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INTERPRETATION OF ENGINE CYCLE-TO-CYCLE* VARIATION BY CHAOTIC TIME SERIES ANALYSIS

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Abstract

In this paper we summarize preliminary results from applying a new mathematical technique- chaotic time series analysis (CTSA)- to cylinder pressure data from a spark-ignition (SI) four-stroke engine fueled with both methanol and iso-octane. Our objective is to look for the presence of "deterministic chaos" dynamics in peak pressure variations and to investigate the potential usefulness of CTSA as a diagnostic tool. Our results suggest that sequential peak cylinder pressures exhibit some characteristic features of deterministic chaos and that CTSA can extract previously unrecognized information from such data.

INTRODUCTION

Cycle-to-cycle variations in internal combustion (IC) engines have long been a subject of investigation since they can have a major impact on emissions and performance [1,2].[†] While several studies have thoroughly examined the statistical variability in parameters such as cylinder pressure, temperature and local velocities; little is known about the basic mechanisms that lead to cycle-to-cycle variations. The presence of these variations, or "cyclic dispersions", is especially noticeable in engines employing fuel-lean conditions (e.g. exhaust gas recirculation, stratified charge, etc.). This is particularly relevant to engines under development which will operate on modern alternative fuels. Improved diagnostics and controls for detecting and quantifying cycle-

to-cycle variations are thus likely to be an important part of future engine design strategies and control systems.

Traditional explanations have sought to interpret the variations as the result of random fluctuations in the equivalence ratio and mixture flow behavior due to the unsteady nature of turbulence in the combustion chamber. However, others have suggested that dynamical instabilities arising from nonlinear combustion kinetics might be the cause. The latter distinction is important because, if correct, it offers the possibility that well-defined deterministic models can be developed to simulate the basic processes involved.

Recently, it has been recognized that many dynamical systems thought to behave randomly are actually deterministic [3-5]. The appearance of randomness can be created by the nonlinear interaction of relatively simple component mechanisms to produce bewildering complexity. Conversely, if one adopts the proper perspective, the complexity is reduced and the apparent randomness is transformed into a more tractable global order. This growing body of scientific knowledge is usually referred to in the literature as "chaos" or "deterministic chaos."

Chaotic time series analysis or CTSA has been developed specifically as a tool to detect and quantify deterministic chaos in sequential experimental measurements. Recent successes in the application of CTSA to diverse types of experimental data indicate this aspect of chaos theory has near-term practical potential [6-9]. Use of the proper analytical methods is important for chaotic data because it has been demonstrated that traditional tools such as Fourier analysis and Box-Jenkins modeling are inadequate [9,10].

Two previous theoretical studies have suggested that cycle-to-cycle variations in SI engines are likely to be the direct result of chaotic processes involving ignition and

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[†]Numbers in brackets designate references at end of paper.

combustion [11,12]. It seems reasonable that engine variables such as peak cylinder pressure should reflect chaotic effects from the latter sources (if they exist) as well as other mechanical and thermal components in the engine. Rather than continue traditional statistical evaluations of experimental data, we decided to test this hypothesis by analyzing cycle-to-cycle peak pressure data using CTSA methods specifically designed to detect deterministic chaos. Both fuel type and engine load were varied to investigate the impact on the results.

BACKGROUND

CTSA - CTSA is typically applied to sequential measurements made at evenly spaced time intervals. The analysis begins by remapping the original data into a new coordinate system, a process known as "embedding". This procedure is based on a fundamental theorem of nonlinear dynamics (Taken's embedding theorem) and produces a "state trajectory" or "phase portrait" that is essentially equivalent to the actual dynamical path of the original system [3,9]. The result of the embedding is the creation of a series of m -dimensional state vectors, p_t , of the following form

$$p_t = [x_t, x_{t+k}, x_{t+2k}, x_{t+3k}, \dots, x_{t+(m-1)k}] \quad (1)$$

where x_t is the original measurement at time t , and k is an integral lag (i.e., number of steps along the time series).

The sequence of state vectors p_t that cover the length of the original time series forms a map of the system's behavior in m -dimensional space. If the trajectory of a system remains confined to a certain region of the embedding space over long time, it is said to be ergodic (i.e., in a dynamical "steady state") and to be following an "attractor". Perturbations to such a system will momentarily displace its path, but it will quickly return to the attractor.

If a dynamical system is nonlinear and behaving chaotically, its attractor will have distinctive characteristics that can be used both to confirm the presence of chaos and to quantify its essential features. Because of their unusual mathematical characteristics (e.g., fractal dimension [13]), chaotic attractors are commonly designated as "strange" attractors.

