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An Experimental Investigation of Pressure Drop of Aqueous Foam in Laminar Tube Flow

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An Experimental Investigation of Pressure Drop of Aqueous Foam in Laminar Tube Flow

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Abstract

This report is the first of two detailing pressure-drop and heat-transfer measurements made at the Foam Flow Heat Transfer Loop. The work was motivated by a desire to extend the application of aqueous foam from petroleum drilling to geothermal drilling. Pressure-drop measurements are detailed in this report; a forthcoming report (SAND85-1922) will describe the heat-transfer measurements. The pressure change across a 2.4-m (8-ft) length of the 2.588-cm (1.019-in.) ID test section was measured for liquid volume fractions between 0.05 and 0.35 and average velocities between 0.12 and 0.80 m/s (0.4 and 2.6 ft/s). The resulting pressure-drop/flow-rate data were correlated to a theoretical model for a Bingham plastic. Simple expressions for the dynamic viscosity and the yield stress as a function of liquid volume fraction were estimated.

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Nomenclature

B	Dimensionless yield stress, Equation (13)
c	Yield stress to wall shear stress ratio = τ_y/τ_w
D	Tube diameter
f	Friction factor, Equation (13)
g	Acceleration due to gravity
h	Vertical distance between pressure measurement ports
L	Distance between pressure measurement ports
m	Exponent = $1/n$
n	Exponent, see Equations (4), (6)
NLINA	nonlinear parameter estimation program
Δp	Measured or total pressure change
Δp_f	Pressure change due to friction
r	Radial position
r_w	Wall or tube radius
r_y	Radial position at which $\tau = \tau_y$
r^+	Dimensionless radial position = r/r_w
Re	Reynolds number = $\rho \bar{u} D / \eta$
S	Sum of squares of differences between data and predictions
u	Local velocity
$\bar{u}$	Average velocity
$u_{\max}$	Maximum velocity
u^+	Dimensionless local velocity = $u/\bar{u}$
δ	Uncertainty
η	Dynamic viscosity
ρ, ρ_l, ρ_g	Density of foam, liquid, and gas constituents, respectively
τ	Local shear stress
τ_w	Wall shear stress
τ_y	Yield stress
ϕ	Liquid volume fraction

An Experimental Investigation of Pressure Drop of Aqueous Foam in Laminar Tube Flow

Introduction

Aqueous foams have many applications that include fire fighting, petroleum drilling, and personnel barriers. This study was motivated by the desire to extend the application of foam for petroleum drilling to geothermal drilling where formation temperatures may exceed 250°C (482°F). Some of the reported advantages of foams as a drilling fluid are:

- Low density produces low bottom hole pressure
- Sand and cuttings fall back very little when circulation stops
- Low loss of circulation into surrounding formation.

To properly size the equipment to deliver foam for a specific application, it is necessary to know the pressure drop as a function of flow rate for various foam consistencies (liquid volume fractions). This report discusses experimental results of pressure drop as a function of flow rate for liquid volume fractions in the range of 0.05 to 0.35. Based upon an assumed model of a Bingham plastic, both the effective viscosity and yield stress have been estimated from the experimental measurements.

A companion report¹ details the results of thermal experiments conducted with aqueous foams. These experiments will provide input to a computer model that will be used to ascertain the suitability of using foam in a high-temperature geothermal environment.

Structure and Properties of Foam

Aqueous foams are two-phase mixtures of a gas (usually nitrogen or air) and a liquid consisting of water plus surfactant. Foam is often characterized by its liquid volume fraction, which is defined as

$$\phi = \frac{\text{liquid volume}}{\text{liquid volume} + \text{gas volume}} \quad (1)$$

Many properties of foam, such as density and viscosity, are functions of the liquid volume fraction.

Foam density is related to liquid volume fraction by

$$\rho = \phi\rho_l + (1-\phi)\rho_g \quad (2)$$

where ρ , ρ_l , and ρ_g are the densities of the foam, the liquid component, and the gas component, respectively.

In the liquid-volume-fraction range of interest, foams are described by Bikerman² as agglomerations of gas bubbles separated by thin liquid films, as opposed to gas emulsions in which the thickness of the interstitial liquid layers is approximately the same as the diameter of the bubble. The ideal foam cell is a pentagonal dodecahedron (Figure 1). The data of David and Marsden³ indicate that cell size is in the range of 100 to 800 μm (0.004 to 0.032 in.). Cell size is strongly dependent on the details of the foam generator and the generation pressure.

