34)$$

w_1 is a constant known inflow, and D is a constant known demand. This is a model of series reservoirs with inflow only into the upper reservoir. Let the problem be to minimize production cost

$$J^E(u, s) = \int_0^{52} c(D - \sum_{i=1}^2 e_i(x_i, u_i))d\tau \quad (3.35)$$

The Hamilton Jacobi equation for this problem can be written

$$\frac{\partial V^\varepsilon}{\partial \tau} + \min_{\substack{u_1, u_2 \\ s_1, s_2}} \left[\varepsilon(w_1 - u_1 - s_1) \frac{\partial V^\varepsilon}{\partial x_1} + (u_1 + s_1 - u_2 - s_2) \frac{\partial V^\varepsilon}{\partial x_2} + c(D - \sum_{i=1}^2 e_i(x_i, u_i)) \right] = 0. \quad (3.36)$$

Discretizing, we obtain

$$x_i \in \{0, \Delta x_i, 2\Delta x_i, \dots, h_i x_i = 1\}$$

$$\frac{\partial V^\varepsilon}{\partial \tau} = \frac{V^\varepsilon(\tau + \Delta \tau, x_1, x_2) - V^\varepsilon(\tau, x_1, x_2)}{\Delta \tau}$$

$$\varepsilon(w_1 - u_1 - s_1) \frac{\partial V^\varepsilon}{\partial x_1} = \left[\varepsilon(w_1 - u_1 - s_1) + \frac{V^\varepsilon(\tau + \Delta \tau, x_1 + \Delta x_1, x_2) - V^\varepsilon(\tau + \Delta \tau, x_1, x_2)}{\Delta x_1} + \varepsilon(s_1 + u_1 - w_1) + \frac{V^\varepsilon(\tau + \Delta \tau, x_1 - \Delta x_1, x_2) - V^\varepsilon(\tau + \Delta \tau, x_1, x_2)}{\Delta x_1} \right]$$

$$(u_1 + s_1 - u_2 - s_2) \frac{\partial V^\varepsilon}{\partial x_2} = \left[(u_1 + s_1 - u_2 - s_2) + \frac{V^\varepsilon(\tau + \Delta \tau, x_1, x_2 + \Delta x_2) - V^\varepsilon(\tau + \Delta \tau, x_1, x_2)}{\Delta x_2} + (u_2 + s_2 - u_1 - s_1) + \frac{V^\varepsilon(\tau + \Delta \tau, x_1, x_2 - \Delta x_2) - V^\varepsilon(\tau + \Delta \tau, x_1, x_2)}{\Delta x_2} \right]$$

Next we set

$$p_{u,s}(x_1, x_2 | x_1, x_2) = 1 - \frac{\varepsilon |w_1 - u_1 - s_1| \Delta \tau}{\Delta x_1} - \frac{\varepsilon |u_1 + s_1 - u_2 - s_2| \Delta \tau}{\Delta x_2} \quad (3.37a)$$

$$p_{u,s}(x_1+\Delta x_1, x_2 | x_1, x_2) = \frac{\varepsilon(w_1 - u_1 - s_1)^+ \Delta \tau}{\Delta x_1} \quad (3.37b)$$

$$p_{u,s}(x_1-\Delta x_1, x_2 | x_1, x_2) = \frac{\varepsilon(u_1 + s_1 - w_1)^+ \Delta \tau}{\Delta x_1} \quad (3.37c)$$

$$p_{u,s}(x_1, x_2+\Delta x_2 | x_1, x_2) = \frac{(u_1 + s_1 - u_2 - s_2)^+ \Delta \tau}{\Delta x_2} \quad (3.37d)$$

$$p_{u,s}(x_1, x_2-\Delta x_2 | x_1, x_2) = \frac{(u_2 + s_2 - u_1 - s_1)^+ \Delta \tau}{\Delta x_2} \quad (3.37e)$$

Since the sum of the $p_{u,s}$ in equations (3.37) is one, equations (3.37) can be interpreted as the transition matrix for a Markov chain for $\Delta \tau$ small enough for all $p_{u,s}$ to be less than one. Substituting the partial derivative expressing and (3.37) into the HJE yields.

$$\begin{aligned} V^E(\tau, x_1, x_2) = & \min_{u_1, u_2, s_1, s_2} \{ V^E(\tau+\Delta \tau, x_1, x_2) p_{u,s}(x_1, x_2 | x_1, x_2) \\ & + V^E(\tau+\Delta \tau, x_1+\Delta x_1, x_2) p_{u,s}(x_1+\Delta x_1, x_2 | x_1, x_2) \\ & + V^E(\tau+\Delta \tau, x_1-\Delta x_1, x_2) p_{u,s}(x_1-\Delta x_1, x_2 | x_1, x_2) \\ & + V^E(\tau+\Delta \tau, x_1, x_2+\Delta x_2) p_{u,s}(x_1, x_2+\Delta x_2 | x_1, x_2) \\ & + V^E(\tau+\Delta \tau, x_1, x_2-\Delta x_2) p_{u,s}(x_1, x_2-\Delta x_2 | x_1, x_2) \\ & + c(D - \sum_{i=1}^2 e_i(x_1, u_i)) \Delta \tau \} \quad (\tau = 52-\Delta \tau, \dots, 0). \end{aligned} \quad (3.38)$$

Equation (3.38) is the Bellman equation [13] for a Markov chain defined by (3.37). If x_1 and x_2 are discretized $\Delta x_1 = 1/2$, $\Delta x_2 = 1$, the six states of the Markov chain (3.26) are

$$\begin{aligned}
 n = 1 &\Rightarrow (x_1, x_2, w, D) = (0, 0, w, D) \\
 n = 2 &\Rightarrow (x_1, x_2, w, D) = (0, 1, w, D) \\
 n = 3 &\Rightarrow (x_1, x_2, w, D) = (1/2, 0, w, D) \\
 n = 4 &\Rightarrow (x_1, x_2, w, D) = (1/2, 1, w, D) \\
 n = 5 &\Rightarrow (x_1, x_2, w, D) = (1, 0, w, D) \\
 n = 6 &\Rightarrow (x_1, x_2, w, D) = (1, 1, w, D) .
 \end{aligned} \tag{3.39}$$

Corresponding to these states are the matrices

$$A_{ii}(u, 0) = \begin{bmatrix} 1 - \frac{(u_1 + s_1 - u_2 - s_2)^+ \Delta \tau}{\Delta x_2} & \frac{(u_1 + s_1 - u_2 - s_2)^+ \Delta \tau}{\Delta x_2} \\ \frac{(u_2 + s_2 - u_1 - s_1)^+ \Delta \tau}{\Delta x_2} & 1 - \frac{(u_2 + s_2 - u_1 - s_1)^+ \Delta \tau}{\Delta x_2} \end{bmatrix} \quad i=1,2,3 \tag{3.40}$$

$$B_{12}(u, s) = B_{23}(u, s) = \begin{bmatrix} \frac{(w - u_1 - s_1)^+ \Delta \tau}{\Delta x_1} & 0 \\ 0 & \frac{(w - u_1 - s_1)^+ \Delta \tau}{\Delta x_1} \end{bmatrix} \tag{3.41}$$

$$B_{21}(u, s) = B_{32}(u, s) = \begin{bmatrix} \frac{(u_1 + s_1 - w)^+ \Delta \tau}{\Delta x_1} & 0 \\ 0 & \frac{(u_1 + s_1 - w)^+ \Delta \tau}{\Delta x_1} \end{bmatrix} \tag{3.42}$$

$$B_{11}(u,s) = \begin{bmatrix} \frac{-|u_1+s_1-w|\Delta\tau}{\Delta x_1} & 0 \\ 0 & -\frac{|u_1+s_1-w|\Delta\tau}{\Delta x_1} \end{bmatrix} \quad (3.44)$$

$$B_{13}(u,s) = B_{31}(u,s) = 0 \quad (3.44)$$

For this two reservoir example, the continuous problem was formulated. Then the HJE was set up and discretized to yield the weakly coupled Markov chain optimization problem. Next the properties of the weakly coupled Markov chain are investigated. In order to simplify the notation, we define the vector u to include u, s . The control being within constraint $u(n) \in U(n)$ implies equations (3.24) - (3.25).

In the weakly-coupled Markov chain (3.26) there are k groups of states: group I_i has n_i states where n_i is the dimension of A_{ii} . Transitions among states in group I_i are dependent on $A_{ii}(u_i) + \epsilon B_{ii}(u_i)$ and are of high probability relative to transitions within states of different groups I_i and I_j . Transitions from states in I_i to states in I_j are dependent on $\epsilon B_{ij}(u)$ and are of low (ϵ) probability. This separation of transitions into high and low probabilities suggests that motion among states within a group is "faster" than motion between states in different groups. For the hydro scheduling problem this corresponds to motion between states in seasonal reservoirs being slower than motion between corresponding states in the weekly reservoirs.

For small ϵ , the behavior within a group can be approximated by the steady-state probability distribution $v_i(u)$ of the fast chain

$$v_i(t+1, u_i) = v_i(t, u_i) A_{ii}(u_i) \quad (3.45)$$

after a "relatively" short time. In other words the probabilities that given states within group I_i will be occupied given some state in I_i is occupied is approximately given by $v_i(u_i)$.

For example, suppose that the policy

$$u_1 = \begin{bmatrix} u(1) \\ u(2) \end{bmatrix} = \begin{bmatrix} u_1(1) & u_2(1) & v_1(1) & v_2(1) \\ u_1(2) & u_2(2) & v_1(2) & v_2(2) \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 0 & 0 \\ 3/4 & 1 & 0 & 0 \end{bmatrix} \quad (3.46)$$

is chosen for the fast chain with transition matrix defined by (3.40):

$$A_{11}(u_1) = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix} \quad (3.47)$$

The steady state probability vector $v_1(u_1)$ satisfies

$$v_1(u_1) = v_1(u_1) A_{11}(u_1), \quad (3.48)$$

$$v_1(u_1) = [1/3 \quad 2/3] . \quad (3.49)$$

Therefore, after a sufficiently long time state 1 (the small reservoir empty) will have been occupied 1/3 of the time and state 2 (the small reservoir full) will have been occupied 2/3 of the time.

Aggregate Markov Chain

The aggregate Markov chain corresponding to the weakly-coupled Markov chain is defined by

$$\Pi(t+1) = \Pi(t) Q(u) \quad (3.50)$$

where

$$Q(u) = I + \epsilon M(u) \quad (3.51)$$

and the elements $m_{ij}(u_1)$ of $M(u)$ satisfy

$$m_{ij}(u_i) = v_i(u_i) B_{ij}(u_i) \underline{1} \quad (3.52)$$

$$\underline{1} = [1 \ 1 \ \dots \ 1] \ . \quad (3.53)$$

The number of states, K , in this chain is the number of groups of states I_i in the weakly-coupled Markov chain. The aggregate chain (3.50) is an approximation for the transition probabilities obtained by aggregating each group of states I_i into a single state.

For the example chain defined by equations (3.40) - (3.44), the groups of states have

$I_1 = \{1, 2\}$ corresponding to large reservoir empty

$I_2 = \{3, 4\}$ corresponding to large reservoir half full

$I_3 = \{4, 5\}$ corresponding to large reservoir full

The aggregation is shown pictorially in Figure 3.1 where the dotted lines represent ϵ -order transition probabilities.

Figure 3.1. Aggregation for the Two Reservoir Example.

The result is the reduction in the order of the chain from 6 to 3.
The aggregate probability transition matrix is

$$Q(u) = I + \epsilon \begin{bmatrix} -4/15 & 4/15 & 0 \\ 0 & -4/15 & 4/15 \\ 0 & \frac{13}{20} & -\frac{13}{20} \end{bmatrix} \quad (3.54)$$

where the inflow m satisfies

$$m = 1.1, \quad (3.55)$$

$$u_1 = u_2 = \begin{bmatrix} 1 & 1/2 & 0 & 0 \\ 3/4 & 1 & 0 & 0 \end{bmatrix}, \quad (3.56a, b)$$

$$u_3 = \begin{bmatrix} 1 & 1 & 3/4 & 0 \\ 1 & 1 & 3/4 & 3/4 \end{bmatrix}. \quad (3.56c)$$

The limiting probability distribution for this aggregate chain is

$$\Pi(u_1, u_2, u_3) = [0 \quad \frac{39}{55} \quad \frac{16}{55}]. \quad (3.57)$$

This implies that in the limit the large reservoir is never empty, is half full 70.9% of the time and is full 29.1% of the time for this inflow and policy.

Aggregation of Markov chains with weak coupling has been studied in [8,9] where the aggregate probabilities are related to the transitional probabilities of the original system under regularity assumptions. Weakly-coupled Markov chains can be viewed as a stochastic analog of discrete-time singularly perturbed systems. The similarities between these two concepts are explored in [10].

In the next section the properties of weakly coupled Markov chains are shown. These results are then used to develop a near optimal control for the original Markov chain problem which can be obtained by solving a control problem for the aggregate chain.

3.4 PROPERTIES OF WEAKLY-COUPLED MARKOV CHAINS

The following assumptions are made in developing the properties of the weakly-coupled Markov chains:

- (A1) For all $u(i) \in U(i)$, $i=1,2,\dots,l$ the weakly-coupled Markov chain has a single ergodic class [13].
- (A2) For each group of states I_i and all $u(j) \in U(j)$, $j \in I_i$ the matrices A_{ii} are regular [14].
- (A3) For all $u(i) \in U(i)$ the aggregate chain has a single ergodic class.

Define

$$p(u) = \lim_{t \rightarrow \infty} p(t, u)$$

Assumption (A1) ensures that under any fixed policy $u(i) \in U(i)$, $i=1,2,\dots,l$, the steady-state probability of the weakly-coupled Markov chain, $p(u)$, exists and is unique. Assumptions (A2) and (A3) are used to relate the steady-state probability $\Pi_i(u)$ of the aggregate chain to the steady state probability of being in group I_i in the original chain.

The relationship between the steady-state probabilities of the weakly-coupled Markov chain and the aggregate and fast chains associated with it is stated as a theorem.

Theorem 3.1

Under assumptions (A1), (A2), (A3), for any policy $u(i) \in U(i)$, $i=1,2,\dots,L$:

Let $\Pi(u)$ be the steady-state probability distribution of the aggregate (slow) chain, defined by the equations

$$\Pi(u) = \Pi(u) Q(u) \quad (3.58)$$

$$\sum_{j=1}^N \Pi_j(u) = 1. \quad (3.59)$$

Then the components $p_i(u)$ of the steady-state probability distribution of the weakly-coupled Markov chain, $p(u)$, satisfy

$$p_i(u) = \Pi_j(u) + O(\epsilon)(v_j^i(u) + O(\epsilon)) \quad i \in I_j, j=1,\dots,N \quad (3.60)$$

where $v_j^i(u)$ are components of $v_j(u)$ defined in (3.45), and

$$\sum_{i \in I_j} v_j^i(u) = 1. \quad (3.61)$$

The proof of this theorem is in Section 3.5.

Theorem 3.1 implies that the probability of being in aggregate state j (the group I_j) is given in the limit by Π_j and the probability of being in a given state $j \in I_j$ given that the occupied state is within group I_j is approximately given by v_j^i . This result is used to determine the near-optimal control.

We now define the control problem (p) as that of finding the feedback policy u solving

$$\min_{u \in U} J_\epsilon(u) \quad (3.62)$$

subject to (3.40) - (3.44) where

$$J_{\varepsilon}(u) = \lim_{T \rightarrow \infty} \frac{1}{1+T} E \sum_{t=0}^T g(s_t, u(s_t)) \quad (3.63)$$

$$U = U(1) \times U(2) \dots \times U(l) , \quad (3.64)$$

$g(i, u(i))$ is the prespecified instantaneous cost defined for $i \in \{1, \dots, l\}$ and $s_t \in \{1, \dots, l\}$ is the state occupied at t .

We first state assumptions required for the well-posedness of this problem.

- (A4) The instantaneous cost $g(s_t, u(s_t))$ is uniformly bounded for $s_t \in \{1, \dots, l\}$, $u(s_t) \in U(s_t)$; that is, there exists K such that

$$|g(s_t, u(s_t))| \leq K$$

for $s_t \in \{1, \dots, l\}$, $u(s_t) \in U(s_t)$.

- (A5) For each $i, j \in \{1, \dots, l\}$, the functions $\text{Prob}(s_{t+1}=j | s_t=i) = a_{ij}(u(i)) + \varepsilon b_{ij}(u(i))$, and $g(i, u(i))$ are continuous with respect to $u(i)$.

- (A6) The sets $U(i)$ of admissible controls for each state i are compact.

This is a problem for which standard results are available; these are summarized in the next theorem.

Theorem 3.2 [15]

Under assumptions (A1), (A4), (A5), and (A6), for $u \in U$ define the system of l equation in $l+1$ unknowns $\gamma \in \mathbb{R}$, $c \in \mathbb{R}^l$

$$\gamma \underline{1} = (A(u) + \varepsilon B(u) - I) c + g(u), \quad (3.65)$$

$$g(u) \triangleq [g(1), u(1), \dots, g(\ell, u(\ell))]'. \quad (3.66)$$

Then:

- (1) A solution (γ, c) always exists.
- (2) If (γ, c) is a solution to (3.65) $\gamma = J_\varepsilon(u)$. The minimum cost may be obtained by solving

$$\gamma \underline{1} = \min_{u \in U} \{A(u) + \varepsilon B(u) - I\} c + g(u). \quad (3.67)$$

- (3) If (γ, c) is a solution to (3.65), $(\gamma, c + b \underline{1})$ is also a solution for any scalar b .
- (4) For any feedback policy $u \in U$ the cost satisfies

$$J_\varepsilon(u) = \gamma = p(u) g(u). \quad (3.68)$$

A direct consequence of Theorems 3.1 and 3.2 is the following corollary.

Corollary 3.1

Under assumptions (A1) - (A6) the cost $J_\varepsilon(u)$ corresponding to any fixed policy $u(i) \in U(i)$, $i=1, \dots, \ell$, may be approximated by

$$J_\varepsilon(u) = J_0(u) + O(\varepsilon) \quad (3.69)$$

where

$$J_0(u) = \sum_{j=1}^N \Pi_j \sum_{i \in I_j} v_j^i(u(i)) g(i, u(i)) \quad (3.70)$$

and Π and v_j are defined (3.58) and (3.45). ■

Proof: (3.69) follows from substituting (3.60) in (3.68).

This corollary suggests the following two-step method to compute the cost $J_0(u)$ under any policy:

- (1) Set $\varepsilon = 0$ and solve for $v_j(u)$ and the cost

$$\sum_{i \in I_j} v_j^1(u(i)) g(i, u(i))$$

- (2) Compute $Q(u)$ using the results of Step 1; then compute $\Pi(u)$ and $J_0(u)$ from (3.70).

As shown in the next theorem, the policy which minimizes $J_0(u)$ is a optimal policy for $J_\varepsilon(u)$.

Theorem 3.3

Let assumptions (A1) - (A6) be satisfied, u^0 be the policy minimizing $J^0(u)$

$$J^0(u^0) = \min_{u \in U} J^0(u) , \quad (3.71)$$

and $u^*(\varepsilon)$ be the policy minimizing $J_\varepsilon(u)$, i.e.

$$J_\varepsilon(u^*(\varepsilon)) = \min_{u \in U} J_\varepsilon(u) . \quad (3.72)$$

Then,

$$\lim_{\varepsilon \rightarrow 0} \{J_\varepsilon(u^0) - J_\varepsilon(u^*(\varepsilon))\} = 0 . \quad (3.73)$$

The proof of this theorem is in Section 3.5. ■

In a particular weakly-coupled Markov chain ϵ is fixed; for chains with sufficiently small ϵ , the policy u^0 obtained by minimizing the zeroth-order cost $J^0(u)$ will yield a near-optimal cost. The policy u^0 is independent of ϵ since for any policy u , $v_j(u)$ is independent of ϵ and the aggregate chain steady-state probability distribution $\Pi(u)$, which is the left eigenvector corresponding to the eigenvalue 0 of $M(u)$, is also independent of ϵ .

Decomposition in Policy Space

Theorem 3.3 defines an optimization problem for the aggregate chain which may be solved to obtain a control which when used yields a cost that is near optimal. The solution to this problem involves computations for the fast chains and an optimization for the aggregate chain. In order to reduce the computations required to perform this optimization, a policy decomposition is defined. The policies u_i $i=1, \dots, K$ for the fast chains i must be aggregated to yield policies for the aggregate chain. Each policy for the aggregate chain is defined by a set of values for a row in the probability transition matrix $Q(u)$. The i th row, $q_i(u_i)$ of $Q(u)$ is only dependent on u_i and not on u_j , $i \neq j$.

From a deterministic viewpoint, one might expect that optimizations on a fast time-scale will yield releases hour-by-hour over a week and that the long optimization will yield the total amount of water to be released over the week. Thus the hourly releases may be aggregated (by summation) to yield the weekly release.

For the Markov chain the analog to this idea is simple to define. Consider the u_i , the i th fast chain and the i th row in the aggregate chain. To be specific, the simple two reservoir example may be considered.

Policy u_1 ,

$$u_1 = \begin{bmatrix} 1 & 1/2 & 0 & 0 \\ 3/4 & 1 & 0 & 0 \end{bmatrix} \quad (3.46)$$

implies there is no spillage and that the rate of release of water from the upper (large) reservoir is 1 when the small reservoir is empty and 3/4 when it is full. Similarly, the rate of release from the lower reservoir is 1/2 when empty and 1 when full. Note that the net rate of change of water level when the small reservoir is empty is $1 - 1/2 = 1/2$. The net rate of change when the reservoir is full is $3/4 - 1 = -1/4$. The stationary probability distribution for the fast chain that corresponds to this policy is

$$v_1(u_1) = [1/3 \quad 2/3] \quad (3.49)$$

Thus the expected change in level in the small reservoir is

$$v_1(u_1) \begin{bmatrix} 1/2 \\ -1/4 \end{bmatrix} \Delta\tau = 0.$$

Thus the net change in level in the small reservoir over a sufficiently long time is 0. The inflow into the large reservoir is

$$m = 1.1. \quad (3.55)$$

Therefore the net change in level for the large reservoir is^{*}

$$ev_1(u_1) \begin{bmatrix} 1.1 - 1 \\ 1.1 - 3/4 \end{bmatrix} \Delta\tau = \frac{4\epsilon}{15}$$

This is the probability of transitioning from empty to 1/2 full in the large reservoir and is $q_{12}(u_1)$. Thus the elements of the row in the aggregate chain which are dependent on u_1 may be interpreted as the expected change in reservoir level. These ideas are discussed in more detail in Section 3.6 (Simulation Results).

^{*}For this example $\Delta\tau=1$.

More insight into the interplay of optimization on the fast and slow time-scales and the policy space decomposition is obtained by considering the deterministic problem of a long term optimization of hydro scheduling by dynamic programming. Figure 3.2 shows the last time stages of this optimization.

Figure 3.2. Dynamic Programming for a Single Reservoir.

The state variable x represents the volume of water in the reservoir. The state is coarsely discretized and the controls considered are the ones which change the reservoir levels between the discrete levels in a week. These controls are dependent on the deterministic (known) inflow. In order to perform dynamic programming for the problem, an instantaneous cost for going from a discrete level at one stage to a discrete level at the next stage must be obtained. There are many possible ways to go from one stage to the next. Due to the fast variations in demand, a significant savings can be obtained by minimizing over these possible trajectories and using the minimum cost as the instantaneous cost for long term optimization. As is shown in Section 3.4, this nested optimization provides a near-optimal solution.

By analogy, the possible rows for the aggregate chain correspond to expected changes in reservoir levels; a fast time-scale optimization may be performed for the fast chains. An example of this is given in Section 3.6.

These ideas may be formally described in terms of the weakly coupled Markov chain. Each fast chain policy u_1 determines a row in the aggregate chain transition matrix $Q(u)$. When more than one policy yields the same row in $Q(u)$ a decomposition in policy space is achieved.

The following definitions are used in the next theorem:

Definition

The sets of policies U_i , $i=1, \dots, K$ for the fast Markov chains (3.45) satisfy

$$U_i = \Delta U(k) \times U(k+1) \times \dots \times U(k+n_i) \quad (3.74)$$

$$k = \sum_{j=1}^{i-1} n_j \quad (3.75)$$

Definition

The subsets $R_i^j \subset U_i$ are defined by

$$(i) \quad \bigcup_j R_i^j = U_i \quad \forall i$$

$$(ii) \quad R_i^j \cap R_i^k = \emptyset \quad \forall (j \neq k), \forall i$$

(iii) $\forall u_1 \in R_i^j$ the corresponding row $q_i(R_i^j)$ of the matrix Q is the same and

$$q(R_i^j) \neq q(R_i^k), \quad j \neq k.$$

The R_i^j are subsets of U with the property that any policy in R_i^j will produce the same row in the aggregate chain. This is equivalent to having the same endpoints for a set of controls in the deterministic dynamic programming example described above.

Theorem 3.4

For all R_i^j , and for all i , consider $u_i^*(R_i^j) \in R_i^j$ which minimizes

$$\sum_{k \in I_i} v_i^*(u_i(k)) g(k, u_i(k)) \quad (3.76)$$

Then the solution to optimization problem for the aggregate chain (3.50) can be found among the policies of the form

$$u = (u_1^*(R_1^j) \dots u_K^*(R_K^j)) \quad (3.77)$$

The proof of this theorem is in Section 3.5.

This theorem states that a policy which minimizes $J_0(u)$ consists of the concatenation of policies for the fast chains which minimize (3.76). One procedure which may be used to set up the policy space decomposition is:

- (1) Define the sets of policies for the fast chains (3.45), $i=1, \dots, K$.
- (2) For each i , partition the set U_i into the sets R_i^j (as defined above). The vector $q_i(R_i^j)$ is the value for the i th row of $Q(u)$ corresponding to any policy $u_i \in R_i^j$. (The sets R_i^j are the decomposition of the policy space U_i).
- (3) For each policy set R_i^j find the policy $u_i^*(R_i^j)$ which minimizes (3.76). Let $g_i^*(R_i^j)$ be the minimum costs corresponding to $u_i^*(R_i^j)$.

- (4) By the time this step is reached, the possible rows $q_1(R_1^j)$ of $Q(u)$ have been found. The minimum cost (3.76) for all policies which yield that row serves as the instantaneous cost for the aggregate chain optimization problem which may be solved by iteration in the reduced policy space.

In this section, the properties of the weakly coupled Markov chain optimization problem have been investigated. The fast Markov chains and the aggregate chain and their optimization problems have been defined. The main ideas have been illustrated by a two reservoir example. The next section contains proofs of the Theorems which have been separated out from this section for continuity of presentation. In Section 3.6 simulation results, illustrating the ideas presented here are presented.

3.5 PROOFS

This section contains proofs of the theorems given in the previous section.

Proof of Theorem 3.1

For each I_i , $j \in I_i$, we can write

$$p_j(u) = \bar{p}_i(u) w_i^j(u)$$

where

$$\bar{p}_i(u) = \sum_{k \in I_i} p_k(u)$$

and $w_i^j(u)$ is the restriction of $p_j(u)$ to I_i . We relate $w_i^j(u)$ and $\bar{p}_i(u)$ to $v_i^j(u)$ and $\pi_i(u)$ respectively.

(i) Relation of $w_1^j(u)$ to $v_1^j(u)$

$e_1(u) \triangleq (w_1^1(u) \dots w_1^{n_1}(u))$ is computed as follows:

Rearrange and partition the vector $p(u)$ of the steady state probability distribution

$$p(u) = (p_1(u) \cdot p_x(u))$$

The matrix of transition probabilities is accordingly partitioned as

$$A(u) + \epsilon B(u) = \begin{bmatrix} A_{11}(u) + \epsilon B_{11}(u) & \epsilon B_{1x}(u) \\ \epsilon B_{x1}(u) & A_{xx}(u) + \epsilon B_{xx}(u) \end{bmatrix}$$

By hypothesis $A(u) + \epsilon B(u)$ is a stochastic matrix; then

$$S_{11}(u) = A_{11}(u) + \epsilon B_{11}(u) + \epsilon^2 B_{1x}(u) (I - A_{xx}(u) - \epsilon B_{xx}(u))^{-1} B_{x1}(u)$$

is also a stochastic matrix [9], and $w_1(u)$ is the solution of

$$w_1(u) = w_1(u) S_{11}(u)$$

$$\sum_k w_1^k(u) = 1$$

Now we compare $w_1(u)$ to $v_1(u)$ defined by (3.45). In order to make this comparison we first need to find a relation between the matrices $A_{11}(u)$ and $S_{11}(u)$. The following lemma gives the conditions under which $A_{11}(u)$ and $S_{11}(u)$ are approximately the same.

Lemma 5.1

For each $u \in U$, we have

$$\|S_{11}(u) - A_{11}(u)\|_2 = O(\epsilon). \quad (3.78)$$

Proof

Define the matrix

$$A_{xx}(u) = \text{diag}(A_{11}(u) \dots A_{i-1,i-1}(u), A_{i+1,i+1}(u) \dots A_{nn}(u))$$

The matrix $I - A_{xx}(u)$ is singular because each $A_{jj}(u)$ ($j=1,2,\dots,i-1,i+1,\dots,n$) is a nonnegative matrix [16]. Therefore,

$$\sigma_{\min}(u) = 0 \quad \forall u \in U$$

where $\sigma_{\min}(u)$ is the minimum singular value of $I - A_{xx}(u)$. The results of [17] (pg. 1.3) imply that

$$\sigma'_{\min}(u) = O(\epsilon) \text{ for each } u \in U$$

where $\sigma'_{\min}(u)$ is the minimum singular value of $I - A_{xx}(u) + \epsilon B_{xx}(u)$ for sufficient small ϵ .

