*Fusion Burn Dynamics in
Dense Z-Pinch (DZP)*

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ABSTRACT

The fusion burn dynamics and energy yield of the dense Z-pinch (DZP) are examined using a profile-averaged, zero-dimensional, time-dependent model. A range of conditions (fuel, line density, voltage, fusion-product heating, enthalpy endloss, density and temperature profiles, current rise rate, electrode impurities) are examined. Magneto-hydrodynamic stability is assumed, and initial conditions are based on those ideally existing after the melting and ionization of a solid fiber of fusion fuel. Plasma conditions required of neutron sources for materials testing ($\dot{S}_n \geq 10^{19} \text{ n/s}$) and for possible commercial power production (ratio of fusion energy yield to energy input, $Q_p \simeq 15$, lower values if reversible recovery of a fraction of the magnetic energy is possible) are described. If $f_B \gtrsim 0.8$ fractional fuel burnup is possible in a nominal 800-ns DT discharge (200-ns current-rise phase at 20 MV/m followed by a 500-ns constant-current crowbarred phase), reactor-relevant values of Q_p may be possible. For the simpler (and shorter) constant-voltage discharge (e.g., no voltage crowbar) the value of Q_p is in the range 5-10 for discharges below 200-ns duration. Smaller levels of fuel burnup, shorter discharges, or generally lower levels of Q_p will require a reversible energy transfer system to meet reactor energy-balance requirements. Imposition of a plasma current rise-time constraint that may be needed for stable plasma operation (e.g., $\dot{I} > 10^{12} \text{ A/s}$) will limit burnup, Q_p , and discharge time to an extent where reversible energy/transfer system will be required to meet reactor energy-balance requirements. The neutron intensity required for DT neutron-source applications occurs for $Q_p \simeq 1$ and pulse rates in the range 10-100 Hz. Maximum values of Q_p for DD and D³He fuels are in the ranges 2-5 and 5-10, respectively, for the above-cited optimistic DT conditions. When current rise-time constraints are imposed in conjunction with thermal-conduction/electron-drift enthalpy endloss and DT fusion-product heating, the erosion and ingestion of a low-Z electrode material (e.g., Li) may have a net beneficial effect on the pinch performance for discharges that are longer than 100-200 ns, with little impact being seen for shorter discharges. Generally, enthalpy endloss appears to be matched adequately to the levels of full charged-particle fusion-product heating; without endloss fusion-product heating would seriously limit fuel burn up and the ability to achieve high-Q discharges. Lastly, the DZP burn model has been applied to simulate a DZP experiment presently becoming operational (ZEBRA), where, depending on $\dot{I}$, discharge time, and initial fiber radius, values of Q_p in the range 0.04 to nearly unity are predicted for this 200-kJ, DD-fueled, ~ 100 -ns discharge.

I. INTRODUCTION

Heating and confinement of high-density plasma by the self magnetic field generated from plasma currents flowing linearly between two electrodes represents one of the earliest plasma confinement schemes considered in the quest for thermonuclear fusion.¹ The formation and heating of such a dense Z-pinch (DZP) by a fast-rising current generally relied on a rapid implosion of a cylindrical current sheath onto the axis of the pinch. These early “classical” pinches were found to be highly unstable,^{1–3} particularly with respect to $m = 0$ (sausage) instabilities. Theoretical predictions using ideal MHD stability theory^{3–4} indicate stability to the sausage mode for current profiles that are sufficiently diffuse. The use of frozen deuterium fibers⁵ to initiate a diffuse-current pinch that would be ohmically heated at nearly constant radius was suggested by Hammel, *et al.*⁶ and achieved experimentally at Los Alamos.⁷ These experiments on nominally the constant-radius, constant-density, diffuse-current pinch, as well as even more recent results,⁸ have shown surprisingly long MHD stable times to both the $m = 0$ sausage and $m = 1$ kink modes. The experiments described in Ref. 5, and subsequently in Refs. 9 and 10, show no $m = 0$ or $m = 1$ activity during most of the current-rise period, with some indication of an $m = 0$ termination only at the end of the $I \simeq 0.25\text{-MA}$ (initial pinch radius $a_0 = 10\text{-}20\text{ }\mu\text{m}$) discharge. Results from the Naval Research Laboratory (NRL)⁸ also show surprisingly long $m = 0$ and $m = 1$ stable times, with the $I \simeq 0.55\text{-MA}$ ($a = 80\text{-}125\text{ }\mu\text{m}$) discharges terminating in a strong $m = 0$ compression only when $\dot{I} \simeq 0$; these latter terminations generated a large neutron pulse from accelerated ions, whereas the thermonuclear neutron yield from the nevertheless hotter (200–300 eV) Los Alamos experiments was considerably less. In both experiments, the predicted^{3,4} fast-growing kink instability was not observed; the absence of the kink instability in both DZP experiments represents the main puzzle and uncertainty for extrapolations to fusion-burn and high-Q (the ratio of fusion energy to input waterline energy, $Q_p = W_F/W_{WL}$) conditions. Experimental activity and progress on the DZP aimed at resolving this uncertainty have recently been summarized in Ref. 11.

While a number of scoping studies of the DZP fusion-reactor embodiment have been reported,^{12–14} the potential of this approach as a small but intense source of fusion neu-

trons for materials testing¹⁵ represents a nearer-term application. A simplified analytic model of the DZP has been developed^{13,16} to examine the tradeoffs between physics requirements, fusion-neutron flux/current, volume available for performing materials tests at a given neutron flux, and, eventually, operational characteristics (i.e., pulse rates, power requirements, heat loads, stresses, erosion, fueling, etc.). The present study revisits and builds on this past work,^{13,15} including recent work by Robson,¹⁷ and identifies optimal burn conditions that maximize the plasma Q-value, Q_p , for a range of plasma and driver (water-filled transmission line and, possibly, a power crowbar) conditions. Specifically, a time-dependent, profile-averaging plasma model is evaluated for equal electron and ion temperatures. Evaluation of plasma performance (e.g., burnup fraction, f_B , and Q-value, Q_p) is conducted by prescribing the pinch voltage needed to achieve a given burn condition, rather than specifying the current wave form, as was done in Ref. 17; in this way, estimates of the power-supply size and energy can be made. Although conditions for radiative collapse¹⁸ of the pinch are sometimes observed computationally, the conditions where the pinch becomes optically thick to Bremsstrahlung and where the density increases to the point of electron degeneracy are generally not encountered in the regime of interest to this study and, therefore, are not modeled. Similarly, assumptions of an inertialess plasma (e.g., rate-of-change of plasma radius is much less than the radial Alfvén speed) and the electron drift velocity being less than the electron thermal speed (e.g., no current-driven instabilities) are retained, but relevant diagnostics are monitored throughout these computations. The electrodes as a source of impurities are largely ignored, but preliminary estimates of the level and time scale for increased electron density from the plasma/electrode interaction are made.

Many of the effects modeled in this study have yet to be achieved in the laboratory and, if achieved, would at best be difficult to measure. The approach adopted by this study is to “turn on” a range of physical processes that may plausibly occur and then to quantify systematically the overall impact on the DZP performance (i.e., Q_p , f_B , discharge time, τ_D). Specifically, this parametric scoping study investigates the impact of energy endloss, density depletion by burnup, fusion-product heating, time dependence of applied voltage, initial plasma radius and/or line density, fusion fuel, and recycle of

fusion reaction products (e.g., T and ${}^3\text{He}$ from the DD reaction), current rise-time constraints, and electrode-originated impurities. After describing in Sec. II the integrated plasma/engineering model used to evaluate the DZP, a DT “point-of-departure” case is developed in Sec. III.A; this point-of-departure case is then used to examine the impact of the above-listed effects on pinch performance in Sec. III.B. The impacts of current rise-time constraints (e.g., $\dot{I} > 10^{12} \text{ A/s}$) and impurities originating from a low-Z (Li) electrode are also examined in Sec. III.B. Section III.C gives preliminary estimates of dynamics that might be expected from a DD experiment presently becoming operational at Los Alamos (ZEBRA). Conclusions and main issues are summarized in Sec. IV.

II. MODEL

A single-particle ($T_e = T_i$), profile-averaged energy balance, along with simplified circuit and mass balances, constitutes the essential elements of the time-dependent model used herein. Plasma MHD stability is implicitly assumed to be provided by resistive effects ($m = 0$ stability at low temperature), viscosity ($m = 0$ stability at high temperature), skin effects (to enhance the effects of viscosity and/or resistivity), or (possibly) axial flow ($m = 1$ stability at high temperature). Inertial effects are ignored (e.g., the radial Alfvén time, $\tau_A = a/v_A$, is small compared to any time scale on which the coupled pinch-circuit dynamics are measured), and the plasma is assumed to be always optically thin to radiation and electronically nondegenerate. The complexity of the initial melting of the solid DT fiber is circumvented in this study by assuming a low-temperature, low-current pinch of radius a_0 has been established at time $t = 0$. Current rise-time and electrode effects are ignored initially in order to determine the ultimate potential of the DZP. Except for plasma temperature, which is given in keV, SI units are used throughout.

A. Plasma Profiles

The DZP is described by a filament of radius $a(t)$ and length ℓ_p , with self-similar density and temperature profiles being described parametrically by

$$n(r) = (1 + \alpha_n)n(1 - x^2)^{\alpha_n} \quad (1)$$

$$T(r) = \frac{1 + \alpha_p}{1 + \alpha_n} T(1 - x^2)^{\alpha_p} \quad , \quad (2)$$

where $x = r/a$ is the normalized plasma radius and $\alpha_p \equiv \alpha_n + \alpha_T$; the following expression for the pressure profile results:

$$p(r) = (1 + \alpha_p)p(1 - x^2)^{\alpha_p} . \quad (3)$$

In these expressions, n and p are volume average values, T is the density-weighted volume average, and axial variations are neglected (i.e., $a \ll \ell_p$). In order to avoid the notational complexity of designating averaged quantities with brackets (e.g., $\langle \rangle$) or bars (e.g., $\bar{n}$, $\bar{T}$, etc.), state variables presented without spatial arguments will always be averaged quantities, as defined above. The profile parameterization given by Eqs. (1) and (2) is not capable of modeling the pressure peaked Kadomstev profile for which an $m = 0$ wall-supported stable profile is theoretically possible; the impact of this profile on pinch performance has been addressed elsewhere.¹⁵

From pressure balance ($\vec{\nabla}p = \vec{j} \times \vec{B}$), the following expressions resulted for the magnetic-field and current-density profiles in the linear DZP:

$$[B_\theta(r)/B_\theta(a)]^2 \equiv b^2 = [1 - (1 + \alpha_p)(1 - y)^{\alpha_p}y - (1 - y)^{\alpha_p+1}]/y \quad (4)$$

$$j(r)/j = \alpha_p(1 + \alpha_p)y^{1/2}(1 - y)^{\alpha_p-1}/2b , \quad (5)$$

where $y \equiv x^2$ and $j = I/\pi a^2$ is the average current density. With the exponents α_n and α_T specified, profile factors for the i^{th} plasma power P_i ($i = \text{ohmic, radiation, or fusion}$ powers, designated respectively by the subscripts OHM, BR, and FUS) of the form

$$g_i = \frac{\int p_i dV_p}{P_i} , \quad (6)$$

are computed for use in the energy balance and circuit equation, where $p_i(r)$ is the local power density, $V_p = \pi a^2 \ell_p$ is the plasma volume, $dV_p = 2\pi r dr \ell_p$, and P_i is the total power.

