DEVELOPMENT OF AN AEROSOL SIZE CLASSIFIER

Progress Report

for Period May 31, 1977 - January 31, 1978

L. J. Forney
Department of Civil Engineering

and

D. G. Ravenhall
Department of Physics

University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

January 1978

Prepared for

DEPARTMENT OF ENERGY
UNDER CONTRACT NO. EE-77-S-02-4319

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

DEVELOPMENT OF AN AEROSOL SIZE CLASSIFIER

The purpose of this research is to characterize the performance of rectangular slit virtual impactors. A mathematical description of two-dimensional jets of ideal fluid impinging upon a void has been developed using conformal mapping techniques. The analysis includes fluid deflecting plates of finite length inclined at arbitrary angles to the incident jet. Particle trajectories were simulated for various theoretically admissible instrument geometries. Salient features of the particle sizing effectiveness of such devices were determined for a range of void space openings $h/w > 1$ combined with jet to plate spacing s/w ranging from 0 to 2.

Visual observations of two-dimensional jets impinging upon a void were conducted with a scaled-up water model. It was found that excess fluid flux into the void is minimized for a range of jet Reynolds numbers of $750 < Re < 1500$. At smaller values of Re wall boundary layers interfere while at larger Re jet core instabilities develop. Modifications were made to an existing prototype virtual impactor to eliminate internal wall losses. Experiments are being performed using mono-dispersed $2.5 \mu m$ DOP particles to determine the magnitude of the internal losses and to fully characterize the instrument response.

Forney, L. J.
Ravenhall, D. G.

DEVELOPMENT OF AN AEROSOL SIZE CLASSIFIER

Progress report to Department of Energy under contract No. EE-77-S-02-4319 for period May 31, 1977 to January 31, 1978.

KEYWORDS: Aerosols / particle size classifier / particle impactor / air pollution.

TABLE OF CONTENTS

Section	Page
1. INTRODUCTION	1
2. THEORY	3
2.1 Ideal Fluid Jet Impinging Upon a Void	3
2.2 Particle Trajectories	4
2.3 Improved Calculations with Solid Plate Impactors	4
3. MODEL STUDIES	5
3.1 Scaled-Up Water Model	5
3.2 Fluid Instabilities	5
4. PROTOTYPE TESTS	15
4.1 Flow System Modifications	15
4.2 Impactor Modifications	16
4.3 Preliminary Results	16
5. PROJECT MANAGEMENT	20
5.1 Project Compliance	20
5.2 Personnel Contributions	20
5.2 Publications	20
REFERENCES	22
APPENDIX A Aerosol Impactors: A study of a Fluid Jet Impinging Upon a Void	
APPENDIX B Aerosol Sizing with a Slotted Virtual Impactor	
APPENDIX C Particle Trajectories in a Two-Dimensional Jet of Ideal Fluid Impinging Upon a Plate	

LIST OF FIGURES

Figure	Page
1. Scaled-up Water Model	6
2. Photograph of Flow Field Illustrating Jet Core Instabilities for $s/w = 3.0$ and $h/w = 2.8$	9
3. Jet Length ℓ/w to Onset of Instability vs. Jet Reynolds Number Re . Solid line is $\ell/w = 1500/Re$	10
4. Percent of Ink Tracer Lost to Void vs. Jet Reynolds Number. Initial Tracer Position is $w/5$ from Impactor Wall	11
5. Schematic of Aerosol Flow System	12
6. Flowmeter Wiring Schematic	13
7. Flow Adapter for Use with Optical Particle Counter	14
8. Side View of Prototype Device	18
9. Device Efficiency and Percent Loss vs. Particle Stokes Number	19

1. INTRODUCTION

Fine suspended atmospheric particles represent a significant and potentially serious form of air pollution because of their size. Small airborne particles, defined here as solid or liquid aerosols less than three microns ($<3\mu\text{m}$) in diameter, are typically formed by a spectrum of physical and chemical processes such as high temperature metallurgical operations or fossil fuel combustion. These processes produce metallic fumes, inert ash, and solid or liquid hydrocarbons. In addition, a large mass fraction of small particulate matter consisting of potentially toxic substances (both soluble and insoluble) is formed by atmospheric photochemical gaseous reactions involving nitric and sulfur oxides in combination with water vapor. These fine particles because of their size remain suspended longer, readily scatter light causing haze and reduced visibility and penetrate the natural defense mechanisms of the human respiratory system leading to retention within the deeper recesses of the lungs.

Until recently, routine methods of sampling airborne particulate matter involved either the use of a Hi-Volume Sampler over a 24 hour period for compliance with ambient air quality standards or a multi-stage cascade impactor in specific studies to determine the particulate mass-size distribution and chemical composition. While the Hi-Volume sampler yields the total particulate mass concentration on a filter and with further analysis the total chemical composition, it fails to distinguish between particles of different sizes, i.e. respirable versus nonrespirable. Likewise, use of the cascade impactor provides the necessary mass-size distribution but involves the following problems:

particles bounce off both treated and untreated collection surfaces resulting in erroneous mass loadings, multiple heavy stages yield lower particulate mass loadings per stage resulting in reduced sensitivity, and gaseous pollutants and vapors react and adsorb on the surface while volatile particulate evaporate. In addition, the fixed geometry of the cascade impactor makes it difficult to calibrate and susceptible to large internal wall losses.

Recently, attempts have been made to size-classify or at least to fractionate suspended particles into respirable and nonrespirable sizes with what are called virtual impactors. These single stage virtual impactors direct particle laden air at a collection surface in a manner similar to one stage of a cascade device. In both cases particles either impact on the collection surface or escape depending on their aerodynamic diameters. The virtual impactor, however, replaces the solid collection surface such as that which occurs in a cascade device with a virtual surface or air void. Thus, large particles impact into a stagnate air void and particle-surface-interaction problems are eliminated.

The purpose of this research is to characterize the performance of the virtual impactor. In particular, the properties of a rectangular slit virtual impactor are being investigated both theoretically and experimentally. In Sec. 2 the work on a mathematical description of the flow field is outlined, Sec. 3 indicates the scope and preliminary results of water model studies while Sec. 4 describes prototype developments. In Sec. 5 the project management including personnel contributions and publications are listed.

2. Theory

Necessary to the understanding of aerosol impactors is a complete mathematical description of the fluid flow field. Having such a description allows one to simulate particle trajectories and to predict the device's particle sizing effectiveness. The plane two-dimensional flow field with large jet Reynolds numbers (ideal fluid) such as that occurring in the present device illustrated in Sec. 4 suggests the use of conformal mapping techniques. Thus, conformal mapping was used to characterize the flow patterns in the present study.

2.1 Ideal Fluid Jet Impinging Upon a Void

Following the preliminary work outlined by Forney (1977), a complete theoretical analysis of a two-dimensional jet of ideal fluid impinging normally upon a void has been made and compared with experimental observations (see Appendix A). Equations describing the streamline positions and fluid velocities were derived and compared with experimental observations on a scaled-up water model. Limitations to the theory were demonstrated and these results were successfully correlated with the onset of changes in the observed flow patterns. In general, it was found that for a given jet spacing s/w there exists a maximum void space $h/w = h_{lim}/w$ beyond which the jet detaches from the fluid deflecting plates. Presumably, the prototype impactor could be operated at larger void space ratios h/w than that allowed by the theoretical analysis; however, one could not predict the properties of these flow fields.

2.2 Particle Trajectories

The theoretical analysis of Forney, Ravenhall and Winn (1978), discussed in Sec. 2.1, was extended to include fluid deflecting plates of finite length inclined at arbitrary angles to the incident jet (see Appendix B). Particle trajectories were simulated for various virtual impactor geometries to determine the potential effectiveness of the device for size-classifying aerosols. Features such as particle collection efficiencies and loss factors are presented for theoretically admissible geometries.

It was found that particle losses were reduced for geometries with larger void widths h/w and that the slope of the efficiency curve may be larger for unconventional deflecting plate angles $<90^\circ$. The latter result is particularly useful since those impactors whose efficiency curves have large slopes represent instruments with superior capabilities for particle size discrimination. More work needs to be done, however, with scaled-up water models to determine the character of flows with fluid deflecting plate angles $<90^\circ$.

2.3 Improved Calculations with Solid Plate Impactors

The computational techniques developed for use with virtual impactors were applied to the simpler solid plate impactor geometry (see Appendix C). It was found that the original calculations of Davies and Aylward (1951) were somewhat in error for deflecting plate angles $\beta = 90^\circ$ and that an unconventional plate angle $\beta \neq 90^\circ$ maximizes the slope of the particle efficiency curves for a given jet-to-plate spacing s/w .

3. Model Studies

3.1 Scaled-Up Water Model

Features of the impactor flow field were observed and recorded using a 10:1 scaled-up Plexiglas water model as illustrated in Fig. 1. The model was immersed and anchored in a 25 gal water tank with the outlet connected directly to a centrifugal pump. Dye was injected into the throat of the model from tygon tubing connecting three stainless steel injector nozzles to a cylindrical reservoir suspended above the tank. Tubing clamps regulated the flow of washable, water-diluted, black India ink into the three injection lines positioned at the model throat in a plane midway between the top and bottom model surfaces.

3.2 Fluid Instabilities

Several trial throat geometries were constructed and tested. It was found that unstable flow would result in the model throat for all operating conditions unless several precautions were taken. A stable flow field was achieved by using a converging throat geometry and a honeycomb flow straightener placed at the throat entrance for the geometry of Fig. 1. Initial attempts to achieve a stable flow field for a symmetric two-dimensional jet impinging upon a void were unsuccessful. Vortices consistently developed along streamlines within the impaction region and this contributed to an intermittent fluid flux into the void for the symmetric case. As a result, it was necessary to operate with one half the flow field as represented in Fig. 1 with the indicated wall along the plane of symmetry on the left of the jet throat in the schematic. Fortunately, this is the same geometry used in the prototype device.

Figure 1. Scaled-up Water Model.

Because excessive fluid flux into the void volume prohibits effective particle sizing with a virtual impactor, a systematic study was conducted on the causes of fluid loss. Two problem areas were identified and their effects were observed with the water model. The first is the existence of a fluid boundary layer along the walls and the second is the onset of fluid instabilities in the jet core at a sufficient distance from the throat. A photo of the latter problem is illustrated in Fig. 2.

Considering the problem of instabilities within the jet core such as that shown in Fig. 2, measurements were made of the length of the streamline ℓ down the jet axis from the throat to the point at which the instabilities developed. The results are indicated in Fig. 3 where the normalized distance ℓ/w is plotted for $0 < Re < 3000$ where $Re (= Q/\nu L)$ is the jet Reynolds number, Q is the water flow rate, ν is the kinematic viscosity and L is the depth of the model perpendicular to the illustration of Fig. 1. From dimensional arguments it can be shown that $\ell/w \propto 1/Re$ while neglecting small changes in device geometry. It is shown in Fig. 3 that provided $h/w \approx \ell/w$, $\ell/w \approx 1500/Re$. This immediately places bounds on the Reynold number of $750 < Re < 1500$ for typical values of the void width $1.0 < h/w < 2.0$.

