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COO-1545-227

Mass Generation and Chiral Symmetry Breaking by Pseudoparticles^{*}

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ABSTRACT

Massless QCD is studied with regard to mass generation and chiral $SU(N_f)$ symmetry breaking from pseudoparticle effects. While mass is generated when there is only one massless quark, and chiral $U(1)$ is always broken, we see no rigorous indication of the breaking of chiral $SU(N_f)$ and mass generation when there are more than one massless quarks in the original theory.

^{*}Work supported in part by the DOE under contract number EY-76-C-02-1545.*000.

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I. Introduction

We study a theory of N_f massless quarks, doublets under color $SU(N)$, gauge invariantly coupled to a Yang-Mills field A_μ . The theory possibly describes an interesting limit in which chiral $SU(N_f) \times SU(N_f)$ is an exact symmetry of hadron dynamics. If conventional ideas of chiral symmetry realization and confinement of quarks are correct, then this theory should exhibit a solution with vacuum symmetry spontaneously broken to non-chiral $SU(N_f)$ and the appearance of $N_f^2 - 1$ massless pseudoscalars.

The theory as stated has no visible mass scale. Thus its solutions are either scale invariant or specified by an arbitrarily assignable scale. The need for renormalization at some non-zero mass μ indicates that the latter is the case. The theory is then defined at least by its off-shell Green's functions in the deep euclidean region, with renormalization group analysis¹ supplying the invariant [as $g \rightarrow 0$, $\mu \rightarrow \infty$]

$$\frac{d}{d\mu} \left[\mu^{\left(\frac{11}{3} N - \frac{2}{3} N_f \right)} e^{-8\pi^2/g^2(\mu)} \right] \approx 0 \quad (1.1)$$

indicating, for μ large,

$$\frac{g^2(\mu)}{8\pi^2} = \left[\frac{\frac{11}{3} N - \frac{2}{3} N_f}{\ln(\mu/\mu_0)} \right]^{-1} \quad (1.2)$$

where μ_0 is the invariant renormalization mass, the single parameter upon which the theory depends. Green's functions are then consistently interpreted as expansions in powers of (μ_0/p) where p is the external momentum; μ_0 is presumably measured by the deviations from naive scaling observed in deep inelastic scattering.

When this theory develops massive confined hadron states---as dogma insists it must, even in the chirally symmetric limit when $m_\pi^2 = 0$ ---these

masses would have to be scaled by μ_0 , the same scale determining deviations from asymptotic freedom. Such a coincidence of mass scales which in principle are quite independent seems a desirable motivating feature of this massless theory.

A possible intermediate step, between asymptotically free massless quarks at short distances and a confinement phase with the features just described, is one in which the quarks acquire a mass dynamically, without help of bare masses and a new mass scale independent of μ_0 . Then it would again be plausible that the hadrons composed of these "dressed" constituents have masses characterized by μ_0 alone.

Thus it is of interest to study the Green's functions of the "massless" theory seeking indications of

- (1) chiral $SU(N_f)$ asymmetric Green's functions, indicating a chirally asymmetric vacuum, $N_f > 1$;
- (2) quark mass generation in the two point functions, $N_f > 1$.

The hope that pseudoparticle field configurations are relevant in these matters was spurred by 't Hooft's² study of one-loop quantum corrections about the classical solution of Belavin, Polyakov, Schwartz and Tyupkin.³ He found the one-loop di-fermion propagator correlation functions of scalar and pseudo-scalar densities, e.g.

$$\langle \bar{\psi}(x) \psi(x) \bar{\psi}(0) \psi(0) \rangle,$$

clearly have $SU(N_f) \times SU(N_f)$ symmetry, but not $U(N_f) \times U(N_f)$. [As a teaching aid he then constructed an effective Lagrangian which reproduces the $SU(N_f) \times SU(N_f)$ tensor structure of the Green's functions in certain limits.] Subsequent analysis and interpretation of Jackiw and Rebbi,⁴ and Callan, Dashen and Gross⁵ exposed the fully gauge invariant vacuum $|\theta\rangle$ [implied by the theory as developed (for example, in a functional formalism) about the

pseudoparticle solution] as an $SU(N_f) \times SU(N_f)$ symmetric, but chiral $U(1)$ broken state, transforming as

$$\begin{aligned} e^{i\alpha Q_5^0} |\theta\rangle &= |\theta + 2N_f \alpha\rangle \\ e^{i\alpha Q_5^i} |\theta\rangle &= |\theta\rangle \end{aligned} \tag{1.3}$$

where Q_5^0 is the chiral $U(1)$ generator and Q_5^i are the chiral $SU(N)$ generators.

It is then interesting to ask whether more complicated pseudoparticle effects could induce the phase transition to a more physical vacuum which has a lower symmetry (as well as a lower energy!) or correspondingly whether these effects can produce Green's functions which have the tensor structure corresponding to a symmetry lower than $SU(N_f) \times SU(N_f)$. [The symmetry properties of the Green's functions and the symmetry properties of the vacuum are of course identical.] For example, the spontaneous breakdown of chiral $SU(N_f)$ would be evidenced by the dynamical appearance of quark mass terms in the propagation functions in a theory with $N_f > 1$ flavors.

In this paper we study pseudoparticle effects, especially with reference to their interaction with fermions and the fermion zero energy modes, and the implications for mass generation and chiral symmetry breakdown.

In Sec.II we discuss the theory as developed about exact pseudoparticle solutions and, more generally, about paths with integer topological quantum number. In Sec.III we discuss approximate minima of the action. In Sec.IV we present a summary and conclusions.