Consequently,

$$\epsilon^2 \| \Lambda_{ix} B_{ix}(u) (I - A_{xx}(u) - \epsilon B_{xx}(u))^{-1} B_{xi}(u) \|_2 = O(\epsilon) \text{ for each } u \in U$$

since

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \{ \epsilon^2 \| \Lambda_{ix} B_{ix}(u) (I - A_{xx}(u) - \epsilon B_{xx}(u))^{-1} B_{xi}(u) \Lambda_{xi}(u) \|_2 \} &\leq \\ &\leq \lim_{\epsilon \rightarrow 0} \| B_{ix}(u) \|_2 \| B_{xi}(u) \|_2 \frac{1}{O(\epsilon)\epsilon} = \text{constant for each } u \in U \end{aligned}$$

since

$$\| S_{ii} - A_{ii} \|_2 \leq \epsilon \| B_{ii} \|_2 + \epsilon^2 \| B_{ix}(u) (I - A_{xx}(u) - \epsilon B_{xx}(u))^{-1} B_{xi}(u) \|_2$$

Equation (3.78) is satisfied and Lemma A1 is proved. ■

Therefore there exists a bounded matrix $C(u)$ such that

$$w_i(u) = w_i(u) (A_{ii}(u) + \varepsilon C(u)) .$$

Define

$$\delta w_i(u) = w_i(u) - v_i(u)$$

where $v_i(u)$ is the solution of

$$v_i(u) = v_i(u) A_{ii}(u)$$

$$\sum_k v_i^k(u) = 1$$

which implies

$$\delta w_i(u) (I - A_{ii}(u)) = \varepsilon C(u) w_i(u) .$$

Then, the fact that the minimum nonzero singular value of $I - A(u)$ is independent of ε and the results of [12] imply that

$$||\delta w_i(u)|| \leq O(\varepsilon) \text{ for any } u \in U .$$

Consequently,

$$w_i(u) = v_i(u) + O(\varepsilon)$$

for each $u \in U$.

Similarly we can show that for all $u \in U$

$$w_j(u) = v_j(u) + O(\varepsilon) \quad \forall_j \quad (j=1,2,\dots,N)$$

Next we relate $\bar{p}(u)$ to $\Pi(u)$.

(ii) Relation of $\bar{p}(u)$ to $\Pi(u)$

Because of assumptions (A1), (A5) and (A7) the result of [8] can be used and

$$\bar{p}_i(u) = \Pi_i(u) + o(\epsilon) \text{ for each } u \in U, \forall i. \quad (3.79)$$

Combining (3.78) and (3.79) we obtain

$$p_j(u) = (\Pi_i(u) + o(\epsilon)) (v_1^j(u) + o(\epsilon)) \text{ for each } u \in U.$$

The proof of Theorem 1 is complete. ■

Proof of Theorem 3.3

We first prove the following. Let

$$f(\epsilon) = \max_{u \in U} \{J_\epsilon(u) - J_0(u)\}.$$

Then,

$$\lim_{\epsilon \rightarrow 0} f(\epsilon) = 0. \quad (3.80)$$

Suppose not, then there exists u' such that

$$\lim_{\epsilon \rightarrow 0} \{J_\epsilon(u') - J_0(u')\} \neq 0.$$

This is a contradiction since

$$J_\epsilon(u') - J_0(u') = o(\epsilon)$$

for any $u \in U$. Therefore, (3.80) holds.

By definition

$$J_\epsilon(u^0) - J_\epsilon(u^*(\epsilon)) \geq 0$$

and

$$J_0(u^0) - J_0(u^*(\epsilon)) \leq 0.$$

Adding and subtracting, we have

$$\begin{aligned} J_\epsilon(u^0) - J_\epsilon(u^*(\epsilon)) &= J_\epsilon(u^0) - J_0(u^0) \\ &\quad - J_\epsilon(u^*(\epsilon)) + J_0(u^*(\epsilon)) \\ &\quad + J_0(u^0) - J_0(u^*(\epsilon)) \end{aligned}$$

which implies

$$\begin{aligned} J_\epsilon(u^0) - J_\epsilon(u^*(\epsilon)) &\leq J_\epsilon(u^0) - J_0(u^0) \\ &\quad - J_\epsilon(u^*(\epsilon)) + J_0(u^*(\epsilon)) \\ &\leq |J_\epsilon(u^0) - J_0(u^0)| \\ &\quad + |J_\epsilon(u^*(\epsilon)) - J_0(u^*(\epsilon))| \\ &\leq 2f(\epsilon) \end{aligned}$$

$$\Rightarrow \lim_{\epsilon \rightarrow 0} \{J_\epsilon(u^0) - J_\epsilon(u^*(\epsilon))\} = 0$$

and the theorem is proved. ■

Proof of Theorem 3.4

Consider the policies

$$u = (u_1^*(R_1^j) \dots u_{i-1}^*(R_{i-1}^j) u_i^*(R_i^j) u_{i+1}^*(R_{i+1}^j) \dots u_N^*(R_N^j))$$

and

$$u' = (u_1^*(R_1^j) \dots u_{i-1}^*(R_{i-1}^j) u_i^*(R_i^j) u_{i+1}^*(R_{i+1}^j) \dots u_N^*(R_N^j))$$

Since

$$u_i^*(R_i^j), u_i^*(R_i^j) \in R_i^j, q_i(u_i) = q_i(u_i^*) = q_i(R_i^j).$$

Therefore, the matrix of the aggregate chain corresponding to the two policies u and u' is the same, i.e.,

$$Q(u) = Q(u')$$

Consequently,

$$\Pi(u) = \Pi(u')$$

By the definition of $u_i^*(R_i^j)$,

$$\sum_{k \in I_1} v_1(u_i^*(k)) g(k, u_i^*(k)) \leq \sum_{k \in I_1} v_1(u_i'(k)) g(k, u_i'(k)),$$

implies that

$$J_0(u) = \sum_{I_1} \Pi_1(u) \sum_{k \in I_1} v_1(u_i^*(k)) g(k, u_i^*(k))$$

$$\leq \sum_{I_1} \Pi_1(u') \sum_{k \in I_1} v_1(u_i'(k)) g(k, u_i'(k)) = J_0(u')$$

Hence, the policy u' does not have to be considered in the optimization problem for the slow Markov chain.

Similar arguments can be used to show that any policy which is not of the form

$$u = (u_1^*(R_1^j) \dots u_N^*(R_N^j))$$

can be deleted in the optimization problem for the slow Markov chain. ■

3.6 SIMULATION RESULTS

In this section, a sample problem is used to illustrate Markov chain time-scale decomposition. The results of the decomposition are compared with those obtained by peak shaving, and the results show close agreement.

The reservoir dynamics model, the demand and inflow models are first described. Then these models are combined to form a weakly coupled Markov chain. This approach to formulating the optimization problem is an alternative to the stochastic approximation method described earlier. Its advantage is that it is simple, may be easily interpreted and compared with other method solutions. Next, the steps in the decomposition of the optimization are described. Finally, the results are compared with a simulation to peak shaving results for the same dynamic system.

Reservoir Dynamics Model

The equation for the reservoir dynamics is,

$$\dot{x} = \frac{\epsilon}{7} (w - u - s)$$

where w , u and s are inflow, release and spillage rates, respectively, and

$$0 \leq u \leq 1$$

$$0 \leq s$$

$$0 \leq x \leq 1$$

The reservoir dynamics can be modeled by a Markov chain of the form

$$p(t+\Delta t) = p(t)[I + A_x \Delta t]$$

where $p \in \mathbb{R}^{N+1}$, $p_i(t)$, $i = (0, 1, \dots, N)$, is the probability that the reservoir is i/N full at time t , $I + A_x \Delta t$ is the transition probability matrix and Δt is the time step. Discretizing the reservoir to 5 levels $(0, 1/4, 2/4, 3/4, 1)$, $N=4$ yields the matrix A_x .

$$A_x = \begin{bmatrix} -\frac{\varepsilon}{7}(w-s-u)^+ & \frac{\varepsilon}{7}(w-s-u)^+ & 0 & 0 & 0 \\ \frac{\varepsilon}{7}(s+u-w)^+ & -\frac{\varepsilon}{7}|w-u-s| & \frac{\varepsilon}{7}(w-s-u)^+ & 0 & 0 \\ 0 & \frac{\varepsilon}{7}(s+u-w)^+ & -\frac{\varepsilon}{7}|w-u-s| & \frac{\varepsilon}{7}(w-s-u)^+ & 0 \\ 0 & 0 & \frac{\varepsilon}{7}(s+u-w)^+ & -\frac{\varepsilon}{7}|w-u-s| & \frac{\varepsilon}{7}(w-s-u)^+ \\ 0 & 0 & 0 & \frac{\varepsilon}{7}(s+u-w)^+ & -\frac{\varepsilon}{7}(s+u-w)^+ \end{bmatrix}$$

$$(a)^+ = \max(a, 0).$$

The elements of this matrix assume that Δt is small enough that it is only possible to transition up one level, down one level or remain at the same level.

The probability to transition from one level to another is proportional to x . Since transition probabilities are positive the matrix is arranged to that probabilities of transition down levels are below the main diagonal and of transitioning up levels are above the main diagonal.

Load Model

The demand model is separated into two time scales, the fast and the slow. The fast scale corresponds to daily and weekly load and has a 7 day periodicity. Figure 3.3 gives the specific fast load characteristics used for this test problem. The smallest time step for the fast demand is 3 hours, which corresponds to the time step, Δt , used for the simulation. The demand pattern shows a high demand during the day, peaking at noon, with lower demands at night and early morning. This demand pattern remains the same during the week (days 1 - 5) and is down slightly for the weekend (days 6 and 7).

The fast load model can be represented as a Markov chain using the equations below.

$$p(t+\Delta t) = p(t) C[I+A_{FL}\Delta t]$$

where

$p_i(t)$ = probability that the fast load level i is occupied at time t .

A_{FL} = NF by NF matrix containing the inverse of the average time spent transferring from one fast load level to another, i.e., net rate.

For the single reservoir test problem, A_{FL} was derived [11] from Figure 3.3 and is given by

$$A_{FL} = \begin{bmatrix} -4 & 8/7 & 20/7 & 0 & 0 & 0 & 0 & 0 \\ 4 & -8 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 & 8 & 0 & 0 \\ 0 & 4 & 0 & -8 & 0 & 0 & 4 & 0 \\ 8 & 0 & 0 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & -8 & 0 & 4 \\ 0 & 0 & 0 & 4 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & -4 \end{bmatrix}$$

Figure 3.3. Daily and Weekly Load Periodicity.

The slow scale corresponds to seasonal load characteristics for the single reservoir model and has a yearly periodicity. Figure 3.4 gives the specific slow load characteristics for this test problem. The smallest time step for the slow demand was 3 months.

Inflow Model

The inflow model followed the same time scale, periodicity, and time step as the slow demand model as is also shown in Figure 3.4.

The slow load model and inflow model can be represented as a Markov chain using the equations below.

$$p(t+\Delta t) = p(t)[I + \epsilon A_{SL} \Delta t]$$

where

$p_i(t)$ = probability that the slow load/inflow combination occupies state i at time t .

A_{SL} = NS by NS matrix containing the inverse of the average time spent transferring from one combination of slow load and inflow to another.

For the single reservoir test problem, A_{SL} was derived [11] from Figure 3.4 and is shown below.

$$A_{SL} = \begin{bmatrix} -4/7 & 0 & 4/7 \\ 4/7 & -4/7 & 0 \\ 0 & 2/7 & -2/7 \end{bmatrix}$$

Figure 3.4. Seasonal Load and Inflow Model.

Cost Function

For the single reservoir problem, the value of water was the cost of displaced thermal generation, $D-u$, as shown in the equation below.

$$\int_0^T C(D-u)dt$$

where

D = total demand

u = total release rate

Since demand was broken into its fast and slow components, the total demand at any one time equals the sum of its corresponding fast and slow parts. For this test problem, the cost function C is assumed to be quadratic. The cost equation above then becomes the following:

$$\sum_{i=1}^{NT} (DF_i + DS_i - u_i)^2 \Delta t$$

where

NT = # time periods Δt in the simulation

DF_i = fast demand at time period i

DS_i = slow demand at time period i

u_i = release rate at time period i

Δt = fast time step

Two Time-Scale Markov Chain

The overall single reservoir problem was modeled as a two time scale Markov chain using each of the Markov chain models described earlier. The number of states that exist for the overall system is equal to the number of reservoir levels allowed (N+1) times the number of slow load/inflow combination (NS) times the number of fast load levels (NF). Each state is characterized by a reservoir level, a slow load/inflow combination and a fast load level. For the single reservoir test problem, there were 120 states. These states are defined (a_i, b_j, c_k) ($i=1, \dots, 5$ ($j=1, 2, 3$) ($k=1, \dots, 8$))) where

$$a_i \in \{0, .25, .5, .75, 1.03\}$$

$$b_j \in \{S_0, S_1, S_2\}$$

$$c_k \in \{0, .6, 1.0, 1.1, 1.25, 1.5, 1.7, 2.0\}$$

The transition probability matrix for the weakly coupled Markov chain is

$$A(u, s) + \epsilon B(u, s) = \begin{bmatrix} \tilde{A}(\epsilon, u, s) & \epsilon E_2(u, s) & 0 \\ \epsilon E_1(u, s) & \tilde{A}(\epsilon, u, s) & \epsilon E_2(u, s) \\ 0 & \epsilon E_1(u, s) & \tilde{A}(\epsilon, u, s) \end{bmatrix}$$

where

$$\tilde{A}(\epsilon, u, s) = \begin{bmatrix} I + A_x \Delta t - \Sigma & 0 & \epsilon \frac{4}{7} \Delta t I \\ \epsilon \frac{4}{7} \Delta t I & I + A_x \Delta t - \Sigma & 0 \\ 0 & \epsilon \frac{2}{7} \Delta t I & I + A_x \Delta t - \Sigma \end{bmatrix}$$

$$E_1(u, s) = \text{diag} \left[\frac{1}{7} (u+s-w)^+ \Delta t, \frac{1}{7} (u+s-w)^+ \Delta t, \frac{1}{7} (u+s-w)^+ \Delta t \right]$$

$$E_2(u, s) = \text{diag}[\frac{1}{7}(w-u-s)^+ \Delta t, \frac{1}{7}(w-u-s)^+ \Delta t, \frac{1}{7}(w-u-s)^+ \Delta t I]$$

and Σ represents the sum of the off diagonal block row matrices. This matrix has the structure of a weakly coupled Markov chain (3.26).

Computation of Subproblem Solutions

The Markov chain algorithm incorporates the use of policy iteration to determine the best release policy for different states of the single reservoir system. Figure 3.5 gives a general flowchart of this Markov chain algorithm.

Step 1 calculates the stationary (steady state) probability distribution $V_i(u_i)$ for the i^{th} fast chain. For the single reservoir problem, each matrix $A_{ii}(u)$ of the block diagonal matrix $A(u)$ is the same and independent of u so that the stationary distributions are all the same, $V_1(u_1) = V_2(u_2) = \dots = V_N(u_N)$ for all fast chains N and need only be calculated once. This steady state distribution V is obtained by using simultaneous equations with the Gaussian elimination method. This steady state distribution is then used to calculate the aggregate matrix $Q(u)$ and the instantaneous cost for each fast chain. A fast chain was characterized by its slow load/inflow level and reservoir level as described earlier.

Step 2 determines the feasible policies $u_i (i=1, (NF \text{ fast load levels}))$, for each fast chain using quadratic programming. Two quadratic programming problems were defined, both with the objective to minimize the cost of thermal demand under the reservoir dynamics constraints as defined earlier. Two additional reservoir dynamic constraints are added to insure the inflow was greater than or equal to outflow (release) when the reservoir was empty or to insure the outflow was greater than inflow when the reservoir was full.

Figure 3.5. Markov Chain Flowchart.

One Quadratic Programming (QP) problem, called the UP case, had the additional constraint that the average increase in the reservoir level must equal a fraction of the actual inflow. The other QP problem, called the DOWN case, had the additional constraint that the average decrease in the reservoir level must equal a fraction of obtainable decrease in reservoir level in time Δt . This step corresponds to policy space decomposition and solution of the fast chain optimization. If the reservoir was full and the UP constraint was to be satisfied or if the reservoir was empty and the DOWN constraint was to be satisfied, the optimal solution could be directly calculated and was to release the amount of inflow. If the inflow was greater than the releases (u) upper bound (1), then the upper bound would be released and spillage, equal to the difference of the inflow and the upper bound, would occur. For all other cases of demand levels, reservoir levels and inflow levels, the QP was solved assuming that spillage would be zero. Figure 3.6 gives the mathematical description of the two QP sub-problems.

Step 3 calculated, for each fast chain and each of its feasible policies found in Step 2, the aggregate row $Q_k(u_1)$ to be used in the policy iteration. The policy iteration method used here is described in [13].

Quadratic Programming Subproblem with Spillage Assumed to be Zero

Minimize

$$\sum_{i=1}^{NF} v_i (DF_i + DS - u_i)^2 \Delta t$$

subject to

$$0 \leq u_i \leq 1 \quad v_i$$

$$u_i \leq m \text{ if reservoir empty } v_i$$

$$u_i \geq m \text{ if reservoir full } v_i$$

UP $\sum_{i=1}^{NF} V_i(m-u_i) = km$ $m - u_i \geq 0 \quad \psi_i$	DOWN $\sum_{i=1}^{NF} V_i(u_i-m) = k(1-m)$ $u_i - m \geq 0 \quad \psi_i$
---	---

Figure 3.6. Quadratic Programming Sub-Problem with spillage assumed to be zero. This problem has separate constraints depending on whether the aggregate chain probability transition is for moving UP a level or DOWN a level.

Step 4 performs the policy iteration to determine the optimal policy, $u_i (i=1, \dots, \text{fast load levels})$ for each fast chain.

Step 5 performs a simulation of the single reservoir system using the results from the policy iteration step. It determines the trajectories of the reservoir level and release levels for a given year time span.

Resulting Control Policies

Table 3.2 gives the policies obtained through the use of the policy space decomposition and optimization. For each aggregate state a fast chain is defined for which an optimal policy is computed via the policy space decomposition. Consider the policy for the aggregate state (.25, 3.0, 0.5). It can be seen that the spillage is $S=0$ and that the rate of water release for electric generation increases as the fast load increases. Spillage occurs only for the case where the reservoir is full and the inflow rate is greater than the maximum generation release rate.

Due to the fact that the problem solved here is one of minimizing the average cost per unit time, an infinite horizon problem, the policies are independent of the initial and final states. These policies also have the advantage that if the demand curves change, they may still be used since they are a feedback on state and are time independent.

TABLE 3.2 SAMPLE SET OF POLICIES OBTAINED VIA POLICY SPACE DECOMPOSITION

AGGREGATE STATE s (RESERVOIR LEVEL, SEASONAL LOAD, INFLOW)	0.0	0.6	1.0	1.1	1.75	1.5	1.7	2.0 +	FAST LOAD LEVELS
(0.0 , 3.00 0.5)	(0,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	P O L I C I E S
(0.0 , 3.25 0.5)	(0,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	(.5,0)	
(0.0 , 3.50 1.1)	(.09,0)	(.69,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.25 , 3.00 0.5)	(.5,0)	(.5,0)	(.52,0)	(.62,0)	(.77,0)	(1,0)	(1,0)	(1,0)	
(.25 , 3.25 0.5)	(.5,0)	(.5,0)	(.52,0)	(.62,0)	(.77,0)	(1,0)	(1,0)	(1,0)	
(.25 , 3.50 1.1)	(.5,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.50 , 3.00 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.50 , 3.25 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.50 , 3.50 1.1)	(.5,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.75 , 3.00 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.75 , 3.25 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(.75 , 3.50 1.1)	(.5,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(1.0 , 3.00 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(1.0 , 3.75 0.5)	(.5,0)	(.5,0)	(.79,0)	(.89,0)	(1,0)	(1,0)	(1,0)	(1,0)	
(1.0 , 3.50 1.1)	(1,.1)	(1,.1)	(1,.1)	(1,.1)	(1,.1)	(1,.1)	(1,.1)	(1,.1)	

Corresponding Trajectories

In order to see how these policies would work in practice, a single reservoir simulation was run using the policies in Table 3.2 for initial reservoir levels of .3, .4, 1.0. The demand and inflow trajectories are shown in Figures 3.3 and 3.4. The reservoir trajectories are shown in Figure 3.7. As can be seen from these trajectories the policies tend to keep the reservoir levels approximately the same over a year.

In order to compare the performance of the policies to a more commonly used method, peak shaving was run for the same demand, inflows and reservoir dynamics. The peak shaving was designed to leave the reservoir .5 full. A comparison of the results of these two methods is shown in Table 3.3. Overall, it can be seen that the policies perform very well relative to peak shaving.

TABLE 3.3 COMPARISON OF MARKOV CHAIN AND PEAK SHAVING SIMULATION RESULTS

CASE #	INITIAL RESERVOIR LEVEL	MARKOV CHAIN		PEAK SHAVING
		FINAL RESERVOIR LEVEL	AVG* COST PER DAY	AVG COST PER DAY
1	.3	.29	13.0	12.9
2	.4	.37	12.9	12.9
3	1.0	.88	12.6	12.9

*Total Cost/364 days

3.7 RESEARCH DIRECTIONS

In this chapter, the application of techniques for time-scale decomposition of the hydro scheduling problem have been described. As stated in Section 3, the purpose of these techniques is to partition (based on naturally present multiple time-scales) a large problem into

Figure 3.7. Reservoir Trajectories for Simulation Run Using Policies Obtained Via Policy Space Decomposition.

subproblems which together may be solved more easily than the original problem.

These methods should provide insight into the original large problem and define subproblems which may be solved by any of a number of optimization techniques.

In order to consolidate and extend the results obtained so far research into three areas is required:

- Computational methods which take advantage of the structure of the Markov chain decomposition are needed. In addition, Markov chain spatial decomposition techniques need to be developed and integrated with the time-scale decomposition already obtained.
- The relation between the concepts obtained through study of time-scale decomposition and current approaches to the operations scheduling problem need to be investigated further. In addition, the exact nature and structure of the multiple time-scale Markov chain model requires further consideration.
- Other elements of the operations scheduling problem should be integrated into the problem formulation treated by temporal decomposition.

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IV. SPATIAL DECOMPOSITION

The time-scale decomposition method described in the previous chapter provides a natural decomposition of the operations scheduling problem. However, the computational complexity involved in carrying out the stochastic dynamic programming (SDP) approach requires further reduction of the problem for algorithmic development. Spatial decomposition methods, and in particular dynamic programming successive approximation (DPSA), have been successful in reducing the amount of computation in the application of dynamic programming to realistic problems. This chapter describes the extension of spatial decomposition methods to the case where uncertainty is present.

The DPSA method has been applied to the hydro scheduling of a system of parallel reservoirs in the presence of random inflows. We analyze the stochastic DPSA algorithm to (a) characterize the control law resulting from its application and (b) extend it to more general reservoir system configurations. It is shown that the scheduling of a system of parallel reservoirs using local feedback control laws is equivalent to a dynamic team problem with partially nested information structure. Based on some team theory concepts we can show that a stochastic DPSA algorithm provides a globally optimal solution within the class of local feedback control laws. The method is optimal for convex cost functions of the control and convex constraint sets. Thus, stochastic DPSA provides a good supoptimal control law for a system of parallel reservoirs.

Local feedback control policies do not form a partially nested information structure for a system of series reservoirs. The implication of this result is that stochastic DPSA algorithm cannot guarantee a globally optimal solution within the class of local feedback control laws. To overcome the difficulties present in the series reservoir problem with local feedback and still have a method that requires less computation than the centralized stochastic control problem, we consider a class of feedback laws with downstream information flow. Both the performance and the amount of computation for this method is bounded on either side by stochastic DPSA and SDP.

Numerical results on a simplified simulation of the hydro scheduling problem indicates that stochastic DPSA performs well compared to SDP for a system of parallel reservoirs. Numerical results on the application of downstream information flow algorithm will be presented in a future report.

This chapter is organized as follows: Section 4.1 reviews methods which have been used for spatial decomposition in the deterministic case. In addition, this section discusses the issues involved in the extension of deterministic methods to stochastic spatial decomposition. Section 4.2 extends the dynamic programming successive approximation method to a system of parallel reservoirs. It is shown that the DPSA solution is equivalent to team theory solution and consequently, produces a globally optimal solution within the class of local feedback control laws. Section 4.3 demonstrates why the application of DPSA to a system of series reservoirs cannot guarantee an optimal solution. This section describes a new algorithm for the spatial decomposition of a system of series reservoirs based on a downstream information flow. Section 4.4 shows some preliminary simulation results. Future research directions in stochastic spatial decomposition are described in Section 4.5.

4.1 BACKGROUND

The curse of dimensionality in dynamic programming is well-known. Several methods have been advocated for reducing dimensionality in dynamic programming. A recent survey article on computational advances in dynamic programming [1] classifies the methods into nine major categories. However, we will limit ourselves only to decomposition techniques and in particular to the dynamic programming successive approximation method [2]. This is done primarily due to the success of DPSA in hydro-scheduling applications [3] and due to its potential for extension to the stochastic case.

The principle objective of applying spatial decomposition techniques is to partition a problem involving many state variables into a sequence of problems each involving one or a few state variables. Since computational requirements usually increase exponentially with the number of state variables, a significant savings can be achieved in this manner. Next, we describe the DPSA algorithm for the deterministic case.

Description of Successive Approximation Method

The DPSA method is most clearly explained when there are as many control variables as state variables. For clarity, therefore, the application of the method to this case will be discussed first, and the extension to other problems will be indicated later in this section.

- (i) Consider a system described by the nonlinear difference equation

$$x(k+1) = g[x(k), u(k), k] \quad (4.1)$$

where

x = state vector, n -dimensional
 u = control vector, n -dimensional
 k = index for stage variable
 g = n -dimensional vector functional;

- (ii) A performance index

$$J = \sum_{k=0}^N L_k[x(k), u(k), k] \quad (4.2)$$

where

J = total cost
 L_k = cost for a single stage;

- (iii) Constraints

$$x \in X(k) \quad (4.3)$$

$$u \in U(x, k) \quad (4.4)$$

where

$X(k)$ = set of admissible states at stage k

$U(x,k)$ = set of admissible controls at state x ,
stage k ;

and (iv) An initial state

$$x(0) = c \quad (4.5)$$

The object is to find a control strategy $\gamma(\cdot,0), \gamma(\cdot,1), \dots, \gamma(\cdot,N)$ such that J in Equation (4.2) is minimized, subject to the system Equation (4.1), the constraint equations (4.3) and (4.4), and the initial condition (4.5). The control sequence $\{u(k)\}$ is then recovered from the strategy by the feedback relationship $u(k) = \gamma[x(k),k]$. Note that the form of Equation (4.1) is very general, and encompasses as special cases the equations describing the operations scheduling problem.

The DPSA Computation Procedure

In order to begin the basic procedure, it is assumed that a nominal state trajectory is given. The sequence of states along this trajectory is denoted as $\{x^0(k)\}$, where $x^0(0) = c$.

The corresponding sequence of controls is denoted as $\{u^0(k)\}$. Now, one of the n state variables, say x_{i_1} , is selected. A dynamic programming problem is then solved in which the sequence of all other states, namely,

$$\{x_i^0(k)\}, i \neq i_1,$$

is held fixed. The problem thus has only one state variable, x_{i_1} . The control vector, on the other hand, still has n components. However, the constraint that the sequence of $(n-1)$ of the states must remain fixed

imposes $(n-1)$ equality constraints on the control vector, so that effectively there is only one control variable as well. The performance criterion remains the same as in Equation (4.2), and the constraints are as in Equations (4.3) and (4.4). The solution to this optimization problem is a new control sequence, $\{u^{(1)}(k)\}$, and a new trajectory $\{x^{(1)}(k)\}$.

Next, a different state variable, say x_{i_2} , is selected and the above optimization procedure repeated. This continues until every state variable has been selected for optimization at least once. Termination occurs when the same control sequence and trajectory are obtained for optimization with respect to any of the state variables.

The advantage of this procedure is that the solution of an n -dimensional dynamic programming problem is reduced to the solution of a sequence of problems of lesser dimension. The computational savings in time and storage for high-dimensional problems can be enormous.

Convergence results for DPSA have been developed for the deterministic case. These results guarantee convergence [4] in the following cases:

- (1) Bounded states, unbounded controls, invertible mapping from controls to states, with certain convexity conditions on the objective function.
- (2) Unbounded states, bounded controls, invertible mapping from controls to states, with similar convexity conditions on the objective function.
- (3) Unbounded controls and states, with quadratic objective function.

Research Issues in Stochastic Spatial Decomposition

The ideal feedback controller for stochastic systems employs a closed-loop strategy, i.e., plans for the use of future measurements in determining the control approach. In general, the design of optimal

closed-loop controllers for a stochastic system is not computationally tractable except for systems with special structure, e.g., those for which the certainty equivalence principle [5] applies. For such systems the optimal closed-loop controls can be computed by assuming that the random disturbances are equal to their expected value and then solving an associated deterministic control problem. An example of such a system is one with linear dynamics, quadratic cost, Gaussian disturbances, and no other constraints (LQG).

The basic strategy of successive approximations is to solve a sequence of single state subproblems, each defined using current nominal solutions for other states. Thus, a natural extension of DPSA to the stochastic case can be done by defining single state subproblems for the stochastic case and assuring that the optimization procedure reduces the performance index at each step. SCI [6] has developed such an algorithm for the hydro allocation with a system of parallel reservoirs. This method essentially replaces the random cost functional with its expected value for all reservoirs except the one being optimized.

In deterministic problems, dynamic programming and DPSA produce the same solution. However, in stochastic control problems, each control decision depends to a large extent on the available data. For example, the stochastic dynamic programming (centralized control) solution cannot, in general, be reconstructed by solving a sequence of problems each involving one or a few state variables. It should be emphasized that the feedback solution generated by a spatial decomposition method is valid even when the trajectories of the system deviate from the nominal used in the initial step of the algorithm.

The basic issue to be resolved in the absence of adequate computational capability is the selection of meaningful control strategies. As a starting point, we characterize the stochastic DPSA algorithm for a system of parallel reservoirs and extend the spatial decomposition methods to a system of series reservoirs. There are other approaches to stochastic spatial decomposition and these will be discussed in Section 4.5.

4.2 STOCHASTIC OPTIMIZATION IN PARALLEL RESERVOIR SYSTEMS

This section characterizes the control policies resulting from the application of spatial decomposition methods to the stochastic optimization of a system of parallel reservoirs. It is shown that under mild assumptions DPSA provides a globally optimal solution in the class of local feedbacks. The parallel reservoir problem is formulated in Subsection 4.2.1. Team-theory concepts are reviewed in Subsection 4.2.2 and the optimality of the DPSA method is discussed in Subsection 4.2.3 and an example is given in Subsection 4.2.4. Subsection 4.2.5 gives a summary of stochastic spatial decomposition for parallel reservoirs.

