Dr. 1827

LA-8512-MS

Informal Report

192
11/5/80

Excluded Flux Analysis of a Field Reversed Plasma

MASTER

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

Excluded Flux Analysis of a Field Reversed Plasma

Michel Tuszewski

DISCLAIMER

This document contains information which was prepared by the University of California, Los Angeles, under contract to the U.S. Department of Energy. The U.S. Government is authorized to reproduce and distribute reprints for government purposes not withstanding any copyright notation that may appear hereon. This document is intended to provide a basis for the development of standards and procedures for the design and construction of fusion reactors. It is not intended to be used as a basis for the design and construction of fusion reactors. The U.S. Government is authorized to reproduce and distribute reprints for government purposes not withstanding any copyright notation that may appear hereon.

EXCLUDED FLUX ANALYSIS OF A FIELD REVERSED PLASMA

by

Michel Tuszewski

ABSTRACT

We describe in this report a simple eigenvalue approach for mapping the vacuum field between an assumed separatrix and a conducting wall. This allows an improved determination of the plasma shape in Field Reversed Experiments (FRX) compared to previous estimates based on excluded flux radius measurements. It is also shown that an array of magnetic field diagnostics located outside the discharge tube cannot resolve features of the plasma shape that have scale length smaller than about a coil radius.

I. INTRODUCTION

In Field Reversal Experiments,¹ a simple but important diagnostic is the excluded flux array which yields the plasma dimensions. The flux ϕ and the B_z component of the magnetic field are measured at various axial (z) positions with loops and probes located between the theta pinch coil and the discharge tube to obtain the excluded flux radius $r_E(z)$, defined as

$$\phi_p = B_p \pi (r_p^2 - r_E^2) = \int_0^{r_p} B_z 2\pi r dr, \quad (1)$$

where $\phi_p = \phi(r_p, z)$, $B_p = B_z(r_p, z)$ and r_p is the radial position of the probes and loops. For straight field lines, neglecting the plasma pressure on open field lines, one can show (see Appendices A and B) that r_E is equal to the separatrix radius r_s . This latter quantity is of great interest since it gives precisely the dimensions of the compact torus. However, in the presence of field curvature (e.g. due to finite plasma length), r_E is either greater or

smaller than r_s depending on whether the field curvature is positive or negative at the given axial position and one has to further analyze the excluded flux data to obtain the plasma (separatrix) profile. We first introduce the potential problem to be solved and present a simple computational approach to its solution. Then, we give numerical results for an idealized example for which some analytical solutions can also be obtained and for some experimental data of the FRX-B experiment.¹ We then discuss the limitations of the excluded flux analysis and finally conclude.

II. SOLUTION TO THE POTENTIAL PROBLEM

We wish to determine the separatrix profile $r_s(z)$ from experimental excluded flux data obtained from flux loops and B_z probes located just inside the theta-pinch conducting wall. We assume that the plasma pressure on the open field lines region between separatrix and conducting wall can be neglected as shown in A. This problem is simply stated in terms of a 2-D potential (the stream of flux function $\psi = \phi/2\pi$) as shown in Fig. 1(a). We introduce dimensionless variables $x = r/r_w$, $y = z/r_w$ where r_w is the coil inner radius. $\psi(x,y)$ is normalized to the wall value $\psi_w(1,y)$. On a given shot and at a given time, we assume ψ_w to be a constant. In the open field lines region between the separatrix ($\psi = 0$) and the coil ($\psi = 1$), $\psi(x,y)$ obeys the vacuum Grad-Shafranov equation,²

$$\Delta^*\psi = x \frac{\partial}{\partial x} \left(\frac{1}{x} \frac{\partial \psi}{\partial x} \right) + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad . \quad (2)$$

Equation (2) is a second order partial differential equation which is to be solved with the following boundary conditions:

$$\psi(1,y) = 1, \quad \psi(x_s(y), y) = 0 \quad , \quad (3)$$

where $x_s(y)$ is the separatrix profile. Solving for ψ , one can obtain $B_z = (1/x)\partial\psi/\partial x$ and in turn the excluded flux radius at any point using

Fig. 1.