II. Exact Results

To begin, consider the euclidean generating functional for paths A_μ classifiable by an integer winding number v ,

$$W_\nu(\eta, \eta^\dagger) = \int [DA]_\nu \int d\psi d\psi^\dagger d\bar{\psi} \exp[\mathcal{L}(A, \bar{\psi}) - \psi^\dagger \not{D} \psi + \psi^\dagger \eta + \eta^\dagger \psi] \quad (2.1)$$

where $\mathcal{L}(A, \bar{\psi})$ contains gauge fixing and ghost terms in addition to the purely gauge field Lagrangian, and $\not{D}$ is the covariant derivative on color isospinor fermions ψ ,

$$\begin{aligned} \not{D} &= \gamma^\mu \partial_\mu - ig \not{A} \\ \not{A} &= \frac{\sigma^a}{2} A_\mu^a \gamma^\mu. \end{aligned} \quad (2.2)$$

The fermionic integration may be decomposed into subspaces within and orthogonal to square integrable zero energy solutions $\psi_0(A)$ and $\psi_\perp(A)$, functionals of A , with

$$\begin{aligned} \not{D} \psi_0 &= 0 \\ \not{D} \psi_\perp &\neq 0 \end{aligned} \quad (2.3)$$

where the Atiyah-Singer index theorem^{6,7} guarantees for gauge fields topologically classifiable with integer winding number ν that ψ_0 exists for $|\nu| \geq 1$.

Then we have

$$\begin{aligned} W_\nu &= \int [DA]_\nu \int d\bar{\psi} \left[\exp \int d^4x \mathcal{L}(A, \bar{\psi}) \right] \text{Det } \not{D}^\perp(A) \\ &\prod_\Lambda \int d^4y \eta^\dagger(y) \psi_{0\Lambda}(y) \int d^4z \psi_{0\Lambda}^\dagger(z) \eta(z) \exp \int d^4w \eta^\dagger(w) \not{D}^\perp \eta(w) \end{aligned} \quad (2.4)$$

where the product is over the zero energy eigenfunction and $\not{D}^\perp(A)$ is the Dirac operator in the subspace perpendicular to the zero energy solutions.

This result is of interest because it is known that

$$n_R - n_L = \nu N_f$$

where $n_{R,L}$ are the number of right or left chirality zero energy solutions. For $|\nu| \geq 1$, this implies that the pseudoparticle contribution to the propagation function vanishes if $N_f \geq 2$. We will comment further on this very general result⁸ below.

To be more specific, let us consider saddle point expansions about some classical extremum A_{cl} . Then the quantum theory developed perturbatively about A_{cl} in terms of small fields A_{qu} with

$$\begin{aligned} A &= A_{cl} + A_{qu} \\ \mathcal{D}_{cl} &= \mathcal{D}(A_{cl}) \\ \mathcal{D}_{cl} \psi_0 &= 0 \\ \mathcal{D}_{cl} \psi_{\perp} &\neq 0 \end{aligned} \quad (2.5)$$

is given by

$$\begin{aligned} W_v &= \int \mathcal{D}A_{qu} \mathcal{D}A_{cl} \mathcal{D}\bar{\eta} \exp \int d^4x \left[\mathcal{L}(A, \bar{\eta}) + ig \frac{\delta}{\delta \eta} P_{\perp} A_{qu} P_{\perp} \frac{\delta}{\delta \eta^{\dagger}} \right] \\ &\quad \text{Det } \mathcal{D}^{\perp} \exp \int d^4x \bar{\eta}^{\dagger} \mathcal{D}_{cl}^{\perp} \eta \\ &\quad \prod_{S=1}^{N_f} \prod_{\Lambda=1}^{|v|} \left[\int d^4x \left(\eta^{\dagger} + ig \eta^{\dagger} D_{cl, \perp}^{-1} A_{qu} \right) f_{o\Lambda}(x) \right]_S \\ &\quad \left[\int d^4y f_{o\Lambda}^{\dagger}(y) \left(\eta + ig A_{qu} D_{cl, \perp}^{-1} \eta \right) \right]_S \end{aligned} \quad (2.6)$$

where $f_{o\Lambda}(x)$ are tabulated zero energy solutions about the pseudoparticle configuration $D_{cl}^{\perp}(A_{cl})$ which have been calculated by 't Hooft² for $|v| = 1$ and by Grossmann⁹ for $|v| \geq 1$ configurations. Again, it is clear that, in any finite order of perturbation theory about the instanton, the propagation function vanishes for $N_f > 1$, $|v| > 0$,¹⁰ consistent with the more general result Eq. (2.4) which obtained for any path integer v classifiable, independent of the "size" of g or A_{qu} .

The special Feynman rules implied by Eq. (2.6) for fermionic interactions associated with zero frequency modes are summarized in Fig.1. At least $|v|$

insertions of Fig.1a and $|v|$ external fermions for each flavor, $|v|N_f$ in all, are required for a nonvanishing Green's function. Additionally, there are the conventional interactions, in the presence of the instanton, implied in $\mathcal{L}(A, \bar{\psi}, \psi)$ and the $\bar{\psi}_L A_{qu} \psi_L$ term. Understood in $\int \mathcal{D}A_{c\ell}$ is the integration over pseudoparticle size, position, and global orientation in color space. It is notable that smearing over these coordinates not only restores translational and color invariance, but also produces non-trivial amplitudes from diagrams which are disconnected at the potential scattering [fixed pseudoparticle position and size] level.