Considering the problem presented by the wall boundary layer, visual observations were made of the percentage loss of an ink tracer positioned at a distance of $0.2 w$ from the left wall of the impactor throat of Fig. 1. These results are plotted in Fig. 4. For small jet Reynold numbers (large boundary layers) a large loss of tracer fluid to the void volume was observed. To quantitatively predict the onset of

fluid loss from boundary layer effects for the conditions of Fig. 4 we note that the boundary layer thickness δ on the left wall of the jet throat of Fig. 1 is given by

$$\frac{\delta}{w} = \frac{5.48}{\sqrt{Re_x}} \quad (1)$$

where x is the throat length and $Re_x = Re(x/w)$ or

$$\frac{\delta}{w} = 5.48 \frac{\sqrt{x/w}}{\sqrt{Re}} \quad (2)$$

where $x/w \simeq 4.0$ for the water model. We now assume that if the fluid has insufficient momentum to follow the jet around the corner, it flows into the void. Thus, choosing $1/2 \delta = 0.2 w$ such that the velocity of the ink tracer is reduced by 20% from its value in the absence of the boundary layer and substituting into Eq. (2) one finds $Re \simeq 900$ for the onset of ink flow into the void. This is represented by the vertical dashed line on the left in Fig. 4. An increase in tracer flux into the void at $Re > 1500$ is also observed in Fig. 4 as a result of jet core oscillations of the type shown in Fig. 2.

Fig. 2. Photograph of Flow Field Illustrating Jet Core Instabilities for $s/w = 3.0$ and $h/w = 2.8$

Figure 3. Jet Length l/w to Onset of Instability vs. Jet Reynolds Number Re . Solid line is $l/w = 1500/Re$.

Figure 4. Percent of Ink Tracer Lost to Void vs. Jet Reynolds Number. Initial Tracer Position is $w/5$ from Impactor Wall.

Figure 5. Schematic of Aerosol Flow System.

Figure 6. Flowmeter Wiring Schematic.

Figure 7. Flow Adapter for Use with Optical Particle Counter.

4. Prototype Tests

4.1 Flow System Modifications

The test section or aerosol holding chamber indicated in Fig. 5 was enlarged to a volume of 1000 liters to stabilize the aerosol concentration. This provided an aerosol residence time of 8 to 10 min for a typical air flowrate of 100 l/min. A flexible rubber glove was also mounted into the side wall of the holding chamber to allow instrument adjustments within the closed tank.

The three rotameters illustrated in Fig. 5 used to regulate the air flow at the two impactor exhausts and an ambient filter sampler respectively were replaced with three Hastings mass flowmeters accurate to 2%. The 0-5 volts DC output from each of the three transducers was connected to a three pole switch and a 0-5 volt DC, 5000 Ω meter as illustrated in the wiring schematic of Fig. 6. The installation of the transducers provides a much more accurate measure of the various air flowrates used to calibrate the impactor.

A Royco Model 245 optical particle counter with a model 507 plug-in digital display was purchased and rack mounted. The counter sampling flowrate of 28.5 l/min exactly matched the impactor flowrate for a typical impactor jet Reynolds number of 1540. This eliminated the need for splitting the aerosol flow at the impactor exhaust. In addition, the remote sensor which is a standard feature of the Royco instrument eliminated long sampling lines and the potential for particle losses. To accommodate the optical particle counter for the purpose of measuring particle efficiency curves, a cone shaped flow adapter was constructed as shown in Fig. 7.

4.2 Impactor Modifications

A number of important modifications were made to the original impactor design to eliminate internal particle losses. The first was to construct two brass filter holders (47 mm diameter) which constituted part of the integral device structure. This eliminated constrictions at both exhausts and their potential for particle losses. These modifications are illustrated in the instrument schematic, Fig. 8. An additional feature which resulted from this modification was the reduction in the size of the void volume below the particle impaction region. This latter change reduces particle residence time in the void volume and decreases particle losses to internal surfaces. In addition, a piece of brass stock was machined and attached to the lower surface of the horizontal adjustable plate making it aerodynamically smooth compared to the previous contour.

Since particles in the original prototype fractionator tended to impact on the inside front face (left wall) of jet throat in Fig. 8) at large particle Stokes numbers and because it would be advantageous to increase the air flow in the void volume, an excess air brass inlet channel was constructed. This provided a sheath of clean air flowing down the inside front face of the void volume and allows one to reduce the bleed flow while maintaining large air flows within the void volume. More experimentation will be done to characterize the effect of this feature on the instrument response.

4.3 Preliminary Results

A typical efficiency curve and loss spectrum were determined with the prototype device. These are shown in Fig. 9. The particles were 2.5 μm

DOP oil drops and the total sampling rate was 30 μ /min. Of the total flow, 5% (1.5 μ /min) was bled into the void volume from the particle laden airstream with no flow into the excess air inlet. Work is currently in progress to fully characterize the instrument response for various void widths and air bleedflows and to determine the influence of the magnitude of the excess air flow on particle losses at large Stokes numbers.

Figure 8. Side View of Prototype Device.

Figure 9. Device Efficiency and Percent Loss vs. Particle Stokes Number.

5. Project Managment

5.1 Project Compliance

The original proposal specified that a portion of the project would consist of field tests conducted by Dr. R. L. Solomon with the prototype device and a commercially available virtual impactor. However, Dr. Solomon left the University shortly after the initiation of the project. At approximately the same time it became apparent that a theoretical approach under development by Professors D. G. Ravenhall and L. J. Forney would provide valuable information concerning the performance of virtual impactors. Thus, the latter effort was substituted for the proposed contribution of Dr. Solomon. Funds to support the numerical work of Dr. Ravenhall, however, were provided by NSF grant PHY75-21590 of the Department of Physics.

5.2 Personnel Contributions

<u>Name</u>	<u>Level</u>	<u>Period</u>
Dr. L. J. Forney	1/2 time	6/1/77-present
Dr. D. G. Ravenhall	1/2 time	6/1/77-12/30/77
Mr. S. S. Lee	2/3 time	8/1/77-present
Mr. W. M. Langlois	1/4 time	8/1/77-present
Mr. M. Jazayeri	NSF Fellowship	6/15/77-8/15/77

5.3 Publications

Forney, L. J., Ravenhall, D. G., and Winn, D. S., Aerosol Impactors:

A Study of a Fluid Jet Impinging Upon a Void, Journal of Applied Physics, 49, No. 3 (1978).

Ravenhall, D. G., Forney, L. J., and Jazayeri, M., Aerosol Sizing with a Slotted Virtual Impactor, Journal of Colloid and Interface Science, 64, No. 3 (1978).

Ravenhall, D. G., and Forney, L. J., Particle Trajectories in a Two-Dimensional Jet of Ideal Fluid Impinging Upon a Plate, Aerosol Science (submitted January 1978).

Forney, L. J., and Ravenhall, D. G., Aerosol Impactors: A Two-Dimensional Jet of Ideal Fluid Impinging Upon a Void, Proceedings of the Eighth U.S. National Congress of Applied Mechanics, Los Angeles, California, June (1978).

REFERENCES

- Forney, L. J., Rev. Sci. Instrum. 47, 1264 (1976).
- Forney, L. J. Final Rept. NSF ENG 75-10303, Dept. of Civil Engr. Rept. UILU-ENG 77-2006 (1977).
- Forney, L. J., Ravenhall, D. G, and Winn, D. S., J. Applied Physics 49, No. 3 (1978).
- Ravenhall, D. G., Forney, L. J. and Jazayeri, M., J. Colloid and Interface Sci. 64, No. 3 (1978).
- Ravenhall, D. G. and Forney, L. J., Aerosol Science, (submitted) (1978).

Appendix A

AEROSOL IMPACTORS: A STUDY OF A FLUID JET
IMPINGING UPON A VOID

by

L. J. Forney¹

D. G. Ravenhall²

D. S. Winn³

University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

J. Applied Physics 49, No. 3 (1978)

¹Department of Civil Engineering

²Department of Physics

³Department of Mechanical Engineering

ABSTRACT

The theoretical analysis of a two-dimensional jet of ideal fluid impinging normally upon a void has been made and compared with experimental observations. Using a scaled-up water model with a fixed jet Reynolds number of 750, general features of the flow such as streamline positions and fluid velocities are in good agreement with theoretical predictions. Moreover, theoretical limitations on the void-space opening ratio b/w for a given jet spacing ratio s/w covering a range from 0 to 2 correlate well with the onset of irregularities in the observed flow pattern. Finally, experimental precautions are outlined necessary to minimize fluid instabilities and concurrent fluid flux into the void space.

I. INTRODUCTION

The importance of aerosol size with regard to aerodynamic properties, toxicity and ability to scatter light has stimulated the development of a technology concerned with particle sizing instrumentation. A class of devices used to size suspended particles are commonly called particle impactors. These instruments utilize the particle inertia in a jet and operate by directing a particle-laden air stream at a collection surface. Thus, the instruments function as aerosol size classifiers since large particles, because of their momentum, impact with the collection surface while smaller particles escape.

Critical to the understanding of inertial particle impactors is a complete description of the fluid flow field. Davies and Aylward¹⁾ derived analytical expressions for the flow field of a two-dimensional jet of ideal fluid impinging normally upon a solid plate, while a review of techniques applicable to similar problems is provided in Birkhoff and Zarantonello²⁾. Davies and Aylward¹⁾ also used their results to derive impactor efficiencies. Similarly, Ranz and Wong³⁾ derived analytical expressions, later modified by Mercer and Chow⁴⁾, for impactor efficiencies using a simple approximation to the actual flow field. More recently, Marple⁵⁾ numerically solved the full Navier-Stokes equations to describe the flow field for both rectangular and round jets impinging upon a solid plate.

The boundary condition or collection surface in the works above was a solid plane upon which the particle-laden fluid jet impinged. However, recent experimental efforts have sought to eliminate this solid collection

surface since particle surface interactions have limited the usefulness of inertial impactors (Hounam and Sherwood,⁶⁾ Conner,⁷⁾ Loo and Jaklevic,⁸⁾ Forney⁹⁾). Attempts have been made in these devices to replace the solid collection plate with a virtual surface bounding a stagnant air void. Little is currently known about the character of these flow fields. Because these new impactor geometries introduce a free streamline bounding the jet and stagnant void and since jet Reynolds numbers are typically large ($> 10^3$), a two-dimensional potential or ideal fluid solution has been explored for a jet impinging normally upon a virtual surface or void. The results of this analysis are compared with features of recorded flow fields using a scaled-up water model.

The equations connecting the various complex planes needed to describe the flow are given in Sec. II, the experimental apparatus and techniques are detailed in Sec. III, and the comparison of theoretical predictions for the flow pattern with those obtained experimentally is shown in Sec. IV. Results are summarized in Sec. V, while a brief account of the computational methods needed to realize the results of Sec. II is given in the Appendix.

II. FLOW THEORY

The shape and desired flow pattern of the device are shown in Fig.