(a) The potential problem; (b) choice of current sources (+) and boundary points (o).

Eq. (1). To solve Eq. (2), various methods are available. One can use a Partial Differential Equation solver which will give a mapping of ψ on a grid.³ Alternatively, one can introduce surface currents on the conducting boundaries and iterate on the current density profiles to satisfy the boundary potential conditions, a physical but mathematically indirect procedure.⁴ These approaches require the use of large computer codes. In the hope of a simpler solution, we

chose instead to define a set of N eigenfunctions ψ_m satisfying Eq.(2) to construct an approximate solution

$$\psi = \sum_{m=1}^N c_m \cdot \psi_m \quad , \quad (4)$$

where the constants c_m are to be determined by requiring ψ to satisfy N boundary conditions; $\psi = 1$ at K locations on the coil boundary and $\psi = 0$ at $N-K$ locations on the separatrix. This set of boundary conditions can be written on a matrix form $AX = B$ where $A = (a_{i,m})_{N,N}$ with $a_{i,m} = \psi_i(x_m, y_m)$, $X = (c_m)_{N,N}$ and $B = (b_i)_{N,1}$ with $b_i = 1$ for $i \leq K$ and $b_i = 0$ for $i > K$. One solves for x by matrix inversion so that the N unknown coefficients c_m are uniquely determined and $\psi(x,y)$ is then known everywhere as seen with Eq. (4). This approach can be subject to serious convergence problems in practice, depending on the choice of the eigenfunctions ψ_m . One often experiences some oscillations similar to those sometimes obtained when fitting a polynomial of order $N-1$ to N points. This was the case for our first two choices for ψ_m 's, from separation of variables of Eq. (2). We used:

$$\psi_m = x I_1(\alpha m x) \cos(\alpha m y) \quad m = 0, 1, 2, \dots \quad (5)$$

where $\alpha = 2\pi l/r_w$ or, by linear combination and differentiation⁵

$$\psi_m = x^2 \sum_{p=0}^m \frac{(-1)^p \left(\frac{x}{2}\right)^{2(m-p)} y^{2p}}{(m-p)!(m-p+1)!2p!} \quad m = 0, 1, 2, \dots \quad (6)$$

These sets of eigenfunctions did not allow convergence (e.g. as N increased, strong oscillations of ψ developed) but, in the second case described by Eq. (6), a single plasma term $\psi = -br^4/4$ was included (effectively a source of ψ "inside" the separatrix) and when solving then the full Grad-Shafranov equation $\Delta^*\psi = -br^2 = -r^2 p'(\psi)$, a rapid convergence was found. Similar convergence problems have been described by Lewis.⁶ He used a combination of polynomials (similar to those of Eq. (6)) and of current sources "inside" the

conducting boundaries for the ψ_m 's to solve a wide range of potential problems. These last two choices of eigenfunctions led to the choice of current loop sources well "inside the conductors" as a set of ψ_m . A given source located at x_m, y_m contributes to

$$\psi_m = \frac{[(x + x_m)^2 + (y - y_m)^2]^{1/2}}{2} \left[\left(1 - \frac{k^2}{2}\right) K(k) - E(k) \right] \quad (7)$$

where $k^2 = 4x_m / [(x + x_m)^2 + (y - y_m)^2]$ and E and K are the complete elliptical functions of argument k. The choice of the locations of the current sources (+) and of the boundary points (o) is not critical provided one uses some empirical rules⁶ such as maintaining about equal separation between sources and points and between successive points ($\Delta_2 \sim \Delta_1$, see Fig. 1(b)) and following similar patterns for boundaries and current source lines. These are not crucial requirements, but a judicious choice of locations is effective in bettering the approximation and minimizing N.