It is interesting to recall¹¹ some examples of such calculations. For $N_f = 1$, the one-loop two-point function contribution from the $v=1$ sector is

$$\langle \psi_j^{*\alpha}(x_2) \psi_i^\beta(x_1) \rangle = K(1) e^{-8\pi^2/g^2} g^{-8} \mu^{20/3} \int d^4x d\rho^{-4/3} f_i^\alpha\left(\frac{x_1-x}{\rho}\right) f_j^{*\beta}\left(\frac{x_2-x}{\rho}\right) \quad (2.7)$$

where

$$K(N_f) = 2^{14+N_f} \pi^{6-2N_f} \exp\left[2N_f \alpha\left(\frac{1}{2}\right) - \alpha(1)\right] \quad (2.8)$$

with α some constants defined in Ref.2, and f_i^α are the two component (lower component) zero energy modes ($v=1$)

$$f_i^\alpha(z) = (1+z^2)^{-\frac{3}{2}} \sigma_2^{\alpha i} \quad (2.9)$$

where αi label Dirac and isospin indices. This corresponds to the instanton

A_μ in "non-singular" gauge

$$A_\mu^a = \frac{2}{g} \frac{\eta_{\mu\nu}^a x_\nu}{x^2 + \rho^2} \quad (2.10)$$

in the Dirac spinor representation with

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix} \quad \psi = \begin{pmatrix} \psi_U \\ \psi_L \end{pmatrix} \quad (2.11)$$

Averaging over global color isospin we have

$$\langle \psi_i^{\alpha*}(x_2) \psi_j^{\beta}(x_1) \rangle = K(1) e^{-8\pi^2/g^2} \mu^{20/3} \frac{1}{2} \delta_{ij} \delta_{\alpha\beta}$$

$$\int d^4x d\rho \rho^{-4/3} \left(1 + \left(\frac{x_1 - x}{\rho} \right)^2 \right)^{-3/2} \left(1 + \left(\frac{x_2 - x}{\rho} \right)^2 \right)^{-3/2} \quad (2.12)$$

where μ is the running renormalization point mass. In four component Dirac formalism this is given by replacing

$$\delta_{\alpha\beta} \longleftrightarrow \left(\frac{1-\gamma_5}{2} \right)^{\alpha\beta}$$

With the help of the Fourier transform

$$\int d^4x e^{ixp} \left[1 + \left(\frac{x}{p} \right)^2 \right]^{-3/2} = \frac{4\pi^2 p^2}{p} e^{-pp} \quad (2.13)$$

one may express this propagator as

$$\langle \psi_i^{\alpha*}(x_2) \psi_j^{\beta}(x_1) \rangle = K(1) e^{-8\pi^2/g^2} g^{-8} \mu^{20/3} \frac{1}{2} \delta_{ij} \left(\frac{1-\gamma_5}{2} \right)^{\alpha\beta} \Gamma\left(\frac{17}{3}\right) 2^{-17/3} \int d^4x e^{-ip(x_1-x_2)} \frac{1}{p^{23/3}} \quad (2.14)$$

so that in momentum space we have the behavior

$$e^{-8\pi^2/g^2(\mu)} g^{-8}(\mu) \frac{\mu^{20/3}}{p^{23/3}} \equiv \frac{\mu_0^{20/3}}{p^{23/3}} \quad (2.15)$$

where μ_0 is the invariant renormalization group mass. This expression reveals the nature of the momentum space instanton amplitudes as an asymptotic expansion in powers of μ_0/p .

Another way of phrasing this is to renormalize g at $\mu = 1/\rho$,¹⁰ and then take these factors into the size integration. The result is

$$\begin{aligned}
\langle \psi_i^{*\alpha}(x_2) \psi_j^\beta(x_1) \rangle &= K(1)^{\frac{1}{2}} \delta_{ij}^{\frac{1}{2}} (1-\gamma_5)^{\alpha\beta} \int \frac{d^4 x d\rho}{\rho^8} e^{-8\pi^2/g^2 \left(\frac{1}{\rho}\right)} g^{-8\left(\frac{1}{\rho}\right)} \\
&\quad \left[1 + \left(\frac{x_1 - x_2}{\rho}\right)^2\right]^{-\frac{3}{2}} \left[1 + \left(\frac{x_2 - x_1}{\rho}\right)^2\right]^{-\frac{3}{2}} \\
&= K(1)^{\frac{1}{2}} \delta_{ij}^{\frac{1}{2}} (1-\gamma_5)^{\alpha\beta} \int \frac{d^4 p e^{ip(x_1 - x_2)}}{p^2} \int \frac{d\rho}{\rho^2} e^{-8\pi^2/g^2 \left(\frac{1}{\rho}\right)} g^{-8\left(\frac{1}{\rho}\right)} e^{-2p\rho}
\end{aligned} \tag{2.16}$$

in which form it is convenient to invoke models of large ρ behavior of the coupling terms.