Figure 1. Pentagonal Dodecahedron

The liquid within the cell interfaces will drain in a gravitational field; hence, the foam has a limited lifetime in a static environment. When a foam sample is placed in a container, the majority of the liquid will eventually collect in the bottom of the container. During this drainage process, the structure of the foam becomes very fragile, almost like a spider web. Figures 2a-c are microphotographs of the time varying structure of a foam sample (shown with a scale with small divisions of 1/64 in. for reference). A foam sample was placed in a container open to the atmosphere. The microscope was focused on the second layer of cells; photographs were taken at times of 3 min 28 s, 12 min 46 s, and 32 min 11 s after the sample was initially drawn. The liquid volume fraction was 0.101 at a test section pressure of 2.58 bar (37.44 psia); after expansion to atmospheric pressure of .84 bar (12.2 psia), the approximate liquid volume fraction was calculated to be 0.035. As time progresses, small cells coalesce into large cells. If the test section were horizontal, then drainage could be an important factor. In a vertical test section, drainage is still taking place, but since the drainage velocity is small in comparison to the bulk velocity, it was felt that drainage did not have a substantial impact on the test results reported here.

Additional foam drainage studies were conducted by Rand and Kraynik.⁴ They found that the time

required for a given fraction of the original liquid to drain from the foam increased with decreasing cell size. For their study, cell size was decreased by increasing the pressure at which the foam was generated.

Figure 2 indicates the wide variation in cell size for the foam of this study. Even if it was possible to produce a foam of uniform cell size, there would be a variation in cell structure at a solid/foam interface. Figure 3 is a microphotograph of a solid/foam interface with the foam stationary. This photograph was obtained by filling a container with foam at atmospheric pressure, placing a microscope slide over the top of the container, and focusing the microscope on the first layer of cells. The dark region is the wall of the container.

For the present study, air is used as the gas; the surfactant is alpha olefin sulfonate.* The volume ratio of surfactant to water is maintained at nominally 0.01 for all tests.

*The surfactant used for all tests in this study is available commercially as Whitconate AOS from Whitco Chemical Company.

(a) After 3 min 28 s

(b) After 12 min 46 s

(Scale)

(c) After 32 min 11 s

Figure 2. Microphotographs of Foam Sample at Different Times After It Was Drawn

Figure 3. Microphotograph of the Foam-Solid Interface

Theoretical Pressure-Drop/Flow-Rate Model

It has been proposed by numerous investigators that aqueous foams exhibit a yield stress.^{5,6} In a pipe-flow geometry, the existence of a yield stress indicates that when the local shear stress is below the yield stress the foam will flow as a solid. Near the centerline where the shear stress goes to zero, foam will move as a plug and the velocity gradient will be zero. In the wall region where the shear stress is largest, a velocity gradient will be present. Some investigators have modeled foams as a Bingham plastic;^{5,6} the stress above the yield stress is linearly proportional to the velocity gradient

$$\tau - \tau_y = -\eta \frac{du}{dr} \quad (3)$$

where τ is local shear stress, τ_y is the yield stress, η is viscosity, u is velocity, and r is radial position.

Other investigators have assumed foam to behave as a "power-law" fluid.⁷ A combination of the "power-law" fluid and the Bingham plastic is the Bulkley-Herschel model

$$\tau - \tau_y = -\eta \left(\frac{r_w}{\bar{u}} \frac{du}{dr} \right)^{n-1} \frac{du}{dr} \quad (4)$$

where $\bar{u}$ is the average velocity, r_w is the pipe radius, and n is an exponent. If τ_y is zero and $n=1$, then the Bulkley-Herschel model reduces to the laminar Newtonian model.

The quantity $(r_w/\bar{u} \, du/dr)$ raised to the exponent $(n-1)$ is dimensionless; hence, η has the units of viscosity (FT/L^2). This approach appears to offer some advantage over that in the literature in which the units of η depend on the value of the "power-law" exponent.

The flow-rate/pressure-drop relationship can be developed from the momentum equation for steady and fully developed pipe flow:

$$\frac{1}{r} \frac{d}{dr} (r\tau) = \frac{\Delta p}{L} \quad (5)$$

where $\Delta p/L$ is the pressure drop per unit length. This equation can be integrated from the centerline to any arbitrary radius to yield

$$\tau = \Delta p r / 2L \quad (6)$$

or

$$\tau = r \tau_w / r_w \quad (7)$$

Equation (4) can be solved for the velocity gradient to yield

$$-\frac{du}{dr} = \left\{ \frac{\tau - \tau_y}{\eta \left(\frac{r_w}{\bar{u}} \right)^{n-1}} \right\}^{1/n} = \frac{\bar{u}}{r_w} \left(\frac{\tau - \tau_y}{\eta \frac{\bar{u}}{r_w}} \right)^{1/n} \quad (8)$$