III. NUMERICAL RESULTS

An ideal FRX example shown in Fig. 2 is first considered. The separatrix was chosen as a straight line ($x_s = 0.5$) for $y < 2$ and as part of a circle for $2 < y < 2.5$. With $N = 8$, a solution was found with a precision of 15%. With $N = 18$ with a precision of 2-3% and with $N = 27$ as in Fig. 2, ψ was mapped everywhere between the boundaries with a precision of about 1%. Fig. 2 displays contours of constant ψ in solid lines for $\psi = 0, 0.2, 0.4, 0.6, 0.8$ and 1.0. The 27 current sources (+) and boundary points (o) are indicated on Fig. 2 as well as two excluded flux radius profiles; one (dotted line) with only the contribution of the plasma by normalizing ϕ and B_z to their respective vacuum values for the finite length coil and the other (long-dashed line) with the contribution of both plasma and finite length coil by normalizing ϕ and B_z to their respective values in the case of straight field lines (e.g. infinite length coil). The expressions for the excluded flux radius in both cases are given in Appendix B. The main features of Fig. 2 are:

Fig. 2.

Numerical solution for an ideal FRX example.

- (1) For $y < 2$ and $y > 3$ where field lines are fairly straight, one can solve analytically as done in Appendix B and find excellent ($< 1\%$) agreement between the approximation and the exact solution.
- (2) The excluded flux radius profiles calculated from values of ψ and B_z at $x_p = 0.964$ (FRX-B parameter) exhibit the following features:
 - (i) $x_E \sim x_s$ where field lines are straight; $x_E = x_0$ for $y < 2$ near the center of the coil and $x_E = 0$ for $3.5 < y < 4.5$ in the vacuum region. In the latter case, the erratic behaviour around 0.1 reflects x_E the random 1% error on ψ and B_z when both plasma and vacuum contributions are considered. However, these errors cancel out when normalizing to the vacuum field (dotted line).
 - (ii) $x_E > x_s$ where field curvature is positive; vacuum curvature for $y > 4.5$ and plasma curvature for $2.5 < y < 3.5$

(iii) $x_E < x_S$ due to the negative plasma curvature around $y = 2$.
 These features are in agreement with the analysis given in Appendix B.

We now consider some experimental data of the FRX-B device. Figure 3 presents results for the shot #1424, at five different times following the main

Fig. 3.

Analysis of the FRX-B shot 1424 at $t = 8, 18, 28, 38$, and $48 \mu s$ after the main theta-pinch discharge. The measured excluded flux radii are indicated with a circle. The calculated excluded flux and separatrix profiles are shown in dotted and solid lines, respectively.

theta-pinch discharge. For this shot, r_w is 12.5 cm and r_p is 10.4 cm. The excluded flux array consists of five stations and the values of r_E at various times are indicated with circles in Fig. 3. The separatrix is modeled by a set of five points connected by straight lines and a portion of parabola for the final section. The initial choice for the five separatrix points is the set of excluded flux radii. For a given separatrix profile, one calculates the excluded flux profile and compares it with the experimental data. One iterates on the separatrix profile until the calculated r_E profile matches the five experimental values. At each time, as given in Fig. 3, one plots the final separatrix profile (solid line) and the corresponding excluded flux radius profile (dotted curve). The latter profile represents only the contribution of the plasma curvature and is obtained with Eq. (B-3). This analysis allows a more accurate description of the plasma dimensions (e.g. volume and length for energy balance). For the case of Fig. 3, the plasma half-length from the separatrix profile is shorter by about 10% than the one inferred from the excluded flux radius profile. The measurements of ψ and B_z have an experimental uncertainty of 1-2%. This causes some error on the values of x_E , in particular, as x_E decreases towards zero; for example, consider Eq. (B-3) and a 1% relative variation for the quantity (ψ_p/ψ_o) (B_o/B_p). In a vacuum region where $x_E = 0$, this variation will result in $\Delta x_E = 0.1$, while at the coil center where $x_E = 0.5$, the same variation will result in $\Delta x_E = 0.02$. For the cases presented in Fig. 3, the errors are small (less than 4%) except for the fifth station, where $x_E \sim 0.2 \pm 0.04$, which gives an uncertainty of about 5% for the plasma (separatrix) half-length. As discussed in Appendix A, the neglect of the plasma pressure on the open field lines in the neighborhood of the fifth station could also contribute to some additional uncertainty.