4.2.1 Problem Formulation

We consider the problem of hydro allocation of a system of two parallel reservoirs. This simplified version of the problem formulated in Section II is considered to facilitate initial discussion and analysis. The dynamics of the two reservoirs are described by

$$x_1(k+1) = x_1(k) - u_1(k) + w_1(k) , \quad (4.6)$$

$$x_2(k+1) = x_2(k) - u_2(k) + w_2(k) , \quad (4.7)$$

where: $x_i(k)$ = level of water in reservoir i at time k ,
 $u_i(k)$ = release of water from reservoir i at time k ,
 $w_i(k)$ = random inflows at reservoir i at time k .

We assume that $w_i(k)$ ($i = 1, 2$) is white Gaussian noise with given mean and covariance, i.e.,

$$\underline{w}(k) = \mathcal{N} \left[\begin{pmatrix} w_1(k) \\ w_2(k) \end{pmatrix} , \begin{pmatrix} \sigma_{11}(k) & \sigma_{12}(k) \\ \sigma_{21}(k) & \sigma_{22}(k) \end{pmatrix} \right] \quad (4.8)$$

$\sigma_{12}(k) = \sigma_{21}(k)$ may be zero or nonzero. The value of $\sigma_{12}(k)$ will probably depend on the geographic location of the two reservoirs (i.e.,

how close to each other the two reservoirs are located). Also, the inflows may be considered to be correlated in time. Then, by augmenting the state, we can bring the equations describing the dynamics of the reservoir into a form similar to Eqs. (4.6)-(4.7).

We assume that at each instant of time the release in reservoir i depends only on the level of water in that reservoir, i.e.,

$$u_1(k) = \gamma_1(x_1(k), k), \quad (4.9)$$

and

$$u_2(k) = \gamma_2(x_2(k), k), \quad (4.10)$$

where $\gamma_i(\cdot, k)$ is the control law used by the i th reservoir. Further,

$$\gamma_i \in \Gamma_i,$$

where Γ_i is the class of admissible control laws for the i th reservoir.

The Optimization Problem (P)

As discussed in Section II, hydro units impact the production cost through the demand constraint. The demand to be supplied by non-hydro units at time k is

$$D(k) = \sum_{i=1}^2 f_i[x_i(k), u_i(k)],$$

where f_i is the generation at the i th reservoir at time k . The use of hydro is optimized by minimizing the demand to be supplied from non-hydro units. Ignoring head effects and assuming generation to be directly proportional to release, we consider the cost function

$$J = E \sum_{k=0}^{N-1} F[D(k) - u_1(k) - u_2(k)], \quad (4.11)$$

where the expectation E is taken with respect to all the random variables. We assume that the stage cost function $F(\cdot)$ is a convex differentiable function of $u_1(k)$ and $u_2(k)$.

The optimization problem (p) can be stated as follows:

$$\text{Minimize } E \sum_{k=0}^{N-1} F[D(k) - \gamma_1(\cdot, k) - \gamma_2(\cdot, k)] \\ \{\gamma_1(\cdot, k), \gamma_2(\cdot, k)\}_{k=0}^{N-1}$$

subject to (4.6)-(4.10).

4.2.2 Team Theory and Information Structure

We will interpret problem (p) as a team consisting of $2N$ members $\{11, 21, 12, 22, \dots, 1N, 2N\}$ where each decision maker acts only once. The order of action is fixed in advance and does not depend on random events. Decision maker ik ($i=1,2$) has the following information

$$z_i(k) \triangleq \begin{bmatrix} y_1(1) = x_1(1) \\ y_1(2) = x_1(2) \\ \vdots \\ y_1(k) = x_1(k) \end{bmatrix}, \quad i = 1, 2 \quad (4.13)$$

For convenience of subsequent discussion, we can write (4.13) as follows:

$$z_i(k) = H_i(k) \lambda_i(k) + \sum_{j=0}^{k-1} d_{ij}(k) u_i(j), \quad (4.14)$$

where

$$\lambda_i(k) = [x_i(0), w_i(0), \dots, w_i(k-1)] , \quad (4.15)$$

$$H_1(k) = \begin{bmatrix} 1 & 0 & & \\ 1 & 1 & 0 & \\ \vdots & \vdots & \ddots & \\ 1 & 1 & & 1 \end{bmatrix}, \quad (4.16)$$

and

$$d_{1j}(k) = [0 \dots 0 -1 -1 \dots -1]^T. \quad (4.17)$$

The matrix $H_1(k)$ is of dimension $k \times (k+1)$ and the k -dimensional vector $d_{1j}(k)$ has j number of zeros.

Equation (4.14) gives the information available to decision maker ik . The first term in (4.14) depends only on the characteristics of the noise and the initial conditions. The second term depends on the decision of players $1j$, $i=1,2$; $j=0,1,\dots,k-1$. For static teams, $d_{1j}=0$, i.e., the information $z_1(k)$ is only a function of the random variable λ_1 and is independent of the action of the other players. In a dynamic team, $d_{1j} \neq 0$ and a player's information is affected by past actions of other players. The static team problem is well understood; dynamic teams can be reduced to static teams under certain information structures. In the rest of this section we will briefly review some team theory concepts and apply them to the parallel reservoir problem in the next section.

We first need some definitions:

Definition 4.1

We say that the decision maker $1j$ is related to the decision maker ik if $d_{1j}(k) \neq 0$ in (4.14). In this case we write

$$(1j) R (ik). \quad (4.18)$$

Definition 4.2

Decision maker ij is a precedent of decision maker kr , $kr \{ij$,
if:

- (a) $(ij) R (kr)$, or
- (b) there exists $\{ig, ip, \dots, kl\}$ decision makers such that
 $(ij) R (ig)$, $(ig) R (ip)$, $(kl) R (kr)$.

Definitions 4.1 and 4.2 can be interpreted as follows. Decision maker ij is related to decision maker ik if the action of ij affects the information of ik . In this case, ij is also called a precedent of ik . Definition 4.2(b) considers the situation where the decision maker ij affects kr through the action of other players $ig, ip, \dots, kl$.

Mathematically, the information available to decision maker ik is the σ -algebra Z_{ik} generated by the action of the decision makers who are precedents of ik . Note that whereas in centralized control Z_{ik} is independent of the choice of the control laws γ_{ij} , in decentralized control Z_{ik} depends in general on the control laws used by all the precedents of ik , i.e., $Z_{ik} = Z_{ik} \{\gamma_{ij}\}$. This is the most important difference between centralized and decentralized control. It is a consequence of the following result [7]. If the data available to a controller m includes the observations and the actions of all controllers prior to it, then the conclusions controller m can draw do not depend on the functional relation (policy, control law) used by the previous controllers to reach their decisions.

In centralized control, at any time k , the controller has available all the past actions and observations, therefore its information does not depend on the policies previously employed. (This statement is taken for granted in the application of dynamic programming to centralized stochastic control problems, where it plays an essential role.)

In decentralized control, agents do not in general have available all the past observations and actions, thus their information depends on the functional relations used by previous agents.

Define

$$\left. \begin{aligned} \alpha_{k+1}(\cdot) &\triangleq \gamma_1(\cdot, k), \\ \alpha_{N+k+1}(\cdot) &\triangleq \gamma_2(\cdot, k), \\ r_{k+1} &\triangleq u_1(k), \\ r_{N+k+1} &\triangleq u_2(k), \\ \xi_{k+1} &\triangleq z_1(k), \\ \xi_{N+k+1} &\triangleq z_2(k). \end{aligned} \right\} \quad k = 0, 1, \dots, N-1 \quad (4.19)$$

The cost function defined by Eq. (4.11) can be written as

$$J(\alpha) \triangleq E\{L[\alpha_1, \alpha_2, \dots, \alpha_{2N}]\} . \quad (4.20)$$

For any control action r_i of player i and any value of his information variable ξ_i , the conditional expected cost is

$$\mathcal{J}_i(r_i, \xi_i) \triangleq E\{L(\alpha_1(\xi_1), \alpha_2(\xi_2), \dots, r_i, \dots, \alpha_{2N}(\xi_{2N}))\} .$$

Definition 4.3

A control policy is defined to be person-by-person optimal if it cannot be improved by changing any one component α_i alone. For any control policy, $\tilde{\alpha}$, let

$$J^p(\alpha_p, \tilde{\alpha}) = J(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_{p-1}, \alpha_p, \tilde{\alpha}_{p+1}, \dots, \tilde{\alpha}_{2N}) ;$$

then $\tilde{\alpha}$ is person-by-person optimal if, for every i ,

$$J(\tilde{\alpha}) = \min_{\alpha_p} J^p(\alpha_p, \tilde{\alpha}) . \quad (4.21)$$

A necessary condition for $\tilde{\alpha}$ to be person-by-person optimal is

$$\left. \frac{\partial \mathcal{L}_i(r_i, \xi_i)}{\partial r_i} \right|_{r_i = \alpha_i(\xi_i)} = 0 , \quad (4.22)$$

for all i and ξ_i .

An important property of static teams is stated in the next theorem.

Theorem 4.1 [8]

For each i let r_i be a real variable, and suppose that for every value of x_1 and x_2 , $L(\cdot)$ is a convex, differentiable function of the control $r = (r_1, r_2, \dots, r_{2N})$; then any person-by-person optimal policy is optimal.

Theorem 4.1 implies that as long as the cost function is convex in the control variable, the person-by-person optimal policy is globally optimal. The theorem remains valid even when the control variable r is constrained to a convex set.

Definition 4.4

An information structure is called partially nested if, whenever decision maker ij precedes decision maker kt , $Z_{ij} \subset Z_{ik}$ for all admissible policies of all decision makers.

The partially-nested information structure has the property that a player can reconstruct the actions of all its precedents once the control laws are fixed. A consequence of this property is the following theorem.

Theorem 4.2 [9]

A dynamic team with partially nested information structure

$$z_i(k) = H_i(k) \lambda_i(k) + \sum_{j=0}^{k-1} d_{ij}(k) u_i(j) ,$$

where j is a precedent of i or j equal to i is equivalent to an information structure in static form

$$\hat{z}_i(k) = H_j(k) \lambda_i(k) ,$$

for any fixed set of control laws.

Remark

The above theorem shows that a dynamic team problem with partially nested information structure can be transformed into a static team.

4.2.3 DPSA Algorithm for Parallel Reservoirs

The DPSA algorithm provides an iterative method for computing local feedback controls. At each step the algorithm fixes all control laws except one and optimizes the problem with respect to the free control laws. The result of the DPSA procedure is a sequence of monotonically decreasing costs, i.e., at the end of j th iteration we have

$$J[\gamma_1^{j+1}(\cdot, k), \gamma_2^j(\cdot, k)] \leq J[\gamma_1^j(\cdot, k), \gamma_2^j(\cdot, k)] \quad (4.23)$$

assuming the control policy with respect to the first reservoir is being updated. When the sequence converges we obtain a person-by-person optimal.

Due to the special structure of the parallel reservoir problem, information $z_i(k-1)$ is contained in information $z_i(k)$ and the σ -algebra

$$Z_{i(k-1)} \subset Z_{ik} . \quad (4.24)$$

Thus the parallel reservoir system with local feedback has partially nested information structure and the optimization problem (p) can be reduced to a static team problem. Since the stage cost function is assumed to be convex in the control u , applying Theorem 4.1, a person-by-person optimal solution to problem (p) is also optimal.

Since DPSA is an algorithm which computes person-by-person optimal, for the parallel reservoir problem with convex cost function, DPSA provides a unique solution in the class of local feedback laws.

4.2.4 Example

This section presents an example illustrating the nature of the decentralized solution and compares it with the linear-quadratic-Gaussian (centralized) solution. The decentralized solution is computed using both DPSA and team theory methods. A more detailed example consisting of a system of parallel reservoirs with nonquadratic cost functions will be treated in Section 4.4.

Consider the two-reservoir problem (Eqs. (4.6) and (4.7)) with the quadratic cost

$$J = E \sum_{k=0}^1 x^T(k) Q(k) x(k) + u^T(k) R(k) u(k) + x^T(2) Q(2) x(2) , \quad (4.25)$$

where

$$x(k) = [x_1(k) \ x_2(k)]^T , \quad u(k) = [u_1(k) \ u_2(k)]^T ,$$

$$Q(0) = Q(1) = R(0) = R(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad Q(2) = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} . *$$

* Assume for simplicity that $x_1(0)$ and $x_2(0)$ are independent and that $\sigma_{12}(k) = \sigma_{21}(k) = 0 \quad \forall k$.

Linear Quadratic Gaussian (LOG) Solution

Define

$$A(0) = A(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B(0) = B(1) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$K(2) = Q(2),$$

$$K(1) = A^T(1) [K(2) - K(2)B(1) [B(1)K(2)B(1) + R(1)]^{-1} B^T(1) K(2)] A(1) + Q(1)$$

$$= \begin{bmatrix} 22/15 & 2/15 \\ 2/15 & 22/15 \end{bmatrix}.$$

Then the LQG solution is given by [10]

$$\begin{aligned} u^*(1) &= [R(1) + B^T(1)K(2)B(1)]^{-1} B^T(1)K(2)A(1)x(1) \\ &= \begin{bmatrix} 7/15 & 2/15 \\ 2/15 & 7/15 \end{bmatrix} x(1), \end{aligned} \quad (4.26)$$

$$u^*(0) = 2/91 \begin{bmatrix} 27 & 1 \\ 1 & 27 \end{bmatrix} x(0). \quad (4.27)$$

DPSA Solution

Define J_1^j as the cost-to-go at stage 1 when optimization is carried with respect to reservoir j . Thus,

$$\begin{aligned} J_1^1 = \min_{u_1(1)} E \{ & x_1^2(1) + x_2^2(1) + u_1^2(1) + u_2^2(1) + x_1^2(2) + x_2^2(2) \\ & + x_1(2) x_2(2) \}. \end{aligned}$$

The minimization in (4.26) is carried out given observations $[x_1(1), x_1(0)]$ and an expected trajectory for x_2 . From (4.26),

$$u_1(1) = \frac{1}{4} [2x_1(1) + E(x_2(2))] . \quad (4.29)$$

Next,

$$J_0^1 = \min_{u_1(0)} E\{x_1^2(0) + x_2^2(0) + u_1^2(0) + u_2^2(0) + J_1^1\} ,$$

and the minimization results in the control

$$u_1(0) = \frac{1}{5} [3x_1(0) + \frac{1}{2} E(x_2(2))] . \quad (4.30)$$

Observing (4.29) and (4.30), by symmetry, the DPSA solution is

$$u^*(1) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} x(1) + \begin{bmatrix} 0 & 1/4 \\ 1/4 & 0 \end{bmatrix} \begin{bmatrix} E(x_1(2)) \\ E(x_2(2)) \end{bmatrix} , \quad (4.31)$$

$$u^*(0) = \begin{bmatrix} 3/5 & 0 \\ 0 & 3/5 \end{bmatrix} x(0) + \begin{bmatrix} 0 & 1/10 \\ 1/10 & 0 \end{bmatrix} \begin{bmatrix} E(x_1(2)) \\ E(x_2(2)) \end{bmatrix} . \quad (4.32)$$

Remark: Notice the feedback solution in (4.31)-(4.32) is different in structure from the LQG solution. In the deterministic case, or in the certainty equivalence method, Eqs. (4.31)-(4.32) reduce to the LQG solution.

Team-Theory Solution

Define the observations at each reservoir to be

$$y_1(k) = x_1(k) , \quad (4.33)$$

$$y_2(k) = x_2(k) , \quad k=0,1,2 \dots \quad (4.34)$$

First, we compute $u_1(1)$ and $u_2(1)$. Use dynamic Eqs. (4.6) and (4.7) to write cost as

$$\begin{aligned} J = & E\{x_1^2(0) + x_2^2(0) + x_1^2(1) + x_2^2(1) \\ & + [x_1(1) - u_1(1) + w_1(1)]^2 + [x_2(1) - u_2(1) + w_2(1)]^2 \\ & + [x_1(1) - u_1(1) + w_1(1)] [x_2(1) - u_2(1) + w_2(1)] \\ & + u_2^2(0) + u_2^2(1) + u_1^2(0) + u_1^2(1)\} . \end{aligned} \quad (4.35)$$

We can write

$$J = E_{y_1(0), y_1(1)} \{E\{J|y_1(0), y_1(1)\}\}$$

for fixed

$$\gamma_1^*(y_1(0)), \gamma_2^*(y_2(0)), \gamma_2^*(y_2(0), y_2(1)) .$$

Then

$$\min_{\gamma_1(1) \in \Gamma_1(1)} E_{y_1(0), y_1(1)} \{E\{J|y_1(0), y_1(1)\}\} ,$$

is equivalent to

$$\min_{u_1(1)} E\{J|y_1(0), y_1(1)\} . \quad (4.36)$$

We use (4.34) to determine $u_1(1)$. Let

$$E\{J|y_1(0), y_1(1)\} = \mathcal{L}_1(1) .$$

Then, setting $\frac{\partial \mathcal{L}_1(1)}{\partial u_1(1)} = 0$ gives

$$\begin{aligned} 4u_1(1) - 2x_1(1) - E\{x_2(1) - u_2(1) \\ + w_2(1)|x_1(0), x_1(1)\} = 0 . \end{aligned} \quad (4.37)$$

This implies

$$u_1(1) = \frac{1}{4} [2x_1(1) + E(x_2(1) | x_1(0), x_1(1))] ,$$

or

$$u_1^*(1) = \frac{1}{2} x_1(1) + \frac{1}{4} E(x_2(1)) . \quad (4.38)$$

By symmetry we find that

$$u_2^*(1) = \frac{1}{2} x_2(1) + \frac{1}{4} E(x_1(2)) . \quad (4.39)$$

Using similar arguments, Appendix A shows that

$$u_1^*(0) = \frac{3}{5} x_1(0) + \frac{1}{10} E(x_2(2)) . \quad (4.40)$$

By symmetry we get

$$u_2^*(0) = \frac{3}{5} x_2(0) + \frac{1}{10} E(x_1(2)) . \quad (4.41)$$

Note that the optimal solution obtained by the team formulation of the problem is the same as the solution obtained by DPSA. This is due to the information structure of the problem. For the sequential reservoir problem, terms like

$$\frac{\partial}{\partial u_1(0)} E\{[\gamma_1^*(y_1(0), y_1(1))]^2 | x_1(0)\} , \quad (4.42)$$

will not be equal to zero and are very difficult to evaluate. The cost, although quadratic in u , will not be in general a convex function of γ . The necessary conditions

$$\frac{\partial \mathcal{L}_1(1)}{\partial u_1(1)} = 0 , \quad \frac{\partial \mathcal{L}_1(0)}{\partial u_1(0)} = 0 , \quad \frac{\partial \mathcal{L}_2(1)}{\partial u_2(1)} = 0 , \quad \frac{\partial \mathcal{L}_2(0)}{\partial u_2(0)} = 0 , \quad (4.43)$$

will provide only local minima, complicating determination of the globally optimal solution.

4.2.6 Summary

This section has characterized the control laws resulting from the application of spatial decomposition to a system of parallel reservoirs. It is shown that the DPSA algorithm provides a global optimal within the class of local feedback laws. The example illustrates the effect of information structure on the control law. Extensions of the DPSA method to a system of series reservoirs requires additional assumptions on the information structure; these are considered in Section 4.3.

4.3 STOCHASTIC OPTIMIZATION IN SERIES RESERVOIR SYSTEMS

In this section we investigate stochastic optimization methods for series reservoir systems. Information in hydro scheduling is itself not an issue. However, the way it is processed affects the nature of the control law. Unlike a system of parallel reservoirs, local feedback control laws in a system of series reservoirs do not form a partially nested information set. Thus, the application of stochastic DPSA cannot guarantee a globally optimal solution within the class of local feedbacks. We examine different information structures to find reduced-order computational methods which give a good suboptimal solution.

The series reservoir problem is formulated in Subsection 4.3.1. Subsection 4.3.2 points out the difficulty in applying DPSA to the problem. Subsection 4.3.3 introduces the feedback laws with downstream information flow, and an algorithm is outlined in Subsection 4.3.4. Subsection 4.3.5 describes how the method applies directly to the hydro-scheduling problem, and a summary is given in Section 4.3.6.

4.3.1 Problem Formulation

Consider a series reservoir system as shown in Figure 4.1. The analysis is carried out for three reservoirs. The results are true for any number of reservoirs.

Figure 4.1. Series Reservoir System.

The dynamics of the system are described by

$$\begin{aligned}
 x_1(k+1) &= x_1(k) - u_1(k) + w_1(k) , \\
 x_2(k+1) &= x_2(k) - u_2(k) + u_1(k) + w_2(k) , \\
 x_3(k+1) &= x_3(k) - u_3(k) + u_2(k) + w_3(k) ,
 \end{aligned}
 \tag{4.44}$$

where: $x_i(k)$ = level of water in reservoir i at time k ,
 $u_i(k)$ = release of water from reservoir i at time k ,
 $w_i(k)$ = random inflow at reservoir i at time k .

We assume that the inflows are spatially and temporally independent.
 However, these assumptions can be relaxed.

The measurements are given by

$$\begin{aligned} y_1(k) &= x_1(k) , \\ y_2(k) &= x_2(k) , \\ y_3(k) &= x_3(k) . \end{aligned} \tag{4.45}$$

For simplicity, consider a quadratic cost of the form

$$J = E\{\mathcal{L}\} ,$$

where

$$\begin{aligned} \mathcal{L} = & x_1^2(0) + x_2^2(0) + x_3^2(0) + x_1^2(1) + x_2^2(1) + x_3^2(1) + x_1^2(2) \\ & + x_2^2(2) + x_3^2(2) + x_1(2) x_2(2) + x_2(2) x_3(2) + x_1(2) x_3(2) \\ & + u_1^2(0) + u_2^2(0) + u_3^2(0) + u_1^2(1) + u_2^2(1) + u_3^2(1) . \end{aligned} \tag{4.46}$$

We want to choose $\{u_i(0), u_i(1) | i=1,2,3\}$ to minimize the cost, using the measurements and subject to the dynamics of the system.

Ideally, we would use all the measurements. If this is computationally possible, then we have a centralized stochastic control problem which can be solved by stochastic dynamic programming. If we cannot handle the amount of computation required to compute a control law based on stochastic dynamic programming, then we have to look for reduced-order computational methods. In the parallel reservoir problem, we considered local feedback controls and showed that dynamic programming successive approximations can be used to achieve a global optimum in the class of local feedback laws. Thus, we begin our investigation by considering local feedback controls.

4.3.2 Local Feedback Laws

This section examines the information structure in a series reservoir system with local feedback. It is shown that the information structure is not partially nested and we are dealing with a nonclassical stochastic control problem. Global optimality cannot be guaranteed for such problems. This leads us to examine alternate feedback laws in Section 4.3.3. The series reservoir problem with local feedback can be stated as follows:

$$\left. \begin{array}{l} \text{Minimize } J \\ \{u_i(0), u_i(1)\} \quad i=1,2,3 \end{array} \right\} \quad (B)$$

subject to (4.44)-(4.46), and

$$\begin{aligned} u_i(0) &= (y_i(0)) , \\ u_i(1) &= \left(y_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) , \end{aligned} \quad (4.47)$$

where

$$y_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \{y_i(0), y_i(1)\} .$$

To analyze the problem, let us assume that we have 3N decision makers (10,20,30,11,21,31) and examine the information structure. Consider decision maker 21. The information available to 21 is

$$z_{21} = \begin{cases} y_2(0) = x_2(0) \\ y_2(1) = x_2(1) \end{cases} \quad (4.48)$$

Using (4.44) we can write (4.48) as follows:

$$z_{21} = \begin{cases} y_2(0) = x_2(0) \\ y_2(1) = x_2(0) - u_2(0) + u_1(0) + w_2(0) \end{cases} \quad (4.49)$$

Since we assume that the decision makers have agreed on the strategy they are using, decision maker 21, based on this information, can exactly recover the control $u_2(0)$ of decision maker 20. However, decision maker 21 cannot recover the action of 10 as he has no access to the measurement $x_1(0)$. Hence, our information equivalent to z_{21} for decision-making purposes is

$$\tilde{z}_{21} = \begin{cases} y_2(0) = x_2(0) \\ \tilde{y}_2(1) = x_2(0) + u_1(0) + w_2(0) \end{cases} \quad (4.50)$$

Equation (4.50) shows that the information of 21 depends on the decision of 10. Consequently, the information structure of problem (B) is not partially nested. Problem (B) is a nonclassical stochastic control problem as defined in [11]. The solution of nonclassical stochastic control problems is in general unknown. The main difficulty in these problems comes from the dependence of one control law on other control laws (in our problem γ_{21} depends on γ_{10}). To show how this dependence affects the solution, we consider problem (B) and optimize with respect to decision maker 10. We assume fixed $\gamma_{20}^*, \gamma_{30}^*, \gamma_{11}^*, \gamma_{21}^*, \gamma_{31}^*$. We write

$$J = E\{\mathcal{L}\} = E_{z_{10}}\{E\{\mathcal{L}/z_{10}\}\} \quad (4.51)$$

for fixed $\gamma_{10}^*, \gamma_{20}^*, \gamma_{30}^*, \gamma_{11}^*, \gamma_{21}^*, \gamma_{31}^*$. Then for fixed $\gamma_{20}^*, \gamma_{30}^*, \gamma_{11}^*, \gamma_{21}^*, \gamma_{31}^*$ and any fixed z_{10}

$$\min_{\gamma_{10}} E_{z_{10}}\{E\{\mathcal{L}/z_{10}\}\} \Leftrightarrow \min_{u_1(0)} E\{\mathcal{L}/z_{10}\} \quad (4.52)$$

Let

$$J_{10} = E\{\mathcal{L}/z_{10}\} \quad (4.53)$$

A necessary condition for optimality is given by

$$\frac{\partial J_{10}}{\partial u_1(0)} = 0 \quad (4.54)$$

To compute (4.54), we use the dynamic equations and write J_{10} as

$$\begin{aligned}
J_{10} = & E\{u_1^2(0) + u_2^2(0) + u_3^2(0) + u_1^2(1) + u_2^2(1) \\
& + u_3^2(1) + (x_1(0) - u_1(0) - u_1(1) + w_1(0) + w_1(1))^2 \\
& + (x_2(0) - u_2(0) - u_2(1) + u_1(0) + u_1(1) + w_2(0) + w_2(1))^2 \\
& + (x_3(0) - u_3(0) - u_3(1) + u_2(0) + u_2(1) + w_3(0) + w_3(1))^2 \\
& + (x_1(0) - u_1(0) + u_1(1) + w_1(0) + w_1(1))(x_2(0) - u_2(0) - u_2(1) \\
& + u_1(0) + u_1(1) + w_1(1)) \\
& + (x_2(0) - u_2(0) - u_2(1) + u_1(0) + u_1(1) + w_2(0) + w_2(1)) \cdot \\
& (x_3(0) - u_3(0) - u_3(1) + u_2(0) + u_2(1) + w_3(0) + w_3(1)) \\
& + (x_1(0) - u_1(0) - u_1(1) + w_1(0) + w_1(1))(x_3(0) - u_3(0) - u_3(1) \\
& + u_2(0) + u_2(1) + w_3(1)) \\
& + (x_1(0) - u_1(0) + w_1(0))^2 + (x_2(0) - u_2(0) + u_1(0) + w_2(0))^2 \\
& + (x_3(0) - u_3(0) + u_2(0) + w_3(0))^2 + x_1^2(0) + x_2^2(0) \\
& + x_3^2(0) \} / \{ \epsilon_{10} \} . \tag{4.55}
\end{aligned}$$

We also need to have a precedence (relation) diagram as defined in Ref. 5. To construct this diagram, we need to write the measurements in terms of the primitive random variables [9] and the controls. In general, we have

$$y_i(j) = x_i(0) - \sum_{\tau=0}^{k-1} u_i(j) + \sum_{\tau=0}^{k-1} u_{i-1}(j) + \sum_{\tau=0}^{k-1} w_i(j) . \tag{4.56}$$

Based on (4.56), the precedence diagram in Figure 4.2 shows how the control action of one player affects the information of another player. Using (4.55) and Figure 4.2, we get from (4.54)

$$\begin{aligned}
& 2u_1(0) + \frac{\partial}{\partial u_1(0)} E\{u_1^2(1)/z_{10}\} + \frac{\partial}{\partial u_1(0)} E\{u_2^2(1)/z_{10}\} + 2u_1(0) \\
& + \frac{\partial}{\partial u_1(0)} E\{u_1^2(1)/z_{10}\} + 2 \frac{\partial}{\partial u_1(0)} E\{u_1(0)u_1(1)/z_{10}\} - 2x_1(0) \\
& - 2E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_1(0)/z_{10}\right\} + \frac{\partial}{\partial u_1(0)} E\{u_2^2(1)/z_{10}\} + 2u_1(0) \\
& - 2 \frac{\partial}{\partial u_1(0)} E\{u_2(1), u_1(0)/z_{10}\} + \frac{\partial}{\partial u_1(0)} E\{u_1^2(1)/z_{10}\} \\
& + 2 \frac{\partial}{\partial u_1(0)} E\{u_1(0), u_1(1)/z_{10}\} - 2 \frac{\partial}{\partial u_1(0)} E\{u_2(1), u_1(1)/z_{10}\} \\
& + 2x_1(0) - 2E\left\{\frac{\partial u_2(1)}{\partial u_1(0)} x_1(0)/z_{10}\right\} + 2E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_1(0)/z_{10}\right\} \\
& + \frac{\partial}{\partial u_1(0)} E\{u_2^2(1)/z_{10}\} + 2E \frac{\partial u_2(1)}{\partial u_1(0)} [x_3(0) - u_3(0) - u_3(1) - u_2(0)]/z_{10} \\
& - E\{x_2(2)/z_{10}\} + E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_1(2)/z_{10}\right\} - E\left\{\frac{\partial u_2(1)}{\partial u_1(0)} x_1(2)/z_{10}\right\} \\
& + E\{x_1(2)/z_{10}\} + E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_1(2)/z_{10}\right\} - E\left\{\frac{\partial u_2(1)}{\partial u_1(0)} x_3(2)/z_{10}\right\} \\
& + E\{x_3(2)/z_{10}\} + E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_3(2)/z_{10}\right\} + E\left\{\frac{\partial u_2(0)}{\partial u_1(0)} x_2(2)/z_{10}\right\} \\
& + x_3(2)/z_{10} - E\left\{\frac{\partial u_1(1)}{\partial u_1(0)} x_3(2)/z_{10}\right\} + E\left\{\frac{\partial u_2(1)}{\partial u_1(0)} x_2(2)/z_{10}\right\} + 2u_1(0) \\
& - 2x_1(0) + 2u_1(0) + 2E\{x_2(0)/z_{10}\} - 2E\{u_2(0)/z_{10}\} = 0. \quad (4.57)
\end{aligned}$$

In (4.57), the expressions $\frac{\partial u_2(1)}{\partial u_1(0)}$ and $\frac{\partial u_1(1)}{\partial u_1(0)}$ appear.