Two measurements found to be especially useful in detecting strange attractors are the correlation dimension and the Kolmogorov entropy [14,15]. Both of these quantities can be estimated from the correlation function (also known as the correlation integral) of the embedded time series data, which is defined by

$$C(r) = 1/N \sum_{j=1}^N \sum_{k \neq j}^N [1/(N-1)] \theta(r - |p_j - p_k|) \quad (2)$$

where N is the total number of trajectory points, $r \geq 0$, $|p_j - p_k|$ is the Euclidean distance between trajectory points j and k , and θ is a step function such that $\theta(s)=1$ if $s \geq 0$ and $\theta(s)=0$ otherwise. Essentially, $C(r)$ represents an average cumulative spatial distribution of the embedded points. The interesting property of $C(r)$ is that for chaotic attractors it scales with r according to a power-law relationship,

$$C(r) \propto r^D \quad (3)$$

over a relatively wide range of r .

The exponent D in Eq (3) is the correlation dimension and reflects the essential dynamical degrees of freedom in the system. Because of the fractal nature of strange attractors, D usually is non-integral.

Kolmogorov entropy, K , reflects the degree of chaos. Specifically, it is a measure of the rate of spreading of nearby points on the attractor with time. Highly chaotic systems exhibit rapid spreading or, in other words, rapid memory loss. K can be estimated (conservatively) from the change in $C(r)$ that results from increasing the embedding dimension, m [9,15]. The basic relationship is,

$$K \approx \ln[C(r) |_{m=n} / C(r) |_{m=n+d}] / (d\tau) \quad (4)$$

where $C(r)$ is evaluated in the region where it scales according to Eq (3), d is an integer > 0 , and τ is the number of unit steps between points in the time sequence (for our data τ is always just 1). K is always positive for chaotic systems.

Theoretically at least, measurement of any chaotic system variable should yield the same dynamical features. For practical reasons, we elected to use the peak cylinder pressure in each power stroke as our variable of interest. The main advantages in using this quantity are that: 1) it can be unambiguously defined from the pressure trace; 2) it is necessary to observe only a portion of the power stroke and none of the exhaust stroke to obtain a measurement for each cycle; and 3) variations over a relatively large number of cycles can be observed, improving the sampling statistics. The main disadvantage of this measurement is that it may tend to mask or filter out small time-scale processes within cycles.

It should also be noted that quantities such as peak cylinder pressure are by definition discrete (i.e., there is no defined

measurement between cycles i and $i+1$). This constrains the data analysis to CTSA methods that are suitable for discrete behavior. In applying Eq (1) the index t then represents the iteration or cycle rather than time. The logistic map [3,4] and the dripping-faucet [6] are good examples of other discrete systems that have been analyzed for chaos.

EXPERIMENTAL

APPARATUS - A single-cylinder Ricardo Hydra Mk.III engine was used to generate the experimental data. The engine was configured in an unthrottled direct-injection, stratified charge (DISC) mode, that was a modification of the MAN-FM spark-assisted design used for research and commercial buses [14-16]. The FM system employs an angled injector to spray fuel tangentially into a piston-bowl insert. Swirl is imparted to the intake air by the port geometry. A multi-electrode spark plug is positioned in the fuel spray path and extends slightly into the bowl. Engine specifics are presented in Table 1.

Table 1. Engine Characteristics

Single cylinder, vertical	
Bore diam. =	94 mm
Stroke =	90 mm
Displacement =	0.625 litre
Comp. Ratio =	13.2 : 1
Cyl. Block:	cast iron with steel wet cylinder liner, separate cooling
Cyl. Head:	aluminum with separate cooling
Comb. chamber:	bowl-in-piston inserts, 304L Stainless
Valvetrain:	belt-driven SOHC
Injection:	Bosch PE4A for modified builds (belt-driven) CAV LR pintle type nozzle with 0.62 mm needle lift high seat volume-unloading delivery valve
Ignition:	Autotronics MSD-6A multi-strike electronic ignition with magnetic trigger, triple electrode spark plug.

As part of another program, our engine was modified to operate with thermal barriers around the piston bowl. A substitute piston of matching bore and stroke was machined to accommodate bowl inserts which created a 0.050-inch air gap between the insert and piston body. A plasma-sprayed coating (0.050 inches thick) of partially stabilized zirconia was applied as a heat-loss barrier.

