SINGLE PASS COLLIDER MEMO

CN-364

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DATE: June 20, 1988

TITLE: DIFFERENCE ORBITS AND BPM-RESOLUTIONS*

SLA -CN--364

DE88 013255

I. Introduction

In the interaction region of the SLC, the information of beam position monitors is used to get crucial information on the beam parameters, like the position, the inclination, or the deflection of the beams at the interaction point. The information is usually extracted from the BPM readings by fitting orbits, using the known twiss parameters at the BPM positions. The quality of these fits depends, however, on the assumed resolutions of the BPMs. This is a particularly serious problem in the interaction region, since BPMs with quite different resolutions and very different values for the beta-function have to be combined [1].

This note presents a method which allows empirical extraction of BPM-resolutions from the orbit-fits. The method has been originally developed in reference [2]. It has been tested by Monte Carlo simulations and successfully exploited for the measurement of spatial resolutions of a time projection chamber.

II. The General Method

Let $\mathbf{s} = (s_1, \dots, s_N)^T$ be a set of N coordinates at which measurements $\mathbf{y} = (y_1, \dots, y_N)^T$ are performed. Let $\sigma_1, \dots, \sigma_N$ be the unknown (gaussian) resolutions of these measurements and let $e_1, \dots, e_N$ be any estimate of these resolutions.

Remark : Mathematically the results for the resolutions will not depend on these estimated resolutions. A better estimate will, however, lead to better numerical stability and to higher statistical precision of the method.

Example : In the example discussed in chapter IV, six BPMs located near the interaction region will be analyzed. Therefore $N = 6$, and $s_1, \dots, s_6$ are the longitudinal coordinates of the BPMs, $y_1, \dots, y_6$ represent horizontal or vertical positions measured by the BPMs, and $\sigma_1, \dots, \sigma_6$ are the BPM resolutions.

Let the physics of the measured quantity be known to be describable by a function f

$$s \mapsto f(s; p_1, \dots, p_n) = f(s; \mathbf{p})$$

* Work supported by the Department of Energy Contract DE-AC03-76SF00515.

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where $\mathbf{p} = (p_1, \dots, p_n)^T$ are n free parameters. These parameters can be determined from the measurements $\mathbf{y}$ (if $N \geq n$) by an error-weighted least squares fit $P: \mathbb{R}^N \rightarrow \mathbb{R}^n$:

$$P: \mathbf{y} \mapsto \mathbf{p}(\mathbf{y}) \quad (2)$$

where the mapping P is defined by the condition

$$\chi^2 = \sum_{i=1}^N \frac{1}{\epsilon_i^2} (y_i - f(s_i; \mathbf{p}))^2 = \text{Min}. \quad (3)$$

Example : Let f describe the path of a charged particle moving perpendicular to a homogeneous magnetic field. In this case, $n = 3$ and p_1, p_2, p_3 will describe the center and the radius of the circular trajectory. The function f is then a parametrization of the circle.

Let $\mathbf{r} = (r_1, \dots, r_N)^T$ be the vector of fit-residuals:

$$r_i = y_i - f(s_i; \mathbf{p}(\mathbf{y})) \quad (4)$$

The width of the (gaussian) distribution of residuals (for many independent measurements) can thus be related to the unknown resolution σ_i of the measurements y_i at s_i :

$$\begin{aligned} \delta r_i^2 &= \sum_{j=1}^N \left(\frac{\partial r_i}{\partial y_j} \right)^2 \sigma_j^2 \\ &= \sum_{j=1}^N \left(\delta_{ij} - \frac{\partial f(s_i; \mathbf{p}(\mathbf{y}))}{\partial y_j} \right)^2 \sigma_j^2 \\ &= \sum_{j=1}^N \left(\delta_{ij} - \sum_{k=1}^n \frac{\partial f}{\partial p_k} \Big|_{(s_i; \mathbf{p}(\mathbf{y}))} \frac{\partial p_k}{\partial y_j} \Big|_{\mathbf{y}} \right)^2 \sigma_j^2 \end{aligned}$$

Let $\mathbf{M} = (m_{ij})$ be the $N \times N$ matrix defined by:

$$m_{ij} = \delta_{ij} - \sum_{k=1}^n \frac{\partial f}{\partial p_k} \Big|_{(s_i; \mathbf{p}(\mathbf{y}))} \frac{\partial p_k}{\partial y_j} \Big|_{\mathbf{y}} \quad (5)$$

Then the σ_j can be in principle obtained from the linear equation

$$\delta r_i^2 = \sum_{j=1}^N m_{ij}^2 \sigma_j^2 \quad (6)$$

by measuring δr_i^2 .