Figure 4.2. Precedence Diagram for Series Reservoir with Local Feedback

It is shown in Appendix A that $\frac{\partial u_1(1)}{\partial u_1(0)} = 0$. However, $\frac{\partial u_2(1)}{\partial u_1(0)} \neq 0$. Moreover, $\frac{\partial u_2(1)}{\partial u_1(0)}$ depends explicitly on the form of the control of decision maker 21. Furthermore, because of the dependence of γ_{21} on γ_{10} , the cost function, although quadratic in u

$$u = (u_1(0) \ u_2(0) \ u_3(0) \ u_1(1) \ u_2(1) \ u_3(1)) ,$$

is not in general convex in γ_{10} . Then, solution of problem (B) through (4.54) is, even if possible, unsatisfactory because the control resulting from (4.54) may lead to a local minimum. Dynamic programming successive approximations would also lead to a local optimum or a singular point or even a singular loop [7]. To overcome the difficulties present in the series reservoir problem with local feedback and still have a method that requires less computations than the centralized stochastic control problem, we consider the following class of feedback laws.

4.3.3. Downstream Information Flow

Consider again the three-reservoir system described by (4.44), with measurements given by (4.45) and assume that the water release in each reservoir takes into account measurements in the reservoirs above with a unit delay. More specifically we assume that

$$u_i(k) = \gamma_i \left(y_i \binom{k}{0} y_{i-1} \binom{k-1}{0} \dots y_2 \binom{k-i+1}{0} \right), \quad k=0,1; i=1,2,3. \quad (4.58)$$

Then we have the following optimization problem

$$\left. \begin{array}{l} \text{Minimize } J \\ \gamma_{10}, \dots, \gamma_{31} \\ \text{subject to (4.44), (4.45) and (4.58).} \end{array} \right\} \quad (C)$$

To analyze problem (C), we consider again six decision makers: 10, 20, 30, 11, 21, 31, and examine the information available to each of them. Consider decision maker ik . This information is

$$z_{ik} = \left\{ \begin{array}{l} y_1(0) = x_1(0) \\ y_1(1) = x_1(1) \\ \vdots \\ y_1(k) = x_1(k) \\ \hline y_{i-1}(0) = x_{i-1}(0) \\ y_{i-1}(1) = x_{i-1}(1) \\ \vdots \\ y_{i-1}(k-1) = x_{i-1}(k-1) \\ \hline \vdots \\ \hline y_1(0) = x_1(0) \\ y_1(1) = x_1(1) \\ \vdots \\ y_1(k-i+1) = x_1(k-i+1) \end{array} \right. \quad (4.59)$$

Using the dynamics of the system, we can write each one of the measurements in (4.59) in terms of the primitive random variables and the decisions of preceding decision makers. We have, for $j < i$, $\tau < k$,

$$y_j(\tau) = x_j(0) - \sum_{\ell=0}^{\tau-1} u_j(\ell) + \sum_{\ell=0}^{\tau-1} u_{j-1}(\ell) + \sum_{\ell=0}^{\tau-1} w_j(\ell) . \quad (4.60)$$

Thus, (4.59) can be written as

$$z_{ik} = \left\{ \begin{array}{l} y_1(0) = x_1(0) \\ y_1(1) = x_1(0) - u_1(0) + u_{1-1}(0) + w_1(0) \\ y_1(k) = x_1(0) - \sum_{\ell=0}^{k-1} u_1(\ell) + \sum_{\ell=0}^{k-1} u_{1-1}(\ell) + \sum_{\ell=0}^{k-1} w_1(\ell) \\ \hline y_{i-1}(0) = x_{i-1}(0) \\ y_{i-1}(1) = x_{i-1}(0) - u_{i-1}(0) + u_{i-2}(0) + w_{i-1}(0) \\ \vdots \\ y_{i-1}(k-1) = x_{i-1}(0) - \sum_{\ell=0}^{k-2} u_{i-1}(\ell) + \sum_{\ell=0}^{k-2} u_{i-2}(\ell) + \sum_{\ell=0}^{k-2} w_{i-1}(\ell) \\ \vdots \\ \hline \vdots \\ \hline y_1(0) = x_1(0) \\ y_1(1) = x_1(0) - u_1(0) + w_1(0) \\ \vdots \\ y_1(k-i+1) = x_1(0) - \sum_{\ell=0}^{k-1} u_1(\ell) + \sum_{\ell=0}^{k-1} w_1(\ell) \end{array} \right. \quad (4.61)$$

Since the decision makers can agree in advance on the decision rule used, decision maker ik can deduce the actions $u_j(\tau)$ ($j < i$, $\tau \leq k$) exactly. This happens because $u_j(\tau) = \gamma_j(y_1(\tau-j+1), \dots, y_j(\tau))$ and $y_j(\tau), \dots, y_1(\tau-j+1)$ are all available to decision maker ik . Consequently, the dynamic team problem (C) can be reduced to a static team problem with the following equivalent information pattern:

$$\bar{z}_{ik} = \left\{ \begin{array}{l} \bar{y}_1(0) = x_1(0) \\ \bar{y}_1(1) = x_1(0) + w_1(0) \\ \vdots \\ \bar{y}_1(k) = x_1(0) + \sum_{\ell=0}^{k-1} w_1(\ell) \\ \text{-----} \\ \bar{y}_{i-1}(0) = x_{i-1}(0) \\ \bar{y}_{i-1}(0) = x_{i-1}(0) + w_{i-1}(0) \\ \vdots \\ \bar{y}_{i-1}(k-1) = x_{i-1}(0) + \sum_{\ell=0}^{k-2} w_{i-1}(\ell) \\ \text{-----} \\ \vdots \\ \text{-----} \\ \bar{y}_1(0) = x_1(0) \\ \bar{y}_1(1) = x_1(0) + w_1(0) \\ \vdots \\ \bar{y}_1(k-i+1) = x_1(0) + \sum_{\ell=0}^{k-i} w_1(\ell) \end{array} \right.$$

for all i and k .

The quadratic static team has a unique solution [12] and the optimal decisions are affine functions of the information. The optimal solution can be computed by necessary conditions similar to (4.54). To find $u_1(0) = \gamma_1(x_1(0))$, for example, we define

$$J_{10} = E\{\mathcal{L}/z_{10}\} = E\{\mathcal{L}/x_1(0)\}, \quad (4.63)$$

and compute $u_1(0)$ from the equation

$$\frac{\partial J_{10}}{\partial u_1(0)} = 0. \quad (4.64)$$

$\frac{\partial J_{10}}{\partial u_1(0)}$ gives an expression similar to (4.57) due to the information structure we have assumed

$$\frac{\partial u_1(1)}{\partial u_1(0)} = 0, \quad (4.65)$$

and

$$\frac{\partial u_2(1)}{\partial u_1(0)} = 0. \quad (4.66)$$

Thus (4.57) reduces to

$$5u_1(0) + E\{u_1^*(1)/x_1(0)\} - 3x_1(0) + 3E\{x_2(2)/x_1(0)\} = 0. \quad (4.67)$$

Equations similar to (4.67) result from the necessary conditions for optimality of $u_1(1)$, $u_2(1)$, $u_1(1)$, $u_2(1)$, $u_3(1)$. These algebraic equations determine the optimal controls. Next we describe an algorithm for the solution of problem (C).

4.3.4 Algorithm

We describe an orthogonal search algorithm which provides the global optimal solution for problem (C). The steps of the algorithm are the following:

- (1) Assume a nominal policy $\{\gamma_{10}^0, \gamma_{11}^0, \gamma_{20}^0, \gamma_{21}^0, \gamma_{30}^0, \gamma_{31}^0\}$.
- (2) Use dynamic programming to update the control policy of reservoir 1. Let the new policy be $\{\gamma_{10}^1, \gamma_{11}^1\}$. In this step, conditional expectations are taken with respect to $x_1(0)$ and $\{x_1(0), x_1(1)\}$.

- (3) Use dynamic programming and the new policy from step 2 to update the policy for reservoir 2. Let the new policy be $\{\gamma'_{20}, \gamma'_{21}\}$. In this step, conditional expectations are taken with respect to $x_2(0)$ and $\{x_2(0), x_2(1), x_1(0)\}$. The policy of reservoir 3 is fixed at $\{\gamma^o_{30}, \gamma^o_{31}\}$.
- (4) Use dynamic programming and the new policies from step 2 and 3 to determine a new control policy, $\{\gamma'_{30}, \gamma'_{31}\}$, for reservoir 3. In this step, conditional expectations are taken with respect to $x_3(0)$ and $\{x_3(0), x_3(1), x_2(0)\}$.
- (5) Repeat steps (2), (3) and (4) until the change in the expected cost resulting from each step to the next is smaller than a predetermined quantity.

Each step in this algorithm is designed to retain a partially-nested information structure. This property together with the convexity of the cost function with respect to u assures that the person-by-person optimal solution is optimal within the class of downstream information flow control laws.

We compare the number of computations required by this algorithm to the number of computations required by stochastic dynamic programming. Suppose we have m reservoirs; assume 10 discrete levels of water in the reservoir, 10 discrete levels of releases, and 10 discrete levels of inflows. Then stochastic dynamic programming requires 10^{3m} computations per stage. Our algorithm requires $10^3 + 10^4 + \dots + 10^{m+2}$ computations per stage per iteration. For $m=10$ we have to compare 10^{30} computations per stage required for stochastic dynamic programming to approximately 10^{12} computations per iteration per stage required from our algorithm. If the convergence is fast, then we have a considerable reduction in the amount of computations. Dynamic programming successive approximation (corresponding to the case of local feedback controls) would require 10^3 computations per stage per iteration, but would not guarantee convergence to the global minimum (within the class of local feedback controls) as pointed out in Subsection 4.3.1.

The solution provided by the algorithm described above requires more computation than dynamic programming successive approximations; however, convergence to a global optimum is guaranteed and the resulting minimum cost is lower than the minimum cost of the stochastic control problem with local feedback.

4.3.5 Summary

Spatial decomposition methods which have played a vital role in the application of dynamic programming to large problems need careful consideration before they can be applied in a stochastic environment. It is shown that the application of DPSA for determining the optimal schedules for a system of series reservoirs may lead to local minima. To overcome this difficulty and still have an approach that requires less computation than the centralized stochastic control problem, we have introduced a class of feedback laws with downstream information flow. An algorithm to find a globally optimal solution within this class has been outlined. Numerical results on the application of this algorithm to the hydro-scheduling problem will be presented in future reports.

4.4 SIMULATION RESULTS

In this section, we present numerical results on the stochastic optimization methods described in Section 4.2. A sample problem in hydro-scheduling is solved for a system of parallel reservoirs using dynamic programming, stochastic dynamic programming and dynamic programming successive approximation.

4.4.1 Description of the Problem

The objective is to generate an optimal monthly release policy for a system of reservoirs given the demand for power and the inflow distribution. The hydro system has the following features:

- Number of reservoirs = 3
- Planning period = 12 months
- Dynamics of the system

$$x_1(k+1) = x_1(k) + 2\omega_1(k) - u_1(k)$$

$$x_2(k+1) = x_2(k) + \omega_2(k) - u_2(k)$$

$$x_3(k+1) = x_3(k) + \omega_3(k) - u_3(k)$$

- Capacity of reservoirs

$$\#1 = 16$$

$$\#2 = 9$$

$$\#3 = 8$$

- Admissible controls:

$$0 \leq u_1 \leq 3$$

$$0 \leq u_2 \leq 3$$

$$0 \leq u_3 \leq 2$$

- Stage unit = 1 month
- Reservoir quantization size: 1
- Demand: see Figure 4.3
- Inflow distribution: see Table 4.1
- Cost Function:

$$J = \sum_{k=0}^{10} \frac{1}{12} [D(k) - u_1(k) - u_2(k) - u_3(k)]^2 + \frac{1}{12} [x_1(11) + x_2(11) + x_3(11)]^2$$

Figure 4.3. Monthly Demand for Power

Table 4.1

 ω_1 , ω_2 and ω_3 Assumed to Have the Same Distribution

$\omega_1(k)$	k=0,1	k=2-8	k=9-10
0	0	1/2	0
1	1/3	1/6	1/3
2	1/3	1/6	1/3
3	1/3	1/6	1/3
$E[\omega_1(k)]$	2.0	1.0	2.0

4.4.2 Numerical Results

The deterministic dynamic programming solution was computed by replacing the inflow distribution by the expected value. A feedback control policy was computed for each of the three different methods — DP, SDP and stochastic DPSA. These policies were evaluated by simulating 50 one-year inflow sequences.

The control policy generated by stochastic DPSA for the fourth (k=3) and fifth (k=7) months is shown in Figure 4.4. These two months correspond to the period when the demand is at its lowest and highest value, respectively. At peak demand, the controls take their maximum permissible value, except when the reservoir level is very low. At low demand, the controls take their maximum permissible value when the reservoir level is high.

A typical reservoir level and control trajectory generated by stochastic DPSA for a simulated random inflow sequence is shown in Figure 4.5. During this simulation, reservoir 1 overflows during stages 1 and 2. The relative frequency-minimum cost plots for SDP and stochastic DPSA is shown in Figure 4.6. The SDP cost has a lower mean (501.4) compared to the mean (506.4) of the stochastic DPSA cost. Both plots were generated for the initial condition ($x_1(0) = 15$, $x_2(0) = 8$, $x_3(0) = 6$).

(a) LOW DEMAND ($k=3$)

(b) PEAK DEMAND ($k=7$)

Figure 4.4. Control Policy Generated by Stochastic DPSA.

Figure 4.5. Reservoir Trajectories with Stochastic DPSA Control Policy.

Figure 4.6. Relative Frequency-Minimum Cost for SDP and Stochastic DP/SA.

Figure 4.7. Cumulative Frequency-Annual Spillage Plot for Reservoir 1.

Figure 4.8. Cumulative Frequency-Minimum Cost Plot for the Parallel Reservoir System.

A cumulative frequency-spillage plot for reservoir 1 is shown in Figure 4.7. An examination of Figure 4.7 shows that, in general, the SDP control policy results in less spillage than the stochastic DPSA policy. Figure 4.8 shows the cumulative frequency-cost plots for DP, stochastic DPSA and SDP. In general, stochastic DPSA results in a cost between DP and SDP.

For the simplified hydro scheduling problem presented in this section stochastic DPSA provides a good suboptimal control law. We are currently expanding the problem to include more general cost functions and log-normal distributions [13] for the inflow. Numerical results based on these additions and the performance of the spatial decomposition methods for series reservoirs will be discussed in future reports.

4.5 RESEARCH DIRECTIONS

The spatial decomposition approach presented in this section covers only a small spectrum of approximations to the stochastic control problem. Future work in this area involves: (1) application of the methods developed in this section to a more detailed hydro-scheduling problem, and (b) development of alternate decomposition methods.

In the parallel reservoir problem we determined optimal controls based only on local information. This situation is the opposite of stochastic dynamic programming where the information is centralized. As an intermediate situation, we can consider a hierarchical structure where a central decision maker gathers all the information and from time to time informs each reservoir about the state of the other reservoirs. Hierarchical control problems under uncertainty have been investigated in the control literature [14]. We will investigate the applicability of the methods to the parallel reservoir problem.

Another intermediate situation we can consider is the case where the decision maker in each reservoir knows its measurement and the sum of the measurements in all other reservoirs (i.e., the water available in the rest of the system). This information suggests that each

reservoir can solve a problem where the rest of the system is aggregated into one state. Aggregation methods have been extensively investigated in deterministic control problems [15]. However, extension of aggregation methods to the stochastic case needs to be investigated.

The series reservoir problem is considerably more difficult than the parallel reservoir problem. The difficulty arises because decisions made in one reservoir affect the state (content of water) of all other reservoirs below. In the deterministic case, model simplification methods have been developed which aggregate all the reservoirs upstream into one reservoir [13]. The analysis of the series reservoir problem we performed shows that extensions of this method to the stochastic case require careful examination.

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V. STOCHASTIC PRODUCTION SIMULATION

This section extends the stochastic production simulation (SPS) method [1] to take into account two uncertainties: random energy in storage units, and random fuel costs [2]. Subsection 5.1 presents an overview of the role of stochastic production costing in the operation scheduling problem. The approaches for treating random storage energy and fuel costs are described in Subsections 5.2 and 5.3, respectively. Subsection 5.4 indicates the areas of further research.

5.1 BACKGROUND

The objective of SPS is to estimate, for each time stage, the production cost and reliability of the generating system, taking into account the uncertainties associated with the production process. The basic idea of SPS is illustrated in Figure 5.1. The four inputs--load, generation capacities, energy in storage units, and fuel costs (generation costs)--summarize the information and uncertainties from new data. The probability distributions (or density functions) of the inputs are used to determine the probability distribution functions of the output and cost of each generating unit. In addition, reliability indices are computed for each unit.

The need for conducting a stage-by-stage simulation arises from the need to compute the transition cost of each stage for each of the sub-problems resulting from the time-scale decomposition of the unified operation scheduling problem. This is illustrated in Figure 1.1. The stage length shown in Figure 1.1 is for illustration; the actual stage length will depend on the time-scale decomposition method used in the long-term optimization models. The stage-by-stage SPS of the large time-scale sub-problem (e.g., multi-year scheduler) utilizes information from the stage-by-stage SPS of smaller time-scale sub-problems. For example, weekly scheduling of hydros requires the knowledge of hourly commitment schedules of thermal units.

Figure 5.1. Scope of Stochastic Production Simulation Model.

Assumptions

The following assumptions are made in SPS:

- (A1) The simulation of the production can be performed on a set of independent successive stages, given multistage resource allocations defined by the schedulers of different sub-problems.
- (A2) All uncertainties affecting the system can be reduced to the following set of random variables: load, generation capacities, energy availability of storage units and fuel costs.
- (A3) All of these random variables can be considered as independent (both time and space independent). This implies that the time autocorrelations of the load, energy availability and fuel cost are to be dealt with at the level of the seasonal management models, in such a way that these variables appear as independent for the stage-by-stage production simulation model.
- (A4) All random variables are adequately defined for each stage by stationary (valid throughout a stage) probability distributions, because their changes are slow enough to be neglected during a stage.

Assumption A2 states that there are basically four uncertainties in this problem. The first two--random load and random generation capacities--are treated by the conventional models of load-duration curve and multistate generating unit availability [3,4]. The other two uncertainties--random energy availability and random fuel costs--need to be considered by specially-developed techniques.

The approaches developed in this research for treating the latter two uncertainties do not simulate all the possible situations one by one, as in an enumeration approach, nor do they sample those situations, as

in a Monte Carlo approach. Rather, it performs direct calculations on the probability distributions of the random variables. Details of these approaches are given in the next two subsections.

5.2 MODELING OF STORAGE UNITS WITH RANDOM ENERGY AVAILABILITY

In order to treat the problem of random energy availability for storage units, the proposed approximation consists of modeling the operation of storage units directly on the load curve, neglecting the interactions with the other random variables (forced outages of thermal generations and fuel costs). This approximation defines a basic aspect of the methodology: The problem of the storage units is no longer directly coupled with the problem of thermal units. The two problems are still coupled through the load curve, i.e., indirectly to the monotonically increasing cost curve of the thermal system. The resulting schedules are only near-optimal as discussed hereafter.

Neglecting the interactions between the fuel costs and the operation of storage units is obviously an acceptable approximation because the dynamics of the fuel costs variations are extremely slow. This approximation implies that no energy will be kept in storage for counteracting potential changes in fuel costs.

Neglecting the interactions between the thermal forced outages and the storage operation is not so obviously acceptable. The optimal operation of storage units is indeed related to the evolution of the system operating cost, which is affected by the thermal outages. However, even though some dispatchers adjust the storage (e.g., hydro) schedules upon occurrence of forced outages, this is not a common practice.

Several types of storage^(*) units are encountered in power systems. For modeling purposes, they can be grouped into the following five categories:

- (i) Seasonal storage (including multiannual reservoirs) without pumping capability.
- (ii) Seasonal storage with pumping capability.
- (iii) Short-cycle storage (weekly and daily reservoirs) without pumping capability.
- (iv) Short-cycle storage with pumping capability.
- (v) Run-of-the-River (ROR) units.

Of these five categories of hydro units, the ROR units, when available, are scheduled for shaving peaks. The optimal scheduling of the rest, for deterministic inflows, could be treated by the methods developed in a DOE contract entitled, "Energy Storage Evaluation Methods" [5]. The extension to the stochastic case is being investigated. This subsection considers the modeling of seasonal storage with random energy availability in the stochastic production simulation.

5.2.1 Modeling of Stochastic Seasonal Storages

Suppose the energy available HEN is specified as a discrete random variable by a mass probability distribution. The possible values of HEN are $y_1 \dots y_k \dots y_K$ (in MWh), with the associated probabilities $\pi_1 \dots \pi_k \dots \pi_K$ such that:

$$\sum_{k=1}^K \pi_k = 1 \quad (5.1)$$

(*) Because hydro units are vastly predominant in the class of storage units, it may happen that the name "hydro" unit be used sometimes in lieu of "storage" unit.

All the values of y_k being mutually exclusive, the result of loading hydro unit, under a peak-shaving policy with all the possible values of HEN is defined by

$$RLDC(x) = \sum_{k=1}^K \pi_k * RLDC_k(x) \quad (5.2)$$

where $RLDC_k(x)$ is the residual load duration curve when the energy available HEN is equal to y_k .

The $RLDC_k(x)$ curve is obtained by the peak-shaving policy, as shown on Figure 5.2. One can see that $RLDC_k(x)$ is defined as follows:

$$RLDC_k(x) = LDC(x+HCAP) \quad \text{if } x > z_k \quad (5.3a)$$

$$RLDC_k(x) = LDC(x) \quad \text{if } x < z_k \quad (5.3b)$$

where z_k is the loading level of the hydro unit such that the available energy $HEN=y$ is exhausted, $LDC(x)$ is the load duration curve and $HCAP$ is the capacity of the unit.

Let the z_k 's be defined such that $z_k \leq z_{k-1}$ for $k = 2, 3, \dots, K$. (5.4)
Then, equation (5.2) can be written as

$$RLDC(x) = \sum_{i=1}^{k-1} \pi_i LDC(x) + \sum_{i=k}^K \pi_i LDC(x+HCAP) \quad \forall k \quad (5.5)$$

$$\text{and} \quad z_k \leq x \leq z_{k-1} \quad (5.6)$$

$$\text{Defining} \quad \alpha = \sum_{i=k}^K \pi_i$$

Figure 5.2. Peak-Shaving by a Storage Unit of Capacity HCAP with an Energy Available $HEN = y_k$.

$$RLDC(x) = \alpha.LDC(x+HCAP)+(1-\alpha).LDC(x) \quad (5.7)$$

with:

$$\alpha = \text{Prob} (H E N > y_k) \quad (5.8)$$

$$z_k = Z(y_k) \quad (5.9)$$

$$z_k < x < z_{k-1} \quad (5.10)$$

where the Z function is implicitly defined by the algorithm illustrated on Figure 5.3. Figure 5.4 shows RLDC(x) for a unit with three possible values of energy $\{y_1, y_2, y_3\}$ with equal probability.

5.3 MODELING OF EFFECTS OF RANDOM FUEL COSTS ON LOADING ORDER OF THERMAL UNITS

5.3.1 Fuel Costs and Loading Order

When the fuel costs are fixed, the economic dispatching of the thermal generation simply consists of sequential loading of the various units (or segments of units) in the order of increasing incremental cost.

If the fuel costs are no longer predictable in a deterministic way, the simulation of the economic dispatch becomes a much more complex task because no single loading order can be used. At present no direct analytical approach is available to the solution of this problem. It is therefore necessary to rely on an efficient enumeration method that minimizes the total amount of calculations involved. Such a method has been designed and is illustrated below for the case of two different fuel types.

Figure 5.3. Components of the RLDC For a Storage Unit with 3 Possible Values y_1 y_2 and y_3 of the Energy Available HEN.

Figure 5.4. RLDC(x) in the Example of a Storage Unit with 3 Possible Values of Energy Available y_1 , y_2 and y_3 Leading to Loading Levels z_1 , z_2 and z_3 .

5.3.2 Magnitude of the Number of Load Sequences

Suppose a system contains I units of type B (burning fuel B) and J units of type A (burning fuel A). The upper limit to the number of possible loading orders is, irrespective of their costs, the total number of permutations of $(I + J)$ units:

$$(I + J)!$$

Usually within those units of one type, the loading order is prespecified in increasing order of their operating costs. The number of possible loading sequences is then

$$N(I, J) = \frac{(I + J)!}{I! \cdot J!} \quad (5.11)$$

Examining equation (5.11), it is easy to see that the number of possible loading sequences is still too high. For $I = 10$ and $J = 10$, $N(I, J) = 1.8 \times 10^5$. Thus a complete enumeration approach is impractical. However, the actual number of loading sequences is far less than $N(I, J)$.

Let $A_j, j=1 \dots J$ be the set of units burning fuel A and $B_i, i=1 \dots I$ be the set of units burning fuel B. We assume that B is the fuel whose cost is random, and A is the symbolic representation of all the other deterministic fuels. We further assume that the units in each category (A and B) are already ranked in increasing cost order. For units B, this is also the order of increasing heat rates.

The number of possible loading sequences is less than $N(I, J)$ for the following reasons:

- (1) the particular values of the terms in the B_i series and in the A_j series can be such that some combinations are not possible if the fuel cost distributions are discrete;

- (2) Even if the distributions of the fuel costs are continuous, their range may be such that other combinations have a zero probability to occur.
- (3) Fixing the relative position of some units may prohibit some loading orders for some other units because of the perfect correlation between the B_i operating costs.
- (4) It is not necessary to examine all loading sequences in order to determine the expected output of a given unit. For instance, the expected outputs of unit A_4 in the following three sequences are identical.

B_1	A_1	B_2	B_3	A_4
A_1	B_1	B_2	B_3	A_4
B_1	B_2	B_3	A_1	A_4

This is true because the expected output of a given unit, say A_j , does not depend on the loading order of the units that have been loaded before A_j .

5.3.3 Implicit Enumeration Algorithm

Based on the previous discussion, an algorithm can be presented for generating the necessary partial sequences. It is convenient to use two indices for denoting the steps of the algorithm because the algorithm will scan the columns of the Ranking Probability Array, and, in each column, it will scan the entries of the various rows. The step of the algorithm will be denoted "Step $j.i$ " when looking at entry i in column j of the Ranking Probability Array. (The Ranking Probability Array has entries p_{ij} 's which indicate the probability that the operating cost of unit A_j is higher than that of B_i .)

"Step 1" Series: Analyze all loading positions of A_1

Step 1.0 Load A_1
1.1 Load $B_1 + A_1$
1.2 Load $B_1 + B_2 + A_1$
1.3 Load $B_1 + B_2 + B_3 + A_1$
.
.
.
1.i Load $B_1 + \dots + B_i + A_1$
.
.
.
1.I Load $B_1 + \dots + B_I + A_1$

"Step 2" Series: Analyze all loading positions of A_2

Step 2.0 Load $A_1 + A_2$
2.1 Load $A_1 + B_1 + A_2$
2.2 Load $A_1 + B_1 + B_2 + A_2$
.
.
.
2.i Load $A_1 + B_1 + \dots + B_i + A_2$
.
.
.
2.I Load $A_1 + B_1 + \dots + B_I + A_2$
.
.
.

"Step j" Series: Analyze all loading positions of A_j

Step j.0 Load $A_1 + A_2 + \dots + A_j$
 j.1 Load $A_1 + A_2 + \dots + A_{j-1} + B_1 + A_j$
 j.2 Load $A_1 + \dots + A_{j-1} + B_2 + A_j$
 .
 .
 j.1 Load $A_1 + \dots + A_{j-1} + B_1 + \dots + B_{j-1} + A_j$
 .
 .
 j.I Load $A_1 + \dots + A_{j-1} + B_1 + \dots + B_I + A_j$
 .
 .
 .

"Step J" Series: Analyze all loading positions of A_J

Step J.0 Load $A_1 + A_2 + \dots + A_J + B_1 + B_2 + \dots + B_I$
 J.1 Load $A_1 + A_2 + \dots + A_{J-1} + B_1 + A_J$
 J.2 Load $A_1 + \dots + A_{J-1} + B_1 + B_2 + A_J$
 .
 .
 J.1 Load $A_1 + \dots + A_{J-1} + B_1 + \dots + B_I + A_J$
 .
 .
 J.I Load $A_1 + \dots + A_{J-1} + B_1 \dots B_I + A_J$

In general, the algorithm will generate $(I+1)*J$ partial sequences.
 This number is less than the total number of loading orders $N(I,J)$.

Table 5.1 illustrates the algorithm for a system of 6 units, 3 units burning B and 3 units burning fuel A. The algorithm has generated 12 partial sequences of various lengths instead of the 20 possible complete sequences (of 6 elements) shown in Table 5.2.

Table 5.1 also shows the various results of the loading process that are to be retained for calculating the expected output of the various units, namely:

- $EA_j(j.i)$ = expected output of unit A_j as computed at Step $j.i$ of the algorithm.
- $EB_1(j.i)$ = same for unit B_1 .

These quantities are determined by the standard procedure involving equivalent or residual load duration curves.

The expected output of B_1 and A_j will be obtained by weighting the above quantities by the associated probabilities:

$$EA_j = \sum_{j.i} WA_j(j.i) * EA_j(j.i) \quad (5.12)$$

$$EB_1 = \sum_{j.i} WB_1(j.i) * EB_1(j.i) \quad (5.13)$$

The probability weights WA_j and WB_j will be determined in the next two subsections.