For $N_f = 2$ (a, etc. labels the flavor index), the leading $v = 1$ contribution to fermionic functions is a scattering amplitude

$$\begin{aligned}
\langle \psi_a^{*i\alpha}(x_1) \psi_b^{j\beta}(x_2) \psi_c^{*k\gamma}(x_3) \psi_d^{\ell\delta}(x_4) \rangle &= (\delta_{ab} \delta_{cd} - \delta_{ad} \delta_{bc}) \\
&\quad K(2) e^{-8\pi^2/g^2} g^{-8} \mu^6 \frac{1}{12} \left\{ \left(\frac{1-\gamma_5}{2}\right)^{\alpha\beta} \left(\frac{1-\gamma_5}{2}\right)^{\gamma\delta} (2\delta_{ij} \delta_{kl} - \delta_{il} \delta_{kj}) \right. \\
&\quad \left. + \frac{j - \ell}{\beta - \delta} \right\} \int \frac{d^4 x d\rho}{\rho^5} \Pi_{i=1}^4 \left[1 + \left(\frac{x - x_i}{\rho}\right)^2\right]^{-\frac{3}{2}}
\end{aligned} \tag{2.17}$$

$$\begin{aligned}
&\equiv T_{iajbkcld}^{\alpha\beta\gamma\delta} K(2) \int \frac{d^4 x d\rho}{\rho^5} \frac{1}{\rho^6} e^{-8\pi^2/g^2 \left(\frac{1}{\rho}\right)} g^{-8\left(\frac{1}{\rho}\right)} \Pi_{i=1}^4 \left[1 + \left(\frac{x - x_i}{\rho}\right)^2\right]^{-\frac{3}{2}} \\
&\equiv \langle x_1 x_2 x_3 x_4 \rangle
\end{aligned} \tag{2.18}$$

Alternatively, we may express this in terms of a Fourier expansion in momentum space, valid for $p \gg \mu_0$,

$$\langle x_1 x_2 x_3 x_4 \rangle = T_{iajbkc}^{\alpha\beta\gamma\delta} K(2) \mu^6 I(x_1 x_2 x_3 x_4)$$

with

$$I(x_1 x_2 x_3 x_4) = (2\pi)^{-16} \int d^4 p_1 d^4 p_2 d^4 p_3 d^4 p_4 e^{-i(p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4)} \tilde{I}(p_1 p_2 p_3 p_4) \quad (2.19)$$

$$I(x_1 x_2 x_3 x_3) = (2\pi)^{-12} \int d^4 p_1 d^4 p_2 d^4 p_3 e^{-i(p_1 x_1 + p_2 x_2 + p_3 x_3)} \tilde{I}(p_1 p_2 p_3) \quad (2.20)$$

$$I(x_1 x_1 x_2 x_2) = (2\pi)^{-8} \int d^4 p_1 d^4 p_2 e^{-i(p_1 x_1 + p_2 x_2)} \tilde{I}(p_1 p_2). \quad (2.21)$$

Using the form (1.13) and

$$\int d^4 x e^{ipx} \left[1 + \left(\frac{x}{\rho} \right)^2 \right]^{-3} = \frac{\pi^2 \rho^3}{2p} (p\rho)^2 K_1(p\rho),$$

we have

$$\tilde{I}(p_1 p_2) = (2\pi)^4 (2\pi^2)^2 \delta^4(p_1 + p_2) |p_1|^{-4} \frac{1}{10}$$

$$\tilde{I}(p_1 p_2 p_3) = (2\pi)^4 (2\pi^2)^3 \delta^4(p_1 + p_2 + p_3) \prod_i \frac{1}{|p_i|} \frac{|p_3|^3}{(|p_1| + |p_2| + |p_3|)^8} \frac{2\sqrt{\pi} \Gamma(8) \Gamma(6)}{\Gamma(7\frac{1}{2})} {}_2F_1 \left(8; \frac{3}{2}; 7\frac{1}{2}; \frac{|p_1| + |p_2| - |p_3|}{|p_1| + |p_2| + |p_3|} \right)$$

$$\tilde{I}(p_1 p_2 p_3 p_4) = (2\pi)^4 2\pi^2 \delta^4(p_1 + p_2 + p_3 + p_4) \prod_i \frac{1}{|p_i|} 2 \times 8! \left(\sum_{i=1}^4 |p_i| \right)^{-8}$$

It is particularly interesting to consider in detail the di-fermion propagation functions $I(x_1 x_1 x_2 x_2)$. Forming the color gauge invariant flavor combinations

$$\begin{aligned}
\sigma &= \bar{\psi}\psi & \eta &= i\bar{\psi}\gamma_5\psi \\
\vec{\pi}_s &= \bar{\psi}\vec{\sigma}\psi & \vec{\pi} &= i\bar{\psi}\gamma_5\vec{\sigma}\psi
\end{aligned}
\tag{2.22}$$

we have, adding instanton and anti-instanton amplitudes with phases $e^{\pm i\theta}$, the di-fermion propagation matrix

$$\langle x_1 x_1 x_2 x_2 \rangle_\theta = e^{-8\pi^2/g^2} g^{-8} \mu^6 \int \frac{d^4 p}{p^4} e^{-ip(x_1-x_2)} \frac{M(\theta)}{5}$$

$M(\theta) =$

	σ	η	π	π^2
σ	$\cos\theta$	$-\sin\theta$	0	0
η	$-\sin\theta$	$-\cos\theta$	0	0
π	0	0	$\cos\theta$	$\sin\theta$
π_s	0	0	$\sin\theta$	$-\cos\theta$

(2.23)

This form clearly indicates the $SU(2) \times SU(2)$ invariance of the Green's functions $[\langle\sigma^2\rangle = \langle\pi^2\rangle, \langle\eta^2\rangle = \langle\pi_s^2\rangle]$ and the chiral $U(1)$ breaking $[\langle\sigma^2\rangle \neq \langle\eta^2\rangle, \langle\pi^2\rangle \neq \langle\pi_s^2\rangle]$.