1A. The jet of fluid incident from the right, of half-width w , is confined by the parallel walls AB and $A'B'$. The void space lies to the left of the plate DE , $E'D'$, whose aperture has half-width h . The end of the jet is a horizontal distance s from the plate. The fluid as it emerges from the jet is bounded on the outside by the free streamlines BC , $B'C'$ and at the void region by the free streamline EFE' . The fluid is assumed to be in contact with the plate over the whole distance DE , $E'D'$, and the apparatus and flow pattern are symmetrical about the central axis FG .

The analysis of the flow pattern uses the standard techniques of ideal fluid flow, as described in Batchelor,¹⁰⁾ for example, and similar to an earlier attempt by Smith.¹¹⁾ The characteristics of the physical device, regarded as singularities and other properties of the stream function in the z -plane, are to be mapped into the simple pattern, as shown in Fig. 1B, in the W -plane (where $dW/dz = v^*$, the complex conjugate of v the fluid velocity). To ensure the correspondence, the W -plane half-width of the parallel walls must equal $u_0 w$, where u_0 is the asymptotic velocity of the incident jet. For definiteness the origin of W is taken to be the point F .

The presence of free streamlines suggests the introduction of the Kirchhoff variable $\Omega = \log(u_0 dz/dW)$. In the Ω -plane the boundaries of the flow pattern become the rectangular shape illustrated in Fig. 1C. The points A , C , A' transform to the origin, and the locations of the sides

depend on the fluid velocity on the boundaries of the flow. Thus since Ω is related to the change of direction θ of the fluid velocity by $\text{Im } \Omega = \theta - \pi$, the fact that the plate is perpendicular to the jet (i.e. $\theta = 3\pi/2$ for ED) means that the height of the rectangle is $\pi/2$. The locations of the right and left sides are not prescribable, but reveal the magnitudes of the velocities of the fluid at the free streamlines.

The W and Ω planes are linked by a subsidiary complex variable ζ , illustrated in Fig. 1D. The parallel region of W inside the two straight boundaries is mapped onto the upper half-plane of ζ using a Schwartz-Christophel transformation. This requires the specification of the position of the points CD, C'D' in ζ , which are defined to be at $\zeta = \pm c'$. The relationship is then

$$\frac{dW}{d\zeta} \propto \zeta(\zeta - c')^{-1}(\zeta + c')^{-1} \quad (\text{II.1})$$

which upon integration and insertion of the width in W of the boundaries, produces the result

$$\zeta^2 = c'^2 [1 - \exp(\pi W/u_o w)] \quad (\text{II.2})$$

For the mapping of the inside of the rectangular region in Ω onto the upper half-plane of ζ , the positions of the vertices E, DC and B are specified in ζ . The point B is put at $\zeta = 1$, setting the scale of the ζ map, and E is put at $\zeta = c$. (From the topology of the ζ -plane it is clear that $0 < c < c' < 1$.) The Schwartz-Christophel transformation linking Ω and ζ is then defined by

$$\frac{d\Omega}{d\zeta} \propto (\zeta^2 - c^2)^{-1/2} (\zeta^2 - c'^2)^{-1/2} (\zeta^2 - 1)^{-1/2} . \quad (\text{II.3})$$

Upon integration of this expression, and application of the boundary condition that the velocity of the emerging jet is u_∞ , the relationship becomes

$$\Omega(\zeta) = \ln R \{F(\phi[\zeta] \backslash \alpha) / F(\chi \backslash \alpha) - 1\} . \quad (\text{II.4})$$

where $R = u_\infty / u_0$. In this expression F represents the incomplete elliptic integral of the first kind.¹²⁾ The modular angle α of this function is defined by

$$\cos \alpha = \left[\frac{c^2(1-c'^2)}{(1-c^2)c'^2} \right]^{1/2} , \quad (\text{II.5a})$$

the argument χ of the constant factor involving F by

$$\tan \chi = \left[\frac{c'^2}{1-c'^2} \right]^{1/2} , \quad (\text{II.5b})$$

and the dependence of the argument ϕ on ζ by

$$\tan \phi = \left[\frac{c'^2}{1-c'^2} \frac{\zeta^2 - 1}{\zeta^2} \right]^{1/2} . \quad (\text{II.5c})$$

The parameter $R = u_\infty / u_0$ can now be related to the condition that the height of the Ω rectangle is $\pi/2$. On manipulation of Eq. (II.4), one obtains

$$\ln R = \frac{1}{2\pi} F(\chi \backslash \alpha) / K'(\alpha) \quad (\text{II.6})$$

where $K'(\alpha)$ denotes the complete elliptic integral $F(\frac{1}{2}\pi \backslash \frac{1}{2}\pi - \alpha)$.¹²⁾ The velocity u_1 of the free streamline $E'FE$ may be obtained in a similar manner, in the form

$$\ln(u_\infty/u_1) = \frac{1}{2\pi} K(\alpha)/K'(\alpha) \quad (\text{II.7})$$

where $K(\alpha) \equiv F(\frac{1}{2}\pi \backslash \alpha)$. The pressures of the incident jet P_0 and the void space P_1 (setting the emerging jet at zero pressure) may be obtained by Bernoulli's law. The application to the moving fluid of Newton's second law produces the equation

$$\rho w u_0^2 = P_1 h + f - P_0 w \quad (\text{II.8a})$$

where f is the (positive) force exerted by the plate. If the emerging jet is detached, f is zero. When the jet is attached, therefore, the relationship provides an inequality that h/w must satisfy, which may be written in the form

$$h/w < h_{\text{det}}/w = (u_\infty^2 + u_0^2)/(u_\infty^2 - u_1^2) \quad (\text{II.8b})$$

For completeness it is observed that the mappings can be generalized in a straightforward way to encompass more complex devices, in which the two segments of the plate ED and E'D' are at a general angle to the jet, and also in which the plate segments are finite rather than of infinite extent. However, we shall not consider these complications in the present work.

Equations (II.2,4) contain implicitly the dependence of $\Omega[=2\pi(u_0 dz/dW)]$ on W . It is thus a simple step, in principle, to relate the flow pattern to the physical dimensions of the apparatus, w , s and h , or in dimensionless terms s/w , h/w . On obtaining z as a function of W in the form

$$\frac{dz}{dW} = u_0^{-1} \exp[\Omega(W)] \quad (\text{II.9a})$$

it follows that for any two positions p_1 and p_2 in the flow field,

$$z(p_2) - z(p_1) = u_0^{-1} \int_{W(p_1)}^{W(p_2)} \exp[\Omega(W)] dW \quad . \quad (II.9b)$$

The connection between Ω and W is achieved, however, by the introduction of the ζ plane and two additional computational parameters c and c' .

The relationships obtained therefore imply the process of choosing c and c' and discovering from Eqs. (II.9) what physical parameters the device possesses, instead of the desired inverse process, of prescribing s/w and h/w , and determining c , c' and other properties of the flow field.

In the absence of any simple reliable approximations to these relationships which would permit an analytic inversion of the algebra, a computational procedure has been adopted. Some details of the numerical analysis are given in the Appendix. Here, it is sufficient to say that a computer program has been constructed which performs the inverse process mentioned. Thus, for allowable values of s/w and h/w , the characteristics of the device and all properties of the flow field can be determined numerically.

III. MODEL STUDIES

The character of the flow field was observed and recorded using a 10:1 scaled-up Plexiglas water model of the proposed geometry as shown in Fig. 2. Tests were conducted at various values of the dimensionless groups s/w and h/w covering a range from 0 - 2 anticipated in particle impactors. Using a dye injection, flow visualization technique, similar to that of Winkler¹³⁾, photographs were taken of the flow field in the throat region of the device at a Reynolds number of $Re = 750$ (e.g., Figs. 3, 6 and 7). The Reynolds number chosen was considered to be sufficiently large to minimize boundary layers effects, yet small enough to reduce turbulence. The boundary layer thickness was here calculated to be roughly .05 w . Moreover, the value of Re chosen was consistent with the results of Marple⁵⁾, who demonstrated that varying Re from 250 to 12,500 has a nominal effect on impactor flow fields utilizing solid boundaries.

Features of the flow field were recorded by immersing and anchoring the model in a 25 gallon water tank with the model outlet connected directly to a centrifugal pump. Dye was injected into the throat of the model from tygon tubing connecting three stainless steel injection nozzles to a cylindrical reservoir suspended above the tank. Tubing clamps regulated the flow of washable, water-diluted, black India ink into the three injection lines positioned at the model throat in a plane midway between the top and bottom model surfaces. Sufficient clarity and contrast of the streamlines was provided in the model by using a fluorescent lamp placed directly beneath the holding tank. Here, the lower surface of the tank was covered with vellum to diffuse the lamp light.

Several trial throat geometries were constructed and tested. It was found that unstable flow would result in the model throat unless precautions were taken to eliminate instabilities. A stable flow field in the water model was achieved in the present study by using a converging throat geometry in combination with a honeycomb flow straightener placed at the throat entrance. Moreover, to eliminate intermittent loss of fluid into the void space, it was necessary to put a wall (see Fig. 2) in the plane of symmetry (line FG of Fig. 1A) of the proposed geometry. None of these modifications limit the applicability of the theory presented, however, since FG of Fig. 1A corresponds to a natural streamline of the flow field and all fluid streamlines are asymptotically forced into parallel alignment at the throat exit by the model geometry of Fig. 2.

IV. RESULTS AND DISCUSSION

The application of the flow theory of Sec. II proceeded largely as a computer experiment. Values of the geometric ratios s/w and h/w suggested by experience with real impactors were inserted into the program. The values of c and c' needed in the ζ -plane to reproduce these values in the z -plane were then obtained, together with all details of the flow pattern. Comparisons were then made between predicted and measured flow properties such as the position of the streamlines. A sample comparison with $s/w=1$, $h/w=1$, is demonstrated in Fig. 3. The relationship between s/w , h/w and c , c' is summarized in Fig. 4. The topology of the ζ -plane requires $0 < c < c' < 1$, which confined allowed values of s/w , h/w to the triangular region shown. For any given s/w , increasing values of h/w corresponded to increasing values of c and c' , and h/w approached a limiting value h_{lim}/w as c , c' tended to 1. Detailed examination showed that near this limit h/w and α , the modular angle of the elliptic integrals of Sec. II, have the following behavior:

$$(h_{lim}-h)/w \approx (1-c'^2)^{1/2}$$

$$\alpha \approx [2(c'-c)/(1-c)]^{1/2} \approx \text{constant}.$$

It is thus apparent that as a function of c' , h/w has a branch point at $c'=1$, and values greater than h_{lim}/w are not allowed by the assumed mappings.

Numerical values of h_{lim}/w as a function of s/w are represented in Fig. 5 by the solid line. The data points (solid circles) were determined from

the scaled-up water model where h/w was increased, for constant s/w , until the flow field showed the onset of a marked irregularity. Here, the form of the irregularity was a kink or short curl on the free streamline at the point $E(E')$ as illustrated by the photograph in Fig. 6.

