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ADIABATIC COMPRESSION OF ELONGATED FIELD-REVERSED CONFIGURATIONS*

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I. INTRODUCTION

The simplest model of plasma dynamics is the adiabatic model. In this model the plasma is assumed to be in MHD equilibrium at each instant of time. The equilibria are connected by the requirement that they all have the same entropy per unit flux, i.e., the equilibria form a sequence generated by adiabatic changes. The standard way of computing such a sequence of equilibria was developed by Grad¹, but its practical use requires a fairly complicated code. It would be helpful if approximately the same results could be gotten either with a much simpler code or by analytical techniques. In Sec. II a one-dimensional equilibrium code is described and its results are checked against a two-dimensional equilibrium code; in Sec. III an even simpler analytic calculation is presented.

II. ONE-DIMENSIONAL ADIABATIC MODEL

Elongated FRC equilibria can very nearly be described as two regions with straight field lines connected by a short transition region with curved field lines (see Fig. 1). This property makes it possible to extract two-dimensional information from one-dimensional calculations. It will be exploited here to obtain an approximate model for adiabatically changing FRC equilibria.

Consider an elongated FRC in a conducting cylinder of radius r_w , as shown in Fig. 1. It has magnetic flux $2\pi\psi_0$ outside of the separatrix and magnetic flux $2\pi\psi_t$ inside the separatrix. It has pressure profile $p(\psi)$, where ψ is the poloidal flux function, and separatrix length l . We restrict our discussion to the case where the pressure vanishes on and outside the separatrix; the pressure is also assumed to rise monotonically from the separatrix to its maximum value, p_m , at the equilibrium vortex point. Imagine now that an initial equilibrium is adiabatically changed by slowly varying either r_w or ψ_0 . Note that reversibility of adiabatic changes makes it possible to consider these two different kinds of changes separately. Under adiabatic changes ψ_t is conserved and the magnetofluid is tied to the field lines. Hence, the condition that entropy per unit flux be conserved reduces to the condition $\mu_f(\psi) = \mu_i(\psi)$ where the subscripts "i" and "f" refer to equilibria before and after the adiabatic change, respectively, and where

$$\mu(\psi) = p(\psi) \left(2\pi\phi \frac{d\psi}{d\psi} \right)^\gamma \quad (1)$$

where γ is the adiabatic exponent. We now make the one-dimensional approximation by writing

$$\oint \frac{d\psi}{B} \approx \frac{2l}{B} \approx \frac{2l}{[2(p_m - p(\psi))]^{1/2}} \quad (2)$$

This approximation eliminates the transition region where the field lines are

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curved and represents the magnetofluid by a cylinder of straight field lines of length ℓ . In general this is a terrible approximation, but for highly elongated FRCs it is not too bad.

If the pressure profile is written in the form $p(\psi) = p_m \beta(\phi)$, where $\phi = \psi/\psi_t$, then entropy conservation becomes a condition relating initial and final β profiles.

$$\frac{\beta_f(\phi)}{[1-\beta_f(\phi)]^{\gamma/2}} = A \frac{\beta_i(\phi)}{[1-\beta_i(\phi)]^{\gamma/2}} \quad (3)$$

where $A = (\ell_i/\ell_f)^\gamma (p_{mi}/p_{mf})^{1-\gamma/2}$. In the case of wall compression, $A = 1$ and the pressure profile does not change shape. The variation of ℓ during wall compression in this approximation is given in Table I. In the case of flux compression $A \neq 1$ and a one-dimensional equilibrium code is needed to vary A until a new equilibrium is found that satisfies radial pressure balance, the average-beta condition, and trapped flux conservation. A modification of the code described in the appendix of Ref. 2 was used here.

To test the accuracy of the one-dimensional approximation, the pressure profiles and separatrix lengths from the one-dimensional code were used in Hewett's two-dimensional FRC equilibrium code.² The correct entropy functions were then computed on the two-dimensional mesh and compared with each other to see if entropy was indeed conserved. We found that if the equilibria had $\ell/r_s > 10$, the one-dimensional results were accurate to about 10%. These requirements on the elongation are satisfied by most experiments.

III. ANALYTICAL MODEL

We consider an elongated FRC equilibrium inside a straight cylindrical flux conserver, as shown in Fig. 1. As in Sec. II, the separatrix is modeled as a cylinder of length ℓ and radius r_s and we consider the adiabatic compression of the FRC by changes in r_w and ψ_0 (e.g. wall and flux compressions, respectively). The energy balance within the separatrix can be written as $dE = dW$, where $E = \int (p/(\gamma-1) + B^2/2\mu_0) dV$ is the total energy within the separatrix volume V and where dW is the work done on the separatrix by the external magnetic field pressure. With the above assumptions and using the average beta condition, we obtain $E = p_m V [1 - (2-\gamma)x_s^2/2]/(\gamma-1)$ and, therefore,

$$dE = E \left[\frac{2dr_w}{r_w} + 2 \frac{dx_s}{x_s} + \frac{d\ell}{\ell} + \frac{dp_m}{p_m} - \frac{x_s dx_s (2-\gamma)}{1 - (2-\gamma)x_s^2/2} \right] \quad (4)$$