B. Energy Balance

The global energy balance for the DZP takes the following form:

$$\frac{d}{dt}(3f_z N k_B T \ell_p) = P_{OHM} + P_{CP} - P_{RAD} - P_{EL} - P_{PdV} , \quad (7)$$

where $N = n\pi a^2$ is the ion line density, $f_z \equiv (1 + Z_{AVE})/2$, $Z_{AVE} = \sum_j n_j Z_j / \sum_j n_j$, and the powers P_i are, respectively, ohmic heating, charge-particle fusion-product heating, Bremsstrahlung loss, plasma enthalpy endloss (axial electron-drift and electron parallel-field conductive transport), and plasma (radial) expansion work. Each term in Eq. (7) is given below:

$$P_{OHM} = g_{OHM} C_{OHM} (\ell_p / \pi a^2) I^2 / T^{3/2} \quad (8)$$

$$P_{CP} = \sum_{ij} f_{cp}^{ij} P_{FUS}^{ij} \quad (9)$$

$$P_{RAD} = g_{BR} C_{BR} n^2 T^{1/2} V_p \quad (10)$$

$$P_{EL} = 3 f_z n k_B T V_p / \tau_{EL} \quad (11)$$

$$P_{PdV} = 2 f_z n k_B T (dV_p / dt) , \quad (12)$$

where $g_i (i = OHM, BR, FUS)$ are the above-described profile factors, f_{cp}^{ij} is the fraction of the total fusion energy released as charged particles from fusion reaction ij (DT, DD, or D^3He) that couples to the plasma, $n = \sum_j n_j$ is the total ion density, τ_{EL} is an effective energy endloss time, and the constants C_{OHM} and C_{BR} are given below:

$$C_{OHM} (ohm \ m \ keV^{3/2}) = \frac{3.2140(10)^{-9} Z_{eff} \ell n \Lambda}{1 + 0.72(Z_{eff} - 1)^2 / Z_{eff}} \quad (13)$$

$$C_{BR} (W/m^3 keV^{1/2}) = 5.35(10)^{-37} Z_{eff} \quad (14)$$

$$Z_{eff} = \frac{\sum_j n_j Z_j^2}{\sum_j n_j Z_j} , \quad (15)$$

and $\ell n \Lambda$ is the Coulomb logarithm (here taken as constant at $\ell n \Lambda \simeq 10$).

The following normalizing parameters for time, temperature, current, and line density are defined:

$$\tau_\Omega = \frac{3}{16} \frac{\mu_o}{C_{OHM}} T_{eq}^{3/2} a_o^2 \quad (16)$$

$$T_{eq} = \frac{16\pi k_B}{\mu_o} \frac{C_{OHM}}{C_{BR} N_o} \quad (17)$$

$$I_{eq} = \frac{16\pi k_B}{\mu_o} \sqrt{C_{OHM}/C_{BR}} \quad (18)$$

$$f_B = 1 - N/N_o \quad , \quad (19)$$

where $N_o = n_s \pi a_o^2$ and a_o are the initial ion line density and plasma radius, with n_s being the (solid) density of the initial fuel mixture immediately after fiber melting and ionization. In these expressions, $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$ and $k_B = 1000e$, where $e = 1.602 \times 10^{-19} \text{ J/eV}$. Applying these normalizations, the energy equation becomes

$$\frac{di}{d\xi} = \frac{(1 - f_B)^{3/2}}{2x^2 i^2 f_z^{1/2}} [f_z h_H - i^2 h_L] - \frac{2}{3} i \frac{d \ln x}{d\xi} \quad . \quad (20)$$

The normalized current, plasma radius, and time are $i = I/I_{eq}$, $x = a/a_o$, and $\xi = t/\tau_n$, respectively, with charged-particle-heating and endloss corrections being expressed in terms of $h_H \equiv 1 + P_{CP}/P_{OHM}$ and $h_L \equiv 1 + P_{EL}/P_{BR}$. These additional heating and loss terms are described quantitatively in the following subsection. The current I_{eq} is the Pease-Braginskii current,^{19,20} which in the absence of fusion-product heating and enthalpy endloss is the current for which the pinch is in pressure equilibrium with radiation loss and ohmic heating ($di/d\xi = dx/d\xi = 0$); in the presence of fusion-product heating and endloss, the Pease-Braginskii current, according to Eq. (20), is modified by the factor $(f_z h_H/h_L)^{1/2}$.

The normalized plasma temperature, $\Theta = T/T_{eq}$, is determined directly from pressure balance

$$NTf_z = \frac{\mu_o}{16\pi k_B} I^2 \quad , \quad (21)$$

which is a statement of Bennett equilibrium, and takes the following normalized form

$$\Theta(1 - f_B)f_z = i^2 \quad , \quad (22)$$

where $\Theta = T/T_{eq}$ is the normalized volume-average, density-weighted temperature. The electron drift velocity, $v_D \equiv I/Ne$, is monitored relative to the speed of light in an average

sense by the following expression:

$$v_D/c = [I_{eq}/(N_o ec)]\Theta/(1 - f_B), \quad (23)$$

where for typical values of $N_o \simeq 3.8(10)^{19} \text{ m}^{-1}$ and $I_{eq} \simeq 1.6(10)^6 \text{ A}$ amounts to $\sim 10^{-3}$; hence, significant burnup would be required before current-driven instabilities appear, provided Θ is not too large.

C. Charged-Particle Heating and Endloss Times

Direct plasma heating by fusion charged products is treated parametrically through the fractions f_{cp}^{ij} . Most recently,²¹ detailed alpha-particle orbit calculations in a DT plasma have shown that the expression $f_\alpha^{DT} \simeq 1 - T/15$ fits well calculational results, where $T(\text{keV})$ is the plasma temperature. The impact on the burn dynamics of using this dependence is examined in Sec. III.B.

Axial endloss of plasma enthalpy is modeled either through the direct electron drift or by electron parallel-field thermal conduction; the electrodes are assumed to sustain the plasma pressure. The electron drift current density is taken as a fraction f_D of $j = nv_{the}$, which is expressed in this study as carrying only the energy of the electron, $\sim \frac{3}{2}k_B T$. With $v_{the} \simeq (2k_B T/m_e)^{1/2}$, the drift time is given by

$$\begin{aligned} \tau_D &\equiv \frac{3f_z n k_B T V_P}{f_D n v_{the} Z_{AVE} (\frac{3}{2} k_B T) \pi a^2} \\ &= (2m_e/k_B)^{1/2} \frac{f_z \ell_p}{f_D Z_{AVE} T^{1/2}} . \end{aligned} \quad (24)$$

Axial thermal conduction is assumed to occur only by electrons in the unmagnetized $[(\omega\tau)_e \leq 1]$, where ω is the electron gyro-frequency and τ is the electron Coulomb collision time] regions of the pinch. Hence, with the parallel-field electron thermal conductivity given by

$$\kappa_{||} = C_K T^{5/2} , \quad (25)$$

where $C_K = 1.04 \times 10^{16}/\ell n\Lambda$, an axial temperature profile of the following form results:

$$T(z)/T_o = (1 - z^2)^{2/7} , \quad (26)$$

where z is the axial distance from the pinch midplane normalized to ℓ_p . The axially averaged temperature is given by

$$T/T_o = \left[\int_0^1 \frac{dz}{(1-z^2)^{2/7}} \right]^{-1} = 1/1.235 , \quad (27)$$

with the mid-plane temperature given by

$$T_o = \left[\frac{7}{16} \frac{Q \ell_p^2}{C_K} \right]^{2/7} , \quad (28)$$

where Q is the net average volumetric heating (ohmic plus charged-particle less radiation) rate. If x_L is the fraction of radius a where $(\omega\tau)_e \leq 1$, with

$$(\omega\tau)_e^2 = \frac{3.66(10)^{51}}{Z_{eff}^4 (\ell n \Lambda)^2} \frac{B^2 T^3}{n^2} , \quad (29)$$

and the thermal-conduction time is given by $\tau_c \equiv 3f_z n k_B T V_p / (q_c \pi a^2 x_L^2)$, where $q_c \pi a^2 \simeq Q V_p$, it follows that

$$\begin{aligned} \tau_c &= \frac{7}{16} \frac{3f_z n k_B T \ell_p^2}{x_L^2 T_o^{7/2} C_K} \\ &= 1.0043 \times 10^{-23} (T/T_o)^{7/2} f_z \left[\frac{\ell_p I}{x_L a} \right]^2 / T^{7/2} , \end{aligned} \quad (30)$$

where the pressure balance [Eq. (21)] has been used. The endloss times, τ_c and τ_D , are compared with Bremsstrahlung loss time,

$$\tau_{BR} = 15.62 \times 10^{12} \frac{a^2 T^{3/2}}{g_{BR} Z_{eff} I^2} , \quad (31)$$

where the constant includes the axial-average profile based on Eq. (26); for $I = 1.6(10)^6 A$, $x_L \simeq 1$ (flat pressure profile), $T \simeq 10 \text{ keV}$, $\ell_p = 0.1 \text{ m}$, $Z_{eff} = 1$, $a \simeq 20 \times 10^{-6} \text{ m}$, and $g_{OHM} \simeq 2.0$, the respective values of $\tau_D/\tau_c/\tau_{BR}$ are $\sim 4/97/38 \times 10^{-9} s$. In order to assure that parallel electron thermal conduction does not exceed the free-streaming limit, the following expression is used to compute the axial endloss time used in the energy balance:

$$\tau_{EL} = \tau_c + \tau_D . \quad (32)$$

Estimates of the rate of electrode erosion are made under the assumption that all endloss energy is returned to the plasma as electrode ions at temperature T after a delay appropriate to the axial thermal transit time of the electrode impurity ion; this simplified model for electrode erosion is described further in Sec. III.B.4.

D. Magnetic-Energy Balance (Circuit Equation)

Rather than forcing the time dependence of the plasma current, the present model specifies the axial electric field, E_p , applied across the pinch. Generally, the condition $E_p \simeq \text{constant}$ accurately reflects the early phase of the coaxial waterline discharge that is assumed here to drive the DZP. An initially large voltage ideally would be applied to the pinch until a specified condition (e.g., average temperature) is reached, after which the (depleted) waterline would be replaced electrically by a lower-(resistive) voltage power supply. Both the (optimistic) “crowbarred” mode of operation as well as the standard constant- E_p mode of operation are evaluated parametrically in this study. The circuit equation for either case is

$$V_p = E_p \ell_p = \frac{d}{dt}(LI) + IR_p, \quad (33)$$

which in normalized form is re-expressed as follows:

$$\epsilon \equiv E_p/E_{eq} = \frac{\frac{3}{8}(1-f_B)^{3/2}f_z}{i^2 x^2} + (1+f_L) \frac{d}{d\xi} \left[i \ell n \frac{r_c}{x} \right], \quad (34)$$

where $r_c = R_c/a_o$ is the return-conductor radius relative to the initial pinch radius, f_L is the ratio of stray (gap) inductance to that of the pinch, and $E_{eq} \equiv \mu_o I_{eq}/2\pi\tau_\Omega$ is a (fifth) normalization parameter. The inductance of the front end of the waterline is given by

$$L \simeq (1+f_L)(\mu_o \ell_p/2\pi) \ell n \left(\frac{R_c}{a} \right), \quad (35)$$

and neglects the inductance associated with the inner regions of the pinch, which contains only $\sim 4\text{-}5\%$ of the total stored magnetic energy. The quantity f_L is estimated from a simplified model of the waterline power supply (Sec. II.F) and reflects primarily parasitic inductances associated with regions of electrical insulation separating the waterline and the discharge chamber vacuum; typically f_L is expected to be in the range 0.2-0.3. While

the crowbar mode of “triggering” power supplies after achieving a given (optimal) pinch condition with the waterline *per se* gives the highest Q-values, the discharge times approach or exceed a microsecond and the current time derivative may actually go through zero, thereby violating an empirical stability “criterion.” Enforcement of a given current rise-time constraint (e.g., $\dot{I} > 10^{12} \text{ A/s}$) generally eliminates this optimal-Q operating mode, with the high-voltage waterline driving a shorter (100-200 ns), more intense burn; both options are examined.

E. Particle Balance (Fuel Burnup)

Particle depletion only by fusion is considered; radial particle transport is assumed to be small, and the electrodes are required to maintain axial pressure balance. The following fusion reactions are considered:

where the energy of each fusion product, $E_j(\text{MeV})$, is given in parentheses. All fusion products are assumed to be lost from the plasma without undergoing second-order reactions, although the impact of the catalyzed DD reaction (e.g., *in situ* burnup of tritium) is considered. If the burnup fraction of the i th species is defined as follows:

$$f_{Bi} \equiv 1 - \frac{N_i}{N_{i0}} , \quad (40)$$

the following expressions for f_{Bi} result for given initial (fractional) concentrations of tritium (f_{TO}), helium-3 (f_{HO}), and deuterium ($f_{DO} = 1 - f_{TO} - f_{HO}$):

$$\frac{df_{BT}}{d\xi} = g_{DT} n_D (1 - f_{BT}) < \sigma v >_{DT} n_n \quad (41)$$

$$\frac{df_{BH}}{d\xi} = g_{DH}n_D(1 - f_{BH}) <\sigma v>_{DH} \tau_\Omega \quad (42)$$

$$\begin{aligned} \frac{df_{BD}}{d\xi} &= g_{DD}n_D(1 - f_{BD}) <\sigma v>_{DD} \tau_\Omega/2 \\ &+ g_{DH}n_D(1 - f_{BH})(f_{HO}/f_{DO}) <\sigma v>_{DH} \tau_\Omega \\ &+ g_{DT}n_D(1 - f_{BT})(f_{TO}/f_{DO}) <\sigma v>_{DT} \tau_\Omega , \end{aligned} \quad (43)$$

where the subscript H refers to ^3He . The total burnup fraction is given by

$$f_B = 1 - f_{DO}(1 - f_{BD}) - f_{TO}(1 - f_{BT}) - f_{HO}(1 - f_{BH}) . \quad (44)$$

It is this expression for the total ion burnup fractions that is used in Eq. (22) for the pressure balance. If the triton produced in the proton branch of the DD reaction is burned *in situ* (e.g., catalyzed DD), then the following term is added to the right-hand side of Eq. (41): $\frac{1}{4}g_{DD}n_D(1 - f_{BD})(f_{DO}/f_{TO}) <\sigma v>_{DD}$.