Integrating Equation (8) from the wall to an arbitrary position r while using Equation (7), the velocity profile can be written as

$$\frac{u}{u_{\max}} = \begin{cases} 1 - \left(\frac{r^+ - c}{1 - c} \right)^{m+1} & r^+ \geq c \\ 1 & r^+ < c \end{cases} \quad (9)$$

where $r^+ = r/r_w$, $m = 1/n$, and $c = \tau_y/\tau_w = r_y/r_w$. The average velocity is found to be

$$\begin{aligned} \frac{\bar{u}}{u_{\max}} &= 1 - \frac{2(1-c)}{m+2} + \frac{2(1-c)^2}{(m+2)(m+3)} \\ &= c^2 + 2c(1-c) \frac{m+1}{m+2} + (1-c)^2 \frac{m+1}{m+3} \end{aligned} \quad (10)$$

Combining Equations (9) and (10), the dimensionless velocity profile can be expressed as

$$\begin{aligned} u^+ &= \frac{u}{\bar{u}} = \\ &= \frac{1 - \left(\frac{r^+ - c}{1-c}\right)^{m+1}}{1 - 2\left(\frac{1-c}{m+2}\right) + 2\frac{(1-c)^2}{(m+2)(m+3)}} \quad r^+ \geq c \\ &= \frac{1}{1 - 2\left(\frac{1-c}{m+2}\right) + 2\frac{(1-c)^2}{(m+2)(m+3)}} \quad r^+ < c. \end{aligned} \quad (11)$$

Figures 4 and 5 present dimensionless velocity profiles obtained from Equation (11). Figure 4 is for the Bingham plastic ($m=n=1.0$, $0 \leq c \leq 1.0$). In dimensionless coordinates, the size of the plug flow region is given by $r^+ = c$. The laminar Newtonian velocity profile is the $c = 0$ curve. As c increases, the magnitude of the dimensionless velocity gradient at the wall also increases. Figure 5 shows the impact of the power-law exponent on the dimensionless velocity profile for two different values of c . As m increases ($n = 1/m$ decreases), the magnitude of the dimensionless velocity gradient at the wall increases. Increasing m also increases the extent of the constant velocity region in the center portion of the pipe. A convenient friction-factor/Reynolds-number relationship can be written as

$$\begin{aligned} fRe &= \\ &= \frac{16}{\left[\left(1 - \frac{B}{fRe}\right)^{m+1} \left[1 - \frac{2\left(1 - \frac{B}{fRe}\right)}{m+2} + \frac{2\left(1 - \frac{B}{fRe}\right)^2}{(m+2)(m+3)} \right] \right]^{1/m}} \end{aligned} \quad (12)$$

where B is a dimensionless yield stress, and the other parameters in Equation (12) are defined as

$$\begin{aligned} f &= \frac{D\Delta p/L}{\rho \frac{\bar{u}^2}{2}} \\ B &= \frac{8\tau_y D}{\eta \bar{u}} \\ Re &= \frac{\rho \bar{u} D}{\eta} \\ c &= \frac{B}{fRe} \\ \tau_w &= \frac{f}{4} \frac{\rho \bar{u}^2}{2} \end{aligned} \quad (13)$$

Figure 4. Dimensionless Velocity Profiles for a Bingham Plastic ($n = 1$, $0 \leq c \leq 1$)

Figure 5. Dimensionless Velocity Profiles for a Power-Law Fluid

Figure 6 presents the results of Equation (12) graphically. For laminar Newtonian fluids, the traditional value $fRe=64$ is obtained. As c increases from 0 to 0.3, fRe increases from 64 to 103. The data shown on Figure 6 will be discussed in a subsequent section.

An important equation used in estimating the viscosity (η) and the yield stress (τ_y) is

$$\bar{u} = \frac{\tau_w r_w}{\eta} \times \left\{ \frac{(1 - \tau_y/\tau_w)^{m+1}}{m+1} \times \left[1 - \frac{2(1 - \tau_y/\tau_w)}{m+2} + \frac{2(1 - \tau_y/\tau_w)^2}{(m+2)(m+3)} \right] \right\}^{1/m} \quad (14)$$

When $\tau_y/\tau_w \ll 1$, u is approximately linear in τ_w . This linearity is more clearly evident for the Bingham plastic model. For the Bingham plastic ($m=1$), Equation (14) reduces to

$$\bar{u} = \frac{\tau_w r_w}{4\eta} \left[1 - \frac{4}{3} \frac{\tau_y}{\tau_w} + \frac{1}{3} \left(\frac{\tau_y}{\tau_w} \right)^4 \right] \quad (15)$$