III. Linear Fits

In this section, the case of special interest - a linear fit - is discussed. In this case the function $f(s; \mathbf{p})$ can be expressed as a linear superposition of known functions $\mathbf{g} = (g_1, \dots, g_n)^T$:

$$f(s; \mathbf{p}) = \sum_{i=1}^n p_i g_i(s) = \mathbf{p}^T \cdot \mathbf{g}(s) \quad (7)$$

Examples:

1. $f(s; \mathbf{p})$ could be a polynomial: $g_i(s) = s^i$.
2. $f(s; \mathbf{p})$ could be a difference orbit which is expressible by the beta-function $\beta(s)$, the betatron phase $\phi(s)$ and the dispersion $\eta(s)$. In this case the linear superposition of the following three functions has to be considered:

$$\begin{aligned} g_1(s) &= \sqrt{\beta(s)} \cos \phi(s) \\ g_2(s) &= \sqrt{\beta(s)} \sin \phi(s) \\ g_3(s) &= \eta(s) \end{aligned} \quad (8)$$

The results of chapter II can be easily specialized to the linear case. From (5) and (7) one gets:

$$m_{ij} = \delta_{ij} - \sum_{k=1}^n g_k(s_i) \frac{\partial p_k}{\partial y_j} \Big|_y = \delta_{ij} - \mathbf{g}^T(s_i) \cdot \frac{\partial \mathbf{p}}{\partial y_j} \Big|_y \quad (9)$$

In order to evaluate the partial derivative $\frac{\partial \mathbf{p}}{\partial y_j}$, the fit P is written down explicitly. Defining the $n \times n$ matrix $\mathbf{A} = (A_{ki})$ and the vector $\mathbf{c} = (c_1, \dots, c_n)^T$ by

$$\begin{aligned} A_{ki} &= \sum_{j=1}^N \frac{1}{e_j^2} g_k(s_j) g_i(s_j) \\ c_k &= \sum_{j=1}^N \frac{1}{e_j^2} y_j g_k(s_j) \end{aligned}$$

or written more compactly as

$$\begin{aligned} \mathbf{A} &= \sum_{j=1}^N \frac{1}{e_j^2} \mathbf{g}(s_j) \cdot \mathbf{g}^T(s_j) \\ \mathbf{c} &= \sum_{j=1}^N \frac{1}{e_j^2} y_j \mathbf{g}(s_j) \end{aligned} \quad (10)$$

condition (3) yields the linear equation

$$\mathbf{A} \cdot \mathbf{p} = \mathbf{c} \quad (11)$$

The fit is hence expressable by

$$\mathbf{p}(\mathbf{y}) = \mathbf{A}^{-1} \cdot \mathbf{c}(\mathbf{y}) \quad (12)$$

Since $\mathbf{A}$ does not depend on $\mathbf{y}$, one gets from (12):

$$\frac{\partial \mathbf{p}}{\partial y_j} = \mathbf{A}^{-1} \cdot \frac{\partial \mathbf{c}}{\partial y_j} = \frac{1}{e_j^2} \mathbf{A}^{-1} \cdot \mathbf{g}(s_j) \quad (13)$$

From (9) one therefore obtains:

$$m_{ij} = \delta_{ij} - \frac{1}{e_j^2} \mathbf{g}^T(s_i) \cdot \mathbf{A}^{-1} \cdot \mathbf{g}(s_j) \quad (14)$$

Finally, defining the matrix $\mathbf{B} = (B_{ij}) \in \mathbb{R}^n \times \mathbb{R}^N$ by

$$B_{ij} = \mathbf{g}_i(s_j) \quad (15)$$

one gets the result

$$m_{ij} = \delta_{ij} - \frac{1}{e_j^2} \left(\mathbf{B}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{B} \right)_{ij} \quad (16)$$

Introducing the diagonal error weighting matrix $\mathbf{E} = (E_{ij}) \in \mathbb{R}^N \times \mathbb{R}^N$ by

$$E_{ij} = \frac{\delta_{ij}}{e_i e_j} \quad (17)$$

one arrives at the compact matrix relation

$$\mathbf{M} = \mathbf{I} - \mathbf{B}^T (\mathbf{B} \mathbf{E} \mathbf{B}^T)^{-1} \mathbf{B} \mathbf{E} \quad (18)$$

where $\mathbf{I}$ is the $N \times N$ unity matrix. Note that in this (linear) case the matrix $\mathbf{M}$ does not any longer depend on the measurements $\mathbf{y}$. Furthermore $\mathbf{M}$ has the property $\mathbf{M}^2 = \mathbf{M}$ and is hence (since $\mathbf{M} \neq \mathbf{I}$) a singular matrix. Also one easily gets from (18) that

$$\text{tr}(\mathbf{M}) = N - n = NDF \quad (19)$$

which means that the diagonal elements of $\mathbf{M}$ reflect the number of degrees of freedom in the fit.