Iso-octane and M85 (85% methanol, 15% unleaded gasoline) were each used as fuels. These fuels were selected to see potential differences in combustion arising from chemical and physical properties (e.g. volatility), without having to significantly alter the engine configuration. Spark assist was provided by a high-energy multi-strike electronic ignition module, fired by a magnetic crankshaft-mounted trigger. This configuration results in a waste spark occurring late in the exhaust stroke, reducing the likelihood of residual gas effects.

DATA ACQUISITION - The basic data acquisition arrangement is depicted in Fig. 1. Cylinder pressure was measured with a Kistler 6123 quartz transducer (0-200 bar) mounted in the cylinder head and connected to a charge amplifier. Crank angle was simultaneously measured with a Data Technology RS23L shaft encoder having two channels in quadrature with 360 counts/revolution and a "top-dead center" (TDC) marker channel. The TDC marker was shifted to a position approximately 10 degrees before crank TDC to facilitate toggling between slow and fast data recording (see below). Additional corroborating measurements of crank position were made using magnetic pickups over flywheel markings.

Cylinder pressure data were recorded with a stand-alone computer-aided measurement and control (CAMAC) crate made by DSP Technology, Inc. Four channels of 12-bit data were stored in 1024 kilobytes of total memory until full. As described previously, we were interested only in identifying the peak pressure during the power stroke, so a method was needed to trigger data storage on and off at the appropriate times. This was accomplished by toggling the CAMAC data sample rate between 10 and 100,000 Hz. The higher rate was toggled on 4 to 5 degrees before TDC and toggled off (back to the lower rate) at about 10 degrees after TDC. By experience we found that this produced a data window that reliably contained identifiable portions of both the rising and falling portions of the pressure trace.

Sample rate toggling was accomplished using a Metrabyte CTM-05 board mounted in the host personal computer which also communicates with the CAMAC. The counter board tallied both TDC pulses to recognize the beginning of each power stroke and shaft encoder degree pulses to recognize the beginning of the high-data-rate window.

Once the peak pressure windows were recorded, it was necessary to scan the pressure trace segments to identify the peak in each cycle. A computer program was developed to perform this task automatically, and extensive testing confirmed its reliability. Using the above facilities, we were typically able to record unbroken sequences of approximately 1300 peak cycle pressures. As we describe in the following section, this amount of data was sufficient to distinguish

Fig. 1. Data acquisition and control system for engine chaos study.

trends, but quantitative uncertainties in the results are rather high. Such uncertainties can be reduced by the use of larger data sets.

Some noteworthy issues regarding the collection of data for the chaos time series analyses include synchronicity of channels and timebases. Specifically, comparison of cycle-to-cycle cylinder pressure data for identical crank positions requires high resolution of crank angle and repeatability of these signals. In addition, the synchronicity of digitized data from channel-to-channel at any given cycle is important and, in our case, was guaranteed by the microprocessor operation within the data acquisition system's analog/digital converters.

Further efforts to expand on the embedding space necessarily require larger-sized datasets. This could be achieved in the present data acquisition system by decreasing the number of channels recorded, decreasing the duration of the high data rate period, increasing the engine speed or adding memory to the CAMAC crate. Difficulties were encountered when some of these tactics were attempted. In particular, there were stray electrical signals which could not be entirely eliminated from the shaft encoder signals, and these were interpreted by the counter/timer board as legitimate pulses. This was aggravated at higher speeds where the counting process deteriorated, leading to shifting of the data window around the pressure peak. Shortening the width of the window was only marginally effective and led to

similar problems with keeping the window 'centered' around the peak.

The data acquisition system functions by sampling and storing data until the total memory is filled. Total memory is aggregated equally among the number of channels, so that decreasing the number of channels (from 4 to 2) should prove helpful (double the number of cycles recorded). However, this requires the engine and data acquisition system to operate stably for twice as long, perhaps ten minutes, as the memory is filled. Usually, the counting process was not able to sustain such an uninterrupted run.

Further investigation efforts may include measurement of the spark-to-peak time, spark-to-initial pressure rise time, or measurement of instantaneous piston velocity at some fixed crank position cycle-to-cycle. Such measurements will present new challenges in resolving cyclic dispersions of dependent parameters.

TESTING SCOPE - Experimental runs with the M85 fuel were made using three basic operating conditions. Peak pressure data were recorded for: 1) 1140 rpm, idling; 2) 1150 rpm, 5-6 ft-lb load; and 3) 1170 rpm, 6-7 ft-lb load. These three runs are designated here by the codes M1, M2, and M3, respectively.