F. Waterline Size and Stray Inductances

A water-filled transmission line provides the initial voltage, V_p , for the time τ_F needed to achieve a target temperature, T_F , after which the pinch voltage is decreased to and subsequently held constant at the value $V_R = IR_p$ existing at time τ_F , where R_p is the pinch resistance. As noted previously, stability-related constraints (e.g., $\dot{I} \geq 10^{12} \text{ A/s}$) may preclude this mode of operation. The approximate waterline model illustrated in Fig. 1 is used to estimate dimensions and associated stray inductances, given the values of V_p and τ_F . In addition to the plasma inductance,

$$L_p = \frac{\mu_o \ell_p}{2\pi} \ln(R_c/a) , \quad (45)$$

the inductances of “horizontal” gap, “vertical” gap, and insulator gap (Fig. 1) are approximated as follows:

$$L_{g1} = \frac{\mu_o \ell_g}{2\pi} \ln \frac{R_1 + \delta_g}{a_o} \quad (46)$$

$$L_{g2} = \frac{\mu_o \ell_g}{2\pi} \ln \frac{R_1 + \delta_g}{R_1} \quad (47)$$

$$L_i = \frac{\mu_o \ell_i}{2\pi} \ln \frac{R_2}{R_1} . \quad (48)$$

Fig. 1. Schematic diagram of water-filled transmission line used to model DZP.

Each of the radii, lengths, and gap thicknesses used in these expressions is illustrated in Fig. 1. Once these inductances are determined, the stray-inductance parameter used in Eq. (35) can then be determined from the following expression:

$$f_L = \frac{L_{g1} + L_{g2} + L_i}{L_p} . \quad (49)$$

The gap lengths, ℓ_g , and thicknesses, δ_g , are fixed at $\ell_g \simeq \delta_g = 0.01$ m. The insulator dielectric strength, $E_i \simeq 12 \times 10^6$ V/m, is used to determine ℓ_i for a given applied voltage, V_p . As is illustrated in Fig. 1, insulation grading is envisaged. The inner and outer radii of the insulator stack are taken as 50% and 75%, respectively, of the inner water-line radius [e.g., $R_1 \simeq R_i/2$ and $R_2 \simeq (3/4)R_i$]. Given that the magnetic-field energy

$$W_B \simeq (1 + f_L)L_p I^2/2 \quad (50)$$

is delivered to the plasma end of the waterline with an efficiency $\eta_{WL} \simeq 0.95$, the total energy stored in the waterline, $W_{WL} \simeq W_B/\eta_{WL}$, is used to determine the waterline capacitance,

$$C_{WL} = 2W_{WL}/V_p^2 = \frac{2\pi\epsilon_w\ell_w}{\ell\ln(R_o/R_i)} , \quad (51)$$

where ϵ_w is the dielectric constant of water. Since the length of the waterline, ℓ_w , is determined by τ_F (e.g., time to achieve the target temperature, T_F) from the following expression:

$$\tau_F = 2\sqrt{L_{WL}C_{WL}} = \frac{2\ell_w}{c}(\epsilon_w/\epsilon_o)^{1/2} , \quad (52)$$

where c is the speed of light and ϵ_o is the permittivity of free space, Eq. (51) determines the ratio R_o/R_i of the inner and outer waterline radii. For the case of constrained voltage (without triggering), τ_F in Eq. (52) is simply replaced by the total discharge time, $\tau_D \leq \tau_F$. A relationship developed empirically by the designers of waterlines,²² which is based on the statistics of breakdown across the waterline coaxial conductors and the need for a given limit on electrode area for a given standoff voltage, completes the model that characterizes the waterline geometry; this latter relationship is given as follows:

$$R_i^{0.83} = \frac{(2\pi\ell_w)^{0.17}(V_p/14 \times 10^6)}{R_o/R_i - 1} . \quad (53)$$

Once the trigger time, τ_F , (or τ_D in the case of constant- V_p operation) is determined, Eqs. (45) through (53) can be evaluated, including the value of the stray inductance parameter, f_L , which is then compared to that originally assumed in computing the discharge transient used originally to estimate τ_F or τ_D .

For times after τ_F when constraints on the magnitude of $\dot{I}$ are not imposed, the pinch voltage is decreased to the resistive voltage at that time and held at that value for the remainder of the discharge. Usually, the majority of the fusion yield occurs after the time τ_F , and a total discharge time, τ_D , is determined either by the maximization of the plasma Q-value or a physical limit (e.g., practical limit of fuel burnup, f_B , electron drift speed relative to c , number of Alfvén times, $N_A \equiv \int_0^{\tau_D} dt/\tau_A$, stability, current-rise rate, etc.). In this way, the characteristics of the (relatively) low-voltage power crowbar needed to sustain the discharge over the period $\tau_D - \tau_F$ can be determined as well as the characteristics of the high-voltage, water-filled transmission line. This approximate, but self-consistent model, therefore, allows the trade offs to be made between τ_D , τ_F , and Q_p for a range of plasma parameters (e.g., T_F , a_o , N_o , V_p profiles, fuel, etc.), including the case where $\dot{I}$ is forced to remain positive and above a critical value.

III. PARAMETRIC RESULTS

As noted previously, this study “tunes in” a range of physical phenomena and operational choices in order to quantify the impact on DZP performance, as measured by Q_p ; no implication is intended that actual control of some of the parameters varied in the study can actually be achieved, or at least varied independently of each other. In the course of conducting this parametric study, a “point-of-departure” or base case is first developed, and described in the following subsection. Using the plasma Q-value as an “object” function, the performance of the DZP for a range of operating conditions (e.g., profiles, endloss, fusion-product heating, fuel choice, applied voltage, line density, current rise rate, electrode erosion, etc.) is evaluated. Generally, a waterline discharge brings the DZP to a trigger temperature, T_F , after which only the plasma resistive voltage, $V_R(\tau_F)$, at time τ_F is maintained by a lower-voltage power supply. After parametrically studying this optimistic triggered or power-crowbarred mode of operation, a second “point-of-departure” case is

defined that is based on a constant- V_p mode of operation, and this constant- V_p mode is also studied parametrically. For either case, the initial conditions assume the formation of a low-temperature ($\Theta \simeq 0.01$), low-current ($i \simeq \Theta^{1/2} = 0.1$) pinch; the fiber melting/ablation process is not modeled and remains an important area of future work.^{23,24} Table I summarizes the essential features of all cases modeled for which burn trajectories are given and described in the following subsections.

The plasma Q-value, $Q_p \equiv W_F/W_{WL}$, is the main object function examined in this study, and some mention of what constitutes an “acceptable” value is warranted at this point. The value of Q_p deemed economically acceptable for a pulsed commercial reactor based on the DZP depends on the degree of recovery of invested magnetic energy by the thermal cycle and/or the magnet energy transfer and storage (ETS) system. While the focus of this study is not the reactor embodiment, a simplified reactor energy balance is described in Sec. IV, which indicates that values of $Q_p \gtrsim 10$ would be needed, depending on the degree of magnetic energy recovery.

A. Development of “Point-of-Departure” Case

Density and temperature profiles characterized by Eqs. (1) and (2) with $\alpha_T = \alpha_n = 1.0$ were arbitrarily adopted for the “point-of-departure” case. These profiles are shown in Fig. 2, along with the possibly more realistic values of $\alpha_n = 1.0$ and $\alpha_T = 0.1$; all quantities are normalized to average values, except for magnetic field, which is normalized to the edge-plasma value. Figure 2 also gives the respective profile factor, g_i . These “self-similar” profiles are assumed to be maintained throughout the discharge period. With these point-of-departure profiles, a constant voltage, V_p , was applied to the pinch, with the dynamic response of the normalized discharge parameters (e.g., $x = a/a_o$, $\Theta = T/T_{eq}$, and $i = I/I_{eq}$ versus $\xi = t/\tau_\Omega$) being shown in Fig. 3. For this DT-fueled Case I, density depletion by burnup, endloss of plasma enthalpy, fusion-product heating, and electrode erosion were nulled. The plasma in this case settles into the Pease-Braginskii equilibrium ($di/d\xi = d\Theta/d\xi = 0$; $\Theta = i = 1$) at a normalized radius, x_{eq} given by

$$x_{eq} = \sqrt{\frac{3}{8\epsilon}} \quad , \quad (54)$$

TABLE I
SUMMARY OF DZP BURN SIMULATIONS^(a)

CASE	FIGURES	FUEL	CAT. ^(b)	TRIG. ^(c) $T_F(keV)$	RADIUS $a_o(\mu m)$	FUEL BURNUP	FP HEATING ^(d)	END- LOSS ^(e)	ELECTRODE EVAPORATION
I	3	DT	NO	NO	20	NO	NO	NO	NO
II	4	DT	NO	YES	20	NO	NO	NO	NO
IIIA ^(f)	5	DT	NO	YES	20	YES	NO	NO	NO
IIIB	5	DT	NO	NO	20	YES	NO	NO	NO
IVA	6	DT	NO	YES	20	YES	NO	YES	NO
IVB	6	DT	NO	NO	20	YES	NO	YES	NO
VA	7	DT	NO	YES	20	YES	YES	YES	NO
VB ^(f)	7	DT	NO	NO	20	YES	YES	YES	NO
VIA	8	DT	NO	YES	20	YES	YES	NO	NO
VIB	8	DT	NO	NO	20	YES	YES	NO	NO
VIIA	15	DT	NO	NO	42.5	YES	YES	YES	NO
VIIB	15	DT	NO	NO	45.0	YES	YES	YES	NO
VIIC	15	DT	NO	NO	40	YES	YES	YES	NO
VIID	15	DT	NO	NO	40	YES	YES	NO	NO
VIII	17	DT	NO	NO	20	YES	YES	YES	YES
IXA	20	DD	YES	NO	20	YES	YES	YES	NO
IXB	20	DD	NO	NO	20	YES	NO	YES	NO
IXC	20	DD	NO	NO	20	YES	YES	YES	NO
IXD	20	DD	YES	NO	25	YES	YES	YES	NO
XA	21	D ³ He	YES	NO	12	YES	YES	YES	NO
XB	21	D ³ He	NO	NO	12	YES	YES	YES	NO
XIA-O ^(g)	25	DD	YES	NO ^(h)	VARIED	YES	YES	YES	NO

(a) All simulations made with $\alpha_n = 1.0$ and $\alpha_T = 1.0$; variations in profile have little impact on burn dynamics or overall pinch performance. Also, the applied voltage for all cases listed is $V_p = 2\text{ MV}$, and the pinch length was fixed at $\ell_p = 0.1\text{ m}$ ($E_p = V_p/\ell_p = 20\text{ MV/m}$).

(b) Tritium generated from DDp branch burned *in situ*.

(c) Voltage $V_p = E_p \ell_p$ triggered (reduced) to resistive voltage once plasma temperature reaches T_F .

(d) When fusion-product (FP) heating is considered, 100% coupling to plasma was assumed. Heating by tritium FP from DDn is allowed only if the catalyzed option is used.

(e) Thermal conduction over entire pinch radius ($x_L = 1.0$). Only Case VIII includes electrode (Li) evaporation and ingression.

(f) Case IIIA provided an initial point-of-departure case for purposes of parametric studies; this role was subsequently shifted to Case VB for the constant- V_p operational mode.

(g) ZEBRA case with ℓ_p reduced from 0.1 to 0.05 m and the discharge time fixed to $\tau_D = \tau_{WL} = 100\text{ ns}$ for a nominal initial plasma radius of $a_o = 15\text{ }\mu\text{m}$. In the series of simulations indicated, both a_o and τ_{WL} were varied.

