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ANL/FPP/TM-114

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ANALYSIS OF THE MULTIPHASE INDUCTOR-CONVERTER BRIDGE

by

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August 1978

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TABLE OF CONTENTS

	<u>Page</u>
Introduction	1
The Operation of the Inductor-Convertor Bridge	1
Power Flow in the m-Phase ICB	2
Calculation of Other System Variables	9
The Special Case of Three-Phase System	13
Conclusions	15
References	15

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	Circuit Diagram for the 3-Capacitor Model IC Bridge	17
2.	Circuit Diagram for an m-Phase ICB	18
3.	Equivalent Diagram of an m-Phase ICB	19
4.	One Phase Equivalent Circuit of an m-Phase ICB ..	20
5.	General Current Waveform of each Equivalent Source in an m-Phase ICB	21
6.	Phase Currents for Two Different Switching Patterns in a 5-Phase ICB	22
7.	Plot of Net Power vs Number of Phases	23
8.	Plot of $\langle p \rangle$ vs α	24
9.	Average Coil Currents, Voltages and Power vs Time	25
10.	Experimental Average Coil Voltage and Current Traces	26

Introduction

The Inductor-Converter Bridge (ICB) is a solid state DC-AC-DC converter system for bidirectional energy transfer between two inductors.³ The active controllability of the ICB makes these circuits useful in many power supply applications where the load current or power demand can change with time. The reversability of the energy flow is useful in applications where the load is a high Q magnet coil and its stored energy can be recovered at the end of each use cycle. The later application will be of particular interest for the magnet coil power supplies of the present and future tokamak devices^{1,3} where energy economy and moderation of the pulsed load on the power grid is a concern.³

This report presents a mathematical model of the general m-phase ICB*. The analysis is useful for gaining insight in the behavior of the ICB and the significance of the parameters involved. It is also useful as a tool for the design of ICB systems. Furthermore, the equations presented can be used to represent the ICB in a closed loop control system with the output being the load quantity of interest.

The Operation of the Inductor-Converter Bridge

The detailed operation of ICB is presented in a previous report.¹ The following is a brief review of ICB operation. Figure 1 shows a three-phase ICB where the storage coil is represented by L_S and the load coil is represented by L_L . At a typical instant during the energy transfer, current i_S and i_L will be flowing in the storage and load coils, respectively. The SCR's of the left-hand and right-hand bridge are fired in the normal inverter sequence. Since the inductor currents are nearly constant for short periods of time, each inductor-inverter combination will appear as a three-phase rectangular wave current source viewed from the three lines: a, b, and c.

*The case of 3-phase and 5-phase has been studied numerically before.¹

It will be shown analytically that the relative advance of the right hand phase currents with respect to the left hand will cause a net energy flow to the load and vice versa. The y-connected capacitors in the middle serve two purposes: [1] they make energy transfer between the two out-of-phase inverters possible (to be explained later), [2] if certain phase shift limits are not exceeded, it can be verified that they produce the reverse voltages to force-commutate the movement of current between elements in the sequence. The elimination of an external counter pulse circuit to turn off the SCR, renders this system inherently more efficient and economical. In practice, the frequency of SCR firing and the capacitor sizes will be such that only a very small fraction of each coil energy is extracted in each bridge cycle. Therefore, by varying the relative inverter phases and/or the frequency in every few cycles, very fine control over the rate of energy transfer in either direction can be achieved.

In order to study the ICB in general, an m-phase ICB is presented in Fig. 2. The SCR's are replaced by ideal switches, within the justification that commutation is always possible for all phase differences of interest.¹

Power Flow in the m-Phase ICB

Viewing each inverter from the m phase-lines, one will see an m-phase rectangular current source system, each phase being $\frac{2\pi}{m}$ radians displaced from the next and the rectangle amplitude being equal to the instantaneous coil current. An equivalent circuit of this system may be drawn as shown in Fig. 3.