IV. LIMITATIONS OF AN EXCLUDED FLUX ANALYSIS

The results shown in Figs. 2 and 3 suggest that the excluded flux radius profile gives only a smoothed version of the plasma (separatrix) shape because the available information at $r_p \sim r_w$ is far from the radius of interest (r_s) and involves an integral over the open field line region ($r_s < r < r_p$) as seen with Eq. (1). Therefore, a given excluded flux radius profile can be consistent with a range of separatrix profiles. In this section, we investigate this question by considering again the ideal case of Fig. 2 and

imposing on the straight portion of the separatrix ($0 < y < 2$) a perturbation of variable amplitude, cosine or Gaussian. Physically, this corresponds to various tearing modes. Dimensionless variables $x = r/r_w$ and $y = z/r_w$ are used and symmetry about $y = 0$ is assumed as in Fig. 2, so that only a half-coil solution is presented for each case. To accommodate the increase in boundary length due to the perturbations, more current sources and boundary points are used to model the separatrix. With $N = 66$ eigenfunctions, a precision from $\sim 1\%$ at worse (Fig. 4(j)) to $\sim 0.1\%$ at best (Fig. 4(a)) is obtained. Each graph of Fig. 4 consists of the equipotential boundaries ($\psi = 1$, the coil, and $\psi = 0$, the separatrix) in solid lines and of the excluded flux radius profile in dotted line from values of ψ and B_z at $x_p = 0.964$.

The fit of r_E to r_s is only a function of the relative values of h , the distance between diagnostics and perturbed separatrix, and δ , the characteristic scale length of the perturbation. These distances are indicated in Fig. 4(e). It is also useful to introduce a mode number m for cosine perturbation as the number of constrictions for an entire coil (e.g. the plasma tears in $m + 1$ fragments and, for example, $m = 2$ for the case of Fig. 4(e)). For the cosine perturbation considered here, $\delta \sim 2/m$. The results given in Fig. 4 show that the excluded flux radius profile follows quite well the separatrix perturbation for $\delta \sim 2$ ($m = 1$, Figs. 4(a) and 4(b)), poorly for $\delta \sim 0.6$ (Figs. 4(c) and 4(d)) and $\delta \sim 0.8$ ($m = 2$, Figs. 4(e) and 4(f)) and not at all for $\delta \sim 0.5$ ($m = 4$, Figs. 4(g) and 4(h)) and $\delta \sim 0.3$ ($m = 6$, Figs. 4(i) and 4(j)). In the last two cases, the excluded flux radius is quite insensitive to an increase of perturbation amplitude. Information on the plasma shape is possible only for $\delta > h$ (h is of order unity for the cases of Fig. 4). Similar results were obtained for $x_s = 0.85$ ($x_s = 0.5$ for the above cases) which brings the separatrix to a minimum distance of the diagnostics array since the inner quartz discharge tube radius is ~ 0.9 in FRX devices. The previous discussion for each mode still applies with a slight improvement in the fit of r_E to r_s relative to the respective cases of Figs. 4 and 5 because of the decrease in h .

Therefore, we conclude that an excluded flux array outside the discharge tube can only resolve a structure on the separatrix which has a scale length greater than about a coil radius. This limits the study of possible tearings to the lowest modes only ($m = 1$ and perhaps $m = 2$) since a typical FRX plasma length is about 4-6 coil radii. However, theoretical studies⁷ indicate that

Fig. 4.

Perturbed separatrices (a) and (b) cosine ($m=1$) perturbation, (c) and (d) Gaussian perturbation, (e) and (f) cosine ($m=2$) perturbation, (g) and (h) cosine ($m=4$) perturbation, (i) and (j) cosine ($m=6$) perturbation.

the most likely tearing states are those of lowest values of m . Furthermore, it is clear that in the analysis of the experimental data of Fig. 3, the separatrix profiles presented are only the smoothest possible choices and that an infinite combination of perturbed separatrices could also satisfy the experimental data. Finally, it appears from the results of this section, that it is not necessary to have a separation between successive stations in the excluded flux array of less than about half a coil radius since it is the minimum resolvable distance.