(h) Discharge time, τ_D , at waterline time, τ_{WL} , which was varied around experimental value of $\tau_{WL} \simeq 100\text{ ns}$.

Fig. 2. Density, temperature, pressure, and current-density magnetic-field profiles used to model DZP, where normalization to average values has been used, except for magnetic field, which is expressed relative to the edge-plasma value. Shown also are the plasma profile factors, g_i .

Fig. 3. Plasma transient for constant voltage (Case I).

and equal to $x_{eq} \simeq 0.46$, where $\epsilon = E_p/E_{eq}$ is given by Eq. (34). The values of Q_p , neutron production rate, $\dot{S}_N$, and neutron yield, S_N , given on Fig. 3 have meaning only in the unlikely circumstance when DT fueling is achieved (i.e., $f_B = 0.0$ for this case).

Figure 4 illustrates the pinch dynamics for the condition when the waterline voltage is maintained only for a time required to achieve a target temperature $T_F = \Theta_F T_{eq}$. At the time τ_F , when the desired temperature is achieved, the waterline is assumed (designed) to be depleted, and a self- or power-crowbar (of unspecified design) is applied to supply the resistive voltage, $V_R(\tau_F)$, existing across the pinch at time τ_F . Fuel burnup effects have not been included in Case II. Knowledge of V_p , $V_R(\tau_F)$, and τ_F , along with the final discharge time, τ_D , allows an estimate of the power-supply requirements (Sec. II.F) to be made. It is seen from Fig. 4 that after some oscillations the Pease-Braginskii condition is approached, but now at a higher value of normalized radius, $x_{eq} \propto 1/\epsilon^{1/2}$, than resulted for Case I, since $\epsilon = E_p/E_{eq}$ is now smaller after time τ_F .

The dynamics depicted in Figs. 3 and 4 change appreciably once fuel burnup, fusion-product heating, endloss, and modification of Z_{eff} plasma pressure by electrode erosion are taken into account. Since a mechanism for *in situ* refueling cannot be envisaged for the time scales of these discharges, fuel burnup is included as an essential part of the point-of-departure case being developed. Figure 5 gives the discharge kinetics when density depletion resulting from DT burnup is included, but fusion-product heating and enthalpy endloss are not; the trigger temperature is $T_F = 8 \text{ keV}$. Shown also on Fig. 5 (and all subsequent figures of this kind) is the burnup fraction, f_B . An optimal value of Q_p is shown for these “triggered” cases, in that once f_B exceeds 0.8-0.9 the cost in ohmic dissipation exceeds the incremental fusion yield from the $f_B \gtrsim 0.8$, reduced-reactivity plasma. Also noted is the large thermal excursion, since the ohmic dissipation is shared by fewer and fewer particles as burnup proceeds (the fusion products are not retained). The example given in Fig. 5 is adopted here as initial “point-of-departure” case for the triggered or crowbarred mode of operation; a second point-of-departure case is developed when V_p is held fixed in order to enforce minimum limits on $\dot{I}$ and to examine the implication of a more realistic (simple) driven system; Figure 5 also includes a constant- V_p (non-triggered) case (Case IIIB).

Fig. 4. Plasma transient for voltage decrease once $T_F > 8$ keV (Case II).

Fig. 5. Plasma transient for voltage decrease once $T_F > 8$ keV and for constant voltage, including density depletion from fusion burnup: initial "point-of-departure" (POD) case (Case IIIA).

The impact of “turning on” enthalpy endloss without electrode erosion, as described in Sec. II.B, is illustrated in Fig. 6 for both triggered and constant- V_p cases. The endloss is dominated by electron parallel-field thermal conduction for this case where the entire plasma column is assumed to conduct [e.g., $x_L = 1.0$ in Eq. (30)]. The combination of density loss by burnup and enthalpy loss by axial conduction accelerates the pinch collapse and fuel burnup, with the simulation depicted as Case IVA on Fig. 6 being terminated (as in all other cases) when a temperature limit of ~ 200 keV is exceeded. Generally, as will be shown, endloss shrinks the timescale of the burn along with a diminution of the plasma Q-value. The high fusion-product power density for the DT case is sufficient to sustain the pinch against axial thermal conduction over the entire plasma cross-section, with only $\sim 20\%$ decrease in Q_p (again, only the fusion product energy is retained in the plasma). Also shown as Case IVB on Fig. 6 is a constant- V_p discharge where, as for the triggered Case IVA, $V_p = 2$ MV ($E_p = 20$ MV/m) and fusion-product heating is not included. Generally, the constant- V_p case achieves somewhat smaller value of Q_p before the fuel-depletion-driven thermal excursion and radius contraction terminate the discharge.

When both alpha-particle heating (100%) and full ($x_L = 1.0$) plasma enthalpy endloss occur, the transients given in Fig. 7 result. Generally, the endloss energy sink ameliorates somewhat the adverse effects of the alpha-particle-driven expansion and premature quench that would result without this added energy sink, allowing plasma Q-values near $Q_p \simeq 20$ to be achieved for the triggered case (Case VA). As for the other cases with burnup (Case III, Fig. 5) or burnup and endloss (Case IVA, Fig. 6), the combination of nominally constant current and diminishing line density, through pressure balance $[\Theta(1 - f_B)f_z = i^2]$, rapidly drives the plasma temperature to the ~ 200 -keV computational cutoff value. As Case VB, Fig. 7 shows the effect of constant- V_p operation generally results in somewhat reduced Q_p and a large reduction in the discharge time scale, τ_D , as well as the generation of a greater portion of the yield during the period when $\dot{I} > 0$.

Cases VA and VB in Fig. 7 are identified as the two “point-of departure” or base cases that are subjected to parametric study in Sec. III.B. These two cases differ in whether or not a triggered mode *versus* constant- V_p mode of operation is invoked. Table IIA gives the

Fig. 6. Point-of-departure case (Case III, Fig. 5) with triggered or constant voltage, including full ($x_L = 1.0$) axial loss of plasma enthalpy (Case IV).

Fig. 7. Point-of-departure case (Case III, Fig. 5) with triggered or constant voltage, including 100% fusion-product heating; this case is adopted here as new "point-of-departure" case (Case VB).

initial conditions assumed for both, Table IIB gives the respective waterline parameters, and Table IIC lists the overall performance parameters. The inevitability of some level of endloss and some degree of fusion-product heating is assumed, although the latter is less certain. The impact of full fusion-product heating without enthalpy endloss can be dramatic, as is shown in Fig. 8; the achievable value of Q_p decreases by over an order of magnitude, as the recompressing plasma column undergoes a rapid expansion once a significant fusion rate is achieved. This expansion causes the fusion rate to diminish rapidly; the consequence is a relatively low burnup fraction ($f_B \simeq 0.1$) and Q-value ($Q_p \simeq 2$). For the power densities envisaged for DT fusion, the maximum level of enthalpy endloss appears to be well matched with the fusion-product heating (Fig. 7) to counter the adverse effects illustrated in Fig. 8 when endloss is “shut off.”

Up to this point all zero-dimensional simulations are based on parabolic density and temperature profiles (α_n and α_T equal to unity). The impact of assuming a relatively flat temperature profile ($\alpha_T = 0.1$) while maintaining a parabolic density profile ($\alpha_n = 1.0$), as is illustrated in Fig. 2, gives burn trajectories that are indistinguishable from these reported; for the constant- V_p Case VB, Q_p decreases from 11.6 to 11.3 when α_T is decreased from the $\alpha_T = 1.0$ point-of-departure case value to $\alpha_T = 0.1$, with discharge time increasing ~ 10 ns from the Case VB value of $\tau_D = 257$ ns for $\dot{I} > 10^{12}$ A/s (277 ns total).

B. Parametric Variations about “Point-of-Departure” Case

The updated point-of-departure Case VB (Fig. 7, Table II) includes fuel burnup, endloss of plasma enthalpy ($x_L = 1.0$), fusion-product heating ($f_\alpha = 1.0$), and constant- V_p operation. After examining the dependence of Q_p and τ_D on plasma radius and applied voltage for the idealized case where endloss and fusion-product heating is not included (but burnup is included, e.g., Case III, Fig. 5) in the following Sec. III.B.1, the parametric dependence of the point-of-departure Case V on V_p and a_o is examined in Sec. III.B.2. The value $V_p = 2$ MV ($E_p = 20$ MV/m) and $a_o = 20$ μ m are taken for the point-of-departure case. Section III.B.3 gives results for the point-of-departure case when a_o is increased to 40 μ m and endloss parameter x_L is varied, again including fusion-product heating, fuel burnup, and constant- V_p operation (e.g., Case VB, Fig. 7). The impact of (Li) electrode

TABLE IIA
DZP INITIAL PARAMETERS

Parameters	Value
Initial fiber	
• Length, $\ell_p(m)$	0.1
• Radius, $a_o(\mu m)$	20.
• Volume density, $n_o(10^{28}/m^3)$	5.50
• Line density, $N_o(10^{19}/m)$	6.91
• Initial inductance, $L_{p_o}(10^{-7}H)$	1.70
Return conductor	
• Length, $\ell_c(m)$	0.1
• Radius, $R_c(m)$	0.1
• External inductance factor, $f_L = L_{ext}/L_p$	0.20
Plasma profiles factors	
• Density, α_n	1.0
• Temperature, α_T	1.0
• Ohmic heating, g_{OHM}	1.57
• Bremsstrahlung, g_{BR}	1.40
• DT reactivity, g_{DT}	1.89
• Axial conduction, g_{CAX}	1.23
Normalization parameters	
• Equilibrium (Pease-Braginskii) current, $I_{eq}(MA)$	1.67
• Equilibrium temperature, $T_{eq}(keV)$	6.3
• Ohmic time, $\tau_\Omega(10^{-9}s)$	29.2
• Initial Alfvén time, $\tau_A(10^{-12}s)$	20.2
• Drift speed parameter, v_D/c	5.0×10^{-4}
• Coulomb logarithm, $\ell n\Lambda$	10.0
Initial condition	
• Normalized temperature, $\Theta_o = T_o/T_{eq}$	0.01
• Normalized current, $i_o = I_o/I_{eq}$	0.1
• Normalized radius, $x_o = a/a_o$	1.0
• Impurity, Z_{eff}	1.0
• D/T ratio	1.0
• Trigger temperature, $T_F(keV)^{(a)}$	8.0
• Driver voltage, $V_p(MV)$	2.0

(a) Only for triggered-voltage point-of-departure Case IIIA, the role of which was subsequently shifted to the constant-voltage Case VB.

TABLE IIB

"POINT-OF-DEPARTURE" CASE (CASE V) WATERLINE PARAMETERS

	Triggered- V_p (Case VA)	Constant- V_p (Case VB)
Fixed parameters:		
• Water dielectric constant, ϵ/ϵ_0	80.0	80.0
• Insulator dielectric strength, $E_{INS}(MV/m)$	12.0	12.0
• Front-end gap length, $\ell_g(mm)$	10.0	10.0
• Front-end gap separation, $\delta_g(mm)$	10.0	10.0
• Front-end transfer efficiency, η_{WL}	0.95	0.95
• Energy transfer and storage efficiency, η_{ETS}	0.0 ^(a)	
Trigger conditions^(b)		
• Trigger temperature, $T_F(keV)$	8.0	—
• Trigger time, $\tau_F(ns)$	171.	—
• Trigger current, $I_F(MA)$	1.86	—
• Before/after voltage, $V_p/V_R(MV)$	2.0/0.11	—
• Plasma radius, $a_F(\mu m)$	35.8	—
• Waterline energy, $W_{WL}(kJ)$	373.3	—
• Fusion energy, $W_F(kJ)$	208.5	—
• Plasma Q-value, Q_p	0.56	—
Water-line geometry^(c)		
• Length, $\ell_W(m)$	2.87	4.65
• Inner radius of insulator stack, $R_1(m)$	0.78	0.39
• Outer radius of insulator stack, $R_2(m)$	1.17	0.59
• Inner radius of waterline, $R_i(m)^{(d)}$	1.56	0.79
• Outer radius of waterline, $R_o(m)^{(d)}$	1.67	0.92
• External inductance factor, $f_L^{(e)}$	0.20	0.20
• Capacitance, $C_{WL}(\mu F)$	0.19	0.14
• Impedance, $Z_{WL}(ohm)$	0.27	0.47
Post-trigger conditions		
• Plasma resistive voltage, $V_R(kV)$	105.0	—
• Plasma current, $I_F(MA)$	1.85	—
• Resistive power, $P_R(GW)$	195.7	—

(a) No recovery of magnetic energy.