The energy flow from left to right in this circuit is identical to the energy flow from the storage to the load coils. Note that in this circuit the phases are identical.* Therefore,

*The only distinction between the phases is the phase on which the transfer begins and the sequence of the phases that follow. It can be shown that the net power is independent of this starting phase.

the net power delivered by either coil is divided equally between the phases. We may now proceed by calculating the power exchange between the two sides for one phase and multiplying the results by m for the total power.

Figure 4 shows the equivalent circuit of one phase. This equivalent circuit is the most basic unit of any m -phase inductor-converter bridge. The current sources are rectangular wave current sources with slowly changing amplitudes. It can be shown that the net power delivered by the left-hand source to the right-hand source is

$$\langle p_{S_a}(t) \rangle = \sum_{n=1}^{\infty} \frac{1}{2n\omega C} (a_{nS_a} b_{nL_a} - b_{nS_a} a_{nL_a}) \quad (1)$$

where the a 's and b 's are the Fourier coefficients of the source side or load side current waveforms, as shown by the subscripts.

The Fourier coefficients are derived from the general waveform of each source as shown in Fig. 5. It can be easily verified that the length of each rectangle in an m -phase system will be $\frac{2\pi}{m}$. α is the phase difference of the source under study and an arbitrary fixed reference phase. t_p is the separation between the positive and the negative rectangles in one period. This parameter depends on the number of phases in the system and, in general, the pattern of SCR firing chosen. Figure 6 shows two different rectangular waveforms that result from two switching patterns in a five-phase system. In general, there will be Integer $[(m+1)/2]$ various switching patterns possible. Of all the possible patterns in an m -phase system, only one produces symmetrical current pulse spacing as shown in pattern (b) of Figure 6. Such a symmetry is only possible if m is odd. The significance of symmetry will become apparent in the power equations. t_p is bound between $\frac{\pi}{m}$ and $\frac{(2m-5)\pi}{m}$ and may vary in steps of $\frac{\pi}{m}$, $m > 2$.* The case of symmetry, where existent, will give $t_p = \frac{(m-2)\pi}{m}$.

*The case of $m = 2$ gives $t_p = 0$ not shown by these expressions.

With the above parameters, the Fourier coefficients of the general waveform will be

$$\left\{ \begin{aligned} a_n &= \frac{2I}{n\pi} \left\{ \left[-\sin \frac{n\pi}{m} \sin n \left(\frac{2\pi}{m} + t_p \right) \right] \cos n\alpha \right. \\ &\quad \left. + \left[\cos n\pi \sin n \left(\pi - \frac{\pi}{m} \right) + \sin \frac{n\pi}{m} \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \sin n\alpha \right\} \\ b_n &= \frac{2I}{n\pi} \left\{ \left[\cos n\pi \sin n \left(\frac{\pi}{m} - \pi \right) - \sin \frac{n\pi}{m} \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \cos n\alpha \right. \\ &\quad \left. - \sin \frac{n\pi}{m} \sin n \left(\frac{2\pi}{m} + t_p \right) \sin n\alpha \right\} \end{aligned} \right. \quad (2)$$

$$\left\{ \begin{aligned} b_n &= \frac{2I}{n\pi} \left\{ \left[\cos n\pi \sin n \left(\frac{\pi}{m} - \pi \right) - \sin \frac{n\pi}{m} \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \cos n\alpha \right. \\ &\quad \left. - \sin \frac{n\pi}{m} \sin n \left(\frac{2\pi}{m} + t_p \right) \sin n\alpha \right\} \end{aligned} \right. \quad (3)$$

where a_n and b_n is defined in

$$i(t) = \sum_{n=1}^{\infty} a_n \sin n \omega t + b_n \cos n \omega t . \quad (4)$$

The relative phase difference between the storage and load sources is $(\alpha_L - \alpha_S)$. However, because the net power is independent of the reference phase and only dependent on the phase difference, we will choose $\alpha_S = 0$ and $\alpha_L = \alpha$ for simplicity. Therefore, the above Fourier components represent the load source, a_{nLa} , b_{nLa} and the storage source Fourier components will simplify to*