For each s/w the onset of jet detachment from the plate $ED(E'D')$, where $h/w = h_{det}/w$, was estimated from Eq. (II.8b) and is represented by the dashed line in Fig. 5. Here, the solid squares of Fig. 5 represent the experimentally observed h_{det}/w marking the onset of jet detachment while Fig. 7 illustrates a typical flow pattern. In all cases it was found that h_{det}/w is considerably larger than h_{lim}/w . Thus, it was independently confirmed that the phenomenon in the flow pattern which limits h/w is not the detachment of the emerging jet from the plate but another breakdown in the flow pattern. (The flow pattern at $h/w \rightarrow h_{lim}/w$ has the free streamline from B to C convex to the plate, as is the normal case. Thus even in the limiting situation, the pressure exerted by the plate on the jet is non-zero everywhere, and the force $\int$ involved in Eq. (II.8a) is definitely not zero.)

Some physical features of the flow pattern, such as $R = u_\infty/u_0$, behaved in a completely regular manner as the limit $h/w \rightarrow h_{lim}/w$ was approached. Figure 8 illustrated the calculated contour (solid line) of R versus s/w at h_{lim}/w . Here, the data presented by the solid circles were determined from the water model by measuring the jet width at h_{lim}/w . The dashed line, shown for comparison, represents the theoretical prediction of Davies and Aylward¹⁾ for the case of ideal jet impingement onto a solid wall ($h/w = 0$).

In searching for the cause of the limitation on h/w , it was found that in the limit $h/w \rightarrow h_{lim}/w$ the position of the bifurcation point F extends indefinitely far into the incoming jet. In Fig. 9 are plotted computed values of the horizontal distance d/w from plate to F as a function of h/w (solid line) for $s/w=1$. An examination of the analytic behavior of the various functions of the flow field near F ($\zeta=0$) and for $c'=1$ confirmed the singular behavior, showing that $d/w \propto (1-c')^{-1/3}$. Also, for a given s/w , as h/w tends to h_{lim}/w , u_1/u_0 tends to 1. Consequently the void-space pressure P_1 approaches P_0 . A possible physical reason for the limit may thus be that the pressure difference across the jet becomes insufficient to turn the full jet. [The flow pattern which extrapolates from this limit, with the start of the free streamline at point F now at infinity in z (between G and A in Fig. 1) appears impossible to realize, and will not be considered further.]

As was pointed out in Sec. III, it was impossible to maintain a stable flow field unless a wall was placed at the plane of symmetry or line FG of Fig. 1A. Hence, all experimental observations were made using this geometry representing one half of the flow field as seen in Fig. 3. To compare experimental observations of the position of the bifurcation point F , a length d' was defined as the distance from the plate to the point on the central streamline which has separated from the center plane by a small length b . For values of b ($\sim 0.05 w$) comparable to the boundary layer thickness for the water model, values of d' were finite even for the limiting flow pattern ($h/w \rightarrow h_{lim}/w$). A calculated contour of d'/w is

indicated by the dashed line in Fig. 9 for $s/w = 1.0$. Although it is expected that the dashed line of Fig. 9 will correlate the position of the bifurcation point for large $h/w \rightarrow h_{lim}/w$, it undoubtedly represents a poor correlation for small h/w with the necessary solid wall along FG of Fig. 1A. This results from the fact that a corner eddy will exist for small h/w . However, potential application for this geometry as a particle fractionator excludes values of $h/w < 1.0$.

To confirm experimental observations of the position of the bifurcation point in the water model, upper and lower bounds of $.045 < b/w < .07$ were estimated for the boundary layer thickness of the water model. These values were introduced into the numerical computations as described above to define a range of values of the length d'_{lim}/w at h_{lim}/w and the results were compared with experimental observations. The results are shown in Fig. 10 which indicate good correlation between experiment (solid symbols) and the range of possible theoretical values.

As was stated earlier, the investigation of the flow pattern has been made largely through a computer program. Some features of the relationships between the normalized physical dimensions s/w , h/w , d/w , d'/w and the ζ -plane parameters c , c' were accessible to analytic approximation. The one quoted earlier, that $d/w \propto (1-c')^{-1/3}$, required only the behaviour of Ω near $\zeta = 0$, so that ϕ , the argument of the incomplete elliptic integral $F(\phi|\alpha)$ in Eq. (II.4), remained close to $\frac{1}{2}\pi$. Thus F could be approximated independently of the value of α . For the normalized physical parameters s/w and h/w , an approximation of $\Omega(\zeta)$ was required

over a much wider range of ϕ , from the origin to the branch point of F . Furthermore, it was evident from the numerical work that α depends on s/w and is not necessarily small. Although considerable effort was expended on searching for a reasonably good analytic approximation to $F(\phi/\alpha)$ and to the whole mapping, the search was unsuccessful, and these reasons make it unlikely that such an approximation exists.

V. SUMMARY AND CONCLUSION

For a jet impinging on a void, the ideal fluid flow pattern has been explored analytically and computationally, and compared with experimental observations on a scaled-up water model. General features such as positions of streamlines are in good agreement. The theoretical limitations on the void-space opening ratio h/w for a given jet spacing ratio s/w correlate well with the onset of instabilities in the observed flow pattern.

The reliability of the ideal fluid flow assumption suggests the further comparison, between theory and experiment, of the efficiency of such particle impactors for sizing suspended particles. It also suggests the possibility of rapid theoretical exploration of design modifications by using the computational methods outlined here. Such work is now in progress.

ACKNOWLEDGEMENTS

The research described was supported in part by an Energy Research and Development grant ERDA-EE-77-S-02-4319 (L.J.F.), a National Science Foundation grant PHY75-21590 (D.G.R.) and a U.S. Environmental Protection Agency traineeship grant EPA-T-900-203 (D.S.W.). We acknowledge with thanks some useful comments by R. L. Schult.

. APPENDIX: COMPUTATION

The computation mentioned in Sec. II involves first the direct evaluation of distances in the z -plane, using Eq. (II.9), for specified values of the ζ -plane parameters c, c' . The elliptic integrals needed in Eqs. (II.4,6) are obtained as follows: for $K(\alpha)$ and for $F(x|\alpha)$, where x is real (α being always real), repeated use of the descending Landen transformation described in Sec. 17.5 of Ref. (12); and for $F(\phi|\alpha)$ when ϕ is complex, the decomposition of it in terms of the real-argument functions given in Eq. (17.4.11) of Ref. (12). The latter procedure provides accurate results even close to the branch point at $\phi = \frac{\pi}{2} + i\ln(\cot \frac{1}{2}\alpha)$, a necessary virtue since that value of ϕ corresponds to the vertex E in the Ω -plane, whose location in z is one of the desired results of the computation. For the integral of Eq. (II.9b), the contour is chosen to be the straight line in the W -plane joining $W(p_1)$ and $W(p_2)$, and the integral is evaluated by Simpson's Rule. This procedure for a given pair of values of c and c' produces values of s/w and h/w . Obvious checks on the accuracy of the calculation, besides the fact that salient features of the W, Ω and ζ -plane correspond properly, are that the z -plane jet width is correctly given as 1 (in units of w), and that the emerging jet width has the value $(= w u_0 / u_m)$ required of an incompressible fluid.

The procedure is inverted by means of two sequential secant-method searches on c and c' , so as to find values which will produce specified

values for s/w and h/w . For examining the behavior of h/w near its limiting value h_{lim}/w , there is great sensitivity to the c and c' values, and the procedure is modified so that h/w and c' are found for specified values of s/w and c .

The programs have been written to operate in time-sharing mode on DEC-10 and CYBER-175 computers.

REFERENCES

1. C. N. Davies and M. Aylward, Proc. Phys. Soc. (London), B64, 889 (1951).
2. G. Birkhoff and E. H. Zarantonello, "Jets, Wakes, and Cavities", Academic Press Inc., New York (1957).
3. W. E. Ranz and J. B. Wong, Ind. and Eng. Chem. 44, 1371 (1952).
4. T. T. Mercer and H. Y. Chow, J. Colloid and Interface Sci. 27, 75 (1968).
5. V. A. Marple, "A Fundamental Study of Inertial Impactors", Ph.D. Thesis, U. of Minnesota, Minn., 1970 (unpublished).
6. R. F. Hounam and R. J. Sherwood, Ind. Hyg. J. 26, 122 (1965).
7. W. D. Connor, J. Air Poll. Control Assoc. 16, 35 (1966).
8. B. W. Loo and J. W. Jaklevic, Lawrence Berkeley Laboratory Pub. No. 2468, 1974 (unpublished).
9. L. J. Forney, Rev. Sci. Instr. 47, 1264 (1976).
10. G. K. Batchelor, "An Introduction to Fluid Dynamics", Cambridge U. Press (1967).
11. T. J. Smith, "Jet of Ideal Fluid Impinging Normally Upon a Void", M.S. Thesis, U. of Illinois, Urbana, Illinois (1977).
12. M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions", Dover Inc., New York (1970).
13. J. P. Winkler, "A Model Study of a Virtual Dichotomous Impactor", M.S. Thesis, U. of Illinois, Urbana, Illinois (1975).

FIGURE CAPTIONS

Figure 1 Transformation planes

Figure 2 Top view of water model.

Figure 3 Photograph of flow field with computed flow pattern superimposed,
for $s/w = 1.0$ and $h/w = 1.0$.

Figure 4 Contours of normalized jet dimensions s/w , h/w on c , c' coordinates.

Figure 5 Correlation of measured and computed jet dimensions at onset
of flow field irregularities.

Figure 6 Photograph of flow field illustrating onset of indeterminate
pattern for jet dimensions $h/w > h_{lim}/w$, $s/w = 1.0$ and $h/w = 1.2$.

Figure 7 Photograph of flow field illustrating a detached jet for jet dimensions
 $h/w > h_{det}/w$, $s/w = 1.0$ and $h/w = 2.6$.

Figure 8 Computed and measured values of $R(= u_m/u_0)$ vs. s/w .

Figure 9 Computed values of bifurcation distances d/w , d'/w vs. void
width h/w , for $s/w = 1.0$ ($h/w =$ dimensionless boundary layer thickness).

Figure 10 Correlation of measured and computed bifurcation distance d'/w
vs. void width for $h/w = h_{lim}/w$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 3

Fig. 9

Fig. 10

Appendix B

AEROSOL SIZING WITH
A SLOTTED VIRTUAL IMPACTOR

By

D. G. Ravenhall¹

L. J. Forney²

M. Jazayeri²

University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

J. Colloid and Interface Sci. 64, No. 3 (1978)

¹Department of Physics

²Department of Civil Engineering

ABSTRACT

A theoretical analysis of a two-dimensional jet of ideal fluid impinging normally upon a void has been extended to include fluid deflecting plates of finite length inclined at arbitrary angles to the incident jet. Particle trajectories were numerically simulated for various theoretically admissible instrument geometries. Salient features of the particle sizing effectiveness of the device were determined for a range of void space openings $h/w > 1$ combined with jet-to-plate spacings s/w ranging from 0 to 2, at two fluid deflecting plate angles $\beta = 60^\circ, 90^\circ$. It was found numerically that internal particle losses were substantially reduced when the void h/w tends toward h_{lim}/w , its theoretical limiting value, and that instrument performance in general improved at smaller β .