5.3.4 Estimating Probability Weights for Outputs of A and B Units

The probability weight $WA_j(j.i)$ to be associated with the output $EA_j(j.i)$ of unit A_j in step $j.i$ of the algorithm is the total probability associated with all feasible permutations in the set of units that are loaded before A_j in the partial sequence generated at Step $j.i$.

TABLE 5.1 LIST OF LOADING SEQUENCES CREATED BY THE
ALGORITHM FOR A SYSTEM OF 6 UNITS

$A_1 A_2 A_3 B_1 B_2 B_3$

<u>m</u>	<u>STEP j.i</u>	<u>PARTIAL LOADING SEQUENCE</u>	<u>ASSOCIATED UNIT OUTPUTS</u>
1	1.0	A_1	$EA_1 (1.0)$
2	1.1	$B_1 A_1$	$EA_1 (1.1) \quad EB_1 (1.1)$
3	1.2	$B_1 B_2 A_1$	$EA_1 (1.2) \quad EB_2 (1.2)$
4	1.3	$B_1 B_2 B_3 A_1$	$EA_1 (1.3) \quad EB_3 (1.3)$
5	2.0	$A_1 A_2$	$EA_2 (2.0)$
6	2.1	$A_1 B_1 A_2$	$EA_2 (2.1) \quad EB_1 (2.1)$
7	2.2	$A_1 B_1 B_2 A_2$	$EA_2 (2.2) \quad EB_2 (2.2)$
8	2.3	$A_1 B_1 B_2 B_3 A_2$	$EA_2 (2.3) \quad EB_3 (2.3)$
9	3.0	$A_1 A_2 A_3$	$EA_3 (3.0) \quad EB_1 (3.0)$
10	3.1	$A_1 A_2 B_1 A_3$	$EA_3 (3.1) \quad EB_1 (3.1)$
11	3.2	$A_1 A_2 B_1 B_2 A_3$	$EA_3 (3.2) \quad EB_2 (3.2)$
12	3.3	$A_1 A_2 B_1 B_2 B_3 A_3$	$EA_3 (3.3) \quad EB_3 (3.3)$

TABLE 5.2 ALL POSSIBLE LOADING SEQUENCES FOR
A SYSTEM OF 6 UNITS A_1 A_2 A_3
 B_1 B_2 B_3

1		B1	B2	B3	A1	A2	A3
2		.	.	A1	B3	A2	A3
3		.	.	.	A2	B3	A3
4		.	.	.	.	A3	B3
5		.	A1	B2	B3	A2	A3
6		.	.	.	A2	B3	A3
7		.	.	.	.	A3	B3
8		.	.	A2	B2	B3	A3
9		.	.	.	.	A3	B3
10		.	.	.	A3	B2	B3
11	A1	B1	B2	B3	A2	A3	
12	.	.	.	A2	B3	A3	
13	.	.	.	.	A3	B3	
14	.	.	A2	B2	B3	A3	
15	.	.	.	.	A3	B3	
16	.	.	.	A3	B2	B3	
17	.	A2	B1	B2	B3	A3	
18	.	.	.	.	A3	B3	
19	.	.	.	A3	B2	B3	
20	.	.	A3	B1	B2	B3	

This total probability can be computed directly by imposing the condition:

$$B_i < A_j < B_{i+1} \quad (5.14)$$

which uniquely defines the terms in the partial sequence without considering the authorized permutations.

Condition (5-14) can be related to the random fuel cost K as follows:

$$HR_i \cdot K < A_j < HR_{i+1} \cdot K \quad (5.15)$$

where HR_i is the heat rate of unit B_i . The conditions for K are thus:

$$\frac{A_j}{HR_{i+1}} < K < \frac{A_j}{HR_i} \quad (5.16)$$

or,

$$\underline{KA}_{ji} < K < \overline{KA}_{ji} \quad (5.17)$$

and the probability for these conditions to be satisfied are directly read on the cumulative distribution ϕ of K :

$$\phi(x) = \text{Prob}(K < x)$$

so that,

$$WA_j(j,i) = \text{Prob}(\underline{KA}_{ji} < K < \overline{KA}_{ji}) = \phi(\overline{KA}_{ji}) - \phi(\underline{KA}_{ji}) \quad (5.18)$$

Similarly, the probability weight $WB(j.i)$ to be associated with the output $EB_j(j.i)$ of unit B_j in Step $j.i$ of the algorithm is the total probability associated with all permutations of units loaded before B_j at this step. This total probability is also the probability that the following conditions be satisfied:

$$A_{j-1} < B_{j1} < A_j \quad (5.19)$$

or

$$\frac{A_{j-1}}{HR_{j1}} < K < \frac{A_j}{HR_{j1}} \quad (5.20)$$

which is similar to Equation (5-15). The probability that such conditions be satisfied can thus be derived from the cumulative distribution ϕ as in Equation (5-18).

$$WB_j(j.i) = \text{Prob}(\underline{KB}_{j1} < K < \overline{KB}_{j1}) = \phi(\overline{KB}_{j1}) - \phi(\underline{KB}_{j1}) \quad (5.21)$$

where

$$\underline{KB}_{j1} = \frac{A_{j-1}}{HR_{j1}} \quad (5.22)$$

and

$$\overline{KB}_{j1} = \frac{A_j}{HR_{j1}} \quad (5.23)$$

5.3.5 Calculation of Expected Production Cost

In order to determine the expected production cost of the A- units, the expected output EA_j of each A- unit is to be multiplied by the operating cost A_j of that unit. Since this operating cost is not random, the

following simple formula applies:

$$PCA = \sum_j A_j \cdot EA_j \quad (5.24)$$

The simple calculation of Equation (5.24) does not hold for the B- units because their fuel cost being random, the amount of energy produced $EB_1(j.i)$ was conditional to the fuel cost K belonging to a specific range:

$$\underline{KB}_{ji} < K < \overline{KB}_{ji} \quad (5.25)$$

It is therefore necessary to associate with each partial energy $EB_1(j.i)$ the expected fuel cost $E[K(j.i)]$ conditional to the constraint (5.25) that applied when the energy $EB_1(j.i)$ was determined.

The conditional distribution of $K(j.i)$ is computed from the p.d.f. of $\phi(x)$ of K :

$$\phi(x) = \frac{d(x)}{dx} \quad (5.26)$$

as follows:

$$\phi_{ji}(x) = \begin{cases} \frac{\phi(x)}{\int_{\underline{KB}_{ji}}^{\overline{KB}_{ji}} \phi(x) dx} & \text{if } \underline{KB}_{ji} < K < \overline{KB}_{ji} \\ 0 & \text{otherwise} \end{cases} \quad (5.27)$$

Then the conditional expectation is given by:

$$E[K(j.i)] = \int_{\underline{KB}_{ji}}^{\overline{KB}_{ji}} x \cdot \phi_{j.i}(x) dx \quad (5.28)$$

The expected production cost of unit B_1 is thus computed as:

$$PCB_1 = \sum_{j,i} E[K(j,i)] \cdot EB_1(j,i) \quad (5.29)$$

The expected production cost for all B- units is

$$PCB = \sum_i PCB_i \quad (5.30)$$

The expected production cost for the entire system is computed as:

$$PC = PCA + PCB \quad (5.31)$$

This does not include any security cost, which would represent, for example, the cost of emergency energy imported from surrounding systems.

5.4 RESEARCH DIRECTIONS

There are two major areas that warrant further research related to the stochastic production simulation:

- The effect of time-serial correlation of random hydro inflows and load needs to be considered. It is no longer valid to simply add the stage-by-stage production costs and obtain the total annual cost. A scheme needs to be devised to properly consider that effect.
- The implicit enumeration procedure can be refined by further exploiting the structure of the Ranking Probability Array.

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VI. MAINTENANCE SCHEDULING AND NUCLEAR REFUELING

6.1 PROBLEM DESCRIPTION

The economic operation of an electric utility system requires the simultaneous solution of all aspects of the operation scheduling problem. However, because of the complexity and dimensionality difficulties, the problem is traditionally decomposed into subproblems: maintenance scheduling and nuclear refueling, production scheduling, unit commitment, and economic dispatch. The interaction between these functions are presented in Figure 6.1 [1].

The nuclear refueling problem occurs on the same time scale as the maintenance scheduling problem. Although improved and more rapid refueling technologies have recently become available [2], present nuclear units must be taken off-line for a significant period of time for refueling. Therefore, most utilities plan long fuel cycles (≥ 1 year) to take advantages of the low operating cost of nuclear units as compared to fossil plants [3]. The advantage of longer fuel cycles is offset, however, by the increased enrichment required, the cost of which increases nonlinearly with increasing fuel energy potential. It is, therefore, necessary to examine the many nuclear cycle lengths possible. A few automated techniques [3-5] have been developed to generate feasible nuclear refueling schedules.

For an optimal nuclear refueling solution, one needs to include in-core management effects, fuel contracts, etc. Appendix D discusses the problem in more detail. For the purpose of this research, it is assumed that the nominal refueling dates are already set and that between refueling dates a nuclear unit can be represented in a manner similar to fossil fuel units, i.e., when available for service, it has a specific heat rate (H Btu/kWh) and fuel cost (F \$/MBtu). Windows can be allowed to move refueling dates back and forth around the nominal values. For this kind of representation of nuclear units, nuclear refueling is not much different

Figure 6.1. Interaction of Unit Commitment, Maintenance and Nuclear Refueling Scheduling, and Economic Dispatch.

from maintenance scheduling. Therefore, in this year's research, we will concentrate on the maintenance scheduling problem. With slight modifications, the results should be applicable to the refueling problem.

The rest of this chapter discusses the maintenance scheduling problem. Basically, maintenance scheduling is the choice of the time for each generating equipment during the period of study at which the routine maintenance service is to be provided to reduce the possibility of equipment failure. The choice of the maintenance dates shall be made such that the utility's objective function is optimized subject to constraints such as system security, maintenance constraints (resource, crew), etc.

6.2 MAINTENANCE SCHEDULING BACKGROUND

In 1977 the maintenance expenses of the privately-owned utilities in the United States reached \$3.81 billion, an increase of \$645 million from 1976* (a jump of 17%). The reliability of system operation and production cost in an electric power system are all affected by the maintenance outage of generating facilities. Additionally, carefully optimized maintenance schedules could potentially defer some capital expenditure for new plants in times of tightening reserve margins, and allow critical maintenance work to be performed which might not otherwise be done. Therefore, maintenance scheduling in the electric utility system is a significant part of the overall operations scheduling problem.

The task of maintenance scheduling involves specifying dates at which manpower is to be allocated to an overhaul of a major functional element or group of elements in a single power plant such that overall system security level is acceptable, costs to the utility are minimized

*Statistics of "Privately Owned Electric Utilities in the United States 1977," U.S. DOE, Energy Information Administration, January 1979.

and all system constraints are met. For the purpose of preventive maintenance scheduling, the important costs are energy production costs and maintenance cost. The latter is of importance if planned outage durations are allowed to vary within given limits dictated mainly by availability of additional manpower either through increased staff or overtime.

Maintenance activities for a generating unit include the following. For a fossil-fueled generating unit, the major maintenance activities include checkup and maintenance of boiler (typically once a year) and turbine (typically once every five years). There are other checkups such as regulator, generator, scrubbers (for those coal units furnished with such), condenser, etc. For nuclear units, reactors form the major component that needs maintenance checkups. Minor maintenance checkup activities for nuclear units include checkups of turbine, condenser, regulator, generator, etc.

A number of uncertainties are involved in dealing with this long-term scheduling problem. These include load uncertainty, fuel supply and price uncertainty, generating unit reliability (forced outages), uncertainty in resource, and crew availability.

A thorough literature review and definition of terms used in maintenance scheduling can be found in Appendix C. Based on the review of literature, one can categorize the existing maintenance scheduling approaches into the following categories:

- (1) Manual Maintenance.— There is no optimization involved in this approach. Maintenance outages are planned in times of expected low load, one unit at a time. It is clear that optimization methods can do better than this approach.
- (2) Preferred Maintenance [6].— The objective of this approach is to minimize deviations of the maintenance schedule from an ideal one (e.g., one determined by the preferred maintenance beginning periods of each unit). Of course, this approach doesn't guarantee optimality.

(3) Levelizing Net Reserve or Maximizing the Minimum Reserve [6-10,16]. -

This approach is aimed at equalizing the capacity in excess of peak load available during each time interval (e.g., a week) to minimize customer interruptions. It is known that this approach cannot guarantee equalizing risk of losing load during all intervals. The reason is that two units having the same total capacity of a third unit don't produce the same LOLP effect. It also is not capable of minimizing the operating costs to the utility. Minimizing the sum of the net reserve squared also appears in the literature.

(4) Minimizing the Risk [11-13,16]. - This methodology is aimed at minimizing the chances of losing the load. There are approximations involved in applying the literature available in this area which have not been proved to produce a schedule with minimum risk. Of course, the method also does not guarantee the cost minimization.

(5) Minimizing Cost [14,15]. - Little research has been carried out in this area, although the idea is clearly in accordance with the objective of maintenance scheduling. Several reasons exist for the lack of methods. First, performing stochastic production costing for a large number of alternatives is very time consuming. Second, because of the many constraints that should be satisfied by a maintenance scheduling methodology, there is not much room for optimization; as a result, being able to obtain a feasible schedule is in many cases satisfactory. Of two references connected with cost minimization, one encounters instability in computations, and the other one is not computationally feasible for maintenance scheduling. They are reviewed in Appendix C.

In summary, one can see that traditionally the objective of maintenance scheduling has been one of the following three: (1) find a feasible schedule that satisfies constraints, (2) obtain a schedule which maximizes the system reliability subject to constraints, and (3) obtain a minimum cost schedule which is feasible. The main concern of the utility industry seems to be reliability in dealing with the problem of maintenance scheduling.

6.3 PROBLEM FORMULATION

The general objective of operation of a utility system is to minimize the operating costs. However, maintenance scheduling involves such a large number of constraints that often the most that can be done is to move the maintenance starting time of a unit within a specified window. The production cost for a reasonably large utility would not probably be too sensitive to moving a unit's maintenance schedule in a window. Also, minimizing risk implicitly reduces utility operating costs in the sense that the risk of not supplying the load would be reduced and therefore the utility wouldn't have to purchase more expensive energy.

In view of these considerations, two problem formulations are presented. These two alternative formulations are minimization of production costs and minimization of risk (maximization of reliability).

In view of these considerations, at this stage we present two problem formulations. These two alternative formulations are minimization of production costs and minimization of risk (maximization of reliability). They are presented next.

(1) Maintenance Scheduling as a Cost Minimization Problem

The problem in this formulation is a cost minimization one. Mathematically it can be described as:

$$\text{Min}_U \sum_{t=1}^T E[C_t(A_1(t), \dots, A_N(t), d_t, U_1(t), \dots, U_N(t))] , \quad (6.1)$$

where: N = number of units in the system

t = time increment (e.g., one week)

T = number of increments in the time horizon

$E[\cdot]$ = expected value

$C_t(\cdot)$ = total cost in increment t

$A_n(t)$ = availability of unit n at time increment t
(state variable)

d_t = demand at time increment t

$U_n(t)$ = decision to put unit n on maintenance at time increment t .

This minimization is carried out subject to the following constraints:

- Reliability (Security) Constraints. - Any feasible maintenance schedule must lead to a probability of losing the load during any time being less than a prespecified value.
- Resource Constraints. - This constraint ensures that the maintenance material, equipment and personnel are available for performing maintenance on a unit planned for checkups and repairs.
- Crew Constraints. - This constraint could be met by a crew being assigned to each power plant. For a situation like this, not more than one unit in that plant can be scheduled for maintenance during the same week.
- Timing Constraints. - Almost all maintenance outages are cyclic in nature. For each unit to be scheduled for maintenance, the earliest and latest time that the maintenance should be started is usually specified.
- Regulation Constraints. - There may be a regulation constraint on maintenance outages. For example, not more than three major units should be on maintenance during the same week to maintain system regulation requirements.
- Transmission Limitation Constraints. - In systems which are divided into distinct geographical areas, it may be necessary to spread maintenance geographically in order to meet transmission line constraints between areas.

The above are the major constraints a maintenance schedule should satisfy. There may be other constraints for a specific utility.

(2) Maintenance Scheduling as a Risk Minimization Problem

The objective of this formulation is to minimize system risk over the time horizon of interest. This is more popular than cost minimization, as one can see in the literature survey (Appendix C). In mathematical terms, it can be described by:

$$\text{Min}_U \sum_{t=1}^T R_t(A_1(t), \dots, A_N(t), d_t, U_1(t), \dots, U_N(t)) , \quad (6.2)$$

where $R_t(\cdot)$ is the system risk at t , and other variables are the same as before.

This minimization is carried out subject to constraints:

- $R_t(\cdot) \leq \bar{R}$ = maximum risk allowed
- Resource constraints
- Crew constraints
- Timing constraints
- Regulation constraints
- Transmission limitation constraints.

6.4 APPROACH

The feasibility of a number of optimization techniques to solve the problem of maintenance scheduling will be studied in the second year of the project. Some of the more promising approaches are discussed below.

For cost minimization problem formulation, the following approach is under investigation.

Dynamic Programming/Branch-and-Bound Approach [1,17]

For a minimization problem, with additive objective function, the functional equation of dynamic programming can be written as the following:

$$f(y) = \min_{y \in (\Omega - y_0)} \{f(y') + c(y', d) \mid t(y', d) = y\}, \quad (6.3)$$

with the boundary condition

$$f(y_0) = K_0,$$

where: $f(y)$ = cost of reaching state y

$c(y', d)$ = the cost incurred when decision d was made with system being in state y'

$t(y', d)$ = state to which system would evolve after decision d was applied to the system at state y'

y_0 = initial state

Ω = set of all feasible states.

The use of the above recursive equation can lead to the optimum solution. However, for some problems pure dynamic programming might not be a feasible approach because of the so called "curse of dimensionality." Maintenance scheduling problem for a sizable system falls under the category of dimensionally large problems. To reduce the size of the problem, a branch-and-bound approach will be used to fathom those states which cannot lead to an optimum solution.

Let P denote the original discrete optimization problem that we need to solve. In order that we be able to use the recursive formulas of this section to solve the discrete optimization problem, we assume that the dynamic program whose functional equation is given in this section represents [18-20] the discrete optimization problem. Let u be an upper bound on the objective function value of an optimal solution to the original discrete optimization problem P . Then, since the dynamic programming problem is a representation of P , it follows that u is also an upper bound on f^* , that is, $u \geq f^*$.

Now let $\ell(y')$ be a lower bound for getting from state y' to Ω (set of final states), then we can state the following proposition.

Proposition 1

If $f(y') + \ell(y') > u$,

then y' cannot lead to an optimal policy. So we can discard ("fathom") state y' and consequently save storage and possibly computational time. For proof, see Morin and Marsten [17].

Simply stated, if the sum of the minimum cost of reaching state y' from the initial state plus a lower bound on the cost of reaching any final state from state y' exceeds an upper bound on the cost of an optimal policy, then no feasible completion of any optimal subpolicy for the state y' could be an optimal policy. Any state $y' \in \Omega$ that satisfies the above criteria is said to be "fathomed" [21,22].

An obvious upper bound u is the objective function value of any feasible solution to the original discrete optimization problem. If the upper bound $u = \bar{u}$ is determined in this manner and we limit ourselves to identifying an optimal policy (instead of the entire set of optimal policies), then the following slightly stronger fathoming criterion can be employed.

Proposition 2 [17]

If $f(y') + \ell(y') \geq \bar{u}$,

then state y' can be fathomed. Some nice features of Proposition 2 include:

- (a) One can verify optimality. If at any point during the recursion all states are fathomed, the recursion can be stopped and concluded that the policy whose objective function value is $\bar{u}$ is an optimal policy.

- (b) We can get ϵ -optimal solutions as follows: Specify $\epsilon \geq 0$, an acceptable value that the objective function value of the feasible solution can be off from the optimum solution. Then, we can use the following fathoming criteria: if $f(y') + \ell(y') \geq \bar{u} - \epsilon$, then discard y' .

This novel method will be applied to the maintenance scheduling problem and is expected to make the problem manageable. The expected incremental generation cost (EIGC) curves will be used to calculate the operating cost of a state.

For the risk minimization formulation, loss-of-load probability (LOLP) and expected-unsupplied-energy (EUE) criteria will be used as objective functions. Computationally efficient methods such as generation availability curves (GAC) will be used to solve the problem.

The EIGC and GAC will be described next [24].

EIGC and GAC consider load as a frame of reference and convolve generation probability functions. This approach is complementary (dual) to Jamouille's method of equivalent-load-duration approach.

Consider the convolution of generators in the reverse loading order, i.e., unit 1 is the most expensive unit. Each unit is characterized as follows:

- Capacity G_1
- Availability p_1
- Forced outage rate $q_1 = 1 - p_1$
- Incremental Cost $C_1(x)$ (mills/kwh) $0 \leq x \leq G_1$

Note 1: The time period considered is the smallest maintenance interval, e.g., a week. Therefore, maintenance can be scheduled explicitly.

Note 2: The method is developed for the general case of a $C_i(x)$ function which models the heat rate curve of the unit. In the diagrams, for the sake of simplicity, we can assume it is a constant.

The first unit has a generation availability curve (GAC1) and the expected cost curve EIGC1 shown in Figure 6.2. The meaning of GAC1 is

Figure 6.2. Unit 1 in the Reverse Loading Order.

clear. If the system has only one unit, the LOLP is equal to $1 - GAC1(x)$ for any load level x . The EIGC curve gives the expected incremental cost of supplying x MW of load. It is the probability of x MW of generation available in the system multiplied by the incremental cost of generating x MW, i.e., $p_1 c_1$ in this case. Note that a certain amount of x MW load is expected to be unserved. This must be kept in mind when interpreting EIGC.

The second unit in the reverse loading order is now added. For simplicity in this example, assume it has the same capacity. The GAC2 curve is given by Eq. (6.4) and illustrated in Figure 6.3:

$$GAC2(x) = q_2 \cdot GAC1(x) + p_2 \cdot GAC1(x - G_2) \quad (6.4)$$

The EIGC2 curve is given by Eq. (6.5) and illustrated in Figure 6.3:

Figure 6.3. Unit 2 Added.

$$EIGC_2(x) = q_2 \cdot EIGC_1(x) + p_2 \cdot c_2(x) + p_2 \cdot EIGC_1(x-G_2) . \quad (6.5)$$

Note that $c_2(x)$ is defined to be zero when x is outside the range of zero and G_2 . Equation (6.4) is straightforward. Equation (6.5) has three terms. The first one represents the condition when unit 2 is out. Therefore, unit 1 must be scheduled to supply the load between zero and G_1 . The expected cost is therefore $q_2 \cdot EIGC_1(x)$. The second term represents the condition when unit 2 is available. Then no matter what happens to unit 1, the expected cost is $p_2 \cdot c_2(x)$ for x between zero and G_2 . The third term applies to loads greater than G_2 where unit 2 has no effect at all except for the probability p_2 .

The third unit can be incorporated in a similar way. Thus, in general,

$$GAC_i(x) = q_i \cdot GAC_{i-1}(x) + p_i \cdot GAC_{i-1}(x-G_i) , \quad (6.6)$$

$$EIGC_i(x) = q_i \cdot EIGC_{i-1}(x) + p_i \cdot c_i(x) + p_i \cdot EIGC_{i-1}(x-G_i) . \quad (6.7)$$

After all units are convolved, we obtain two curves, GACS and EIGCS, for the system. It is possible to add a cost component which accounts for the cost of unserved energy. Let this penalty cost be CP mills/kwh. The expected unserved energy for x MW of load is given by

$$EUE(x) = \int_0^x [1 - GACS(x)] dx .$$

The probability of not serving x MW of load is

$$PLOL(x) = 1 - GACS(x)$$

Since the incremental cost of EUE is CP mills/kwh, the total expected incremental cost of x MW of load is given by

$$TEIC(x) = EIGCS(x) + CP \cdot [1 - GACS(x)] .$$

The function $TEIC(x)$ is the incremental cost function so very useful in many optimization problems. By integrating it, one can obtain the expected generation cost curve $EGC(x)$:

$$EGC(x) = \int_0^x TEIC(u) du .$$

The two curves $TEIC$ and EGC are illustrated in Figure 6.4. Other valuable information can be derived from $EIGCS$, e.g., the total expected production cost for a given LDC can be computed by convolving the LDC with the $EIGCS$. However, it is not possible to compute the energy production by units. The LOLP for a given LDC can be computed by convolving the LDC with $GACS$.

6.5 RESEARCH DIRECTIONS

The following items need further research:

- **Solution Methodologies** -- A number of possible solution approaches are mentioned earlier in this chapter. Research is needed to identify which of these approaches or any other approach is suitable for solving the maintenance scheduling problem.

Figure 6.4. $EGC(x)$ and $TEIC(x)$.

- Identification of the "Best" Objective Function and Appropriate Solution Method(s) -- For each of the appropriate objective functions, an (or more than one) appropriate solution method will be identified. The output of this effort will be the answer to the question of which objective function and the corresponding solution approach is more appropriate under what conditions.
- Interface With Overall Problem of Operation Scheduling -- The objective of this portion of the research is to make sure that the approach taken in maintenance scheduling is consistent with the overall problem and that the proper interface is possible.

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VII. SHORT-TERM RESOURCE SCHEDULING

The short-term resource scheduling problem concerns the hourly decisions of unit commitment. The unit commitment function in operations scheduling has typically referred to the determination of which generating units are to be running and available for dispatch. The time scale of this problem is typically hour-by-hour over a day or a week. Included in this function are four other short-term scheduling decisions, namely:

- Scheduling of energy-limited interchange
- Scheduling of hydro
- Scheduling of energy storage
- Scheduling of controllable load (load management).

In this report, the following key aspects of the short-term scheduling problem are addressed:

- Complete formulation of the unit commitment problem including incorporation of stochastic effects.
- Integration of unit commitment with other parts of short-term scheduling problem.
- Development of an approach to the solution of the unit commitment problem.

This section is organized as follows. Section 7.1 summarizes the current state-of-the-art in solving the unit commitment problem. Section 7.2 presents the unit commitment problem formulation developed by SCI including the development of stochastic models for load and forced outage and requirements on spinning reserve and fast-start reserve. Section 7.3 presents a computational procedure for solving the stochastic unit commitment problem using dynamic programming. A major computational issue also considered in Section 7.3 is the reduction of the dimensionality of the state-space used in the dynamic programming algorithm. Combinatorial approaches, e.g., branch-and-bound, and other static prescheduling procedures are suggested as candidates for alleviating the dimensionality

problem. Section 7.4 discusses the interface of the unit commitment problem with the hydro scheduling problem. A successive optimization procedure is proposed which fully integrates the short-term scheduling problem into a unified operations scheduling procedure. Finally, Section 7.5 discusses future directions of research and development.

7.1 RECENT RESULTS IN THE LITERATURE

This section describes some recent approaches to the short-term scheduling problem. The papers discussed do not comprise a comprehensive survey, but are intended to provide an overview of recent advances in the field and to compare the merits of various approaches. Note that few authors have considered the integration of unit commitment problem into the overall operations scheduling problems and fewer still have incorporated uncertainty into the formulation.

The approaches can be grouped in terms of computational approach as follows:

- Priority Ordering and Heuristic Methods
- Dynamic Programming
- Combinatorial Methods.

Priority lists are typically developed by ranking units according to the sum of their incremental cost and start-up costs, where the start-up costs are averaged over a nominal operating time. Because of the heuristic nature of the priority list and related approaches, the commitments obtained are suboptimal. The general research trend in recent years has been toward the development of dynamic and combinatorial approaches. Due to the effectiveness of the priority list in reducing the dimensionality of the short-term scheduling problem, it has remained as a popular operational technique and serves as a valuable benchmark for assessing the viability of more sophisticated methods.

With the increasing availability of low-cost computing power, the dynamic and combinatorial approaches are expected to gain prominence in operational use. Frequently these methods are used in combination with each other, often with a priority ordering. Some of these approaches have been demonstrated successfully in power system application, e.g., the COMMIT program developed by SCI [1]. This program basically implements a forward dynamic programming procedure assuming a partial priority ordering on the thermal units. The approach incorporates requirements on spinning reserve and allows for transmission losses. Minimum up-time and down-time are also accounted for in the COMMIT procedure.

Advances in the development of static combinatorial methods have been reported by Dodu [2] and Dillon [3]. In Ref. 2 the unit commitment problem is formulated including hydropower and active flow constraints on the network. The problem is solved using a combination of branch and bound and linear programming. Several simplifying assumptions are made, e.g., linear cost of thermal power and no stochastic effects. The authors report having applied their technique to example problems with networks as large as 200 nodes.

In Ref. 3 the authors use mixed integer programming to solve the unit commitment problem. This requires that they use a linear model for generation cost. A significant feature of their problem formulation is the development of two probabilistic requirements for spinning reserve. The first is a requirement on fast-start units and the second is a requirement on units with longer start-up times (on the order of several hours). This apparently unique representation of reserve requirements is an advance in the development of stochastic commitment programs which reflect realistic operating objectives. They report solving problems with 16 units.

Muckstadt and Koenig [4] have combined combinatorial and dynamic methods. The problem is formulated using piecewise constant incremental costs for the thermal units. The basic concept of their computational

approach is to use a Lagrangian relaxation method to decompose the problem into separate dynamic subproblems for each individual unit, i.e., a "spatial" decomposition. The Lagrange multipliers are updated by a search through a branch-and-bound tree. The authors use a subgradient method to perform this search. The authors do not consider uncertainty in their formulation. They report solving problems with 15 generators.

Another approach combining combinatorial and dynamic methods has been developed by Morin and Marsten [5] and applied to unit commitment by Yamayee [6]. The problem is formulated in Ref. 6 using a quadratic thermal cost model. The basic idea is to solve the problem directly using dynamic programming while using lower bounds on the cost-to-go and upper bounds on the optimal schedule to fathom states from the dynamic programming search. This approach can be generalized easily to the stochastic case using stochastic dynamic programming. This method is applied to an example with 24 units and is discussed further in Section 7.3.