One may wonder if $|v| > 1$ pseudoparticle effects, and/or higher order n-point functions, might not reveal some further breaking, in one loop, of the residual $SU(N_f) \times SU(N_f)$ symmetry. The detailed investigation of Ref.11 indicates that that is not the case.

This section has displayed a number of fermion Green's functions calculated in one-loop expansion about pseudoparticle configurations with $|v| = 1$. They are consistently interpreted as off-shell expansions in an asymptotically free phase of the theory.

In this phase it is clear that chiral $U(1)$ is broken, that it is broken by instanton effects, and that it is broken in the vacuum:

$$\langle \sigma^2 \rangle_\theta \neq \langle \eta^2 \rangle_\theta \quad Q^5 |\theta\rangle \neq 0$$

Whether or not there is a zero mass isoscalar pseudoscalar meson in the theory (required or not by celebrated theorem or conjecture) associated with this breaking of $U(1)$, is obscure to us. It seems difficult to us to prove the existence or non-existence of a state in the spectrum of this theory, in its present stage, and we will not comment further on this interesting question.

A consequence of the Green's function structure is that pseudoparticle effects ($|v| > 1$) of the type we have considered here do not enter into $e^+e^- \rightarrow$ hadrons annihilation. This is measured by the absorptive part of

$$\langle \bar{\psi}(x) \gamma_\mu \psi(x) \bar{\psi}(0) \gamma_\nu \psi(0) \rangle$$

which vanishes in the instanton sectors with $|v| > 0$. For $|v| = 1$, this is merely due to the space-time structure of the one-loop amplitude, which supports only scalar and pseudoscalar amplitudes. For $|v| > 1$, the leading n -point functions contains $|v|N_f$ quark-antiquark densities and hence makes no contribution to the current-current correlation function.

For similar reasons there are no pseudoparticle corrections to the β -function calculation, at least at the one-loop level.

For $N_f > 1$, it is clear that no mass is generated in one-loop in the pseudoparticle sector. For $N_f = 1$, a mass is generated, reflecting chiral $U(1)$ breaking, and it is given asymptotically by Eq.(2.14)

$$m(p) \approx \frac{\mu_o^{20/3}}{p^{17/3}}$$

in the large momentum limit. This mass generation should not be confused with the breaking of chiral $SU(N)$, $N > 1$.

For $N_f > 1$, one might ask if pseudoparticle effects of higher order in the perturbation theory might generate a mass. The implication of Eq.(2.14) is that this is not the case for any paths with integer non-zero winding number. This result is non-perturbative. It has nothing to do with the size of g or the size of the quantum oscillations. It relies purely on the technical assumptions (integer $|v| > 0$) which provide the zero modes.

In the next section we study some possible loopholes in this argument. It is clear, for example, that nothing yet prevents the $v = 0$ sector from generating a mass, from configurations which are in some sense, in this non-linear theory, suppositions of pseudoparticles and anti-pseudoparticles.

These and other possibilities will be considered below.

Section III. Dilute Gas Approximations

In this section we would like to consider the effect of various approximate minima of the action. The approximate minima that we will be considering are sums of single instantons and anti-instantons, widely separated. First we note that it is very important in which gauge these individual instantons and anti-instantons are written, since the sum of instantons and anti-instantons, each in one gauge, is not related to a sum of instantons and anti-instantons, each in another gauge, by an overall gauge transformation. The significance of this is apparent if we consider an instanton-anti-instanton pair each in the singular gauge ($A \rightarrow r^{-3}$ as $r \rightarrow \infty$) and each in the nonsingular gauge ($A \rightarrow r^{-1}$ as $r \rightarrow \infty$). In the nonsingular gauge the action density contains cross terms involving the instanton and anti-instanton which decrease at infinity like r^{-4} and thus the integral over all space of this term diverges logarithmically even when the pair is widely separated. In the singular

gauge, however, the cross terms decrease like r^{-8} at infinity and therefore the cross terms are integrable. The action density due to an instanton or anti-instanton in the singular gauge has an $(r-z_i)^{-2}$ singularity which is integrable provided that the instanton and anti-instanton are separated. The cross terms in the action decrease with increasing separation between the instanton and anti-instanton. In this sense the widely separated configuration is an approximate minimum of the action.

We should mention in passing the possibility of mixed configurations with an instanton in the singular gauge and an anti-instanton in the non-singular gauge. This configuration gives integrable cross terms and therefore an approximate minimum of the action. In a configuration with many instantons and anti-instantons only one can be in the non-singular gauge, but it may be any one of them. It would appear therefore that when we are dealing with N objects we now have $N + 1$ configurations instead of one, thus greatly increasing the entropy of these dilute gas configurations.¹²

The singularity structure of instantons in the singular gauge requires careful study. This can be seen in a calculation of the topological density $F_{\mu\nu}^a F_{\mu\nu}^{*a}$, from one singular instanton (singularity at z_μ). This configuration gives in addition to the smooth term a delta function $\delta^4(x-z)$. Including this δ -function term changes the topological charge from one to zero.¹³ Of course, this situation is not a total surprise since $F\tilde{F}$ is invariant only under non-singular gauge transformations but not under singular ones.

The standard solution to this problem is to write A_μ using sections; that is, near the point z_μ we write A_μ in the non-singular gauge so that the behavior is smooth at z_μ , and for r large we write A_μ in the singular gauge and then we patch these two solutions together in some intermediate region with a gauge transformation. This procedure is somewhat complicated and it is often times more convenient simply to exclude a region around the singularity.