I. INTRODUCTION

Fine suspended atmospheric particles represent a potentially harmful form of air pollution. Small airborne solid or liquid particles of a few microns in diameter or less are formed directly by a spectrum of physical and chemical processes. A significant mass fraction of suspended particulate matter, for example the nitrate or sulfate component, is also formed indirectly in atmospheric processes involving such major gaseous pollutants as nitric and sulfur oxides. These small particles, because of their size, remain suspended longer, readily scatter light and penetrate the natural defense mechanisms of the human respiratory system. Because of the recognized toxicity of airborne particulate matter and the important role submicron particles play in the evolution of air pollution, research has been conducted on instrumentation for the purpose of monitoring the particle size distribution or chemical composition of suspended particles.

Recently, much interest has focused on devices capable of fractionating suspended particles into a respirable (less than roughly $3\text{ }\mu\text{m}$ in diameter) and a nonrespirable size class (larger particles) for subsequent mass or chemical analysis.¹⁾ This is most easily accomplished with what are called particle impactors. These instruments direct a particle laden air stream toward a collection surface where large particles because of their inertia are collected while smaller sizes escape. Since conventional impactors utilize a solid, treated or untreated, collection surface, particles will rebound, reentrain and agglomerate on these surfaces leading to erroneous results. Hence several recent research efforts have attempted to replace the solid collection surface with a slowly pumped stagnant air void in instruments that are termed virtual impactors.^{2),3),4),5)}

Critical to the understanding of virtual impactors is a complete description of the fluid flow field and particle trajectories. Recently, Forney, Ravenhall and Winn⁶⁾ have made a theoretical analysis of the fluid properties of a two-dimensional jet of ideal fluid impinging normally upon a void and compared their results with experimental observations. This paper generalizes the theory and introduces particles into the flow field for a variety of geometries. This provides useful information concerning the theoretical operating characteristics of these instruments such as particle efficiency curves and loss factors.

The flow theory and its limitations for arbitrary geometries are described in Sec. II, mathematical techniques used to determine particle trajectories are outlined in Sec. III, while the results concerning typical features of the particle efficiency and loss spectrums are presented in Sec. IV. In Sec. V a brief summary is given.

II. FLOW THEORY

The methods used to obtain the two-dimensional flow-pattern of the assumed ideal fluid involve only minor extension of the previous work.⁶⁾ Summarizing, the physical device shown in the z -plane of Fig. 1 is symmetrical about the real axis. Parallel walls AB, A'B' define the incident jet, of half-width w and with throat velocity u_0 , which impinges on two plates E_1E_2 , $E'_1E'_2$ of length p held at an angle β to the incident jet axis. The end of the jet is a distance s , measured parallel to the axis, from the inner edge of the plate, whose opening has half-width h . There is a free streamline E'_2FE_2 , with velocity u_1 , at the boundary between the moving fluid and stagnant fluid in the void to the left of the plates. The emerging jet is defined by free streamlines BCDE, B'C'D'E', with asymptotic velocity u_∞ . To obtain the simple velocity potential $W(dW/dz = v^*$, where v is the fluid velocity), and to realize the fluid velocities exhibited in the Ω -plane [where $\Omega = \ln(u_0 dz/dW)$] as shown in Fig. 1, the subsidiary ζ -plane is introduced, and related to them by means of Schwartz-Christoffel transformations. The generalizations from the previous work of having a finite plate inclined at a general angle β to the jet require that the points CD and E_2 be distinct, and that the rectangle in Ω have height $\pi - \beta$. The Ω plane thus contains three separate parameters c , c' and e , where $0 < c < c' < e < 1$. The analytic relationships among the variables are no more complicated than the previous analysis,⁶⁾ however, and they are listed without further derivation. The relationship between W and ζ involves now the parameter e , or

$$\zeta = e^2 (1 - \exp(\pi W/u_0 w)), \quad (1)$$

while that between Ω and ζ is unchanged:

$$\Omega = \ln R \{F(\phi(\zeta)/\alpha)/F(\chi/\alpha) - 1\}, \quad (2)$$

where $R = u_\infty/u_0$. The modular angle α of F , the incomplete elliptic integral of the first kind,⁷⁾ is defined by

$$\cos \alpha = \{c'^2(1 - c'^2)/(c'^2(1 - c'^2))\}^{1/2}, \quad (3)$$

the argument $\phi[\zeta]$ of this function by

$$\tan \phi[\zeta] = \{c'^2(\zeta^2 - 1)/((1 - c'^2)\zeta^2)\}^{1/2} \quad (4)$$

and the argument χ of the constant factor involving F by

$$\sin \chi = c'. \quad (5)$$

Some physical ζ -plane quantities of interest are obtained by examining these relationships in particular regions. Thus

$$\ln R = \ln(u_\infty/u_0) = (\pi - \beta) F(\chi|\alpha)/K'(\alpha) \quad (6)$$

and

$$\ln(u_\infty/u_1) = (\pi - \beta) K(\alpha)/K'(\alpha), \quad (7)$$

where $K(\alpha) = F(\frac{\pi}{2}|\alpha)$ and $K'(\alpha) = F(\frac{\pi}{2}|\frac{\pi}{2} - \alpha)$ are the complete elliptic integrals of the first kind.⁷⁾ The angle of the freely emerging jet, β_{em} , is

$$\beta_{em} = \pi - \ln R F(\mu|\frac{\pi}{2} - \alpha) / F(\chi|\alpha) \quad (8)$$

with the argument μ given by

$$\sin \mu = \{c'^2(1 - e^2)/((1 - c'^2)e^2)\}^{1/2} \quad (9)$$

From these relationships fluid pressures, etc., at the various asymptotic regions of the flow may be calculated. Physical dimensions are obtained by using the detailed properties of the transformations. Thus the distance in the z -plane between points P_1 and P_2 is

$$z(P_1) - z(P_2) = u_0^{-1} \int_{W(P_1)}^{W(P_2)} \exp(\Omega[\zeta(W)]) dW \quad (10)$$

where the brackets in the integrand indicate the analytic dependences that determine the integration to be carried out in the W -plane. The contour chosen was the straight line joining $W(P_1)$ and $W(P_2)$. The z -plane lengths

characterizing the device, s/w , h/w and p/w , can be calculated for any set of ζ -plane parameters c , c' , e , and a prescribed plate angle β .

The numerical procedures adopted to invert the process, and obtain the c , c' , e values which will produce prescribed values of s/w , h/w and p/w have been described and justified previously. Most of the results reported here concern the special case in which the emerging jet remains in contact with an infinitely long plate as opposed to one of finite length. This occurs when $e = c'$, in which case the inversion process necessary to characterize the dimensions of the device is only two-dimensional as before. Since the objective of the present work is the calculation of particle separation efficiencies for the general device, only such details of the flow characteristics as are needed for that purpose will be described.

Because of the existence of free streamlines bounding the jet and from dimensional arguments, one would expect instabilities to develop at some critical length ℓ as measured from the jet exit.⁹⁾ Thus, the length ℓ of the central streamline from the point at the same velocity potential as B to the point at the same potential as E_2 has been computed for various geometries (see Sec. IV).

III. PARTICLE TRAJECTORIES

In fluid moving with velocity $\vec{v}_f$, a particle at the point $\vec{x}$ with velocity $\vec{v} = d\vec{x}/dt$ is assumed to obey the equation of motion

$$m \frac{d\vec{v}}{dt} = K(\vec{v}_f(\vec{x}) - \vec{v}) \quad (11)$$

where m is its effective mass and the force term represents Stokes' Law. The vectors are all two-dimensional. For particles initially moving with the fluid in the incident jet, this equation defines a trajectory which may (a) enter the void space, (b) intersect the plate E_1E_2 , or (c) continue with the emerging jet. Which of these contingencies occurs depends on the initial position across the jet y_0 (where $0 > y_0 > -w$), on the constants K and m and of course on the fluid flow pattern. In the units used to describe that pattern, w and u_0 the trajectory is defined in terms of a dimensionless independent variable $\tau = u_0 t/w$ by the equations

$$\begin{aligned} d(\vec{x}/w)/d\tau &= \vec{v}/u_0, \\ d(\vec{v}/u_0)/d\tau &= (wK/mu_0) (\vec{v}_f/u_0 - \vec{v}/u_0). \end{aligned} \quad (12)$$

The one parameter which characterizes the particle is thus the Stokes number $\psi = mu_0/wK$. For the simplest assumptions, that $m = \frac{4}{3} \pi a^3 \rho$, where a is the particle radius and ρ its density, and that $K = 6\pi\eta a$ where η is the fluid viscosity, then $\psi = \frac{2}{9} \rho u_0^2 a^2 / w\eta$. We define, as is customary, the variable $\psi^{1/2}$,

$$\psi^{1/2} = a \left(\frac{2}{9} \rho u_0^2 / w\eta \right)^{1/2} \quad (13)$$

so that $\psi^{1/2}$ is a dimensionless parameter proportional to the particle radius.

Particle trajectories for a given device configuration and flow pattern are obtained by integrating Eq. (12) numerically, using table look-up and interpolation for $\vec{v}_f(\vec{x})$. For a given $|y_0|/w$ and ψ , the destination (a),

(b) or (c) of the particle as described above was determined. Assuming a uniform population across the incident jet of particles of a given ψ , two separate fractional efficiencies, E_V and E_T , may be defined and calculated. E_V is the fraction of particles entering the void, and is numerically equal to the value of $|y_0|/w$ for which the trajectory hits the point E_2 , the inner edge of the plate. E_T is the fraction of particles which are intercepted by the void or by the plate, so that they are not present in the emerging jet. E_T is equal to the value of $|y_0|/w$ when the trajectory intersects E_1 , or for a plate of infinite length, when the trajectory asymptotically intersects the plate. Implicit in this definition is the assumption that $|y_0|$ is a monotonic and increasing function of ψ . While not necessarily true for the solid-surface impactor⁸, this assumption obtains for all trajectories examined in the present work. Some trajectories which define salient features of E_V and E_T are shown in Fig. 5, for a typical flow pattern. The trajectory labelled T_1 defines the value of ψ above which E_V is equal to one, while T_2 does the corresponding thing for E_T . The loss factor L ,

$$L = E_T - E_V, \quad (14)$$

represents the fraction of particles of a given ψ hitting the plate, and thus not accessible to easy detection with the device. For a given flow pattern, these quantities depend on $\psi^{1/2}$ in the manner indicated in Fig. 6.