Figure 6. Dependence of the Friction-Factor/Reynolds-Number Relationship Upon the Yield-Stress/Wall-Shear-Stress Ratio

Analysis of the Data

Pressure-drop measurements were taken for liquid volume fractions ranging between 0.05 and 0.35 and average velocities between 0.12 and 0.80 m/s (0.4 and 2.6 ft/s). The liquid volume fraction range is that of practical interest for geothermal drilling applications; the velocity range is limited by the maximum and minimum possible mass flow rates of air or the liquid. The velocity range also corresponds to that of practical interest; the field application, however, has a larger pipe radius and, therefore, will have larger Reynolds numbers. Since foam is very viscous, the maximum Reynolds number for the data of the present study is less than 100; the corresponding hydrodynamic entry length is less than five tube diameters or 130 mm (5 in.). Therefore, the flow can be considered laminar and hydrodynamically fully developed everywhere between the pressure measurement ports.

The total pressure drop across a 2.44-m (8.00-ft) length of the 3.00-m (9.83-ft) long by 25.88-mm (1.019-in.) ID test section is measured with a differential pressure transducer; a separate differential pressure transducer is used in conjunction with an atmospheric pressure gauge to measure the static pressure at the center of the test section length. This static pressure measurement is used to determine various foam properties including the liquid volume fraction. Preliminary experiments with the test section horizontal indicated that liquid drainage affected the test results; hence, the test section is oriented vertically. A more detailed description of the experimental apparatus is found in the Appendix.

The total pressure change consists of hydrostatic and frictional components. The frictional pressure drop is given by

$$\Delta p_f = \Delta p + \rho gh \quad (16)$$

where Δp_f is the frictional pressure drop, Δp is the total (or measured) pressure drop, ρ is the foam density, g is the acceleration due to gravity, and $h(>0)$ is the vertical distance between the pressure ports. The frictional pressure drop is related to the wall shear stress through the momentum balance given by Equation (6):

$$\tau_w = \Delta p_f r_w / 2L$$

The average foam velocity is found from the mass flow rate of the air, the volumetric flow rate of the liquid plus its density, and the cross-sectional area of the tube via the continuity equation.

The average velocity versus wall shear stress data for the entire range of liquid volume fractions and mass flow rates are shown in Figure 7 and given in Table 1. The curves included on Figure 7 will be explained later.

The relative uncertainty in the frictional pressure drop can be expressed as

$$\frac{\delta(\Delta p_f)}{\Delta p_f} = \left[\frac{(\delta \Delta p)^2}{\Delta p_f^2} + \frac{(\delta(\rho gh))^2}{\Delta p_f^2} \right]^{1/2} \quad (17)$$

The most important source of uncertainty is the variability in the pressure-drop measurements, which is

approximately 5% of the measured value. Hence, the relative uncertainty in the frictional pressure drop is proportional to the ratio $(\Delta p / \Delta p_f)$. This ratio is tabulated in Table 1. This source of uncertainty is the smallest for small values of the total pressure drop. Small values of Δp often occur when the frictional and hydrostatic pressure drops are approximately equal. This condition exists for the midrange liquid volume fractions at certain flow rates (see Table 1). Large uncertainties result if the hydrostatic pressure drop is much larger than the frictional pressure drop. The foam is relatively dense for $\phi = 0.35$, resulting in a larger value for $|\Delta p / \Delta p_f|$. Consequently, the uncertainty associated with $\phi = 0.35$ is large, on the order of 20%.