The approximate minima of the action that we are considering in this section play their most important role in the winding number zero sector, that is a dilute gas of equal numbers of instantons and anti-instantons. In the non-zero winding number sectors we already have a number of exact results that greatly restrict the allowed Green's functions. In the winding number zero section the Atiyah-Singer theorem is no longer of any value and we will have to rely on direct calculations, in discussing the zero modes.

We will again in this section only consider the fermion part of the theory. The fermion part of the Lagrangian is

$$\mathcal{L}_F = \bar{\psi} \gamma_\mu \left(\partial_\mu - ig \sum_i A_\mu^{a,i}(x-z_i, \rho_i) \frac{\sigma^a}{2} \right) \psi \quad (3.1)$$

where $A_\mu^{a,i}$ are the various instantons or anti-instantons located at points z_i with size ρ_i . Any fluctuation about these background field configurations would be treated as part of an interaction as in Sec.I. As in the previous sections, a great deal of the physics will be controlled by the possible zero fermion modes. We are therefore led to consider the existence of solutions

$$\gamma_\mu \left(\partial_\mu - ig \sum_i A_\mu^{i,a}(x-z_i, \rho_i) \frac{\sigma^a}{2} \right) \psi_0 = 0. \quad (3.2)$$

First we will argue that in the lowest order dilute gas, where there are N_+ instantons and N_- anti-instantons, there are $N_+ N_-$ right-handed zero mode and $N_+ N_-$ left-handed zero modes. By "lowest order dilute gas approximation" we mean that if we have a function centered at z_i , $F(z-z_i)$, and another centered at z_j , $G(z-z_j)$, we will neglect products of the form FG for $i \neq j$ since the functions fall off rapidly, and the centers z_i, z_j are widely separated by assumption.

It is clear that if $\phi_0^{(i)}(x-z_i, \rho_i)$ is a solution of

$$\gamma_\mu \left(\partial_\mu - i g A_\mu^{i,a}(x-z_i, \rho_i) \right) \phi_o^{(i)}(x-z_i, \rho_i) = 0, \quad (3.3)$$

then it is a solution of (3.2) in the lowest order dilute gas approximation, and we therefore have the prescribed number of zero modes since (3.3) has N_f right- (left-) handed zero modes if $A_\mu^{i,a}$ is an instanton (anti-instanton). These zero modes can be treated explicitly in the functional integral, and we find

$$W_\nu = \sum_{N_+ = 0}^{\infty} \sum_{N_- = 0}^{\infty} \delta_{N_+ - N_-, \nu} \prod_{i=0}^{N_+} \prod_{j=0}^{N_-} \prod_{\Lambda_+ = 1}^{N_f} \prod_{\Lambda_- = 1}^{N_f} e^{-(8\pi^2/g^2)(N^+ + N^-)}$$

$$\int \mathcal{D}A_\nu \mathcal{D}\psi_\perp^\dagger \mathcal{D}\psi_\perp \mathcal{D}\Phi \left[\int \phi_{o\Lambda_+}^{(i)\dagger}(x_+) \eta(x_+) \eta^\dagger(y_+) \phi_{o\Lambda_+}^{(i)}(y_+) dx_+ dy_+ \right]$$

$$\times \left[\int \phi_{o\Lambda_-}^{(j)\dagger}(x_-) \eta(x_-) \eta^\dagger(y_-) \phi_{o\Lambda_-}^{(j)}(y_-) dx_- dy_- \right] \exp \int d^4x \mathcal{L}(A, \psi_\perp, \psi_\perp^\dagger, \eta, \eta^\dagger \Phi). \quad (3.4)$$

Let us consider in a certain sense the simplest situation, $N_f = 1$ and $\nu = 0$ sector. Then there is one right-handed and one left-handed zero modes, and W_0 takes the form

$$W_0 = e^{-16\pi^2/g^2} \int \mathcal{D}A \mathcal{D}\psi_\perp \mathcal{D}\psi_\perp^\dagger \mathcal{D}\Phi \left\{ \left[\int \phi_o^{(1)\dagger}(x_1) \eta(x_1) dx_1 \right] \left[\int \eta^\dagger(y_1) \phi_o^{(1)}(y_1) dy_1 \right] \right.$$

$$\left. \left[\int \phi_o^{(2)\dagger}(x_2) \eta(x_2) dx_2 \right] \left[\int \eta^\dagger(y_2) \phi_o^{(2)}(y_2) dy_2 \right] \right\} \exp \left[\int d^4x \mathcal{L} \right]. \quad (3.5)$$

We see that the lowest point function to which this can contribute is the four point function. We therefore conclude that there are no propagator contributions to quark masses from the $\nu = 0$ sector coming from a dilute gas of instantons and anti-instantons in the lowest order approximation. We also note that in the more physical situation with $N_f = 2$ the lowest point function is eight.

Let us now consider how these conclusions might change if we attempted to improve the lowest order dilute gas. We assume that the exact fermionic eigenfunctions and eigenvalues of the equation

$$\gamma_{\mu} \left(\partial_{\mu} - ig \sum_j A_{\mu}^{a,j}(x-z_j, \rho_j) \frac{\sigma^a}{2} \right) \psi_{\alpha}(x) = E_{\alpha} \psi_{\alpha}(x) \quad (3.6)$$

are continuously connected to the dilute gas solutions for $|z_i - z_j| \gg \rho_{i,j}$ with

$$\begin{aligned} \lim_{|z_i - z_j| \rightarrow \infty} \psi_{\alpha} &= \phi_{\alpha} \\ \lim_{|z_i - z_j| \rightarrow \infty} E_{\alpha} &= \lambda_{\alpha} \end{aligned} \quad (3.7)$$

We are particularly interested in whether the $\lambda = 0$ mode remains a zero mode.