*These Fourier components would be valid exactly if the coil currents were constant at all times. However, since the coil currents are nearly constant over one bridge period, the Wentzel-Kramers-Brihouin (WKB) approximation² is used to treat the coil currents as constants, and the Fourier components will become as shown.

$$\left\{ \begin{aligned} a_{nSa} &= \frac{-2 i_S}{n\pi} \sin \frac{n\pi}{m} \sin n \left(\frac{2\pi}{m} + t_p \right) \end{aligned} \right. \quad (5)$$

$$\left\{ \begin{aligned} b_{nSa} &= \frac{2 i_S}{n\pi} \left[\cos n\pi \sin n \left(\frac{\pi}{m} - \pi \right) - \sin \frac{n\pi}{m} \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \end{aligned} \right. \quad (6)$$

Substitution of these Fourier components in the average power equation will yield

$$\langle p_{Sa} (t) \rangle = \sum_{n=1}^{\infty} \frac{4 i_S i_L}{n^3 \pi^2 \omega C} \left[1 - \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \sin^2 \frac{n\pi}{m} \sin n \alpha \quad (7)$$

The total net power delivered by the m storage sources or the storage coil will be m times the above or

$$\langle p_S (t) \rangle = \sum_{n=1}^{\infty} \frac{4 i_S i_L m}{n^3 \pi^2 \omega C} \left[1 - \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \sin^2 \frac{n\pi}{m} \sin n \alpha \quad (8)$$

Several important conclusions may be immediately drawn from the net power equation.

- (1) The t_p , which represents the effect of the inverter switching pattern, for maximum net power can be found by equating the quantity in the brackets to its maximum value of 2:

$$\left[1 - \cos n \left(\frac{2\pi}{m} + t_p \right) \right] = 2$$

$$\cos n \left(\frac{2\pi}{m} + t_p \right) = -1 \Rightarrow \frac{2n\pi}{m} + n t_p = n\pi, n = \text{odd}$$

$$t_p = \frac{(m-2)\pi}{m}.$$

Note that this is the same as the value of t_p for symmetrical switching pattern. Therefore, the symmetrical switching pattern leads to the shortest energy transfer time. Since energy lost in the transfer cycle is directly related to the products of current, thyristor voltage drop, and transfer time, the symmetrical pattern is inherently more efficient. And, since even-phased systems do not possess a symmetrical pattern, they are likewise less efficient. The net power for an odd-numbered ICB with symmetrical switching pattern reduces to:

$$\langle p_S \rangle (t) = \sum_{n=1}^{\infty} \frac{4}{n^3} \frac{i_S i_{Lm}}{\pi^2 \omega C} [1 - \cos n \pi] \sin^2 \frac{n\pi}{m} \sin n\alpha. \quad (9)$$

- (2) The contribution of the harmonics of the square wave currents in the power equation is attenuated by $\frac{1}{n^3}$. For a symmetrical switching pattern, the bracketed term of Eq. (9) goes to zero for even values of n . Therefore, even harmonics do not contribute net power. The next possible harmonic to contribute is $n = 3$ which can at the most deliver 3.7% net power, compared to the fundamental

frequency. In fact, it can be shown that at least 95% of the net power is delivered by the fundamental frequency. It should also be noted that asymmetrical switching patterns not only reduce the fundamental frequency power, but also allow power delivery by even harmonics; e.g., up to 12.5% of the fundamental is delivered by the second harmonic. Power delivery by higher harmonics with their associated magnetic coupling losses and EMI is anticipated to be undesirable.