Figure 7. Relationship of Average Velocity to the Wall Shear Stress

Table 1. Summary of Pressure-Drop Measurements

ϕ	Δp psi	Δp_f psi	τ_w psi	u ft/s	temp °F	$ \Delta p/\Delta p_f $
0.055	0.917	1.112	0.00294	0.43	63.9	0.825
0.054	0.908	1.099	0.00291	0.44	64.1	0.826
0.055	1.304	1.500	0.00396	0.72	65.8	0.869
0.056	1.284	1.486	0.00393	0.72	65.8	0.864
0.053	1.621	1.812	0.00479	0.94	61.2	0.896
0.053	1.573	1.762	0.00466	0.94	61.4	0.893
0.053	2.073	2.264	0.00599	1.20	60.6	0.916
0.054	2.028	2.222	0.00587	1.18	60.6	0.913
0.051	2.978	3.166	0.00837	1.79	60.9	0.941
0.051	3.067	3.255	0.00860	1.80	60.8	0.942
0.053	3.271	3.469	0.00917	2.11	59.6	0.943
0.053	3.272	3.467	0.00916	2.13	59.7	0.944
0.104	0.272	0.635	0.00168	0.41	67.7	0.428
0.101	0.253	0.606	0.00160	0.42	68.5	0.417
0.101	0.614	0.969	0.00256	0.54	66.5	0.634
0.100	0.598	0.951	0.00251	0.55	66.5	0.629
0.102	0.683	1.042	0.00275	0.64	67.6	0.655
0.102	0.660	1.019	0.00269	0.64	67.4	0.648
0.107	0.772	1.149	0.00304	0.77	67.6	0.672
0.107	0.783	1.159	0.00306	0.77	67.8	0.676
0.109	1.166	1.550	0.00410	1.27	63.8	0.752
0.109	1.186	1.569	0.00415	1.26	63.7	0.756
0.108	1.290	1.670	0.00441	1.52	63.4	0.772
0.108	1.406	1.787	0.00472	1.51	63.5	0.787
0.107	1.997	2.375	0.00628	1.92	65.3	0.841
0.107	1.986	2.365	0.00625	1.91	65.3	0.840
0.104	2.466	2.836	0.00750	2.28	65.5	0.870
0.103	2.369	2.734	0.00723	2.29	65.6	0.866
0.105	2.735	3.110	0.00822	2.60	60.0	0.879
0.105	2.681	3.055	0.00808	2.60	60.1	0.878
0.158	-0.054	0.498	0.00132	0.47	57.7	0.108
0.148	0.012	0.529	0.00140	0.50	57.6	0.023
0.158	0.184	0.737	0.00195	0.82	62.8	0.250
0.155	0.186	0.728	0.00193	0.84	62.3	0.255
0.152	0.546	1.078	0.00285	1.20	56.9	0.506
0.152	0.499	1.032	0.00273	1.19	57.0	0.484
0.156	0.796	1.344	0.00355	1.51	57.6	0.592
0.156	0.806	1.354	0.00358	1.51	57.6	0.595
0.154	1.044	1.584	0.00419	1.84	61.7	0.659
0.154	1.058	1.599	0.00423	1.84	61.7	0.662
0.156	1.470	2.018	0.00533	2.56	55.2	0.728

(continued)

Table 1 (concluded)

ϕ	Δp psi	Δp_r psi	τ_w psi	u ft/s	temp °F	$ \Delta p/\Delta p_r $
0.159	1.414	1.975	0.00522	2.52	55.4	0.716
0.197	-0.305	0.380	0.00101	0.50	66.1	0.803
0.196	-0.324	0.360	0.00095	0.50	66.3	0.900
0.204	-0.218	0.492	0.00130	0.63	63.6	0.443
0.197	-0.173	0.512	0.00135	0.65	63.7	0.338
0.204	-0.179	0.515	0.00136	0.75	68.0	0.350
0.205	-0.221	0.492	0.00130	0.75	68.1	0.449
0.205	-0.090	0.623	0.00165	0.97	66.9	0.144
0.198	-0.060	0.632	0.00167	0.97	66.9	0.095
0.198	0.136	0.828	0.00219	1.16	60.1	0.164
0.198	0.137	0.827	0.00219	1.16	60.0	0.166
0.206	0.151	0.869	0.00230	1.34	60.0	0.174
0.206	0.108	0.828	0.00219	1.33	60.2	0.130
0.204	0.239	0.951	0.00251	1.47	57.6	0.251
0.204	0.202	0.915	0.00242	1.47	57.6	0.221
0.206	0.262	0.979	0.00259	1.65	60.7	0.268
0.205	0.277	0.995	0.00263	1.65	60.7	0.278
0.207	0.411	1.134	0.00300	1.90	56.4	0.362
0.205	0.427	1.142	0.00302	1.93	56.0	0.374
0.257	-0.634	0.259	0.00069	0.56	66.3	2.448
0.252	-0.586	0.291	0.00077	0.57	66.3	2.014
0.249	-0.520	0.349	0.00092	0.69	54.2	1.490
0.250	-0.491	0.380	0.00100	0.68	55.0	1.292
0.250	-0.454	0.419	0.00111	0.82	51.6	1.084
0.250	-0.432	0.439	0.00116	0.82	51.6	0.984
0.253	-0.381	0.499	0.00132	0.85	64.6	0.764
0.252	-0.365	0.512	0.00135	0.85	64.9	0.713
0.250	-0.321	0.547	0.00145	1.06	64.5	0.587
0.247	-0.324	0.537	0.00142	1.07	64.5	0.603
0.245	-0.162	0.692	0.00183	1.33	62.6	0.234
0.244	-0.147	0.705	0.00186	1.34	64.5	0.209
0.247	-0.085	0.775	0.00205	1.77	61.0	0.114
0.248	-0.110	0.754	0.00199	1.76	61.2	0.146
0.356	-1.032	0.204	0.00054	0.70	70.7	5.059
0.357	-1.112	0.129	0.00034	0.70	70.6	8.620
0.357	-0.979	0.260	0.00069	0.84	62.3	3.765
0.349	-0.928	0.304	0.00080	0.98	62.3	3.003
0.354	-0.954	0.278	0.00073	0.97	63.4	3.432
0.346	-0.942	0.260	0.00069	1.04	64.0	3.623
0.343	-0.910	0.282	0.00074	1.04	64.1	3.227
0.349	-0.915	0.296	0.00078	1.23	62.0	3.091
0.352	-0.910	0.313	0.00083	1.24	61.4	2.907