Additional parameters defined are a measure of the total number of lost particles represented by

$$A = \int_0^\infty L(\theta) d\theta \quad (15)$$

where $\theta = \psi^{1/2}$. Also of interest, as measures of the ψ discrimination of the device are: first, the relative dispersion S_V and S_T of the efficiencies E_V and E_T , calculated as

$$S = (\psi_{84}^{1/2} - \psi_{16}^{1/2}) / (0.68 \psi_{50}^{1/2}) \quad (16)$$

where for example $\psi_{84}^{1/2} = \psi^{1/2}(E = 0.84)$ and secondly, $\psi_{50}^{1/2}$ which represents

a parameter proportional to the particle cutoff diameter. It is understood that Eq. (15) represents only a first approximation to particle loss on the jet deflecting plate since a uniform particle distribution was assumed in the definition of A. Equation (16) represents an estimate of the relative variation of the parameter $\psi^{1/2}$ about the cutoff diameter $\psi_{50}^{1/2}$ for a given efficiency curve. Formally, S in Eq. (16) is an estimate of the quantity $d(\ln\theta)/dE$ evaluated at $\theta = \psi_{50}^{1/2}$. Since it is desirable to maximize the normalized slope of the efficiency curve ($= dE/d(\ln\theta)$) for an impactor, one seeks to minimize the quantity S.

IV. RESULTS AND DISCUSSION

The potential flow theory outlined in Sec. II and the computer programs which realize it involve deflecting plates of finite length p inclined at a general angle β to the incident jet, as illustrated in Fig. 1. The introduction of two new adjustable dimensions into the design adds considerable flexibility, but is a conceptual complication. The basic aim of the device is to produce a deflection of the jet sudden enough that particles have difficulty in following the fluid and are projected into the void. Decreasing the plate angle β is a direct way to increase the deflection of the flow and, as will be seen, it improves the efficiency of the device. Making the plate length finite, at least for lengths $p \geq w$, affects the flow pattern appreciably only at some distance from the inner plate edge, and is thus expected to have much less effect on the efficiency. Calculation with a situation generalized from that illustrated in Fig. 5, with the parameters $p/w = 1$, $\beta = 60^\circ$, $s/w = 1$ and $h/w = 1.65$, confirm this surmise theoretically. The results of the rest of the paper are therefore restricted to the effects of varying the plate angle ($\beta = 90^\circ$ and 60°) for plates of infinite length ($p/w \rightarrow \infty$, $e = c'$). Consequently, features of the analysis such as the relationship between the geometric ratios s/w , h/w and the ζ -plane parameters c , c' are identical to the previous results⁶⁾ for $\beta = 90^\circ$, while differing somewhat for $\beta = 60^\circ$, as seen by comparison of Fig. 2 and Fig. 2 of Ref. 6. In particular, for any β there exists a theoretical, maximum limiting value of the dimensionless void width $h/w \rightarrow h_{lim}/w$ as c , c' tend to 1. This feature may be observed in Fig. 2 by examining any contour of fixed s/w , and physical reasons for it were given in Ref. 6. The dependence of h_{lim}/w on s/w for the two plate angles considered is shown in Fig. 3. It is seen that

as the deflecting plate angle β decreases, larger void widths are theoretically achievable i.e., the deflecting plate is more easily able to accommodate the tendency for the jet to detach from the plate at larger h/w . The effect of changing β is less marked, however, on the emergent fluid velocities, which are similar in magnitude for all jet to plate spacings s/w as shown in Fig. 4. Features of the fluid flow field such as streamline positions and the existence of a bifurcation point at F (see Fig. 1) on the streamline bounding the void within the jet throat are illustrated in Fig. 5 for the case of $\beta = 60^\circ$, $s/w = 1$ and $h/w = h_{lim}/w = 1.67$.

Using the methods described in Sec. III, particle trajectories for a range of device configurations were determined numerically. For each value of the Stokes number ψ , the starting values of y_0/w needed so that the trajectory either hits E_2 , the edge of the plate, or asymptotically intersects the plate, were determined by successive trials. Assuming a uniform population of particles across the jet, these values then are the efficiencies E_V and E_T . The two limiting trajectories T_1 and T_2 of Fig. 5 correspond to values of ψ for which E_V and E_T are equal to 1 respectively. It is clear from that figure that for $h/w < 1$, the trajectory T_1 cannot be realized, and that E_V is < 1 for all ψ , an undesirable property. (The area A , Eq. (15), tends to ∞ as $h/w \rightarrow 1$, and is infinite for $h/w < 1$.) Consequently, device configurations considered have been limited to $h/w > 1$.

Figure 6 illustrates a typical plot of efficiencies and loss factor for one device configuration. The curves of E_T , E_V and L are parabolic for small ψ^2 , a numerical result which may be verified analytically by considering properties of the solution to Eqs. (12) as $\psi \rightarrow 0$. This result is in contrast to conventional solid collection surfaces ($h/w = 0$) with a fluid stagnation point, where the fraction of particles collected on the surface (efficiency curve) intersects the abscissa at a critical value of $\psi > 0$.⁸⁾

This marked difference in behavior may be attributed to the quite different flow patterns obtained in the two situations, with the present limitation $h/w > 1$. Another consequence of the difference is the result, mentioned in the last section, that $|y_o|$ is found to be a monotonic and increasing function of ψ for the cases considered, whereas for the solid collection-surface device it may be double-valued. (Presumably $|y_o|$ would be double-valued for the present device, for $h/w \ll 1$, but such a value is difficult to handle numerically, and is not interesting for the reasons given above). Another noteworthy feature of Figure 6 is that the loss factor goes to zero at the finite value of ψ for which $E_v = 1$, for typical operating conditions with $h/w > 1.0$.

Several properties of the various design configurations for $\beta = 90^\circ$ are summarized in Figs. 7 and 8. The input to these plots involves numerical values of the normalized jet length l/w , the variation of the efficiencies S_v , S_T , and the area A of the loss factor L . The last three quantities are obtained by hand calculation from graphs like that of Fig. 6 for many pairs of values of s/w and h/w . Contours of constant values of any of these quantities, in the plane of s/w and h/w , were then obtained by graphical interpolation.

From experimental observations and dimensional arguments one would expect a laminar jet bounded by free streamlines to develop instabilities at a sufficient distance from its source. This property places limits on the design configuration for a given air flowrate. Experiments⁹⁾ indicate a breakdown in the flow pattern of the two-dimensional jet at $l/w \sim 1500/Re$ where l is measured along the centerline of the jet as described in Sec. II, the Reynolds number $Re = u_o w / \nu$, where $u_o w = Q$ the air flowrate per unit depth of the device, ν is the kinematic viscosity ($= \eta / \rho_a$), and ρ_a is the density

of air. Thus, because normal conditions dictate $Re > 750$, contours of constant l/w are provided as a design criterion in Fig. 7 up to $l/w = 2$. Contours of constant A covering a factor of three in magnitude are also included in Fig. 7. Because it is desirable to minimize A , it is apparent from Fig. 7 that given a choice of either fixed s/w or fixed l/w , the optimum position on the plot is always located on the h_{lim}/w contour. Figure 8 indicates contours of constant dispersion S_T and S_V . Comparing Figs. 7 and 8, it is seen that the configurations which possess smaller A values also have smaller values of S_V , but somewhat larger values of S_T . It is normally desirable to minimize S (particularly S_T) but the small variation ($\sim 30\%$) in the magnitude of S_T over large regions of Fig. 8 suggest that the minimization of A , to reduce internal particle losses, is a more important design consideration. From now on, therefore, attention is concentrated on the configurations for which $h/w = h_{lim}/w$.

Particle sizing characteristics of the device at the limiting void width $h/w = h_{lim}/w$, obtained in the manner described for two deflecting plate angles $\beta = 60^\circ, 90^\circ$ over a range of jet lengths $.75 < l/w < 2.25$, are shown in Figs. 9-12. The jet length l/w is related to the jet-to-plate spacing in Fig. 9. Figure 10 illustrates variations in the properties of the particle loss factor. It is apparent from Fig. 10 that smaller β and larger s/w improves instrument performance by greatly reducing particle losses to the jet deflecting plates. The dispersion S_T of the efficiency curve E_T is reduced somewhat also, as indicated in Fig. 11. Figure 12 demonstrates variations in the critical Stokes number $\psi_{50}^{1/2}$ for $E = 0.5$ over the same range of s/w . It is seen that the critical Stokes number for E_T decreases as the jet to plate spacing s/w is reduced, the same trend as was found by Davies and Aylward.⁸⁾

Dimensions and operating characteristics of sample devices at the two angles for specified values of l/w are listed in Table I. These values exemplify the use of Figs. 9-12. They suggest that the use of a plate angle less than 90° provides improved device properties. All of the numerical results reported in this paper were performed in time-sharing mode on the University of Illinois CYBER-175 computer.

V. SUMMARY AND CONCLUSION

General expressions are presented to describe the ideal fluid flow pattern for a jet impinging on a rectangular void. These expressions allow computation of flow patterns for fluid deflecting plates of finite length p/w inclined at arbitrary angles β to the incident jet. Particles were introduced into the flow field to determine the relative effectiveness of various geometries for size-classifying suspended particles. Salient features of particle collection efficiencies and loss factors are presented for an important range of theoretically admissible geometric ratios s/w and h/w representing the jet to plate spacing and void width, respectively. It was found that in general the device operates most efficiently when the void width $h/w = h_{lim}/w$, the theoretical limiting value, and for plate angles less than 90° . Experiments with water and air models are in progress to examine these theoretical suggestions.

ACKNOWLEDGEMENT

The research reported here was supported in part by the National Science Foundation under grant PHY75-21590 (D.G.R.) and under a summer fellowship grant SMI76-83542 (M.J.), and also by the Energy Research and Development Agency under grant ERDA-EE-77-S-02-4319 (L.J.F.).

FIGURE CAPTIONS

- Figure 1. Transformation planes (see Sec. II).
- Figure 2. Contours of normalized jet dimensions s/w , h/w on c , c' coordinates for $\beta = 60^\circ$.
- Figure 3. Computed values of maximum void width h_{lim}/w vs. jet to plate spacing s/w .
- Figure 4. Computed values of $R(=u_\infty/u_0)$ vs. s/w .
- Figure 5. Flow pattern for $h/w = 1.67$, $s/w = 1.0$, and $\beta = 60^\circ$. Trajectories of injected particles defining the values of $\psi^{1/2}$ at which E_V and E_T are equal to one are indicated by the dashed lines T_1 and T_2 .
- Figure 6. Fractional efficiencies and particle losses for a typical device geometry with $\beta = 90^\circ$ where E_V is the fraction of particles entering the void, E_T is the fraction intercepted by plate and void, and $L(=E_T - E_V)$ is the fraction hitting plate (loss factor).
- Figure 7. Contours of constant ℓ and A on a h/w , s/w plot for $\beta = 90^\circ$. ℓ represents distance along jet axis while A is the area under the loss factor.
- Figure 8. Contours of relative dispersion S_V and S_T of efficiencies E_V and E_T on a h/w , s/w plot for $\beta = 90^\circ$.
- Figure 9. Jet length ℓ/w vs. s/w at $h/w = h_{lim}/w$ for two plate angles .
- Figure 10. Computed values of A and L_m for two plate angles β at the limiting void width $h/w = h_{lim}/w$. A represents the total number of particles lost while L_m is the maximum value of the loss factor.

Figure 11. Computed values of relative dispersion S_T and S_V of efficiency curves E_T and E_V at two plate angles β for $h/w = h_{lim}/w$.

Figure 12. Computed values of $\psi_{50}^{1/2}$ representing a parameter proportional to the cutoff particle diameter for $h/w = h_{lim}/w$ and two plate angles β .