V. CONCLUSIONS

We have presented a simple numerical approach to analyze the excluded flux data of FRX experiments. Neglecting plasma pressure on open field lines, one solves the vacuum Grad-Shafranov equation between two equipotential surfaces, the conducting wall where $\psi = 1$ and the separatrix where $\psi = 0$. The plasma profile inside the separatrix is not needed for this analysis. One finds by iterative procedure a separatrix profile that is consistent with the excluded flux measurements. It is shown, however, that there is a multiple solution to this problem since the excluded flux array cannot resolve a characteristic scale length of the separatrix profile smaller than about a coil radius. This limits in particular the study of tearing to the very lowest modes. A more refined analysis could include plasma models inside as well as outside the separatrix.

ACKNOWLEDGMENTS

I wish to thank many members of the CT program at Los Alamos Scientific Laboratory for several valuable discussions, in particular, W.T. Armstrong and R.K. Linford, who also suggested this problem. I also gratefully acknowledge the descriptions of their respective numerical codes by C. Ekdahl, T. Hewitt, K. Lewis and L. Mann.

REFERENCES

1. W.T. Armstrong, R.K. Linford, J. Lipson, D.A. Platts and E.G. Sherwood, (submitted to Physics of Fluids, 1980).
2. G. Bateman, MHD Instabilities, (MIT Press, Cambridge, 1978) p. 66.
3. D.A. Baker and L.W. Mann, Proc. of Second Topical Conference on Pulsed High- β Plasmas, Garching by Munich, (1972), p. 69.
4. C. Ekdahl, private communication, 1980.
5. H. Weitzner, J. Hammer and H. Berk, Bull. Am. Phys. Soc. 24, 954 (1979).
6. R. Lewis, Jour. Appl. Phys., 37, 2541 (1966).
7. J.M. Finn and P.K. Kaw, Physics of Fluids 20, 72 (1977).

APPENDIX A
PLASMA PRESSURE ON OPEN FIELD LINES

To estimate the contribution to the excluded flux of the plasma pressure on open field lines, we restrict ourselves for simplicity to the 1-D case of straight field lines. We consider the central part of the plasma, where the separatrix radius is r_s and the end of the coil, where the separatrix radius is zero. Equation (1) gives

$$B_p \pi (r_p^2 - r_E^2) = 2\pi \int_{r_s}^{r_p} B_z r dr \quad , \quad (A-1)$$

where $B_p = B_z(r_p)$, r_p is the radial position of the probes and flux loops, and where one has used the relation $\phi(r_s) = 0$. The radial pressure balance can be written as

$$B_z = B_p(1 - \beta)^{1/2} \quad , \quad (A-2)$$

where $\beta(r)$ is the ratio of plasma pressure to external magnetic pressure $B_p^2/2\mu_0$. Using Eq. (A-2) in Eq. (A-1), one obtains

$$r_E^2 = r_p^2 - 2 \int_{r_s}^{r_p} (1-\beta)^{1/2} r dr \quad . \quad (A-3)$$

For the case of no plasma pressure on open field lines, $\beta = 0$ and Eq. (A-3) yields $r_E = r_s$. For $\beta(r) \neq 0$, $r_E > r_s$ and we define the contribution of the plasma pressure by

$$\Delta r_E = r_E - r_s \quad , \quad (A-4)$$

where r_E is obtained with Eq. (A-3). We first estimate Δr_E by assuming a rigid rotor profile for the plasma on open field lines, as shown in Fig. A-1(a). The plasma pressure profile is

Fig. A-1.