From the assumed relationship between the average foam velocity and the wall shear stress, Equation (14), the viscosity and yield stress for a given nominal liquid volume fraction can be estimated. A nonlinear parameter estimation program (NLINA)⁸ was used to fit the experimental data to the model relationship, Equation (14). NLINA determines the values of the parameters that produce the minimum sum of squares of the differences between the model and the experimental data. Although there are three unknown parameters [viscosity, yield stress, and the power-law exponent (m or n)], all three cannot be determined independently because their sensitivity coefficients are not independent. The exponent, therefore, is specified and the other two parameters (viscosity and yield stress) are estimated. Numerical calculations indicated that the exponent, n , and the viscosity, η , are coupled. The preferred value for the exponent is that which results in the minimum sum of the squares of the errors.

The sum-of-squares error, S , is given in Table 2. The variation of S with m is very small, and there is no clear value of m which results in a minimum value of S for all liquid volume fractions. Since $n = 1$ results in a much simpler model (Bingham plastic model), unity is preferred for the exponent. The insensitivity of S to the assumed exponent, n , indicates that this experiment cannot be used to estimate n . Velocity profile measurements would be helpful but would be more complicated than the pressure drop measurements. Incidentally, data for $\phi = 0.35$ is deleted from Table 2 because of the limited range of the data and large relative uncertainty.

Figures 8 and 9 show the variation of the Bingham plastic ($n=1$) viscosity and yield stress with the nominal liquid volume fraction. Viscosity has an apparent exponential variation, whereas yield stress has an apparent logarithmic variation with liquid volume fraction. Utilizing these assumed variations, the following relationships were determined using NLINA for a Bingham plastic with variable liquid volume fraction:

$$\eta = 0.4304e^{-6.838\phi} \text{ N}\cdot\text{s}/\text{m}^2 \quad (18)$$

$$= 0.008989e^{-6.838\phi} \text{ lb}_f\cdot\text{s}/\text{ft}^2$$

$$\tau_y = 0.3773\phi^{-0.8994} \text{ Pa} \quad (19)$$

$$= 5.472 \times 10^{-5}\phi^{-0.8994} \text{ psi}$$

where ϕ is the liquid volume fraction. Equations (18) and (19) should be restricted to the liquid volume range of the present study ($0.05 \leq \phi \leq 0.35$). These equations are also limited to foam similar to that utilized for the experiments reported here; i.e., the water/surfactant ratio must be equal to that used (0.01) and the methods of foam generation should be similar. For the present study, the absolute pressure ranged between 170 kPa (25 psia) and 400 kPa (60 psia). The foam temperature was essentially room temperature for all tests; temperature dependence of these properties has not been considered.

Table 2. Sum-of-Squares Error for Various Exponents

$m=1/n$	Nominal Liquid Volume Fractions				
	0.05	0.10	0.15	0.20	0.25
0.5	0.02907	0.13107	0.04918	0.07477	0.15208
0.9	0.02903	0.13096	0.04888	0.07444	0.15167
1.0	0.02903	0.13095	0.04884	0.07440	0.15149
1.1	0.02902	0.13095	0.04881	0.07437	0.15157
1.5	0.02901	0.13095	0.04873	0.07429	0.15147
2.0	0.02901	0.13096	0.04869	0.07425	0.15141
2.5	0.02900	0.13097	0.04867	0.07424	0.15139
3.0	0.02900	0.13099	0.04866	0.07424	0.15138
4.0	0.02900	0.13101	0.04867	0.07425	0.15139
10.0	0.02900	0.13109	0.04874	0.07434	0.15149

Figure 8. Variation of the Bingham Plastic Viscosity With Liquid Volume Fraction

Figure 9. Variation of the Bingham Plastic Yield Stress With Liquid Volume Fraction