Three runs were also made with iso-octane at: 1) 1105 rpm, idling; 2) 1105 rpm, 6-7 ft-lb load; and 3) 1190 rpm, 7-8 ft-lb load. Coarse measurements of fuel consumption rate were made for the various tests. Observed flowrates ranged from 0.51-0.78 ml/s

for the methanol fuel and 0.26-0.41 ml/s for the iso-octane, over the load range. There was no adjustment of fuel rack position between the loaded conditions for either fuel. However, the fuel consumption is also influenced by engine speed, which could be kept constant within ± 50 rpm. It is not believed that significant differences in fuel flow rate existed at the loaded cases which might have affected the charge density or other mixture conditions. In the last run the iso-octane injection volume was increased to be roughly equivalent to run M3 in terms of combustion energy input per cycle. The iso-octane runs are designated by the codes 11, 12, and 13, respectively.

ANALYSIS OF RESULTS

The peak pressure patterns recorded for each of the test conditions described previously are depicted in Figs. 2 and 3. No analysis is necessary to see that there are major differences among them. Perhaps most noticeable is the large number of partial burns (indicated by the low pressure spikes) that occurred during idle operation for both fuels. Frequent incomplete burns while idling appears to be characteristic of many direct-injection spark ignition engines and probably results from inadequate fuel-air

Fig. 2. Peak pressure vs. cycle number for M85 runs.

Fig. 3. Peak pressure vs. cycle number for iso-octane runs.

mixing and incomplete propagation. The higher temperatures and higher overall fuel-air ratio that occur with increased engine load tend to reduce the occurrence of incomplete combustion.

As expected, peak pressure statistics also reveal clear differences. As shown in Figs. 4 and 5 and Table 2, the probability distributions for both fuels indicate a trend toward higher average peak pressure and reduced variance with increased engine load. Runs M2 and M3 exhibited the smallest cycle-to-cycle variability.

As previously described, CTSA begins by embedding the original data in a state space created from values at lagged time (or in this case cycle) intervals. A preliminary approximation to the complete embedding that is often useful for screening purposes is to make two-dimensional plots of the time series data like those illustrated in Figs. 6 and 7. In these plots, x-axis values are the measured peak pressure for cycle i and y-axis values are the measured peak pressure for cycle $i+10$. The lagging value of 10 is not critical in this case; other values ranging from 1 to 50 produce essentially the same results.

Again in Figs. 6 and 7 we see distinctive patterns produced by each fuel and operating condition. The large frequency of partial burns during idling tends to result

Fig. 4. Frequency distributions of cyclic peak pressures for M85 runs.

in scattered points that fill most of a rectangular "box"; i.e., the pressure limits defined by total lack of combustion and the pressure at complete combustion. The upper right corner of the box has a higher density of points reflecting the greater probability of pairs of lagged cycles that both have good combustion. Differences between the fuels are indicated by slight differences in the point scatter and density within the box.

At higher load conditions the points tend to cluster near the upper right corner of the box, and occasional partial burns form vertical and horizontal projections. Again more subtle differences in the shape of the main cluster and projections distinguish the various non-idling cases. It is clear that some change in the quality of combustion

Fig. 5. Frequency distributions of cyclic peak pressures for iso-octane runs.

occurred when the fuels were switched. We surmise that physical properties and/or combustion kinetics may account for our observations, and suggest that the DISC engine may be inherently preferential to methanol.

Note that completely random pressure variations would produce the "shotgun" pattern typical of Gaussian processes. The fact that such patterns are not observed indicates that nonrandom processes are involved. This is corroborated by the lack of a characteristic bell-shape in the frequency distributions.

If we went no further, the patterns in Figs. 6 and 7 could be used alone or in conjunction with the frequency statistics to distinguish patterns of cycle-to-cycle variation. Additional information is gained,

Table 2. Statistical Summary of Peak Pressure Measurements

Run	Average	Max	Standard Min	Standard Deviation	Skewness
M1	479.6	593.8	304.2	51.5	-17.3
M2	617.5	647.0	533.2	14.0	-27.4
M3	676.0	717.8	298.8	17.9	-213.
11	483.8	576.7	318.4	49.6	-10.4
12	608.7	654.3	332.0	27.6	-65.5
13	634.1	732.4	316.4	53.5	-46.7

Fig. 6. Plot of cycle "i" peak pressure vs. lagged cycle "i+10" peak pressure for M85 runs.

Fig. 7. Plot of cycle "i" peak pressure vs. lagged cycle "i+10" peak pressure for iso-octane runs.

however, by making higher dimensional embeddings. As is demonstrated below, the actual number of dynamical degrees of freedom involved can be much higher than 2, and two-dimensional plots can thus mask important dynamic details.