- (3) The influence of the number of phases (m) on the net power can be closely approximated by $m \sin^2 \frac{\pi}{m}$. A plot of this function is presented in Fig. 7. This plot indicates that, for all other parameters held constant, the three-phase system has a significant net power advantage over other choices; e.g., 32% higher than five-phase system. Note that the final preference of the three-phase system cannot be based on this conclusion alone. For example, economy may dictate the highest operating voltages tolerable by the SCR's. In this case the capacitance (C) should also be varied along with (m).
- (4) The advance angle of the load bridge with respect to the storage bridge (α) is the way of controlling the direction and the rate of energy flow in the system. For α positive (load leading) the net power delivered by the storage coil $\langle p_s \rangle$ is positive and vice versa. The angle of maximum net power in general may be found by maximizing $\langle p_s \rangle$ with respect to α :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \left[1 - \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \sin^2 \frac{n\pi}{m} \cos n \alpha = 0. \quad (10)$$

Figure 8 presents the plot of net power as a function of α with the number of phases m as the parameter. The

superiority of $m = 3$ in net power when all other parameters are held constant is again evident. It is seen that the net power peaks at a lower angle as the number of phases is increased. The ripples on the curves are due to the contribution of the harmonics. Note that these ripples are also present when $m = 3$. However, 99.5% of the net power is delivered by the fundamental frequency which renders this curve almost purely sinusoidal.

- (5) ω is another parameter by which the rate of energy flow, but not flow direction, in the system may be controlled. Note that this parameter controls the power linearly; whereas power is a nonlinear function of α . If control by α only is chosen, the value of operating ω should be chosen based on other circuit considerations, as will be mentioned later.
- (6) The phase capacitance C appears in the power equation as $\frac{1}{C}$. For the highest net power, C should be made as small as possible. This would also be desirable from the standpoint of capacitor costs which can be a major part of system costs. However, for a given ω , lowering the value of C will increase the peak voltages experienced by the bridges. This increase will be of the order of

$$v_C = \frac{i \Delta t}{C}$$

where i is the instantaneous current in the storage or the load coil, Δt is the interval of time when the capacitor is being charged and C is the phase capacitance. The allowable peak voltages are limited by the maximum insulation capability of the circuit. Therefore, it would seem that for the best design, the combination of the highest ω allowable by the SCR turn

off times, the highest SCR breakover voltage, and the lowest phase capacitance C would be desirable.

- (7) Since the total energy in the system is conserved in our lossless model, it can be shown that the maximum net energy transfer occurs when

$$I_S = i_L = I_0 \sqrt{2} \quad ,$$

where I_0 is the storage current when all of the energy is stored.

Calculation of Other System Variables

As the energy is transferred from one coil to the other, the currents in the coils will change with time. The time function of the coil currents is found below from the average power equation. For simplicity of notation, the constant k which can be computed to any arbitrary number of significant figures is introduced.

$$k \triangleq \sum_{n=1}^{\infty} \frac{4m}{n^3 \pi^2 \omega C} \left[1 - \cos n \left(\frac{2\pi}{m} + t_p \right) \right] \sin^2 \frac{n\pi}{m} \sin n\alpha$$

$$\begin{cases} \langle p_S \rangle = k i_S i_L = - \frac{d}{dt} \frac{1}{2} L_S i_S^2 \\ \langle p_S \rangle = k i_S i_L = \frac{d}{dt} \frac{1}{2} L_L i_L^2 \end{cases}$$

$$\begin{cases} i_S + \frac{k}{L_S} i_L = 0 \\ i_L - \frac{k}{L_L} i_S = 0 \end{cases} \quad (11)$$

$$(12)$$

$$i_S + \frac{k^2}{L_S L_L} i_S = 0 \quad (13)$$

$$i_S = A e^{j k t / \sqrt{L_S L_L}} + B e^{-j k t / \sqrt{L_S L_L}} \quad (14)$$

Using the boundary conditions

$$\begin{cases} i_S|_{t=0} = I_0 \\ i_S|_{t=0} = 0 \end{cases} ,$$

we will obtain

$$i_S(t) = \frac{I_0}{2} \left(e^{j k t / \sqrt{L_S L_L}} + e^{-j k t / \sqrt{L_S L_L}} \right) \quad (15)$$

or

$$i_S(t) = I_0 \cos \frac{k}{\sqrt{L_S L_L}} t \quad (16)$$

Also

$$i_L(t) = I_0 \sqrt{\frac{L_S}{L_L}} \sin \frac{k}{\sqrt{L_S L_L}} t \quad (17)$$

Equations (16) and (17) describe the coil currents in one complete lossless energy transfer from the source to the load, by their first quarter cycle. Beyond this interval, the storage current equation indicates a reversal of current which will not be allowed by the bridge SCR's in the circuit and the energy transfer will be terminated.