Note that none of the above methods deals directly with the stochastic nature of the problem. Ref. 3 does present a procedure that takes into account forced outage probabilities in setting spinning reserve. However, the authors do not include the effect of forced outage on cost nor do they include uncertainties in demand. The formulation presented below incorporates all the uncertainties of interest.

7.2 PROBLEM FORMULATION

The formulation presented here will be limited to the unit commitment problem; that is, the determination of which thermal units (nuclear and fossil) are to be running and available for dispatch. Interchange will also be included in the subproblem. The interaction with hydro, energy storage and load management is discussed in Section 7.4.

7.2.1 Mathematical Formulation

This section contains a mathematical formulation of the unit commitment problem.

Decision Variables

The basic decision variables for the unit commitment problem are:

(1) the decisions to put the thermal units on-line, (2) the thermal unit generation (if on-line), and (3) the imported energy. In addition, the availability of the thermal units will depend on whether the unit is on forced outage. The following variables can be defined for time period k , thermal unit i

$$G_i(k) = \text{generation}$$

$$B_i(k) = \text{Binary variable describing decision to put unit on-line this period}$$

$$= \begin{cases} 1, & \text{decision to put unit on-line} \\ 0, & \text{decision to make unit unavailable} \end{cases}$$

$$C_i(k) = \text{Binary random variable describing forced outage of unit}$$

$$= \begin{cases} 0, & \text{if unit out} \\ 1, & \text{otherwise.} \end{cases}$$

The availability of the unit, $A_i(k)$, for period k is then given by

$$A_i(k) = B_i(k) C_i(k) ,$$

where

$$A_i(k) = \begin{cases} 1, & \text{if unit is available} \\ 0, & \text{otherwise.} \end{cases}$$

The interchange for period k contract j will be represented by the variable $CI_i(k)$.

State Equations

Various costs and constraints of the unit commitment problem are dependent on integrated values of the decision variables — namely, the accumulated up and down-time of the thermal units and the total imported power over some time interval. These integrated values can conveniently be described by state equations.

The cumulative up-time of unit i up to time $k+1$ satisfies the equation

$$XU_i(k+1) = (XU_i(k) + 1) A_i(k) .$$

Similarly, the cumulative down-time is given by

$$XD_i(k+1) = (XD_i(k) + 1) (1 - A_i(k)) .$$

The cumulative energy imported under contract j , $XI_j(k)$, satisfies

$$XI_j(k+1) = XI_j(k) + GI_j(k) .$$

The initial values of $XU_i(k)$, $XD_i(k)$ and $XI_j(k)$ must be set appropriately.

Constraints

The following constraints apply to this problem:

- (1) The generation is zero if the unit is unavailable

$$G_i(k) = 0 \quad \text{if} \quad A_i(k) = 0 .$$

- (2) The generation is between certain bounds if the unit is available

$$G_i(k) \in [GMIN_i, GMAX_i] \quad \text{if} \quad A_i(k) = 1 .$$

(3) Minimum down-time

$$XD_i(k) \geq XDMIN_i \quad \text{if} \quad \beta_i(k) = 1 ,$$

where $\beta_i(k)$ is change in status of unit i given by

$$\beta_i(k) = A_i(k+1) - A_i(k) .$$

(4) The maximum change in generation from period k to $k+1$ may be limited

$$|G_i(k+1) - G_i(k)| < GDMAX_i$$

if

$$A_i(k+1) = A_i(k) = 1 .$$

(5) The interchange energy and cumulative energy may be constrained

$$GIMIN_j \leq GI_j(k) \leq GIMAX_j ,$$

$$XIMIN_j \leq XI_j(k) \leq XIMAX_j .$$

In addition to these constraints, there are the system-wide constraints of meeting load and having required reserve. The load equation is

$$\sum_i G_i(k) + \sum_j GI_j(k) = L_k , \quad (7.1)$$

where L_k is the net load (averaged over the time interval) to be supplied by the thermal units and interchange. It is assumed that there are emergency interchange contracts to react to loss-of-load situations — the cost of these contracts can be set to control the expected unserved energy. (Note also the reserve requirement below.)

Reserve Requirement

"Fast" and "slow" reserve requirements are considered in this initial problem formulation. The "fast reserve" is the reserve capacity available

within, say, ten minutes of a forced outage. This capacity is drawn from a variety of sources:

- Excess capacity of on-line thermal units available within ten minutes;
- "Excess capacity" (EXCAP) storage units, e.g., hydro and pumped storage;
- Thermal units which may be brought on-line within ten minutes (RFAST(k)), e.g., gas turbines.

The objective of the requirement on "fast" reserve is to insure that the system has adequate reserve to carry its load with a given pre-set probability in the event of random forced outages. The constraint may be written as follows:

$$\Pr \left\{ \sum_{\text{units } i} \min[G_i(k) + \text{GDFMAX}_i, \text{GMAX}_i] A_i(k) + \sum_{\text{contracts } j} \text{GIMAX}_j + \text{EXCAP}(k) + \text{RFAST}(k) > L(k) + \Omega_k \right\} \geq 1 - \delta_f . \quad (7.2)$$

In this equation, GDFMAX_i represents the reserve available from units which are already committed. The quantity Ω_k is an additional "safety margin" which may be required as part of the operating policy. The fast reserve risk level is denoted by δ_f .

A second probabilistic constraint dealing with "slow" reserve may also be imposed. The "slow" reserve capacity is defined here initially as the reserve system capacity available within the next planning period, e.g., an hour. The division between "fast" and "slow" is somewhat arbitrary; an appropriate classification for the units may depend on the needs of a particular utility. The slow reserve is drawn from the same sources as fast reserve plus those uncommitted units which can be brought on-line within an hour ($\text{RSLOW}(k)$). The purpose of the slow reserve requirement is to insure that the load during the next period can be met with a given probability using units other than the fast reserve units. The slow reserve requirement may be expressed as:

$$\Pr \left\{ \sum_i \min(G_i(k) + GMAX_i, FMAX_i) A_i(k+1) + \sum_j GIMAX_j + EXCAP(k+1) + RFAST(k+1) + RSLOW(k+1) > L(k+1) + \Omega_{k+1} \right\} \geq 1 - \delta_s . \quad (7.3)$$

The risk level, δ_s , for the slow reserve requirement would typically be chosen smaller than the fast reserve risk level to reflect the greater operator flexibility in responding to forced outage on the longer time scale.

The formulation of the reserve requirements assumes that the operator will take the following action after a forced outage:

- First, an emergency response is taken which may include committing gas turbines and other reserve capacity.
- Second, after the situation has stabilized additional less expensive units are brought on-line as quickly as possible to reduce the risk of an additional shortfall and allow the shutdown of the more expensive reserve units.

The separation of the reserve requirement into fast and slow components enhances the capability of the operating system to respond quickly to a forced outage without incurring prohibitive costs.

Costs

The costs include the generation cost, $C_i(G_i(k))$; the start-up costs, $STC_i(XD_i(k))$; the shut-down costs, $SHC_i(XU_i(k), \beta_i(k), G_i(k))$ and the interchange costs, $CI_j(GI_j(k))$. The total expected cost to be minimized is then

$$\begin{aligned}
& \left. \begin{aligned}
& \sum_k^E \left\{ \sum_{\text{units } i}^E [C_i(G_i(k)) + STC_i(XD_i(k), \beta_i(k)) \right. \\
& \quad \left. + SHC_i(XU_i(k), \beta_i(k), G_i(k))] + \sum_{\text{contracts } j} CI_j(GI_j(k)) \right\}
\end{aligned} \right\} \quad (7.4)
\end{aligned}$$

The stochastic optimization is performed both with respect to random load and random forced outage. The next subsection presents the formulation of the required stochastic models.

7.2.2 Stochastic Models

The load and the outage of the thermal units are stochastic. The average load for each period can be modeled as a discrete-time Markov process of the form:

$$L(k+1) - \bar{L}(k+1) = \alpha(L(k) - \bar{L}(k)) + e(k),$$

where: $L(k)$ = actual average load during period k ,

$\bar{L}(k)$ = a priori expected average load at period k ,

$e(k)$ = a zero mean white sequence with known covariance, Q

α = damping factor, $0 < \alpha < 1$.

The above model allows for the presence of temporal correlation in the random part of the load. Small values of the parameter α indicate high frequency variations in the load; large values correspond to low frequency variations. For a real power system, the parameters Q and α must be identified empirically.

We propose a Markov chain model for unit outage and repair. The units are assumed to have a constant conditional failure rate:

$$P(C_1(k) = 0 | C_1(k-1) = 1) = p_1 ,$$

$$P(C_1(k) = 1 | C_1(k-1) = 1) = 1 - p_1 .$$

The model for repair is somewhat more complex. We assume that once the unit is down ($C(1) = 0$) it is in one of several "failure states," according to the type of failure that occurs.

Corresponding to each failure state there is an associated repair procedure, perhaps consisting of a number of stages and a time-to-completion of each stage of the repair procedure. This model will yield detailed information on unit down-time. Figure 7.1 is an illustration of a simple example of this model. The model has two basic independent failure modes. One type of failure is a simple failure which is always repaired in one hour. The second failure is more complex requiring either 12 or 18 total hours for repair.

In general, each unit has an associated set of failure states $\{f_1^1, \dots, f_1^r\}$. These states are such that $f_1^j(k) = 1$ if the unit is operating and $f_1^j(k) = 0$ if the unit is in the j th failure state. In the example of Figure 7.1 the failure states are denoted by I, II, II', and II". A certain subset of the failure states, say $\{f_1^1, \dots, f_1^r\}$, are accessed directly from the "working state" with a given probability each time period:

Figure 7.1. Example Markov Chain Model for Unit Failure and Repair

$$P(f_1^j(k) = 0 | C_1(k-1) = 1) = p_1^j ,$$

$j = 1, 2, \dots, r$ where

$$\sum_{j=1}^r p_1^j = p_1 .$$

In the example these failure states correspond to the states I and II. The dynamics of the failure states will not be explicitly written here but must include the time lags associated with repair. Also, we note that variable $C(i)$, the aggregate variable indicating unit failure, is given by:

$$C_1(k) = \prod_{\substack{\text{failure states} \\ j}} f_1^j(k) .$$

The above Markov chain model for unit failure may be too complex for direct incorporation into an optimization procedure for unit commitment; however, it can serve as the basis for procedures which utilize statistics of the process, e.g., mean time-to-repair.

This section has presented a formulation of the unit commitment which realistically reflects operator objectives and incorporates uncertainty, and which will be the basis for our further work. The next section describes a general combinatorial/dynamic programming approach to solving the unit commitment problem.

7.3 COMPUTATIONAL APPROACH TO UNIT COMMITMENT

The basic output of the unit commitment problem is the specification on an hour-by-hour basis of which units are to be running and available for dispatch. In order to evaluate the cost of alternative commitment policies it is necessary to calculate the generation of each unit; however, this generation is not a true output since the actual generation of the individual units is determined on a much shorter time scale, typically by

Automatic Generation Control (AGC) system. Since the load is uncertain, the generation of the individual units will also be stochastic and will be determined in a closed-loop manner. The unit commitment will, of course, make sure that load will be met with sufficient probability.

Thus, the major uncertainty associated with unit commitment is unit availability. Both unit repair and outage probability models are important, as discussed in Section 7.2.2. To incorporate these statistics in a full closed-loop feedback policy would be unrealistic since that would require a state space of at least 2^N where N is the number of units. This is much too large for typical utilities. Furthermore, most outages are infrequent events and thus the tremendous computational burden of a full feedback solution would be unwarranted. Instead, we propose an open-loop feedback approach as illustrated in Figure 7.2. The inputs to the problem are the load statistics, the outage probabilities and the expected unit availabilities. By the latter, we mean the most probable trajectory of availabilities of the units given the information known at the initial time. The information would include the availabilities of the units at the initial times plus any knowledge of projected completion or start of maintenance over the time horizon. Given this

Figure 7.2. Sliding Window Approach to Unit Commitment.

information, we solve the unit commitment problem assuming the units are available as specified over a period of time MT where T is typically 24 hours and M is 7, so that the time horizon is a week. Note that we still plan to take the outage probabilities into account in the demand (Eq. (7.1)), reserve requirements (Eq. (7.2)), and the cost (Eq. (7.3)). Typically, as shown in Figure 7.2, the open-loop problem is re-solved daily so that new information on load and availability can be incorporated. If an unforeseen outage occurs that seriously effects the commitment, then the open-loop problem would be re-run. Such an event would be the forced outage of a large unit at the start of the day. Clearly, the early repair of a large unit could also require the re-solving of the open-loop problem. The problem is solved with a one-week time horizon to remove the effect of terminal conditions from the problem.

To summarize, the computational approach described above constitutes an open-loop feedback policy for short-term scheduling. At each iteration, the procedure yields a sequence of commitments which minimize the expected generation cost subject to the load and probabilistic reserve requirements, thereby hedging against forced outages and random peaks in the load. Finally, new information is incorporated periodically allowing the procedure to respond to random events.

Dynamic Programming

Dynamic programming (DP) is a prime candidate for the solution of the unit commitment problem. SCI has previously developed a computer program, COMMIT, which applies DP to a deterministic version of the unit commitment problem. In the remainder of this subsection, we will describe the COMMIT procedure and discuss how it can be modified to solve the more general problem given in Section 7.2.

The COMMIT procedure employs a single-state dynamic program for unit commitment. A priority ordering is assumed so that a single state variable, x_k , can describe the state of the system. The state dynamics are described by:

$$x_{k+1} = x_k + u_k$$

where u_k is the number of units newly committed or shut down at stage k .

The incremental cost for the thermal units is assumed to be piecewise linear in the COMMIT approach. The economic dispatch is computed by an iterative technique which loads each unit until incremental costs are equal subject to the unit generation capacity constraints and the requirement that the load be met. Transmission losses are also accounted for in the dispatch algorithm using penalty factors for each unit. This approach can be extended to accommodate an expected value cost function.

Reserve requirements are modeled by the policies:

- (1) Hourly spinning and fast-start reserve requirements are used;
- (2) All available but uncommitted gas turbines are used as fast-start reserve.

The current formulation of reserve requirements in COMMIT is not probabilistic but can be generalized to be consistent with the approach used in Section 7.2 of this report.

The main difficulty in extending the COMMIT approach is the assumption of the priority ordering. The priority list concept can be incorporated directly in a stochastic unit commitment procedure. It has the strong advantage of providing a very substantial reduction in the dimensionality of the state space in the dynamic optimization. However, the development of other techniques which do not compromise the optimality of the commitment strategy is preferable. These techniques will now be described.

Combinatorial (e.g., branch-and-bound) and other static optimization methods may be combined with dynamic programming to alleviate the computational burden associated with pure dynamic programming. The underlying idea is to use branch-and-bound or some other technique to discard states which cannot lead to an optimal solution. This concept was apparently

first considered in detail by Morin and Marsten [5] and was applied by Yamayee, et al [6] to the unit commitment problem. Details of this approach have been described in Section 6. However, these procedures described there do not constitute a complete set of bounding techniques. Additional methods may be required to provide adequate reduction of the dimensionality for problems of realistic size.

Prescheduling Procedures

The development of suitable procedures for bounding the state-space of the dynamic programming search is an area for continued research. In conjunction with the branch-and-bound approach, it may also be desirable to develop procedures for "prescheduling" certain units. By a prescheduling procedure we mean a static optimization which will determine in advance of the dynamic optimization, subsets of units which will be definitely committed at certain times and other units which will definitely not be committed. Dillon [3] reports using a heuristic prescheduling procedures apparently based on load characteristics and computational experience. However, the development of theoretically justifiable procedures which are of general applicability have not been developed and would be of tremendous value in reducing dimensionality.

7.4 INTERFACE WITH HYDRO PROBLEM

The problem formulation in the previous section ignored hydro generation. If hydro generation is added to the system, then this generation can be used to help meet both the demand and the reserve requirements. Thus, if hydro is included, then the load equation (7.1) must be modified to:

$$\sum_{\substack{\text{thermal} \\ \text{units} \\ i}} G_i(k) + \sum_{\substack{\text{contracts} \\ j}} GI_j(k) + \sum_{\substack{\text{hydro} \\ \text{units} \\ \ell}} GH_\ell(k) = L_k . \quad (7.5)$$

The reserve equation (7.2) already contains a term reflecting the excess capacity of the hydro units. In addition, the water balance and reservoir constraints must be included in the formulation (see Section 7.1).

The unit commitment problem can be reformulated to highlight the thermal and hydro components: Minimize the thermal and interchange costs

$$E \left\{ \sum_k \left[\sum_{\substack{\text{thermal} \\ \text{units} \\ i}} TC(G_i(k), A_i(k), XT_i(k)) + \sum_{\substack{\text{contracts} \\ j}} CI_j(GI_j(k)) \right] \right\}, \quad (7.6)$$

subject to the load requirements

$$\sum_{\substack{\text{thermal} \\ \text{units} \\ i}} G_i(k) + \sum_{\substack{\text{contracts} \\ j}} GI_j(k) + \sum_{\substack{\text{hydro} \\ \text{units} \\ \ell}} GH_\ell(k) = L(k), \quad (7.7)$$

the fast reserve requirements

$$\Pr \left\{ \sum_{\substack{\text{thermal} \\ \text{units} \\ i}} FTHRES_i(k) + \sum_{\substack{\text{contracts} \\ j}} GIMAX_j(k) + \sum_{\substack{\text{hydro} \\ \text{units} \\ \ell}} EXCAP_\ell(k) > L(k) + \Omega_k \right\} \geq 1-\delta \quad (7.8)$$

and the thermal and hydro system constraints

$$(\underline{G}, \underline{A}, \underline{XT}) \in THCON, \quad (7.9)$$

$$(\underline{U}, \underline{XH}) \in HYCON, \quad (7.10)$$

where TC denotes the thermal cost terms (both generation and start-up), XT_i denotes all the thermal unit states for unit i, GH_ℓ is the hydro generation, for hydro unit ℓ and FTHRES includes all the fast thermal

reserve terms. $\underline{G}$ denotes the thermal generation vector for all units and all times, i.e., $\underline{G} = (G_1(1), G_2(1), \dots, G_{\text{NUNITS}}(K))$; $\underline{A}$, $\underline{XT}$, $\underline{U}$, and $\underline{XH}$ are similarly defined for the thermal commitment, thermal state, hydro outflow, and hydro state vectors, respectively. THCON and HYCON are, respectively, the sets of allowable thermal and hydro operations. The slow reserve constraint has been omitted to simplify the presentation; it can be included analogously to the fast reserve constraint.

The technique we propose to use to solve this problem is the method of successive optimization. In this method, the unknowns of the problem are divided up into several sets and then the original problem is solved successively each time with respect to the variables in a different set — the other variables held constant. It can be shown, under suitable convexity assumptions, that this technique converges to the true optimum Refs. [1,2]. In applying successive optimization to the unit commitment problem, the variables are divided into two sets — namely, the commitments and thermal states ($\underline{A}$, $\underline{XT}$) and the hydro states and controls ($\underline{U}$, $\underline{XH}$); the thermal generation, $\underline{G}$, is always allowed to be variable. The method proceeds as follows:

- Step 1: Initialize the hydro variables to some feasible solution.
- Step 2: Solve problems (7.6)-(7.10) for the thermal commitments $\underline{A}$ keeping the hydro variables ($\underline{U}$, $\underline{XH}$) fixed at their values from the previous step.
- Step 3: Solve problem (7.6)-(7.10) for the hydro generation keeping the commitments $\underline{A}$ and thermal states $\underline{XT}$ fixed at their values from Step 2. This is a continuous problem (all discrete variables fixed), and therefore much easier to solve than the original problem.
- Step 4: Repeat Steps 2 and 3 until convergence is obtained.

The above procedure is guaranteed to converge since the cost will be reduced at each optimization. However, it will, in general, yield a sub-

optimal solution since the set of commitments $\underline{A}$ is not convex. The resulting solution should, however, be close to optimal because the problems solved in Steps 2 and 3 are basically orthogonal. This assumption will be investigated.

The use of successive optimization has the following important advantages:

- (1) The problem solved in Step 2 is the unit commitment problem without hydro. The hydro generation can be interpreted as reducing the system load.
- (2) The problem solved in Step 3 can be reformulated to be just a function of hydrogeneration as follows. Given the commitments $\underline{A}$, the optimal allocation of thermal generation as a function of net TI (thermal + interchange) generation is a straightforward convex nonlinear programming problem (see Refs. 3 and 4) for solution techniques) for the deterministic case. Thus, for this case, one can obtain the system cost for all levels of TI generation. This cost can be denoted

$$TC'_k \left\{ \sum_{\substack{\text{thermal} \\ \text{units} \\ i}} G_i(k) + \sum_{\substack{\text{contracts} \\ j}} GI_j(k) \right\}$$

for time k . Using (7.5), this equation becomes

$$TC'_k \left\{ L_k - \sum_{\substack{\text{hydro} \\ \text{units} \\ l}} GH_l(k) \right\}$$

which is only a function of hydro generation. The introduction of stochastic effects somewhat complicates this problem.

However, the stochastic production costing techniques described in Section V can be used to obtain the $E\{TC'_k\}$ curve. The resulting hydro optimization problem is of the same form as that which exists at other time scales (see Section III) and,

thus, the sophisticated methods that have been developed for these problems can also be applied to the short-term scheduling problem.

- (3) Energy storage and load management can easily be included into this problem by considering them as third and fourth sets of variables in the successive optimization procedure.

7.5 FUTURE DIRECTIONS

The work described in this section was limited to formulating the unit commitment problem and developing concepts for its solution. There are clearly many issues that must be addressed. The following areas are presently under study:

- (1) The development and substantiation of procedures to preschedule some of the units to limit the size of the state space in the dynamic programming problem.
- (2) The calculation of the expected cost given load and availability uncertainties.
- (3) The effectiveness of branch-and-bound procedures to reduce the state.
- (4) The convergence of the successive optimization method for the unit commitment problem.

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APPENDIX A

DERIVATION OF SPATIAL DECOMPOSITION RESULTS

This appendix derives expressions for some of the partial derivatives which were used in Section 4.3.2, and the team theory control laws used in Section 4.2.4.

Local Feedback

At first we show that under local feedback structure

$$\frac{\partial u_1(1)}{\partial u_1(0)} = 0$$

and

$$\frac{\partial u_2(1)}{\partial u_1(0)} \neq 0.$$

We have from the local feedback structure

$$u_1(0) = \gamma_1(y_1(0)) \quad (A.1)$$

$$u_2(0) = \gamma_2(y_2(0)) \quad (A.2)$$

$$u_1(1) = \gamma_3(y_1(0), y_1(1)) \quad (A.3)$$

$$u_2(1) = \gamma_4(y_2(0), y_2(1)) \quad (A.4)$$

and

$$z_{11} = \begin{cases} y_1(0) = x_1(0) & (A.5) \\ y_1(1) = x_1(1) = x_1(0) - u_1(0) + w_1(0) & (A.6) \end{cases}$$

$$z_{21} = \begin{cases} y_2(0) = x_2(0) & (A.7) \\ y_2(1) = x_2(1) = x_2(0) - u_2(0) + u_1(0) + w_2(0) & (A.8) \end{cases}$$

Since we assume that the decision makers have agreed on the strategy they are using, decision maker 11, based on his information, can exactly recover the control $u_1(0)$ of decision maker 10. Consequently, an information equivalent to z_{11} for decision making purposes is

$$\bar{z}_{11} = \begin{cases} y_1(0) = x_1(0) \\ \bar{y}_1(1) = x_1(0) + w_1(0) \end{cases} \quad (\text{A.9})$$

Using (A.9), (A.3) can be written as

$$u_1(1) = \gamma_3(z_{11}) = \gamma_3(y_1(0), \bar{y}_1(1)) = \gamma_3^1(x_1(0), w_1(0)) \quad (\text{A.10})$$

From (A.10) we conclude that

$$\frac{\partial u_1(1)}{\partial u_1(0)} = \frac{\partial \gamma_3^1(x_1(0), w_1(0))}{\partial u_1(0)} = 0 \quad (\text{A.11})$$

An information equivalent to z_{21} for decision making purposes is given by (7), namely

$$\bar{z}_{21} = \begin{cases} y_2(0) = x_2(0) \\ \bar{y}_2(1) = x_2(0) + u_1(0) + w_2(0) \end{cases} \quad (\text{A.12})$$

Using (A.12), (A.4) can be written as

$$u_2(1) = \gamma_4(y_2(0), \bar{y}_2(1)) = \gamma_4^1(x_2(0), u_1(0), w_2(0)) \quad (\text{A.13})$$

From (A.13) we conclude that

$$\frac{\partial u_2(1)}{\partial u_1(0)} = \frac{\partial \gamma_4^1(x_2(0), u_1(0), w_2(0))}{\partial u_1(0)} \neq 0 \quad (\text{A.14})$$

Downstream Information Flow

Consider now the downstream information flow. Under this information structure

$$u_1(0) = \gamma_1(y_1(0)) \quad (\text{A.15})$$

$$u_2(0) = \gamma_2(y_2(0)) \quad (\text{A.16})$$

$$u_1(1) = \gamma_3(y_1(0)y_1(1)) \quad (\text{A.17})$$

$$u_2(1) = \gamma_5(y_2(0)y_2(1)y_1(0)) \quad (\text{A.18})$$

and

$$z_{11} = \begin{cases} y_1(0) = x_1(0) \\ y_1(1) = x_1(1) = x_1(0) - u_1(0) + w_1(0) \end{cases} \quad (\text{A.19})$$

$$z_{21} = \begin{cases} y_2(0) = x_2(0) \\ y_2(1) = x_2(1) = x_2(0) - u_2(0) + u_1(0) + w_2(0) \\ y_1(0) = x_1(0) \end{cases} \quad (\text{A.20})$$

The proof that

$$\frac{\partial u_1(1)}{\partial u_1(0)} = 0$$

is similar to the local feedback case. As before, we assume that the decision makers have agreed on the control strategy. Consequently, decision maker 21, based on his information, can exactly deduce the actions $u_1(0)$ and $u_2(0)$ of decision makers 10 and 20. Therefore, an information equivalent to z_{21} for decision making purposes is

$$\tilde{z}_{21} = \begin{cases} y_2(0) = x_2(0) \\ \tilde{y}_2(1) = x_2(0) + w_2(0) \\ y_1(0) = x_1(0) \end{cases} \quad (\text{A.21})$$

Using (A.21), (A.5) can be written as

$$\begin{aligned} u_2(1) &= \gamma_5(\tilde{z}_{21}) = \gamma_5(y_2(0), \tilde{y}_2(1), y_1(0)) \\ &= \gamma_5^1(x_2(0), x_1(0), w_2(0)) \end{aligned} \quad (\text{A.22})$$

From (A.22) we conclude that

$$\frac{\partial u_2(1)}{\partial u_1(0)} = \frac{\partial \gamma_5^1(x_2(0), x_1(0), w_2(0))}{\partial u_1(0)} = 0 \quad (\text{A.23})$$

Consider now the team theory control laws used in Section 4.2.4. We want to find $\text{Min}_{u_1(0)} E\{J|x_1(0)\}$ when $\gamma_2(y_2(0))$, $\gamma_1(y_1(0), y_1(1))$ and $y_2(y_2(0), y_2(1))$ assume their optimal values.

Using the dynamic equations (4.6) and (4.7) we can write the cost as

$$\begin{aligned} J &= E\{x_1^2(0) + x_2^2(0) + (x_1(0) - \gamma_1(y_1(0)) + w_1(0))^2 \\ &\quad + (x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) + (\gamma_1(y_1(0))))^2 + \\ &\quad + [\gamma_2^*(y_2(0))]^2 + [\gamma_1^*(y_1(0), y_1(1))]^2 + \\ &\quad + [y_2^*(y_2(0), y_2(1))]^2 + \\ &\quad + (x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) - \gamma_2^*(y_2(0), y_2(1)) + w_2(1))^2 \\ &\quad + (x_1(0) - \gamma_1(y_1(0)) + w_1(0) - \gamma_1^*(y_1(1), y_1(0)) + w_1(1))^2 \end{aligned}$$

$$\begin{aligned}
& + (x_1(0) - \gamma_1(y_1(0)) + w_1(0) - \gamma_1^*(y_1(0), y_1(1)) + w_1(1)) \cdot \\
& (x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) - \gamma_2^*(y_2(0), y_2(1)) + w_2(1)) \quad (A.24)
\end{aligned}$$

Let

$$E\{J|x_1(0)\} = \mathcal{L}_1(0)$$

Then setting $\frac{\partial \mathcal{L}_1(0)}{\partial u_1(0)} = 0$ gives

$$\begin{aligned}
& 2 \gamma_1(y_1(0)) + 2 \gamma_1(y_1(0)) + \frac{E\{(\gamma_1^*(y_1(0), y_1(1)))^2 | y_1(0)\}}{\partial u_1(0)} \\
& - 2 x_1(0) + 2 \gamma_1(y_1(0)) - 2 x_1(0) + 2 E\{\gamma_1^*(y_1(1), y_1(0)) | x_1(0)\} \\
& + 2 \gamma_1(y_1(0)) \cdot \frac{\partial}{\partial u_1(0)} E\{\gamma_1^*(y_1(0), y_1(1)) | x_1(0)\} \\
& - E\{x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) - \gamma_2^*(y_2(0), y_2(1)) + w_2(1) | x_1(0)\} \\
& - \frac{\partial}{\partial u_1(0)} E\{(\gamma_1^*(y_1(0), y_1(1))) (x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) \\
& - \gamma_2^*(y_2(0), y_2(1)) + w_2(1)) | x_1(0)\} \quad (A.25)
\end{aligned}$$

Since the information structure is partially nested, the last term in (A.25) is equal to zero and

$$\frac{\partial}{\partial u_1(0)} E\{[\gamma_1^*(y_1(0), y_1(1))]^2 | y_1(0)\} = 0 \quad (A.26)$$

Therefore,

$$\frac{\partial \mathcal{L}_1(0)}{\partial u_1(0)} = 0$$

gives

$$\begin{aligned}
& 6 u_1(0) - 4 x_1(0) + 2 E\{\gamma_1^*(y_1(0), y_1(1)) | x_1(0)\} \\
& - E\{x_2(0) - \gamma_2^*(y_2(0)) + w_2(0) - \gamma_2^*(y_2(0), y_2(1)) + w_2(1) | x_1(0)\} = 0
\end{aligned}$$

or

$$6 u_1(0) - 4 x_1(0) + E\{x_1(1) | x_1(0)\} + \frac{1}{2} E\{x_2(2)\} - E(x_2(2)) = 0$$

or

$$6 u_1(0) - 4 x_1(0) + x_1(0) - u_1(0) - \frac{1}{2} E(x_2(2)) = 0 .$$

APPENDIX B

OPTIMAL GENERATION SCHEDULING USING THE MAXIMUM PRINCIPLE, AGGREGATION, AND SUCCESSIVE APPROXIMATIONS

This appendix describes an aggregation/disaggregation technique based on successive approximations and dynamic programming. The method is used to compute optimal aggregate releases and then disaggregate these to obtain releases for each reservoir, where an individual reservoir's constraints are taken into account.