Resolution of the higher dimensional details can be accomplished by performing the correlation calculation [Eq (2)] at successively higher embedding dimensions. This process is continued until the estimated correlation dimension, D , and Kolmogorov entropy K , converge; i.e., the estimates no longer change with increasing embedding dimension. This procedure is important not only because it defines convergent D and K values, but because it distinguishes chaotic processes from nonchaotic ones (e.g., Gaussian processes). For nonchaotic data, D and K change without limit as m is increased [17,18].

It is our experience that the convergence of D and K can be difficult to determine when data sets are small (e.g., < 10,000 points) and the number of degrees of freedom is relatively large (e.g., $D > 10$). This limitation basically results from the data set being too small to "fill" the embedding space sufficiently to fully represent the attractor statistics. As is discussed below, this was a shortcoming in our data, and merits further improvement.

Figure 8 illustrates correlation integral results for run M2 at successively higher embedding dimensions between 14 and 42 (with a constant embedding lag of 10). Note that log coordinates are used for both axes,

so that linear regions are indicative of fractal scaling. Furthermore, the apparent convergence of the slopes to a constant value indicates the presence of chaotic structure.

Figures 9 and 10 illustrate the convergence of the correlation dimension and estimated Kolmogorov entropy for run M2 over an even greater range in embedding dimension. Note that the dimension and entropy are apparently varying about constant values (@ 15.7 and 0.28, respectively) for embedding dimensions above about 34. The correlation dimension is relatively high compared to estimates for many other systems reported in the chaos literature [see 5, 8, 11, and 12, for example], but it does not seem unreasonable for such a complex system. However, we caution the reader that the limited extent of our data sets (<1400 points in all cases and <1000 in one case) makes reliable identification of the convergence values very difficult, especially at such high apparent correlation dimensions.

Table 3 summarizes our best estimates of the correlation dimensions and Kolmogorov entropy for each of the six data sets. There appear to be changes in D and K for each fuel as engine load is increased. This suggests that the basic mechanisms involved in fuel combustion are changing with load and being reflected in the attractor properties. Higher D and K values suggest the presence of added components; lower values suggest the reverse.

Attractors with such high dimensions obviously can not be clearly visualized in 2 or 3 dimensions. We must be satisfied with

Fig. 8. Correlation integral results for run M2 (M85) with increasing embedding dimension.

Fig. 9. Estimated correlation dimension vs. embedding dimension for run M2 (M85).

Fig. 10. Estimated Kolmogorov entropy vs. embedding dimension for run M2 (M85).

**Table 3. Summary of Estimates
for Correlation Dimension
and Kolmogorov Entropy**

Run	D	K
M1	13	0.21
M2	15	0.28
M3	15	0.27
I1	17	0.49
I2	16	0.35
I3	15	0.49

only projections, cutaway sections, or numerical descriptions. One alternative for visually comparing different attractors is to use their $C(r)$ plots. Recall that $C(r)$ represents the attractors average cumulative distribution in state space. Direct comparisons among $C(r)$ plots then represent comparisons of state space distribution. Such a comparison at constant embedding dimension is shown for the M85 test runs in Fig. 11. Note that the location and extent of the linear portion of each embedded data set varies.

The apparent high dimensionality of our results (i.e., high D) suggests that the observed peak pressure variation is due to the collective effect of a number of nonlinear components. This might explain the observation that large variations in the combustion time (the expected effect of changing the fuel properties and engine load) changed

the total dimensionality by a maximum of only about 30%. Other aspects of the chaotic structure may thus be due to engine design features such as gear lash and cylinder wall thermal conductivity. If this is true, then computer models such as those developed by Daily [11] will need to include more engine design details in order to more closely approximate the observed behavior.

An alternative explanation for the apparent high dimension is that it results from noise in the pressure measurement. Such an effect has been observed previously for other types of data. This might be investigated through the application of signal processing techniques to remove noise components from the data.

CONCLUSIONS

Our general observations support the conclusion that cycle-to-cycle peak pressure variations in four stroke SI engines exhibit some of the characteristic features of deterministic chaos. Specifically, the peak pressure data exhibit fractal scaling and positive Kolmogorov entropy. The results of this study should be considered tentative due to the limited length of the data sets evaluated.

Subsequent studies should include other types of engines and other variables that are likely to reflect the essential elements of cycle-to-cycle dynamics. In addition, much larger data sets covering tens of thousands of cycles are needed to clearly establish

Fig. 11. Comparison of correlation integral results for M85 runs at constant embedding dimension.

quantitative measures and demonstrate the consistency of the chaotic patterns.

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