In principle the resolutions σ_i follow from solving equation (6), although the numerical instabilities might be severe.

IV. Data Analysis

Beam position measurements were taken for the electron beam during a time period of 7 minutes. About 100 single beam pulses were recorded for all BPMs starting in the North-RTL downstream to the interaction region.

4.1. Beam Quality

While the accelerator conditions were generally stable during the time of data taking, significant pulse to pulse variations were observed in the interaction region. This is illustrated in fig. 1, where x and y positions measured by BPM235 (micro FB69) are plotted as a function of time. The variations are especially large in the horizontal direction. Note that BPM235 is positioned in front of the final quadrupole-triplet, i.e. at a point where the beta-function is large. The BPM is therefore very sensitive to orbit fluctuations. The six large vertical orbit excursions apparent in the lower half of fig. 1 can be uniquely explained by large energy fluctuations as measured in the beam switchyard. These data points were not considered in the analysis below.

4.2 Cuts and Corrections

At least part of the fluctuations observed in the interaction region can be clearly explained by orbit variations far upstream. In order to show this, the readings from BPMs in the beam-switchyard and in sector 2 of the linac were fitted using the known twiss parameters. This procedure yields a high precision measurement of the "injection" parameters x, x', y, y' (and $\frac{\delta E}{E}$ in the switchyard) at two reference points: the position of BPM801 at the end of sector 30 in the linac and BPM201 at the beginning of sector 2 in the linac. A cut in the relative energy variation of $\pm 0.2\%$ was applied to the data for the following data analysis.

Fig. 2 shows that the horizontal positions measured by BPM235 (corrected for a finite but small residual dispersion induced from the data) are strongly correlated to the fitted x parameters at the two reference points in the linac. In fig. 3 the distribution of x -readings of BPM235 is shown

1. for the raw data with $\left| \frac{\delta E}{E} \right| \leq 0.2\%$ (fig. 3a)
2. after correction for residual dispersion (fig. 3b)
3. after correction for the jitter in x at BPM801 (fig. 3c)
4. after simultaneous correction for the x, x' jitters at BPM801 (fig. 3d).

As can be seen, the variation of x measured at BPM235 decreases from $372\mu\text{m}$ to $206\mu\text{m}$ from step 1 to 3, while step 4 doesn't lead to any further improvement. Estimating the BPM resolutions to be about $50\mu\text{m}$ one can conclude that 70% of the (quadratic) noise contributions at BPM235 could be taken out by identified upstream fluctuations.

Similar procedures were used to unfold the known noise contributions for all BPMs in the interaction region. The cleaned data were used for the extraction of BPM resolutions. By this method the numerical stability of the methods described in chapter III can be improved.

4.3. Analysis of BPM Resolutions

Applying the methods of chapter III, the resolutions of the BPMs in the interaction region can now be extracted. The orbit fits include $N = 6$ BPMs in a region which is non-dispersive. The fit is therefore a superposition of the $n = 2$ functions g_1, g_2 defined in (8). The two parameters p_1, p_2 determine the relative weight of the "cosinlike" and "sinlike" trajectories, or equivalently, the initial betatron phase, ϕ_0 , and the amplitude of the betatron oscillation.

Table 1 shows the result for the horizontal resolutions, where constant estimates were used in the fit. Note that the errors given for the unfolded resolutions are coupled. One can decrease this coupling by iteration, using the unfolded resolutions as new estimates. Table 2 shows the results after three iterations.

The statistical and systematic precision is not large enough to measure directly all resolutions. Some σ_z^2 may turn out to be negative (see tables 1 and 2). In this case only upper limits for σ_z can be derived.

Applying the cleaning-corrections described above, one can further improve the numerical stability of the results. Table 3 gives the final horizontal resolutions using the corrected data. The results are compatible with the numbers in table 2.

Similarly, table 4 shows the vertical resolutions after applying the iterations. The cleaning of the data (though it has been applied) has little effect here, since no large vertical fluctuations of the orbit are present.