(b) Condition when temperature equals T_F and waterline voltage of V_p reduced to resistive voltage at that time, and then held at that value for the remainder of the discharge.

(c) Refer to Fig. 1.

(d) These gap spacings, R_o-R_i , predicted by the J. C. Martin formulation²² are ~50% those used in actual designs, which conservatively include factors of safety.

(e) $f_L = 0.20$ assumed.

TABLE IIC
"POINT-OF-DEPARTURE" CASE (CASE V) PERFORMANCE

	Triggered- V_p (Case VA)	Constant- V_p (Case VB)
Discharge times		
• Waterline drive, $\tau_F(ns)$	171.1	277.5
• Crowbar time, $\tau_D - \tau_F(ns)$	424.3	0.0
• Time for $dI/dt < 10^{12} A/s$ (ns)	172.6	256.8
Fuel burnup, f_B		
• For $dI/dt < 10^{12} A/s$	0.022	0.041
• Maximum	0.99	0.99
Plasma Q-value, $Q_p = W_F/(W_B + W_{OHM})$		
• For $dI/dt < 10^{12} A/s$	0.61	5.5
• Maximum	21.80	11.6
Normalized plasma radius, $x = a/a_o^{(a)}$	0.008	0.003
Fusion energy, $W_F(MJ)^{(a)}$	9.7	9.8
Ohmic dissipation, $W_{OHM}(kJ)^{(a)}$	118.0	60.7
Field energy, $W_B(kJ)^{(a)}$	446.9	843.8

(a) Conditions at point of maximum Q-value.

Fig. 8. Point-of-departure case (Case V, Fig. 7) with triggered or constant voltage, but without axial loss of plasma enthalpy, while maintaining 100% fusion-product heating (Case VI).

evaporation and ingression into the plasma is investigated in Sec. III.B.4. Lastly, the DD and $D^3\text{He}$ fuel cycles are examined in Sec. III.B.5 for $V_p = 2 \text{ MV}$ and variable initial plasma radius, a_o .

1. DT Burn Dynamics without Fusion-Product Heating or Endloss

a. Triggered Operation. Triggered operation describes an idealized mode whereby the full pinch voltage is applied only for a time τ_F needed to achieve a desired pinch temperature, T_F , after which the system is crowbarred and the resistive voltage at time τ_F is maintained for the remainder of the discharge time, τ_D . While Table II gives results for the triggered mode under final point-of-departure conditions (e.g., including fusion-product heating and endloss), the parametric dependence of Q_p on T_F and a_o was examined first for even more idealized circumstances when endloss and fusion-product heating were turned off (Case III, Fig. 5); fuel burnup, however, is included. The dependence of Q_p , f_B , τ_F , and τ_D on the trigger temperature, T_F , is shown on Fig. 9. The dynamics of the pinch is typical of that shown for Case IIIA in Fig. 5. Shown also on Fig. 9 is the value of Q_p at the time when $\dot{I}$ falls below 10^{12} A/s for the near optimal value of $T_F \simeq 8 \text{ keV}$; Fig. 10 shows the dependence of Q_p , τ_D , and τ_F as a function of V_p . The dependence of Q_p on V_p and T_F for this $a_o = 20\text{-}\mu\text{m}$ case can be explained by the dynamics illustrated in Fig. 5; the nearly equilibrium pinch, upon recompression and achievement of a Pease-like current, undergoes a slow compression and rapid ohmic heating as the line density decreases because of burnup [Eq. (22), with a nominally constant current]. Triggering at a low temperature for a given V_p leads to a “fizzle”-like condition, where for the times considered the burnup is low and the ohmic dissipation is large; low- Q_p discharges result. Above a given trigger voltage, significant burnup is achieved for ever-decreasing discharge times, leading to a rapid increase in Q_p . At or above this threshold-like value of $T_F \simeq 6\text{--}8 \text{ keV}$, the pinch performance is relatively insensitive to V_p (Fig. 10), which primarily determines the discharge time scale. For all of these high- Q triggered cases, most of the fusion yield occurs when $\dot{I} < 10^{12} \text{ A/s}$.

b. Untriggered (Constant- V_p) Operation. While reactor-relevant Q -values ($Q_p \gtrsim 15$) are predicted for the triggered mode of operation, the required discharge times approach a microsecond, and most of the fusion yield occurs when $\dot{I} \leq 10^{12} \text{ A/s}$, with

Fig. 9. Dependence of Q-value, burnup fraction, trigger time, and discharge time on trigger temperature for Case IIIA conditions (Fig. 5).

the current derivative actually becoming negative. The question of stability of discharges where $N_A \geq 2.3 \times 10^4$, coupled with the uncertain means by which such discharges can be "crowbarred" for times on the order of $\tau_D - \tau_F \simeq 500\text{-}600 \text{ ns}$, has led to the adoption of the constant- V_p mode of operation for all subsequent studies. This subsection continues with the idealized assumption of no endloss or fusion-product heating. Case IIIB in Fig. 5

Fig. 10. Dependence of Q-value, trigger time, and discharge time on initial voltage for Case IIIA conditions (Fig. 5).

gives a typical burn trajectory for this $V_p = 2\text{-MV}$, $a_o = 20\text{-}\mu\text{m}$ case. The dependence of Q_p and τ_D on V_p ($a_o = 20 \mu\text{m}$) and a_o ($V_p = 2 \text{ MV}$) is shown on Figs. 11 and 12, respectively. Relative to the triggered mode of discharges, these shorter $a_o = 20\text{-}\mu\text{m}$ constant- V_p discharges predict a reduction in the Q-value by a factor of approximately two, and, like the triggered- V_p mode, Q_p is relatively insensitive to V_p ; the applied voltage sets the time scale rather than the Q-value. All of the $a_o = 20\text{-}\mu\text{m}$ discharges shown in Fig. 11 achieve the projected Q-value with $\dot{I} \geq 10^{12} \text{ A/s}$.

Fig. 11. Dependence of Q-value and discharge time on initial voltage for Case IIIB (constant-voltage) conditions (Fig. 5).

Unlike V_p , the value for initial fiber radius has a dramatic impact on the Q-value achieved as well as on that part of Q_p and τ_D where $\dot{I} \geq 10^{12}$ A/s. As seen from Fig. 12, increasing a_0 initially increases Q_p and τ_D . As a_0 is increased above $\sim 30 \mu m$ for these constant- V_p (2 MV) discharges, the discharges become sluggish, $\dot{I}$ falls below a critical value (10^{12} A/s, or even becomes negative), and eventually the discharge becomes too massive to heat; the value of Q_p plummets. This dependence of Q_p on a_0 is examined further for the more realistic point-of-departure case where endloss and fusion-product heating are included (Case VB, Fig. 7).

2. DT Burn Dynamics with Fusion-Product Heating and Plasma Enthalpy Endloss (Updated Point-of-Departure Case). Since it seems unlikely that plasma enthalpy endloss will not be a factor in determining the dynamics of an $\ell_p = 0.05$ - 0.10 -m pinch, the point-of-departure case adopted endloss at a maximum value ($x_L = 1.0$)

Fig. 12. Dependence of Q-value and discharge time on initial plasma radius for Case IIIB (constant-voltage) conditions (Fig. 5). Also shown is the dependence of Q-value on initial plasma radius when full fusion-product heating and plasma enthalpy endloss are included (Case VB, Fig. 7).

as an operating condition. As shown in Fig. 6 for Case IVB, endloss without fusion-product heating leads to a rapid collapse of the pinch and an accelerated burn, with the fusion rate, $\dot{S}_N \propto (1 - f_B)^2 < \sigma v > / x^2$, never turning over during the simulation period (set by T exceeding 200 keV or f_B exceeding 0.99) because of the increased

density, despite the thermal excursion and reduced reactivity. Reactor-relevant values of Q_p (~ 18 for Case IVB, Fig. 6) result. Inclusion of fusion-product heating without endloss or an equivalently effective means to drain off the excess heat from the pinch leads to an uncontrolled expansion, a fusion quench, and seriously degraded Q -values ($Q_p \lesssim 0.8$), as is shown in Fig. 8 for Case VIB under otherwise identical conditions. Both fusion-product heating and plasma enthalpy endloss, along with constant- V_p operation with fuel burnup, were adopted as a new, more realistic point-of-departure Case VB (Fig. 7). The parametric dependence of this case on V_p and a_o is reported in Figs. 13 and 14, respectively. Generally, the sensitivity of Q_p to V_p for the constant- V_p ($a_o = 20 \mu m$)

Fig. 13. Dependence of Q -value and discharge time on voltage for Case VB (Fig. 7) conditions (constant-voltage, full end-loss, full fusion-product heating).

Fig. 14A & B. Dependence of Q-value and discharge time on initial plasma radius for Case VIB (Fig. 7) conditions for two values of applied (constant) voltage.

mode of operation is greater when endloss and fusion-product heating are included than when these effects are not included (Fig. 11). The point-of-departure case exhibits Q_p values in the range 10-12 for discharge times of $\tau_D = 200$ ns ($N_A \simeq 30,000$); the value of Q_p is approximately halved if the $\dot{I} > 10^{12}$ A/s constraint is imposed, with little impact on τ_D (Fig. 13). Most cases in this regime exhibit nearly total burnup, and the decreasing Q_p with increasing V_p is attributed to greater ohmic dissipation in the more restricted (more

Fig. 14B.

compressed) plasma. The dependence of Q_p on a_o for two values of V_p (2 MV and 5 MV) given in Fig. 14 shows an increase in Q_p to values as high as $Q_p \approx 16$ ($\tau_D = 500$ ns) until a critical initial pinch radius is reached (apparently independent of V_p) where ohmic heating is insufficient to induce a thermal runaway, and a "fizzle" occurs; this behavior is shown in Fig. 15, which gives a range of burn transients near $a_o \approx 40 \mu m$ (40.0, 42.5, and 45 μm) as Case VIIA, VIIB, and VIIC, with Case VIID showing the adverse effect for $a_o = 40 \mu m$ and $V_p = 2$ MV of shutting-off endloss while maintaining full fusion-product heating. The

dramatic difference in increasing a_o from $42.5 \mu m$ (thermal runaway, high burnup) to $45.0 \mu m$ (a “fizzling” equilibrium, low-temperature, low-burnup pinch) is noted. A comparison of the $a_o = 40\text{-}\mu m$ cases (Cases VIIC and VIID) illustrates the importance of endloss in shedding fusion-product heat for these larger-radius plasmas.

3. DT Burn Dynamics with Full Fusion-Product Heating and Variable Plasma Enthalpy Endloss. The endloss parameter x_L [Eq. (30)], while in principle dependent on radial field profiles, was equated to unity for the point-of-departure case. Without plasma enthalpy loss in the presence of fusion-product heating, the Q_p -limiting, alpha-particle expansion of the kind depicted in Fig. 15 for Case VIID occurs. The dependence of Q_p on x_L is shown on Fig. 16 for the $a_o = 40\text{-}\mu m$, $V_p = 2\text{-}MV$ Case VIIC; values of $x_L \lesssim 0.5$ are needed to provide a level of energy drain that is sufficient to shed the full ($f_\alpha = 1.0$) alpha-particle heating rate to an extent needed to reverse the adverse fusion-product-driven expansion. Again, the impact of constraining the system to $\dot{I} > 10^{12} A/s$ more than halves the value of Q_p , with little impact on τ_D , which for this larger-radius pinch is in the 400-500 ns range.

4. Electrode Impurities. Impurity influx from the electrodes has not been modeled in the simulations made up to this point. If the electrode material is liquid lithium, the ion transit time from the electrode to the pinch midplane at $T_{eq} \simeq 6 \text{ keV}$ is $\tau_i \sim 120 \text{ ns}$ for $\ell_p = 0.1 \text{ m}$; hence, according to Fig. 13, voltages in excess of $\sim 5 \text{ MV}$ may be desirable to assure that $\tau_D \lesssim \tau_i$ in order that the electrode material have minimal impact on the burn. In order to examine this supposition quantitatively, the following electrode ingress model was developed and implemented.

The conduction energy delivered to each electrode in a given time increment is assumed to appear as ionized electrode material in the vicinity of the electrode but at the mean plasma temperature existing at that time. The electrode material, for the purposes of this calculation, is liquid lithium metal. The ionized electrode material created from each electrode is assumed to join the zero-dimensional plasma and contribute to the Z_{eff} after it has moved one-fourth of the pinch length at the thermal velocity $v_i \simeq (2k_B T/m_i)^{1/2}$; hence, a lithium impurity ion created at time t contributes to the Z_{eff} of the bulk

Fig. 15. Plasma transients for constant-voltage cases, with full endloss and fusion-product heating (Case VB, Fig. 7), at plasma radii near the critical "fizzle" conditions shown in Fig. 14. Also shown is the transient for the larger initial-radius case ($a_o = 40 \mu\text{m}$) when endloss is eliminated.