(a) Rigid rotor profile. (b) Truncated rigid rotor profile. (c) Curve of the rigid rotor parameter K as function of r_s/r_w for 2-D equilibria.

$$\beta(r) = \text{sech}^2 K \left(\frac{2r^2}{r_s^2} - 1 \right) \quad , \quad (\text{A-5})$$

where $K = -B(0)/B_p$ is arbitrary for 1-D case. However, with the constraint of a 2-D equilibria,¹ K is uniquely specified for a given r_s with

$$\frac{\tan hK}{K} = 1 - \left(\frac{r_s}{r_w} \right)^2 \quad , \quad (\text{A-6})$$

where r_w is the conducting wall radius. A graph of K as a function of r_s/r_w is shown in Fig. A-1(c). Using Eqs. (A-3) to (A-6) with $r_w = 12.5$ cm, and $r_p = 10.4$ cm (FRX-A and B values), one obtains Δr_E as function of r_s , as given in Fig. A-2(a). We note that, for $r_s \sim 5-7$ cm (range of the observed experimental values), Δr_E is only about 10% of r_s . This value is an upper bound on Δr_E because of the rapid end loss and thermal conduction on open field

Fig. A-2.

Contribution of Δr_E of the open field lines plasma pressure to the excluded flux radius as a function of separatrix radius r_s for: (a) rigid rotor profile, and (b) truncated rigid rotor profile.

lines which decrease β well below the rigid rotor profile value of Eq. (A-5). Experimental measurements¹ suggest that, outside the separatrix, the plasma pressure drops quite rapidly on a scale length of about one ion gyroradius ρ_i . We therefore consider the more realistic β profile of Fig. A-1(b) with

$$\beta(r) = \beta_s \left(1 - \frac{r - r_s}{\rho_i} \right) , \quad (A-7)$$

where β_s is $\beta(r_s)$ as given by Eq. (A-5). For this β profile, $r_w = 12.5$ cm, $r_p = 10.90$ and $\rho_i = 0.4$ cm, the graph of Δr_E as function of r_s is given in Fig. A-2(b). For $r_s \sim 5-7$ cm, $\Delta r_E < 0.1$ cm, which is a negligible increase.

We consider now the end of the coil region. For the model of Fig. A-1(b), one expects that the plasma lost in the thin central annular region $r_s, r_s + \rho_i$ will exhaust in a jet of radius $\sim (2r_s\rho_i)^{1/2}$ by conservation of flux. Pressure

probe measurements on FRX-A¹ confirm this radial dependence. We approximate this jet by the following profile

$$\beta(r) = 1 - \frac{r^2}{2r_s \rho_i} \quad , \quad (\text{A-8})$$

assuming a maximum value of unity for the peak β on axis. Using Eq. (A-8) in Eqs. (A-3) and (A-4), one obtains

$$\Delta r_E = \left(\frac{2r_s \rho_i}{3} \right)^{1/2} \quad , \quad (\text{A-9})$$

where r_s and ρ_i refer to the central region plasma. For $r_s = 6$ cm and $\rho_i = 0.4$ cm, Eq. (A-9) gives $\Delta r_E \sim 1.3$ cm. The experimental uncertainty of 1-2% on the measurements of ϕ and B_z gives to r_E a random value of 1 to 2 cm in this end region so that the contribution of the plasma pressure (1 cm) cannot presently be resolved experimentally. In the intermediate region where the separatrix radius drops towards zero and where one measures typically an excluded flux radius of 2 to 3 cm, the open field line plasma pressure could account for ~ 10 -30% of the excluded flux radius.

APPENDIX B
EXCLUDED FLUX AND SEPARATRIX RADII

The relationship between the excluded flux radius r_E and the separatrix radius r_S , at any axial position, can be obtained with Eq. (1). Using $\Psi = \phi/2\pi$ and dimensionless variable $x = r/r_w$ and $y = z/r_w$, we have

$$\psi_p = \int_0^{x_p} B_z x dx = \int_{x_s}^{x_p} B_z x dx = \frac{\bar{B}}{2} (x_p^2 - x_s^2) \quad , \quad (B-1)$$

where $\bar{B}$ is the radial averaged B_z field value at a given y . Equation (1) yields then

$$x_L^2 = x_s^2 + \left(1 - \frac{\bar{B}}{B_p}\right) (x_p^2 - x_s^2) \quad . \quad (B-2)$$

One distinguishes three cases depending on the magnetic field curvature as shown in Fig. B-1. Using Eq. (B-2) and noting that x_p is greater than x_s for any value of y , one obtains for the different cases; (a) $x_E < x_s$ for negative curvature, (b) $x_E = x_s$ for straight field and, (c) $x_E > x_s$ for positive curvature

Fig. B-1.