B.1 INTRODUCTION

Consider the problem of minimizing the cost functional

$$J = \int_0^T C(\alpha(t)) dt , \quad (B.1)$$

$$\alpha(t) = D(t) - \sum_{i=1}^N u_i(t) , \quad (B.2)$$

where $D(t)$ is a load demand for a given power system and $u(t) \in \mathbb{R}$ for $t \in [0, T]$ is the generation output of the hydro units. The part of the demand not met by hydro units must be met by thermal units. This is where cost is incurred. The objective is to allocate hydro generation such that the thermal generation cost is minimized subject to constraints:

$$\dot{x} = Cu + Dr , \quad (B.3)$$

$$x_{\min} \leq x(t) \leq x_{\max} , \quad (B.4)$$

$$u_{\min} \leq u(t) \leq u_{\max} , \quad (B.5)$$

where $r(t) \in \mathbb{R}^P$ for $t \in [0, T]$ are inflows to the reservoirs, $x(t) \in \mathbb{R}^N$ for $t \in [0, T]$ are the state variables representing the amounts of stored water in the reservoirs, and C and D are matrices of appropriate dimension describing series and parallel interconnections of the reservoirs.

The presence of state and control constraints make the Dynamic Programming algorithm a natural choice to solve this problem. However, as the order of (B.3) becomes large, the computational burden of Dynamic Programming makes the optimal scheduling policy unattainable.

We have developed an algorithm that eliminates much of the computational burden without sacrificing significant accuracy. The algorithm consists of two types of decompositions:

- (1) Spatial, and
- (2) Temporal.

We now discuss these independent decompositions in detail.

B.2 SPATIAL DECOMPOSITION

The idea of a spatial decomposition is to reduce our high-order optimization into a series of one-dimensional problems that converges to the optimal. While the maximum principle cannot easily handle the state constraints of this problem, it does serve as a tool that enables us to determine which states we may aggregate together.

Applying the maximum principle to our problem, we obtain a Hamiltonian function given by:

$$H(x, u, p, t) = C(\alpha(t)) + \langle p, Dr + Bu \rangle, \quad (B.6)$$

$$p \in \mathbb{R}^N$$

where p is the costate vector. Necessary conditions for optimality are the following:

$$(i) \quad \frac{\partial H}{\partial p_i} = \dot{x}_i, \quad (B.7)$$

$$(ii) \quad \frac{\partial H}{\partial x_i} = -\dot{p}_i = 0 \quad (B.8)$$

$$\Rightarrow p_i = \text{constant} \quad \forall_i$$

$$(iii) \quad u^* = \underset{u_{\min} = u^* \leq u_{\max}}{\operatorname{argmin}} H(x, p, u, t). \quad (B.9)$$

From (B.8) and (B.9), when u is within constraints we obtain

$$\nabla_u H = 0 \Rightarrow \frac{\partial C}{\partial \theta} \underline{1} = B^T p, \quad (B.10)$$

where

$$\underline{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Since B is always nonsingular, the values of p can be determined uniquely. The following lemma establishes a special structure of B .

Lemma

Assuming no feedback flows in the reservoir network, then there exists an ordering of state variables such that B is upper triangular, and $b_{ij} \in \{-1, 0, 1\}$.

Proof: By construction

For every reservoir we assign a "price level" (P.L.). Reservoirs the furthest "downstream" (i.e., do not flow into any other reservoirs) are at P.L. 1. The next set of reservoirs "upstream" are at P.L. 2, and so forth, until you have assigned every reservoir a P.L.

Sequentially assign state variables to all reservoirs of P.L. 1, then P.L. 2, and so forth. The order within a given price level is arbitrary. The resulting B matrix will always be upper triangular.

The following example serves to illustrate this property.

Example

$$x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} r + \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} u$$

Since B is upper triangular, this implies B^T is lower triangular.

From the construction of B , the trailing principle submatrix corresponding to P.L. 1 = -I which implies

$$\frac{\partial C}{\partial \alpha} = -p_1 \quad \forall i \in \text{P.L. 1}, \quad (\text{B.11})$$

$$p_1 = p_j = \text{constant} \quad \forall i, j \in \text{P.L. 1}. \quad (\text{B.12})$$

Likewise,

$$\forall x_1 \in \text{P.L. M}, \quad p_1 = M \cdot p_1. \quad (\text{B.13})$$

From [1,2] it has been shown that the costate variables have the physical meaning of "water price" (equivalent thermal cost that is replaced by the water $x_1(t)$).

The water at P.L. M has M turbines to go through so that x_1 , $i \in \text{P.L. M}$ has a "water price" that is M times higher than that of P.L. 1.

The aggregation proposed here is actually valid for a different problem, where there are no state constraints.

Remarks

- (1) From maximum principle, the costate relations hold even when the control constraints are encountered.
- (2) When state constraints are encountered, the usefulness of the maximum principle as a solution technique has known limitations.
- (3) A differential volume of water on the same P.L. will contribute the same amount of hydro power.

Thus, an aggregate model consisting of price levels as state variables retains all necessary dynamics but reduces the dimensionality of the system.

We now go through a three-stage state aggregation process motivated by our results from the maximum principle.

Rather than work with our general system model, we will follow a specific example through the aggregation process:

which is dynamically modeled as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (\text{B.14})$$

Stage 1

Obtain one aggregate state for every price level

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (\text{B.15})$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad (\text{B.16})$$

$$\begin{aligned} \dot{y}_1 &= v_2 - v_1, & 0 \leq y_1 &\leq x_{1\max}, \\ & & 0 \leq v_1 &\leq u_{1\max}, \\ \dot{y}_2 &= v_3 - v_2 + r_2, & 0 \leq y_2 &\leq x_{2\max} + x_{3\max}, \\ & & 0 \leq v_2 &\leq u_{2\max} + u_{3\max}, \\ \dot{y}_3 &= r_1 - v_3, & 0 \leq y_3 &\leq x_{4\max}, \\ & & 0 \leq v_3 &\leq u_{4\max}. \end{aligned} \quad (\text{B.17})$$

The result is that all parallelism is removed. The system is now one serial chain.

Stage 2

Create an equivalent parallel system, the purpose of which is to isolate control variables

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (\text{B.18})$$

$$\begin{aligned} z_1 &= r_1 + r_2 - v_1, & z_2 &\leq z_1 \leq z_2 + x_{1\max}, \\ z_2 &= r_1 + r_2 - v_2, & z_3 &\leq z_2 \leq z_3 + x_{2\max} + x_{3\max}, \\ z_3 &= r_1 - v_3, & 0 &\leq z_3 \leq x_{4\max}. \end{aligned} \quad (\text{B.19})$$

Stage 3

Again, the three reservoirs in parallel constitute one "price level," so we can aggregate them

$$\omega = [1 \ 1 \ 1]z, \quad (\text{B.20})$$

$$\dot{\omega} = \underbrace{3r_1 + 2r_2}_{r_\omega} - \underbrace{v_1 - v_2 - v_3}_{u_\omega}. \quad (\text{B.21})$$

The aggregate optimization problem becomes

$$\min_{u_\omega} \int_0^T C(\alpha(t)) dt, \quad \alpha(t) = D(t) - u_\omega(t) \quad (\text{B.22})$$

$$\text{s.t.} \quad \omega = r_\omega - u_\omega,$$

$$0 \leq \omega \leq 3x_{y\max} + 2x_{2\max} + 2x_{3\max} + x_{1\max},$$

$$0 \leq u_\omega \leq \sum_i u_i \max.$$

We now have an optimal control sum and optimal aggregate state. We must now disaggregate w and u_w back to our original state variables so that the control may be implemented.

B.3 DISAGGREGATION AND SUCCESSIVE APPROXIMATIONS

There are two distinct sets of disaggregations

$$(i) \quad \omega \rightarrow z$$

$$(ii) \quad y \rightarrow x,$$

since $z \rightarrow y$ is a nonsingular transformation.

For $\omega \rightarrow z$, we define the secondary constraints as

$$z_1 + z_2 \leq \omega \leq z_1 + z_2 + x_{4\max},$$

$$2z_3 + z_1 \leq \omega \leq z_1 + 2z_3 + x_{2\max} + x_{3\max},$$

$$2z_2 + z_3 \leq \omega \leq 2z_2 + z_3 + x_{1\max},$$

(B.24)

$$v_1 + v_2 \leq u_w \leq v_1 + v_2 + u_{4\max},$$

$$v_2 + v_3 \leq u_w \leq u_3 + v_3 + u_{1\max},$$

$$v_1 + v_3 \leq u_w \leq v_1 + v_3 + u_{2\max} + u_{3\max},$$

and the primary constraints as

$$z_2 \leq z_1 \leq z_2 + x_{1\max},$$

$$z_3 \leq z_2 \leq z_3 + x_{2\max} + x_{1\max},$$

(B.25)

$$0 \leq z_3 \leq x_{4\max},$$

$$0 \leq v_i \leq v_{i\max}, \quad i = 1, 2, 3.$$

By inspection, satisfying the secondary constraints implies satisfying the primary constraints but not vice versa.

We now use these constraints for a successive approximations disaggregation.

Stage 1

Using the constraints (B.25), let

$$r_1 = v_3, \quad r_1 + r_2 = v_2,$$

which implies $z_3 = z_3(0)$ and $z_2 = z_2(0)$ for all t . Solve the following one-dimensional minimization

$$\min_{v_1} \int_0^T C(\alpha(t)) dt, \quad (B.26)$$

subject to

$$\alpha(t) = D(t) - (\partial r_1 + r_2 + v_1^u),$$

$$z_1^0 = r_1 + r_2 - v_1^0,$$

(B.27)

$$z_2(0) \leq z_1^0 \leq z_2(0) + x_{1\max},$$

$$0 \leq v_1^u \leq u_{1\max}.$$

Now let $r_1 = v_3$ and $v_1 = v_1^0$ and solve

$$\min_{v_2} \int_0^T C(\alpha(t)) dt, \quad (B.28)$$

subject to

$$\alpha(t) = D(t) - (v_1^0 - v_2^0 - r_1) ,$$

$$\dot{z}_2^0 = r_1 + r_2 - v_2^0 ,$$

(B.29)

$$z_3(0) \leq z_2^0 \leq z_3(0) + x_{2\max} + x_{3\max} ,$$

$$0 \leq v_2^0 \leq u_{2\max} + u_{3\max} .$$

Finally, let $v_2 = v_2^0$, $v_1 = v_1^0$, $z_2 = z_2^0$, and $z_1 = z_1^0$ and solve

$$\min_{v_3^0} \int_0^T C(\alpha(t)) dt , \quad (B.30)$$

subject to

$$\alpha(t) = D(t) - (v_1^0 + v_2^0 + v_3^0) ,$$

$$z_3^0 = r_1 - v_3^0 ,$$

(B.31)

$$0 \leq z_3 \leq x_{4\max} ,$$

$$0 \leq v_3 \leq u_{4\max} .$$

Stage 2

Use the trajectories z_1^0 , z_2^0 , z_3^0 , u_1^0 , u_2^0 and u_3^0 in (B.24) and see which constraints are satisfied. For every constraint in (B.24) satisfied, replace the primary constraint by its corresponding secondary constraint in the sequential optimization procedure of stage 1.

Stage 3

Evaluate

$$E = \int_0^T (u_w - v_1 - v_2 - v_3) dt . \quad (B.32)$$

If $E < \delta$ (design specification) then stop and proceed to the next stage. If not, go back to stage 1 and perform a new iteration based on the v_1^0 's and z_1^0 's.

Stage 4

One z is determined, y is uniquely determined

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} . \quad (B.33)$$

Stage 5

We now have to disaggregate y to x . For this problem,

$$\begin{aligned} \dot{x}_2 &= u_4 - u_2 , \\ \dot{x}_3 &= r_2 - u_3 , \end{aligned} \quad (B.34)$$

with primary constraints

$$\begin{aligned} 0 \leq x_2 \leq x_{2\max} , \quad 0 \leq u_2 \leq u_{2\max} , \\ 0 \leq x_3 \leq x_{3\max} , \quad 0 \leq u_3 \leq u_{3\max} , \end{aligned} \quad (B.35)$$

and secondary constraints

$$\begin{aligned}
 x_3 &\leq y_3 \leq x_3 + x_{2\max} , \\
 x_2 &\leq y_2 \leq x_2 + x_{2\max} , \\
 u_2 &\leq v_2 \leq u_2 + u_{3\max} , \\
 u_3 &\leq v_2 \leq u_3 + u_{2\max} .
 \end{aligned}
 \tag{B.36}$$

We now proceed in a fashion completely analogous to Stage 1. Let $u_4 = u_2^0$, which implies $x_2 = x_2(0)$ and solve

$$\min_{u_3} \int_0^T C(\alpha(t)) dt ,
 \tag{B.37}$$

where

$$\alpha(t) = D(t) - \underbrace{(u_1^0 + u_4^0 + u_4^0 + u_3^0)}_{\text{known}} ,$$

$$\dot{x}_3 = r_2 - u_3^0 ,
 \tag{B.38}$$

$$0 \leq x_3^0 \leq x_{3\max} ,$$

$$0 \leq u_3^0 \leq u_{3\max} .$$

Then let $u_3 = u_3^0$ and solve

$$\min_{u_2^0} \int_0^T C(\alpha(t)) dt ,$$

$$\alpha(t) = D(t) - (u_1 + u_2^0 + u_3^0 + u_4) ,
 \tag{B.39}$$

$$\dot{x}_2^0 = u_4^0 - u_2^0 ,$$

$$0 \leq x_2^0 \leq x_{2\max} ,$$

$$0 \leq u_2 \leq u_{2\max}$$

Stage 6

Substitute x_2^0 , x_3^0 , u_2^0 , and u_3^0 into (B.36). If they are satisfied, then replace the corresponding primary constraint. Evaluate

$$E_Y = \int_0^T |v_2 - u_1 - u_2| dt . \quad (B.41)$$

If $E_Y < \gamma$ (design specification), then stop. Otherwise, go back to stage 5 and perform another iteration.

We have then completed the disaggregation and have the desired u_1 , u_2 , u_3 , and u_4 .

Advantages of the algorithm:

- (1) Only solve one-dimensional problems.
- (2) Optimal sum obtained for arbitrary $C(\alpha(t))$.
- (3) Asymptotic analysis enables you to establish convergence rates for the successive approximations.
- (4) Disaggregation procedure enables you to form a corridor around your optimal sum that restricts the amount of state space to be discretized.

B.4 TEMPORAL DECOMPOSITIONS

The spatial decompositions allow us to solve a series of one-dimensional problems. The temporal decomposition takes each one-dimensional problem and further reduces the computational burden of Dynamic Programming. This is possible due to the multiple-time-scale nature of the demand.

A typical demand function might be of the form

$$D(t) = D_s(t) + D_w(t) + D_d(t) , \quad (B.42)$$

where $D_d(t)$, $D_w(t)$, and $D_s(t)$ are nearly periodic daily, weekly and seasonal demands, respectively. An example of such a demand is given in Figure B.1.

The basic design idea is to assume a control of the form

$$u(t) = u_s(t) + u_w(t) + u_d(t) , \quad (B.43)$$

and to solve a daily optimal scheduling problem parameterized on all weekly and seasonal control variables. Then this solution is imbedded into the weekly optimization problem which is solved parameterized on the

Figure B.1. Typical Demand Function

seasonal control. Finally, this set of solution is imbedded into the seasonal scheduling problem and optimized over the entire year. This idea is similar to [3,4], and is related to spatial dynamic programming [5]. A simple example will serve to explain this basic idea.

Consider

$$J = \int_0^T (D(t) - u(t))^2 dt , \quad (B.44)$$

$$D(t) = a_0 + a_1 \sin\left(\frac{2\pi t \epsilon}{24}\right) + a_2 \sin\left(\frac{2\pi t}{24}\right) . \quad (B.45)$$

The time scale t is in hours and the interval T is three months, long enough to incorporate a season.. Hence

$$D_s(t) = a_0 + a_1 \sin\left(\frac{2\pi t \epsilon}{24}\right) , \quad (B.46)$$

$$D_d(t) = a_2 \sin\left(\frac{2\pi t}{24}\right) , \quad (B.47)$$

and $\epsilon = 1/90$ is considered a small parameter representing the day-to-season ratio. For a single reservoir, the constraints are

$$\frac{dx}{dt} = r(t) - u(t) ,$$

$$0 \leq u(t) \leq u_{\max} , \quad t \in [0, T] , \quad (B.48)$$

$$0 \leq x(t) \leq x_{\max} , \quad t \in [0, T] .$$

If we let $k = 0, 1, \dots, 90$ be the index for the sequence of days in the season, then the cost (B.44) can be expressed as

$$J = \sum_{k=0}^{90} \int_{24k}^{24(k+1)} (D(t) - u(t))^2 dt , \quad (B.49)$$

and letting $u(t) = u_d(t) + u_s(t)$

$$J = \sum_{k=0}^{90} \int_{24k}^{24(k+1)} (D_d(t) - (u_s(t) + u_d(t)))^2 dt - 2 \int_{24k}^{24(k+1)} D_s(t)(u_s(t) + u_d(t))dt + C(k) , \quad (B.50)$$

where

$$C(k) = \int_{24k}^{24(k+1)} (D_s^2(t) + 2D_d(t) D_x(t)) dt ,$$

does not affect the optimal choice of u . The effective cost to be minimized is

$$J = \sum_{k=0}^{90} \int_{24k}^{24(k+1)} (D_d(t) - (u_s(t) + u_d(t)))^2 dt - 2 \int_{24k}^{24(k+1)} D_s(t)(u_s(t) + u_d(t))dt , \quad (B.51)$$

Note that

$$\int_{24k}^{24(k+1)} D_d(t)dt = 0 , \quad k = 0, 1, \dots, 90 , \quad (B.52)$$

and thus we require that

$$\int_{24k}^{24(k+1)} u_d(t) = 0 , \quad k = 0, 1, \dots, 90 . \quad (B.53)$$

For $24k \leq t < 24(k+1)$, $k=1, \dots, 90$, we can express the seasonal demand in a MacLauren series around $\epsilon = 0$

$$D_s(t) = D_s(24k) + \epsilon \gamma(t) + O(\epsilon^2) . \quad (B.54)$$

Likewise, we require $u_s(t)$ to have a similar form over the same periods of time

$$u_s(t) = u_s(24k) + \epsilon \gamma(t) + O(\epsilon^2) . \quad (B.55)$$

Substituting (B.54) and (B.55) into (B.50) and letting $\epsilon \rightarrow 0$, we obtain the reduced effective cost

$$J = \sum_{k=0}^{90} \int_{24k}^{24(k+1)} (D_d(t) - (u_s(24k) + u_d(t)))^2 dt - 48.0 D_s(24k) u_s(24k) . \quad (B.56)$$

The second term of this cost is constant over each day k . Thus, it need only be optimized on a daily basis. The integral terms of the sum correspond to the cost incurred optimizing the daily control for a given constant seasonal control.

Thus, we can solve the daily scheduling problem parameterized on $u_s(24k)$. Imbed this solution set into (B.56) and solve a computationally faster daily allocation problem.

First, express the state equation in daily transitions

$$\int_{24k}^{24(k+1)} \dot{x} dt = \int_{24k}^{24(k+1)} r(t) dt - \int_{24k}^{24(k+1)} u_s(t) dt , \quad (B.57)$$

$$x(24(k+1)) = x(24k) + \bar{r}(k) - 24u_s(24k) . \quad (B.58)$$

Uniformly quantize state space into q values and define a qxq state transition matrix P where p_{ij} corresponds to a transition in water level from x_i to x_j , where

$$0 = x_1 \leq x_2 \leq \dots \leq x_q = x_{\max}.$$

If x_Δ is the maximum reservoir level changing in one day, then the matrix P will have the form

$$\begin{bmatrix} p_{11} & p_{12} & & & p_1 & & & \\ p_{21} & p_{22} & & & & & 0 & \\ & & p_{23} & & & & & \\ p_{\Delta 1} & & & & & & & p_{q-\Delta, q} \\ & & & & & & p_{q-1, q-1} & p_{q-1, q} \\ 0 & & & p_{q, q-\Delta} & & & p_{q, q-1} & p_{qq} \end{bmatrix}, \quad (B.59)$$

where "0" means impossible transitions. This matrix in turn implies a matrix S of forcing terms necessary to achieve the desired state transitions. For simplicity, let us assume $r(t)$ is constant for all t . Then, from (B.56), p_{ij} implies $u_s^{ij}(24k)$ necessary for this daily transition. Thus, for every u_s^{ij} (drop dependence on k) we solve the daily allocation problem with fixed initial and end points.

$$J_{ij}(u_s^{ij}) = \min_{u_d} \int_{24k}^{24(k+1)} (D_d(t) - u_s^{ij} - u_d(t))^2 dt, \quad (B.60)$$

subject to

$$\begin{aligned} \dot{x} &= r - u_s^{ij} - u_d(t), \\ x(24k) &= x_i, \quad x(24(k+1)) = x_j, \\ -u_s^{ij} &\leq u_d(t) \leq U_{\max} - u_s^{ij}, \\ 0 &\leq x(t) \leq X_{\max}. \end{aligned} \quad (B.61)$$

This is our fast subproblem. When solved for every p_{ij} we have found the "optimal daily cost" to make the transition $x_i \rightarrow x_j$. We identify this matrix as $\Delta J(u_s^{ij})$. Now our optimal scheduling problem reduces to determining the seasonal allocations $u_s(24k)$ in daily time scale k to minimize

$$\bar{J} = \sum_{k=0}^{90} \Delta J(u_s(24k)) - 48 D_s(24k)(u_s(24k)) , \quad (B.62)$$

subject to

$$x(24(k+1)) = x(24k) + 24r - 24u_s(24k) , \quad (B.63)$$

$$0 \leq x \leq x_{\max} ;$$

$$0 \leq u \leq u_{\max} .$$

To illustrate this, set up the grid for days k and $k+1$ for a three-state system. For ease of notation, we assume that D_s and U_s are evaluated on the daily time scale, where $J(i,k+1)$ is the cost to go from state i at day $k+1$. To find, for example, $J(2,k)$,

$$J(2,k) = \min_{u_s^{21}} \{ (\Delta J(u_s^{21}) - 48D_s(k)(u_s^{21})) + J(1,k+1) \} , \quad (B.64)$$

$i = 1, 2, 3 .$

The advantages of this algorithm over the usual dynamic programming procedure are the following:

- (1) Array sizes — if there are NQ state quantizations then for the unperturbed problem you need two matrices of dimension $2160 \times NQ$. In the above algorithm you need one $NQ \times NQ$ array, two $NQ \times 24$ arrays and two $NQ \times 90$ arrays.
- (2) Speed — if we had to find the optimal daily cost parameterized on every day-to-day state transition, then this algorithm would be speedwise inferior to the unperturbed solution. Fortunately, the $\Delta J(u_s^{ij})$ matrix has some nice properties. First, it possesses the same number of nonzero elements as (B.59). And, second, the elements of the cost matrix along a given (sub, super) diagonal defined by $i-j = \text{constant}$, will be equivalent except for transitions involving boundary states. Thus, this array may be filled up with a minimum account of computation.

To give you an idea of the array size savings let $NQ = 10$. Then (B.44) requires 43,200 storage locations, while the two-time-scale method requires 2,300 locations. A numerical example is given below.

Example

Consider the cost

$$J = \int_0^T (D(t) - u(t))^2 dt , \quad (B.65)$$

with demand

$$D(t) = 1 + 0.3 \sin\left(\frac{2\pi t \epsilon}{6}\right) + 0.5 \sin\left(\frac{2\pi t}{6}\right) , \quad (B.66)$$

and constraints

$$\frac{dx(t)}{dt} = 0.2 - u(t) , \quad (B.67)$$

$$0 \leq x(t) \leq 2.4 ,$$

$$0 \leq u(t) \leq 0.4 ,$$

with t in four-hour periods and T equal to 15 days.

State space was quantized in uniform increments of 0.1. Using a backward dynamic programming algorithm, an optimal cost of

$$J^* = 65.030051$$

was attained. Using our two-time-scale method the cost was increased by 0.236%. If we did not use our fast parameterized costs $J(u_s^{ij})$ in (B.60) and instead just neglected all fast daily demand, the problem reduces to

$$J = \sum_{k=0}^{15} 6(D(k) - u_s(k))^2 ,$$

$$x(k+1) = x(k) + 1.2 - 6u_s(k) ,$$

$$0 \leq x(k) \leq 2.4 ,$$

$$0 \leq u_s(k) \leq 0.4 .$$

The solution to this problem has a cost that is 9.15% more than J^* . In the plots that follow, this will be referred to as the "coarse" solution and the optimal will always be the solid line curve.

Figure B.2. Demand Function

Figure B.3. Optimal Control vs. Two Time Scale Control

Figure B.4. Optimal State vs. Two Time Scale State

Figure B.5. Optimal Control vs. Coarse Control

Figure B.6. Optimal State vs. Coarse State.

B.5 CONCLUSIONS

By applying both aggregation and perturbation techniques, a computationally efficient dynamic programming algorithm has been developed to solve the optimal hydro/thermal scheduling problem. By using the maximum principle, we found that the co-states of certain states were equivalent, enabling us to combine these dynamics and obtain reduced order models and optimization problems. The multiple time scale nature of the demand function enabled us to reduce the computational burden even further. Using a simple perturbation technique we solved a sequence of embedded parameterization problems to obtain our optimal solution using a computationally efficient algorithm.

Many of the limitations and pitfalls of this algorithm have yet to be fully understood. However, it sheds some hope on the future in terms of the ability to solve high order dynamic programming problems simply.

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APPENDIX C

MAINTENANCE SCHEDULING TERMINOLOGY AND LITERATURE REVIEW

This appendix gives definition of terms commonly used in maintenance scheduling. A comprehensive review of literature follows the definitions.

C.1 DEFINITION OF TERMS

Before discussing the existing literature, a terminology of maintenance scheduling (MS) is presented.

- Preferred Maintenance: This is the nominal starting date of maintenance for a given generating unit.
- Preventive Maintenance: Regular routine or planned checkups and repair of generating equipment to avoid (reduce) forced outage probability of the generating facility.
- Corrective Maintenance Scheduling: Maintenance built into the system at the design stage.
- Time Horizon: Time horizon is defined as the span of time considered in a solution of the maintenance scheduling study (e.g., a year). Also referred to as scheduling horizon.
- Time Increment: Maintenance scheduling of units are normally done in discrete time, rather than continuously. A typical time increment is one week.
- Unit: This term, as used here, refers to any device for which an outage results in a loss of generating reserves. The term includes generators, boilers, turbines, auxiliary equipment and tielines. In the most general sense, the term includes any maintenance activity which (1) results in a decrease in reserves, or (2) requires manpower from a limited source, or (3) must be scheduled and reported. Sample data for a generating unit is presented in Table A.1.
- Reserve Loss: A reserve loss is generally associated with each unit or device. This value, in megawatts, is the decrease in reserves during maintenance. Since the term "unit" is applied to any maintenance activity, this value is not necessarily the rating of a generating unit, and can represent, for example, deratings due to maintenance on boilers or auxiliary equipment.

TABLE A.1 A SAMPLE DATA FOR AN OUTAGE

Unit-----	Red River 1			
Crew Name-----	Red River			
Reserve lost during maint.-----	200 mw			
Outage duration-----	4 weeks			
Firm (past) dates-----	4-17-68			
	4-24-69			
Flexible (future) dates-----	within 60-44 weeks			
	within 62-48 weeks			
	within 60-40 weeks			
Inhibited periods-----	20 weeks starting 10-1-79			
	20 weeks starting 10-1-71			
	20 weeks starting 10-2-72			
Resource requirements-----	<u>Name</u>	<u>Wk 1</u>	<u>Wk 2</u>	<u>Wk 3</u>
				<u>Wk 4</u>
	Mechanics	30	40	40
	Welders	20	20	10
	Heavy Crane	0	0	1
	System Reg.	1	1	1
	Area 1 Maint.	200	200	200
				200

Source: W. R. Christiaane; "A Program for Calculating Optimal Maintenance Schedules Recognizing Constraints," PICA Proceedings, pp. 230-239, 1973

- Gross Reserve: The gross reserve in megawatts for a given week is the total (in a time increment week) installed generating capacity available during the week less the peak load forecast for the week.
- Net Reserve: The net reserve in megawatts for a given week is the gross reserve for the week less the total reserve lost because of the maintenance scheduled for that week.
- Minimum Net Reserve: The minimum net reserve is the lowest of all the weekly net reserves over the maintenance scheduling horizon (e.g., year).
- Crew of Plant: A crew is assigned to each unit or device, i.e., maintenance activity. For example, a single crew may be responsible for maintaining all the units in a given plant or the units in a given area. As a result, only one of these units can be maintained in a given week. No units or devices on the same crew can be maintained simultaneously.
- Maintenance Outage: Generally speaking, an outage is considered cyclical. Occurrences of an outage (planned outage) are of two types, firm or flexible.