Fig. 16. Dependence of Q-value and discharge time and degree of endloss (x_L) for large-initial-radius case. Figure 15 (Case VIIC and Case VIID) gives the transients for $x_L = 1.0$ and 0.0 , respectively.

(average) at a time $t + \ell_p/4v_i$. The rate of impurity production, expressed per unit of plasma volume and using the definition of endloss time, τ_{EL} [Eq. (32)], becomes

$$dn_i/dt = f_z n/\tau_{EL} . \quad (55)$$

Because of the delay in entering the plasma associated with the ion transit time, the fraction of the total electrode material actually computed to be part of the bulk plasma, f_{ev} , will be less than that actually ablated up to that time. The remainder of the ablated material is approximated as an inward-moving "cloud" at each end of the pinch with

a spectrum of thermal speeds characteristic of the past temperature and endloss-power history; equilibration within this cloud is not modeled at this level of analysis.

Although highly approximate, this electrode-ablation, plasma-contamination model gives an indication of the discharge time scale on which the higher-Z electrode material would move into the central parts of the pinch and, by increasing Z_{eff} and Z_{AVE} , affect pressure and energy balance through the usual terms. The point-of-departure Case VB (Fig. 7) used to generate Fig. 13 was used to examine these electrode effects. Fusion-product heating (100%) and full enthalpy endloss ($x_L = 1.0$) were enforced. Figure 17 gives the full burn kinetics for this Case VIII, and Fig. 18 gives time history of Q_p for this constant $V_p = 2\text{-}MV$ case with and without electrode evaporation ingression.

The difference between $Q_p(\dot{I} \geq 10^{12} \text{ A/s})$ and $Q_p(\text{unconstrained})$ for $V_p = 2.0 \text{ MV}$ is shown on Fig. 18; when no electrode effects modeled and $\dot{I} > 10^{12} \text{ A/s}$, the value of Q_p is actually improved by the electrode material entering the plasma. The time dependence of Z_{eff} and f_{ev} are also shown on Fig. 18. Not until well into this discharge does the inventory of electrode material in plasma actually begin to match that which is evaporated ($f_{ev} \rightarrow 1$), but before this can happen, the pinch collapses, the electrode erosion rate for a brief time dramatically increases, but this material has not had sufficient time to stream into the core plasma. The general improvement of Q_p with electrode evaporation and ingression included is shown for a range of discharge voltages in Fig. 19. The improvement in discharge performance when electrode material is allowed to enter the pinch for these constant- V_p cases results from a delicate balance between voltage and pressure balance, with the arrival of electrode impurities into the pinch actually preventing the current from decreasing dramatically, as is shown in Fig. 18.

Generally, inclusion of these electrode effects compresses the discharge time while increasing the Q-value significantly, particularly if the $\dot{I} > 10^{12}\text{-A/s}$ constraint is applied (Fig. 18). The dependence of Q_p on V_p is shown on Fig. 19 when electrode evaporation is included, again showing the enhancement of Q_p for lower values of V_p (larger discharge times); for $V_p \gtrsim 4 \text{ MV}$, the discharge times are less than or comparable to the ion (Li) axial transit time, and little impact on Q_p is seen (Fig. 19). Also, in this higher voltage region the $\dot{I} \geq 10^{12} \text{ A/s}$ constraint returns.

Fig. 17. Plasma transient for point-of-departure case (Case VB, Fig. 7) with electrode evaporation (Case VIII).

Fig. 18. Time dependence of Z_{eff} , fraction of electrode evaporation entering plasma, f_{ev} , and plasma radius, a/a_o , with (Case VIII, Fig. 17) and without (Case VB, Fig. 7) Li-electrode evaporation.

5. **Fusion Fuel (DD and D^3He).** The impact of using of the less-reactive, “advanced” fuels in the form of the 50/50 DD or D^3He fuel cycles was examined briefly. Direct application of the point-of-departure parameters (Table II) proved somewhat uninteresting for the D^3He fuel, however; in order to enhance performance, the initial plasma radius for D^3He was decreased from $a_o = 20 \mu m$ to $10 \mu m$. The need to achieve the higher temperature on a reasonable (e.g., sub-microsecond) time scale dictated the smaller-radius D^3He fibers. These tradeoffs deserve further examination, and the DD and D^3He cases shown in Figs. 20 and 21 under otherwise point-of-departure conditions are included here as examples of possibilities.

a. **DD Fuel Cycle.** Figure 20 illustrates the burn dynamics for four DD-fueled cases. Cases IXA and IXC most closely replicate the point-of-departure case. As is shown in Sec. IV, Q_p is proportional to E_F/T for the same burnup, and the decrease in

Fig. 19. Dependence of Q-value and discharge time on applied (constant) voltage under Case VB (Fig. 7) conditions, but with (Li) electrode evaporation.

energy yield per fusion, E_F , as well as increase in fusion temperature, contribute to the decreased Q-value ($Q_p \simeq 3-5$), despite the high burnups predicted. The DD Case IXC is uncatalyzed, since both the energy and DT fusions associated with the triton fusion product [Eq. (36)] have been assumed to be lost from the pinch ($E_F \simeq 3.65$ MeV/fusion).

If the case of catalyzed DD fusion is considered ($E_F \simeq 9.96 \text{ MeV/fusion}$), both the DT fusion and retention of all fusion-product energy is assumed; this situation is shown in Fig. 20 as Case IXA. As for the DT Case VIB, rapid fusion-product-heating-induced expansion of the pinch occurs without endloss, and Q_p decreases dramatically to ~ 0.5 ; inclusion of plasma enthalpy endloss for $x_L \simeq 1.0$ [Eq. (30)] reverses this situation, with plasma collapse and an increase in Q_p to ~ 5.0 resulting. When no fusion-product heating is allowed (Case IXB, Fig. 20), the ohmic heating alone is not sufficient to maintain the plasma temperature against full endloss ($x_L = 1.0$) for the low values of DD reactivity, and the Q_p value plummets. Partial (uncatalyzed) fusion product heating is sufficient to reverse this situation (Case IXC, Fig. 20). As shown for Case IXD, slight increases in a_o (from 20 to 25 μm) also cause a “fizzle” condition and low values of Q_p .

The dependence of Q_p and τ_D on a_o for these constant- $V_p(2 \text{ MV})$ DD discharges is shown on Fig. 22, which also shows the impact of uncatalyzed reactions with and without the ^3He fusion-product heating. As for the DT case, a critical radius exists beyond which a non-heating “fizzle” occurs; as noted previously, Case IXD of Fig. 20 gives the burn dynamics for the “fizzle” case. The DD case is examined further in Sec. III.C under ZEBRA conditions, where the discharge time, τ_D , is fixed by the waterline time constant, $\tau_{WL} \simeq 100 \text{ ns}$.

b. $D^3\text{He}$ Fuel Cycle. The case for a 50/50 $D^3\text{He}$ fuel is shown in Fig. 21 as Case X; the catalyzed Case XA (tritons from the DDp branch are assumed to escape the pinch), and the Case XB with tritium fusion-product burnup show little difference. In order to achieve temperatures of interest to the $D^3\text{He}$ reaction on a reasonable timescale, values of a_o below the point-of-departure case ($a_o = 20 \mu m$) were used; both $T_{eq} \propto 1/a_o^2$ and $\tau_\Omega \propto 1/a_o$ scaling parameter increase with decreasing radius, with $a_o \simeq 12 \mu m$ being optimal for the $V_p = 2\text{-MV}$, catalyzed case. The combination of reduced time scale (e.g., smaller a_o) and rapid increase in $\langle \sigma v \rangle_{DH}$ at temperatures above $\sim 25 \text{ keV}$ relative to $\langle \sigma v \rangle_{DD}$ gives the very rapid burn dynamics reported in Fig. 21. As indicated in Fig. 23, high Q_p values are possible for catalyzed $D^3\text{He}$ for discharge times that are below 200 ns. As found for the DT and DD fuel cycles, a critical initial fiber radius above $\sim 12 \mu m$ is found, above which a low- Q_p “fizzle” is predicted.

Fig. 20. Plasma transient for DD fuel (Case IX).

- A. Point-of-departure (Case VB) conditions (Fig. 7) with catalysis.
- B. No fusion-product heating (non-catalyzed).
- C. With fusion-product (non-catalyzed).
- D. Case IXA conditions, but a_0 increased to $25 \mu m$ ("fizzle").

$a_0=20.0$ $T_{eq}=6.4$ $I_{eq}=1.68$ $\tau_0=29.5$

$a_0=25.0$ $T_{eq}=4.1$ $I_{eq}=1.68$ $\tau_0=23.6$

Fig. 21. Plasma transient for catalyzed and non-catalyzed D³He Cases (Case X).

Fig. 22. Dependence of Q-value and discharge time on initial plasma radius for constant-voltage, catalyzed DD-fueled burns.

C. Predictions of ZEBRA Burn Dynamics

The ZEBRA dense Z-pinch experiment is a 200-kJ, $\tau_{WL} \simeq 100$ -ns waterline-driven experiment presently in the checkout phase at Los Alamos. Table III gives the general parameters for ZEBRA, along with the parameters of a complimentary experiment at the Naval Research Laboratory, and Fig. 24 gives a schematic illustration of the ZEBRA device. This section describes simulation results using simplified zero-dimensional model and the ZEBRA-like conditions described in Table IV.

As for the DT cases described in Sec. III.B., the ZEBRA simulations “turn on” certain physical phenomena to examine the relative impact on the dynamics of a pinch operated under ZEBRA conditions. All ZEBRA cases are described under Case XI

Fig. 23. Dependence of Q-value and discharge time on initial plasma radius for constant-voltage, catalyzed D^3He -fueled burns.

in Fig. 25. The nominal conditions apply $V_p = 2$ MV to an $a_0 = 15\text{-}\mu m$, $\ell_p = 0.05\text{-}m$ DD fiber for a time $\tau_{WL} = 100$ ns, after which the voltage is triggered down (or up if the pinch is highly resistive at that time) to the resistive voltage existing at the time $t = \tau_{WL}$. As

TABLE III
DENSE Z-PINCH PROOF-OF-PRINCIPLE EXPERIMENTS

	Naval Research Laboratory (ZFX)	Los Alamos National Laboratory (ZEBRA)
Pinch length, $\ell_p(m)$	0.05	0.05
Pinch current, $I(MA)$	2.0	1.4
Waveform	Sinusoidal	Quasi-Sinusoidal
Quarter-period, $\tau_{WL}(ns)$	630	$\lesssim 100$
Initial current rise, $\dot{I}(10^{12} A/s)$	5	25
Voltage, $V_p(MV)$	0.75	3.0
System energy, $W_{WL}(kJ)$	350	200

Fig. 24. Schematic drawing of ZEBRA dense Z-pinch experiment at Los Alamos.

TABLE IV
CONDITIONS USED TO MODEL ZEBRA EXPERIMENT (Case XI).

Fuel	DD (catalyzed)
Initial density, $n_s(10^{28}/m^3)$	5.5
Fiber length, $\ell_p(m)$	0.05
Return conductor radius, $R_c(m)$	0.03
Initial plasma conditions	
• $x = a/o$	1.0
• $\Theta = T/T_{eq}$	0.01
• $i = I/I_{eq}$	0.10
Voltage, $V_p(MV)^{(a)}$	2
Waterline time constant, $\tau_{WL}^{(a)}(ns)$	100
Profiles, $[1 - (r/a)^2]^{\alpha_{n,T}}$	
• Density, α_n	1.0
• Temperature, α_T	1.0
Waterline constants	Table II

(a) Voltage V_p is applied at time $t = 0$, held for time $t = \tau_{WL}$, and then decreased to the resistive voltage $V_R = (IR_p)_{t=\tau_{WL}}$ and held fixed at that value.

for all previous cases examined, the time dependence of $x = a/a_o$, $\Theta = T/T_{eq}$, $i = I/I_{eq}$, $N_A = \int_0^t dt/(a/v_A)$, f_B , Q_p , $\dot{S}_N$, and $S_N = \int_0^t S_N dt$ are monitored. All cases pertain to the catalyzed DD reaction. Case XIA given in Fig. 25 begins with no endloss, fusion-product (FP) heating, or burnup when the voltage V_p is applied for an indefinite period of time (e.g., $\tau_{WL} = \infty$); the pinch under these conditions simply settles into the $\Theta = i = 1.0$, $x_{eq} = \sqrt{3/8\epsilon}$ equilibrium. This ideal equilibrium shifts when $\tau_{WL} = 100$ ns [i.e., $V_p = (IR_p)_{\tau_{WL}}$ at time $t = \tau_{WL}$], as is shown in Case XIB. Inclusion of burnup in Case XIC shows little impact for the low-reactivity DD pinch for the times examined ($t < 2 \mu s$). Case XID includes endloss, which shifts somewhat the long-time equilibrium. The inclusion of fusion-product heating with $f_\alpha = 1.0$ leads to somewhat elevated temperatures and a small shift in time scales and reaction effects (e.g., fuel depletion, compression, added heating, etc.), but the nominal long-time equilibrium is more or less similar to those reported for previously described subsets of Case XI.