REFERENCES

1. T. B. Dzubay and R. K. Stevens, Environ. Sci. Technol. **9**, 663 (1975).
2. R. F. Hounam and R. J. Sherwood, Am. Ind. Hyg. Assoc. J. **26**, 122 (1965).
3. W. D. Conner, J. Air Pollut. Contr. Assoc. **16**, 35 (1966).
4. B. W. Loo, J. M. Jaklevic, and F. S. Goulding, Lawrence Berkeley Laboratory Pub. LBL-3854, Berkeley, CA (1975).
5. L. J. Forney, Rev. Sci. Instrum. **10**, 1264 (1976).
6. L. J. Forney, D. G. Ravenhall and D. S. Winn, J. Appl. Physics **49**, No. 3 (1978).
7. M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions," Dover, Inc., New York (1970).
8. C. N. Davies and M. Aylward, Proc. Phys. Soc. (London) **B64**, 889 (1951).
9. D. S. Winn, M.S. Thesis, University of Illinois (1977).

TABLE CAPTION

Dimensions and particle-sizing characteristics of some sample devices. β is the angle of the plate to the incident jet (see Fig. 1), l/w is the normalized jet length, s/w the jet-to-plate spacing, h_{lim}/w the limiting opening of the void space. L_m is the maximum value of the loss factor, and A the area of the loss factor. $\psi_{50}^{1/2}$ is the value of the Stokes parameter for 50 percent efficiency, and S the dispersion of the efficiency for entry into the void space (V) or to be intercepted by the void space or the plate (T).

TABLE I

	$\beta = 90^\circ$		$\beta = 60^\circ$	
l/w	1.0	1.5	1.0	1.5
a/w	1.14	1.48	.93	1.13
h_{lim}/w	1.25	1.57	1.60	1.95
$\psi_{50}^{1/2} (E_V)$	.63	.59	.51	.50
$\psi_{50}^{1/2} (E_T)$	.45	.50	.38	.42
L_m	.64	.53	.67	.56
A	.227	.129	.136	.095
S_V	1.4	1.1	1.0	.92
S_T	.70	.79	.53	.66

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Appendix C

PARTICLE TRAJECTORIES IN A TWO-DIMENSIONAL JET
OF IDEAL FLUID IMPINGING UPON A PLATE

by

D. G. Ravenhall¹

L. J. Forney²

University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

submitted to Aerosol Science (1978)

¹Department of Physics

²Department of Civil Engineering

Abstract

Properties of particle inertial impactors assuming a two-dimensional ideal fluid are derived using conformal mapping techniques and improved numerical methods. Results of sequential calculations of one-dimensional contour integrals are matched with analytical expressions for fluid properties near the stagnation point. Particle trajectories are numerically simulated for a range of jet-to-plate spacings and fluid deflecting plate angles. These results are compared with the calculations of Davies and Alyward (1951) for a deflecting plate angle $\beta = 90^\circ$. It was found that an unconventional deflecting plate angle $\beta \neq 90^\circ$ exists for a given jet-to-plate spacing which maximizes the slope of the particle efficiency curves.

I. INTRODUCTION

The detection of suspended atmospheric particles is important in the determination of air quality. Sulfates, nitrates and organic agglomerates are predominately man-made and constitute a significant portion by weight of total suspended particles in urban atmospheres. These small particles, because of their size, readily scatter light, remain suspended longer and penetrate the natural defense mechanisms of the human respiratory system. Because of the recognized toxicity of fine suspended particles, much research has focused on improvements in available instrumentation capable of monitoring particle size distributions.

A widely used technique for collecting airborne particles is known as particle impaction. Instruments commonly called aerosol impactors utilize the particle inertia in a jet by directing a particle-laden air stream at a collection surface. These instruments function as aerosol size classifiers because the larger particles with more momentum impact with the collection surface while smaller particles escape. To complement numerous experiments conducted to calibrate inertial particle impactors of various geometrics, it is useful to provide a complete theoretical description of the fluid flow field, including the simulation of particle trajectories.

Davies and Aylward (1951) derived the first analytical expressions for particle impactor flow fields using conformal mapping techniques by considering a two-dimensional jet of ideal fluid impinging normally upon a solid plate. They numerically simulated particle trajectories in the flow field to derive particle collection efficiencies and other impactor properties. Similarly, Ranz and Wong (1952) and Mercer and Chow (1968)

derived analytical expressions for impactor efficiencies using a simple approximation to the actual flow field. More recently, Marple (1970) solved the full Navier-Stokes equation numerically to describe the flow field for rectangular and round jets impinging upon a solid plate.

In the present paper ideal fluid flow is assumed as was done by Davies and Aylward. However, in the present work an accurate technique using sequential calculation of one-dimensional contour integrals is employed to map the fluid flow fields. The method is more accurate than that of Davies and Aylward, who used a numerical relaxation procedure, and the differences in the results of both techniques are demonstrated. The present technique is also used to generalize the Davies and Aylward results to provide predictions of impactor performance for jet deflecting plates inclined at arbitrary angles with respect to the incident jet. Theoretical predictions of optimum geometries for the impactor performance are thus provided. Section II outlines the theory, Section III gives the details of the computation procedure, Section IV the results and discussion, while Section V gives a brief summary.

II. THEORY

The theory of the assumed ideal fluid flow follows the complex-variable mapping procedure of Davies and Aylward. The desired two-dimensional device is shown as the z -plane in Fig. 1A. It is symmetrical about the axis EF , and comprises a jet of fluid, asymptotic velocity u_0 , confined by parallel walls AB , $A'B'$ a distance $h = 2w$ apart, incident upon two half-plates ED , ED' held at an angle β to the jet axis. The emerging fluid is bounded by the free streamlines BC , $B'C'$ having fluid velocity u_f . The other customarily quoted dimension is the horizontal distance s between the edge of the jet B and the plate vertex F . For plate angles other than 90° a different and more physically meaningful dimension is the related quantity s' , the perpendicular distance from B to the plate. To relate these physical quantities, as in Davies and Aylward, the W , Ω and ζ planes are introduced. W is the velocity potential such that $dW/dz = v_f^*$ the complex conjugate of the fluid velocity v_f , and the boundaries of the device form a horizontal strip in the W plane as shown in Fig. 1B. $\Omega = \ln(u_0 dz/dW)$ is the Kirchhoff variable, in which plane the boundaries of the flow become the infinitely long rectangle shown in Fig. 1C. These two variables are connected by mapping each of them into the upper half-plane of the subsidiary variable ζ , as shown in Fig. 1D. The analytic relationships are slightly different from those of Davies and Aylward because of the inclusion of a general plate angle but they follow straightforwardly from Schwartz-Christoffel transformations. Thus

$$\zeta^2 = 1 - \exp(\pi W/u_0 w), \quad (1)$$

$$\Omega = \left(\frac{\pi - \beta}{\pi} \right) \ln \frac{\zeta^2 - b + [(\zeta^2 - 1)(\zeta^2 - b^2)]^{1/2}}{\zeta^2 + b + [(\zeta^2 - 1)(\zeta^2 - b^2)]^{1/2}}, \quad (2)$$

where b , necessarily > 1 , is the location of the point B in ζ . The velocity ratio $\alpha = u_f/u_0$ is related to b by

$$\ln \alpha = (1 - \beta/\pi) \ln [(b + 1) / (b - 1)]. \quad (3)$$

For the special case $\beta = \pi/2$ Davies and Aylward have given analytic expressions, in parametric form, for the free streamline and other quantities. For the case of general β these relationships are not so straightforward, and in any case the procedure adopted in the present work, described in the next section, avoids the explicit need for such expressions.

For investigating the efficiency of the device as a particle size classifier, it is necessary to explore the trajectories of particles introduced into the incident jet. A particle of mass m when moving with velocity $\vec{v}$ at point $\vec{x}$ is assumed to be subject only to the Stokes force law, so that it obeys the two-dimensional vector equation of motion

$$m d\vec{v}/dt = K(\vec{v}_f(\vec{x}) - \vec{v}), \quad (4)$$

where $\vec{v}_f(\vec{x})$ is the fluid velocity at $\vec{x}$. In terms of the dimensionless variable $\tau = tu_0/w$ the equations to be solved can be conveniently written as

$$\begin{aligned} d(\vec{x}/w)/d\tau &= \vec{v}/u_0 \\ d(\vec{v}/u_0)/d\tau &= [\vec{v}_f/u_0 - \vec{v}/u_0]/\psi \end{aligned} \quad (5)$$

The Stokes parameter $\psi = mu_0/Kw$ characterizes the particle size. With the approximation that $K = 6\pi\eta a$, where a is the particle radius and η the viscosity, ψ takes the form $a^2(\frac{2}{9}\rho u_0/\eta w)$, where ρ is the particle density. For the initial condition that the particle is moving with

the fluid a distance y_0 from the axis at a point well upstream in the incident jet, the four coupled equations (5) must be solved to determine whether the particle strikes the plate or continues with the emerging jet. Referred to velocity components perpendicular and parallel to the plate, so that sufficiently far into the emerging jet $v_{f\perp} \approx 0$, it is necessary to determine if the trajectory has intersected the plate, or, by making an analytic solution to Eqs. (5) in this limit, to determine if it will do so asymptotically. This procedure defines a value of y_0 , as a function of ψ , for particles which do hit the plate according to this criterion.

The assumption that the incident particles are distributed uniformly over the incident jet leads to a fractional efficiency $E(\psi)$ for particle impaction. For the case that $|y_0|$ is a monotonically increasing function of ψ , $E = |y_0|/w$, and if $|y_0|$ is monotonically decreasing (as can occur when β is $\pi/3$, for example) then $E = 1 - |y_0|/w$. For the intermediate case when $y_0(\psi)$ is double-valued, $E = |y_{01} - y_{02}|/w$, where y_{01} and y_{02} are the two values. It is apparent that as a function of ψ , E is not analytic and need not have a continuous derivative. To characterize the sharpness of the discrimination, it is thus convenient to use, besides the dispersion or relative variation of E , defined as

$$S_E = [\psi(E=0.84) - \psi(E=0.16)]/[0.68\psi(E=0.5)], \quad (6)$$

also a similar quantity for the dispersion of $|y_0|$ itself:

$$S_y = [\psi(|y_0|/w=0.84) - \psi(0.16)]/[0.68\psi(0.5)]. \quad (7)$$

III. COMPUTATION

The method employed by Davies and Aylward to obtain a map of fluid velocities was to derive, for the special case $\beta = \pi/2$, a pair of parametric equations for the shape of the free streamline BC in the z -plane. Together with the straight borders AB, DE, and EF, these were then made the boundaries for a closed region inside which the Laplace equation for the velocity potential was solved by a numerical two-dimensional relaxation procedure. Such a method is inaccurate in its representation of the free-streamline boundary by points on a square mesh, and somewhat cumbersome in going again numerically over relationships already derived analytically. The procedure adopted in the present work was designed for a more complicated problem with less tractable analytic relationships (Forney, 1977; Forney, Ravenhall and Winn, 1978; Ravenhall, Forney and Jazayeri, 1978). It is described in some detail, however, since it is believed to be responsible for the differences between the present and the earlier calculations. The geometry of the z -plane is obtained by performing numerical complex arithmetic contour integrals in the W -plane, exploiting the analytic relationships (1) and (2) and the definition $\Omega = \ln(u_0 dz/dW)$, in the form

$$z(P_1) - z(P_2) = u_0^{-1} \int_{W(P_1)}^{W(P_2)} \exp(\Omega[z(W)]) dW. \quad (8)$$