We can consider treating this problem by standard perturbation theory. In such a treatment one separates out one of the terms in $\sum_i A_{\mu}^{(i)}$ and treats the rest as a perturbation. Then we have

$$(\cancel{D} + \cancel{V}) \psi_{\alpha} = E_{\alpha} \psi_{\alpha} \quad (3.8)$$

where

$$\begin{aligned} \cancel{D} &= \gamma_{\mu} \left(\partial_{\mu} - ig A_{\mu}(x-z_1, \rho_1) \right) \\ \cancel{V} &= -ig \gamma_{\mu} \sum_{i \neq 1} A_{\mu}(x-z_i, \rho_i). \end{aligned} \quad (3.9)$$

For notational simplicity let us consider Eq.(3.8) with only two terms

$$(\cancel{D} + \cancel{V}_1 + \cancel{V}_2) \psi_{\alpha} = E_{\alpha} \psi_{\alpha} \quad (3.10)$$

$$\cancel{V}_i = -ig \gamma_{\mu} A_{\mu}(x-z_i, \rho_i). \quad (3.11)$$

Standard iterative perturbation theory for this problem with two potentials can be developed in several ways. For this purpose let us define $\varphi_{\alpha}^{(i)}$

$$(\not{\partial} + \not{V}_i) \varphi_\alpha^{(i)} = \lambda_\alpha^{(i)} \varphi_\alpha^{(i)} \quad i = 1, 2 \quad (3.12)$$

$$p_\alpha^{(i)} = \varphi_\alpha^{(i)} \varphi_\alpha^{(i)*} \quad (3.13)$$

Then

$$\psi_\alpha = \varphi_\alpha^{(1)} \langle \varphi_\alpha^{(1)} | \psi_\alpha \rangle + \frac{1}{E_\alpha - (\not{\partial} + \not{V}_1)} (1 - p_\alpha^{(1)}) \not{V}_2 \psi_\alpha \quad (3.14)$$

with

$$E_\alpha = \lambda_\alpha + \frac{\langle \psi_\alpha^{(1)} | \not{V}_2 | \psi_\alpha \rangle}{\langle \varphi_\alpha^{(1)} | \psi_\alpha \rangle} \quad (3.15)$$

or equivalently with (1) and (2) interchanged. For $\alpha = 0$, $\lambda_0 = 0$ we have the standard results

$$\psi_0^{(0)} = \varphi_0^{(1)}$$

$$E_0^{(0)} = \langle \varphi_0^{(1)} | \not{V}_2 | \varphi_0^{(1)} \rangle = 0$$

$$\psi_0^{(1)} = \psi_0^{(1)} - \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 \varphi_0^{(1)}$$

$$E_0^{(1)} = E_0^{(0)} - \langle \varphi_0^{(1)} | \not{V}_2 \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 | \varphi_0^{(1)} \rangle = 0$$

$$\psi_0^{(2)} = \psi_0^{(2)} + \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 \frac{1}{\not{\partial} + \not{V}_1} \varphi_0^{(1)}$$

$$E_0^{(2)} = E_0^{(1)} + \langle \varphi_0^{(1)} | \not{V}_2 \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 | \varphi_0^{(1)} \rangle = 0. \quad (3.16)$$

Of course, this can be continued to all orders, and E_0 remains zero since it always involves the matrix elements of odd numbers of γ matrices between states of the same chirality. A similar expansion can be made with the roles of the potentials $\not{V}_1$ and $\not{V}_2$ interchanged. Thus we find two solutions ψ_α

that go to $\varphi_\alpha^{(1)}$ and $\varphi_\alpha^{(2)}$ in the limit $\gamma_2 \rightarrow 0$, or the limit $\gamma_1 \rightarrow 0$, respectively. They remain zero modes to all order in perturbation theory.

An alternative approach to the problem is to note that Eq.(3.14) can be written

$$\psi_\alpha = \varphi_\alpha^{(1)} \langle \varphi_\alpha^{(1)} | \psi_\alpha \rangle + \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1 + \gamma_2)} (1 - P_\alpha) \gamma_2 \varphi_\alpha^{(1)} \langle \varphi_\alpha^{(1)} | \psi_\alpha \rangle. \quad (3.17)$$

Then repeated use of the identity,

$$\frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1 + \gamma_2)} = \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1)} + \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1)} \gamma_2 \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1 + \gamma_2)}, \quad (3.18)$$

will generate (3.16) once again. Consider now the identity

$$\frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1 + \gamma_2)} = \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_2)} + \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_2)} \gamma_1 \frac{1}{E_\alpha - (\cancel{\partial} + \gamma_1 + \gamma_2)}. \quad (3.19)$$