The net power may now be shown as an explicit function of time:

$$\langle p_S \rangle (t) = \frac{1}{2} k I_o^2 \sqrt{\frac{L_S}{L_L}} \sin \frac{2k}{\sqrt{L_S L_L}} t. \quad (18)$$

The average coil voltages as a function of time may also be calculated directly from the currents.

$$v = L \frac{di}{dt}$$

$$\Rightarrow \left\{ \begin{array}{l} v_S (t) = k I_o \sqrt{\frac{L_S}{L_L}} \sin \frac{k}{\sqrt{L_S L_L}} t \end{array} \right. \quad (19)$$

$$\left\{ \begin{array}{l} v_L (t) = k I_o \cos \frac{k}{\sqrt{L_S L_L}} t \end{array} \right. \quad (20)$$

We can now show the functional dependence of the average coil voltages on the ratio of the two coil currents. This relationship for the load coil $\langle v_L \rangle$ is derived as follows:

$$\left\{ \begin{array}{l} \frac{i_L}{i_S} = \sqrt{\frac{L_S}{L_L}} \frac{\sin \frac{k}{\sqrt{L_S L_L}} t}{\cos \frac{k}{\sqrt{L_S L_L}} t} \end{array} \right. \quad (21)$$

$$\left\{ \begin{array}{l} \langle v_L \rangle = k I_O \cos \frac{k}{\sqrt{L_S L_L}} t \Rightarrow \cos^2 \frac{k}{\sqrt{L_S L_L}} t = \frac{\langle v_L \rangle^2}{k^2 I_O^2} \end{array} \right. \quad (22)$$

$$\frac{\sin \frac{k}{\sqrt{L_S L_L}} t}{\cos \frac{k}{\sqrt{L_S L_L}} t} = \sqrt{\frac{L_L}{L_S}} \frac{i_L}{i_S} \Rightarrow \frac{\sin^2 \frac{k}{\sqrt{L_S L_L}} t}{\cos^2 \frac{k}{\sqrt{L_S L_L}} t} = \frac{L_L}{L_S} \frac{i_L^2}{i_S^2}$$

$$\Rightarrow \frac{\sin^2 \frac{k}{\sqrt{L_S L_L}} t + \cos^2 \frac{k}{\sqrt{L_S L_L}} t}{\cos^2 \frac{k}{\sqrt{L_S L_L}} t} = \frac{L_L \frac{i_L^2}{i_S^2} + L_S}{L_S}$$

$$\Rightarrow \frac{1}{\cos^2 \frac{k}{\sqrt{L_S L_L}} t} = \frac{L_L \frac{i_L^2}{i_S^2} + L_S}{L_S} \quad (23)$$

substituting for $\cos^2 \frac{k}{\sqrt{L_S L_L}} t$ in the above from Eq. (22),

$$\frac{k^2 I_o^2}{\langle v_L \rangle^2} = \frac{L_L \frac{i_L^2}{i_S^2}}{\frac{L}{S}} = L_S$$

or

$$\langle v_L \rangle = \frac{k I_o}{\sqrt{\frac{L_L}{L_S} \frac{i_L^2}{i_S^2} + 1}} \quad (24)$$

This equation for a three-phase and a five-phase system with α as the parameter has been plotted from numerical computations in a previous report.¹

The Special Case of Three-Phase System

For a three-phase system, the net power equation as a function of transfer angle, α , will reduce to

$$\langle p_S \rangle = \sum_{n=1}^{\infty} \frac{12}{n^3 \pi^2 \omega C} \frac{i_S i_L}{\omega C} [1 - \cos n\pi] \sin^2 \frac{n\pi}{3} \sin n\alpha \quad (25)$$

Note that for a three-phase system, only one switching pattern which is also symmetrical exists. This equation predicts a

maximum net power at $\alpha = 90^\circ$, consistent with the experimental and numerically computed results.