The curves for the various nominal liquid volume fractions on Figure 7 were determined by substituting the above viscosity and yield-stress relationships into the Bingham plastic average-velocity/wall-shear-stress relationship [Equation (15)]. If the yield stress is very small compared to the wall shear stress, the relationship between the average velocity and the wall shear stress is nearly linear; the curves of Figure 7 exhibit this approximately linear region. The theoretical relationship is curved in the region in which the

yield stress is greater than about 35% of the wall shear stress. The yield stress for a particular nominal liquid volume fraction is represented on Figure 6 by the point at which a curve intersects the x-axis (the wall-shear-stress axis). Since the experimental data falls in the linear portion, the extent of the curved region is not well defined, and hence the viscosity is known with greater certainty than the yield stress. The range of practical interest is also in the linear region; therefore, the uncertainty in the yield stress should not be too important for the pressure-drop predictions for field applications. It is expected that Equations (15), (18), and (19) can be used to extrapolate to higher wall-shear-stress values of field applications without difficulty.

Since the curves on Figure 7 were determined for the nominal liquid volume fractions, much of the scatter from these lines can be accounted for by the scatter in the actual liquid volume fraction. For example, most of the data of Figure 7 for a liquid volume fraction of 0.05 lies above the solid line. This bias is because the actual liquid volume fraction was consistently above the nominal liquid volume fraction. The actual liquid volume fraction is always within 0.01 of the nominal value. The actual liquid volume fractions for the data are given in Table 1.

Equations (18) and (19) were also used in conjunction with Equation (13) to determine the data points on Figure 6. The scatter at large c values results primarily from the relatively large uncertainty associated with $\phi = 0.35$ and the lower flow rates for $\phi = 0.25$.

Conclusions

Pressure-drop measurements from the Foam Flow Heat Transfer Loop have been presented. Experimentally determined average-velocity versus wall-shear-stress data can be correlated with the stress-strain model for a power-law, yield-stress fluid. The correlation is equally as good for exponents (n) between 0.25 and unity; the Bingham plastic model ($n=1$) is chosen for its simplicity. Velocity-profile measurements would be useful in determining the best model for the stress-strain relationship for foam. For the Bingham plastic model, viscosity has an exponential variation with liquid volume fraction, whereas yield stress varies logarithmically with liquid volume fraction. Further work is needed to determine the effect of such quantities as foam-generator pressure, weight-percent surfactant, and temperature on viscosity and yield stress.

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APPENDIX

Description of the Experimental Apparatus

A schematic of the experimental apparatus is shown in Figure A-1. The apparatus was designed primarily for heat-transfer foam flow experiments. The test section is basically a counterflow heat exchanger consisting of a 3.00-m (9.84-ft) long by 2.588-cm (1.019-in.) ID stainless-steel tube with wall thickness of 0.6033 cm (0.2375 in.) and concentric within a 5.458-cm-ID steel (nominal 2-in. IPS schedule 20) pipe. For heat-transfer measurements, hot water is circulated through the resulting 1.664-cm (0.655-in.) annulus; for pressure-drop measurements, however, the hot water loop is inactive. Foam flows vertically downward through the inner tube.

The hot water loop consists of a 15-kW electric heater, two circulation pumps in series, and a turbine flow meter. Water temperature is controlled by varying the amount of electrical power supplied to the heater. Thermocouples are used to measure the temperature at the inlet and the outlet of the heater. The

volumetric flow rate is typically around 0.5 ℓ/s (8 gpm). The hot water enters the annulus at a right angle through two pipes that are 180° apart; the outlet has a similar configuration. Individually calibrated RTDs (resistance temperature devices) in each of the inlets and outlets of the test section annulus are used to measure the hot water temperatures, which are typically around 50°C (120°F) for heat-transfer tests. Due to the large capacity rate of the water flow compared to that of the foam flow, the foam boundary condition is nearly isothermal. For "cold flow" pressure-drop tests, the hot water loop is inactive; the temperatures of the foam and the water in the inactive hot water loop are about room temperature (see Table 1).

Figure A-1. Schematic of the Experimental Apparatus

Air is combined with a water/surfactant mixture in the correct proportions to produce foam with the desired liquid volume fraction. Air is drawn from the large building supply tanks, which are maintained at 6.89 bars (100 psi). The mass flow rate is determined by the combination of a sonic flow nozzle and an upstream pressure regulator. Provided the flow is sonic at the throat, the mass flow rate of the air is held constant regardless of downstream pressure fluctuations. A series of nozzles manufactured by Flow-dyne Engineering, Inc., with diameters ranging 0.343 mm to 1.40 mm (0.0135 in. to 0.055 in.) provide for a range of mass flow rates.