Case XIE is adopted here as a “point-of-departure” case for ZEBRA simulations, with overall performance parameters listed in Table V; generally $\dot{I}$ falls below $10^{12} A/s$

Fig. 25. Plasma transients for a range of ZEBRA conditions (Case XI A-O).

$a_0 = 5.0$ $T_{eq} = 102$ $I_{eq} = 1.68$ $\tau_0 = 118$.

$a_0 = 7.5$ $T_{eq} = 45.4$ $I_{eq} = 1.68$ $\tau_0 = 78.6$.

$$a_0=10.0 \quad T_{eq}=25.5 \quad I_{eq}=1.68 \quad \tau_0=59.0$$

TABLE V
ZEBRA PARAMETERS COMPUTED FOR CASE XIE ($f_\alpha = 1.0$) AND
CASE XIH ($f_\alpha = 1 - T/15$).^(a)

	$f_\alpha = 1$	$f_\alpha = 1 - T/15$
Alpha-particle energy retention		
Normalization factors		
• Initial plasma radius, $a_o(\mu m)$	15	15
• Equilibrium current, $I_{eq}(MA)$	1.68	1.68
• Ohmic time, $\tau_\Omega(ns)$	39.3	39.3
Post-trigger voltage, $V_R = [IR_p]_{t=\tau_{WL}}(MV)$	0.63	0.73
Radius upon triggering, $a(\mu m)$	8.1	7.6
Fusion energy upon triggering, $W_F(kJ)$	7.7	8.1
Waterline energy, $W_{WL}(kJ)$	153	150
Post-trigger power, $P_{OHM}(TW)$	0.98	1.13
Pinch performance when $\dot{I} < 10^{12} A/s$		
• Q-value, Q_p	0.041	0.043
• Time (ns)	100	100
• Burnup fraction, f_B	0.0020	0.0021
Waterline parameters (Fig. 1)		
• Length, $\ell_W(m)$	1.68	1.68
• Insulator stack		
– inner radius, $R_1(m)$	0.41	0.40
– outer radius, $R_2(m)$	0.62	0.60
• Waterline		
– inner radius, $R_i(m)^{(b)}$	0.82	0.80
– outer radius, $R_o(m)^{(b)}$	0.91	0.89
• Impedance, $Z_{WL}(ohm)$	0.50	0.52
• Capacitance, $C_{WL}(nF)$	76.6	74.9
• Strong inductance factor, f_L	0.34	0.34

(a) For fixed and computed input, refer to Tables I and II.

(b) These gap spacings, R_o-R_i , predicted by the J. C. Martin formulations²² are ~50% those used in actual designs, which conservatively include factors of safety.

upon triggering at $t = \tau_{WL}$, with a Q-value of $Q_p \simeq 0.04$ being reported at that point. Table V also lists the computed waterline parameters needed to achieve this performance.

The impact of setting $\tau_{WL} = \infty$ is shown as Case XIF, which leads to significant DD burnup, fuel depletion, and overheating at $t \simeq 240 ns$; Q-values near the maximum $Q_p \simeq 4$ are predicted. The impact of reducing V_p from the 2 to 1 MV for $\tau_{WL} = \infty$ is shown under Case XIG; the extended time scale leads to interesting burn dynamics caused

initially by a fuel-depletion-driven overheating, a reduction in the DD reaction rate and fusion-product heating, and a slow compression caused by density and enthalpy endloss.

For all ZEBRA cases examined so far, the fusion-product heating efficiency was fixed at $f_\alpha = 1.0$. While the correlation $f_\alpha = 1 - T/15$ found by Lovberg²¹ applies strictly only to the 3.56-MeV alpha particle from the DT reactions, this expression is nevertheless applied to the (catalyzed) DD fuel in Case XIH; this case should be compared to the point-of-departure ZEBRA Case XIE; integrated results are compared in Table V. The impact of a temperature-dependence f_α is seen to be minimal.

All cases examined up to this point have been for $a_o = 15 \mu m$. More heating and the achievement of significant burnup on a somewhat longer time scale is expected if a_o is reduced. Cases XI I-N give the pinch dynamics for a range of a_o values for otherwise point-of-departure conditions, but with $f_\alpha = 1 - T/15$. For very small values of a_o (e.g., Case XI I), the plasma is so resistive that, upon triggering from $V_p = 2 MV$ to $V_R = IR_p$ at $\tau_{WL} = 100 ns$, the voltage may actually increase. Generally, increasing a_o compresses somewhat the time scale where significant values of Q_p and $\dot{S}_n$ are predicted. The impact of increasing τ_{WL} from 100 to 200 ns is illustrated as Case XI O, which by the time voltage triggering occur, the highly compressed (and heating) plasma is so resistive that V_p must actually be increased in accordance to the computational algorithm adopted here.

Figure 26 summarizes these a_o and τ_{WL} variations, showing that Q_p (again for DD) at the time when $\dot{I}$ falls below $10^{12} A/s$ actually maximizes at a (significant) value in the range 0.4-0.5 for $a_o \simeq 5-10 \mu m$. The time when $\dot{I}$ falls below $10^{12} A/s$ is also shown, and for $a_o \gtrsim 8-10 \mu m$ is equal to τ_{WL} ; for a_o value below $\sim 8-10 \mu m$, the plasma is more resistive and the current derivative falls through this (assumed) critical range before the time for triggering time, τ_{WL} , is reached. These very resistive plasmas do not heat sufficiently by the time $\dot{I} \leq 10^{12} A/s$, and Q_p falls, unless V_p increases ($V_p = 2 MV$ for all these variable- a_o cases).

DISCHARGE TIME, $\tau_D(I \geq 10^{12} \text{ A/s})$ (ns)

Based on the assumption that DZPs can be stably formed and heated to 8-10 keV, with possible excursions subsequently to considerably higher temperatures, over a period of time in some cases approaching a microsecond (tens of thousands of radial Alfvén transit times), high-yield ($Q_p \simeq 20$) bursts of DT fusion energy may be possible from fibers of initial radius $a_o \simeq 20 \mu m$ and length $\ell_p \simeq 0.1 m$. Even for the DD fuel cycle, substantial amounts of neutron production at lower Q-values may be possible under conditions that differ little from those expected in experiments presently becoming operational.¹¹ The impact of axial loss of plasma enthalpy, fusion-product heating, plasma profiles, electrode ablation and ingression, and a range of operational conditions or choices (e.g., V_p , N_o , n_s , a_o , T_F , etc.) on the overall “per-shot” performance of the DZP was examined, with these effects taken together generally having a net benefit on the performance of the point-of-departure DT case (Case VB, Fig. 7) adopted for this study.

The more optimistic projections made herein hinge sensitively on the ability to achieve $\sim 90\%$ thermonuclear burnup of the DT fiber in times on the order of 500-700 ns for the idealized triggered mode of operation and 200-400 ns for the constant- V_p mode of operation. These $Q_p \lesssim 20$ conditions prevail when the current rise rate can have arbitrarily small values; restricting $\dot{I} \geq 10^{12} \text{ A/s}$ gives a constant-voltage discharge with Q -values that re-optimize to values that are about two times below the highest value predicted for the longer, triggered discharges.

Given a firmer prediction or verification of MHD stability characteristics of the DZP, examination of the pulsed technology requirements of this potentially economic and simplified DT fusion energy source is warranted. By invoking an efficient transfer and recovery of magnetic energy, even advanced fuels like DD and $D^3\text{He}$ offer a similar potential of high engineering Q -value, Q_E , with energy transfer and storage efficiencies in the range $\eta_{ETS} \simeq 0.8-0.9$ for DD and $\eta_{ETS} \simeq 0.5$ for $D^3\text{He}$ being required to bring the effective plasma Q -value, $Q_p^* = Q_p/(1 - \eta_{ETS})$, into the range above ~ 15 considered desirable for an economic power reactor. Similarly, shorter DT burns (few 100 ns) with reduced burnup ($f_B \simeq 0.05-0.10$) may achieve reactor-relevant values of Q_p^* by invoking efficient transfer and recovery of magnetic energy. This issue is examined quantitatively after first addressing the magnitude of Q_p more generally.

Ohmic dissipation can play a role in setting the time scale and Q -value for a given discharge. For example, the fusion yield is $W_F \simeq N_o \ell_p E_F f_B$, and the energy investment is $W_{WL} \simeq \frac{1}{2} L_p I^2 (1 + 2R_p \tau_D / L_p)$. Hence, with the energy yield per fusion, E_F , expressed in the same units as the nominal plasma temperature, T , the following approximate expression for $Q_p = W_F / W_{WL}$ results:

$$Q_p \simeq \frac{(E_F/T) f_B}{4 \ln(R_c/a) + 3\zeta_D} \quad , \quad (56)$$

where $\zeta_D = R_p \tau_D / L_p$. With $E_F \simeq 17,600 \text{ keV}$, $T \simeq 10 \text{ keV}$, $f_B \simeq 0.8$, $\ln R_c/a \simeq 8$, it follows that $Q_p \simeq 41/(1 + 3\zeta_D)$ for the DT fuel cycle. The general behavior of Q_p with a_o (or N_o) shown on Fig. 14 is explained by the reduction in ohmic dissipation (i.e., reduced ζ_D) because of the reduced discharge time scale. Also, Eq. (56) illustrates explicitly the decrease in Q_p expected for advanced fuels, which require higher temperatures for E_F

values equal to ($E_F^{D^3He} = 18.3 \text{ MeV}$) or less than ($E_F^{DD} = 3.65 \text{ MeV}$, without tritium burnup, 9.96 MeV with tritium burnup) the DT fuel cycle ($E_F^{DT} = 17.6 \text{ MeV}$).

Since Q_p has served as the primary object function in this study, a somewhat more detailed system energy balance is developed here to examine the reactor relevancy. The magnitude of the required value of $Q_p = W_F/W_{WL}$ will depend on a number of system efficiencies that are summarized in a simplified power-plant energy flow diagram given in Fig. 27. The energy W_B is transferred from the energy transfer and storage system (ETS) with a forward efficiency η_{ETS}^+ [same as η_{WL} previously adopted as 0.95 in Eq. (51)]. The fraction f_{BT} of the energy $\eta_{ETS}^+ W_{WL}$ actually delivered to the pinch is assumed recovered by the thermal cycle along with the neutron and alpha-particle energies, W_N and W_α , respectively. The remaining field energy, $(1 - f_{BT})\eta_{ETS}^+ W_{WL}$ is assumed to be delivered back to the ETS system with an efficiency η_{ETS}^- . The thermal energy W_{TH} is converted to a total electrical output W_{ET} with nominal efficiency η_{TH} . After skimming off the ETS makeup energy, $W_{WL}[1 - (1 - f_{BT})\eta_{ETS}^+\eta_{ETS}^-]$ and other auxiliary power needs (pumps, "housekeeping" power, controls, etc.), $f_{AUX}W_{ET}$, the electrical energy W_E is available for revenue. The fraction $\epsilon \equiv 1/Q_E$ of W_{ET} that must be recirculated is then

$$\epsilon = f_{AUX} + \frac{1}{\eta_{TH}} \cdot \frac{1 - (1 - f_{BT})\eta_{ETS}^+\eta_{ETS}^-}{Q_p(0.8M_N + 0.2) + \eta_{ETS}^+ f_{BT}}, \quad (57)$$

where $M_N \simeq 1.2$ accounts for neutron energy multiplication in a tritium breeding blanket. Defining $\epsilon' = (\epsilon - f_{AUX})\eta_{TH}$ and $Q'_p = Q_p(0.8M_N + 0.2)$, the following expression relates Q'_p to the recovery efficiency, η_{ETS}^- :

$$Q'_p = \left[\frac{1 - \epsilon'\eta_{ETS}^- f_{BT}}{\epsilon'} \right] - \left[\frac{(1 - f_{BT})\eta_{ETS}^+}{\epsilon'} \right] \eta_{ETS}^- . \quad (58)$$

For typical values $\epsilon \lesssim 0.2$ ($Q_E \gtrsim 5$), $f_{AUX} \simeq 0.05$, $M_N \simeq 1.2$, $\eta_{ETS}^+ \simeq 0.95$, and $\eta_{TH} \simeq 0.40$, $Q'_p = 16.7$ ($Q_p = 14.4$) for no magnetic-field-energy recovery either by the ETS system or the blanket system; if $f_{BT} = 1$ (full recovery of magnetic energy by the blanket), then $Q'_p = 15.7$ ($Q_p = 13.5$). Recovery of 50% of the initially invested magnetic energy ($\eta_{ETS}^- = 0.5$) for the above-listed parameters with $f_{BT} = 0$ lowers the requirement on Q'_p to 8.8 ($Q_p = 7.5$). Hence, values of Q_p around 10-15 promise an efficient DT-fueled

Fig. 27. Simplified DZP-based reactor energy balance leading to Eq. (57).