The results are summarized in table 5. Since BPMs 1 and 3 (both North and South) are hardwarewise identical, they are expected to have identical resolutions. In addition the horizontal and vertical resolutions should agree. Table 5 therefore indicates some systematic inconsistency: The vertical resolutions are compatible with a constant around $10\mu\text{m}$. The horizontal resolutions, however, tend to be slightly worse ($20\mu\text{m}$ to $30\mu\text{m}$). Since the vertical orbit was much more stable during the time of data taking, it is expected that the systematic errors are smaller for σ_y than for σ_x . Therefore a resolution of $10\mu\text{m}$ seems to be the best estimate for BPMs 1 and 3 (both North and South). More data must be taken to draw more solid conclusions.

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REFERENCES

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- [2] T. Lohse, *Fit-Residuals and Detector Resolution*, ALEPH-note, CERN (1987).

FIGURE CAPTIONS

Fig 1: BPM 3 North x and y orbit excursions versus time.

Fig 2: BPM 3 North x correlations with upstream BPMs.

Fig 3: BPM 3 North x distribution corrected in steps (see text).

Fig 1

Fig 1

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Fig.3

BPM NAME	PPM NUMBER	ESTIMATES e_i (μm)	RESIDUALS r_i (μm)	σ_z^2 (μm^2)	σ_z (μm)
4 NORTH	205 X	20	43 ± 5	3533 ± 871	59 ± 7
3 NORTH	235 X	20	34 ± 2	2096 ± 652	46 ± 7
1 NORTH	240 X	20	13 ± 1	-815 ± 176	-
IP SOUTH	555 X	20	28 ± 4	939 ± 446	31 ± 7
1 SOUTH	260 X	20	46 ± 5	4234 ± 1168	65 ± 9
3 SOUTH	265 X	20	34 ± 3	1093 ± 364	33 ± 6

TABLE 1: σ_z with no iteration or BPM correction

BPM NAME	BPM NUMBER	ESTIMATES e_i (μm)	RESIDUALS r_i (μm)	σ_z^2 (μm^2)	σ_z (μm)
4 NORTH	205 X	53	47 ± 4	2764 ± 624	53 ± 6
3 NORTH	235 X	47	39 ± 3	2343 ± 496	48 ± 5
1 NORTH	240 X	6	1 ± 1	-464 ± 260	-
IP SOUTH	555 X	38	32 ± 3	1392 ± 334	37 ± 4
1 SOUTH	260 X	56	52 ± 7	3353 ± 954	58 ± 8
3 SOUTH	265 X	33	26 ± 2	1180 ± 323	34 ± 5

TABLE 2: σ_z iterated without BPM correction

BPM NAME	BPM NUMBER	ESTIMATES e_i (μm)	RESIDUALS r_i (μm)	σ_x^2 (μm^2)	σ_x (μm)
4 NORTH	205 X	46	41 ± 3	2048 ± 369	45 ± 4
3 NORTH	235 X	26	18 ± 1	661 ± 237	26 ± 5
1 NORTH	240 X	12	4 ± 1	-132 ± 164	≤ 9 (90% c.l.)
IP SOUTH	555 X	35	31 ± 3	1213 ± 276	35 ± 4
1 SOUTH	260 X	40	34 ± 6	1486 ± 672	39 ± 9
3 SOUTH	265 X	28	24 ± 2	887 ± 231	30 ± 4

TABLE 3: σ_x with iteration and BPM correction

BPM NAME	BPM NUMBER	ESTIMATES e_i (μm)	RESIDUALS r_i (μm)	σ_y^2 (μm^2)	σ_y (μm)
4 NORTH	205 Y	51	50 ± 3	2542 ± 342	50 ± 3
3 NORTH	235 Y	4	2 ± 1	20 ± 123	4 ± 14
1 NORTH	240 Y	6	5 ± 1	40 ± 147	6 ± 12
IP SOUTH	555 Y	37	37 ± 4	1396 ± 327	37 ± 4
1 SOUTH	260 Y	5	1 ± 1	-64 ± 271	≤ 17 (90% c.l.)
3 SOUTH	265 Y	16	12 ± 1	247 ± 384	16 ± 12

TABLE 4: σ_y with iteration and BPM correction

BPM NAME	BPM NUMBER	σ_x (μm)	σ_y (μm)
4 NORTH	205	45 ± 4	50 ± 3
3 NORTH	235	26 ± 5	4 ± 14
3 SOUTH	265	30 ± 4	16 ± 12
1 NORTH	240	≤ 9 (90% c.l.)	6 ± 12
1 SOUTH	260	39 ± 9	≤ 17 (90% c.l.)
IP SOUTH	555	35 ± 4	37 ± 4

TABLE 5: Summary of final BPM resolutions