For any selected pair of points P_1, P_2 in the W -plane, this relationship determines their separation in z . The points chosen may be those defining characteristic dimensions, such as B and E, or they may be a rectangular grid in W defining streamlines and equipotentials, but they

can in fact be any prescribable locations. (Accompanying the values of z is of course the fluid velocity, obtained from the integrand at the end-points). The procedure becomes inaccurate near the stagnation point E , where the integrand is large. For obtaining device dimensions it is not necessary to include E , since its location may be calculated from that of any other point on the plate. Thus for a given value of the ζ -plane parameter b , the value of s/w may be obtained by choosing $P_1 = B$, i.e. the value of W for $\zeta = b$, from Eq. (1), and P_2 to be any point on the line DE . The value of b which yields a desired s/w , for a given β , is found by iteration of this procedure using Newton's method. The value of b needed to give a prescribed $\alpha = u_f/u_o$ may be obtained directly from Eq. (3). For the z -plane velocity map needed to calculate particle trajectories, a mesh of points was chosen in the W -plane which was approximately rectangular far away from E , and a straight line along the lower boundary ABC , but which was deformed so that the closest point to E was $1/10$ of a mesh width from it. The z -coordinates of these mesh points were determined sequentially by a Simpson-Rule evaluation of Eq. (8), and recorded in a table with the corresponding values of the fluid velocity for later look-up. The region in the z -plane adjacent to E , and not covered by the table values, was approximated by the familiar analytic expression for fluid velocity near a corner with included angle β , scaled to match the closest tabulated point.

Compared to the procedure of Davies and Aylward, the method used here has the following advantages: it treats the free streamline exactly, rather than approximating it by rectangular mesh-points in the z -plane;

sequential calculation of one-dimensional integrals permits considerable reduction of integration steps and mesh sizes, and enlargement of the size of the region considered into the incident and emerging jets. It may thus be verified that the final results are independent, to the needed accuracy, of these computational considerations.

The particle trajectories were obtained by Runge-Kutta step-by-step integration of Eqs. (5). The fluid velocity at $\vec{x}$ was obtained by third-order interpolation from the four closest tabulated points, conceptually quite straightforward but for the case $\beta \neq \pi/2$ a computationally somewhat intricate process. As with the fluid-velocity table, all of the computational parameters involved, including the starting x-value and the distance covered of the emerging jet, were varied until the resulting values of y_0 and E were independent of them.

IV. RESULTS AND DISCUSSION

For the case $\beta = \pi/2$ explored in detail by Davies and Aylward, the results of the present investigation are compared in Figs. 2 and 3. The primary result of the trajectory calculation, the quantity $|y_0|/w$, Fig. 2, is seen to depart most from the earlier work near the extreme values $|y_0|/w = 1$, which may be attributed to their poor treatment of the free streamline, and near $|y_0|/w = 0$, where present experience shows that extreme care is needed for trajectories which pass near the stagnation point. The resulting efficiencies $E(\psi)$ for several values of α , Fig. 3, lack the "shoulders" at small E of the earlier work. The evident regularity in shape of these curves of $E(\psi)$ for different α is confirmed by the fact that up to $\alpha = 1.6$ the dispersion S_E , Eq. (6), is remarkably independent of α . For $\alpha = 2.5$, however, $|y_0|/w$ is double-valued over part of its range in Fig. 2, qualitatively the same result as was obtained by Davies and Aylward. This behavior provides the clue to possible improvement of the device characteristics, as is now shown.

The double-valuedness of the $\alpha = 2.5$ curve in Fig. 2 involves the crossing of particle trajectories having the same ψ but different y_0 . This is impossible physically since the trajectories are not analytic functions of z , and it may be understood with the following observations. There are two competing influences tending to deflect the particle trajectory. There is the Stokes-Law force on the particle in the direction of the emerging jet. Particles which start closer to the edge of the jet must move through more fluid to reach the plate, and from this influence, they require larger ψ to do so. This leads to a positive slope for $|y_0(\psi)|$. The other influence is the variation in

fluid velocity across the jet. From this effect, particles near the edge of the incident jet acquire a larger velocity than u_0 , making them harder to deflect, while those near the center of the jet have smaller velocity than u_0 and are easier to deflect. Under the latter influence, particle trajectories may cross and the curve $|y_0(\psi)|$ may have regions of negative slope. The latter influence becomes relatively stronger for larger α . This suggests that a more complicated interplay between these influences may be produced by extending the scope of the geometry, i.e. by using plate angles other than 90° . For $\alpha = 1.6$, plots of $|y_0|/w$ and E against ψ are shown in Fig. 4 for three angles 90° , 75° and 60° . At 60° $|y_0|/w$ decreases monotonically with ψ , the opposite behavior from that at 90° . The efficiency curves E by definition are monotonically increasing, and are rather similar at these two angles. The 75° case, almost intermediate between them, produces an almost vertical $|y_0|/w$ curve, and consequently a much sharper $E(\psi)$. The dispersion S_E is only 0.018, compared to the 90° value of 0.289.

To explore the dependence on β of this improved ability to discriminate particle sizes, it is desired to calculate contours of constant S_y and S_E for ranges of the velocity ratio α and the plate angle β . For simplicity of computation and interpolation, we use the quantity S_y , for which only three values of ψ , associated with $|y_0|/w = 0.84$, 0.50 and 0.16, need be found. Since $|y_0|/w$ is a smooth function, the use in interpolation of its "dispersion" S_y is more meaningful. Except where the $|y_0|/w$ curves are double valued, in which case S_y and S_E are both small, $S_E = |S_y|$, so that attribution of S_E values is accurate except very near the minimum. These contours, for ranges of values of α and β , are shown in Fig. 5. As indicated earlier, for $\beta = 90^\circ$ and $\alpha \geq 1.6$

the dispersion S_E is almost constant. This now generalizes into the property that for $\alpha \gtrsim 1.6$ and $90^\circ \gtrsim \beta \gtrsim 60^\circ$ contours of constant S_E are all roughly parallel to the α axis. From Fig. 5, the value of β needed to produce the sharpest possible discrimination may be read off as a function of the desired α . For values of α less than 2, it is achieved by using a plate angle $\beta = 75^\circ$. The dashed contours shown also in Fig. 5 give s'/w , the perpendicular distance from the edge of the jet to the plate, a dimension of more direct utility than α in constructing a device. It may be estimated that for $\beta = 90^\circ$, only at $s'/w (=s/w) = 0.65$ is the size discrimination optimal.

Since the present work modifies some details of earlier calculations, it is of interest to repeat a comparison, made previously by Marple, in which streamlines are obtained by solving the full Navier-Stokes equations. For the case $\beta = 90^\circ$ and $s/w = 1.0$, i.e. Marple's case $s/h = 0.5$, the streamlines obtained in the present work are superimposed on those of Marple in Fig. 6. Obvious differences in the behavior near the solid walls are the expected consequence of the inclusion of a boundary layer in the Navier-Stokes solution. The better treatment of the upper boundary, corresponding to the free streamline, in the present work probably has little effect on the particle trajectories. The efficiency E of particle collection plotted against the more frequently used size parameter $\psi^{1/2}$ is shown in Fig. 7 for the potential flow and the Navier-Stokes calculations, together with the approximation of Mercer and Chow. Apart from an overall small shift in position, the first two efficiencies differ qualitatively only at $|y_0|/w \gtrsim 1$, an expected consequence of the boundary layer. The previous marked difference of Davies and Aylward at $|y_0|/w \gtrsim 0$ has now disappeared.

The dispersion S_p is respectively 0.114, 0.117, and 0.303 for the potential flow, Navier-Stokes and Mercer-Chow calculations, emphasizing the overall similarity of the first two methods.

CONCLUSIONS

Improved computational techniques are used to characterize the performance of particle inertial impactors. Considering ideal fluid flow, an accurate conformal mapping technique using sequential calculation of one-dimensional contour integrals is employed to map streamlines and velocity fields. Using these numerical methods with the incorporation of an analytic expression for fluid properties near the stagnation point, particle trajectories are accurately simulated for a variety of impactor geometries to predict instrument efficiencies. The theoretical results indicate that an optimum jet deflecting plate angle exists for a given jet-to-plate spacing at which the slope of the particle collection efficiency curve is maximized. In general, optimum performance is predicted for unconventional plate angles $\beta \neq 90^\circ$.

ACKNOWLEDGMENTS

The research described was supported by National Science Foundation grant PHY75-21590 (D.G.R.) and Energy Research and Development Agency grant ERDA-EE-77-S-02-4319 (L.J.F.)

REFERENCES

- Davies, C. N. and Aylward, M. (1951), Proc. Phys. Soc. (London) B, 64, 889.
- Forney, L. J. (1977), Final Rept. NSF ENG75-10303, Rept. UILU-ENG-77-2006,
Dept. of Civil Engr., University of Illinois, Urbana, Illinois.
- Forney, L. J., Ravenhall, D. G. and Winn, D. S. (1978), J. Applied Physics,
49, No. 3.
- Marple, V. A. (1970), Ph.D. thesis, Dept. of Mechanical Engr., University
of Minnesota, Minneapolis, Minnesota.
- Mercer, T. T. and Chow, H. Y. (1968), J. Coll. and Interface Sci., 27, 75.
- Ranz, W. E. and Wong, J. B. (1952), I & E Chem., 44, 1371.
- Ravenhall, D. G., Forney, L. J. and Jazayeri, M. (1978),
J. Coll. and Interface Sci., 64, No. 3.

FIGURE CAPTIONS

- Figure 1. Transformation planes.
- Figure 2. Variation of Stokes number vs. starting coordinate at onset of particle impact for plate angle $\beta = 90^\circ$.
- Figure 3. Particle collection efficiency vs. Stokes number for $\beta = 90^\circ$.
- Figure 4. Variation of starting coordinate and particle collection efficiency vs. Stokes number at plate angles $\beta = 60^\circ$, 75° and 90° for $\alpha = 1.25$ (for $\beta = 90^\circ$, $E = |y_0|/w$).
- Figure 5. Contours of constant dispersion S_y on β vs. α plot. Dashed lines correspond to fixed jet-to-plate spacing s'/w (for $\beta = 90^\circ$, $s'/w = s/w$).
- Figure 6. Streamlines for rectangular impactor at $s/w = 1.0$. Solid lines represent potential flow solution while dashed lines represent Navier-Stokes results ($Re = 3000$) after Marple (1970).
- Figure 7. Comparison of particle efficiency curves for $s/w = 1.0$ and $\beta = 90^\circ$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7