Water is drawn from a 570 ℓ (150 gal) tank that is pressurized to a maximum of 6.89 bars (100 psi) with building air. The water volumetric flow rate is controlled with a cavitating venturi from Fox Valve Development Corporation; due to the constant upstream pressure, the cavitating venturi provides a constant flow rate similarly to the sonic nozzle on the gas side. Surfactant is supplied from a 200 ℓ (55 gal) barrel with a variable-stroke positive-displacement pump. The pump has two adjustable stroke heads with a maximum capacity of 3.5 ml/s (3.4 gph) per head. The volumetric flow rate of surfactant is always maintained at 0.01 of the water volumetric flow rate. The water and surfactant pass through a static mixer (TAH Corporation) to blend the two constituents. The liquid flow passes through one of two parallel turbine meters. These meters are manufactured by Flow Technologies, Inc. The larger capacity meter has a range of 0 to 0.158 ℓ /s (0 to 2.5 gpm); the other, with a range of 0 to 0.0315 ℓ /s (0 to 0.5 gpm), provides greater accuracy at lower flow rates. The range of liquid flow rates is 0.00315 to 0.0632 ℓ /s (0.05 to 1.00 gpm).

The gas and liquid flows are mixed immediately upstream of the foam generator. Experience suggests that vertical foam flow with the air injected at the bottom of the foam generator and the liquid injected horizontally will produce foam of acceptable quality. Steel wool in the air line prior to the point at which the air is injected into the liquid flow provides a fairly uniform pressure at this point. The foam generator is a packed bed of copper wool in a vertical circular tube with an ID of 3.468 cm (1.365 in.). The packing density

is 0.56 g/cm³ (34.7 lb_m/ft³) and the packing length is 11.43 cm (4.5 in.).

After leaving the generator, the foam enters the test section. The test section is mounted vertically in a 22 cm by 23 cm (8 in. by 8.5 in.) cross-section by 1.9-cm (0.75-in.) thick wood box. The vertical orientation minimizes foam drainage problems; the foam flows downward through the test section. The cavity between the test section and the box is filled with silica aerogel insulation, which is available from W. R. Grace and Co., Davison Chemical Division.

The foam inlet and outlet to the test section are connected to nylon bushings that seal the annulus; the foam flows through a hole in the bushing that has the same diameter as the test section. The seals are held via a flexible bracket to the adjoining pieces, which are threaded into the bracket.

The pressure change over a 2.4-m (8.0-ft) length of the test section is measured with a Validyne, Inc., variable-reluctance differential pressure transducer. Two transducers are available with ranges of 0 to 0.0862 bars (0 to 1.25 psid) and 0 to 0.22 bars (0 to 3.2 psid). A 0 to 6.89 bars (0 to 100 psi) Gould, Inc., differential pressure transducer is used to measure the static pressure at the center of the test section. All pressure transducers were calibrated at Sandia National Laboratories. The 1.59-mm (0.0625-in.) diameter stainless-steel tubes used for pressure ports penetrate the outer pipe and are held in place with gland nuts. The connection with the inner tube is sealed with an O-ring at the bottom of a counterbored hole. The pressure port has a diameter of 1.0 mm (0.040 in.).

Due to the long lines between the pressure ports and the transducers, foam tends to back up into the lines, thereby producing erroneous readings. An elaborate valving scheme was designed to flush foam from the pressure lines immediately before taking measurements. The schematic of the valving arrangement is shown in Figure A-2. Valves 2, 4, and 6 are three-way on/off valves. In one position, the lines are connected to the building air supply via a pressure regulator; in the other open position, the lines are connected to the pressure transducers. Valves 1, 3, and 5 seal off the test section during temperature measurements. Valve 7 protects the low-range pressure transducer from being over-ranged.

Figure A-2. Schematic of Valving Arrangement for Flushing Pressure Measurement Lines

Individually calibrated RTDs are used to measure the foam temperature at the inlet and outlet to the test section (one each). Type K thermocouples are used to measure various temperatures, such as the ambient temperature, the hot water temperature, and the temperature of the air flow at the sonic flow nozzle.

An HP 1000 series computer is used for data acquisition. Pressure readings are averaged over approximately 15 scans taken during a 1 to 1.5 min interval. The remaining measurements are scanned during the three pressure measurement intervals and are, therefore, averaged over approximately 45 scans. (The pressures upstream and downstream of the sonic nozzle and the static test section pressure are measured with the same transducer).

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