To obtain dW , we write $dW = F_r dr_s + F_z d\ell$, where F_r and F_z are the radial and axial forces exerted on the separatrix. By neglecting the curved field line region near $z = \ell/2$, as shown in Fig. 1, we obtain $F_r = -2\pi r_s p_m$. By using the control surfaces S_{1-5} indicated in Fig. 1, and by expressing the forces on each surface by means of the Maxwell stress tensor, the axial equilibrium relation reduces to $F_z = -F_{z1} - F_{z2}$. With $F_{z1} = p_m \pi (r_w^2 - r_s^2)$ and $F_{z2} = -p_m \pi r_w^2 (1 - x_s^2)^2$, we obtain $F_z = -p_m \pi r_s^2 (1 - x_s^2)$. Combining these expressions for F_r and F_z with the definition of dW yields

$$dW = -p_m V \left[2 \frac{dr_w}{r_w} + 2 \frac{dx_s}{x_s} + (1 - x_s^2) \frac{d\ell}{\ell} \right] \quad (5)$$

The trapped magnetic flux inside the separatrix can be written as $2\pi\psi_t = \pi r_w^2 B_w x_s^3/24$ with $t = x_s/\int_0^1 (1-\beta)^{1/2} du$, where $u = 2r^2/r_s^2 - 1$. We assume

that f is only a function of x_s , or equivalently, that the change in pressure profile $\beta(u)$ only comes from varying x_s . Then, the relations $\psi_t = \text{constant}$ and $p_m \propto B_w^2$ can be used to obtain

$$\frac{d}{p_m} = 2(3-\epsilon) \frac{dx_s}{x_s} - 4 \frac{dr_w}{r_w} \quad (6)$$

where $\epsilon = (x_s/r) (df/dx_s)$. The relation $dE = dW$ with Eqs. (4)-(6) gives

$$\frac{d\ell}{\ell} = \frac{(4-2\gamma)}{\gamma} \frac{dr_w}{r_w} + \frac{2(3-\gamma-\epsilon)}{\gamma} \frac{dx_s}{x_s} - \frac{(1+\epsilon-\gamma\epsilon)}{\gamma} \frac{d\langle\beta\rangle}{\langle\beta\rangle} \quad (7)$$

where $\langle\beta\rangle = 1-x_s^2/2$.

For wall compression, x_s is constant and integrating Eq. (7) with $\gamma = 5/3$ gives the ℓ scaling in Table I. Assuming that the plasma is isothermal within the separatrix, using the constancy of the particle inventory $N = \langle\beta\rangle n_m \pi r_s^2 \ell$ with $p_m = n_m T_m$, and using radial pressure balance and magnetic flux conservation, we obtain the rest of Table I. These scaling laws are independent of the pressure profile, as was also obtained in Sec. II.

For flux compression, r_w is constant but Eq. (7) cannot be readily integrated unless ϵ is a constant as x_s varies. It has been shown³ that two limiting cases of elongated FRC equilibria are given by sharp-boundary pressure profiles that contain the largest and smallest amount of ψ_t . These are the high-flux sharp-boundary and low-flux sharp-boundary models. For these, the values of ϵ are 0 and -1, respectively, and Eq. (7) can be integrated to recover known results³ for $\gamma = 5/3$. For arbitrary diffuse pressure profiles, it can be shown that $\sqrt{2} \leq f \leq 2/x_s$, where the upper and lower limits correspond to the low flux and high flux sharp-boundary models, respectively. From this inequality, we obtain $-1 \leq \epsilon \leq 0$, which indicates that the scaling laws of flux compression for diffuse profiles are bounded by the two sharp-boundary models. The numerical results of Sec. II show that ϵ is approximately a constant as x_s varies for a given initial $\beta(\phi)$. Even with different initial β profiles, we find that ϵ nearly always lies between -0.2 and -0.3. Therefore, most diffuse profiles scale in flux compression in a way similar to the high-flux sharp-boundary profile for which $\epsilon = 0$. We approximate ϵ by the constant value -0.25 for diffuse profiles and neglect the small variations of less than 10% in the coefficients of Eq. (7) due to departures of ϵ from this value. This approximation is well justified within the one-dimensional model of this work. Then integrating Eq. (7) with the relations $N = \text{const.}$, $\psi_t = \text{const.}$, $p_m \propto B_w^2$, $V \propto x_s^2 \ell$, and $\gamma = 5/3$ we obtain Table II. Figure 2 shows the comparison between the analytic scaling for ℓ and that obtained by the calculation in Sec. II in the case of a typical diffuse profile. The adiabatic compression of diffuse profile FRCs can thus be quite accurately described by simple formulae.

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Figure 1: A field-reversed configuration inside a cylindrical flux conserver.

Figure 2: FRC length as a function of x_s for flux compression. (a) High flux sharp-boundary. (b) Low flux sharp-boundary. (c) One-dimensional compression code. (d) Typical diffuse profile formula from Table II.

TABLE I

Adiabatic Wall Compression Scaling Laws for $\gamma = 5/3$

r	T_e	n_e	B_e
$r_w^{-2/5}$	$r_w^{-8/5}$	$r_w^{-12/5}$	r_w^{-2}

TABLE II

Adiabatic Flux Compression Scaling Laws for $\gamma = 5/3$

	High-flux Sharp Boundary	Typical Diffuse Profile	Low-flux Sharp Boundary
r	$x_s^{-8/5} \langle R \rangle^{-3/5}$	$x_s^{-19/10} \langle R \rangle^{-7/10}$	$x_s^{-14/5} \langle R \rangle^{-1}$
T_e	$x_s^{-12/5} \langle R \rangle^{-2/5}$	$x_s^{-13/2} \langle R \rangle^{-3/10}$	$x_s^{-16/5}$
n_e	$x_s^{-18/5} \langle R \rangle^{-2/5}$	$x_s^{-39/10} \langle R \rangle^{-3/10}$	$x_s^{-24/5}$
B_e	x_s^{-3}	$x_s^{-13/4}$	x_s^{-4}