The three cases of magnetic field curvature: (a) negative curvature, $\bar{B}/B_p > 1$; (b) straight field, $\bar{B}/B_p = 1$; and (c) positive curvature, $\bar{B}/B_p < 1$.

curvature. Therefore, generally, excluded flux and separatrix radii have different values. The above results are helpful in understanding the features of Figs. 2 and 3 and show that the value of the excluded flux radius depends on the magnetic field curvature. This curvature can be present either in the vacuum magnetic field (e.g. finite length theta-pinch coil) or because of the presence of a plasma of arbitrary shape. We can choose to include both contributions by normalizing ϕ and B_z to their respective values in the case of straight field lines (e.g. no curvature) or to keep only the contribution of the plasma to the excluded flux by normalizing ϕ and B_z to their respective values in the vacuum field configuration. Experimentally the latter choice is made since only the plasma features are of interest; a vacuum shot is used as a normalization for ϕ and B_z to obtain the excluded flux radius at each station. Using Eq. (1) for a plasma shot (ψ_p, B_p) and a vacuum shot (ψ_o, B_o) for which $x_E = 0$, one obtains

$$x_E = x_p \left(1 - \frac{\psi_p}{\psi_o} \cdot \frac{B_o}{B_p} \right)^{1/2} . \quad (B-3)$$

Equation (B-3) gives the excluded flux radius profile due to the plasma only and is used for the dotted curve of Fig. 2. Using the straight field lines values of $\psi_o = x_p^2$ and $B_o = 2$ in Eq. (B-3), one obtains

$$x_E = x_p \left(1 - \frac{\psi_p}{\psi_o} \cdot \frac{2}{B_p} \right)^{1/2} . \quad (B-4)$$

Equation (B-4) now includes the contribution of both plasma and vacuum field and is used for the long dashed curve of Fig. 2. For the 1-D case of straight field lines, Eq. (2) can be solved with the boundary conditions given by Eq. (3). One obtains

$$\psi = \frac{x^2 - x_s^2}{1 - x_s^2} \quad \text{and} \quad B_z = \frac{2}{1 - x_s^2} . \quad (B-5)$$

In Fig. 2, for $y < 2$ where $x_s = 0.5$ and for $y > 3$ where $x_s = 0$, one can use Eq. (B-5) with $\psi = \text{constant}$ to find an excellent agreement between exact and approximate solutions for the field lines. Using Eq. (B-5) in Eq. (B-4), one obtains $x_E = x_s$ as expected for straight field. Furthermore, for the FRX-B experimental value of $x_p = 0.964$, one can use Eq. (B-5) to get values of ψ_p/ψ_0 and B_p/B_0 as function of x_s , which are the respective contributions to the excluded flux radius of flux loops and B_z probes, respectively. These values are given in Table B-1. One observes from Table B-1 that most of the information for the excluded flux radius comes from the B_z probes, which reflects the proximity of the flux conserving coil. This suggests using only one flux loop as close to the coil radius as possible in practice (e.g. the coil itself could be used) to get the relative variation of ψ_w from shot to shot and an array of magnetic probes. Because of the proximity of the straight wall which implies $\partial B_z/\partial r \rightarrow 0$ as $r \sim r_w$, the radial location of the B_z probes is not critical, in the neighborhood of r_w . One obtains then the excluded flux radius as given with Eq. (B-3), where x_p refers to the flux loop radius and $\psi_p/\psi_0 \sim \psi_w/\psi_0$ being a constant at a given time.

TABLE B-1

x_s	0	0.5	0.6	0.7	0.8	0.9
ψ_p/ψ_0	1	0.975	0.957	0.927	0.865	0.676
B_p/B_0	1	1.333	1.563	1.961	2.778	5.263