The average coil currents, voltages, and power equations for the three-phase experimental inductor-converter bridge built at Argonne National Laboratory are shown below:

$$\left\{ \begin{array}{lll} i_S = 100 \cos 0.5609 t & \text{A} & (26) \\ i_L = 100 \sin 0.5609 t & \text{A} & (27) \\ v_S = 224.4 \sin 0.5609 t & \text{V} & (28) \\ v_L = 224.4 \cos 0.5609 t & \text{V} & (29) \\ \langle P_S \rangle = 11218 \sin 1.1218 t & \text{W} & (30) \end{array} \right.$$

These equations are derived from the general case with the following specifications:

$$I_O = 100 \text{ A}$$

$$\alpha = 90^\circ$$

$$\omega = 4084 \text{ rad/s}$$

$$C = 10^{-4} \text{ F}$$

$$L_S = L_L = 4 \text{ H.}$$

A plot of these equations appears in Fig. 9. Figure 10 shows the same voltages and currents obtained experimentally. Note that the initial current in the storage coil is somewhat higher than the final current in the load. This is due to the losses in the circuit not accounted for in the equations. However, the

analytical prediction of these current and voltage waveforms are sufficiently verified by the experiment.

Conclusions

The analysis presented in this report is based on the time average behavior of the ICB system. The instantaneous coil and capacitor voltages caused by each bridge SCR firing will have to be computed or derived separately. The coil current equations, however, can be assumed instantaneous with very good accuracy because the coils, having large inductances, will respond to the average of the high frequency voltage waveforms across them.

This analysis also neglected system losses. Losses will occur in the SCR's during their turn-on, conduction, commutation, and also in other circuit components.

The average equations are useful to gain insight in the system behavior and the significance of each parameter. These equations can be used to specify the circuit components, number of phases, and the frequency. The exact value of each component and their current and voltage ratings will then require computation of the instantaneous voltages and currents.

The average equations of this report are also useful in developing a control system for the ICB. These equations can mathematically characterize the ICB in a control loop with the load current requirement as an input and the delivered current as an output.

REFERENCES

1. R. L. Kustom et al., "The Use of Multiphase Inductor-Convertor Bridges as Actively Controlled Power Supplies for Tokamak Coils," Argonne National Laboratory Report ANL/FPP/TM-78 (April 11, 1977).
2. I. N. Sneddon, "Encyclopaedic Dictionary of Mathematics for Engineers and Applied Scientists," Pergamon Press (1976).

3. H. A. Peterson et al., "Superconductive Inductor-Convertor Units for Pulsed Power Loads," Proceedings of International Conference on Energy Storage, Compression and Switching, Asti-Torino, Italy (November 1974).
4. T. J. Bauer, "Three-Phase Superconducting Inductor-Convertor Bridge Network," Unpublished report, Undergraduate Research Participation Program of the Argonne Center for Educational Affairs, Argonne National Laboratory, Argonne, Illinois (December 1976).

Fig. 1. Circuit Diagram for the 3-Capacitor Model IC Bridge.

Fig. 2. Circuit Diagram for an m -phase ICB

Fig. 3. Equivalent Diagram of an m-phase ICB

Fig. 4. One Phase Equivalent Circuit of an m-phase ICB

Fig. 5. General Current Waveform of each Equivalent Source in an m -phase ICB

Fig. 6. Phase Currents for Two Different Switching Patterns in a 5-Phase ICB

Fig. 7. Plot of Net Power vs Number of Phases

Fig. 8. Plot of $\langle p \rangle$ vs α

Fig. 9. Average Coil Currents, Voltages, and Power vs Time

Fig. 10. Experimental average coil voltage and current traces.