DZP reactor. Equation (57) is parametrically evaluated on Fig. 28; for a given value of $Q_E (\gtrsim 5$, as generally dictated by economics), efficient recovery of the invested magnetic energy (i.e., large values of η_{ETS}^-) favors $f_{BT} \rightarrow 0$ (no thermal recovery) and *vice versa*. Hence, for many of the cases considered in this parametric study, an economically viable DT fusion power plant may be possible with only modest levels of reversible magnetic energy recovery.

In conclusion, high- Q DT burns in DZPs should be possible if these $\ell_p \simeq 0.1\text{-m}$, $a_o \simeq 20\text{-}\mu\text{m}$ pinches can be held for times approaching a reasonable fraction of a microsecond ($N_A \gtrsim 10^4$) under conditions of rapidly decreasing line density caused by significant fuel burnup ($f_B \gtrsim 0.9$) and for small, if not negative, values of $\dot{I}$. While fusion-product heating acting alone will destroy the pinch before significant burnup, yield, and Q_p can be achieved, self-heating in the presence of plasma enthalpy endloss has contravening effects, with little

Fig. 28. Dependence of engineering Q-value, Q_E , on plasma Q-value Q_p for a range of magnetic energy recovery efficiencies, η_{ETS} , and thermal recovery fractions, f_{BT} , for $\eta_{ETS}^+ = 0.95$, $M_N = 1.2$, $f_{AUX} = 0.05$, and $\eta_{TH} = 0.40$.

impact on the most favorable ($Q_p \simeq 20$) extended-burn (i.e., triggered) pinch performance. Ingression of low-Z (Li) electrode material may have a net beneficial effect, which is seen in the context of this simplified model as an extension of the high-Q operating regime where $\dot{I} > 10^{12}$ A/s; other more subtle effects of higher-Z impurity influx along the pinch axis during the later, high-temperature phases of the discharge may have positive implications for MHD stability in the fusion-producing phase of the 200-400 ns discharge. Achievement of reactor-relevant Q-values for the DD and D³He fuel cycle will require reversible energy transfer/storage (ETS) with efficiencies in the range $\eta_{ETS} = \eta_{ETS}^+ \eta_{ETS}^- = 0.5-0.8$. The application of ETS systems to the DT fuel cycle would reduce the requirements on burn time and burnup by a factor 3-4. Lastly, the DT point-of-departure case for $Q_p \simeq 1$ generates $S_N \simeq 10^{17}$ neutrons (few percent burnup), which if pulsed at a rate $\sim 10-100$ Hz, would provide a DT fusion source with some interest for fusion nuclear materials testing,^{15,25} albeit the pulsed nature and related (neutron) flux-compression effects give some concern to materials scientists.²⁵

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NOMENCLATURE

$a(m)$	average plasma radius
$a_F(m)$	plasma radius at trigger of crowbar
$a_o(m)$	initial ($t = 0$) average plasma radius
$B_\theta(T)$	poloidal azimuthal magnetic field
BR	Bremsstrahlung Radiation
b	normalized magnetic field, $B_\theta(r)/B_\theta(a)$
$C_{BR}(W/m^3 keV^{1/2})$	Bremsstrahlung coefficient [Eq. (14)]
$C_{OHM}(ohm\ m\ keV^{3/2})$	resistivity coefficient [Eq. (13)]
$C_K(W/m\ keV^{7/2})$	electron thermal conductivity coefficient, $1.04 \times 10^{16}/\ell n\Lambda$
$c(m/s)$	speed of light
$C_{WL}(F)$	waterline capacitance
DZP	Dense Z-pinch
$E_{eq}(V/m)$	normalizing electric field, $\mu_o I_{eq}/2\pi r\Omega$
$E_F^{ij}(keV/fusion)$	fusion energy value for reaction ij
$E_p(V/m)$	axial electric field, V_p/ℓ_p
$e(c, J/eV)$	electron charge, 1.602×10^{19}
ETS	Energy Transfer and Storage
f_α	fraction of alpha-particle energy deposited in plasma
f_{AUX}	fraction of total electrical power supplying auxiliary power needs
f_B	total fusion-fuel burnup fraction
f_{BT}	fraction of magnetic energy transferred to waterline recovered by thermal cycle
f_{B_i}	burnup fraction of i^{th} fuel species
f_{CP}^{ij}	fraction of fusion power from reaction ij in charged particles and retained by plasma
f_D	ratio of electron draft current density to nv_{the}
f_L	ratio of stray inductance to plasma inductance [Eq. (49)]
f_Z	pressure/energy correction for impurities, $(1 + Z_{AVE})/2$
f_{ev}	fraction of electrode impurity evaporated that "joins" bulk plasma

FP	Fusion Product
FUS	Fusion
g_i	plasma power profile factor, $\int P_i dV_p / P_i$ ($i = OHM, FUS, BR$)
h_H	fusion-product-heating correction, $1 + P_{CP} / P_{OHM}$
h_L	endloss correction, $1 + P_{EL} / P_{BR}$
$I(MA)$	plasma current
$\dot{I}(MA/s)$	plasma current rate ($\geq 10^{12}$ A/s assumed as a stability “criterion”)
$I_{eq}(A)$	normalization (Pease-Braginskii) current, [Eq. (18)]
i	normalized current, I / I_{eq}
$I_F(A)$	plasma current at trigger of crowbar
$j(A/m^2)$	average plasma current density, $I / \pi a^2$
$j(r)(A/m^2)$	local (radial) plasma current density
$k_B(J/keV)$	Boltzmann constant, $10^3 e$
$L_{gi}(h)$	inductance of i^{th} gap
$L(h)$	plasma inductance
$L_i(h)$	insulator inductance
$\ell n \Lambda$	Coulomb logarithm ($\simeq 10$.)
$\ell_W(m)$	length of waterline (Fig. 1)
$\ell_i(m)$	length of an insulation stack (Fig. 1)
$\ell_p(m)$	plasma length
MHD	Magnetohydrodynamic
m	azimuthal MHD mode number
$m_e(kg)$	electron mass
$N(1/m)$	line density, $\pi a^2 n$
$N_i(1/m)$	line density of i^{th} species
$N_{io}(1/m)$	initial line density of i^{th} species
$N_o(1/m)$	initial line density, $\pi a_o^2 n_o$
N_A	integrated number of radial Alfvén times, $\int_o^t dt / \tau_A$
$n(1/m^3)$	volume average ion or electron density
$n(r)/1/m^3$	local (radial) ion or electron density

$n_s(1/m^3)$	solid fiber density ($5.5 \times 10^{28} m^{-3}$)
$P_F(MW)$	fusion power
$P_{CP}(MW)$	charged-particle power
$P_{EL}(MW)$	endloss power
$P_{OHM}(MW)$	ohmic power
$P_{PdV}(MW)$	expansion power
$P_{RAD}(MW)$	radiation power (Bremsstrahlung only, BR)
$p(Pa)$	volume-averaged plasma pressure
$p(r)(Pa)$	local (radial) plasma pressure
Q_E	engineering Q-value
$Q(MW/m^3)$	net volumetric heating, $(P_{OHM} + P_{CP} - P_{BR})/V_p$
Q_p	plasma Q-value, W_F/W_{WL}
Q'_p	corrected plasma Q-value, $Q_p(0.8M_N + 0.2)$
$q(W/m^2)$	electron thermal-conduction heat flux
RAD	radiation
$R_c(m)$	return conductor radius (Fig. 1)
$R_p(ohm)$	plasma resistance, $\eta \ell_p / \pi a^2$
$R_{1,2}(m)$	radii of insulator stack (Fig. 1)
$R_{i,o}(m)$	radii of waterline (Fig. 1)
r_c	normalized return conductor ratio, R_c/a_o
S_N	total neutron yield
$S_N(1/s)$	rate of neutron production
$T_F(keV)$	plasma temperature at which waterline is triggered (crowbarred)
$T_{i,e}(keV)$	density-weighted, volume average plasma temperature ($i = \text{ion}$, $e = \text{electron}$, $T_i = T_e \equiv T$)
$T(r)(keV)$	local (radial) ion on electron temperature
$T_{eq}(keV)$	temperature normalization [Eq. (17)]
$t(s)$	time
$V_p(m^3, V)$	average plasma volume, $\pi a^2 \ell_p$, or applied pinch voltage
$V_R(V)$	resistive voltage at trigger of crowbar

$v_D(m/s)$	electron drift velocity, j/ne
$v_i(m/s)$	electrode impurity thermal speed
$v_{the}(m/s)$	electron thermal speed, $\sqrt{2k_B T/m_e}$
$W_B(J)$	magnetic-field energy [Eq. (50)]
$W_C(J)$	recirculating energy
$W_E(J)$	net electrical energy
$W_{ET}(J)$	total electrical energy
$W_F(J)$	integrated fusion energy yield
$W_N(J)$	neutron energy yield
$W_{WL}(J)$	waterline energy, W_B/η_{WL}
x	normalized plasma radius, r/a
x_L	fraction of plasma cross-sectional area carrying electron thermal conduction heat flux
x_{eq}	normalized equilibrium plasma radius, $\sqrt{3/8\epsilon}$
y	normalized plasma area, x^2
Z_{AVE}	$\sum_j^1 n_j Z_j / \sum_j n_j$
Z_{eff}	$\sum_j^1 n_j Z_j^2 / \sum n_j Z_j$
Z_j	atomic number of j^{th} species
$Z_{WL}(ohm)$	waterline impedance
z	normalized pinch axial dimension
α_n	density profile factor
α_T	temperature profile factor
α_p	pressure profile factor, $\alpha_n + \alpha_T$
ϵ	normalized plasma voltage, E_p/E_{eq} , or $1/Q_E$
ϵ'	corrected recirculating power fraction, $(\epsilon - f_{AUX})\eta_{TH}$
ϵ_o	permittivity of free space
ϵ_W	dielectric strength of water (~ 80)
$\kappa_{ }(W/m\ keV)$	parallel-field electron thermal conductivity
$\eta(ohm\ m)$	plasma resistivity
η_{TH}	thermal-conversion efficiency
η_{WL}	waterline transfer efficiency

$\eta_{ETS}^{+,-}$	energy transfer and storage (ETS) efficiency to and from waterline load
η_{ETS}	global ETS efficiency $\eta_{ETS}^+ \eta_{ETS}$
$\langle \sigma \nu \rangle_{ij} (m^2/s)$	fusion reactivity of reaction ij
ζ_D	ratio of discharge time to ohmic time, $R_p \tau_D / L_p$
ξ	normalized time, t/τ_Ω
$\mu_o(h/m)$	permeability of free space, $4\pi \times 10^{-7}$
$(\omega\tau)_e$	product of electron gyrofrequency and Coulomb collision time [Eq. (29)]
Θ	normalized temperature, T/T_{eq}
$\tau_c(s)$	axial thermal conduction time [Eq. (30)]
$\tau_D(s)$	discharge time
$\tau_F(s)$	time when plasma temperature reaches T_F and crowbar is triggered
$\tau_{WL}(s)$	waterline time constant
$\tau_A(s)$	radial Alfvén time
$\tau_\Omega(s)$	time normalization [Eq. (16)]
$\tau_{BR}(s)$	Bremsstrahlung loss time [Eq. (31)]
$\tau_{EL}(s)$	endloss time [Eq. (32)]