We can iterate alternately with (3.18) and (3.19) in (3.17) and generate a new expansion of ψ_0

$$\psi_0^{(0)} = \varphi_0^{(1)}$$

$$E_0^{(0)} = \langle \varphi_0^{(1)} | \gamma_2 | \varphi_0^{(1)} \rangle = 0$$

$$\psi_0^{(1)} = \psi_0^{(0)} + \frac{1}{\cancel{\partial} + \gamma_1} \gamma_2 \varphi_0^{(1)}$$

$$E_0^{(1)} = E_0^{(0)} - \langle \varphi_0^{(1)} | \gamma_2 \frac{1}{\cancel{\partial} + \gamma_1} \gamma_2 | \varphi_0^{(1)} \rangle = 0$$

$$\psi_0^{(2)} = \psi_0^{(1)} + \frac{1}{\cancel{\partial} + \gamma_1} \gamma_2 \frac{1}{\cancel{\partial} + \gamma_2} \varphi_0^{(1)}$$

$$E_0^{(2)} = E_0^{(1)} + \langle \varphi_0^{(1)} | \gamma_2 \frac{1}{\cancel{\partial} + \gamma_1} \gamma_2 \frac{1}{\cancel{\partial} + \gamma_2} \gamma_2 | \varphi_0^{(1)} \rangle = 0. \quad (3.20)$$

Nevertheless, E_0 remains zero to all orders, as before. This expansion was used in Ref.14 to expand the propagator $(\not{\partial} + \not{V}_1 + \not{V}_2)^{-1}$ as follows.

Iterating (3.19) in (3.18) once with $E_\alpha = 0$, we find

$$\frac{1}{\not{\partial} + \not{V}_1 + \not{V}_2} \cong \frac{1}{\not{\partial} + \not{V}_1} - \frac{1}{\not{\partial} + \not{V}_1} \not{V}_2 \frac{1}{\not{\partial} + \not{V}_2} = \frac{1}{\not{\partial} + \not{V}_1} \not{\partial} \frac{1}{\not{\partial} + \not{V}_2}. \quad (3.21)$$

This formal approximation of the quark propagator in the presence of an instanton and anti-instanton ($\not{V}_1$ and $\not{V}_2$) is entirely equivalent to our earlier expansion, and must support the same zero mode.

It should be noted from Eqs.(3.15) and (3.17) the exact nonlinear equation for E_0 ,

$$E_0 = \langle \varphi_0^{(1)} | \not{V}_2 \frac{1}{E_0 - (\not{\partial} + \not{V}_1 + \not{V}_2)} \not{V}_2 | \varphi_0^{(1)} \rangle, \quad (3.22)$$

has the solution $E_0 = 0$, consistent with the expansion result.

There remains the possibility that a level shift of the dilute gas zero mode occurs as the result of a wholly non-perturbative mechanism akin to the double well problem in the one-dimensional Schrödinger equation. In that problem it is known that the presence of a soliton (for $t \rightarrow it$) signals tunneling which splits the two lowest levels by an amount which can't be calculated by the methods employed here. These two lowest levels are essentially the symmetric and antisymmetric combinations of ground states of the individual wells. Therefore, when the wells are widely separated, the wavefunction does not approach the ground state of one of the wells, as we have assumed in our dilute gas perturbation treatment.

On the other hand, there is a closely related problem in which such a pathology does not occur: consider when $\not{V}_1$ and $\not{V}_2$ are both instantons.

Then they have $\nu = 2$, and hence have zero modes either by an appeal to the Atiyah-Singer theory or by observing that exact zero modes calculated by Grossman⁹ tend in the large separation limit to the dilute gas zero modes.

This leaves the question of the physical implication of a dilute gas of instanton and anti-instanton in a rather unsettled state. All of the approaches existing in the literature^{12,14} appear to rest in a fundamental way on perturbation theory, and thus must (though many don't) deal with the problem presented by the zero modes.

It remains a serious challenge to demonstrate whether and how the zero modes are lifted by non-perturbation effects.

IV. Summary and Conclusions

We have studied pseudoparticle effects in massless QCD with N_f flavors and asked whether, for $N_f > 1$, there is evidence of mass generation and the breakdown of $SU(N_f) \times SU(N_f)$ to non-chiral $SU(N_f)$.

There is, of course, no controversy concerning the breakdown of $U(N_f) \times U(N_f)$, or of the generation of a mass when $N_f = 1$; indeed, we display results showing these effects.

However, we fail to find the breaking of chiral $SU(N_f)$, either from loop expansions about exact pseudoparticle solutions, or from functional paths with non-zero integer topological number. This seems to suggest that the important action is in the $\nu = 0$ sector, in this regard.

We then study dilute gas configurations in this sector, and again see indication that the zero modes which block mass generation for $N_f > 1$ persist here, although there are some delicate questions of convergence which may allow evasions of these considerations in some models.

In this paper we have consistently begun with massless QCD, $N_f > 1$, sought mass generation essentially via dimensional transmutation, and failed within our framework.

We have not attempted to put in a mass and asked whether in some sense this mass could reproduce or bootstrap itself; indeed, this procedure, we

believe, can succeed, and we intend to study it in a future publication.

If indeed such a procedure succeeds, then one must ask, what is the nature of the theory as this bootstrapped mass, now a parameter of the theory, is taken to zero; we believe that this limit, unfortunately, takes one back to the $SU(N_f) \times SU(N_f)$ symmetric vacuum, for $N_f > 1$. This procedure, of course, does work in the case $N_f = 1$, where one explicitly sees the non-perturbative development of a mass term which persists as the explicitly introduced mass is turned off. Details of this calculation will be reported shortly.

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Figure Captions

- 1a) Propagation functions of a quark from x to y via the zero energy modes in the presence of an instanton (or anti-instanton) of size ρ located at z .
- 1b) The interaction of two quarks and a gluon at the point w in the presence of an instanton at z with size ρ . One of the quarks propagates with the zero modes while the other propagates via the orthogonal modes.

$$f_{0\Lambda}(x-z;\rho) f_{0\Lambda}^\dagger(y-z;\rho)$$

(a)

(b)