- Firm Maintenance: This term describes an occurrence of an outage which has already taken place or is prescheduled to begin at a specified time.
- Flexible Maintenance: The term describes the occurrences of an outage which is to be scheduled by the maintenance scheduling technique. A minimum and maximum time since the last occurrence are specified for each flexible outage.
- Resource: A resource is a general term applied to a pool of manpower, equipment or other quantity which is limited supply during each time increment and is shared by two or more outages. For example, a single resource pool may represent, (a) all the men in a particular union craft, or (b) two or three heavy cranes required for some outages, (c) the major units in the system used for regulation (d) the maximum number of megawatts of capacity which can be taken out for maintenance in one area of the system.
- Frozen Time: Number of time increments after the maintenance ending during which some given generators cannot begin the maintenance.
- Forced Outage (FO): A forced outage is one which requires that the turbo-generator and/or boiler be taken out of service at once or as soon as thereafter as possible. This include cases where the cause of the outage is such that the equipment is not removed from service until the off-peak period of the same day or the following weekend and cases where damage occurs during shutdown and startup. The time during which the unit is unavailable because of forced outage is forced outage time.
- Time Exposed to Forced Outage: The time exposed to forced outage during a period is the time on forced outage plus the service time (time in operation).
- Forced Outage Rate: The forced outage rate for a unit is the ratio of forced outage time and the time exposed to the forced outage.
- Loss-of-Load-Probability (LOLP): This is a well understood measure of generation system reliability. LOLP is the probability that the system load (MW) exceeds the system's available generation (MW). Normally 260 weekdays' peak load and available generation data are used to calculate this reliability measure. The reason weekends are excluded from this calculation is that weekend peak loads are usually below the system available generation. It should be noted that LOLP does not calculate the amount of energy lost (not supplied by the utility) during the period LOLP was calculated. It also doesn't say how much of the load was not served. All it says is that load is exceeding the generation.

- Expected Unsupplied Energy (EUE): This quantity signifies the expected amount of energy which cannot be supplied by the utility during some period. It is more time consuming to obtain, than LOLP which is rather fast. But if one uses the load duration curve (LDC) for calculating the system reliability measures, obtaining LOLP and EUE involves about the same amount of computational efforts.
- Operating Costs: Operating cost consists of two components, namely variable O&M costs and fuel costs burnt in producing electric energy. This quantity for a generating unit producing electric energy is calculated by multiplying the amount of energy produced or expected to be produced from the unit by the sum of per unit energy variable O&M and per unit energy production cost of the generating facility.
- Production Cost: Production cost is the major component of a generating unit's operating cost. It is the cost of burnt fuel in producing a given amount of electric energy by generating facility.
- Maintenance Costs: Maintenance cost in the context of long term operation scheduling problem consists of two components. The first one is a fixed maintenance cost which is required for a generating facility regardless of how often or how long the facility is used to generate electric energy. This component is normally given as a constant for a given unit. The second component is dependent on the usage of the facility. Normally this component is given in dollars per unit energy produced (e.g., \$/MWh). This cost includes wear and tear due to frequency of startups and shutdowns, the duration and level of operation of the generating facility, etc. Maintenance costs could be variable if one allows the maintenance duration to vary meaning that over time work is allowed or additional staff are hired.

After having established terminology definitions, a relatively comprehensive review of the literature will be presented next.

C.2 LITERATURE REVIEW

The intent of the literature review is to make a comprehensive assessment of the state-of-the-art. Each source reviewed will be referenced first. Then the paper will be described. After the review is complete, a critical look of the existing literature will be presented.

1. K. Hara, M. Kimura, N. Honda, "A Method for Planning Economic Unit Commitment and Maintenance of Thermal Power Systems," IEEE Transactions on PAS, Vol. PAS-85, No. 5, pp. 427-436, May 1966.

This paper combines unit commitment and yearly overhaul (maintenance scheduling) together. The method is more or less a complete enumeration approach. Startup cost of a unit is considered constant, but failure rate of units are combined with the startup costs, that is:

$$C_{is} = \sum_{j=1}^{\ell} K_j [(q_j - q_{j-1}) + a_j T q_j]$$

ℓ = Number of groups

i = Time in level

K = Starting up cost of a unit in group j

j = Number of units of group j on interval i

a = Failure rate (per day)

T = Length of time interval (hour)

For reliability, it uses quadratic cost function for loss of load and for over-supply, it also uses a quadratic cost function. It ignores or does not recognize items such as maintenance duration, flexible maintenance, fixed maintenance, nominal maintenance dates and maintenance windows. It models load for each time interval, T , as a normal distribution. The problem is formulated as cost minimization.

The objective function consists of fuel cost, startup cost, over-supply cost and under-supply cost. Fuel cost is calculated using the idea of equal incremental cost operation of all units in operation. Startup costs are constant. Over- and under-supply costs are set to be proportional to the square of deviation from the load level. None of the maintenance scheduling constraints are included in the formulation. Relaxation is used as a method of solving the problem. This paper is more appropriate for unit commitment than for maintenance scheduling.

2. W. R. Christiaane and A. H. Palmer, "A Technique for the Automated Scheduling of the Maintenance of Generating Facilities," IEEE Transactions on PAS, Vol. PAS-91, pp. 137-144, January/February 1972.

This paper describes a computer program developed by Stagg Systems Inc. to perform maintenance scheduling for a time horizon of up to a year. It is a good paper for description of the problem. The time increment used is one week. For each generating unit that requires maintenance, a window within which the maintenance of the particular generating unit should start is used. The method of approach is maximizing the minimum reserve. For each week the peak load and capacities available are used to calculate the net reserve for the week. A branch-and-bound approach is used to obtain the optimum maintenance schedule. Constraints such as crew, maximum and minimum time between outages, and minimum reserve for each week are included in the approach. A decision tree is formed. At each point on the decision tree a bound for the objective function is calculated.

A simple Branch-and-Bound approach is used. A simple description of the approach is as follows: Let's say that one obtained a feasible schedule with a minimum set reserve of 90 MW. The objective is a better schedule meaning one with a higher minimum reserve. If during any week in the process, one is forced to create a net reserve of less than 90 MW, one knows that he should step back in the decision tree, reserve a preceding decision and "branch" down another path in the tree. Eventually one shall either (1) exhaust all alternative paths down the tree, or (2) reach the end of the tree and generate a schedule with a minimum net reserve higher than 90 MW, say 120 MW. Then 120 MW will be used as a bound on the minimum net reserve. The process will be repeated until successive searches cannot find a schedule with a higher minimum net reserve.

No stochastic effects are included in this approach. The method has not been shown to be able to handle large systems for a scheduling period of one year (52 weeks).

3. L. L. Garver, "Adjusting Maintenance to Levelize Risk," Paper T 72 154-8, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, pp. 2057-2063, September/October 1972.

This paper presents the methodology used very concisely and to the point. It is not a good paper for description of the maintenance scheduling problem. The method has been developed for use in generation planning process.

The paper presents a five step method which incorporates reliability considerations into existing maintenance scheduling methods. The procedure is as follows:

1. Compute the capacity outage table for all the units in the system as would be done for a loss-of-load probability calculation [1].
2. Prepare to estimate the effective capability of each unit by fitting a straight line to a semi-log plot of the capacity outage table [2].
3. Compute the effective load-carrying capability of each unit using the fitted straight line [2].
4. Compute the equivalent load for each maintenance period (e.g. week). This is the single magnitude which if encountered each day in the period would result in the same total risk as the varying loads actually forecast. Equivalent load is that single load which if encountered the same number of times as the varying daily peak loads, would produce the same risk for the interval (week).
5. Schedule the maintenance to level, throughout the year, the equivalent load plus the effective capabilities of the units on maintenance, in place of leveling the reserve.

The procedure outlined in this paper may not result in the maintenance plan that best levelizes risk. The deviation from the best plan may result from two causes; (1) the error in the calculation of load carrying capabilities (resulting from the approximation method used), and (2) the failure to readjust the load carrying capability of units as the maintenance plan is developed. The use of the approximation method to obtain load carrying capabilities may or may not be justified depending upon the reduction in computation time and confidence desired in the resulting plan.

The failure to readjust the load carrying capability of the units as the maintenance plan is developed appears to be more serious. The load carrying capability of a generating unit is dependent upon the relative size and forced outage rates

of all the other units available to the system as well as its own size and forced outage rate. Thus, the load carrying capability of a unit for which maintenance is being planned is not constant throughout the year. It is different in each maintenance period that has different combinations of units already scheduled for maintenance.

4. W. R. Christiaane, "A Program for Calculating Optimal Maintenance Schedules Recognizing Constraints," PICA Proceedings, pp. 230-239, 1973.

This paper describes a computer program for obtaining annual maintenance overhauls schedules for generating facilities. It gives a good description of maintenance scheduling problem. No stochastic effects are included in the methodology. The criterion for scheduling maintenance is net reserve. This is an extension of paper No. 2 reviewed in this literature review. Optimization objective is to maximize the minimum reserve over the scheduling time horizon. Three constraints, namely, resource requirements, crew constraints, and timing constraints are implemented in the computer program presented in the paper.

A brief description of the approach follows:

- The maintenance for each crew is scheduled independently of the other crews. The crews are arranged in order, according to the total megawattweeks of flexible maintenance assigned to the crew. The total for each crew includes the maintenance on all the units or devices assigned to the crew. The crews with the large totals are scheduled first.

For a given crew, the flexible outages on the first unit assigned to the crew are scheduled sequentially. Then the flexible outages for the next unit assigned to the crew are scheduled, and so on. When all the units on the crew have been scheduled, the next crew is scheduled. The periods selected for each flexible outage will depend on the current levels of net reserve and the crew and resource availability resulting from the outages already scheduled.

As each flexible outage is considered in the scheduling process, the weeks are listed in order of preference for the outage. The order depends on the current level of net reserves and the duration of the flexible outage. The weeks are ranked according to the minimum reserve which would occur during the outage, assuming the outage

begins in that week. The weeks in the list are tried until a feasible week is found. The feasibility checks include the three basic constraints already noted. After a feasible week is found, the outage is scheduled for that week. After all flexible outages for the unit have been scheduled, the flexible outages for the next unit on the crew are scheduled. When the entire crew has been scheduled, the next crew is begun.

If no feasible week can be found for a particular flexible outage, the algorithm steps back to the last flexible outage scheduled on the crew. This last outage is re-scheduled to the next preferred week in the list for this outage, and the algorithm moves forward again. If, when the algorithm steps back, there are no weeks left, the algorithm steps back once again to the next to the last outage. If the stepping-back process reaches the first flexible outage, no further stepping-back is possible. Then it is established that no feasible schedule exists for the flexible outages, since all combinations of weeks have been exhausted.

The approach is totally deterministic, no stochastic effects are included in the approach.

5. A. D. Patton and J. Ali, "Comparison of Methods for Generation Maintenance Scheduling," IEEE PES Summer Meeting, Paper C 72 452-1, San Francisco, California, July 1972.

Compares different maintenance scheduling methods. A conclusion of this paper is that the choice of scheduling algorithm is not critical. The objective used in this paper is minimization of risk.

- This method is an extension of Graver's equalizing risk technique (No. 3). However, the use of "effective load carrying capability" is avoided. Units are allocated in accordance with their effect on reliability, and the LOLP is minimized by trying different schedules. Beside using the LOLP as a measure of risk it also approximates that capacity on maintenance can be treated as a fictitious load. This approximation, which is often attempted in order to avoid the need for building a new availability table after the withdrawal of a unit, would lead to wrong results particularly, as mentioned above, when units with high outage rate are involved.

6. H. H. Zörn, V. H. Quintana, "Generator Maintenance Scheduling Via Successive Approximations Dynamic Programming," IEEE Trans. on PAS, Vol. PAS-94, pp. 665-671, March/April 1975.

The maintenance problem is formulated as a dynamic programming problem. The objective function is a cost function with components as expected energy production cost, security penalty cost (cost of unsupplied energy), and penalty cost of violating relaxed constraints. Successive approximation in dynamic programming is applied to reduce the "curse" of dimensionality problem. For this purpose units of similar characteristics are grouped together and dynamic programming for each group is used to obtain an improved maintenance schedule. While optimizing the objective function for seeking the optimal maintenance of one group, the maintenance dates of units in other groups are fixed. The procedure by no means guarantees optimality because (1) of the discrete nature of the problem and as such the objective function is not convex and (2) successive approximation does not guarantee global optimality. Also the solution may not be feasible at all. Uncertainties are not included in the methodology (it is a deterministic procedure).

7. J. F. Dopazo, H. M. Merrill, "Optimal Generator Maintenance Scheduling Using Integer Programming," IEEE Trans. on PAS, Vol. PAS-94, pp. 1537-1545, September/October 1975.

The maintenance problem is formulated as a zero-one integer programming problem. The objective function is linear and the constraints are equalities and inequalities. A simple 3 unit, 4 week time horizon example problem is used to show the method of approach. The results of computer studies using the methodology is reported. Three different case studies with different objectives (Minimum Change from Existing Schedule, Maximize Minimum Reserve and Minimum Cost Schedule) are reported. The minimum cost is not operating cost. The constraints included in the formulation are (1) each unit must be maintained once, (2) duration of maintenance, that is, when maintenance starts it should be completed, (3) sequence of maintenance and interval between successive maintenance dates, (4) resource constraints in the form of limit on the amount of capacity that can be taken out on maintenance.

The method can handle a host of constraints. A branch-and-bound approach is presented to solve the integer program. The solution procedure uses Balas implicit enumeration algorithm [3].

8. J. C. Kohli, J. Sharma, and M. P. Dave, "Optimal Preventive and Corrective Maintenance," Paper C 75 146-6, IEEE PES Winter Meeting, New York, New York, January 26-31, 1975.

The problem of maintenance scheduling is formulated as 0-1 integer program with equality and inequality constraints, in this paper. Several constraints such as time between successive maintenances, precedence constraints, resource (megawatts, manpower) constraints, and exclusion (some units may not be maintained simultaneously, e.g., no two units of the group power plant or no two large units should be maintained during the same period) constraints are included in the methodology. The constraints are divided into two categories: (1) deterministic constraints, (2) chance constraints mainly because of resource availability [5]. These chance constraints are converted to equivalent deterministic constraints using the method of reference [6]. The solution approach is a direct tree search method. Basically it is an enumeration method.

9. H. H. Zörn, V. H. Quintana, "Several Objective Criteria for Optimal Generator Preventive Maintenance Scheduling," Paper F 76 401-0, IEEE PES Summer Meeting, Portland, Oregon, July 18-23, 1976.

This paper presents a comparison of the performance of maintenance scheduling techniques with different objective functions. Objective functions included in this paper are based on (1) generation operating cost, (2) reliability indices (maximizing minimum reserve, levelizing reserve, levelizing risk), (3) deviation from a desired schedule and/or constraint violation penalties. This paper does not provide the reader with a good understanding of the maintenance scheduling problem. No description of maintenance problem elements are given.

10. G. T. Egan, T. S. Dillion, K. Morsztyn, "An Experimental Method of Determination of Optimal Maintenance Schedules in Power Systems Using the Branch-and-Bound Technique," IEEE Trans. on Systems, Man and Cybernetics, Vol. SMC-6, No. 8, pp. 538, 547, 1976.

The problem of maintenance scheduling is formulated as an integer program. The paper very briefly reviews the literature in maintenance scheduling (MS) problem. A number of reliability oriented objective functions are presented (Maximize Minimum Reserve, Minimize the Sum of the Square of Reserves, Minimize the Sum of the Individual probabilities of loss-of-load for such interval, and levelizing the risk). Constraints included in the approach are: (1) maintenance periodicity, (2) manpower (this is a resource) constraint, (3) other resource constraints. Other constraints are also described. A computer program which implements the approach is described and the flow chart is presented. A simple three unit-three interval example is used to illustrate the branch-and-bound approach used to solve the problem.

11. T. S. Dillon, G. T. Egan, "The Application of Combinational Methods to the Problems of Maintenance Scheduling and Unit Commitment in Large Power Systems," Department of Electrical Engineering, Monash University, Australia.

Unit commitment and maintenance scheduling due to their common on and off property are put in the same perspective and integer programming with branch-and-bound method is devised to solve both. Sample examples are presented.

12. H. Khatib, "Maintenance Scheduling of Generating Facilities," IEEE Trans. on PAS, Vol. PAS-98, pp., September/October 1979.

This paper critically reviews the following reliability oriented maintenance scheduling (MS) approaches: Levelizing Reserve (Maximizing

the minimum reserve), Levelizing Risk, and Annual Risk Minimization. It does not provide a good perspective of the problem.

It describes a new reliability oriented approach to solving the problem. The method is based on Interruption Duration Index (I.D.I.). The objective is to levelize (equalize) the I.D.I. measure over the scheduling period. For each time interval energy supplied from "mating" of load duration curve (LDC) and the cumulative capacity table (obtained by convolving probability modes for generating units). This energy is subtracted from the energy requirement (area under LDC) to obtain energy lost. This difference divided by the peak demand for the time interval results the expected hours per time interval the load is lost. This is done for all time intervals of the MS period. Then units are scheduled for maintenance to levelize Interruption Duration. Duration of maintenance, resource and crew constraints are checked as the procedure for levelization is continued. The order of scheduling maintenance is such that units with larger effective size are scheduled first. The computer program based on this method is reported and a sample case study using the computer model is presented.

13. L. F. Escudero, J. W. Horton, and J. E. Scheiderich, "On Maintenance Scheduling for Energy Generators," Paper A 80 011-7, presented at IEEE PES Winter Meeting, New York, New York, February 3-8, 1980.

Maintenance scheduling (MS) problem is formulated as a 0-1 integer programming problem.

The paper gives a fairly good explanation of the constraints involved in the MS problem. Effectively two different objective functions are used to formulate the problem.

- The objective function is to minimize deviations of the MS from the ideal schedule (determined by the preferred maintenance beginning periods of each generator),

- Levelizing the net reserve. This objective could also be attained by maximizing the minimum reserve.

A computer program that performs maintenance scheduling based on the above objective functions described and numerical results are presented. The most powerful applications of the program, according to the authors, are in finding the schedules which levelizes the net reserve.

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APPENDIX D

NUCLEAR REFUELING

Early in the development of commercial nuclear power and its incorporation into electric power systems, it was recognized that the unique characteristics and constraints associated with the management of nuclear fuel would require the development of new methods to plan the operation of nuclear units for overall economic optimization of mixed hydro-nuclear-fossil systems. The economic optimization of power system operation in the past was determined primarily by fossil unit incremental operating costs based upon instantaneous fuel costs and heat rates. Nuclear fuel costs, however, are time dependent, nonlinear functions of the physical and economic parameters of several batches of fuel in residence in the reactor core, with different insertion and removal dates. Thus, it becomes impossible to uniquely specify instantaneous nuclear incremental costs, and it becomes necessary to consider several operating cycles to determine nuclear fuel parameters and their associated costs.

Because nuclear fuel costs have traditionally been less than fossil-fuel costs, early attempts to incorporate nuclear units into existing optimization models simply treated the nuclear units as base-loaded when available, and assumed annual refuelings and equilibrium cycles to effectively decouple nuclear fuel cycle considerations from power system optimization. This allowed the nuclear unit to be treated as a fixed energy "fossil plant" with very low fuel cost. Individual reactor fuel cycle decisions were then suboptimized on the basis of the operating requirements of the particular nuclear unit, i.e., it was possible to regard these units as entities separated from the rest of the systems [1].

The complexity of a nuclear refueling problem will be made clear in this portion of the report. In the next subsection, nuclear fuel cycle is presented. Following this presentation is operational constraints on nuclear unit operation. System implications based on the fuel cycle and

operational constraints requirements follow the discussion. Present utility practices and procedures are presented in a latter subsection. Finally, a description of refueling and maintenance scheduling completes this appendix.

NUCLEAR FUEL CYCLE

In contrast to the rather straightforward economic treatment of fossil fuel costs characterized by a simple purchase-transportation-storage-combustion cycle, the nuclear fuel cycle, as shown in Figure D.1 for a reference light water reactor (LWR) system, is quite complex.

The nominal lead and lag times for a reload batch of fuel are approximately as follows (prior to (-), or after (+) fuel irradiation):

<u>Fuel Cycle Step</u>	<u>Time (Months)</u>
Delivery of uranium concentrations	- 4.0
Conversion of U_3O_8 to UF_6	- 4.0
Enrichment	- 3.0
Fabrication	- 1.5
Irradiation Cycle	~ 12.0
Shipping of spent fuel	+ 6.0
Reprocessing	+ 7.0
Conversion of UNH_3 to UF_6	+ 7.0

Utilities are also required by NRC to contract for enrichment services eight to ten years before actual delivery. Typical irradiation times for one batch of fuel range from several months to a few years, depending upon the particular fuel management scheme chosen. As a result of these time considerations, the economic feasibility and desirability of making significant changes in fuel design and operation must be carefully evaluated, taking into account both contractual obligations with various suppliers and the economics of overall system operation through several subsequent cycles.

Figure D.1. Fuel Cycle Flow Diagram.

The cost, FC, of nuclear fuel for a single reactor over N cycles is a nonlinear, non-separable function of the energy produced during each irradiation cycle:

$$FC = FC(E_1, E_2, \dots E_N) \quad (1)$$

The energy produced during cycle c is given by $E_c = P \cdot L_c \cdot T_c$ where P is the rated power, L_c is the capacity factor, and T is the length of the cycle. E_c is uniquely determined by the state of the fuel in the reactor at the beginning of cycle c, and by the nuclear fuel management decision variables - batch fractions, f, and enrichment, ϵ .

Thus:

$$E_c = F[x(c), f_c, \epsilon_c] \quad (D.2)$$

Here, $x(c)$, the state of the reactor, is defined as the spatial distribution of all uranium and plutonium isotopes and fission products. The nonlinearity of (D.1) arises from the fact that while the cycle energy is approximately linear as a function of enrichment, the cost of the enrichment is nonlinear. The non-separability is due to the batch loading of reactor fuel, i.e., at each refueling interval, only a fraction of the fuel is removed. Thus, a typical core of nuclear fuel during an operating cycle will consist of several distinct batches of fuel with differing enrichments, residence times and remaining reactivities (energy potential). The specification of fuel enrichment for a single lot of fuel therefore requires energy determinations for several operating cycles, taking into account the interactions between different batches of fuel.

OPERATIONAL CONSTRAINTS

The general constraints associated with nuclear energy production relate primarily to the management of the fuel within the reactor, called "in-core" fuel management. Typical in-core fuel management decision variables include [2]:

- (1) The number and location of irradiated fuel assemblies to be replaced - (fraction of batch);

- (2) The pattern in which the remaining fuel assemblies are to be shuffled to new locations to maximize efficient and economic use of existing assemblies;
- (3) Optimal enrichment for each reload assembly (ϵ).

The methods for varying the fuel energy potential for a particular cycle include:

- changes in batch size (number of assemblies removed),
- changes in enrichment,
- power cost down (increased energy potential results from operating the unit at less than rated power), and
- early refueling.

In general, there are multiple combinations of enrichment and batch fraction which will produce a desired cycle energy. These combinations, however, are limited by engineering constraints placed on reactor operation, and the least cost combination must be chosen from this constrained set. Typically, the physics and engineering constraints associated with reactor operation include steady state and transient limits for fuel operating and accident limits [3]. Most optimization models [1] consider only the physical constraints related to maximum power peaking in a fuel rod or assembly, and maximum fuel burnup arising from material limitations for maintaining fuel assembly integrity. The modeling of these physical constraints within the optimization of the two fuel management variables of interest - enrichment and batch fraction - poses a severe problem. The difficulty is to strike the proper balance between accuracy and computational speed.

SYSTEM IMPLICATIONS

Although nuclear fuel and generation characteristics are quite different from those of fossil fuel units, the operation of nuclear units on many

utility systems has been similar to that of fixed-energy fossil units. The relative cost differential between fossil and nuclear units dictated that the nuclear units be operated base loaded when available. Refueling of these units carried out annually, either in the fall or spring months, when system demand was lowest. As the amount of nuclear capacity on a utility system increases [3], however, the operational modes of the nuclear plant must change. If, for example, the amount of nuclear capacity exceeds minimum system demand [2] the nuclear units can no longer operate entirely in a base load mode. The question of inter-nuclear incremental fuel costs now has to be addressed to determine which nuclear units should be base loaded. Another difficulty is the problem of scheduling refueling outages as the number of nuclear units increases. Since, for reliability considerations [4], two units at the same site should not be refueled simultaneously or successively, and large generating units should not be taken out of service during peak demand periods, it is no longer possible to maintain annual refueling for all reactors. The net result is that cycle energies and cycle lengths can no longer be easily specified. These considerations further complicate nuclear fuel cycle economics, and create a need for utility optimization models which can effectively couple nuclear fuel management decisions with overall system operating decisions.

UTILITY PROCEDURES AND PRACTICES

The material presented in this section is mainly from Refs.[1] and [5].

The planning needs and practices of nuclear utilities vary widely in sophistication, depending upon the amount of nuclear capacity installed or on order, overall system size, in-house expertise, and management policy. The information presented here is not intended to be an all-inclusive or comprehensive documentation of nuclear utility planning practices. Rather, the intent is to describe the wide range of approaches taken by some utilities in planning the integration of nuclear plants into their system.

The three principal activities in a utility related to nuclear production scheduling are: nuclear fuel purchasing and associated services

(for example: fabrication, enrichment, conversion); core analysis and nuclear fuel management; and system planning. A detailed description of a large utility's in-core and out-of-core fuel management program is contained in Ref. [5] which provides considerable insight into the scope and complexity of the utility fuel management function.

The first activity, often termed "out-of-core" fuel management, includes contracting for all services and purchases relating to steps along the nuclear fuel cycle before and after irradiation. Because of the hard constraints associated with many of the lead times for nuclear fuel processing services, it is important to closely coordinate this activity with in-core management and the operation of the system. New contracts, for enrichment services, must now be firmed up eight to ten years in advance, and must specify year-by-year enrichment requirements (i.e., specify cycle energy requirements eight to ten years in advance). This places increased emphasis on the need for long-range, integrated planning. In one case, an individual utility's experience in probabilistic planning methods [5] led to the formation of a utility-managed and operated Separative Work Administration Pool (SWAP) as a means of reducing the risk of enrichment orders and enhancing the flexibility of member utilities in meeting changing power system needs. Many utilities have also made use of mathematical programming techniques [7] to determine uranium requirements and optimal contract bids. Increasing attention is being paid to the need for a more integrated approach to out-of-core fuel management planning, especially the coordination of these services with other system requirements so as to increase utility flexibility when faced with unexpected departures from scheduled operation.

The second important utility activity closely related to nuclear production scheduling is concerned with the analysis and management of fuel in the reactor. The extent to which an individual utility performs this type of analysis in-house depends on one or more of the factors mentioned at the beginning of this section. In many cases, reactor manufacturers perform this function because of the nature of the fuel warranties. Some utilities with a large nuclear commitment prefer to handle most of the fuel

management and core analysis functions often in conjunction with a fuel management consulting firm in order to take advantage of flexibility in fuel contracts or changes in system operation. The core analysis function is concerned with accurately tracking and predicting the performance of the fuel in the reactor. Detailed two- and three-dimensional, multi-neutron group computer codes are used because of the need to accurately simulate spatial effects relating to neutron flux distributions and the presence of control poisons (neutron absorbers) in the core. The fuel management function, however, is a broader activity concerned with the economic evaluation of alternatives for minimizing the fuel cycle cost of each reactor over several cycles while meeting individual reactor and system requirements. Examples of typical fuel management options examined include early refueling, cycle stretchout, derating the unit because of regulatory requirements, and changing the enrichment or fraction of fuel replaced for a particular cycle. The nuclear simulator programs used in this kind of analysis are not as detailed as those used in core analysis. Typically, a utility first performs several multi-cycle fuel management evaluations, and from this "scoping" analysis chooses two or three fuel management strategies for a more detailed analysis.

In most utilities, the fuel management function is separated from the system planning and production scheduling groups. However, most utilities recognize the need for close coordination between these two functions. The evaluation of early refueling, for example, must first determine the economic penalty of underutilization of the energy potential in the core. This must then be compared with any system benefits resulting from early refueling, such as enhancing system reliability during a peak load period. The evaluation must also be carried out over several operating cycles because the state of the core at the end-of-cycle is changed. Close coordination is required between the fuel management and system planning departments. No unified methods are currently in use by utilities to automate this procedure in order to evaluate all alternatives on a consistent basis.

The third activity related to nuclear production scheduling is that of operations planning. The exact nature of this function varies widely

from utility to utility, but usually involves the scheduling of nuclear refueling and determination of cycle energy requirements and nuclear capacity factors. In some instances, long-term planning for capacity additions is also performed. Once the nuclear cycle requirements are established, based upon simulation of the power system over a several year period, planning and optimization of the nuclear fuel cycle can be performed. This may be an iterative activity, especially where several reactors and common fuel cycles are involved.

One of the most important and increasingly difficult functions to perform is the determination of nuclear refueling dates when there are several reactors on the system. Under these circumstances, it is usually not possible to fuel all reactors on an annual basis within system and area reliability constraints. In many cases, because the nuclear units represent the largest capacity blocks on the system, refueling schedules are still calculated manually based on estimated capacity factors and rules of thumb regarding margins and planned outages. Although several models [4, 8, 9] have been developed to automate maintenance and refueling schedules, most utilities still plan their refueling outages in a fairly crude manner. This is partly because the early operating history of nuclear reactors has been replete with lengthy and unplanned outages, thus reducing the need for precise refueling programs.

REFUELING PROBLEM

The decoupling of nuclear refueling scheduling from the other modules of the mid-range problem is an acceptable approximation for present power systems. This is because most utilities have only a few nuclear units on-line, and the many constraints on refueling schedules limit the number of feasible refueling strategies. Although improved and more rapid refueling technologies have recently become available [10], present nuclear units must be taken off-line for a significant period of time for refueling. Therefore, most utilities plan long fuel cycles (> 1 year) to take advantage

of the low operating cost of nuclear units as compared to fossil plants [11]. The advantage of longer fuel cycles is offset, however, by the increased enrichment required, whose cost increase nonlinearly with increasing fuel energy potential. It is, therefore, necessary to examine the many nuclear cycle lengths possible. A few automated techniques [4, 9, 11] have been developed to generate feasible nuclear refueling schedules.

From the earlier discussion, it is obvious that the nuclear refueling problem is a complex one. For an optimal nuclear refueling solution, one needs to include in-core management effects, fuel contracts, etc. For the purpose of this research, it was assumed that the nominal refueling dates are already set and a nuclear unit can be represented similar fossil fuel units, i.e., when available for service, they have constant heat rates (H Btu/kWh) and fuel cost is constant (F \$/MBtu) also. Windows were allowed to move refueling dates back and forth. For this kind of representation of nuclear units, nuclear refueling is not much different from maintenance scheduling. Therefore, in this research, we will concentrate on the maintenance scheduling problem. With slight modifications, the results should be applicable to the refueling problem.

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